Heavy quark potentials and spectral functions

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- Introduction : Quarkonia dissolution at high temperatures
- Static quarks and color screening at high temerature
- Meson spectral functions and MEM
- Lattice calculation of quarkonium correlators and spectral functions
- Relation of the quarkonium correlators to the heavy quark transport
- Conclusions and Outlook

Physics at High Baryon Density, Trento, May 29-June 2, 2006

Heavy Quarkonia in QGP

$m_c,\ m_b \gg \Lambda_{QCD}$ ⇒ non-relativistic approximation $(-\frac{\nabla_R^2}{m} + V(R))\psi(R) = E\psi(R)$

Quark Gluon Plasma

⇓

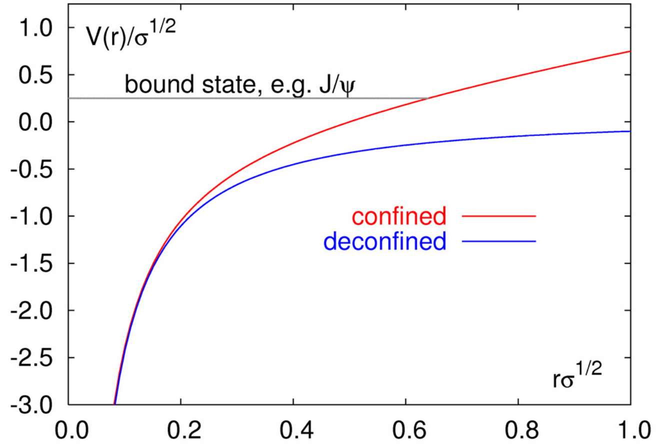
Chromo-electric (Debye) screening:

$$V(r) = -\frac{4}{3}\frac{\alpha_s}{r}\exp(-m_D r), m_D \sim \sqrt{4\pi\alpha_s}T$$

⇓

Quarkonium Suppression

Matsui, Satz, PLB 178 (86) 416



Hierarchy in binding energy ⇒

Sequential Suppression

Karsch, Mehr Satz, ZPC 37 (88) 617

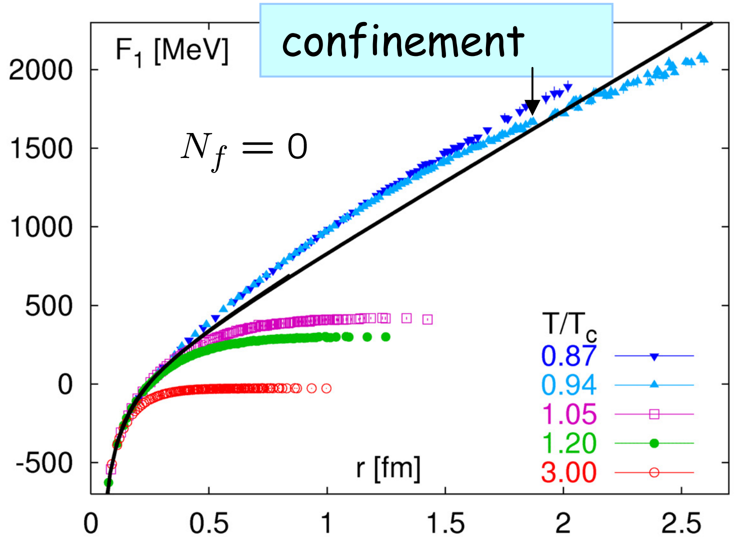
Digal, P.P, Satz, PRD 64 (01) 094015

$c\bar{c}$	$\psi'(2S)$	$\chi_c(1P)$	$J/\psi(1S)$	
$b\bar{b}$		$\Upsilon''(3S),\ \chi_b'(2P)$	$\chi_b(1P)$	$\Upsilon(1S)$
$\langle r^2\rangle^{1/2}$ fm	0.9	0.7	0.4	0.2

