Conical Emission in Asymmetric Nucleus-Nucleus Collisions at FAIR

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Outline

Shock Waves and Mach Cones in Heavy Ion Collisions?

- Probing strongly interacting matter by extracting sound velocity c_s from particle emission angle.

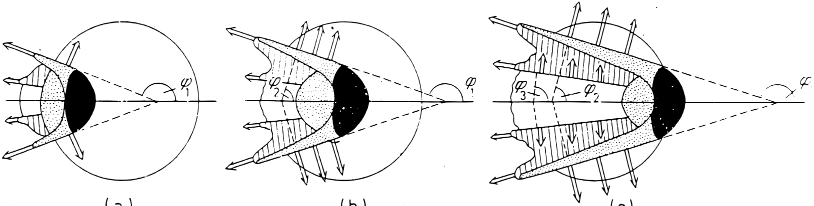
  Mach cone formula: $\theta_{\mathrm{MC}} = \cos^{-1} \frac{c_s}{v}$

  Sound vel. in medium: $c_s^2 = \frac{\partial p}{\partial e}$
- Probing impact of phase transition: small c_s may lead to large emission angles.
- Compare results from hydrodynamics to those from microscopic transport models $\Rightarrow$ underlying process for conical emission:
  - **Hydrodynamics:** Shock wave formation (*Mach cone*).
  - **Transport models:** Binary nucleon-nucleon scattering processes obeying momentum conservation.
- How does medium viscosity effect shock waves and conical emission?

Strong Compression Waves in HIC

Prediction of **strongly compressed shock waves in high-energy nucleus nucleus collisions** resulting in collective *sideward emission* of produced nucleons first studied in:

(calculations in the past non-relativistic; low beam energies $E_{\mathrm{lab}} < 1A$ GeV)



(a) diving stage

(b) penetration stage

(c) final stage

from [**??**]

[1] H. G. Baumgardt, E. Schopper, H. Stöcker, J. Hofmann, W. Greiner *et al.*, Z. Phys. A **273** (1975) 359.

[2] J. Hofmann, H. Stöcker, U. W. Heinz, W. Scheid and W. Greiner, Phys. Rev. Lett. **36** (1976) 88.

[3] H. Stöcker, J. A. Maruhn and W. Greiner, Phys. Rev. Lett. **44** (1980) 725.

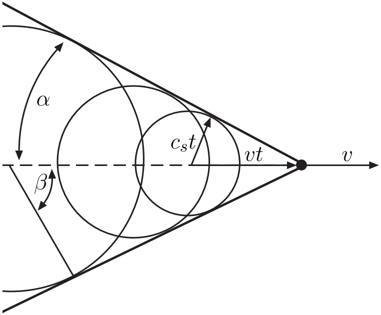
Mach Cones in Classic Theory

Fast particle in medium acts as perturbation source → coherent interference of spherical sound waves propagating with $c_s \Rightarrow$ Mach cone.

Mach cone formula:

- Emission angle:
  $\theta_{MC} = \cos^{-1} c_s/v = \beta$
- Opening angle:
  $\alpha = \sin^{-1} c_s/v$

v is velocity of perturbation source.



Sound velocity from EoS:

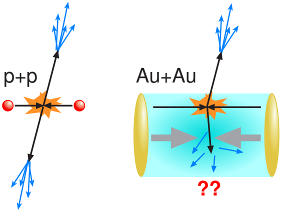
$c_s^2 = \partial p/\partial e$

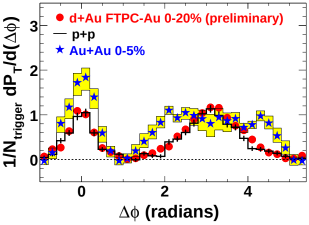
The Mach cone itself moves with $v_{sh} = c_s$ in direction of the particle.

Mach cones or Mach cone-like shock waves in nuclear matter should result in conical particle emission due to transverse deflection of matter.

Mach Cones from Supersonic Parton Jets in HIC

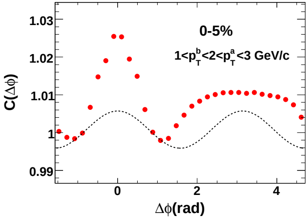
Conical emission observed in RHIC data 2- and 3-particle analysis may result from fast particle-jets evoking Mach cones in the evolving medium:





$4 < p_T^{trig} < 6$ GeV/c, $0.15 < p_T^{ass} < 4$ GeV/c

[6] F. Wang [STAR], Nucl.Phys. A774, 129.



[7] A. Adare *et al.* [PHENIX], PRC **78** (2008) 014901.

Are there a Mach cones in nuclear matter?

- Di-jet events: High-p_T trigger particles on near side; lower p_T associated particles on away side.
- No Mach cone signal in raw data.
- Subtraction of elliptic flow background leads to double-peaked structure at $\Delta\phi \simeq \pi \pm 1.1$.

