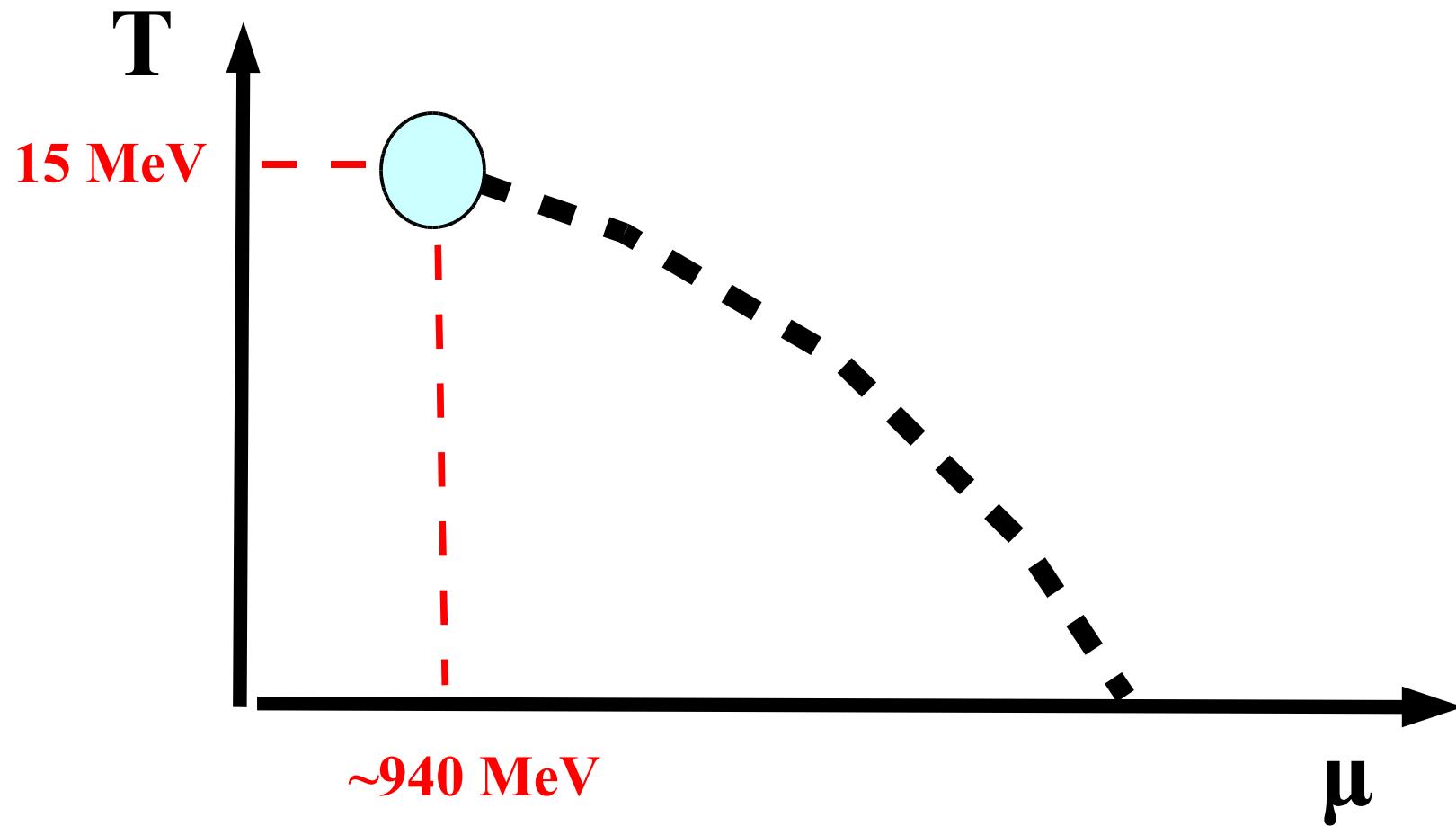


# Fluctuations and Correlations

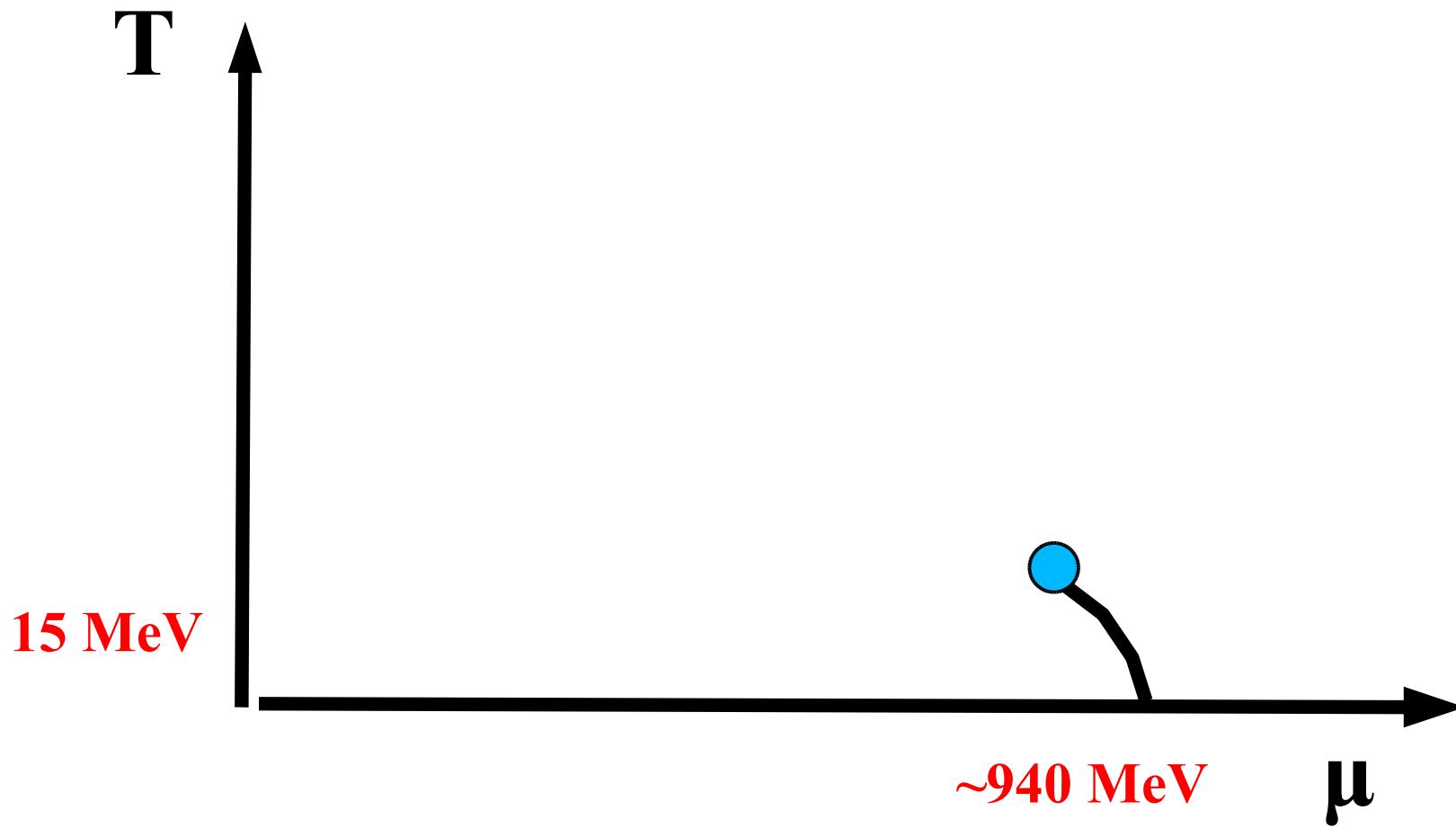
Everything has been said!

But not by everybody.....