temperature

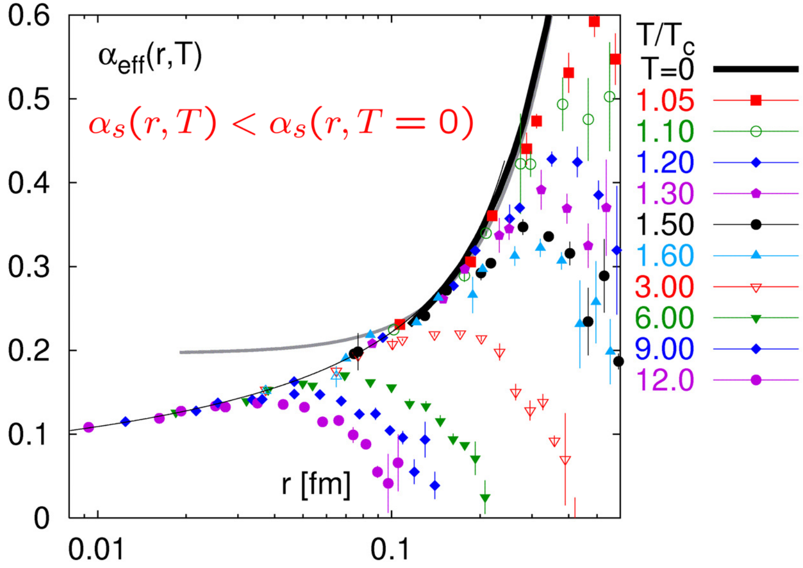
Free energies of static charges

Kaczmarek, Karsch, P.P., Zantow,PRD70 (2004) 074505

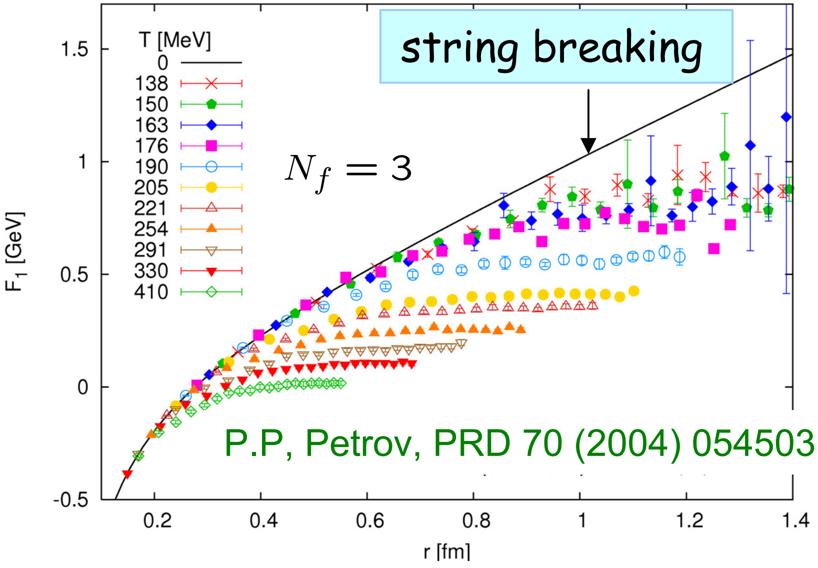


confinement

$N_f = 0$



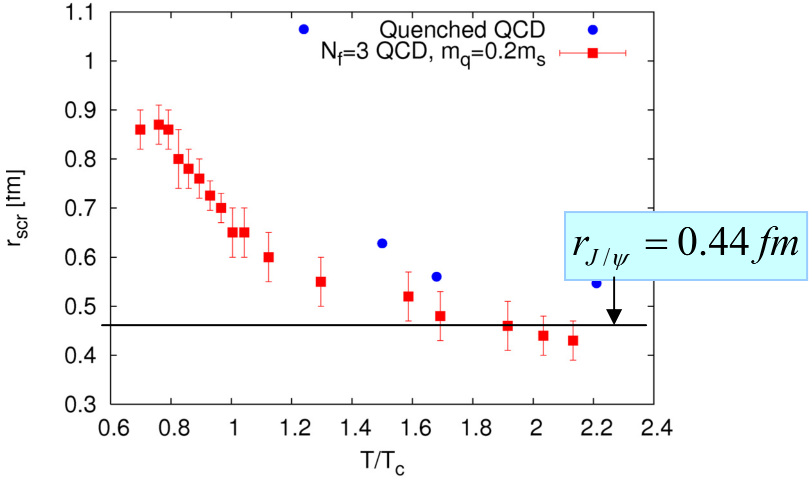
$\alpha_s(r,T) < \alpha_s(r,T=0)$



string breaking

$N_f = 3$

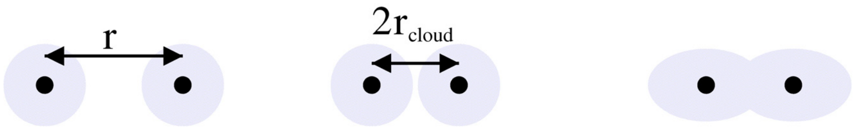
P.P, Petrov, PRD 70 (2004) 054503



$r_{J/\psi} = 0.44\,fm$

We would expect that J/ψ is dissolved at $1.2T_c$

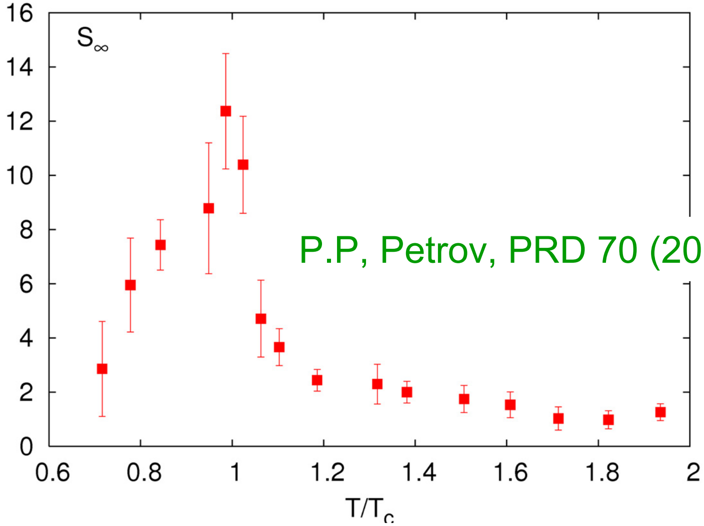
Entropy and internal energies of static charges



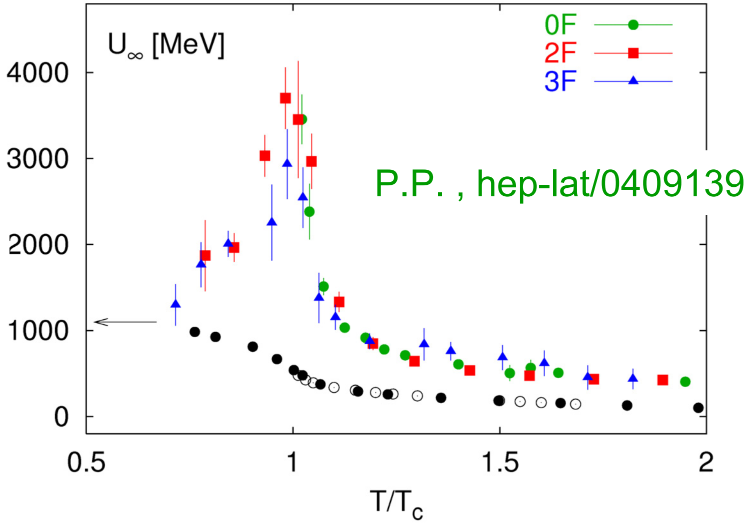
LO : $V(r) \simeq -\frac{4}{3} g^2 \frac{e^{-m_D r}}{4\pi r} \simeq F_1(r,T) \simeq U_1(r,T),$ $\quad m_D = gT\sqrt{1+N_f/6}$

NLO: $F_1(r,T) = -g^2 C_F \frac{e^{-m_D r}}{4\pi r} - \underset{\uparrow}{\frac{C_F m_D g^2}{4\pi}}$ $\quad U_1(r,T) = F_1(r,T) + TS_1(r,T)$

entropy contribution at order $\mathcal{O}(g^3)$



P.P, Petrov, PRD 70 (2004) 054503

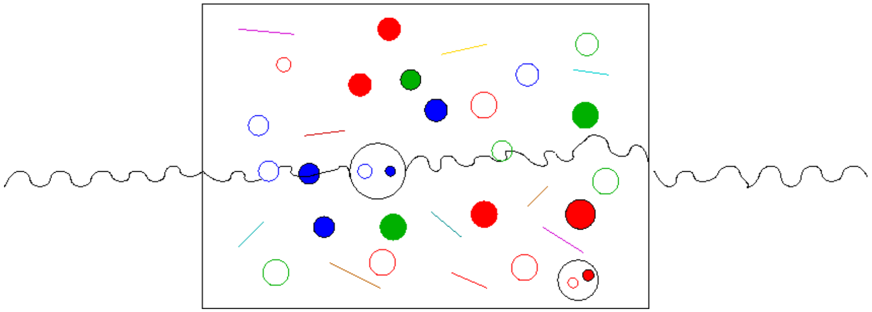


P.P. , hep-lat/0409139

Screening cannot be understood in terms of medium modification of the 2-body potential, there is significant entropy production associated with it !

