The Critical Region of the QCD Phase Transition

Mean field vs. Renormalization group

B.-J. Schaefer[1] and J. Wambach[2,3]

[1]Institut für Physik • KFU Graz

[2]Institut für Kernphysik • TU Darmstadt [3]GSI • Darmstadt

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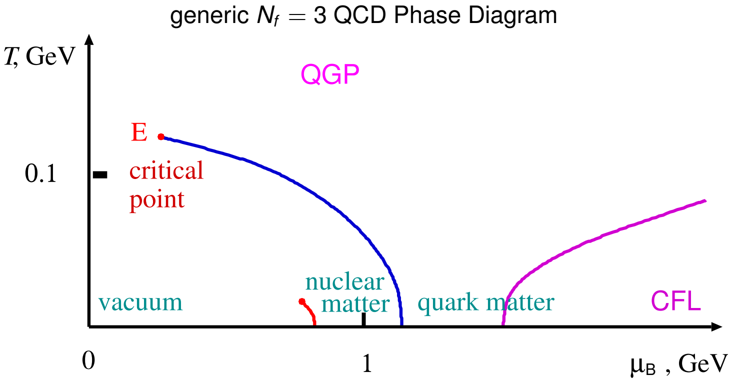
Outline

Motivation/Introduction



lattice at $\mu = 0$: crossover
eff. models at $T = 0$: 1st-order
$\rightarrow \exists$ critical end point E (cf. as in water)

lattice at $\mu \neq 0$: “sign problem”
idealization $\rightarrow$ chiral limit
sharp distinction

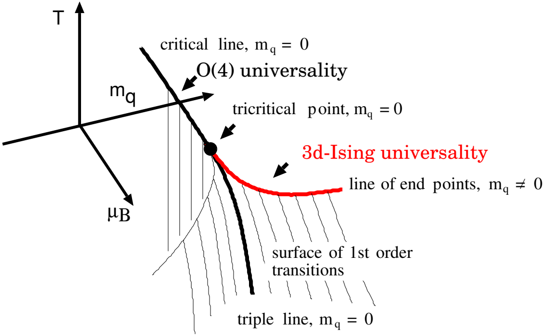
location of crit. point unknown

Motivation/Introduction

3D-view (T, μ_B, m_q)
of $N_f = 2$ QCD phase diagram:

$m_q = 0$: $O(4)$-symmetry
4 modes critical $\sigma, \vec{\pi}$

$m_q \neq 0$: no symmetry remains
only one critical mode σ (Ising)
($\vec{\pi}$ massive)



Landau-Ginzburg potential: order parameter $\vec{\phi} = (\sigma, \vec{\pi})$

$$\Omega(T, \mu; \phi) \sim a(T, \mu)\vec{\phi}^2 + b(T, \mu)\vec{\phi}^4 + c\vec{\phi}^6 + m\sigma$$

2nd order line: $a(T_c, \mu_c) = 0 \to O(4)$ universality ; tricritical point: $b(T_c, \mu_c) = 0$

What are the sizes of the critical regions?

Motivation/Introduction

Ginzburg criterion: size of crit. region $\leftrightarrow$ break down of mean-field theory

Landau-Ginzburg potential for 2nd order phase transition

$$\Omega(T,\mu;\phi) \sim d(\vec{\nabla}\phi)^2 + a' t\phi^2 + b\phi^4 \qquad ; \qquad t = (T - T_c)/T_c$$

Ginzburg-Levanyuk temperature τ_{GL}

$$|t| \sim \frac{T_c^2}{a' d^3} b^2 \equiv \tau_{GL} \sim m_q^{4/5}$$

size depends on microscopic dynamics
universality not applicable
He^4 λ-transition: $\tau_{GL} \sim 10^{-15}$
$O(2)$ spin model: $\tau_{GL} \sim 0.3$

Size of crit. region shrinks as $m_q \to 0$ ($\tau_{GL} \sim b^2$) ; tricritical $b \to 0$

Mean-field approximation

$N_f = 2$ Quark-Meson model

$$\mathcal{L} = \bar{q}\,(i\partial\!\!\!/ - g(\sigma + i\gamma_5\vec{\tau}\vec{\pi}))\,q + \frac{1}{2}(\partial_\mu\sigma\partial^\mu\sigma + \partial_\mu\vec{\pi}\partial^\mu\vec{\pi}) - \frac{\lambda}{4}(\sigma^2 + \vec{\pi}^2 - v^2)^2 - c\sigma$$

partition function:

$$\mathcal{Z}(T,\mu) = \int \mathcal{D}\bar{q}\mathcal{D}q\mathcal{D}\sigma\mathcal{D}\vec{\pi}\exp\left\{ i\int_0^{1/T} dt d^3x\,(\mathcal{L} + \mu\bar{q}\gamma_0 q)\right\}.$$

Mean field approx.: $\sigma \to \langle\sigma\rangle \equiv \phi, \pi \to \langle\pi\rangle = 0$, integrate quark/antiquarks

Grand canonical potential

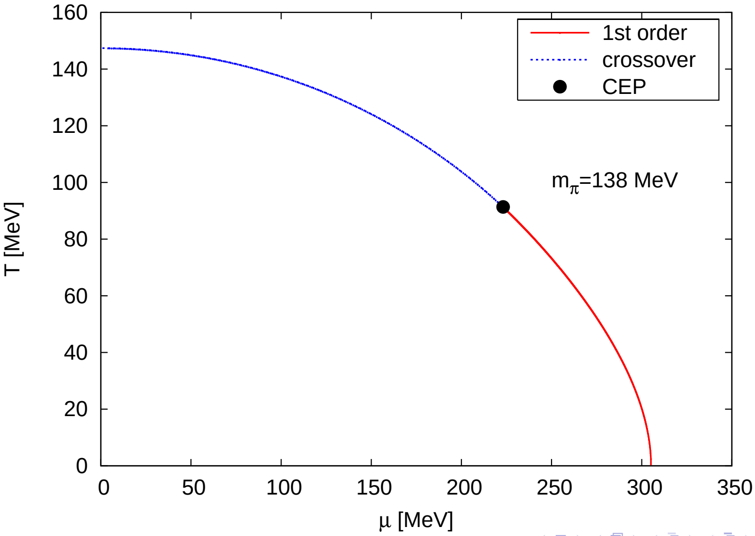
[Rischke et al.]