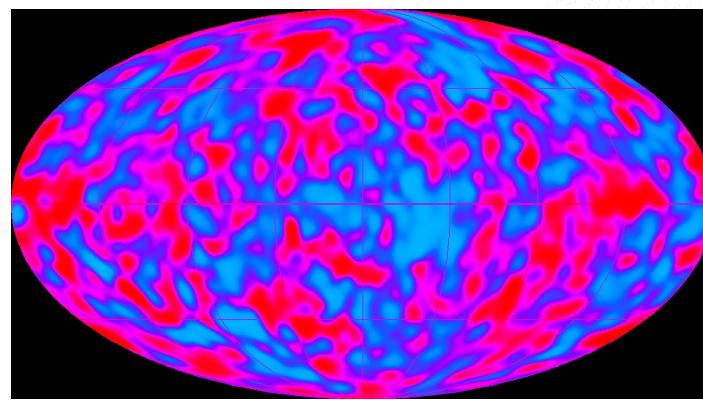
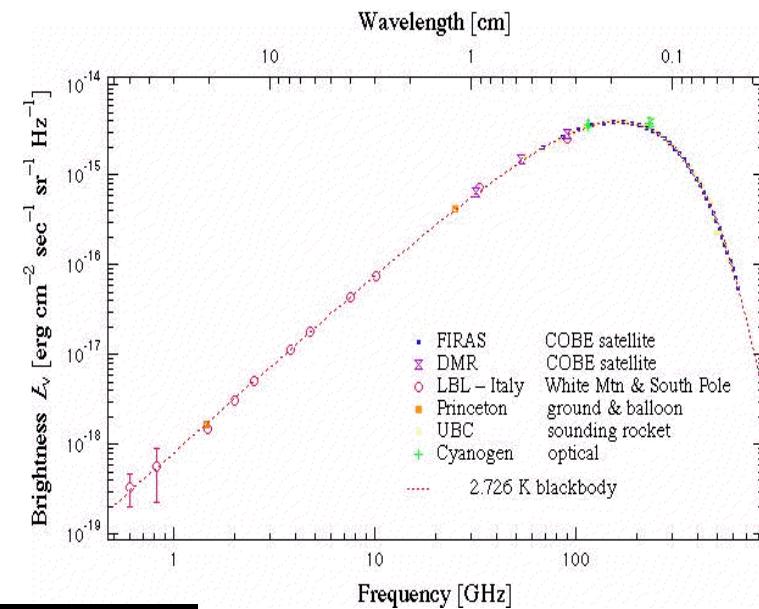
# The QCD Phase Diagram from experiment



# The QCD Phase Diagram from experiment



# The mother of all thermal spectra and fluctuations



Fluctuations at the  
level of  $10^{-5}$  !!!

# Fluctuations in thermal system

e.g. Lattice QCD

$$Z = \text{Tr} [\exp(-\beta(H - \mu_Q Q - \mu_B B - \mu_S S))]$$

Mean :  $\langle X \rangle = T \frac{\partial}{\partial \mu_X} \log(Z) = -\frac{\partial}{\partial \mu_X} F \quad X = Q, B, S$

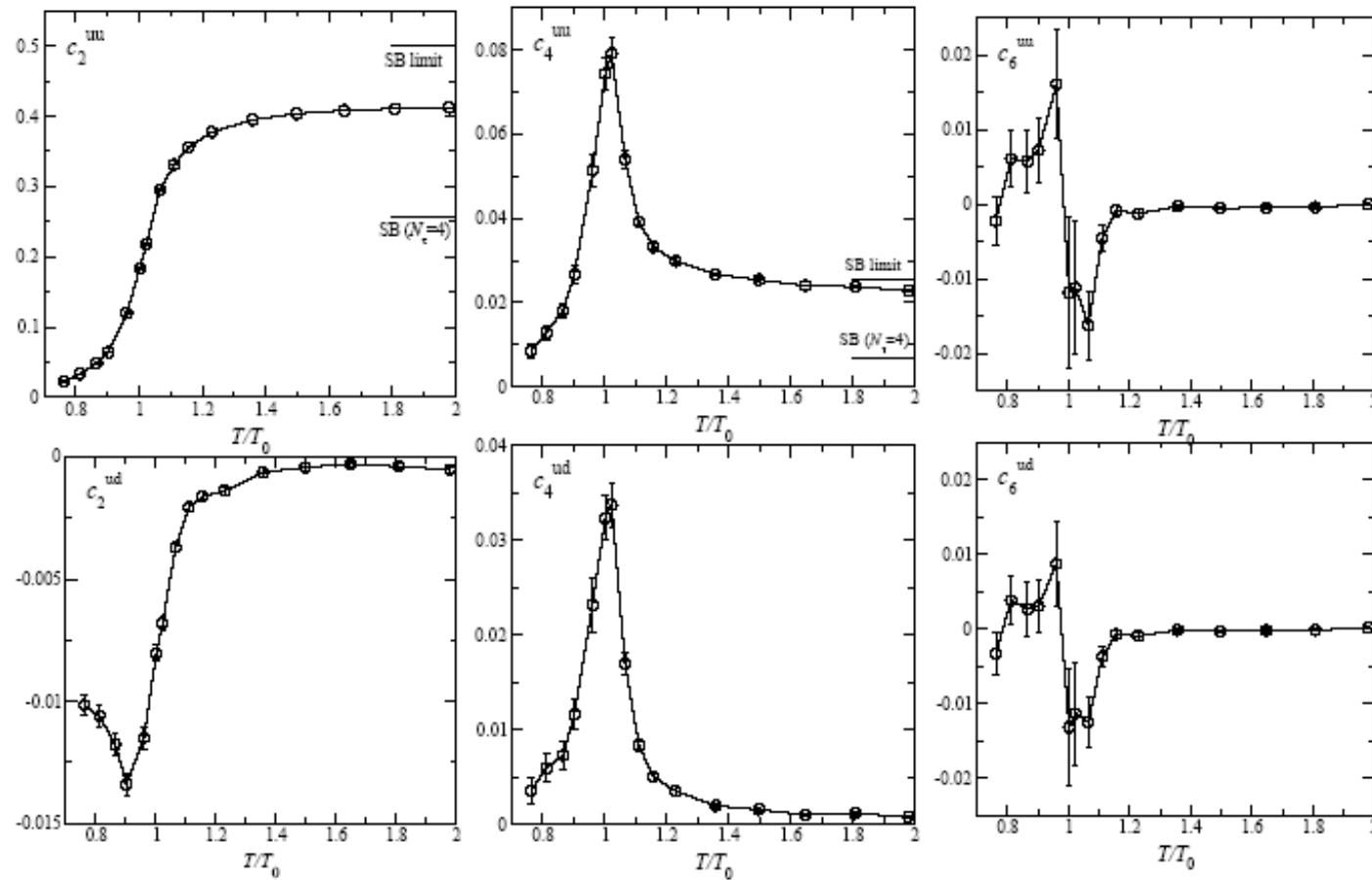
Variance:  $\langle (\delta X)^2 \rangle = T^2 \frac{\partial^2}{\partial \mu_X^2} \log(Z) = -T \frac{\partial^2}{\partial \mu_X^2} F$

Co-Variance:  $\langle (\delta X)(\delta Y) \rangle = T^2 \frac{\partial^2}{\partial \mu_X \partial \mu_Y} \log(Z) = -T \frac{\partial^2}{\partial \mu_X \partial \mu_Y} F$

Susceptibility:  $\chi_{XY} = -\frac{1}{V} \frac{\partial^2}{\partial \mu_X \partial \mu_Y} F = -\frac{1}{V} \frac{\partial}{\partial \mu_X} \langle Y \rangle$

# Lattice-QCD susceptibilities

$$\frac{\chi(T, \mu_q)}{T^2} = 2c_2 + 12c_4\left(\frac{\mu_q}{T}\right)^2 + 30c_6\left(\frac{\mu_q}{T}\right)^4 + \dots$$



