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In this section we summarize the principle physical concepts describing the reaction dynamics in the beam energy range between 2 and 30 AGeV, relevant for the CBM experiment. A detailed discussion of the various models presently available, the degrees of freedom, ranging from hadrons and hadronic resonances over phenomenological strings towards quark and gluons and respective dynamical concepts considered therein for the various reaction stages together with the possible options of the models is given in section “Description of the Models”.

3.3.1 Hydrodynamic type models

Hydrodynamic models rely on the assumption of instantaneous local equilibration. In multi-fluid versions this holds within each fluid, which additional source terms describe the transport among the different fluid components. Then the equations of motion are given by the conservation laws of conserved currents and that of the energy-momentum tensor, which in the relativistic version read:

$$\begin{aligned} \partial_\mu J^\mu_{a,\alpha} &= C_{a,\alpha} \\ \partial_\mu T^{\mu\nu}_\alpha &= F^\nu_\alpha \end{aligned}$$

Here a denotes the different conserved charges (el. charge , baryon number, strangeness, ...), while α denotes the different fluids. The E-M-tensor of each fluid is determined through the pressure P_α and the energy-density ε_α as given by an equation of state (EoS)

$$T^{\mu\nu}_\alpha = (\varepsilon_\alpha + P_\alpha) u^\mu_\alpha u^\nu_\alpha - g^{\mu\nu} P_\alpha,$$

where u^μ_α is the local four-velocity of the fluid α.

3.3.1.1 One-fluid Hydrodynamics

In this case the source terms on the r.h.s. are zero implying instantaneous local stopping and equilibration. Thereby the instantaneous stopping leads to the formation of shock fronts. This model permits to incorporate the EoS and depending on the EoS to deal with phase-transitions. This way collective phenomena like flow can be described. It excludes transparency effects and possible super cooling phenomena.

3.3.1.2 Multi-fluid Hydrodynamics

The multi-fluid model permits the exchange of particles and energy among the different fluids, expressed through the source terms, the particle fluxes $C_{a,\alpha}$ and the friction forces F^{μ}_{α} among the fluids on the r.h.s. of the hydrodynamics equations of motion. Besides the EoS these transport coefficients are to be determined as the model inputs. These type of models relax the assumption of local equilibrium and permit to describe various kind of non-equilibrium phenomena such as partial stopping, under cooling or super heating in phase-transitions or delaying chemical or phase-conversion processes by corresponding transfer rates. Thus the different fluids can locally attain different flow values, temperatures and chemical compositions.

3.3.1.3 Three-fluid model

The three-fluid model of [1] assumes one fluid for each incoming nucleus and a third fluid created through the collisions among the nucleons of the two incident fluids. Different model EoS can by implemented. The coupling terms C_{α} and F^{μ}_{α} are presently estimated by binary collisions with the option to scale their value by a common factor in order to mimic multi-particle collisions relevant at high densities.

First results of the model were presented in [2] employing a simple two-phase model-EoS. See also DynamicalTrajectories.

3.3.1.4 Non-equilibrium chemistry

The flavor kinetic model of Barz. et al. [3] assumes a two-phase scenario where each phase consists of a number of constituents, the quarks and gluons in the QGP-phase and of baryons and mesons in the hadronic phase. A bag-model EoS is assumed for the QGP, while the hadronic phase is described by free mesons and a Walecka-type EoS for the baryons. The latter is important in order to furnish a proper phase transition at high baryon densities. Global thermal and pressure equilibrium in an isotropically expanding radial flow model with a linear (Hubble like) radial velocity profile is assumed. The radial expansion is generated self-consistently through the pressure, which lead to a predicted flow velocity of 0.5 c prior to experimental data. The core of the model are chemical rate equations which take the generic form

$$< rate >=< forward\ rate > (1-\exp[(\mu_r-\mu_l)/T]). \tag{3.1}$$

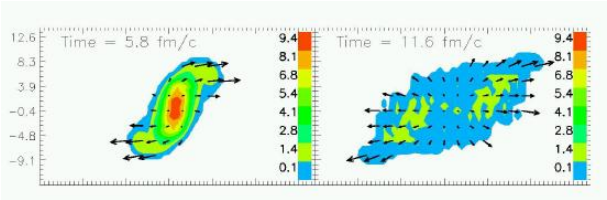


Figure 3.1: Au on Au at 10 AGeV: density contours; arrows indicate flow velocities.

Here μ_r and μ_l denote the sum of the chemical potentials of the particles on the right, respectively left side of a rate balance. E.g for $q+\bar{q} \rightleftharpoons \pi+\pi$ one has $\mu_l = \mu_q + \mu_{\bar{q}}$ and $\mu_r = 2\mu_\pi$ (note only in equilibrium $\mu_q + \mu_{\bar{q}} = 0$).

These rates describe the density changes of the different constituents, both, due to collisions, and through phase-conversion rates. Within each phase standard binary collision and decay rates determine the forward rate for each process, while for the phase-transition rates assumptions lent from the string fragmentation are taken to determine the forward rates. The rate equations fully comply with detailed balance, they are generic since the rate is determined by the off-set in chemical potentials which for each particle follow from its density and the underlying EoS. If the chemical potentials become disfavored in one phase relative to the other the system is drive to the other phase.

The model accounts for radial flow, and permits delayed conversion rates between the two phases and this way super-cooling effects. The generic result of this study is that independent of the detailed assumptions finally the conversion rates become large and after phase-conversion the system finds itself in near chemical equilibrium. Besides conservation law constraints the final composition is blind with respect to detailed composition within the plasma phase, e.g. the ratio of strange to non-strange quarks. Chemical freeze-out emerges automatically through the rate equations.

3.3.1.5 Freeze-out

The hydrodynamic simulations are terminated by a freeze-out procedure, as originally applied to high-energy physics almost 50 years ago [4]. This is still an open and not fully settled issue which is actively discussed. Intuitively clear and easily applicable, however, the method suffers from violating the energy conservation [5]. To remedy the situation, Cooper and Frye [2] proposed their own recipe which was not free from other problems either, since in part it gave negative contributions to the particle spectrum. This negative contribution corresponds to out-frozen particles returning to the hydrodynamic phase. Various further remedies were then discussed in the literature [6] which all suffer from certain deficiencies depending whether the freeze-out happens from a space-like surface (emission into vacuum) or from a time-like hyper-surface (global dilution of the system). Thus, the transition from the highly collisional dynamics to the collision-less one is subtle and most simplified kinetic treatments are hardly justifiable.

3.3.2 Kinetic transport models

Kinetic transport models aim at a microscopic description of the collision dynamics of the constituents of the system in terms of classical transport concepts. Thereby one distinguishes molecular type approaches from Boltzmann equation type schemes.

3.3.2.1 Molecular dynamic approaches (QMD, RQMD)

In these approaches one splits the interaction among the constituents into a smooth long-range part which is explicitly treated by Newtonian forces among the constituents and a short-range components which is treated by a stochastic collision term. The latter obeys the Pauli principle for fermions or includes the corresponding Bose-Einstein factors for bosons. For more details concerning the collision term see next sub-section. In order to deal with smooth phase-space distributions each particle is considered as a Gaussian wave-packet of given width in coordinate space.

3.3.3 Boltzmann equation type approaches

These approaches are based on a microscopic transport equation of Boltzmann-Uehling-Uhlenbeck (BUU) type

$$\underbrace{(\frac{\partial}{\partial t}+\frac{\vec{p}}{m}\frac{\partial}{\partial \vec{x}})}_{\text{free motion}} f(\vec{x},\vec{p},t) - \underbrace{\{U_{\rm pot}, f\}}_{\text{mean field}} = \underbrace{C(f(\vec{x},\vec{p},t))}_{\text{Collision Term}} \tag{3.2}$$

with a collision term C and mean potential U_{pot}.

Here all particles are supposed to be classical on-shell particles. For two-body scattering the the gain and loss parts of collision term are given by the differential cross-section

$$C \propto \int \frac{d^3p_1}{(2\pi)^3}\frac{d^3p_2}{(2\pi)^3}\frac{d^3p_3}{(2\pi)^3}\frac{d\sigma}{d\Omega}(\underbrace{(1-f)f_1(1-f_2)f_3}_{\text{gain}} - \underbrace{f(1-f_1)f_2(1-f_3)}_{\text{loss}})$$

Here the collision term is local, i.e. all spatial coordinates in the four involved distribution functions are the same as on the l.h.s.. Although all simulation techniques pretend to solve the same Boltzmann equation, the different methods lead to physically different results.

3.3.3.1 Different statistical ensembles

The construction of sufficiently smooth Pauli factors and mean field potentials requires special smoothing concepts. This can either be provided by averaging over a large number N of similar events (parallel ensemble method) or by introducing test particles where a large number N of test particles represents one physical particle.

3.3.3.2 Parallel ensemble:

Here the microscopic two-body collisions simulating the collision term happen at finite (two-body) impact parameter. Two prescriptions were used in the literature:

1) the scattering angle in the binary collision is correlated to the impact parameter, e.g. like in classical scattering theory according the differential cross section (e.g. hard sphere scattering). In this case a) angular momentum is conserved microscopically, b) it leads to finite delay-time on the average and c) the collisions thus contribute to the EoS in a non-trivial fashion.

2) the scattering angles are unrelated to the binary impact parameter. Now microscopically angular momentum conservation is violated and the collision term generally does not contribute to the EoS.

3.3.3.3 Test-particle method

In the limit of a large number N of test particles this method simulates a local collision term, since the test particle cross sections scale like $1/\sqrt{N}$ relative to the physical cross sections. Thus this method solves the Boltzmann Uehling Uhlenbeck equation as given above and the collision term does not contribute to the EoS. Also here two different implementations of the collision term are in use:

a. binary scattering initiated according to the relative impact parameter and

b. scattering according to the phase-space occupations of the particles in a local cell.

The recently introduced latter concept opens the perspective for non-trivial collision terms with finite spectral width where processes 1 <–> 3 and multi-particle collisions can occur (see below).

3.3.3.4 The gradient terms

The gradient term encoded in a Poisson bracket determines the conserved currents and contributes to the EoS.

In case of momentum dependent potentials this term is insufficient, and further gradient terms have to appear in the BUU-Eq in order to recover its Galilei (Lorentz) invariance through a corresponding back-flow term. Thus the treatment of momentum dependent forces is non-trivial.