[4] J. Adams *et al.* [STAR Collaboration], Phys. Rev. Lett. **95** (2005) 152301;

[5] J. G. Ulery [STAR Collaboration], Nucl. Phys. A **774** (2006) 581;

Ideal Relativistic Hydrodynamics

- Conversation of energy, momentum and net baryon number:

$$\partial_\mu T^{\mu\nu} = 0 \ , \tag{1}$$

$$\partial_\mu N^\mu = 0 \ ; \quad \mu, \nu = 0, \ldots, 3 \tag{2}$$

- Energy-momentum tensor in ideal hydrodynamics:

$$T^{\mu\nu} = (e + p) u^\mu u^\nu - p g^{\mu\nu} \ , \tag{3}$$

$$N^\mu = n u^\mu \tag{4}$$

with $u^\mu = \gamma(1, \mathbf{v})$; $\gamma = (1 - v^2)^{-1/2}$; $g^{\mu\nu} = diag(1, -1, -1, -1)$

- 6 independent variables, system of only 5 equations
  ⇒ additional equation needed: **Equation of State** (EoS), $p(e, n)$
- Second input requirement is **initial condition** defining the configuration at the start of the hydrodynamic description.

In our study the total collision process from $t = 0$ is described by hydrodynamics.

Solving Hydrodynamics

Equations of motion:

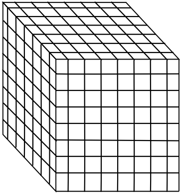
$$\begin{aligned}\partial_t E + \nabla \cdot (E\vec{v}) &= -\nabla \cdot (p\vec{v}) \ , \\ \partial_t \vec{M} + \nabla \cdot \left(\vec{M}\vec{v}\right) &= -\nabla p \ , \\ \partial_t R + \nabla \cdot (R\vec{v}) &= 0\end{aligned}$$

SHASTA [**??**] algorithm is used to solve the hydrodynamic equations on a 3-dimensional Eulerian grid in the computational frame.

[8] D. H. Rischke, S. Bernard and J. A. Maruhn, Nucl. Phys. A **595** (1995) 346.

Parameters: cell size: $dx = 0.2$ fm (typically 200^3 cells)
time step: $dt = 0.08$ fm/c

Ground state values: $e_0 = 146.52$ MeV/fm^3
$n_0 = 0.159$ fm^{-3}

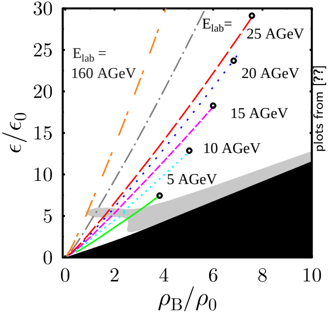


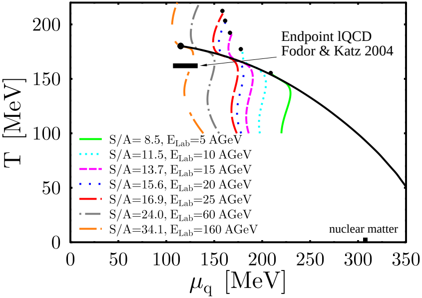
Equation of State: $p(e, n)$

EoS derived from a chiral hadronic SU(3) Lagrangian incorporating:

- the complete set of baryons from lowest flavour $SU(3)$ octet,
- multiplets of scalar, pseudo-scalar, vector and axial-vector mesons.

EoS offers a **realistic description of the properties of nuclei** with the nuclear matter equilibrium point at $\rho_0 = 0.15\ \mathrm{fm}^{-3}$ (e.g. binding energy, single particle energy spectra, charge radii) and of multiplets of spin-0, spin-1 and spin-1/2 particles.





[9] P. Papazoglou, D. Zschiesche et al., Phys. Rev. C **59** (1999) 411;

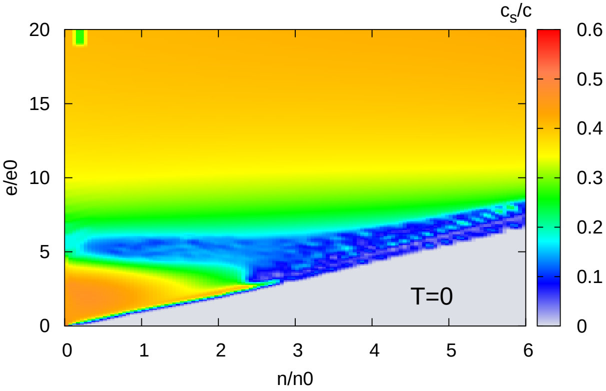
[10] D. Zschiesche, G. Zeeb and S. Schramm, J. Phys. G **34** (2007) 1665;

[11] J. Steinheimer et al., Phys. Rev. C **77** (2008) 034901

Velocity of Sound in the Phase Diagram

Calculating c_s from EoS $p(e,n)$:

$$c_s^2 = \left.\frac{\partial p}{\partial e}\right|_{s/n} = \frac{n}{e+p}\left.\frac{\partial p}{\partial n}\right|_e + \left.\frac{\partial p}{\partial e}\right|_n$$