Meson correlators and spectral functions

Spectral (dynamic structure) function



Example : virtual photon

$$R(\omega) = \frac{\sigma_{e^+e^-\to\mu^+\mu^-}}{\sigma_{e^+e^-\to hadrons}} = \sigma(\omega)/\omega^2$$

$$\frac{dW}{d\omega d^3p} = \frac{5\alpha_{em}^2}{27\pi^2}\frac{1}{\omega^2(e^{\omega/T}-1)}\sigma(\omega,\vec{p},T)$$

$$\frac{D^>(\omega)-D^<(\omega)}{2\pi} = \frac{1}{\pi}\operatorname{Im} D_R(\omega) = \sigma(\omega)$$ ➡ What are the excitations (dof) of the system ?

$$G(\tau,\vec{p},T) = \int d^3x\, e^{i\vec{p}\cdot\vec{x}} \left\langle J_H(\tau,\vec{x}) J_H{}^+(0,0)\right\rangle, \quad J_H(\tau,\vec{x}) = \overline{\psi}(\tau,\vec{x})\Gamma_H\psi(\tau,\vec{x})$$

$$\Gamma_H = 1,\ \gamma_5,\ \gamma_\mu,\ \gamma_5\cdot\gamma_\mu$$

quenched approximation is used !

$$\boxed{G(\tau,T) = D^>(-i\tau)}$$

⇧ Imaginary time ⇧ Real time

$$\boxed{G(\tau,T) = \int_0^\infty d\omega\sigma(\omega,T)\frac{\cosh(\omega(\tau-1/(2T))}{\sinh(\omega/(2T))}}$$

Reconstruction of the spectral functions : MEM

$$G(\tau, T) = \int_0^\infty d\omega \sigma(\omega, T) \cdot K(\omega, T)$$

$\mathcal{O}(10)$ data and $\mathcal{O}(100)$ degrees of freedom to reconstruct

Bayesian techniques: find $\sigma(\omega, T)$ which maximizes $P[\sigma|DH]$

data

Prior knowledge

H :

$\sigma(\omega, T) > 0$ ⇒ Maximum Entropy Method (MEM)

Asakawa, Hatsuda, Nakahara, PRD 60 (99) 091503, Prog. Part. Nucl. Phys. 46 (01) 459

$$P[\sigma|DH] = \exp(-\frac{1}{2}\chi^2 + \alpha S)$$

Likelyhood function

Shannon-Janes entropy:

$$S = \int_0^\infty d\omega[\sigma(\omega) - m(\omega) - \sigma(\omega) \ln \frac{\sigma(\omega)}{m(\omega)}]$$

$m(\omega)$ - default model

$m(\omega \gg \Lambda_{QCD}) = m_0 \omega^2$ -perturbation theory

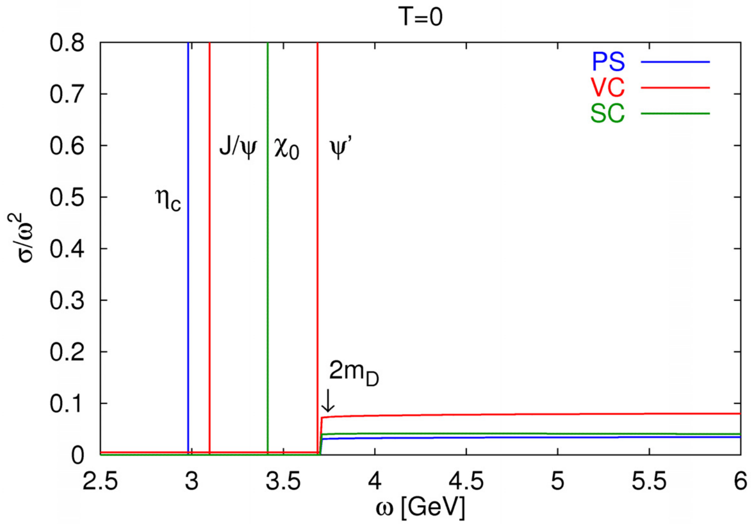
Heavy quarkonia spectral functions

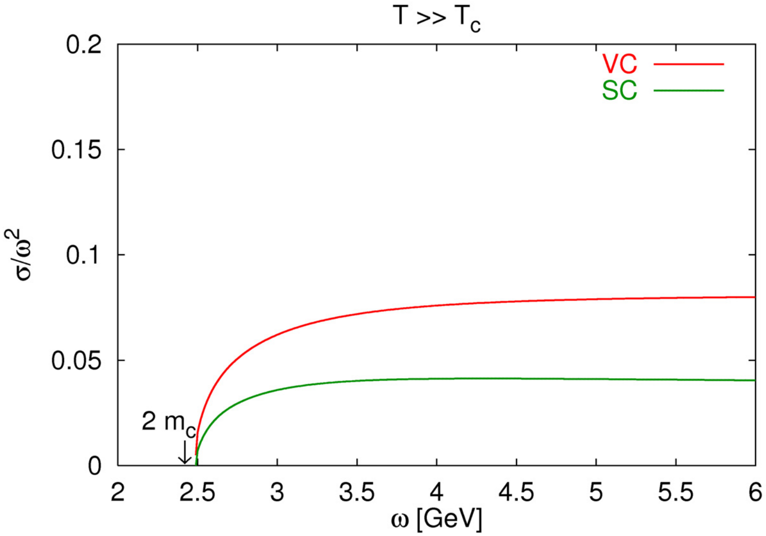
γ_5 : Pseudo – scalar(PS) $\rightarrow \eta_c\ (^1S_0)$

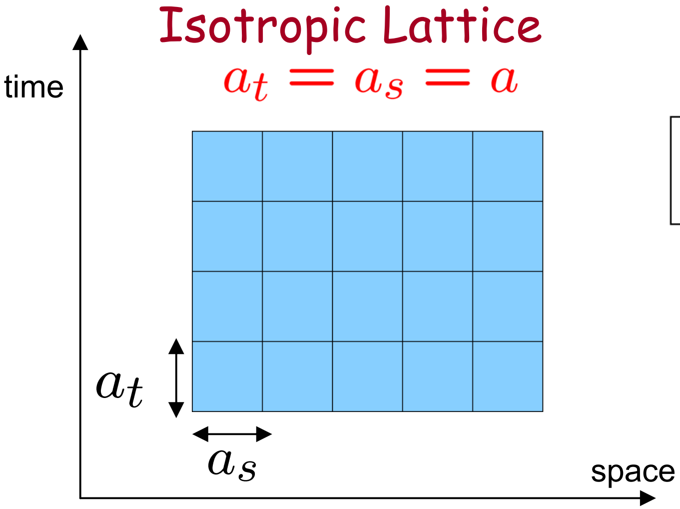
1 : Scalar(SC) $\rightarrow \chi_{c0}\ (^3P_0)$