$$\Omega(T,\mu) = -\frac{T\ln\mathcal{Z}}{V} = \frac{\lambda}{4}(\langle\sigma\rangle^2 - v^2)^2 - c\langle\sigma\rangle + \Omega_{\bar{q}q}(T,\mu)$$

with

$$\Omega_{\bar{q}q}(T,\mu) = -2N_cN_fT\int\frac{d^3k}{(2\pi)^3}\left\{\ln(1 + e^{-(E_q-\mu)/T}) + \ln(1 + e^{-(E_q+\mu)/T})\right\}$$

Phase diagram in MF approximation



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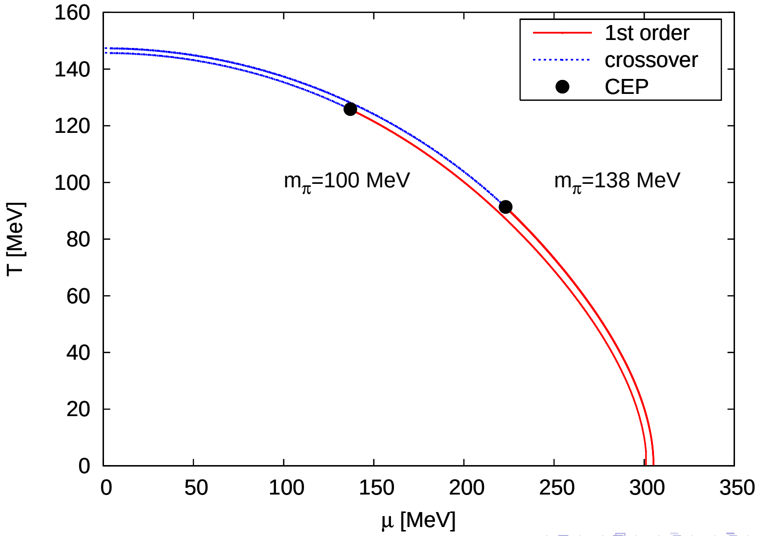

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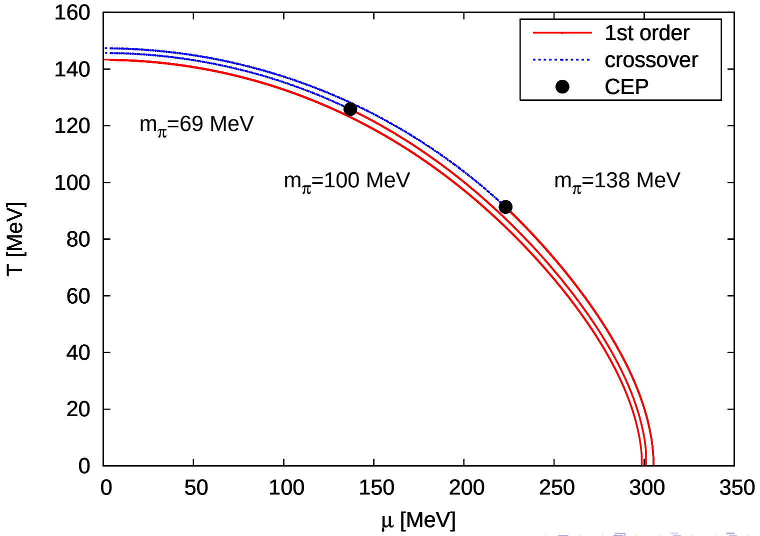
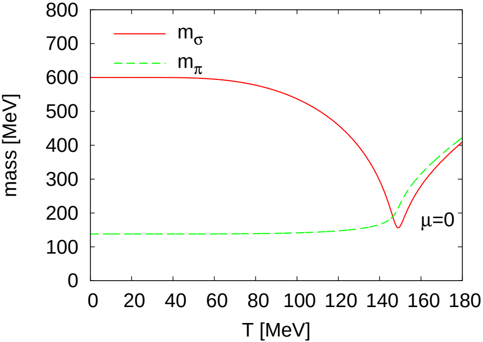
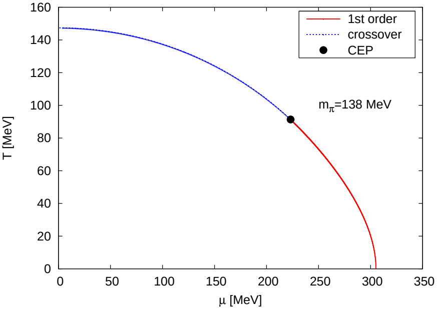


In-medium meson masses ($m_\pi = 138$ MeV)

CEP location: $T_c \sim 91$ MeV , $\mu_c \sim 223$ MeV

T-dependence





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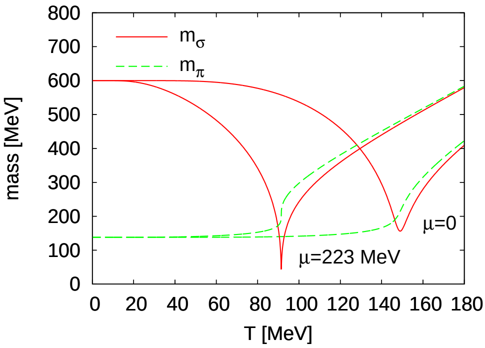