3.3.4 Resonance transport and dynamical spectral functions

Although the basic concepts for a proper transport treatment of broad resonances were already layed out in the book of Kadanoff and Baym (1962) [7], considerable conceptual progress was achieved only very recently. It amounts to perform a systematic first order gradient approximation [8, 9, 10, 11] to the KB-Equations which provides a generalized transport Eq. (written in covariant notation) determining the space-time changes of the four-phase-space distribution function $F(X,p) = f(X,p)A(X,p)$ as

$$v^\mu \partial_\mu F(X,p) + \{\mathrm{Re}\,\Sigma^R, F\} + \{\pm i\Sigma^{-+}, \mathrm{Re}\,G^R\} = C_{\mathrm{loc}}$$

and an Eq. for the retarded Green's function in terms of the retarded self-energy Σ^R

$$\left(G^R(X,p)\right)^{-1} = \left(G_0^R(p)\right)^{-1} - \Sigma^R(X,p)$$

which determines the spectral function through the imaginary part of the retarded Green's function

$$A(X,p) = -2\mathrm{Im}\,G^R(X,p).$$

In thermal equilibrium $f(X,p)$ becomes a Fermi-Dirac or Bose-Einstein distribution in the particles energy p_0. The evolution of F is governed by the generalized transport Eq.. Together with the retarded equation this defines a generalized quantum transport scheme which is void of the usual quasi-particle assumption. The space-time evolution is completely determined by the initial values of the Green functions at time zero for each space point. Thus the evolution is "*Markovian*". Within its validity range this transport scheme is capable to describe slow space-time evolutions of particles with broad damping width, such as resonances, within a transport dynamics, now necessarily formulated in the four-dimensional phase-space.

3.3.4.1 Generalized collision term C_{loc}

Coming from the usual on-shell Boltzmann or Boltzmann-Uehling-Uhlenbeck collision term, each occurring three-momentum distribution function together with its momentum integration is simply to be replaced by its four-momentum analog, thus

$$f(X,\vec{p})\frac{d^3\vec{p}}{(2\pi)^3} \Longrightarrow F(X,p)\frac{d^4p}{(2\pi)^4} = f(X,p)A(X,p)\frac{d^4p}{(2\pi)^4}.$$

Alongside the normally occurring two-body cross-sections have to be replaced by the corresponding T-matrix expressions providing the proper off-shell extensions. For genuine momentum dependent T-matrices the collision term has a finite virial due to the interactions at finite distances and therefore the collision term contributes to the conservation laws in a non-trivial fashion. Within field theory model applications with local couplings one simply has to evaluate the corresponding self-energies for the collision term

$$C_{\rm loc} = \underbrace{-i\Sigma^{-+}(X,p)(A(X,p) \mp F(X,p))}_{\rm gain} - \underbrace{F(X,p)(-i\Sigma^{+-}(X,p))}_{\rm loss}$$

where $-i\Sigma^{\pm\mp}(X,p)$ (alias $-i\Sigma^{><}(X,p)$) are the self-energy correlation functions determining the gain and loss rates. In this case the local collision term drops out of the conservation laws.

3.3.4.2 Detailed balance and 2PI-method

Detailed Balance is guaranteed if all self-energies of the different particles are generated from a given set of closed diagrams by opening the respective propagator lines of the dynamical particles. Thereby the generating closed diagrams have to have the property that they are two-particle irreducible (2PI) [17,18]. This 2PI-method guarantees [19,20]

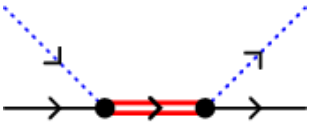
a. detailed balance for the collision term

b. conserved currents arising from the gradient terms (see below)

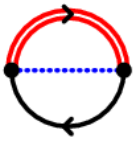
c. to avoid double counting for multi-particle processes (see below)

even after gradient approximation [11] which defines the quantum transport equations discussed above.

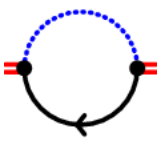
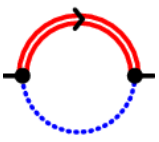
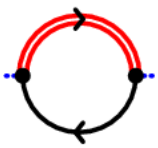
As an illustrative example we discuss the scattering of pions (blue dotted line) on nucleons (full black line) through an intermediate Delta (1232) resonance. Here the scattering amplitude

$T_{\pi N} =$  (3.3)

is formulated by a phenomenological $\pi N\Delta$ vertex. To lowest order the corresponding closed 2PI diagram defining the Φ-functional and through functional variations the self-energies and thus also the collision term is given by

$\Phi =$  (3.4)

Here all three lines denote full self-consistent propagators. The respective self-energies are obtained by opening a corresponding propagator line, i.e.

$\Sigma_\Delta =$  ; $\Sigma_N =$  ; $\Pi_\pi =$  . (3.5)

The example is non-trivial with respect to the on-shell Boltzmann concept as it replaces the binary πN-scattering by an intermediate resonance which itself is treated dynamically within the off-shell transport equations. Cutting these diagrams vertically visualizes the squares of the corresponding tree-level transition amplitudes entering the collision terms for the three species.

The concept to deal with dynamical, i.e. non-on-shell spectral functions, further avoids singular or even mathematically pathological terms as they notoriously appear in higher order perturbative expressions (cf. subsection 2.2.5).

However it should be stated that the here discussed 2PI-concept to close self-consistent two-point function schemes on a certain diagram level is not free from conceptual difficulties. While as a positive achievement Noether currents are conserved on the expectation value level, such currents are no longer conserved on the higher order correlator level conserved. This comes about due to the partial resummation implied by solving the dynamical equations of motion, be it on the Schwinger-Dyson or Kadanoff-Baym level or the here discussed off-shell transport scheme, which leads to a violation of Ward-Takahashi identities. A famous example is the violation of the Goldstone theorem in the spontaneously broken phase, where the self-consistent Hartree-Fock approximation leads to propagators for the Goldstone-bosons which have a non-vanishing mass! Other problems concern the polarization tensor of vector mesons, which, if coupled to conserved currents, should be four-transversal. However the self-consistent 2PI scheme violates this condition. Cure can be obtained from higher order vertex equations such as the Bethe-Salpeter ladder resummations. Such extensions are i) numerically such demanding that they are presently untractable and ii) they spoil the self-consistent concept where one expects all dynamical quantities to be determined self-consistently. Indeed the symmetry preserving self-energies resultu=ing from higher order vertex equations do not take part in the self-consistent Dyson scheme. Recently suggested projection methods [21, 22] lead to other difficulties in form of kinematical singularities, either at energy zero or on the light cone. These lead to spurious (infra-red) modes which seriously corrupt the self-consistent dynamics. Thus, the treatment of vector mesons or even gauge bosons in self-consistent schemes is still unsettled and requires serious conceptual efforts before one comes to reliable schemes.

3.3.4.3 Gradient terms

More subtle than the collision terms are the first order gradient terms given by the two Poisson brackets. Indeed both contribute to the conservation laws. Thereby the first Poisson bracket furnishes the so-called drag-flow. In the quasi-particle limit it accounts for the dressing of the particles by the dragged matter cloud as to form a quasi-particle with a non-trivial dispersion relation with a corresponding in-medium group velocity that can be expressed by an effective mass. This change in flow is just compensated by the second Poisson bracket through the polarization of the medium. The latter thus forms a back-flow component. Only the coherent play of both Poisson brackets restores the conserved Noether currents and thus recovers e.g. Galilei (Lorentz) invariance [11]. Thus a system of N particles is moved with its total mass $M = Nm$ rather than by N times the effective mass!

Since the first Poisson bracket involves space-time and momentum gradients directly acting on the distribution function F this term has an easy classical interpretation where the motion of the corresponding particle is subjected to a force which generally is momentum dependent. A generalization of this concept to the four-momentum picture is straight forward, since it just amounts to establish the corresponding characteristic curves of the homogeneous first-order partial differential equation. For the second Poisson bracket term on the other hand the derivatives of the distribution function appear only implicitly through the self-energy with the result that they affect momenta other than the momentum externally entering the transport equation. This has to be such, since the discussed term describes the reaction of the surrounding matter on the particle moving through the matter. However this term escapes an immediate description in terms of test particles, such that a simulation algorithm could not yet be established for the

exact quantum kinetic equation (3.3).

Guided by equilibrium relations Botermans and Malfliet [15] suggested a simplification of this second Poisson term, cf. [8, 9, 10]

$$\{-i\Sigma^{-+}, \mathrm{Re}\, G^R\} \underset{\mathrm{BM}}{\Longrightarrow} \{f(X,p)\Gamma(X,p), \mathrm{Re}\, G^R\} \qquad \text{with} \quad \Gamma(X,p) = -2\mathrm{Im}\Sigma^R, \tag{3.6}$$

formally valid up to second order gradient terms. Here the distribution function $f(X,p)$ directly appears, while Γ is the damping width. The advantage of this substitution is that now the Poisson-bracket derivatives directly act on the distribution function f and the term amends a test-particle simulation [10,9]. The price to be payed is that then the conservation laws are slightly modified, since instead of the spectral function A rather the entropy-spectral function $A_\mathrm{S} = \frac{1}{2}A\Gamma^2$, as introduced in ref. [16], enters the conserved current expression

$$J^\mu_{\mathrm{BM}}(X) = \int \frac{d^4p}{(2\pi)^4} e v^\mu f(X,p) A_\mathrm{S}(X,p)(X,p) \tag{3.7}$$

The BM-substitution accounts for part of the back-flow. In the quasi-particle limit both spectral functions converge to the same δ-function at the quasi-particle energy [16, 8]. A further merit of the BM-substitution is that for certain collision terms an entropy current can be derived which fulfills an exact H-theorem, for details see ref. [8]. Recent first numerical applications with this substitution [9, 10, 12] were performed.

Once one starts to include the real part of the self-energies in particlura beyond mean-field approximations one has to face the problem of renormalization in the context of self-consistent schemes. This has been an unsettled problem for decades. Recent progress in this context was achieved by the work of van Hees and Knoll [13] and follow-up work by Blaizot et al. [14].

3.3.4.4 Beyond binary collisions

With increasing energy inelastic processes open which lead to the creation of new particles, either through a kind of bremsstrahlung process, like $NN \to NN\gamma$ or $NN \to NN\pi$ or other production mechanisms, like $\bar{p}p \to 5\pi$. Such processes were introduced on the basis of measured cross-sections. However the inclusion of the appropriate backward reaction caused problems and they were commonly neglected. Recently arguments were given which indeed date back to the time of Planck, when he wrote down his radiation law. They state that the equilibration rate is determined by the fastest rate. Hence, if the forward rate is strong also the backward rates should become strong as to furnish the proper equilibrium within the equilibration time set by the forward rate. Thus processes of the type "3 to 2" or even "5 to 2" particles can become important and have be included.

There are several suggestions to properly include such “beyond binary” processes. Not all of them lead to a consistent picture.

Phenomenological quasi-free ansatz for multi-particle processes

One option is to ignore the internal dynamics of the production process itself and to assume that it happens at space-time scales shorter than relevant for the description of the dynamics. Thus, the multi-particle interaction vertex is assumed to be essentially point-like and the picture is that of a quasi-free collision process. Then the multi-particle process rate can be described by Fermi’s golden rule with a transition-matrix T for the process, e.g. extracted from a measured cross-section for a binary entrance channel, or given by the point-like transition vertex from the corresponding lowest order self-energy

diagram. Detailed balance can then be enforced by the 2PI-method described above, just by closing the self-energy diagram with the external line and then opening any other particle line in order to obtain the self-energies for the other particles involved in the process.

This method

a. rests on measured cross-section which in part are scarcely available or based on theoretical guesses;

b. obeys detailed balance by construction;

c. is mostly limited to on-shell particles;

d. cannot properly describe the production of soft particles, particles that are soft on the scale of the collision rate, where the Landau-Pommeranchuk effect suppresses the particle production compared to the quasi-free production prescription.