γ_μ : Vector(VC) $\rightarrow J/\psi\ (^3S_1)$

$\gamma_5\gamma_\mu$: Axial – Vector(AX) $\rightarrow \chi_{c1}\ (^3P_1)$



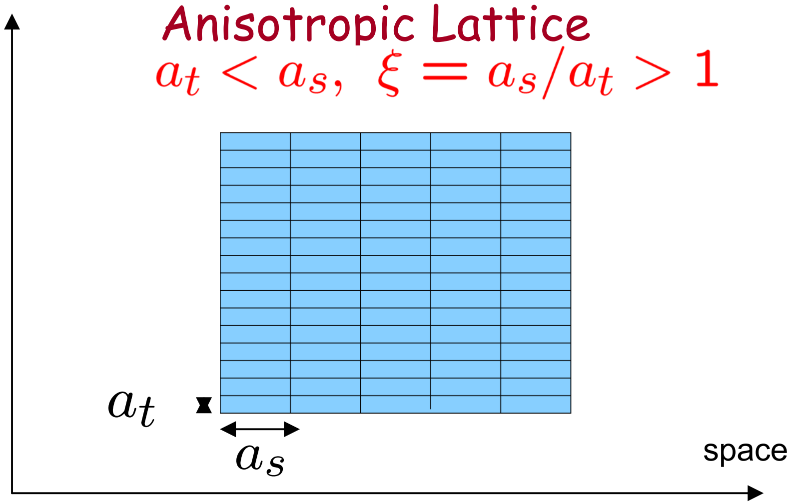




Isotropic Lattice

$a_t = a_s = a$

$$T = \frac{1}{N_t a_t}$$



Anisotropic Lattice

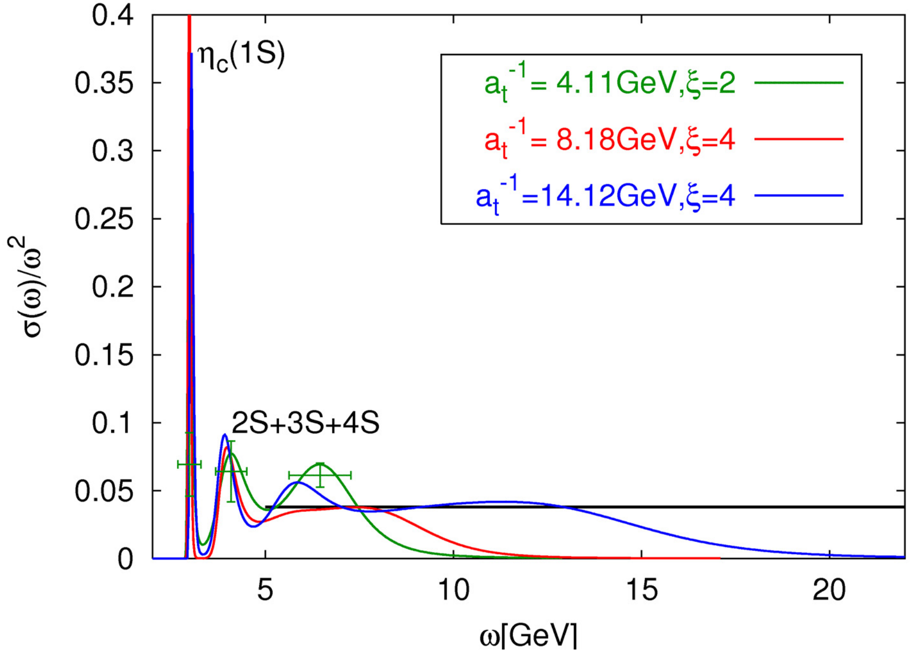
$a_t < a_s,\ \xi = a_s/a_t > 1$

Charmonia spectral functions at T=0

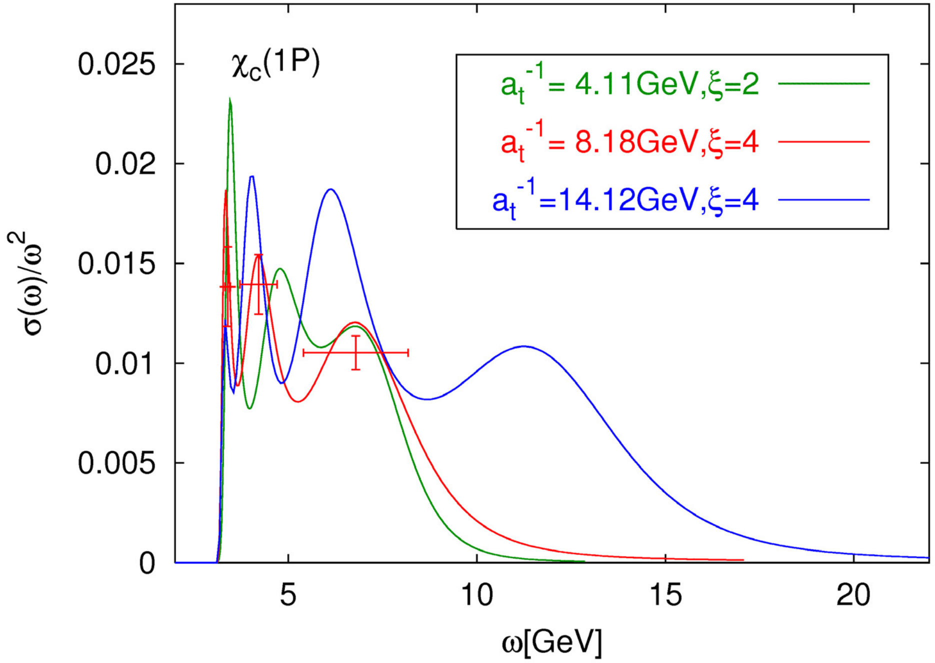
Anisotropic lattices: $16^3 \times 64, \xi = 2$ $16^3 \times 96, \xi = 4$, $24^3 \times 160, \xi = 4$
$L_s = 1.35 - 1.54$fm, #configs=500-930;
Wilson gauge action and Fermilab heavy quark action

Jakovác, P.P., Petrov, Velytsky, hep-lat/0603005

Pseudo-scalar (PS) → S-states



Scalar (SC) → P-states



For $\omega > 5$ GeV the spectral function is sensitive to lattice cut-off ;
In the SC channel even the ground state is poorly resolved ;

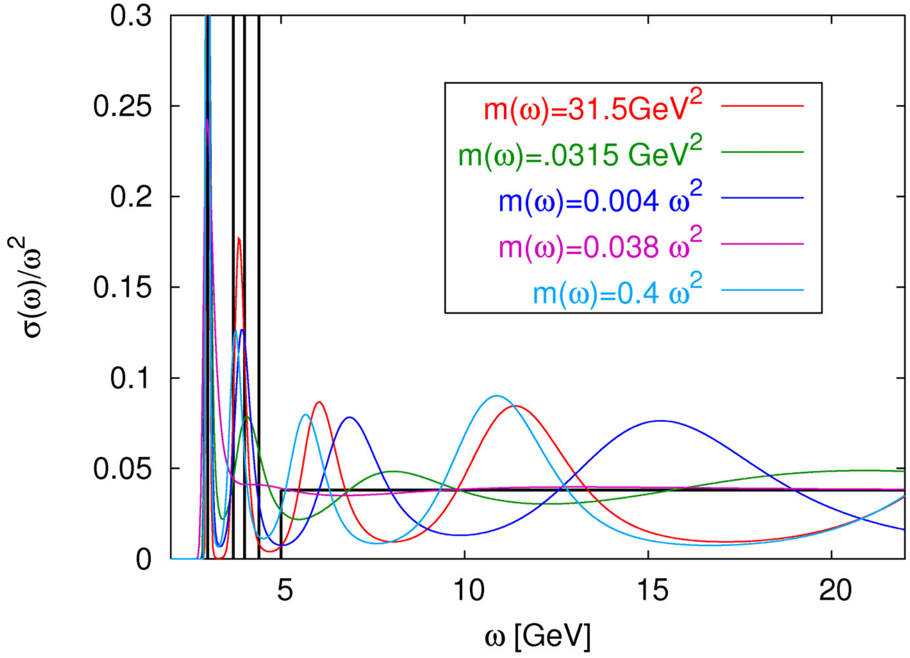
Charmonia spectral functions at T=0 (cont'd)