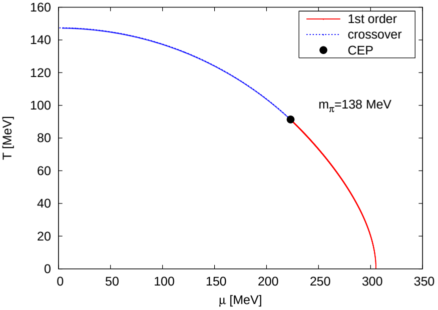

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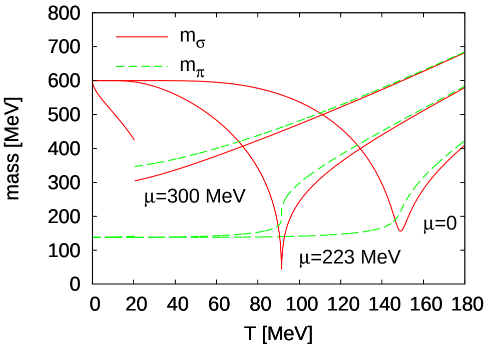

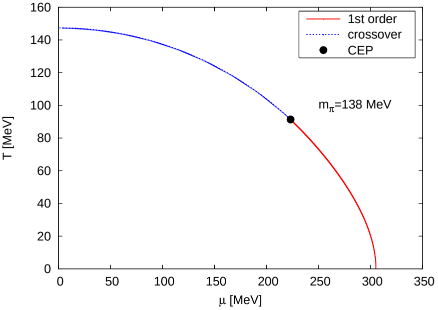

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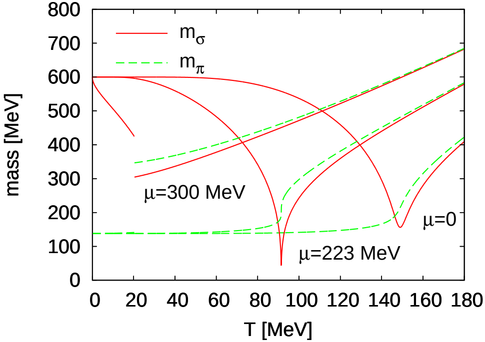
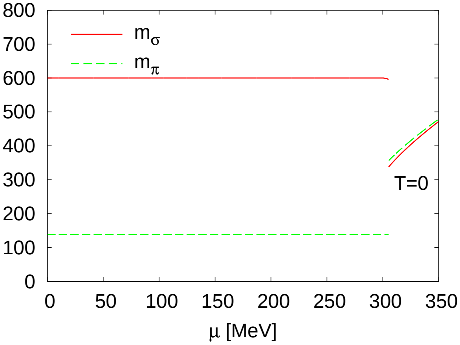


μ-dependence



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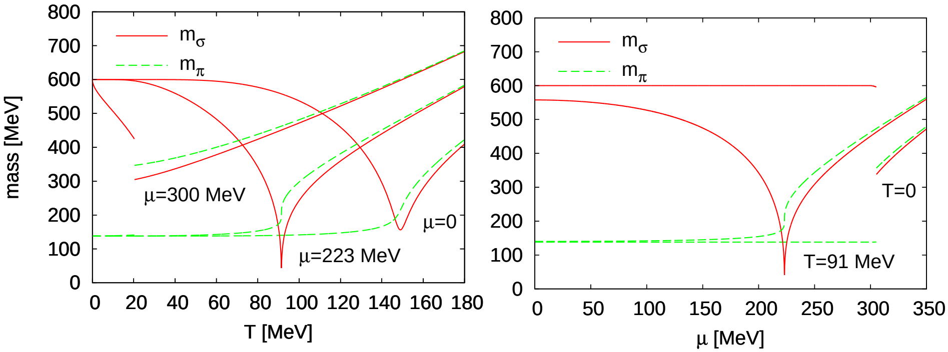
T-dependence μ-dependence



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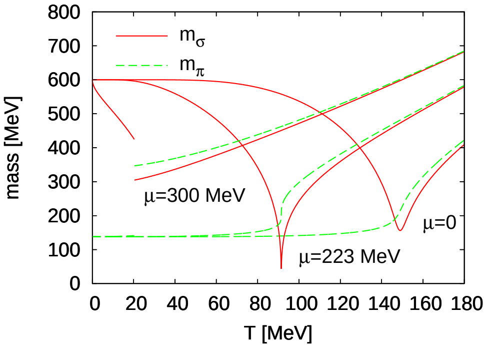
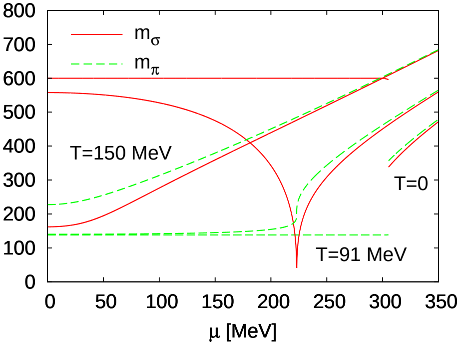