Intermediate resonances

A second option is to describe the process in a sequential scheme where in a first step an intermediate resonance is formed which then subsequently decays.

This involves

a. to deal with broad resonances in a dynamical way;

b. to include decay processes ("1 to 2" particle processes) even if they are kinematically forbidden under on-shell conditions and its inverse process, the fusion ("2 to 1" particle);

c. to fit existing measured cross-sections within this picture.

This strategy obeys detailed balance if all processes are described by self-energies derived from closed diagrams within the 2PI method.

Off-shell transport

The intermediate resonance picture can directly be generalized to to a general off-shell transport concept. This has the advantage that through the off-shellness of normally stable particles the bremsstrahlung (and its inverse) are automatically included (e.g. through a $N \rightarrow N\gamma$ process). This strategy approximately also accounts for the Landau-Pommeranchuk suppression which in this picture arises from the damping widths (e.g. collision rate) of the particles.

Higher order processes within the quasi-particle picture

Attempts to construct higher order collision terms within the quasi-particle approximation merely by adding some perturbative diagrams of higher order for the self-energies run into serious conceptual difficulties. The problem is very subtle and plagued with singular expressions which notoriously appear in perturbation theory at higher orders!

To be specific: For the construction of the collision term the imaginary parts of the self-energies are the key quantities. If a higher order diagrams contains genuine lower order self-energy insertions, one definitely encounters expressions where the absolute square(!) of the perturbative retarded propagators (i.e. propagators with zero damping width) appear. If then the corresponding self-energy insertion does not vanish at the on-shell condition, the expression contains squares of singular distribution functions (i.e. squares of delta- and principle-value-functions) which are mathematically ill defined and which through the integration within the collision term thus lead to diverging collision rates! It is well known

that (a) such problems cannot be solved within perturbation theory, but (b) rather require special partial resummations techniques to infinite order, in order to come to regular and physically meaningful expressions.

Indeed the problem is tightly connected with the irreducibility features required for the kernel of a dynamical equation of motion, in this case for the collision term of the transport equation. The transport equations are self-consistent dynamical equations to the extent that the dynamical quantities calculated in one time step enter as input for the next time step. Thus, the solution of such equations generates higher order terms out of its kernel, namely the collision term (and also the driving self-energies in the Poisson brackets). Therefore the collision term by itself has to obey specific irreducibility criteria: it has to be void of any process that can implicitly be generated through the solution of the transport equation! Restricted to the quasi-particle limit these are precisely the processes, namely intermediate on-shell propagation, that lead to the above stated mathematical difficulties.

Let me add a general note on the irreducibility concept. It is precisely through the formulation of equations of motion with IRREDUCIBLE kernels that one avoids such singular structures. Therefore this issue is indeed completely settled for the case of the self-consistent solutions of the Dyson (or Kadanoff-Baym) equations, when the proper self-energy is entirely a functional of the self-consistent propagator. There the precise rule is that the kernel of the Dyson equation, i.e. the self-energy, has to be derived from a 2-particle irreducible (2PI) functional ([17], [19], i.e. it has to be void of any self-energy insertion. The step towards transport is achieve by a consistent gradient approximation of the Kadanoff-Baym equations (cf. [11]). Still then the 2PI-rule applies.

If, however, as frequently addressed, one tries to restrict the dynamical description to **on-shell** (or better **quasi**-)particles, one seriously spoils the 2PI irreducibility concept! Then one carefully has to separate self-energy terms that are treated explicitly from those generated implicitly through terms of lower order. This implies the necessity of so-called z-factors for the quasi-particle strength on the one hand and an implicit treatment of the complementary background terms. As yet there is no formulation in the literature where such a separation has thoroughly been addressed for the higher order terms such that (a) the irreducibility properties are appropriately formulated for the quasi-particle picture while (b) at the same time physically meaningful and non-singular expressions emerge.

3.3.5 String fragmentation

Phenomenological strings are a tool to cope with the situation that with increasing energy partonic degrees of freedom (quarks and gluons) are release and form the main dynamical components then. The transition from the partonic phase to the hadronic one is frequently furnished by phenomenological string-fragmentation models. They generally rely on models developed in high energy physics such as the Lund string model [23], in order to describe the particle production in pp or $\bar{p}p$ or e^+e^- collisions. Optionally the models include collective string effects such as the color-rope picture [24, 25]. They normally ignore the associated space-time evolution and solely operate in momentum space. Possible space-time effects for the reaction are usually mocked up by a phenomenological formation time that prevents created particles from immediate interactions with other ones. The standard implementation uses quark–quark or quark–anti-quark collisions to form the strings which then decay through pair creation into new $q\bar{q}$-pairs or directly into hadrons or hadron resonances. Void of driving potentials so far the inverse reactions are not included. Thus the concept violates detailed balance which is an essential ingredient to avoid over-population of phase space and to drive the systems towards equilibrium in the event of long overall reaction times. Further details may be given in the model descriptions.

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3.4 Dynamical models in detail

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Ultra-relativistic heavy ion collisions offer the unique opportunity to probe highly excited dense nuclear matter under controlled laboratory conditions. One of the objectives of this research field at the interface of high–energy and nuclear physics is the creation and study of super-dense matter. The compelling driving force for such studies is the expectation that an entirely new form of matter may be created from such reactions. That form of matter, called the Quark Gluon Plasma (QGP), is the QCD analogue of the plasma phase of ordinary atomic matter. However, unlike such ordinary plasmas, the deconfined quanta of a QGP are not directly observable because of the fundamental confining property of the physical QCD vacuum. What is observable are hadronic and leptonic residues of the transient QGP state. There is a large variety of such individual probes. Leptonic probes, $\gamma, e^+e^-, \mu^+\mu^-$ carry information about the spectrum of electromagnetic current fluctuations in the QGP state; the abundance of quarkonia $\Psi, \Psi', \Upsilon, \Upsilon'$ (also observed via l^+l^-) carry information about the chromoelectric field fluctuations in the QGP. The arsenal of hadronic probes, $\pi, K, p, \bar{p}, \Lambda, \Xi, \Omega, \phi, \rho, \ldots$ provide information on the quark flavour chemistry and baryon number transport. Theory suggests that with decays such as $\rho \to e^+e^-$ the properties of the hadronization and chiral symmetry breaking can be indirectly studied. Quantum statistical interference patterns in $\pi\pi$, KK, pp, $\Lambda\Lambda$ correlations provide somewhat cloudy lenses with which the space-time geometry of hadronic ashes of the QGP can be viewed. The detailed rapidity and transverse momentum spectra of hadrons provide barometric information of pressure gradients during the explosive expansion of the QGP drop.

The central problem with all the above probes is precisely that they are all indirect messengers. If we could see free quarks and gluons (as in ordinary plasmas) it would be trivial to verify the QCD prediction of the QGP state. However, nature chooses to hide those constituents within the confines of colour neutral composite many body systems – hadrons.

The QGP state formed in nuclear collisions is a transient rearrangement of the correlations among quarks and gluons contained in the incident baryons into a larger but globally still colour neutral system with however remarkable theoretical properties. The task with heavy ion reactions is to provide experimental information on that fundamental prediction of the Standard Model.

3.4.1 Hydrodynamics

Nuclear fluid dynamics (NFD)

Dumitru, Stoecker, ...
Nuclear fluid dynamics (NFD) is a dynamical model in which a phase transition can explicitly be incorporated (see e.g. [**?**, **?**, **?**, **?**, **?**, **?**, **?**] for details). This is possible since the equation of state (including a phase transition) is a direct input for the calculations. However, NFD is an idealised continuum description based on local equilibrium and energy–momentum conservation. Therefore it is very well suited to study kinematic observables such as collective flow. Since NFD is a macroscopic kinetic theory it is not directly applicable to the study of hadron abundances and particle production. However, NFD calculations predict (local) temperatures and chemical potentials which can be used, e.g. by chemical equilibrium calculations of hadron abundances, to study particle production. Different observables predicted by nuclear fluid dynamics will be discussed in section **??**.

In the ideal fluid approximation (i.e. neglecting off-equilibrium effects), the EoS is the *only* input to the equations of motion that relates directly to properties of the matter under consideration. The EoS

influences the dynamical evolution of the system, and final results are uniquely determined. The initial condition can be chosen from two colliding nuclei (in a full 3D calculation with up to three fluids) or an equilibrated QGP or hadronic matter with prescribed temperature and chemical potential and velocity/flow profiles (for simpler, more schematic calculations). The time-evolution is then studied until hadronic freeze-out for which a decoupling (freeze–out) hyper-surface needs to be specified.

However, the ideal fluid ansatz is only a rough approximation. In the parton cascade study [**?**] for example, large deviations from even the Navier Stokes fluid approach were found.

Relativistic Three Fluid Model

Ivanov, Russkikh, Toneev
http://theory.gsi.de/ mfd/

A direct way to address thermodynamic properties of the matter produced in these reactions consists in the application of hydrodynamic simulations to nuclear collisions. However, finite nuclear stopping power, revealing itself at high incident energies, makes the collision dynamics of non-equilibrium character and prevents us from the application of conventional hydrodynamics especially at the initial stage of the reaction. Since the resulting non-equilibrium features are quite strong, the introduction of viscosity and thermal conductivity does not help to overcome this difficulty, because by definition they are suitable for weak non-equilibrium situations. A possible way out is taking advantage of a multi-fluid approximation to heavy-ion collisions.

Unlike to conventional hydrodynamics, where a local instantaneous stopping of projectile and target matter is assumed, a specific feature of the dynamic 3-fluid description is a finite stopping power resulting in a counter-streaming regime of leading baryon-rich matter. Experimental rapidity distributions in nucleus–nucleus collisions support this counter-streaming behaviour, which can be observed for incident energies between a few up to 200A GeV. The basic idea of a 3-fluid approximation to heavy-ion collisions [1,2] is that at each space-time point $x = (t, \mathbf{x})$ the generally non-equilibrium distribution function of baryon-rich matter, can be represented as a sum of two distinct contributions, $f_{bar.}(x,p) = f_p(x,p) + f_t(x,p)$, initially associated with constituent nucleons of the projectile (p) and target (t) nuclei. In addition, newly produced particles, populating the mid-rapidity region, are associated with a fireball (f) fluid described by the distribution function $f_f(x,p)$. Therefore, the 3-fluid approximation is a minimal way to simulate the finite stopping power at high incident energies. Note that both, the baryon-rich and fireball fluids may consist of any type of hadrons and/or partons (quarks and gluons), rather than only nucleons and pions.

To justify the term “fluids” it is assumed that constituents within each distribution are locally equilibrated, both thermodynamically and chemically. This assumption relies on the fact that intra-fluid collisions are much more efficient in driving a system to equilibrium than inter-fluid interactions. As applied to the fireball fluid, this assumption requires some additional comments, related to the concept of a finite formation time. During the proper formation time τ after production, the fireball fluid propagates freely, interacting neither with itself nor with the baryon-rich fluids. After this time interval, the fireball matter starts to interact with both itself and the baryon-rich fluids and, as a result, thermalizes locally.