Rule of thumb:

$$c_n \sim \langle X^n \rangle \\ X = B, Q, S, \dots$$

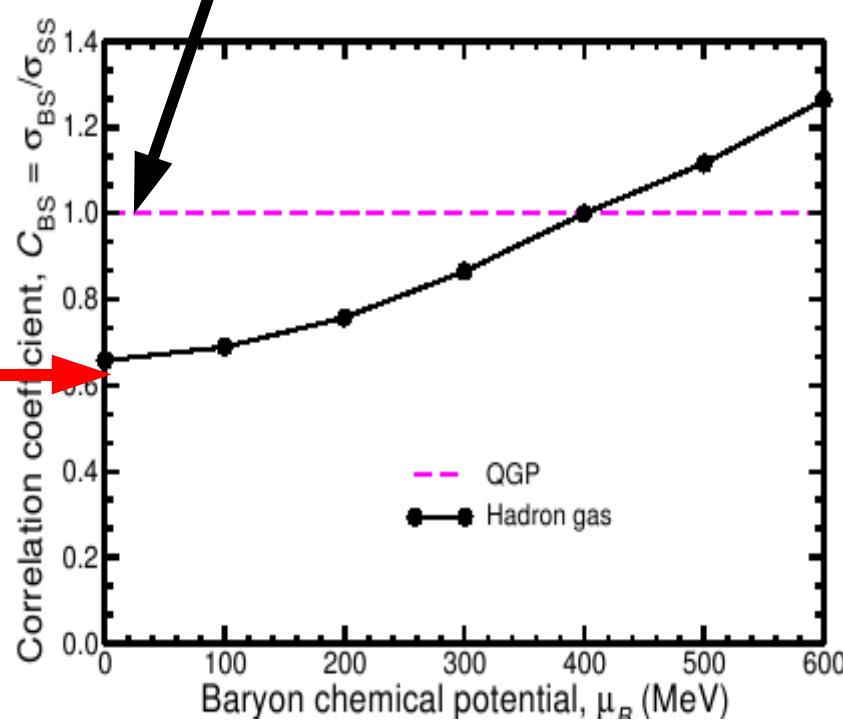
Alton et al, PRD 66 074507 (2002)

# E-by-E observables

- Multiplicity fluctuations
  - interesting centrality dependence at top SPS energies
- Charge fluctuations
  - Resonance gas at RHIC
  - no sensitivity at SPS
- Transverse momentum fluctuations
  - some signal at SPS & RHIC (mostly “jets”)
- Ratio ( $K/\pi$ ) fluctuations
  - statistical at top SPS, possible signal at low SPS

# Something new: $\langle BS \rangle$ , $\langle QS \rangle$

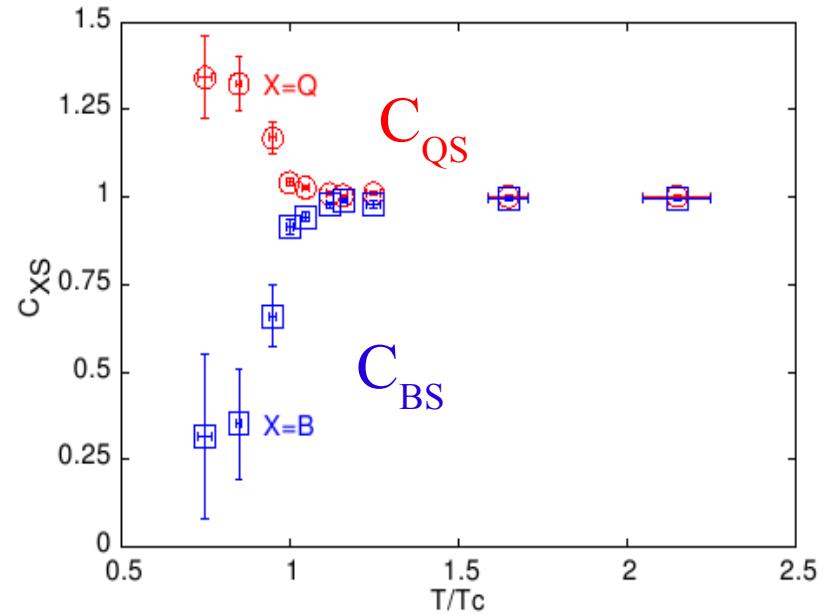
Independet quarks and  
LATTICE QCD for  $T > 1.1 T_c$



Bound state  
QGP

$$C_{BS} = -3 \frac{\langle BS \rangle}{\langle S^2 \rangle}$$

$$C_{QS} = -3 \frac{\langle QS \rangle}{\langle S^2 \rangle}$$



V.K. Majumder, Randrup PRL95:182301,2005

Gavai,Gupta, hep-lat/0510044

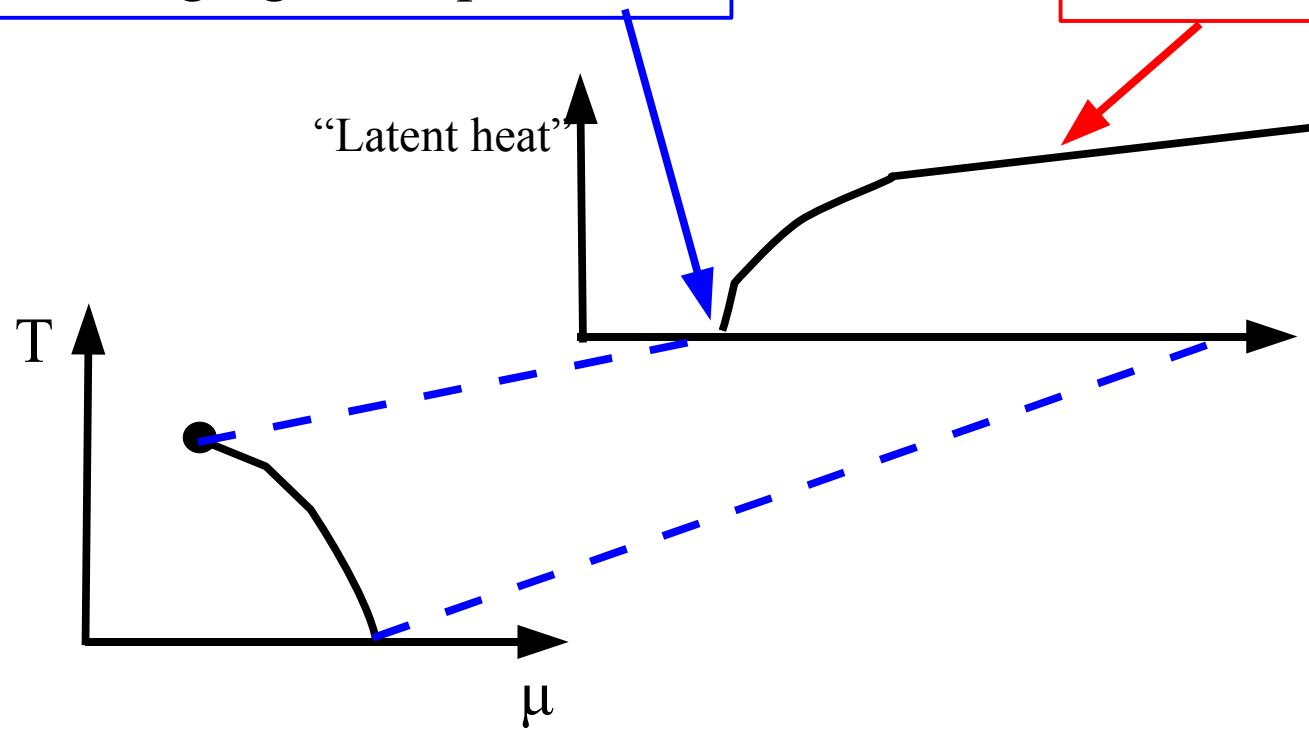
# First order or second order?