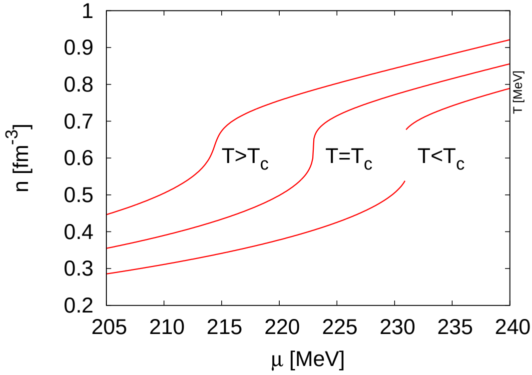
μ-dependence

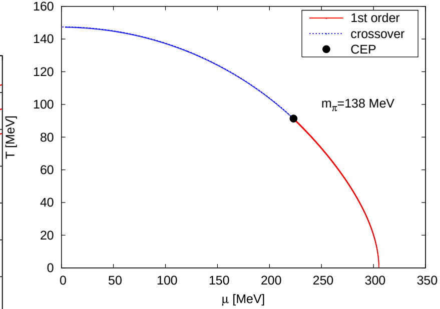


→ potential flattens in radial direction

Quark-number density $n_q(T,\mu)$ (MF)

CEP: $T_c \sim 91$ MeV
$T = T_c \pm 5$ MeV





$$n_q(T,\mu) = -\frac{\partial\Omega(T,\mu)}{\partial\mu}$$

$$\chi_q(T,\mu) = -\frac{\partial^2\Omega(T,\mu)}{(\partial\mu)^2}$$

Quark-number susceptibility $\chi_q(T, \mu)$ (MF)

CEP: $T_c \sim 91$ MeV
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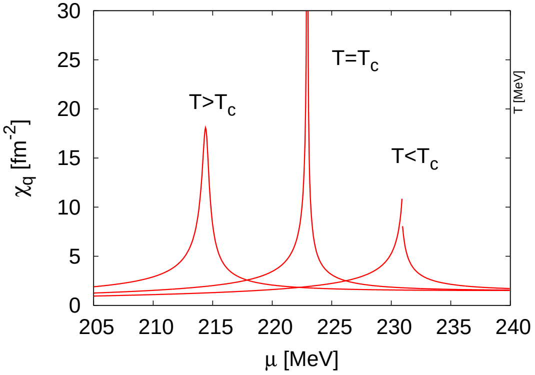
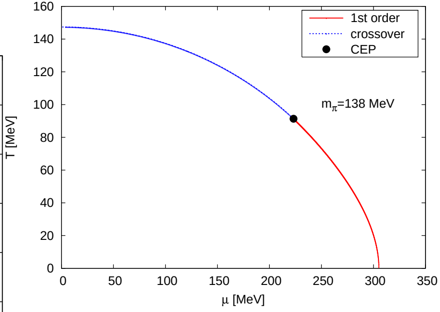




$$n_q(T, \mu) = -\frac{\partial \Omega(T, \mu)}{\partial \mu}$$

$$\chi_q(T, \mu) = -\frac{\partial^2 \Omega(T, \mu)}{(\partial \mu)^2}$$

Quark-number susceptibility $\chi_q(T,\mu)$ (MF)

$$\chi_q \sim |g - g_c|^{-\epsilon} \quad ; \quad g = T, \mu$$

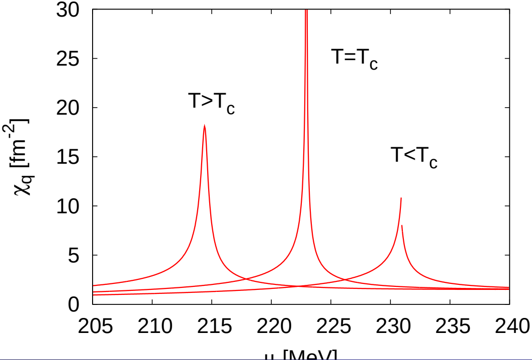
(isothermal) compressibility

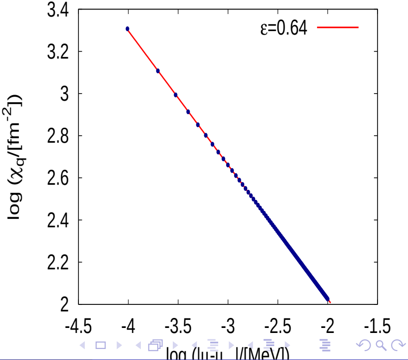
$$\kappa_T \equiv -\frac{1}{V}\left(\frac{\partial V}{\partial P}\right)\bigg|_{T,N} = \frac{\chi_q}{n_q^2}$$

if χ_q large $\rightarrow$ easy to compress

$\rightarrow$ interaction attractive
(or weakly repulsive)

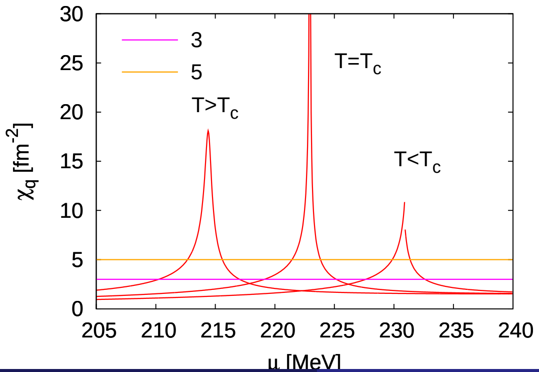
(cf discussion: Walecka model
if $m_\sigma \rightarrow 0$)
crit. exp. $\epsilon = 2/3$ (mean field)





Quark-number susceptibility $\chi_q(T,\mu)$ (MF)