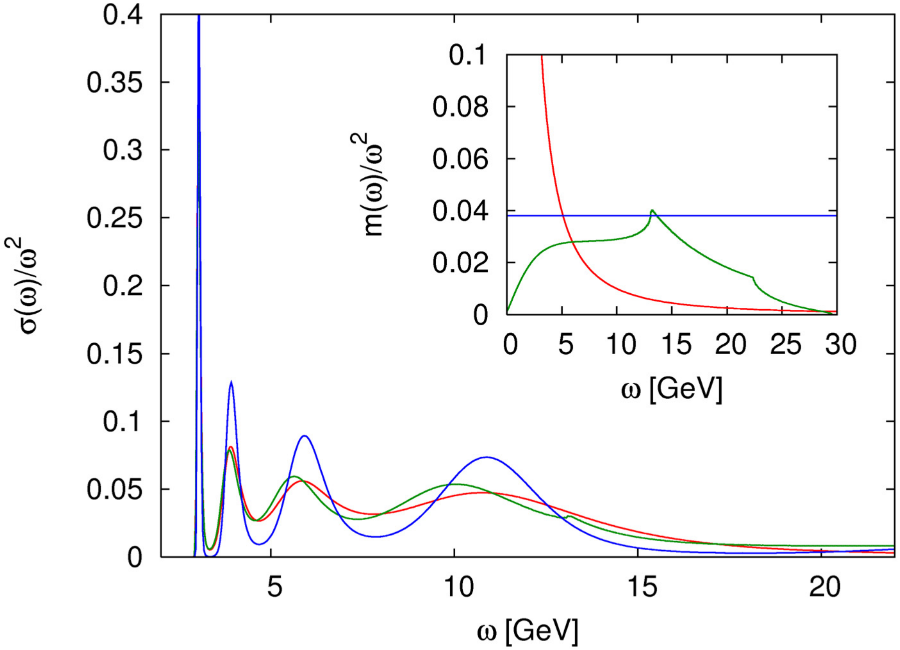
What states can be resolved and what is the dependence on the default model ?

Reconstruction of an input spectral function :

Lattice data in PS channel for:

$$a_t^{-1} = 14.12\text{GeV},\ N_t = 160$$





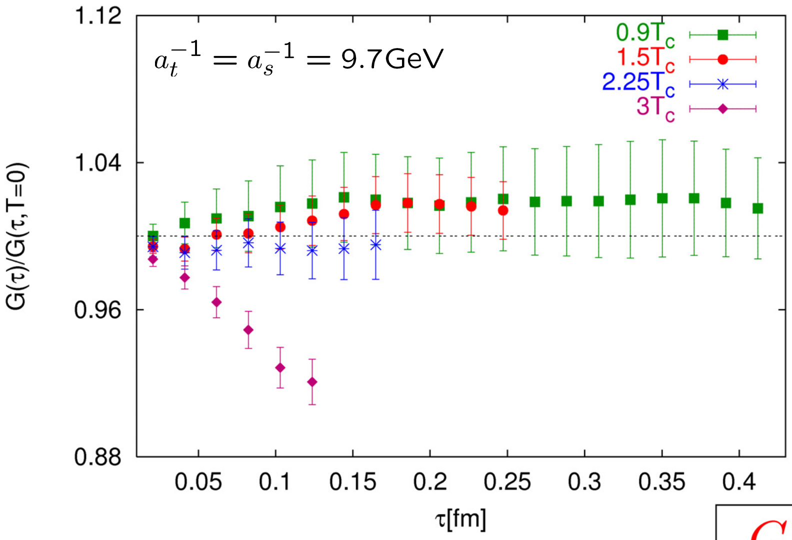
Ground states is well resolved, no default model dependence;

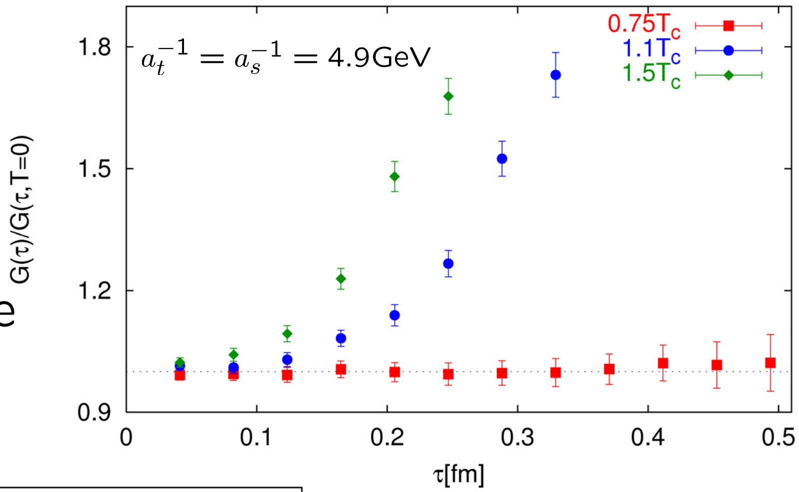
Excited states are not resolved individually, moderate dependence on the default model;

Strong default model dependence in the continuum region, $\omega > 5$ GeV

Charmonia correlators spectral functions at T>0

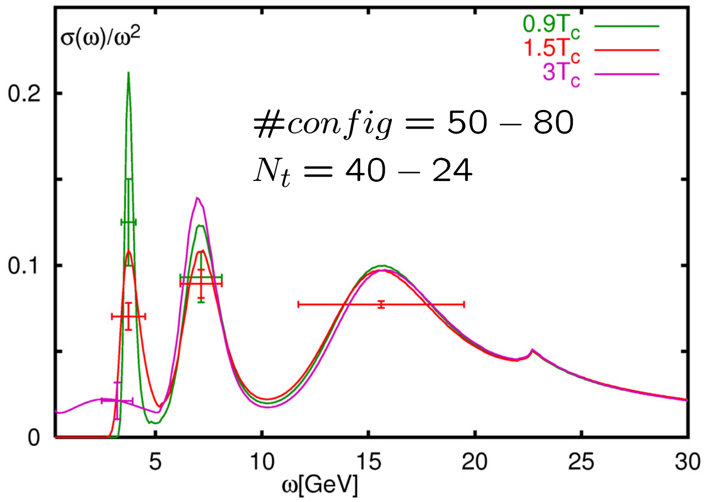
Datta, Karsch, P.P , Wetzorke, PRD 69 (2004) 094507