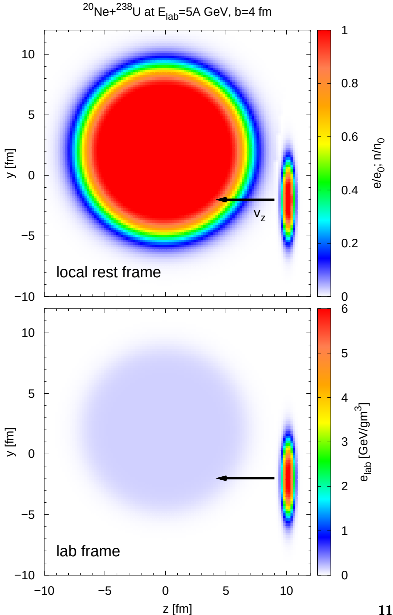
Initial Conditions

- High-energy collision of a small nucleus with a heavier one, e.g., Ne+U in a *fixed target*-setup.
- (3+1)-dim. hydrodyn. description realized in the local rest frame of the target nucleus.
- Ground state energy and net baryon number density of nuclei given by *Woods Saxon* potential

$$e(r) = \frac{e_0}{(1 + e^{(r - r_0)}/d)}$$

with $r_0 = 1.25 \cdot A^{1/3}$ fm, $d = 0.54$ fm.

- Velocity of projectiles in CM system: $v_{\mathrm{CM}} = -\left(1 - 1/\gamma_{\mathrm{CM}}^2\right)^{1/2}$, with $\gamma_{\mathrm{CM}} = (E_{\mathrm{lab}}/(2M_N) + 1)^{1/2}$.
- Boost into rest frame of heavy target nucleus.
- Projectile is Lorentz-contracted in beam-direction.



^{20}Ne+^{238}U at E_{lab}=5A GeV, b=4 fm

Freeze-Out I. – Free Streaming Freeze-Out

Hydrodyn. evolution stopped at t_f when head shock has traversed entire target nucleus. We use two different methods for **isochronous** decoupling into hadrons:

I: Free-Streaming Freeze-Out:
Produce nucleons ($M_N = 0.9$ GeV) exclusively:

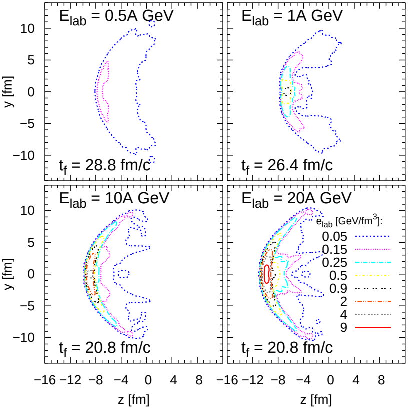
- At t_f set $p = 0 \rightarrow$ nucleons decouple instantaneously.
- Each fluid cell emits baryons accordant to its net baryon number n.
- Nucleons emitted with the velocity of the fluid element carry a *kinetic energy*

$$E_{\rm kin} = (\gamma - 1) M_N .$$

- Compute *emission angle* (angle between beam axis and the particle's total momentum) via

$$\theta_{\rm lab} = \cos^{-1}\left(\frac{v_z}{\sqrt{v_x^2 + v_y^2 + v_z^2}}\right).$$

En. dens. $e_{\rm lab}$ in reaction plane at t_f:



Shock wave shoots through entire target, leaving no fragments in collision zone.

Freeze-Out II. – Cooper-Frye Freeze-Out

II: Cooper-Frye Freeze-Out:
Second freeze-out approach maps the hydrodynamic fields to hadrons via Cooper-Frye equation:

$$E\frac{dN}{d^3p} = \int\limits_{\sigma} f(x,p)p^{\mu}\, d\sigma_{\mu}. \tag{5}$$

Fermi $(+)$ or Bose $(-)$ distribution corresponding to particle species:

$$f(x,p) = \frac{g}{(2\pi)^3}\frac{1}{e^{-\beta(p^{\mu}u_{\mu})} \pm 1}. \tag{6}$$

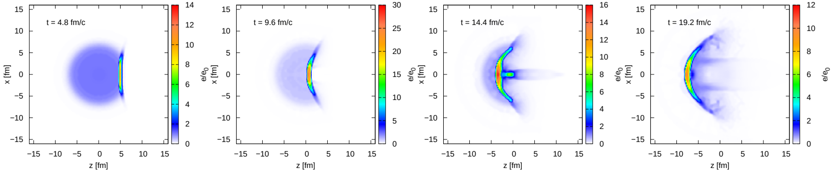
$d\sigma_{\mu} = (d^3x, \vec{0})$ is normal vector on hypersurface for isochronous freeze-out.

Particle vector information is transferred into UrQMD as afterburner. Subsequent hadronic cascade calculations incorporate final state effects, e.g. rescatterings of the particles and resonance decays.

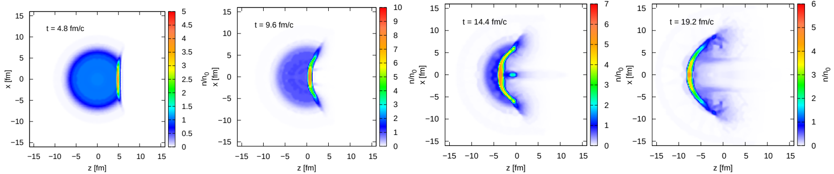
[12] H. Petersen, J. Steinheimer, G. Burau, M. Bleicher and H. Stöcker, Phys. Rev. C **78** (2008) 044901.