- diverges only @ CEP
- finite everywhere else
- height decreases for decreasing μ towards T-axis
- For T below CEP: discontinuous $\rightarrow$ 1st order



ratio: $R_q := \chi_q/\chi_q^{free}$

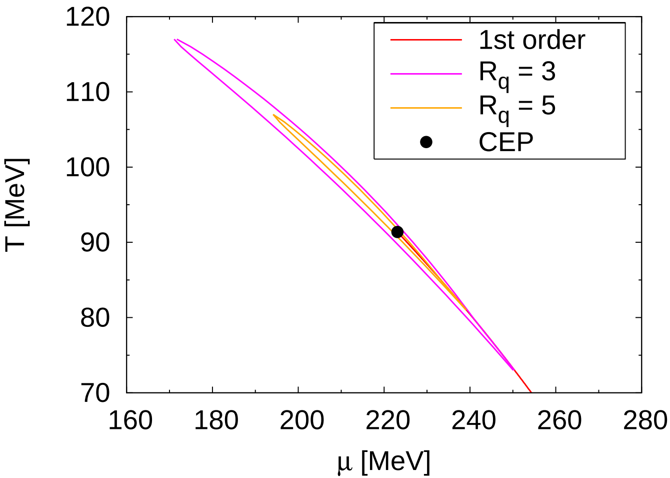
χ_q^{free}: massless free quark gas

$$\chi_q^{free}(T,\mu) = N_c N_f \left(\frac{\mu^2}{\pi^2} + \frac{T^2}{3}\right)$$

e.g. $R_q = 3$ or $R_q = 5$

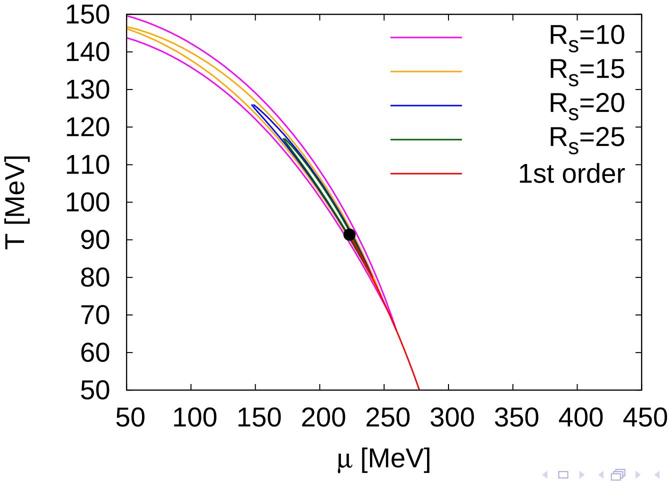
Critical region (MF)

$$R_q = \chi_q(T,\mu)/\chi_q^{free}(T,\mu)$$



Critical region w/ scalar susceptibility χ_σ (MF)

- $\chi_\sigma = 1/m_\sigma^2$: zero-momentum projection of scalar propagator
- encodes all fluctuations of order parameter
- define: $R_s = m_\sigma^2(0,0)/m_\sigma^2(T,\mu)$



Renormalization Group approaches

$\Gamma_k[\phi]$ scale dependent effective action ; $t = \ln(k/\Lambda)$; R_k : regulator

different realizations:

1. exact RG

ERG (averaged action) [Wetterich et al.]

$$\partial_t \Gamma_k[\phi] = \frac{1}{2} tr \left[\partial_t R_k \left(\frac{1}{\Gamma_k^{(2)} + R_k} \right) \right] \quad ; \quad \Gamma_k^{(2)} = \frac{\delta^2 \Gamma_k}{\delta\phi\delta\phi}$$

2. proper-time RG

PTRG [Liao et al.]

$$\partial_t \Gamma_k[\phi] = -\frac{1}{2} \int_0^\infty \frac{d\tau}{\tau} \left[\partial_t R_k(\Lambda^2 \tau) \right] tr \exp\left(-\tau \Gamma_k^{(2)} \right)$$

3. ...

RG flow equations

- Quark-Meson model

$$\Gamma_{k=\Lambda} = \int d^4x \left\{ \bar{q}[\partial\!\!\!/ + g(\sigma + i\vec{\tau}\vec{\pi}\gamma_5)]q + \frac{1}{2}(\partial_\mu\sigma)^2 + \frac{1}{2}(\partial_\mu\vec{\pi})^2 + V(\sigma^2 + \vec{\pi}^2) \right\}$$

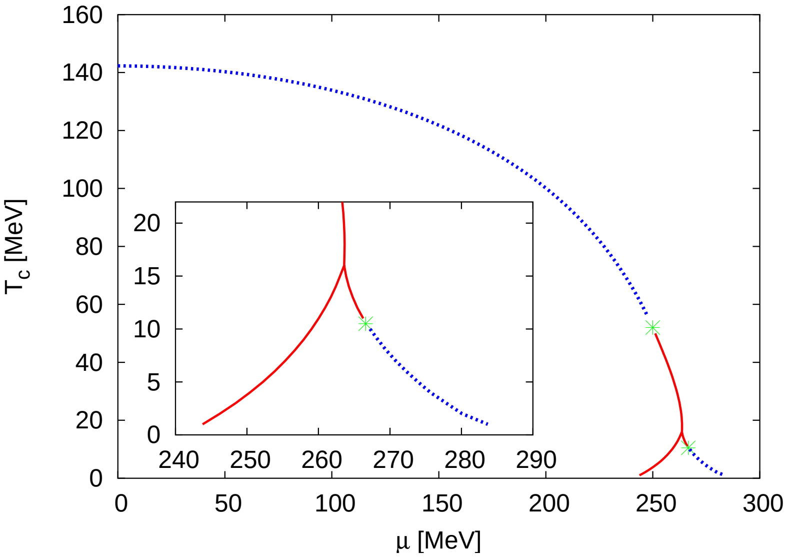
flow for grand canonical potential

$$\partial_t \Omega_k(T,\mu) = \frac{k^4}{12\pi^2} \left[\frac{3}{E_\pi} \coth\left(\frac{E_\pi}{2T}\right) + \frac{1}{E_\sigma} \coth\left(\frac{E_\sigma}{2T}\right) - \frac{2N_cN_f}{E_q} \left\{ \tanh\left(\frac{E_q-\mu}{2T}\right) + \tanh\left(\frac{E_q+\mu}{2T}\right) \right\} \right]$$

$$E_\pi^2 = 1 + 2\Omega_k'/k^2\,, \qquad E_\sigma^2 = 1 + 2\Omega_k'/k^2 + 4\phi^2\Omega_k''/k^2\,, \qquad E_q^2 = 1 + g^2\phi^2/k^2$$