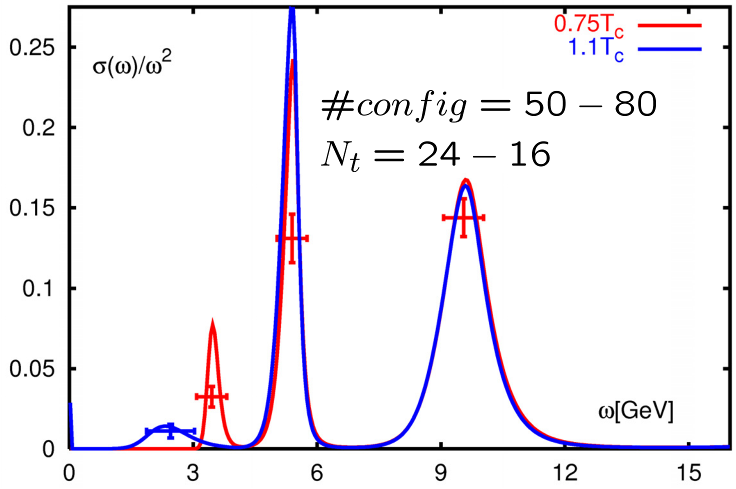


no change in $\sigma(\omega, T)$

$$\boxed{G(\tau,T)/G(\tau,T=0)=1}$$



1S (J/ψ, η_c) exists at $1.5T_c$



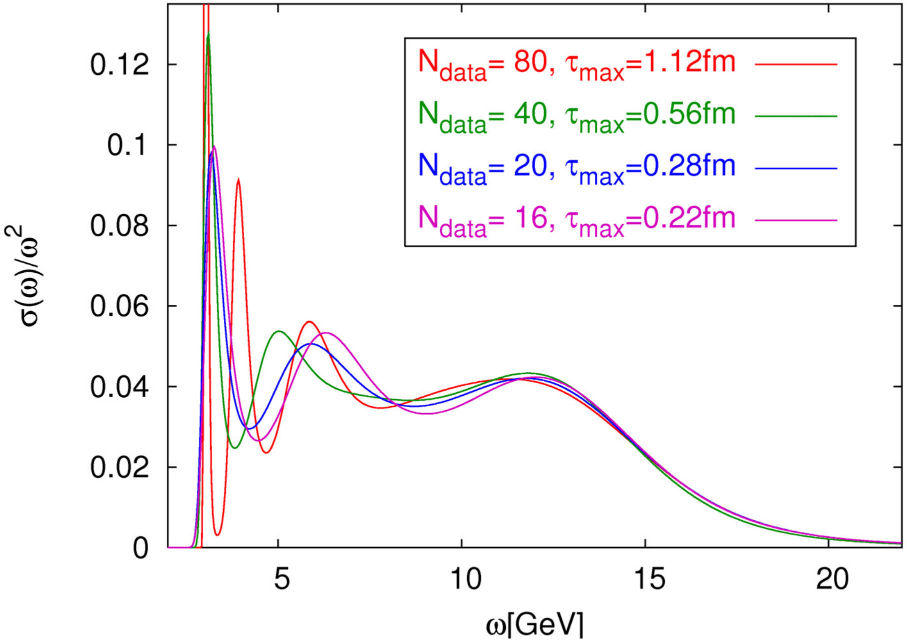
1P (χ_c) is dissolved at $1.1T_c$

Charmonia spectral functions in PS channel at T>0

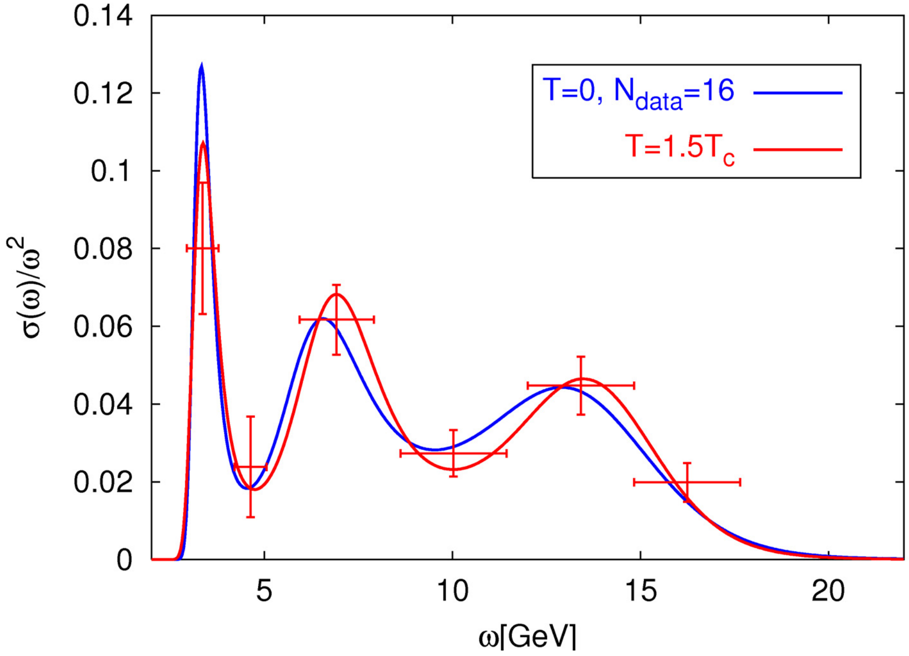
$$T = 1/(N_t a) \quad \leftrightarrow \quad \tau_{max} = 1/(2T)$$

$$\text{PS, } 24^3 \times N_t,\ a_t^{-1} = 14.12 \text{ GeV},\ \xi = 4$$

$T = 0, \quad N_t = 160$



$T = 1.5T_c, \quad N_t = 32$



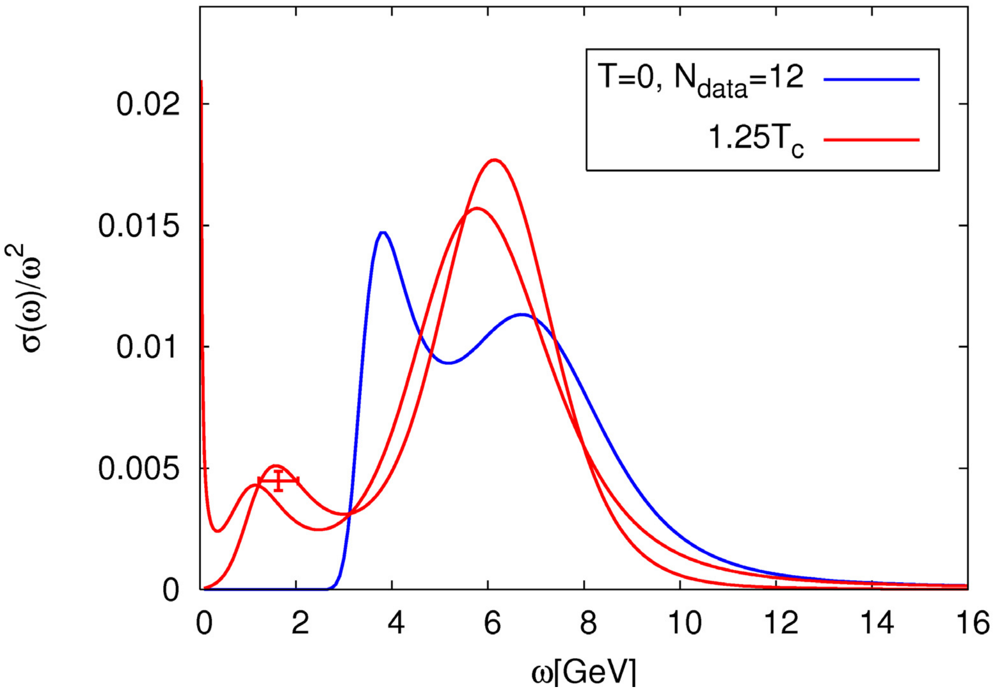
ground state peak is shifted, excited states are not resolved when $\tau_{max}, \quad N_{data}$ become small