The main unknowns of the present approach can be briefly summarised as follows: the equation of state (EoS) and “cross sections”. The EoS is an external input to the calculation and thus can be varied. Our goal is to find an EoS which in the best way reproduces *the largest body of available observables*. The “cross sections” are equally important. They determine friction forces between fluids and hence the nuclear stopping power. In principle, friction forces are EoS dependent, because medium modifications, providing a nontrivial EoS, also modify cross sections, and should be externally supplied together with the EoS. However, at present we have at our disposal only a rough estimate of the friction forces cf.

Ref. [3] based on experimental inclusive proton–proton cross sections. Therefore, we have to fit the friction forces to the stopping power observed in proton rapidity distributions.

The hydrodynamic treatment of heavy-ion collisions is an alternative to kinetic simulations. The hydrodynamic approach has certain advantages and disadvantages. Lacking the microscopic feature of kinetic simulations, it overcomes their basic assumption, i.e. the assumption of binary collisions, which is quite unrealistic in dense matter. It directly addresses the nuclear EoS that is of prime interest in heavy-ion research. Naturally, we have to pay for these pleasant features of hydrodynamics: the treatment assumes that the non-equilibrium stage of the collision can be described by the 3-fluid approximation. The basic reason for introduction of the 3-fluid approximation is simulation of the finite stopping power which is important at the formation stage of the initial hot and dense blob of nuclear matter. In this sense, it is an alternative to constructing this initial blob by means of either various kind kinetic transport models or model assumptions. It is worthwhile to mention that we have to proceed to the hydrodynamic treatment, if we are going to study the quark-gluon phase, since the EoS (with the phase transition) is a direct input of this approach.

3.4.2 Non-equilibrium models

In order to connect the theoretical thermodynamic properties of a QGP with experimental data on finite nuclear collisions, many non-equilibrium dynamical effects must also be estimated. Transport theory is the basic tool to address such problems. Non-equilibrium effects are certain to arise from the rapid time-dependence of the system (even the use of the term “state” seems questionable), finite size effects, inhomogeneity, N-body phase space, particle/resonance production and freeze-out and collective dynamics. Such microscopic and macroscopic (hydrodynamical) models attempt to describe the full time-evolution from an assumed initial state of the heavy ion reaction (i.e. the two colliding nuclei) up to the freeze-out of all initial and produced particles after the reaction. Hydrodynamical models neglect most of these effects by making the assumption that the initial condition can be assumed to be in local thermal equilibrium and that local equilibrium is maintained during evolution. Fireball models simply parameterise final spectra and abundances via freeze-out parameters, e.g. $T, \mu_B, \vec{v}_f$. However, the initial condition in nuclear collisions is a coherent state $|AB\rangle$ of two quantal ($T = 0$) nuclear systems. A non-equilibrium quantum evolution of $|AB\rangle$ introduces complex high order Fock-State components. A key dynamical assumption is that decoherence occurs rapidly during the early phase of the collision yielding a mixed state density matrix (with $S = \mathrm{Tr}\rho \ln \rho > 0$). There is no theorem to insure that ρ evolves to a local equilibrium form $\exp(-u_\mu p^\mu / T)$ at any time during the reaction. That can only be tested via a transport theory approximation to the evolution equations. The question of the form of the initial state $\rho(\tau_0)$ must still be addressed, but once that is specified, transport theory can reveal if local equilibrium is achieved and what observables are least sensitive to uncertainties in $\rho(\tau_0)$.

Depending on the most convenient basis for expanding $\rho(\tau_0)$, transport theory assumes different forms. At low energies the initial ensemble is most conveniently described in terms of mesons and baryons. Here hadronic transport theory is appropriate. At collider energies, pQCD minijet processes are expected to produce a high density mostly gluonic gas. In that regime parton cascade models are more appropriate.

3.4.2.1 Parton cascades

Parton cascade models evolve partonic degrees of freedom. They are therefore mostly applied to study the initial compressional and the high density phase of ultra-relativistic heavy ion collisions (collider energies, $\sqrt{s} \geq 200$ GeV). These models all contain the general structure [**?**]:

1. Initialisation: the nucleons of the colliding nuclei are resolved into their parton substructure ac-

cording to the measured nucleon structure functions and yield the initial parton distributions.

2. Interaction: parton interactions as described by perturbative QCD are used to model the evolution of the ensemble of partons during the course of the collision. This includes multiple scatterings together with associated space-like and time-like parton emission processes before and after each scattering. The sequence of scatterings is, however, incoherent and the neglect of quantum interference effects is questionable.

3. Hadronization: partons are recombined or converted via string fragmentation into final hadron states.

The propagation is performed on straight lines – soft non-perturbative collective field effects have so far been neglected. On the other hand, hadronization has to be modelled by brute force to mock up confinement in the final reaction stage.

3.4.2.2 Hadronic transport models

Hadronic transport models treat relativistic heavy-ion collisions as sequences of binary/N-body collisions of mesons, baryons, strings and their constituents, diquarks and quarks. The real part of the interaction can be obtained in principle from G-Matrix calculations, with the in-medium self-energy and the imaginary part is modelled via hard scattering cross sections. Three main elements form the general structure of these models

1. Initialisation: either with a Fermi-gas ansatz or within a self-consistent approach of minimising the respective Hamiltonian. In BUU type models one-body phase space distributions are represented by so called test-particles which can be of point-like [46, 44] or Gaussian form [45] while in Quantum Molecular Dynamics [47] each individual particle is represented by a Gaussian wave packet.

2. Propagation: propagate the constituents (hadrons, i.e. centroids of wave packets or test-particles representing the hadronic phase space distributions) according to the equations of motion of the respective model (straight lines in the CASCADE case).

3. Collision term: parameterised or tabulated cross sections and decay widths for all the baryons and mesons included.

For high beam energies most models include particle production via string formation – either using the Lund [**?**, **?**, **?**] or a pomeron exchange scheme [**?**]. Partonic degrees of freedom are not treated explicitly and therefore these models do not include a phase transition. However, some models contain further speculative scenarios such as colour-ropes [**?**, **?**], breaking of multiple-strings [**?**] or decay of multi-quark droplets [**?**] which clearly go beyond hadronic physics.

Hadronic transport models are critical for assessing the influence of ordinary or exotic hadronic phenomena on the observables proposed to search for a QGP. They therefore provide a background basis to evaluate whether an observable shows evidence for non-hadron physics.

Degrees of freedom

The energy range where the various models are applicable is essentially determined by the degrees of freedom which are excited. If the excitations, e.g. of particles carrying strangeness are abundant enough to influence the overall reaction dynamics they must explicitly be included when a model is applied to

this energy regime. If, however, the energy of the reaction lies just in the vicinity of the threshold of a mesonic or baryonic excitation such an excitation is extremely rare and does not influence the overall reaction dynamics. It can safely be neglected for practical purposes. To be applicable in a certain energy range, a model must only include the *relevant* degrees of freedom.

However, the production of rare probes in the vicinity of the corresponding threshold leads in many cases to particular physical insight. The reason is simply that rare probes, produced at subthreshold energies have in many cases no change to reach chemical equilibrium with the surrounding medium. They carry therefore more information on the conditions of their production stage. A typical example are subthreshold K^+ mesons produced at SIS energies. They originate to most extent from the high density phase of the reactions and, once produced, interact only weakly with the surrounding medium. Due to strangeness conservation K^+ mesons cannot be absorbed by a medium which carries no strangeness. An analogous situation is expected for open charm $(D,\bar{D})$ production at CBM energies. Such subthreshold particles can generally be treated *perturbatively* which means to neglect their feedback on the overall reaction dynamics [65]. This allows to artificially enhance the corresponding production cross sections in the simulations and to collect the necessary statistics. Otherwise one would produce one subthreshold event in e.g. one of about $10^3 \div 10^5$ heavy ion collisions which would make the investigation of such processes practically impossible. In this method, the dynamical degrees of freedom, e.g. nucleons, Δ resonances, and pions, are not affected by the production of rare probes, and the produced particle is given a probability determined by the ratio of its production cross section to the total two-body scattering cross section. After being produced, these rare particles also undergo elastic and inelastic scattering as well as propagate in mean-field potentials.

At centre-of-mass energies of about $\sqrt{s} \geq 4$ GeV corresponding roughly to the AGS regime of about 10 AGeV laboratory energy strings start to be excited. This is also the energy range where elastic and inelastic collisions start to dominate the reaction dynamics and the nuclear mean field becomes less important. Do to the lack of a mean field pure cascade models should not be used below 10 AGeV. But even when applied to the ultra-relativistic regime where the initial phase of a reaction is dominated by binary collisions, a pure cascade description has to rely on the assumption that the system is already sufficiently dilute in its expansion phase and final state interactions can be neglected. Moreover, cascade models will never allow to address subtle effects such as off-shell transport of broad resonances. For completeness figures 3.2 and 3.3 depict the lowest SU(3) baryon multiplets, i.e. the baryon octet and decouplet as well as the lowest pseudoscalar (ps) and vector (v) meson nonets. These are the basic degrees of freedom taken into account in string based models as asymptotic states of the string decays. When the models are extended to the charm sector the SU(3) multiplets have to be extended to the corresponding SU(4) multiplets [33].

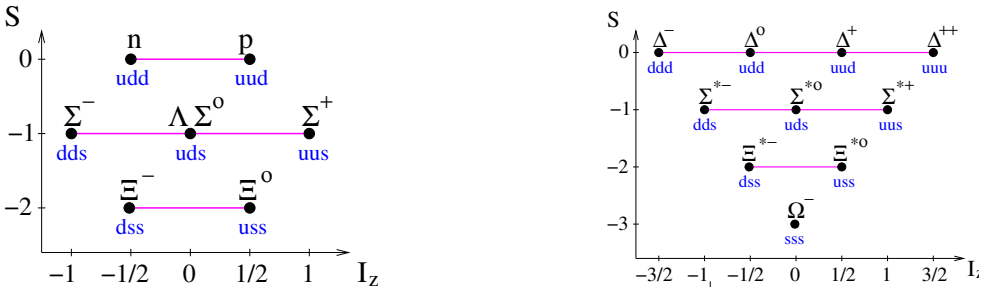

Figure 3.2: The lowest SU(3) baryon multiplets, octet (left) and decouplet (right).

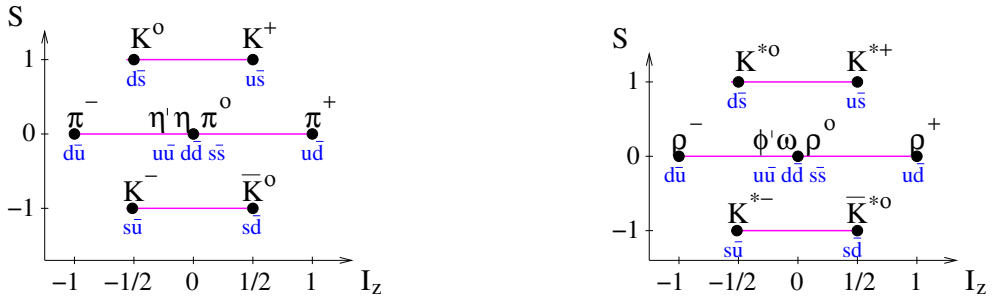

Figure 3.3: The lowest SU(3) pseudoscalar (left) and vector (right) meson nonets.