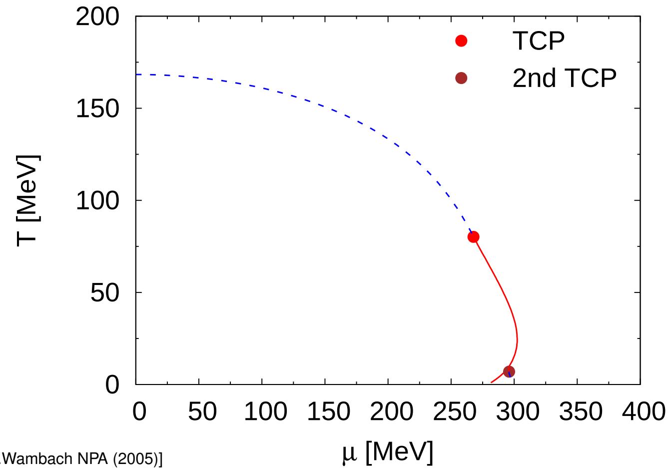
- $\phi \sim \langle \bar{q}q \rangle$
- quark fluctuations: chiral symmetry breaking
- meson fluctuations: chiral symmetry restoration

Phase diagram: (RG)



Phase diagram: $m_q \sim 370$ MeV (RG)

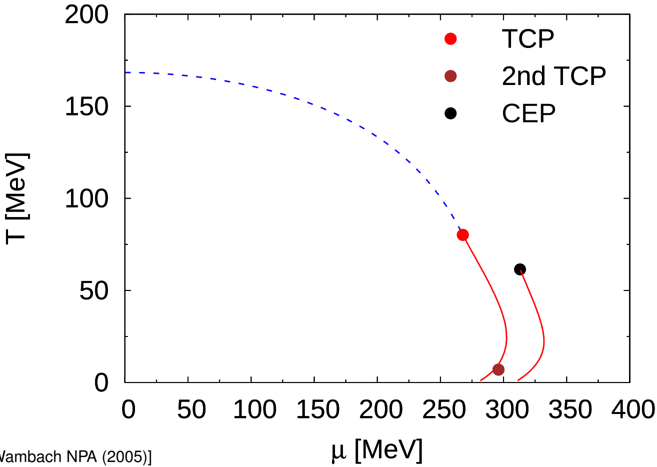
TCP: $T_c \sim 80.2$ MeV 2. 'TCP': $T_c \sim 8$ MeV



[BJS,J.Wambach NPA (2005)]

Phase diagram: $m_q \sim 370$ MeV (RG)

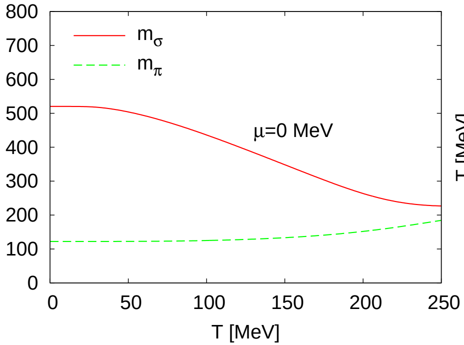
TCP: $T_c \sim 80.2$ MeV 2. 'TCP': $T_c \sim 8$ MeV CEP: $T_c \sim 61.5$ MeV

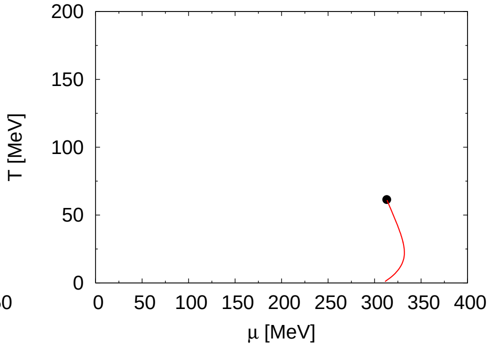


[BJS,J.Wambach NPA (2005)]

CEP:

$T_c \sim 61.5$ MeV, $\mu_c \sim 313$ MeV





In-medium meson masses (RG)

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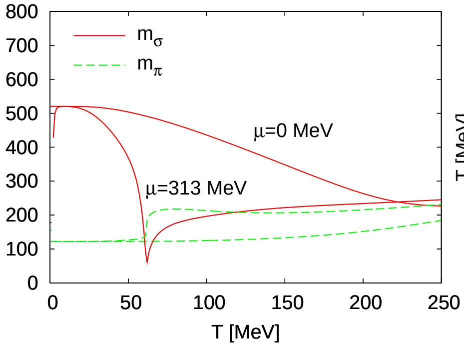