no temperature dependence in the PS spectral functions within errors

Jakovác, P.P., Petrov, Velytsky, hep-lat/0603005

Charmonia spectral functions in SC channel at T>0

SC, $16^3 \times N_t$, $a_t^{-1} = 8.18$ GeV, $\xi = 4$

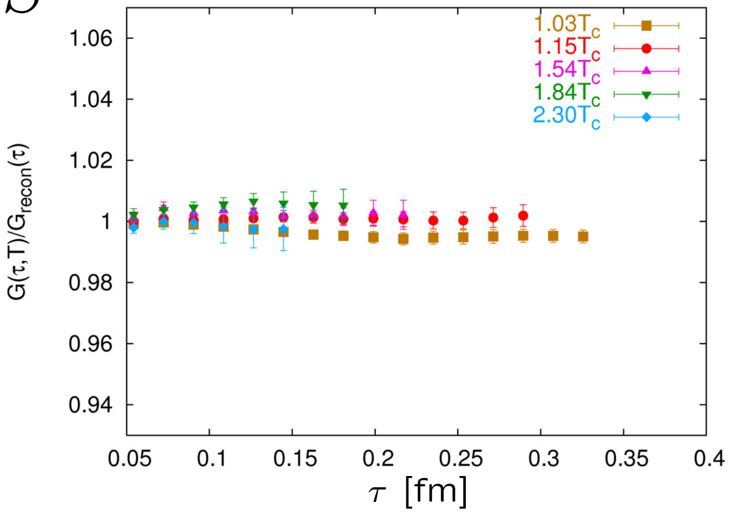


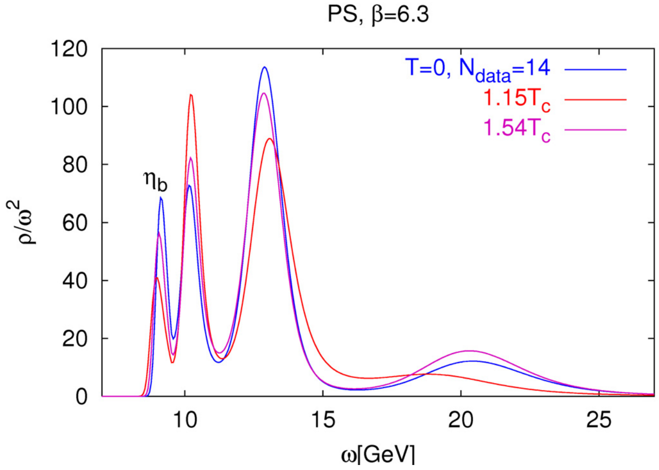
significant changes in the spectral functions (melting of 1P state ?) small statistical errors but significant dependence on the default model

Bottomonia spectral functions on anisotropic lattices

Jakovác, P.P., Petrov, Velytsky, hep-lat/0509138

$1S$

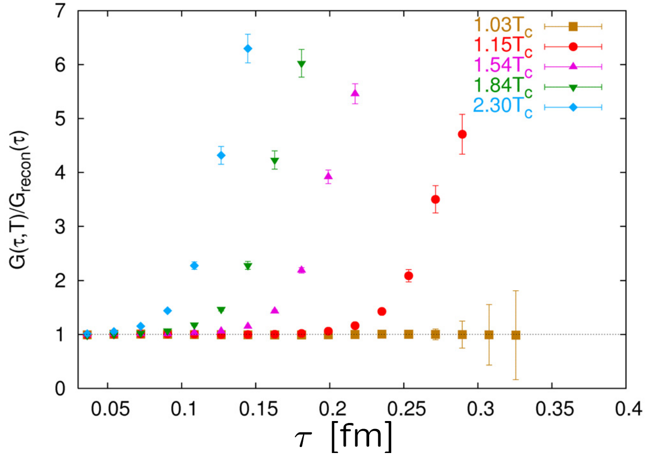


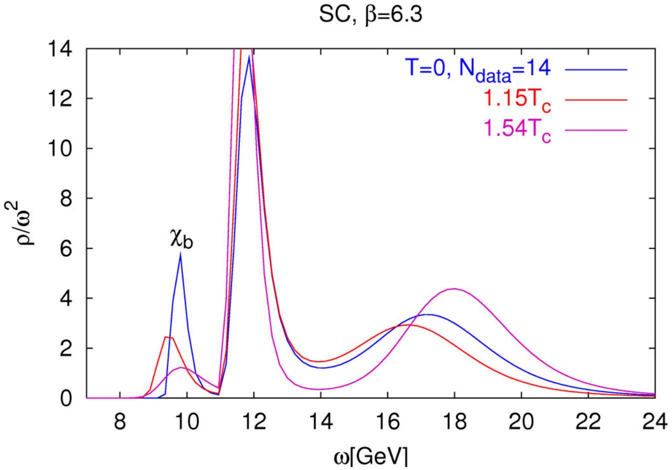


1S states are dissolved only at :

$T > 3T_c$

$1P$





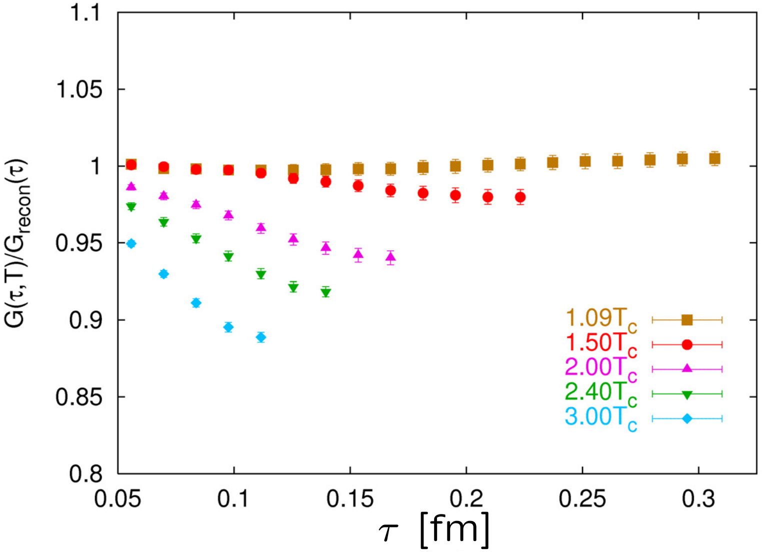
1P states are dissolved at :

$1.15T_c < T < 1.54T_c$

expected χ_b survive till $\sim 1.5T_c$

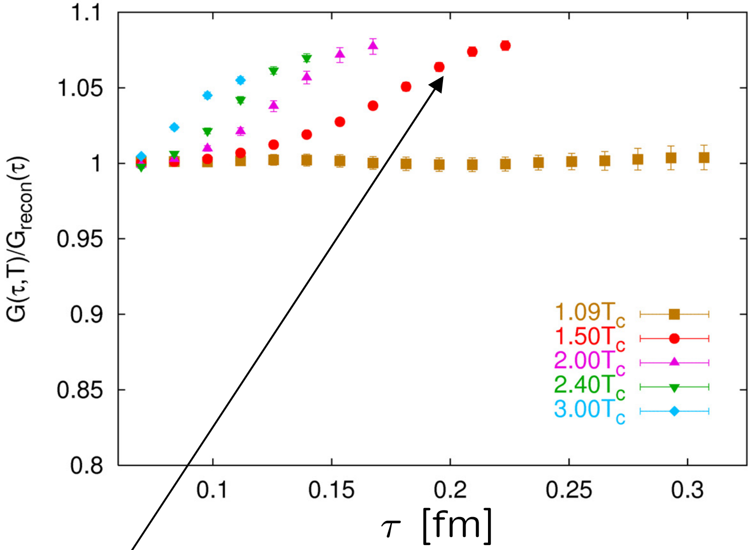
Vector correlator and heavy quark diffusion