Potentials and mean fields

The real part of the optical nucleus-nucleus potential, respectively the mean field dominates the reaction dynamics at low and intermediate energies. Models in particular designed for low and intermediate relativistic energies contain generally a more sophisticated treatment of the mean field than models designed for the ultra-relativistic regime. The nuclear mean field is obtained by the integration of an effective two-body interaction over the occupied states, i.e. the local phase space distributions. In the medium this two-body interaction is given by the *G*-matrix (or in-medium T-matrix). The *G*-matrix has to be derived from microscopic Brueckner-Hartree-Fock (BHF) [36, 39] or relativistic Dirac-Brueckner-Hartree-Fock calculations [37, 38, 34, 35]. To perform such calculations for arbitrary non-equilibrium phase space configurations as they occur in heavy ion reaction is presently out of range. One possibility is therefore to apply the results of BHF/DBHF calculation for infinite nuclear matter in a local density approximation (LDA) to heavy ion reactions. Attempts to go beyond the LDA and to incorporate non-equilibrium features on the level of the effective interaction have e.g been made in [41, 40]. However, the density and momentum dependence of the *G*-matrix is rather complex and therefore in many transport models simpler effective interactions such as Skyrme forces [42] or parameterisations of the non-linear Walecka model of Quantum Hadron Dynamics [43, 44] are used.

The mean field depends in general on the local density ρ, the momentum relative to the surrounding medium p and, when isospin-dependent forces are used, on the local isospin asymmetry β. One has further to distinguish between non-relativistic potentials U, e.g. Skyrme forces, and a relativistic treatment where self-energies Σ of different Lorentz character (scalar and vector) arise, e.g. Walecka type models. Note that even if self-energies do not explicitly depend on momentum, in the relativistic case a momentum dependence arises due to Lorentz forces generated by the vector component of the self-energy.

XXXX connection to hydro eos XXXXXXXXX

3.4.2.3 Model realizations

In the following a compilation of state-of-the-art transport models is given.

Quantum-Molecular-Dynamics (IQMD):

J. Aichelin, Ch. Hartnack
http://www-subatech.in2p3.fr/ theo/qmd/

The isospin quantum molecular dynamics model (IQMD) [73] is a semiclassical transport model treating the interactions of nuclear matter on a microscopic N-body level. Nucleons, deltas and pions interact via Skyrme-type two and three body forces and via binary collisions. The collisions allow for elastic and inelastic channels. Its actual application is majorly focused on the actual SIS18 physics.

Quantum-Molecular-Dynamics ((R)QMD):

A. Faessler, C. Fuchs, ...

The Tübingen relativistic quantum molecular dynamics (R)QMD transport code [4] is in principle similar to the IQMD model. It is relativistic, however, not covariantly formulated. In the Tübingen QMD and IQMD models only pions are included as explicit mesonic degrees of freedom. Heavier mesons such as $K, \eta, \rho, \omega, \ldots$ are treated perturbatively. These models are in particular suited to study subthreshold meson production at SIS energies. They have extensively been applied to kaon production at subthreshold energies [5, 6] and in the case of QMD (Tübingen) also to vector meson and dilepton production [7]. For the latter case the Tübingen model has been extended to include all nuclear resonances with masses below 2 GeV, in total 11 N^* and 10 Δ resonances [7].

BUU (GiBUU):

A. Larionov, U. Mosel, O. Buss, K. Gallmeister, T. Leitner

The Giessen BUU model (GiBUU) [48, 49, 50, 51] explicitly propagates 9 N^* and 9 Δ resonances with mass below 2 GeV. Apart from that, the model propagates the $S = -1$ baryons $Y = \Lambda,\ \Sigma$ and 19 Y^* resonances. Also the cascades and charmed baryons are included. In the mesonic sector, the following particles are propagated: π, η, ρ, σ, ω, η', ϕ, η_c, J/ψ, K, $\bar{K}$, K^*, $\bar{K}^*$. The baryon-baryon (meson-baryon) collisions below $\sqrt{s} = 2.6$ (2) GeV are treated within the resonance scenario, while at higher invariant energies the string model is applied. The model includes optionally (at SIS energies) the nucleon and kaon mean fields. The nucleon potential in the local rest frame (l.r.f.) of nuclear matter has a Skyrme-like form with a momentum dependent part added separately. The nucleon energy in the l.r.f. is then represented in the Lorentz invariant way by keeping only the scalar potential. The actual calculation of the scalar potential is performed selfconsistently, since the nucleon potential in the l.r.f. depends on its momentum. The mean field potentials of the nonstrange baryonic resonances are put equal to the nucleon mean field, while the hyperonic potentials are rescaled by a factor of 2/3 according to the fraction of the nonstrange quarks. The GiBUU model contains a larger set of the baryonic resonances than other transport models (excepting the Tübingen QMD model) and consequently leads to higher pion numbers in vacuum. Medium corrections to the cross sections $NN \leftrightarrow NR$ and $NN \leftrightarrow NN\pi$ reduce the pion number in medium. The in-medium reduced cross sections are implemented (optionally) in GiBUU. They are computed with the Dirac masses from the NL2 model [52]. In particular, the $NN \leftrightarrow N\Delta$ matrix element is given by the one-pion exchange model — same as in the calculations of Dmitriev et al. [53], but with replacement of the vacuum Δ and nucleon masses by the Dirac ones. This leads to a strong in-medium reduction of the cross section [50]. The GiBUU model is suitable not only for heavy-ion collisions and hadron-nucleus reactions, but also for photon-, electron- and neutrino-induced reactions. This gives the possibility to test the same dynamical part of the model with various physical initial conditions. A new numerical realization of the model [54] is currently being tested. The results presented here are based on the old version desribed in [48, 49, 50, 51].

BUU (BRoBUU)

H.W. Barz, Gy. Wolf, M. Zétényi, B. Kämpfer

The BRoBUU computer code for heavy-ion collisions is developed in a Budapest-Rossendorf cooperation. This code solves the Boltzmann-Ühling-Uhlenbeck equation in the quasi-particle limit [74]

$$\frac{\partial F}{\partial t}+\frac{\partial H}{\partial \mathbf{p}}\frac{\partial F}{\partial \mathbf{x}}-\frac{\partial H}{\partial \mathbf{x}}\frac{\partial F}{\partial \mathbf{p}}=C, \quad H=\sqrt{(m_0+U(\mathbf{p},\mathbf{x}))^2+\mathbf{p}^2}$$

for the one-body distribution function F(**x**,**p**,t) of a certain hadron species. This equation is applied to the motion of the different hadrons, each with mass m_0 in a momentum and density dependent mean field U. This scalar mean field U is chosen in such away that the Hamiltonian H equals $H=\sqrt{m_0^2+\mathbf{p}^2}+U^{nr}$ with a usually in a non-relativistic manner calculated potential U^{nr}. Different particles species (each decsribed by a corresponding distribution F) are coupled by the collision integral C which also contains the Ühling-Uhlenbeck terms responsible for Pauli blocking and Bose enhancement in the collision and particle creation and annihilation processes. The coupled set of Boltzman-Ühling-Uhlenbeck equations is solved by using the parallel-ensemble test-particle method. This method transforms the partial differential–integro equations into a set of ordinary differential equations (looking like equations of motion) for a large number of test particles simulating the ensemble averaging process for the respective function F.

Recently theoretical progress has been made in describing the in-medium properties of particles. In the medium particles have a finite life time which is described by the width Γ in the spectral function of the particles. The spectral function can significantly change during the heavy-ion collision process and can be simulated by an ensemble of test particles with different masses. The change of the spectral function is given by time variation of the test particle mass m [75, 76]. For bosons this additional equation reads

$$\frac{dm^2}{dt} \approx (\frac{\delta}{\delta t}Re\Sigma^{ret}+\frac{m^2-m_0^2-Re\Sigma^{ret}}{\Gamma}\frac{\delta}{\delta t}\Gamma),$$

where $Re\Sigma$ is related to the mean field U, and $\delta/\delta t$ stands for the comoving time derivation. This equation ensures that resonances are propagated towards their vacuum spectral functions at freeze-out. In particular, this technique, allowing for a consistent propagation of broad resonances, is applied in the BRoBUU code for calculating the di-electron emission of ω and ρ mesons in the 1 GeV region.

The BRoBUU code propagates in the baryon sector 24 Δ and N^* resonances and π,η,σ,ω and ρ mesons as well. In addition, the strange particles Λ, Σ and $K^{\pm}$ are propagated, however their production processes are treated by a perturbative method so that they do not effect the dynamics of the collision. Baryons propagate in the mean field. Strange baryons feel 2/3 of the baryon field. Nonstrange mesons are not effected by a potential, but a potential may be easily added if needed. Various sets of mean fields for kaons are available [77]. The nonstrange mesons are produced via resonance decays. This means that the reactions $NN \leftrightarrow NR$ and $mN \leftrightarrow R$ are implemented in the code (with R denoting any baryon resonance and m denoting any meson). Parameters are best fitted to available data [78]. For K^+ production and ϕ meson production cross sections parameters are taken from [79] and [80], respectively. Production and absorption cross section of K^- mesons are measured to a large extent, but for $NY \leftrightarrow NNK^-$ processes one has to rely on theoretical predictions [81].

Relativistic BUU (Texas A&M/Stony Brook):

L.-W. Chen, C.-M. Ko, ...

The Relativistic Boltzmann-Uehling-Uhlenbeck (RBUU) approach has been used by several groups to devellop transport models for the intermediate relativistic energy regime.

The RBUU model developed by the Texas A&M and Stony Brook groups is a covariant microscopic transport model for heavy ion collisions at SIS/GSI energies [59,57,60,61]. This model includes simutaneously the effects of mean field, two-body collisions, and the Pauli blocking for fermions. The covariant RBUU transport equation is sovled by the test-particle method and the one-body phase space distributions are represented by point-like test-particles. In this model, only the nucleon, $\Delta(1232)$ resonance, and pion are treated explicitly and the isospin dependence is neglected.. Besides undergoing elastic and inelastic two-body scatterings, these particles also propagate in mean-field potentials. For nucleons, their potential is taken from the nonlinear Walecka model used in Ref. [59] or the effective chiral Lagrangian of Ref. [62]. The Δ resonance is assumed to have the same mean-field potential as the nucleon, while the mean-field potential for the pion is neglected.