Second order:

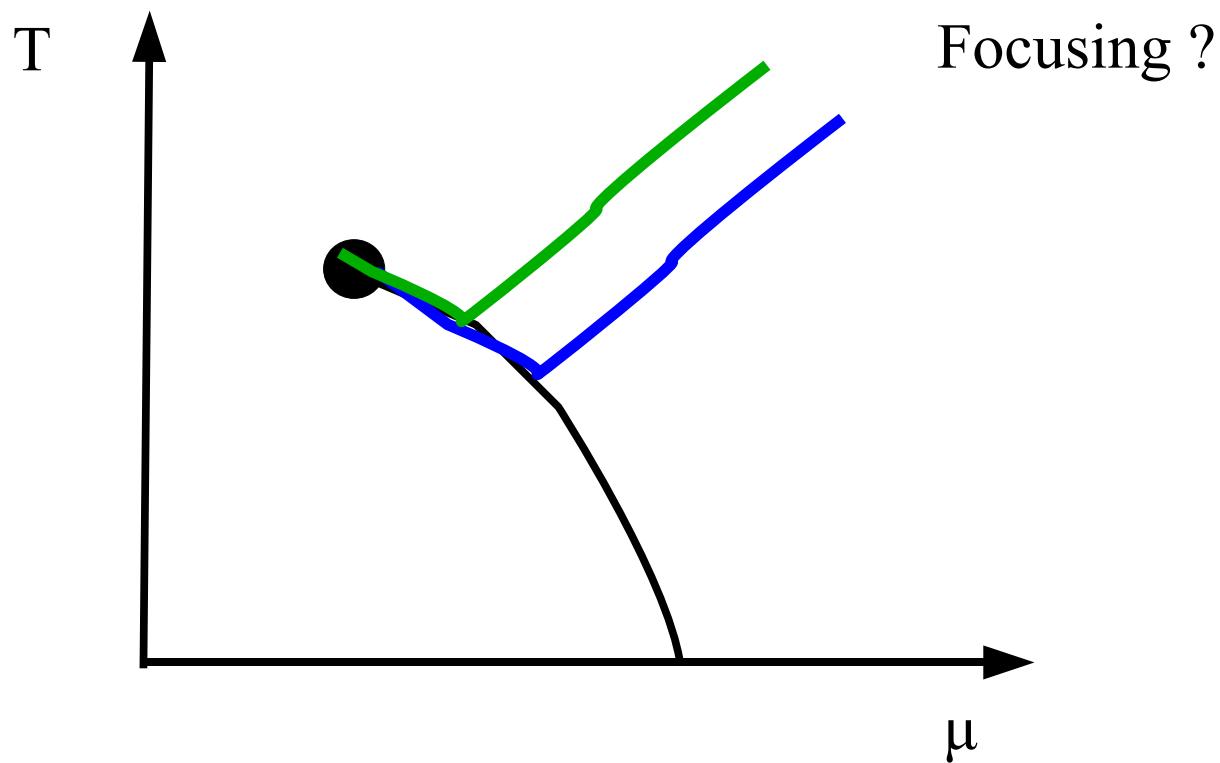
- Critical fluctuations
- Diverging Susceptibilities

First order:

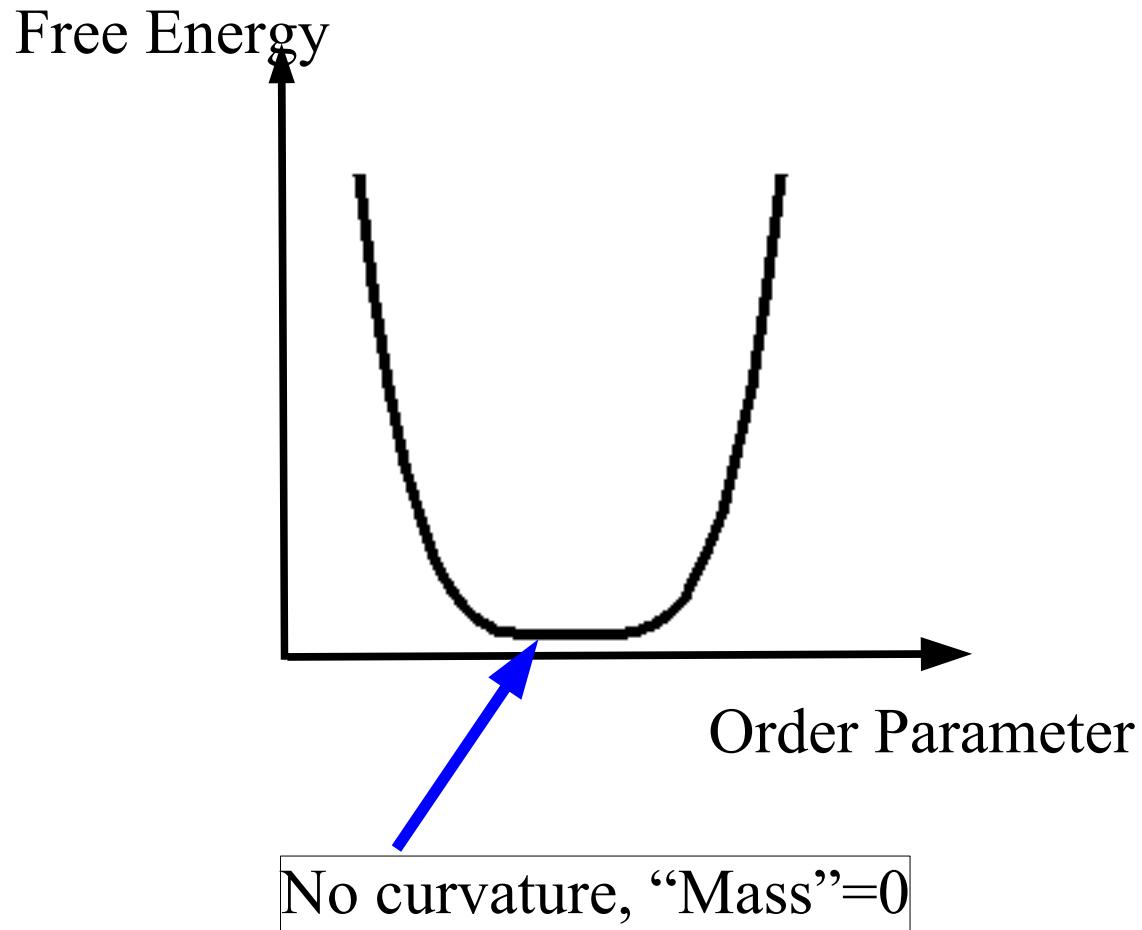
- Phase coexistence, bubbles
- Spinodal instabilities



# First or second order?



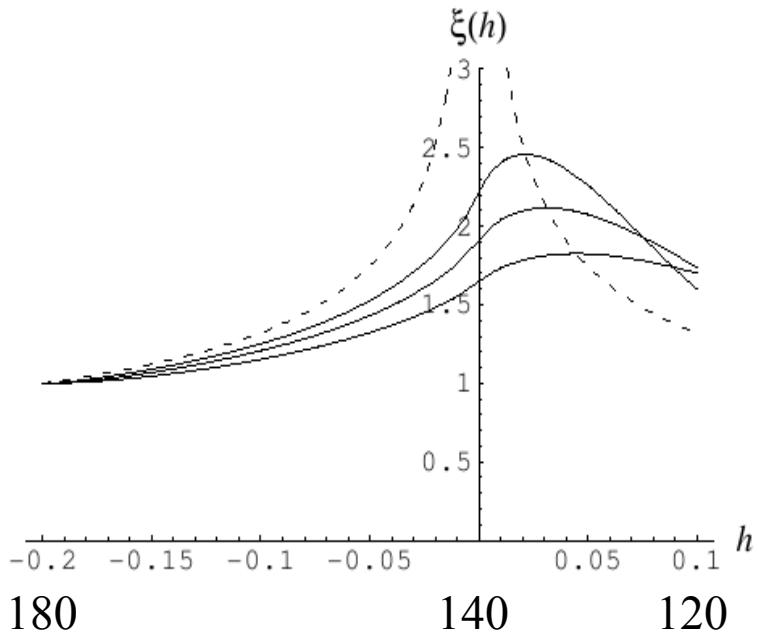
# Second order



- Fluctuation of order parameter at all scales
- Diverging susceptibilities  $\sim 1/(\text{“Mass”})^2$
- Diverging correlation length  $\sim 1/(\text{“Mass”})$
- Universality
- Critical slowing down !