Reaction Process I. - Ne+U,$E_{\rm lab} = 10A$GeV,$b = 0$

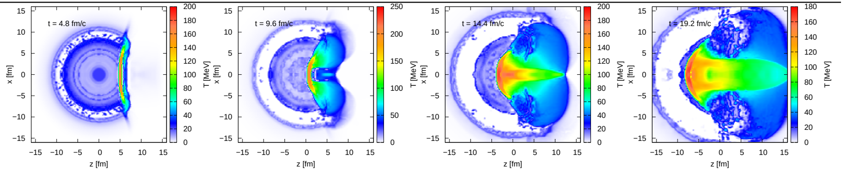
quantities in computational frame e/e_0; n/n_0; T [MeV] at $t \approx 5$; 10; 15; 20



e_{CF}



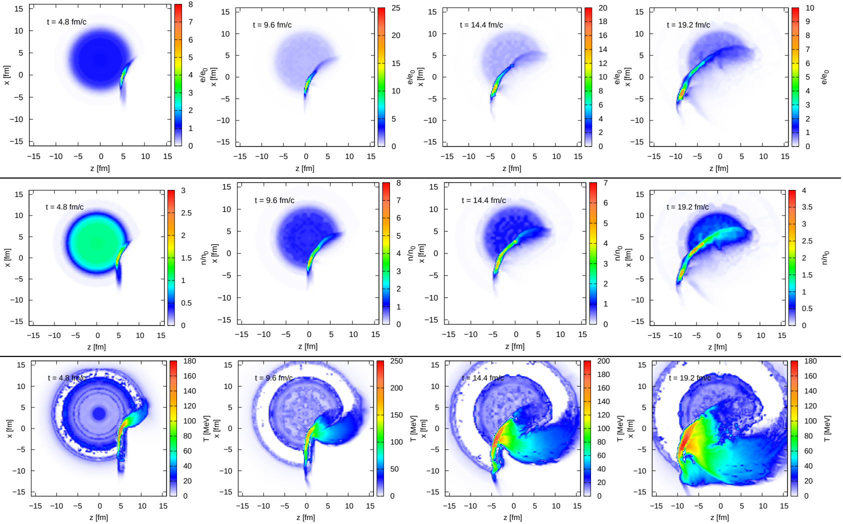
n_{CF}



T_{CF}

Reaction Process II. - Ne+U, $E_{\mathrm{lab}} = 10A$GeV, $b = 7$ fm

quantities in computational frame e/e_0; n/n_0; T [MeV] at $t \approx 5;\ 10;\ 15;\ 20$



e_{CF}

n_{CF}

T_{CF}

Hydrodyn. System in Phase Diagram and c_S

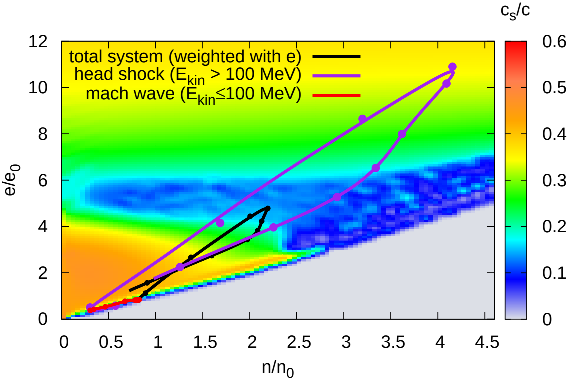
Subdivide total system in:

- Head Shock: $E_{\mathrm{kin}} > 100$ MeV
- Mach Wave: $E_{\mathrm{kin}} \leq 100$ MeV

Energy-weighted **mean values**:

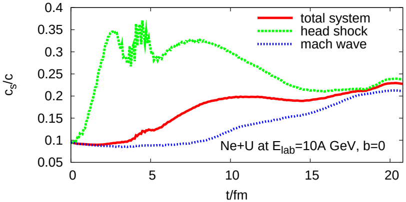
$$\langle O_m \rangle = \frac{\sum\limits_{i,j,k} O_m^{i,j,k} e^{i,j,k}}{\sum\limits_{i,j,k} e^{i,j,k}}$$

with $O_m = (e,\ n,\ c_s)$.