This RBUU model allows one to investigate consistently the medium effects on hadron properties through the change of the scalar and vector potential. Kaons together with its partners (hyperons or antikaons) are produced in this model from pion-baryon and baryon-baryon reactions, i.e., $\pi B \to KY$ and $BB \to BYK$. Antikaons are produced not only from pion-baryon and baryon-baryon reactions, i.e., $\pi B \to K\bar{K}B$ and $BB \to BBK\bar{K}$, but also from the pion-hyperon reactions $\pi Y \to \bar{K}N$ [63], where Y denotes either Λ or Σ. Their cross sections are taken either from predictions of the boson-exchange model or from the empirical values as in Ref. [60]. Annihilation of produced antikaons is included via the inverse reactions of pion-hyperon reactions, i.e., $\bar{K}N \to \pi Y$, as other absorption reactions involve the rarely produced kaons and are thus unimportant. However, the annihilation of kaons is neglected as it has little effect on kaon production [64]. Because of the small production probabilities of kaons, hyperons, and antikaons in heavy-ion collisions at SIS energies, the above discussed reactions are treated perturbatively. For Λ and Σ, their mean-field potentials are taken to be $2/3$ of the nucleon potential according to their light quark content. For kaon and antikaon, their mean-field potentials are obtained from the chiral Lagrange including both scalar and vector interactions [60]. Recently, the similar perturbative method has been also extended to invetigate the subthreshold production of multistrange baryon Ξ [66].

Relativistic BUU (Catania-Munich-Tübingen):

M. di Toro, C. Fuchs, T. Gaitanos, H. Wolter,...

The RBUU model developed by the Catania-Munich-Tübingen groups is a fully covariant transport model for heavy ion collisions at SIS/GSI energies. In contrast to most other BUU models this model is based on a representation of the one-body phase space distributions by covariant Gaussian test-particles [8] instead of point-like test-particles. The collision integral incorporates elastic and inelastic channels (Δ and N^* resonance production with 1- and 2-pion final channels). The resonances feels the same mean field potential as the nucleons, and the pions are propagated under the influence of the Coulomb potential, however, they strongly interact with the hadronic environment via re-absorption processes. In the strangeness sector, only the positive charged (K^+) kaons were been considered in the same way as in Ref. [6]. The RBUU approach has been recently extended for an appropriate description of the isovector part of the nuclear equation of state [9,10]. It has been shown that a fully covariant formulation is essential for the understanding of the Lorentz structure of the symmetry energy at supra-normal densities [9, 10, 11, 12]. In particular one can directly probe the contribution of different isovector mesons: competition between an isovector, vector repulsive ρ and an isovector, scalar attractive δ field, as recently suggested [13,14]. The major genuine relativistic effects, that cannot be revealed in other transport codes where only the relativistic kinematics is included, are:

- The different structure if the contributions of the vector and scalar Lorentz components to the self-energies of protons, neutrons and of the various isospin states of other hadrons ($\Delta^{\pm,0,++}$, $N^{+,0}, \Lambda, \Sigma^{\pm,0}$). This implies an important modification of the standard numerical treatment of the collision term when inelastic processes are included, in particular with respect to the energy conservation. This has been discussed in detail in [12].

- The Lorentz boosting of the vector field contribution to the "magnetic force" acting on the baryons. In ref. [9] it has been clearly shown that in neutron-rich systems this effect will lead to large differences in neutron/proton flows at high transverse momenta.

A Relativistic Transport (ART):

Bao-An Li, C.-M. Ko,...

ART is a hadronic transport model which has in particular been designed for the AGS energy range [67, 68]. It consists of a hadronic cascade supplemented by a Walecka type mean field. Particle production is not described via string excitations but by an explicit treatment of the various hadronic reaction channels. Cross sections are based on parameterisations of available data or determined within the resonance model.

The ART model includes baryon-baryon, baryon-meson, and meson-meson elastic and inelastic scatterings. It treats explicitly the isospin degrees of freedom for most particle species and their interactions, making it suitable for studying isospin effects in heavy ion collisions [69]. Since it includes mean-field potentials for nucleons and kaons, the ART model can also be used for studying the effect due to the hadronic equation of state. Resonances such as ρ and Δ are formed from pion-pion and pion-nucleon scattering, respectively, with cross sections given by the standard Breit-Wigner form, and they also decay according to their respective widths. The masses and widths of resonances are taken to be their values in the vacuum, i.e., effects due to possible modifications in dense hadronic matter are neglected.

For baryon-baryon scatterings, the ART model includes the following inelastic channels: $NN \leftrightarrow N(\Delta N^*)$, $NN \leftrightarrow \Delta(\Delta N^*(1440))$, $NN \leftrightarrow NN(\pi\rho\omega)$, $(N\Delta)\Delta \leftrightarrow NN^*$, and $\Delta N^*(1440) \leftrightarrow NN^*(1535)$. In the above, N^* denotes either $N^*(1440)$ or $N^*(1535)$, and the symbol (ΔN^*) denotes a Δ or an N^*, Also included are reaction channels relevant for kaon production, i.e., $(N\Delta N^*)(N\Delta N^*) \rightarrow (N\Delta)(\Lambda\Sigma)K$. For meson-baryon scatterings, the ART model includes the following reaction channels for the formation and decay of resonances: $\pi N \leftrightarrow (\Delta N^*(1440)\ N^*(1535))$, and $\eta N \leftrightarrow N^*(1535)$. There are also elastic scatterings such as $(\pi\rho)(N\Delta N^*) \rightarrow (\pi\rho)(N\Delta N^*)$. As an example, the cross section for the elastic scattering of $\pi^0 N$ is evaluated by including heavier baryon resonances with masses up to 2.0 GeV/c^2 as intermediate states using the Breit-Wigner form but neglecting interferences between the amplitudes from different resonances [67]. The ART model further includes inelastic reaction channels such as $\pi N \leftrightarrow (\pi\rho\eta)\Delta$ and kaon production channels such as $(\pi\rho\omega\eta)(N\Delta N^*) \leftrightarrow K(\Lambda\Sigma)$. Kaon and antikaon elastic scatterings with nucleons as well as inelastic channels for antikaons, such as $\bar{K}(N\Delta N^*) \leftrightarrow \pi(\Lambda\Sigma)$, are included [70] using parameterized experimental data [71]. Also included are kaon production channels involving three-body final states, $(\pi\rho\omega)(N\Delta N^*) \rightarrow K\bar{K}N$ [70]. Because of the difficulty associated with the three-body kinematics, the inverse kaon annihilation reactions of the above channels are neglected.

For meson-meson interactions, the ART model includes both elastic and inelastic $\pi\pi$ interactions, with the elastic cross section consisting of ρ meson formation and the remaining part treated as elastic scattering. Kaon production from inelastic scatterings of light mesons is included via the the reactions $(\pi\eta) \leftrightarrow K\bar{K}$ and $(\rho\omega)(\rho\omega) \leftrightarrow K\bar{K}$. Kaon or antikaon elastic scatterings with mesons in the SU(2) multiplets except the pion are included using a constant cross section of 10 mb [67], while the kaon-pion elastic scattering is modeled through the K^* resonance [72].

A Multi-Phase Transport (AMPT):

Che-Ming Ko, Bao-An Li, Zi-Wei Lin, Subrata Pal, and Bin Zhang
http://nt3.phys.columbia.edu/people/zlin/AMPT/

AMPT [15, 16, 17] is a Monte Carlo transport model for heavy ion collisions at relativistic energies. It uses the Heavy Ion Jet Interaction Generator (HIJING) for generating the initial conditions, the Zhang's Parton Cascade (ZPC) for modelling the partonic scatterings, and A Relativistic Transport (ART) model for treating hadronic scatterings. The default version of AMPT treats the initial condition as strings and minijets and uses the Lund string fragmentation model as in HIJING, while the string melting version of AMPT treats the initial condition as partons and uses a simple coalescence model to describe hadronization.

Hadron String Dynamics (HSD):

E. Bratkovskaya, W. Cassing
http://www.th.physik.uni-frankfurt.de/ brat/hsd.html

The Hadron-String Dynamics (HSD) transport approach is a covariant microscopic transport model developed to simulate - relativistic heavy-ion collisions - proton-nucleus reactions - pion-nucleus reactions in the energy range from SIS to RHIC.

General ideas:
The HSD transport approach [18, 19, 20] provides the numerical test-particle solution of a coupled set of relativistic transport equations for particles with in-medium self-energies. It is based on quark, diquark, string and hadronic degrees of freedom. High energy inelastic hadron-hadron collisions in HSD are described by the FRITIOF string model (including PYTHIA) whereas low energy hadron-hadron collisions are modelled based on experimental cross sections. The transport approach is matched to reproduce the nucleon-nucleon, meson-nucleon and meson-meson cross section data in a wide kinematic range. HSD takes into account the formation and multiple rescattering of leading pre-hadrons and hadrons. The major aim of HSD is - within a single transport model - to gain an understanding about the nuclear dynamics, the creation of dense and hot hadronic matter and the modification of hadron properties in a medium.

Ultra-relativistic Quantum Molecular Dynamics (UrQMD):

H. Stoecker, M. Bleicher, S. Soff, ...
http://www.th.physik.uni-frankfurt.de/ urqmd/

The Ultra-relativistic Quantum Molecular Dynamics model [21, 22] is a microscopic model used to simulate (ultra)relativistic heavy ion collisions in the energy range from BEVALAC and SIS up to AGS, SPS and RHIC. Main goals are to gain understanding about the following physical phenomena within a single transport model: - Creation of dense hadronic matter at high temperatures - Properties of nuclear matter, Delta & Resonance matter - Creation of mesonic matter and of anti-matter - Creation and transport of rare particles in hadronic matter. - Creation, modification and destruction of strangeness in matter - Emission of electromagnetic probes

Quark-Gluon String Model (QGSM):

Amelin, L. Bravina, C. Fuchs, S. Toneev, E. Zabrodin,...