# Second order

correlation length  $\sim 1/m_\sigma$

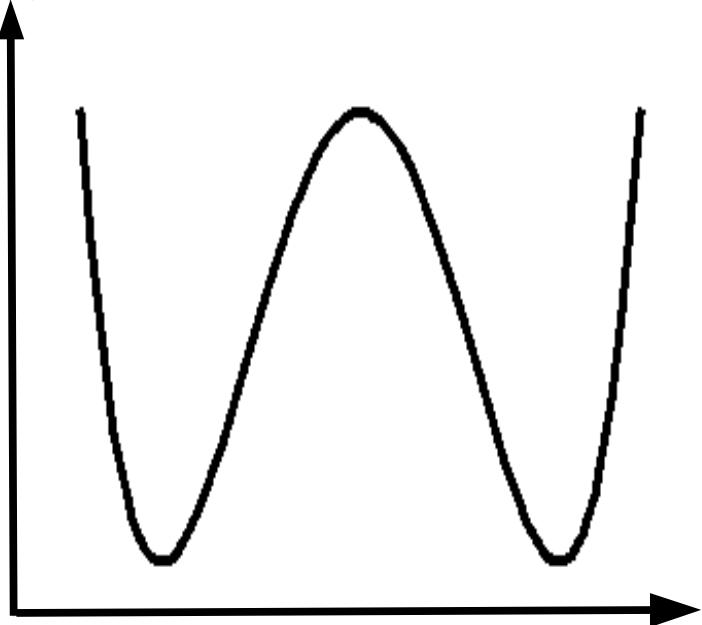


- Critical slowing down
- limited sensitivity on model parameters
- Max. correlation length 2-3 fm
- Translates in **3-5%** effect in  $p_t$ -fluctuations

Bernikov, Rajagopal, hep-ph/9912274

# First order

Free Energy



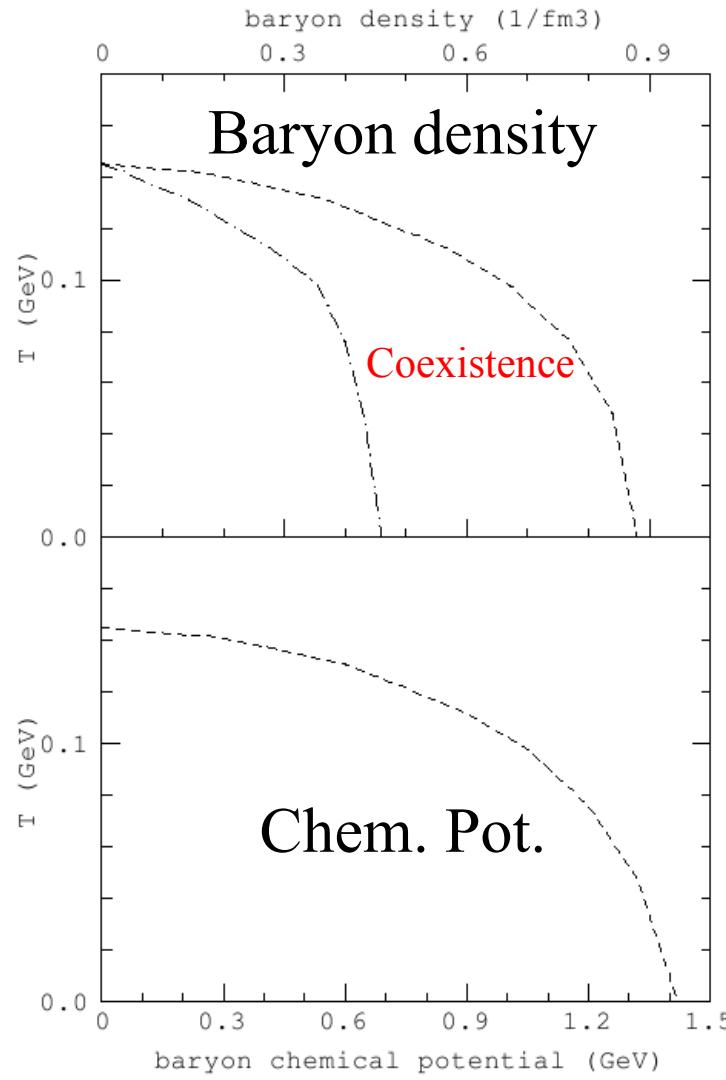
Order Parameter

- Phase coexistence
- “Bubble” formation
  - Spatial fluctuations of order parameter
  - definite length scale
- Specific heat
- Dynamics: Spinodal instability

# First order

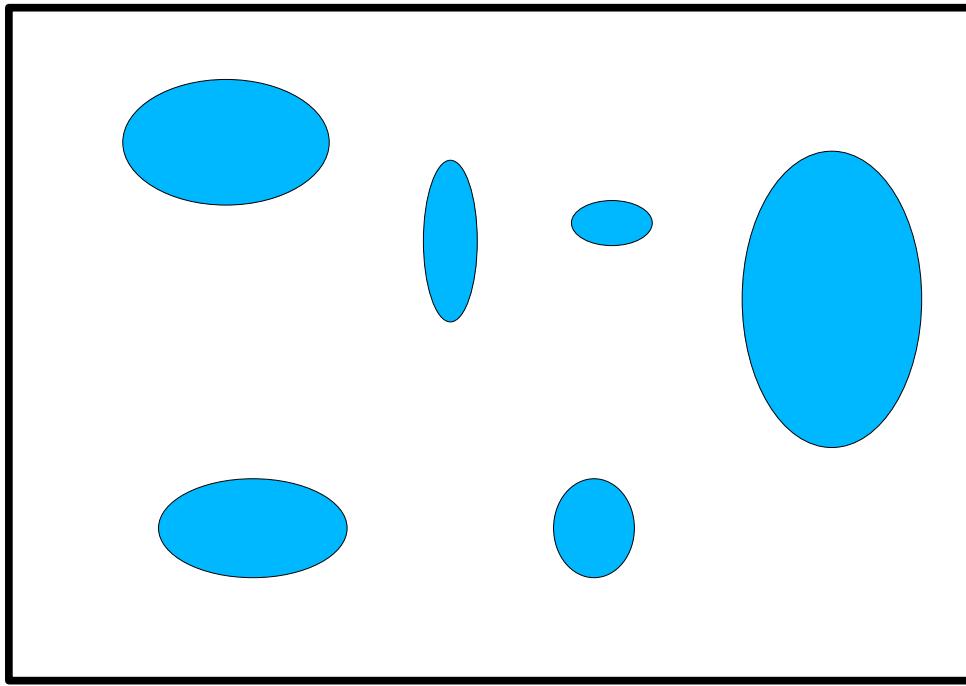
What are the phases?