Pseudo-scalar (η_c)



1S charmonium states survies

Vector (J/ψ)



P.P., Petrov, Velytsky, Teaney, hep-lat/0510021

Vector current is conserved ⟶ fluctuations of charm number

$$\sigma_V^{ii}(\omega) = F^2_{J/\psi}(T)\delta(\omega^2 - m^2_{J/\psi}(T)) + \frac{1}{4\pi^2}\omega^2\sqrt{1 - \frac{4m_D^2(T)}{\omega^2}} + \chi_s(T)\left(\frac{T}{M}\right)\omega\delta(\omega)$$

$$\frac{1}{3}\chi_s(T)\frac{T}{M}\omega \cdot \frac{1}{\pi} \cdot \frac{\eta}{\omega^2+\eta^2}$$

Interactions

Free streaming :
Collision less Boltzmann equation

Effective Langevin theory

$$\eta = \frac{T}{M}\frac{1}{D} \qquad \partial_t N_c + D\nabla^2 N_c = 0$$

Transport contribution to the Euclidean correlators

$$t_{\text{transport}} \simeq M/T^2 \gg 1/T \gg 1/M$$

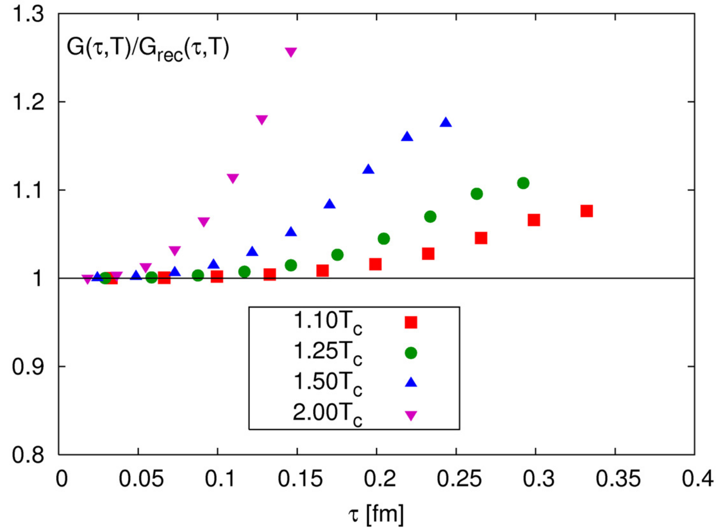
$$G_{JJ}(\omega) = G_{JJ}^{\text{low}}(\omega) + G_{JJ}^{\text{high}}(\omega)$$

P.P. and D. Teaney, hep-ph/0507318

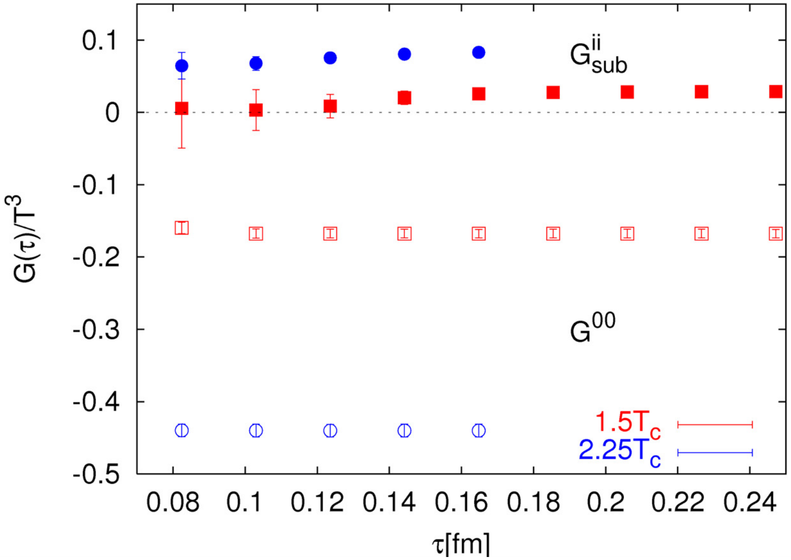
$$\sigma_{JJ}^{\text{low}} = \frac{1}{3}\chi_s(T)\frac{T}{M}\omega \cdot \frac{1}{\pi} \cdot \frac{\eta}{\omega^2+\eta^2}$$

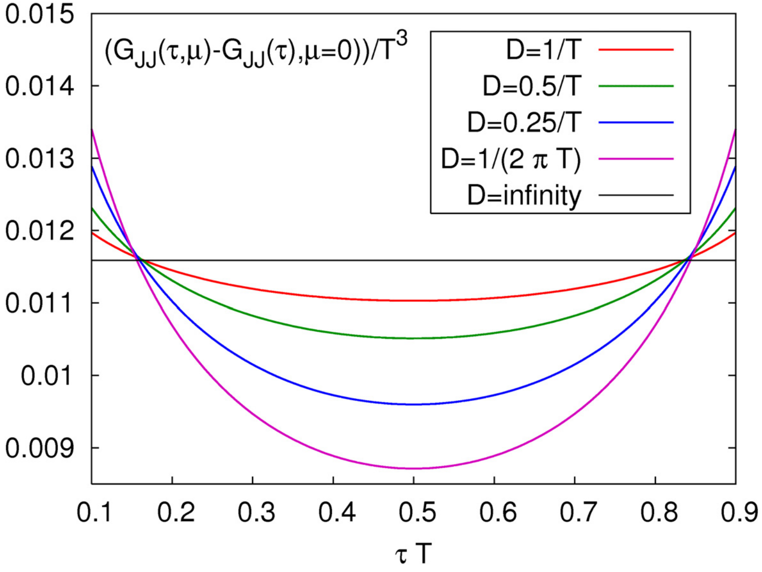
$$\chi_s = \int \frac{d^3p}{(2\pi)^3} \exp(-\sqrt{p^2+M^2}/T)$$

$$G_{JJ}^{\text{low}}(\tau) \simeq \chi_s(T)\frac{T}{M} \qquad G_{00}^{\text{low}}(\tau) \simeq -\chi_s(T)$$



Lattice data (Datta et al,) : $\frac{T}{M} \simeq 5.8(4)$





Conclusions

- Screening at high temperature cannot be understood purely in terms of modification of inter-quark forces, there is significant entropy generation by static charges. Potential model approach need to be revisited

- 1S charmonia states (η_c, J/ψ) survive till unexpectedly high temperatures
- indications for melting of 1P charmonia states (χ_{c0}, χ_{c1})
- indications for melting of 1P bottomonia states (χ_{b0}, χ_{b1}) Unexpected !

- Euclidean correlators calculated on the lattice are sensitive to transport contribution to the spectral functions
  Better lattice data are required but no indication for *DT<1* from the current lattice data