The Quark-Gluon String Model (QGSM) [23, 24, 25, 26] incorporates partonic and hadronic degrees of freedom and is based on Gribov-Regge theory (GRT) [27] accomplished by a string phenomenology of particle production in inelastic hadron-hadron collisions. To describe hadron-hadron, hadron-nucleus and nucleus-nucleus collisions the cascade procedure of multiple secondary interactions of hadrons was implemented. The QGSM incorporates the string fragmentation, formation of resonances, and rescattering of hadrons, but simplifies the nuclear effects neglecting, e.g., the mean fields or evaporation from spectators. As independent degrees of freedom the QGSM includes octet and decuplet baryons, octet and nonet vector and pseudoscalar mesons, and their antiparticles. The momenta and positions of nucleons inside the nuclei are generated in accordance with the Fermi momentum distribution and the Woods-Saxon density distribution, respectively. Pauli blocking of occupied final states is taken into account. Strings in the QGSM can be produced as a result of the colour exchange mechanism or, like in diffractive scattering, due to momentum transfer. The Pomeron, which is a pole with an intercept $\alpha_P(0) > 1$ in the GRT, corresponds to the cylinder-type diagrams. The *s*-channel discontinuities of the diagrams, representing the exchange by *n*-Pomerons, are related to the process of $2k\,(k \leq n)$ string production. If the contributions of all *n*-Pomeron exchanges to the forward elastic scattering amplitude are known, the Abramovskii-Gribov-Kancheli (AGK) cutting rules [28] enable one to determine the cross sections for $2k$-strings. Hard gluon-gluon scattering and semi-hard processes with quark and gluon interactions are also incorporated in the model [29]. The inclusive spectra in the QGSM have automatically the correct triple-Regge limit for the Feynman variable $x \to 1$, double-Regge limit for $x \to 0$, and satisfy all conservation laws. The particular stages of the collision model, namely (i) initialisation of interacting projectile and target nuclei, (ii) string formation via inelastic nucleon-nucleon (hadron-hadron) interaction, (iii) string fragmentation, i.e. hadronization, and (iv) hadron-hadron rescattering, are solved basically by Monte Carlo simulation techniques.

model	baryons		mesons		pot.	strings
	dynamical	perturbative	dynamical	perturbative		
QMD	Δ, N^* with $M \leq 2$ GeV	Λ, Σ	π	$K^+, \eta, \rho, \omega, \phi$	$U(\rho, p)$	NO
IQMD	$\Delta(1232)$	Λ, Σ	π	$K^\pm$	$U(\rho, p)$	NO
BRoBUU	Δ, N^* with $M \leq 2.2$ GeV	Λ, Σ	$\pi, \eta, \sigma, \rho, \omega$	$K^\pm, \phi$	$U(\rho, p)$	NO
GiBUU	see text	Λ, Σ	see text	$K^\pm$	$U(\rho, p)$	YES
RBUU[1]	$\Delta(1232), N^*(1440)$	Λ, Σ	π	K^+	$\Sigma(\rho, \beta)$	NO
RBUU[2]	$\Delta(1232)$	Λ, Σ, Ξ	π	$K^\pm$	$\Sigma(\rho)$	NO
ART	$\Delta, N^*(1440), N^*(1535)$	–	$\pi, K^\pm, \eta, \rho, \omega$	–	$\Sigma(\rho)$	NO
AMPT	SU(3)	–	SU(3) (ps & v)	–	NO	YES
HSD					$\Sigma(\rho)$	YES
UrQMD					$U(\rho)$	YES
QGSM	SU(3)	–	SU(3) (ps & v)	–	NO	YES

Table 3.1: Hadronic degrees of freedom which are explicitly (dynamical) or perturbatively included in the default versions of the various models. The treatment of the nuclear mean field is indicated by U (non-relativistic) and Σ (relativistic). ([1]Catania/Munich/Tübingen, [2]Texas A&M/Stony Brook)

Model	E_{min}	E_{max}
QMD	$\geq$ 100 MeV	$\sim$3-4 GeV
IQMD	$\geq$ 100 MeV	$\sim$2 GeV
BRoBUU	$\geq$ 100 MeV	$\sim$4 GeV
GiBUU	$\geq$ 100 MeV	$\sim$40 GeV
RBUU[1]	$\geq$ 100 MeV	$\sim$2 GeV
RBUU[2]	$\geq$ 100 MeV	$\sim$2 GeV
ART	$\geq$ 100 MeV	AGS
AMPT	$\geq$ SPS	RHIC
HSD	$\geq$ 100 MeV	RHIC
UrQMD	$\geq$ 100 MeV	RHIC
QGSM	$\geq$ 10 GeV	RHIC

Table 3.2: Energy range in which the different models are applicable. ([1]Catania/Tübingen/Munich, [2]Texas A&M/Stony Brook)

Table 3.1 gives a summary of the treatment of the mean field and the hadronic degrees of freedom which are at present included in the transport models listed below. Table 3.2 summarises the energy range where the various models are safely applicable. SPS corresponds thereby to the top SPS energy of about 160 AGeV and RHIC to the top RHIC energy given by $\sqrt{s} = 200$ AGeV in the centre-of-mass system.

3.4.2.4 Benchmarks

This subsection addresses the question how consistent the results of present transport models are. Differences occur due to the use of different physical input such as elementary cross sections or the explicitley included DOFs. This concerns the number of baryonic and mesonic resonances which are included and elementary cross sections which are not constrained by data. There one has to rely on model assumptions and these assumptions differ in the various codes.

The type of the model, i.e. BUU or QMD should not be of relevance as long as 1-body observables are

considered. However, the corresponding simulation codes are complex and sometimes based on different numerical and methodical solution techniques. Although physical observables should be independent on such technical questions one has to exclude them as possible sources of uncertainties as far as possible. This was the major goal of two workshops held in Trento 2001 and 2003 where most groups doing transport model calculations at SIS and higher energies participated. In a first round of homeworks the default versions of the codes were compared, in a second round further specifications were made for a detailed comparison at SIS energy range. The results of the second round have been published in [30]. Fig. 3.4 shows exemplarily the result of the benchmark test for the pion (π^+, π^-) production in central (b=1 fm) Au+Au reactions at 0.96 and 1.48 AGeV and Ni+Ni reactions at 1.93 AGeV.

For this comparison a nuclear mean field corresponding to a soft EoS (K$\sim$200 MeV) was applied and in most models a constant Δ width $\Gamma_\Delta = 120$ MeV, respectively a constant lifetime $\tau = 1/\Gamma_\Delta$ has been used. A constant resonance life time is unphysical and not used in the default versions of the codes but it simplifies such a comparison to some extent, in particular when predictions for pion production are considered. There is an overall agreement in the predictions of the models on the pion rapidity distributions at 1 AGeV, while at higher energies the discrepancies increase. Partially this is due to the different number of included resonances: here in QMD and IQMD only the $\Delta(1232)$ and $N^*(1440)$ resonances have been included, while the Giessen RBUU includes additionally the $N^*(1535)$ and GiBUU also higher lying resonances. However, the GiBUU calculations were performed with the in-medium $NN \leftrightarrow NR$ and $NN \leftrightarrow NN\pi$ cross sections which leads to a reduction at the corresponding pion yields. Figure 3.5 compares the rapidity distributions of protons and produced hadrons ($\pi^\pm$, $K^\pm$ and $\bar{p}$) in central

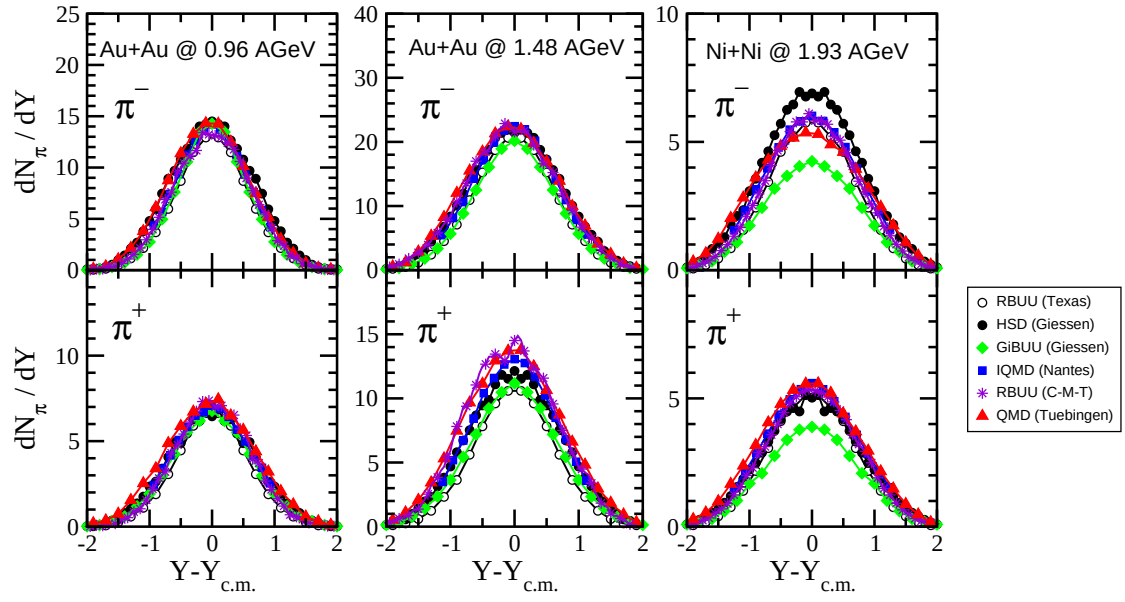

Figure 3.4: $\pi^\pm$ rapidity distributions in central (b=1 fm) Au+Au reactions at 0.96 and 1.48 AGeV and Ni+Ni reactions at 1.93 AGeV from various transport models: GiBUU (Giessen, full diamonds), RBUU (Texas, open circles), RBUU (Catania-Munich-Tübingen, stars), HSD (Giessen, full circles), IQMD (Nantes, full squares) and QMD (Tübingen, full triangles).

(b=1 fm) Pb+Pb reactions at 40 AGeV. Predictions from various transport models are compared: AMPT, HSD, UrQMD.

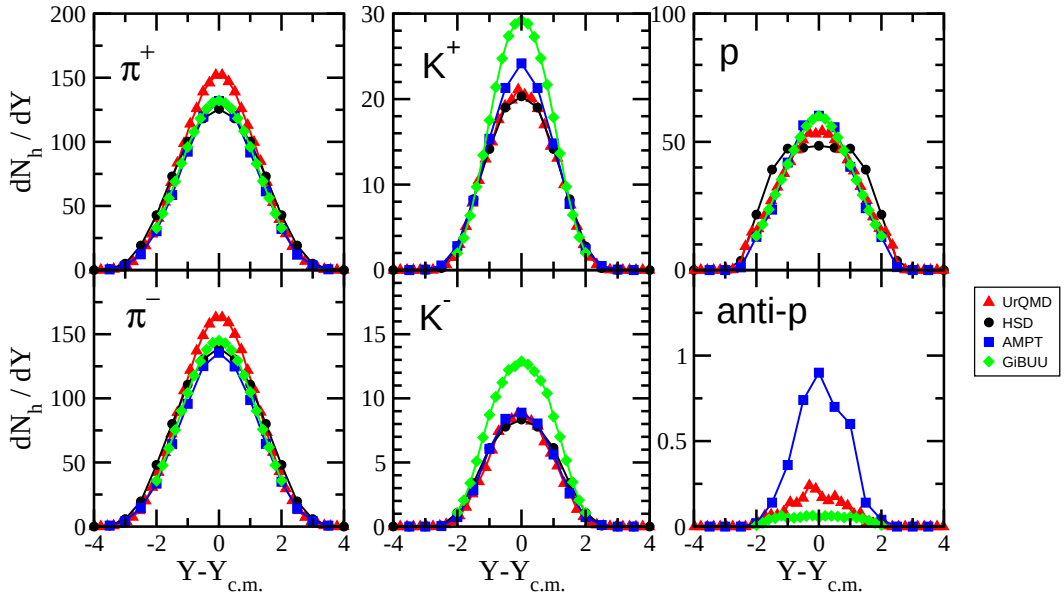

Figure 3.5: $\pi^{\pm}$, $K^{\pm}$ and p, $\bar{p}$ rapidity distributions in central (b=1 fm) Pb+Pb reactions at 40 AGeV from various transport models: GiBUU (Giessen, full diamonds), HSD (full circles), UrQMD (full triangles) and AMPT (full triangles).