# “One” order parameter



P. Braun-Munzinger and J. Stachel,  
Nucl.Phys.A606:320-328,1996

# Baryon number fluctuations

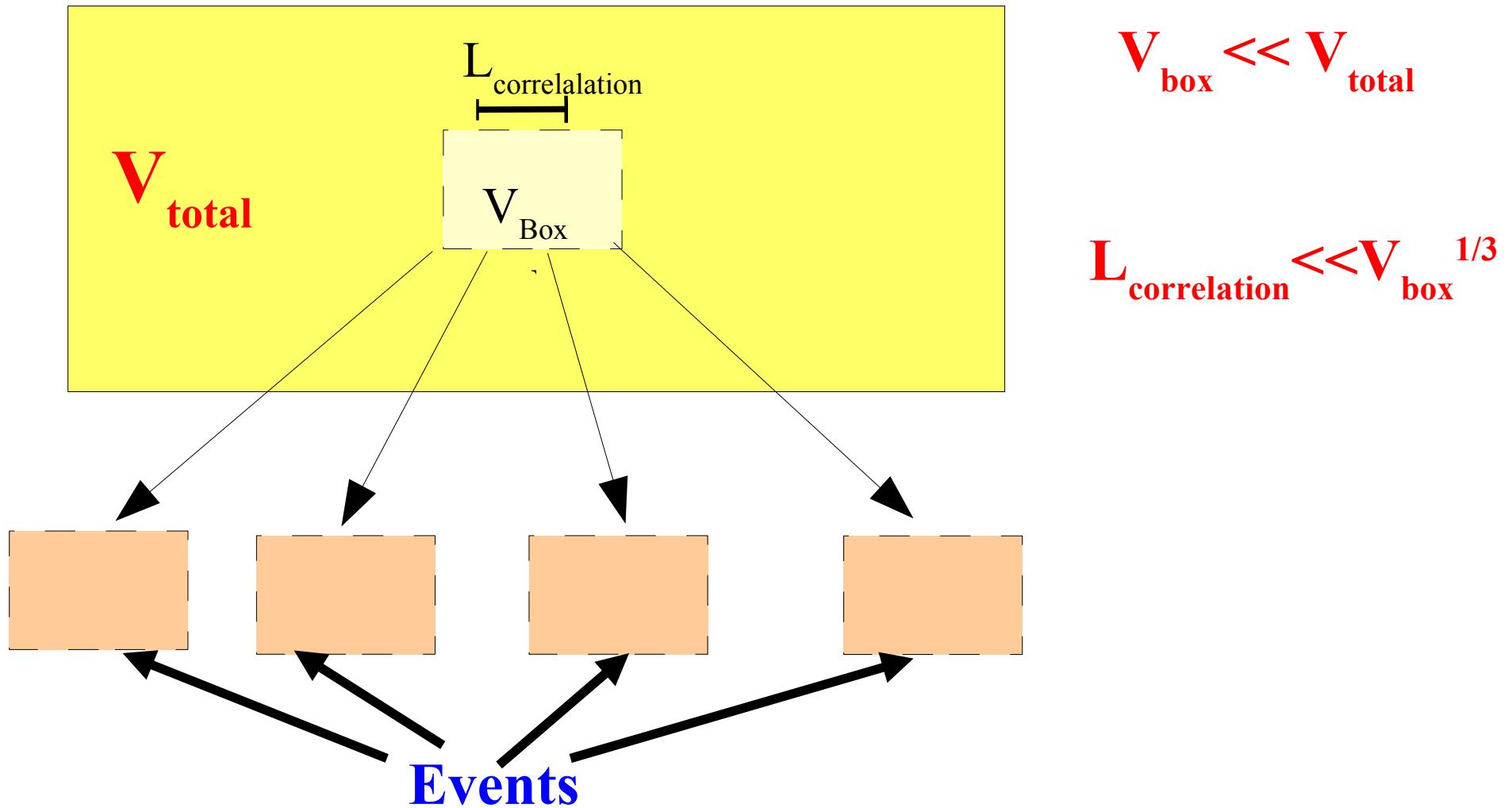


Strong spatial fluctuations

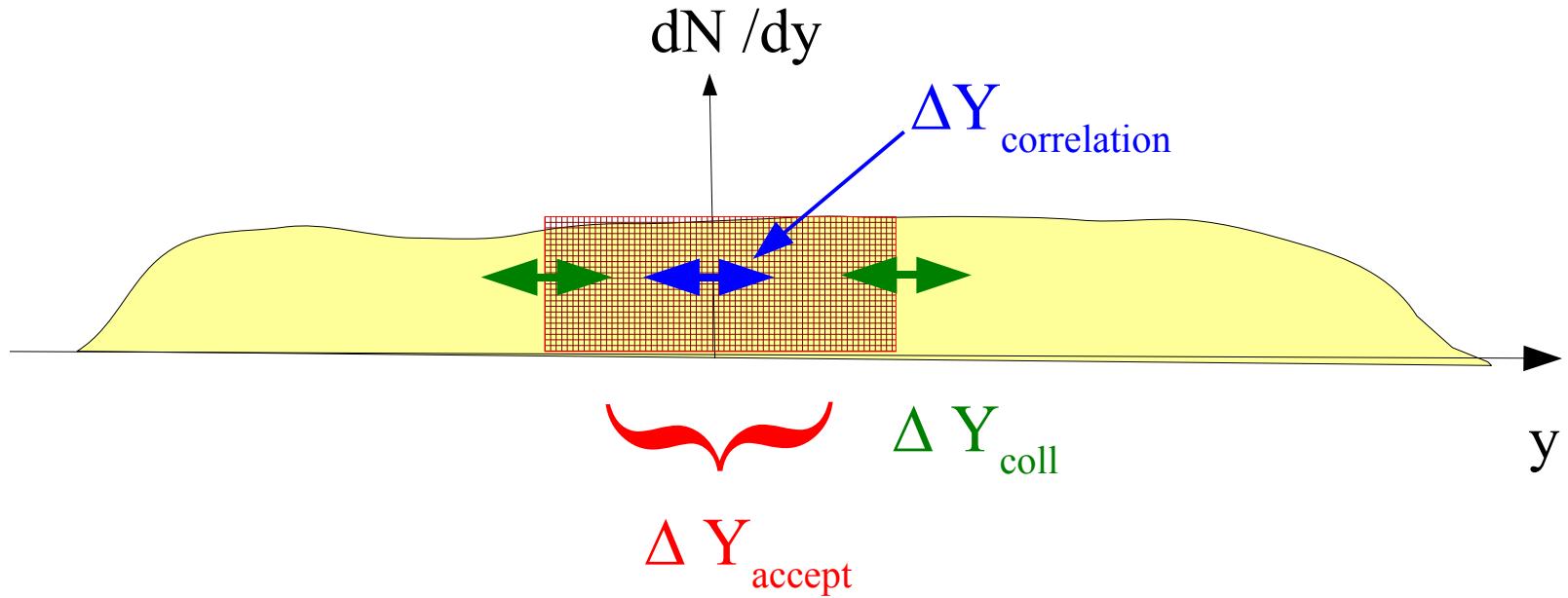
If  $V_{\text{domain}} \ll V$ , small effect  
on integrated Baryon Number  
fluctuations

$$\frac{\langle (\delta N)^2 \rangle}{\langle N \rangle} \approx \left( 1 + \frac{(\Delta \rho)^2}{4 \bar{\rho}^2} \right)$$

# “Charge” Fluctuations



# “Charge” fluctuations



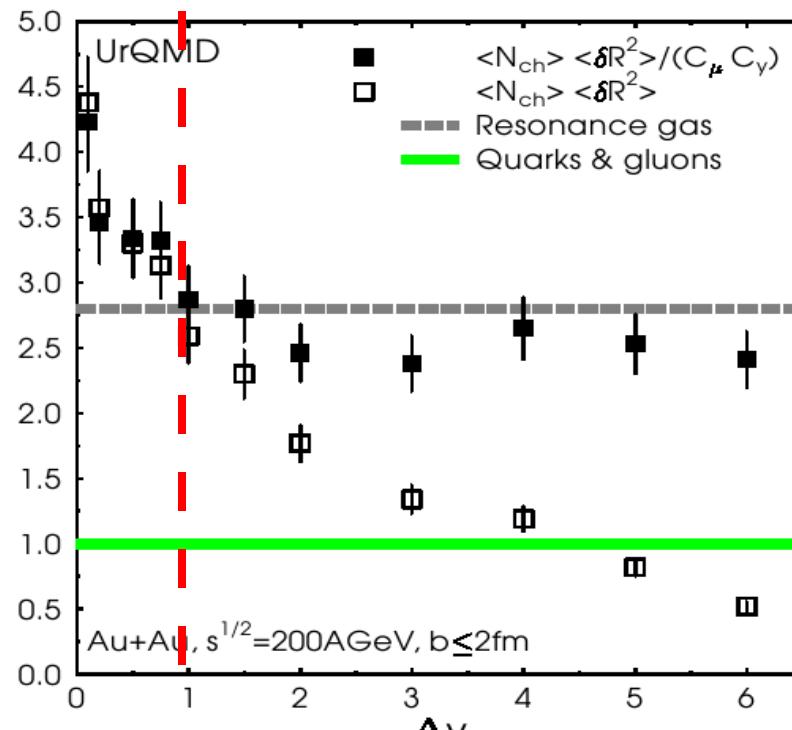
Condition for “charge” fluctuations:

- 1)  $\Delta Y_{\text{correlation}} \ll \Delta Y_{\text{accept}}$  **(catch the physics)**
- 3)  $\Delta Y_{\text{total}} \gg \Delta Y_{\text{accept}} \gg \Delta Y_{\text{coll}}$  **(keep the physics)**

# Charge conservation

$$C_\mu = \tilde{R}_{\Delta y}^2 = \frac{\langle N_+ \rangle_{\Delta y}^2}{\langle N_- \rangle_{\Delta y}^2}$$