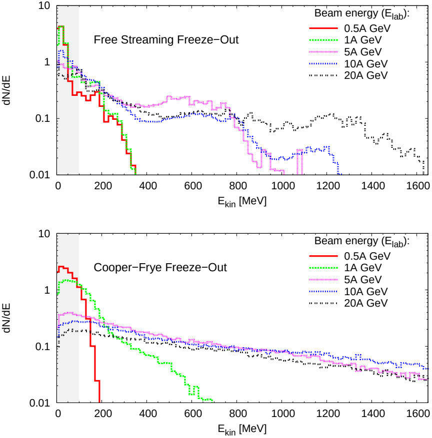
Ne+U at 10A GeV, $b = 0$

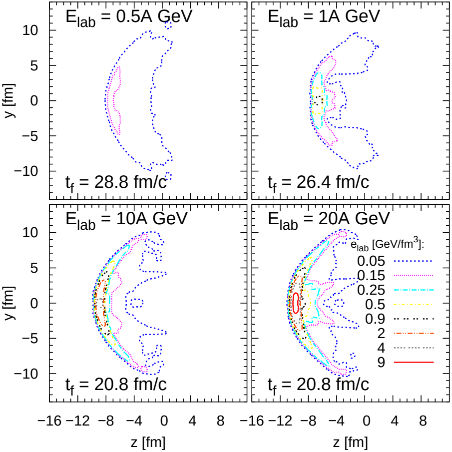
Mean sound velocity in the Mach shock wave over whole evolution: $\langle c_s \rangle \simeq 0.135$.
With $v_{\mathrm{sh}} = 1$ the corresponding emission angle is $\theta_{\mathrm{lab}} \simeq 82^\circ$.
Slower shock front $\Rightarrow$ smaller θ_{lab}.



Energy Spectra of Emitted Nucleons: Ne+U $(b = 0)$

Kinetic energy of emitted nucleons.





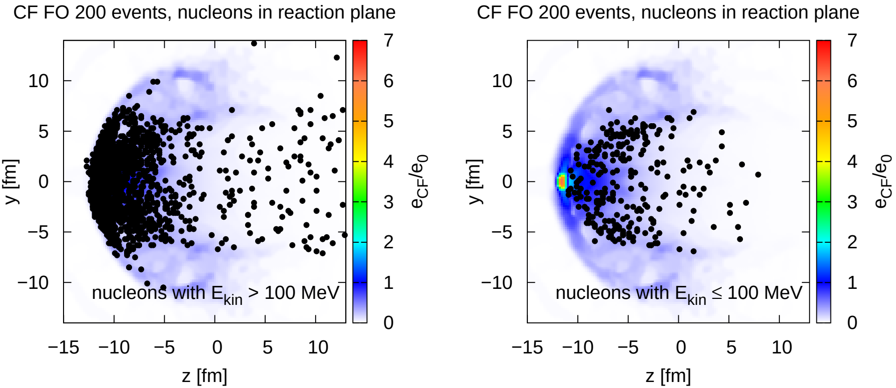
Classification of emitted Nucleons:

Nucleons with $E_{\text{kin}} > 100$ MeV originate from hot and dense **head shock**.

Nucleons with $E_{\text{kin}} < 100$ MeV originate from **Mach shock wave** behind the leading shock front. 100 MeV corresponds to nucleon velocity $v = 0.43$.

Spatial Distribution of Nucleons at t_f: Cooper-Frye

Particles in reaction plane after FO (Ne+U, $E_{\mathrm{lab}} = 5A$ GeV, $b = 0$):

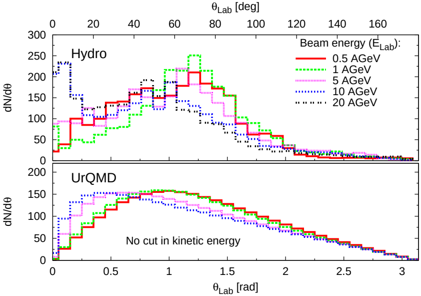


Nucleons with $\underline{E_{kin} > 100\text{ MeV}}$ have their origin principally in hot and dense region around the head shock.

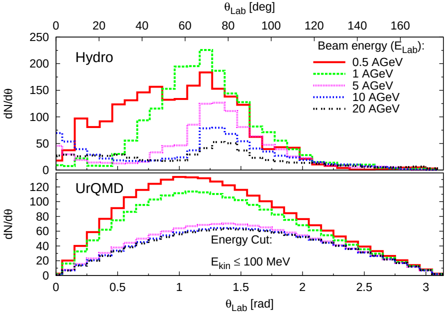
Nucleons with $\underline{E_{kin} \leq 100\text{ MeV}}$ originate from more dilute regions behind the head shock wave (**Mach shock wave** → emission under Mach angles).

Angular Distribution of Emitted Nucleons, $b = 0$

No cut in kinetic energy:



Nucleons with $E_{\mathrm{kin}} \leq 100$ MeV:



Contributions from both the head shock and from the Mach shock wave.

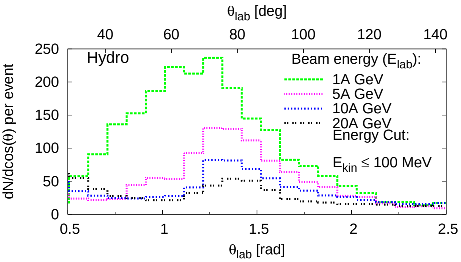
Contributions from Mach shock wave only. $\Rightarrow$ Peak at $\theta_{\mathrm{lab}} = 0$ vanishes.

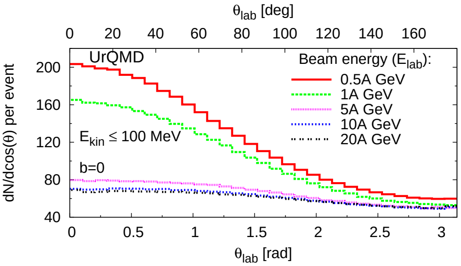
Hydro: Distinct Peak in region $68^\circ \leq \theta_{\mathrm{lab}} \leq 80^\circ \rightarrow$ **Mach shock formation**.
UrQMD: Broad peak corresponds to (boosted) isotropic emission. Underlying mechanism: momentum conserving **binary nucleon-nucleon scattering**.