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3.5 Major Challenges

C. Greiner, W. Cassing

The theoretical description of relativistic nucleus-nucleus collisions is a formidable task since the time evolution of a complicated many-body quantum system has to be followed from the initial state of colliding nuclei - in their correlated groundstates - via intermediate partonic phases (with rather unknown properties) incorporating dynamical hadronization with nontrivial hadronic spectral functions to a final expansion stage dominated by hadronic resonant scattering. Whereas the dynamics of a low density hadron gas with moderate collision rates and/or self energies appears to be sufficiently under control, even the dynamics of a high density hadron gas with roughly 1 hadron per fm^3 is not well understood in view of an eigenvolume of a pion of about 1 fm^3 or nucleon of about 2 fm^2. What are the properties of the hadrons at high baryon density and/or temperature (below T_c)? Do vacuum hadronic resonance states survive in this environment? What are the scattering rates if chiral symmetry is partially restored? What are the contributions from multiple scattering processes? All these open questions are directly linked to the viscosity of a high density hadron gas as well as to the pressure generation in the hadronic phase.

The situation is even worth for temperatures above T_c. Here the effective degrees of freedom are much under debate. How far in temperature (energy density) do hadronic (color neutral) correlators survive? What is the role of gluons in the partially deconfined phase? Do gluons all become very massive? Do quarks become also very massive? What is the role of colored resonance states ($q\bar{q}$) above T_c? What are the transport coefficients in the strongly coupled plasma like shear and bulk viscosity? What is the role of collective modes and dynamical instabilities in the thermalization of the system? To what extent do multi-parton interactions play a role *etc*.? Furthermore, how can the hadronization process (or any phase transition) be described dynamically without decreasing entropy or violating conserved currents? The list of unsettled questions is even much longer and will need experimental information for at least partial answers.

3.5.1 Towards a consistent off-shell transport approach

Any interacting (or decaying) particle is characterized by a spectral function of finite width, i.e. it cannot appropriately be described by a quasi-particle of infinite lifetime. This phenomenon is well known since about half a century [1], but actual solutions in terms of transport calculations have only been presented since the year 2000 [2] and are still of matter of discussion especially with respect to self energies $\Sigma^{>,<,ret}$ that determine the mean-field and off-shell propagation dynamics as well as the mutual collision rates (cf. Section 2). We recall that only in case of local self energies the equation of state is affected by the collision terms (cf Section 2). However, such an approximation is questionable and will have to be refined in future. For the further discussion we, nevertheless, will adopt this approximation and point out those problems that arise independently.

In principle, the Kadanoff-Baym equations - derived in lowest order from the two-particle irreducible (2PI) action $\Gamma[G]$ - provide a convenient starting point but the resulting transport equations do not directly allow for a test-particle solution due to the back-flow problem (cf Section 2) and have to be solved on an eight-dimensional lattice in phase space including a nonlocality in space-time. The question arises, if further approximations to these equations - like the gradient expansion in phase space - do perform well enough in case of inhomogenuos systems with strong coupling. Furthermore, is the Botermas-Malfliet (BM) substitution [3] for the backflow-term accurate enough to allow for a convenient test-particle simulation of the transport equation?

Some of these questions have been already addressed in Refs. [4,5] for the case of a scalar relativistic field

theory with strong coupling. Here the numerical studies on a fixed momentum grid have shown that the gradient expansion in time is a 'reasonable' approximation even for large coupling. This goes in line with the experience in transport studies in the quasi-particle limit which involves also a first order gradient expansion in space-time. Furthermore, the BM approximation for the backflow-term was found to hold very well in case of the scalar Φ^4-theory. Consequently, the Φ^4 model example allows for a convenient test-particle solution - including dynamical spectral functions - when allowing for $1 \leftrightarrow 3$ transition rates (apart from the conventional $2 \leftrightarrow 2$ scattering processes) to achieve proper chemical equilibrium. Note, however, that presently no related proof exists for the case of strongly coupled Yang-Mills systems and that the exact Noether current is conserved only up to first order in the phase-space gradients!

In spite of the apparent (possible) validity of relativistic transport equations in BM approximation further problems pop up in case of low mass quanta with a large spectral width. This comes about as follows: The energy integral of the spectral function is normalized to unity since it is directly connected to the equal-time commutator of the fields which provides the quantization of the theory and the particle interpretation accordingly. In case of a large spectral width the spectral function $A(M^2)$ is nonvanishing for space-like invariant mass squared ($M^2 < 0$); this fraction of the spectral function physically describes t-channel scattering processes whereas the time-like sector corresponds to s-channel processes. Note that space-like parts of the spectral function do not violate micro-causality! Consequently a particle interpretation has to be attributed to 'field quanta' that (depending on the environment (e.g. temperature)) may change from t- to s-channel processes. Presently it is not known how to realize this even in off-shell transport where only s-channel processes are treated dynamically in a more consistent way, i.e. fractions of the spectral function moving within the light-cone.

Whereas in the high temperature limit the notion of a weakly interacting quasi-particle might be valid, i.e. a well defined pole in the propagator with small negative imaginary part, the situation is not clear for the strong QGP (sQGP) where a finite spectral function not necessarily might arise from a well defined pole structure in the lower imaginary plane but simply due to momentum-dependent interactions of the field quanta. Apart from the gauge-fixing problem it is also not clear if a 'parton propagator' posesses a positive definite spectral function if the quanta are partially confined. We mention that presently no transport theory is defined in case of systems with 'ill defined' propagators or spectral functions that may become negative!

3.5.2 Consistent treatment of many-body processes

Whereas the dynamics of low density fermion or bose systems are dominated by 2-body scattering (or $1 \leftrightarrow 2$ reactions) this no longer holds true in case of high density systems with an average separation $d = \rho^{-1/3}$ which is shorter than the average mean-free path λ of the constituents. In this case $2 \leftrightarrow 3$, $2 \leftrightarrow 4$ or $3 \leftrightarrow 4$ etc. processes are expected play a dominant role. In fact, the importance of multi-meson fusion channels for baryon-antibaryon production has been pointed out in Refs. [7, 8, 9] for nucleus-nucleus collisions from AGS to RHIC energies. Furthermore, the significance of $2 \leftrightarrow 3$ has been demonstrated in Ref. [6] for partonic equilibration at RHIC energies. In addition, the notion of a partonic liquid for the sQGP [10, 11] implies that the higher order interaction rates dominate the 2-body scattering processes. Since a strongly coupled QGP is also expected to be produced at FAIR energies [12] it is thus mandatory to develop theoretical concepts that allow for an adequate description of many-body processes. This holds true also for the issue of thermal and chemical equilibration where many-body processes are involved simply by 'detailed balance'. Note that 'detailed balance' is a necessary requirement for transport theories to describe properly the approach to the 'physical' equilibrium state.

The present knowledge may be summarized as follows: Starting from $2 \rightarrow n$ ($n \geq 1$) processes, which are partly fixed by experimental data, the inverse reactions $n \rightarrow 2$ can be fixed by detailed balance if the transition probability can be described by a characteristic matrix element $\mathcal{M}^2$ times the final state

n-body phase space [9]. As demonstrated in the latter work this recipe holds for on-shell as well as off-shell processes provided that Fermion Pauli-blocking or Bose-enhancement do not play a major role. The actual task, however, is tedious! Any two-body initial channel has to be decomposed into the final channels with n different particles of fixed masses and quantum numbers such that 'detailed balance' can be applied riguously. This easily leads to hundreds to different final channels in $2 \rightarrow n$ reactions; a problem that has to be solved for all two-body scattering processes $i + j$ where i, j denote the degrees of freedom involved in the system. Furthermore, the solution of the off-shell dynamics requires the knowledge of the off-shell matrix elements $\mathcal{M}$ in a wide dynamical regime! The present strategy to employ fitted on-shell matrix elements might not work in all cases!

A numerical solution for $2 \leftrightarrow n$ ($n \geq 1$) processes in case of high density systems is available by now. It is denoted as the 'in-cell method' originally proposed by Babovsky et al. [13] and employed for many-body processes in Ref. [9, 6]. This method is Lorentz invariant and also applicable in case of high density systems where geometrical collision criteria either break down or violate micro-causality [9, 6]. In principle the 'in-cell method' also should work for many-body transitions $k \leftrightarrow l$ with $k, l \geq 3$. This has not be proven numerically so far due to the large CPU requirements, but should be possible for actual applications in a couple of years.

While there is - in principle - some systematic strategy to solve the $2 \rightarrow n$ ($n \geq 1$) scattering problem (as outlined above) the situation is worse for $m \rightarrow n$ ($n \geq 1$) for $m \geq 3$ since there is no experimental information on processes with distinguished 3 particles in the initial state. One might employ perturbation theory to evaluate such matrix elements; it is worth a try. Note, however, that in the strong coupling regime perturbation theory is essentially an uncontrolled exercise! Without guidance by sensitive experimental data the problems might not be solvable.

3.5.3 Phase transitions

In hydrodynamical descriptions of many-body systems a local equilibrium is assumed which allows to follow the time evolution of the system as a function of thermodynamical Lagrange parameters such as inverse temperature T^{-1}, chemical potential μ or pressure P. In this framework (of ideal fluid dynamics with vanishing viscosity) the time evolution is fully determined by conservation laws substituted by an equation of state (EoS) that might involve first or second order phase transitions. However, out of equilibrium no Lagrange parameters can properly be defined for a strongly interacting system such that the notion of a phase-transition - especially for finite systems as those of interest here - is not well defined. Accordingly, the description of phase transitions in off-shell transport provides a major challenge for the future.

There are a few limiting cases where actual solutions are possible: This holds for systems in approximate equilibrium that are governed by the EoS (as in ideal hydrodynamics). In this case one may model the density/temperature dependence of the interaction such that the total energy density $\varepsilon(T, \mu)$ exhibits a first (or second order) phase transition. This strategy holds for local interactions and can be employed in transport calculations if the degrees of freedom do not change at the phase transition point (or line). Unfortunately, this limiting case does not apply for the problem of interest, i.e. the deconfinement of the elementary degrees of freedom that are bound in hadrons (in vacuum).

The solution of the problem has to include transitions between the different degrees of freedom: off-shell partons $\leftrightarrow$ off-shell hadrons in the vicinity of the transition border (thermodynamically in T and μ). This can be accomplished by local (in space-time) transition rates which either can be calculated in some model (NJL or extended versions like PNJL) or put in by hand with an educated guess (as in PHSD where the transition matrix elements peak around the phase boundary). Energy- and momentum conservation can easily be satisfied in off-shell dynamics due to the finite spectral width of all degrees of freedom, however, the entropy might change significantly when partons merge to off-shell hadrons. This fact is due

to a change of the bag pressure (parton condensates) that enters the pressure as well as its derivative wrt. temperature, i.e. the entropy. Note that the condensates also enter the energy density of the many-body system. Presently only approximate techniques exist that deal with a dynamical hadronization scenario that conserves the currents as well avoids a decrease of entropy in the phase transition from the sQGP to the interacting hadronic gas. We will have to rely on some modeling in comparison to experimental data in order to find out what 'confinement' actually means.

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