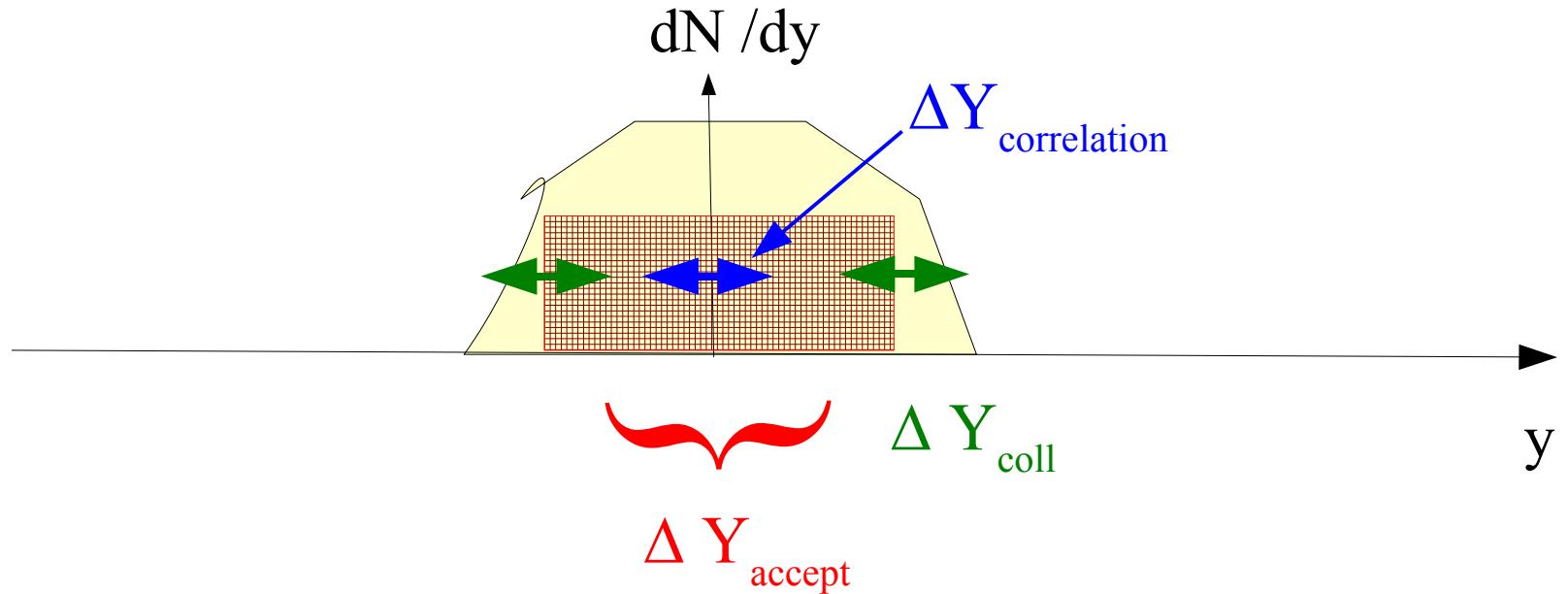
$$C_y = 1 - P = 1 - \frac{\langle N_{ch} \rangle_{\Delta y}}{\langle N_{ch} \rangle_{total}}$$



Correlations (meson decay)

Bleicher et al.  
Phys.Rev.C62:061902,2000

# Charge fluctuations at SPS



Condition for “charge” fluctuations:

1)  $\Delta Y_{\text{correlation}} \ll \Delta Y_{\text{accept}}$  **(catch the physics)**

3)  $\Delta Y_{\text{total}} \gg \Delta Y_{\text{accept}} \gg \Delta Y_{\text{coll}}$  **(keep the physics)**

# Correlation length?

Theory:  $\langle \rho(x) \rho(y) \rangle$  Correlation in **coordinate** space

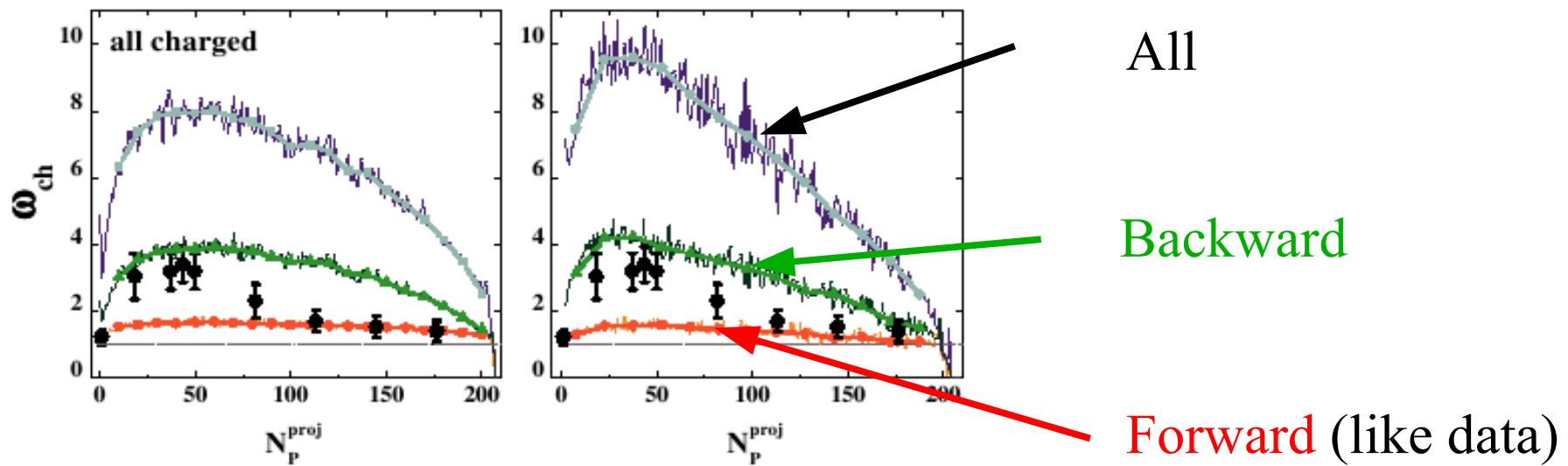
Experiment:  $\frac{dN}{dp_1 dp_2} \sim \langle \tilde{\rho}(p_1) \tilde{\rho}(p_2) \rangle$  Correlation in **momentum** space

Flow helps maybe....

Better: Calculation of momentum space correlations

# Dynamics, event selection ... (or why a symmetric detectors are good)

Konchakovski et al, nucl-th/0511083



- Fluctuations are sensitive to dynamics (mixing of projectile and target material?)
- Event selection/trigger affects fluctuations → large Acceptance!

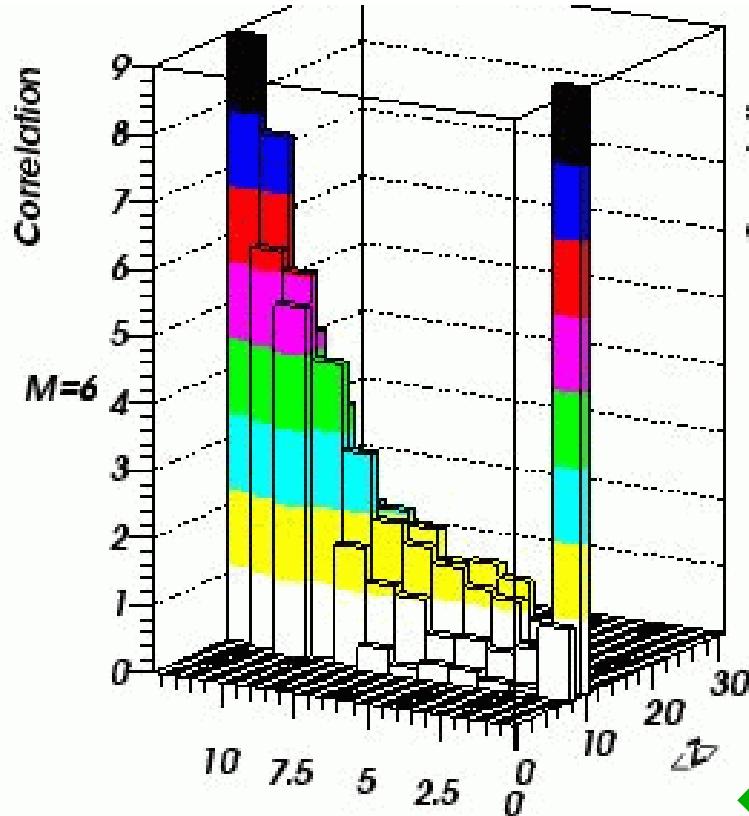
# Things to do!