Remove Signal from Isotropic Emission

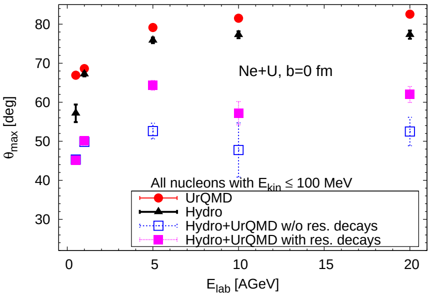
Remove sinusoidal signal coming from mapping the 3-dimensional spherical segments to 2-dimensional (detector) surface by plotting $dN/d\cos\theta_{\mathrm{lab}}$.

Hydro: Mach shock signals remains.
UrQMD: Flat particle distribution, boosted in beam direction dependent on beam energy.

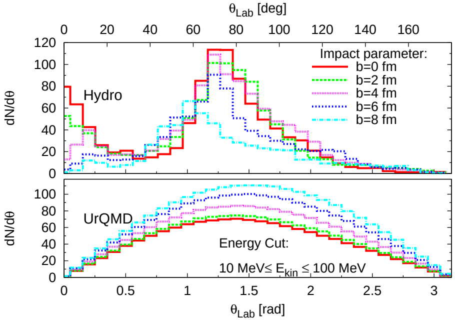


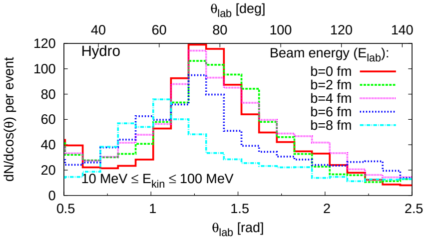


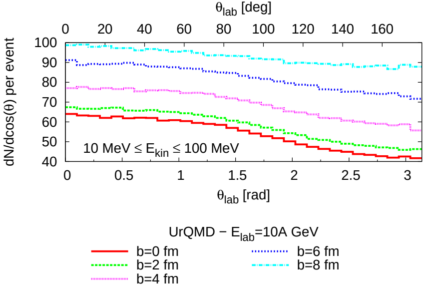
Computed Emission Angles:



Ne+U, $5A$ GeV, $b \neq 0$: Particle Emission Pattern







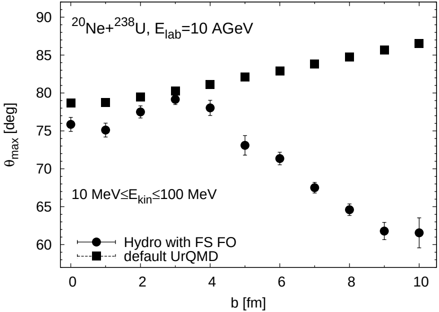
Ideal Hydro: Decreasing $\theta_{\max}$ with higher impact parameter due to tilted Mach shock wave.

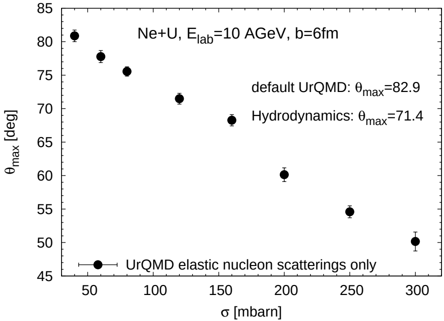
UrQMD (Viscous Hydro): $\theta_{\max} \rightarrow 90^{\circ}$, i.e. isotropic emission of particles from non-perturbed target nucleus.

→ **Particle emission angle $\theta_{\max}$ sensitive to viscosity of nuclear matter.**

Medium Viscosity from Particle Emission Angle

Ne+U, $E_{\rm lab} = 10A$ **GeV,** $b \neq 0$
Viscosity of nuclear matter can be estimated from comparison of emission angle at large impact parameters from (ideal) Hydrodynamics and UrQMD calculations:



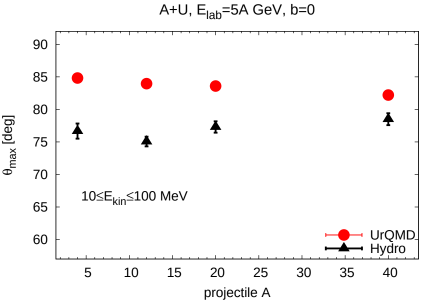


Big difference in $\theta_{\rm lab}$ for higher impact parameters ⇒ **Particle emission angle signals medium viscosity.**

Modified UrQMD with nucleon-nucleon interactions restricted to elastic scatterings only.
Increasing cross section (i.e. smaller viscosity) leads to smaller $\theta_{\rm lab}$.

Dependence on Projectile Size

- There is distinct conical emission for all projectiles up to ^{40}Ca.
- Maximum of emission angle in laboratory frame $\theta_{\max}$ stays nearly constant.
- With increasing projectile size (nearly symmetric systems) the leading head shock strengthens substantially ⇒ no matter left on its back side in which a Mach shock wave could evolve ⇒ no conical emission of reaction products.