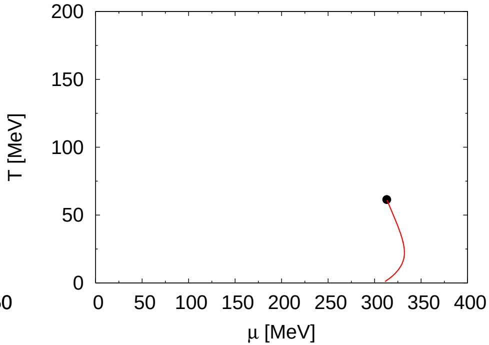

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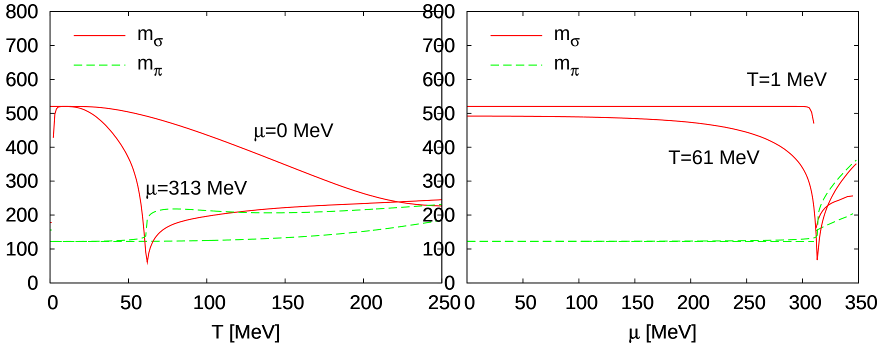

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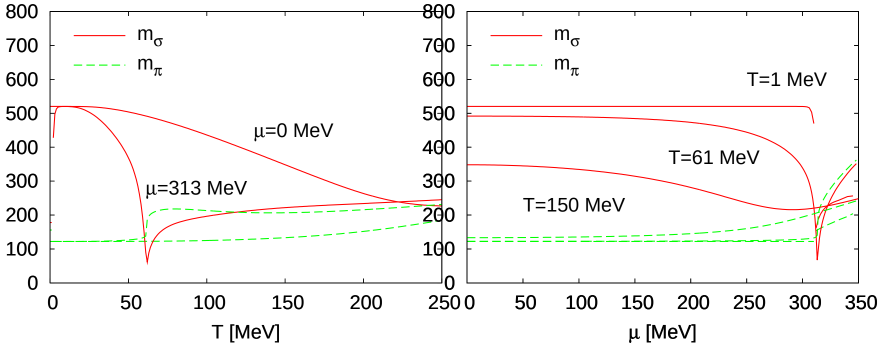

Quark-number density $n_q(T, \mu)$ (RG)

RG: tricritical point (TCP) and critical point (CEP)

TCP: $T_c \sim 80.2$ MeV
$T = T_c \pm 5$ MeV

CEP: $T_c \sim 61.5$ MeV
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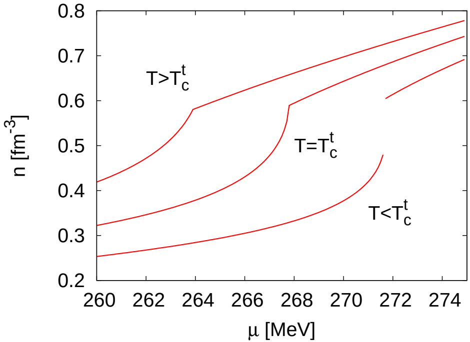

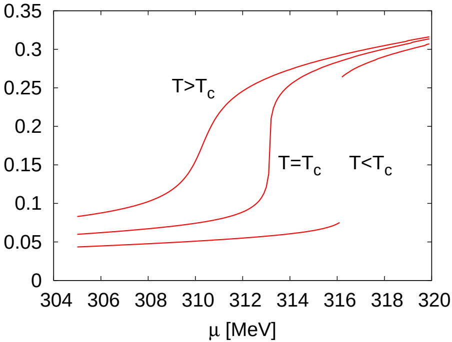

Quark-number susceptibility χ_q (RG)

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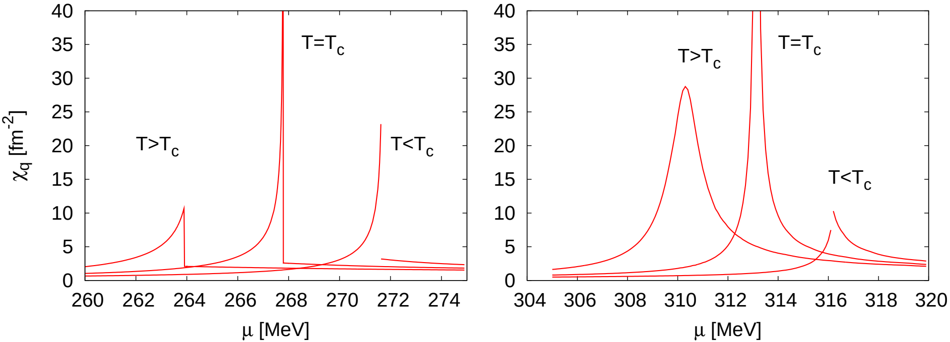


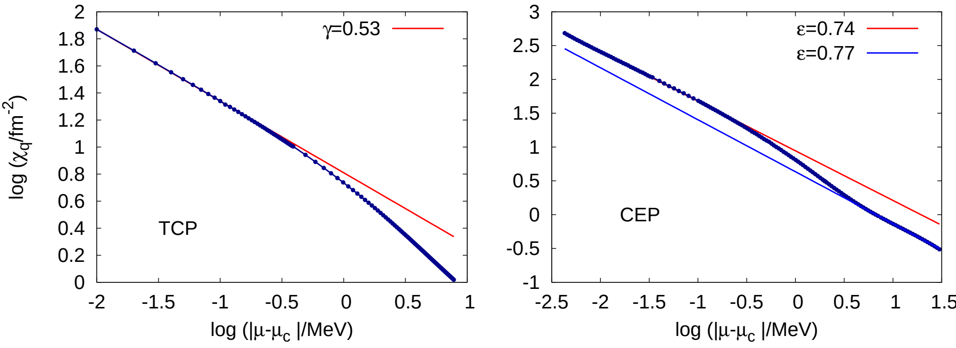
Critical exponents

$$\chi_q \sim |\mu - \mu_c|^{-\gamma}$$

TCP: $\gamma = 0.5$ (Gaussian)