- Characterize the Phases
  - what are useful order parameters
- Test observables using static and dynamical models
  - Effects are small, comparable with 'trivial ones' such as quantum statistics, dynamics etc.
  - Only a well chosen observable / set of observables will prevent us from seeing Poisson
    - e.g. can we live without neutrons?
  - CONSERVATION LAWS

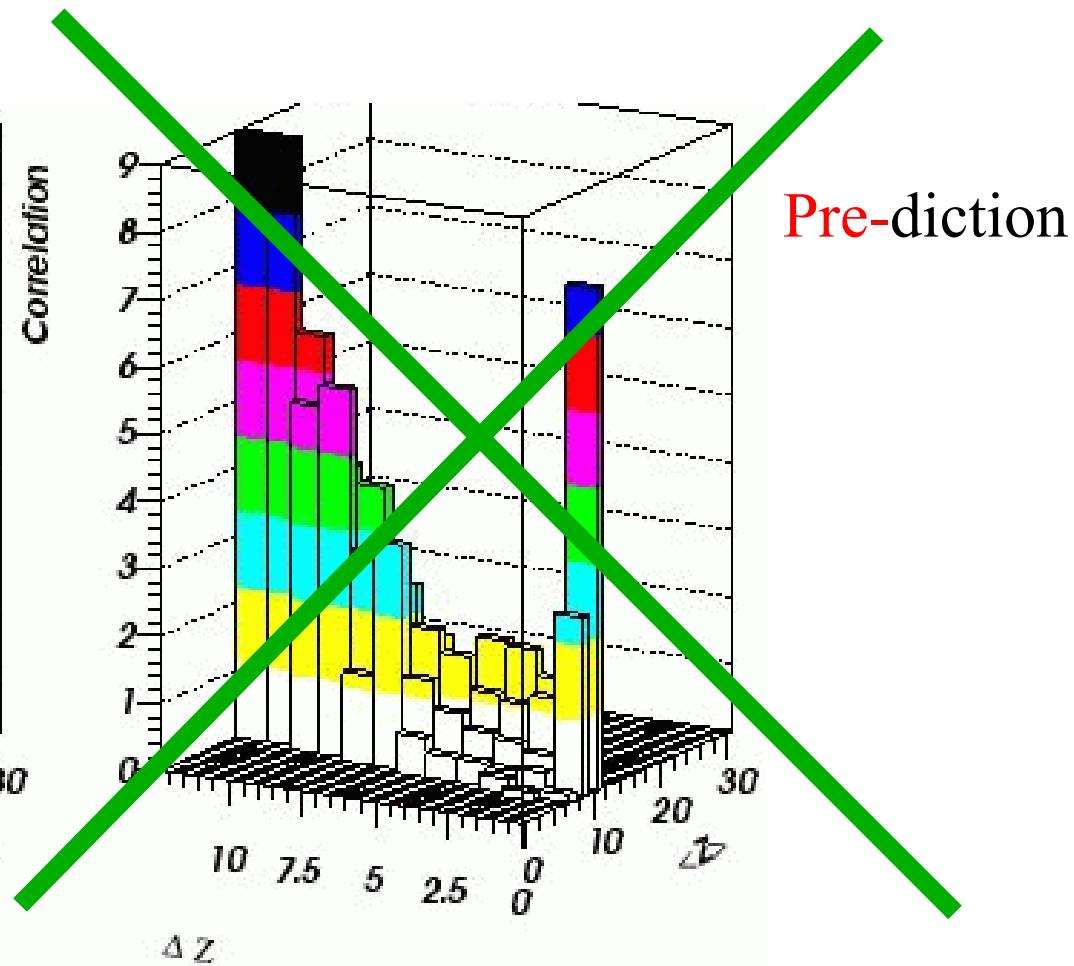
# Spinodal decomposition in nuclear multifragmentation

occurs!

Data speak for themselves!



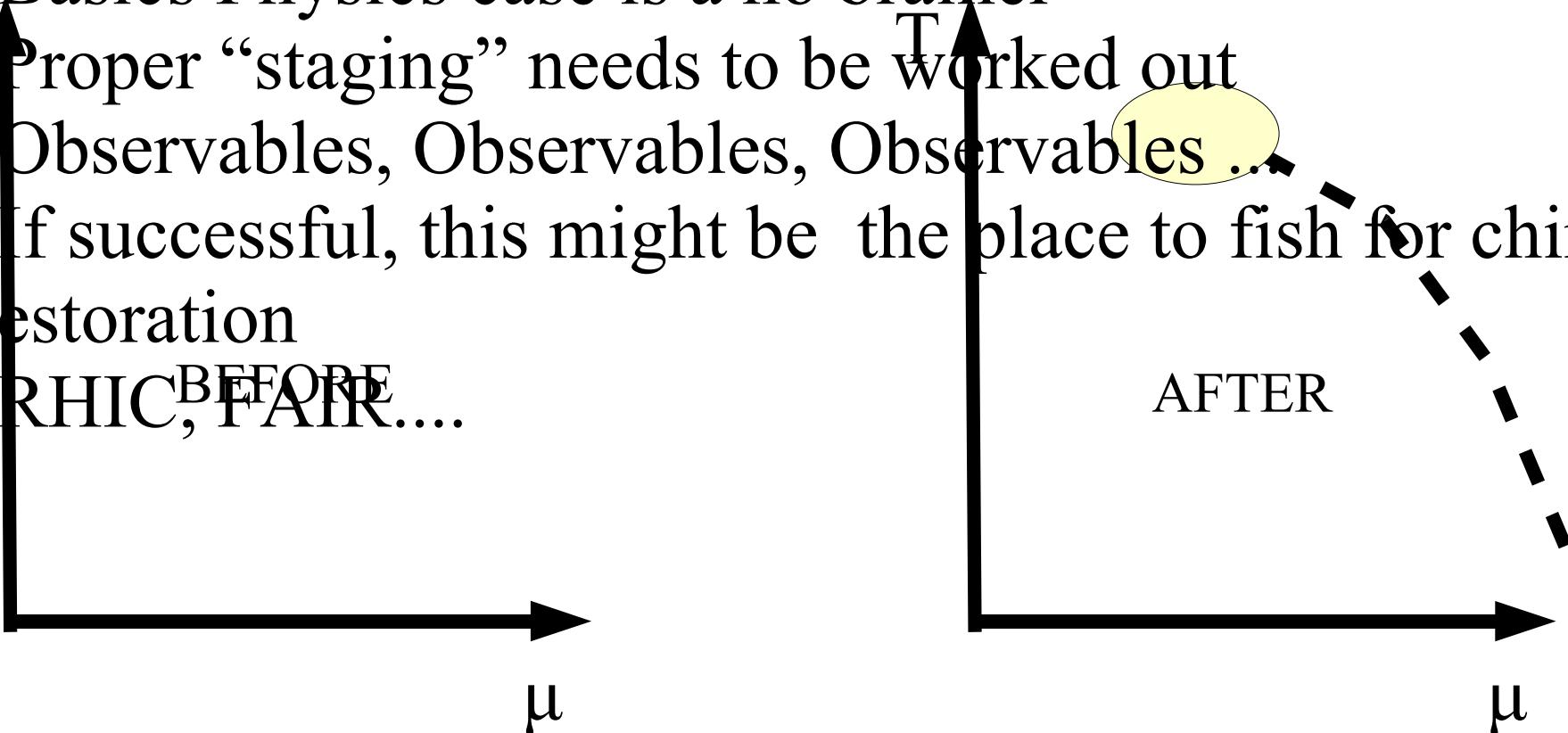
Experiment (INDRA @ GANIL)  
Borderie et al, PRL 86 (2001) 3252



Theory (Boltzmann-Langevin)  
Chomaz, Colonna, Randrup, ...

# This is NOT a one shot deal!

- Basics Physics case is a no brainer
- Proper “staging” needs to be worked out
- Observables, Observables, Observables ...
- If successful, this might be the place to fish for chiral restoration
- RHIC<sup>BEFORE</sup>, FAIR<sup>AFTER</sup>....



# The End