Conclusions

- Calculations show that there is a conical emission of reaction products in high-energy collisions of unequally sized nucleons.
- The emission angle can provide insight on the sound velocity of nuclear matter.
- The emission angle stays constant over a wide range of beam energies and projectile sizes.
- Results from collisions with non-vanishing impact parameter can show if a shock wave creation or nucleon-nucleon scattering is the underlying process for conical emission.
- Measuring the emission angle at large impact parameters may be used to estimate the viscosity of nuclear matter.

Conclusions

Thank you for your attention.

Details of Cooper-Frye Freeze-Out Implementation

[??] H. Petersen, J. Steinheimer, G. Burau, M. Bleicher and H. Stöcker, Phys. Rev. C **78** (2008) 044901 [arXiv:0806.1695].

Monte Carlo sampling of Cooper-Frye equation with the following steps:

back

1. Particle numbers of species i

$$N_i = n_i \gamma V_{\mathrm{cell}} = \int d^3p \; f_i(x,p) \gamma V_{\mathrm{cell}}$$

with $f_i(x,p)$ the lrf equilibrium distribution fct. Momentum integration leads to particle number density

$$\text{Pions: } n_\pi = \frac{g_\pi m_\pi^2 T}{(2\pi)^2} \sum_{k=1}^{\infty} \frac{1}{k} K_2\left(\frac{k m_\pi}{T}\right);$$

$$\text{other: } n_i = \frac{g_i m_i^2 T}{(2\pi)^2} e^{\mu/T} K_2\left(\frac{m_i}{T}\right)$$

with $\mu = B\,\mu_B + S\,\mu_S$.

2. Average total number of particles in cell:

$$\langle N \rangle = \sum_i N_i$$

3. Total number of particles emitted from cell obtained from Poisson distribution $P(N) = \langle N \rangle^N / N! \exp\left(-\langle N \rangle\right)$.

5. Particle type chosen according to $N_i / \langle N \rangle$.

6. I_3 component of isospin is randomly distributed (UrQMD assumes full isospin symmetry), where isospin components that lead to the desired value of the total charge are favoured.

7. 4-momenta of particles generated according to Cooper-Frye formula. For baryons and strange mesons the chemical potentials for baryon number and strangeness taken into account.

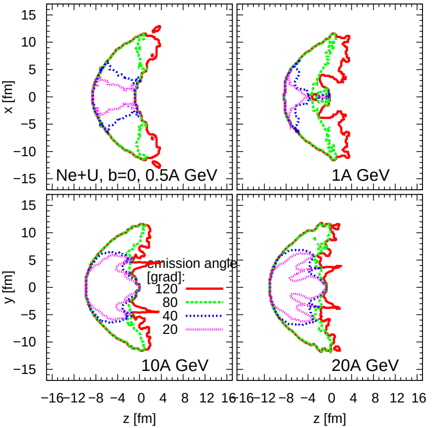
8. Particle vector information is transferred back into UrQMD. Subsequent hadronic cascade calculations incorporate final state effects, e.g. rescatterings of the particles and resonance decays.

Steps pursued on random cells until initial net baryon number is achieved. Charge and strangeness also conserved in each event separately. Energy conservation fulfilled for mean values averaged over several events.

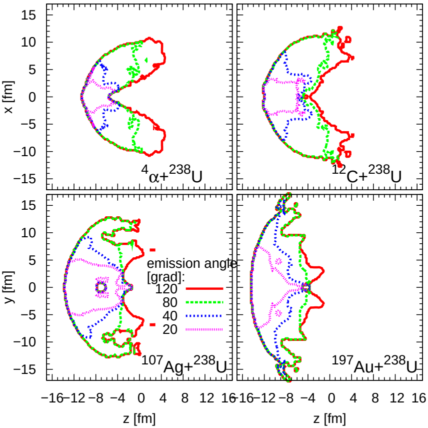
Spatial Distribution of Nucleons at t_f: Free Streaming

Distribution of emission angles in reaction plane with Free Streaming Freeze-Out:

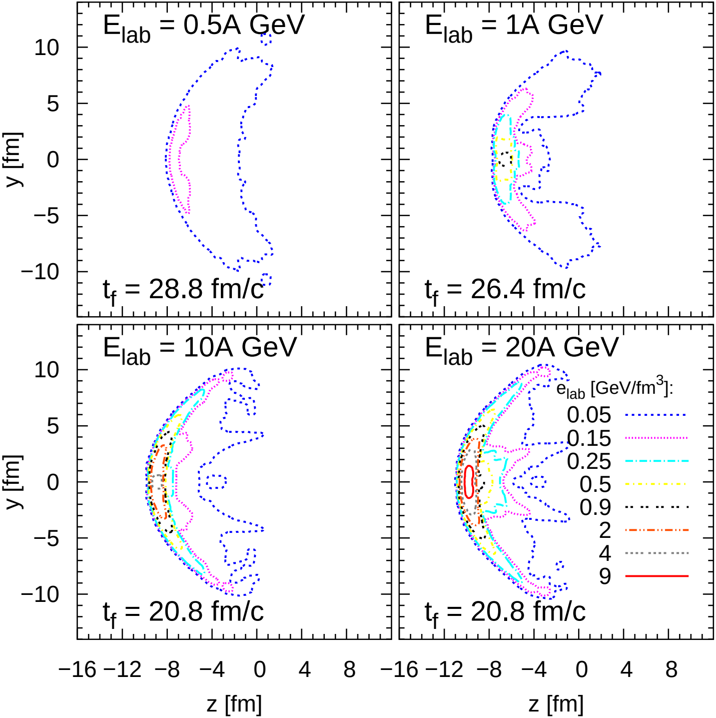
Ne+U, different beam energies $b = 0$ fm:

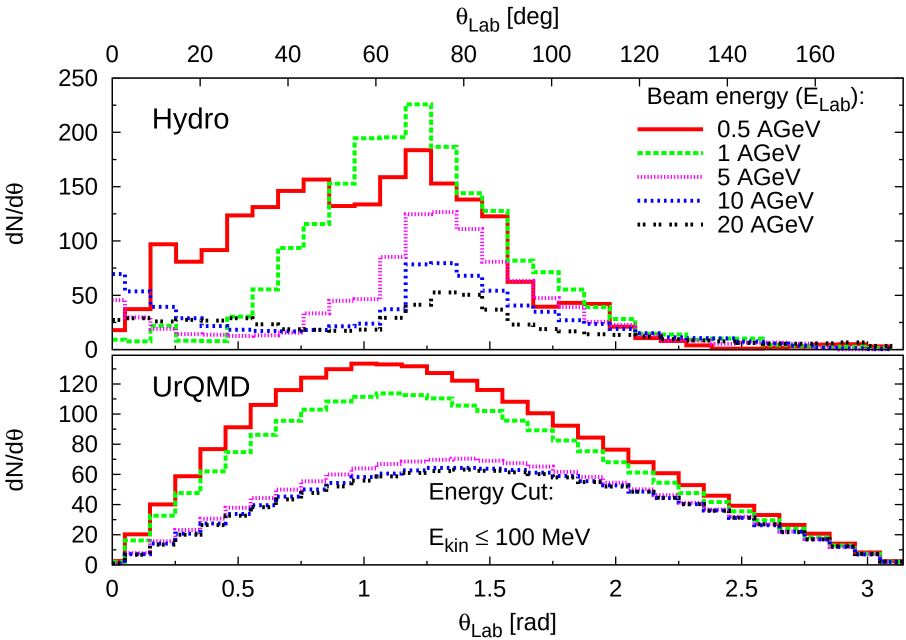


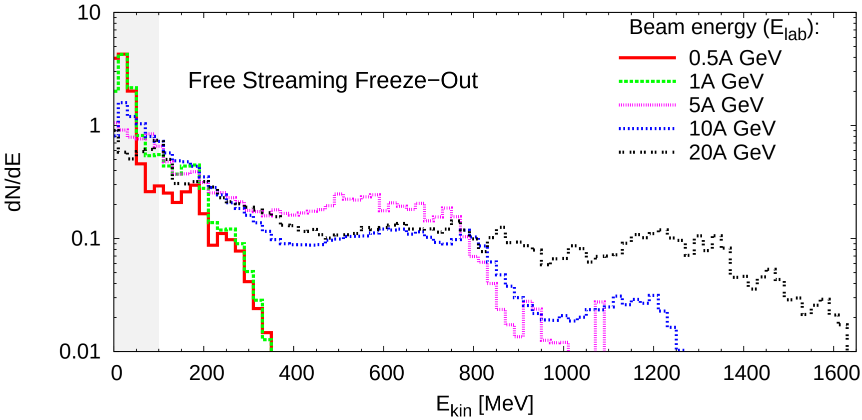
Different projectiles at 5A GeV, $b = 0$ fm:



Nucleon emission from head shock under small angles. Particles with larger emission angle originate from outer regions of the Mach shock wave.


E_lab = 0.5A GeV
t_f = 28.8 fm/c
E_lab = 1A GeV
t_f = 26.4 fm/c
E_lab = 10A GeV
t_f = 20.8 fm/c
E_lab = 20A GeV
t_f = 20.8 fm/c
e_lab [GeV/fm^3]:
0.05
0.15
0.25
0.5
0.9
2
4
9
y [fm]
10
5
0
−5
−10
z [fm]
−16
−12
−8
−4
0
4
8


θ_Lab [deg]
0
20
40
60
80
100
120
140
160
250
200
150
100
50
0
dN/dθ
Hydro
Beam energy (E_Lab):
0.5 AGeV
1 AGeV
5 AGeV
10 AGeV
20 AGeV
UrQMD
120
100
80
60
40
20
0
dN/dθ
Energy Cut:
E_kin ≤ 100 MeV
0
0.5
1
1.5
2
2.5
3
θ_Lab [rad]


Free Streaming Freeze−Out
Beam energy (E_{lab}):
0.5A GeV
1A GeV
5A GeV
10A GeV
20A GeV
dN/dE
10
1
0.1
0.01
0
200
400
600
800
1000
1200
1400
1600
E_{kin} [MeV]