CEP: MF: $\epsilon = 2/3$

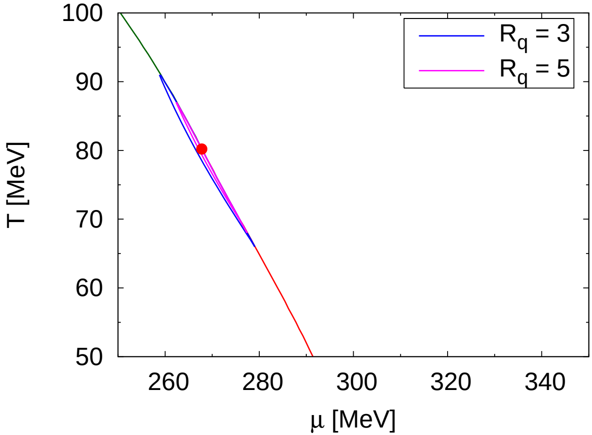
3D Ising: $\epsilon = 0.78$



MF: tricritical exponents different from bicritical exponents

Critical region with quark number susceptibility χ_q

- Size of crit. region shrinks as $m_q \to 0$ ($\tau_{GL} \sim b^2$) tricritical $b \to 0$



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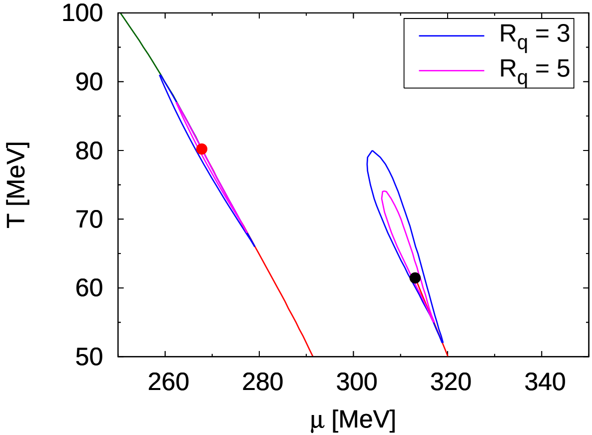
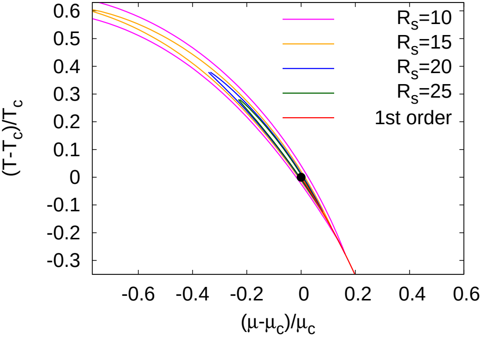


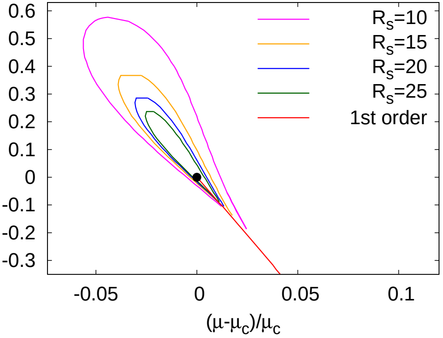
Comparison with scalar χ_σ : MF↔RG

$$R_s = m_\sigma^2(0,0)/m_\sigma^2(T,\mu)$$

MF

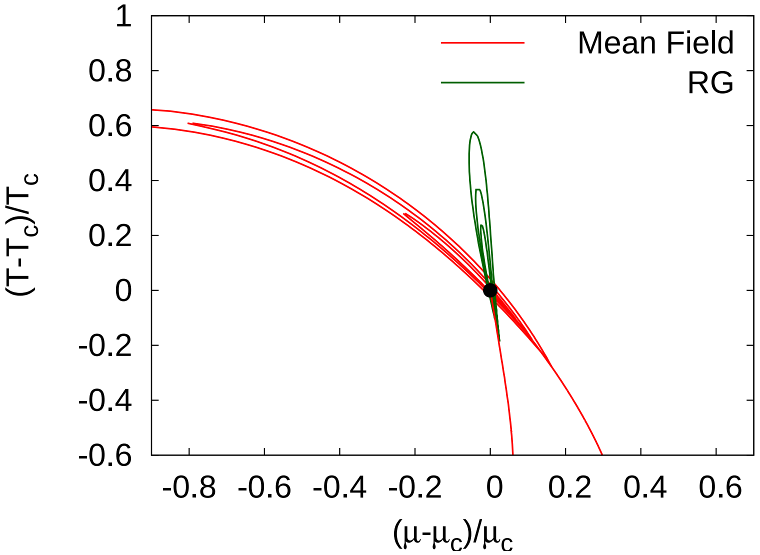


RG



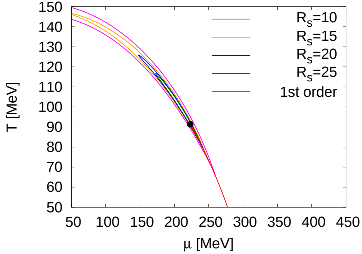
critical region w/ RG more compressed

Comparison with scalar χ_σ : MF↔RG

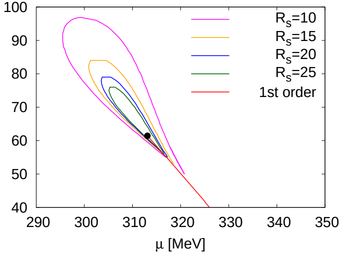


Summary

Mean-field



RG approach



- MF: only CEP found RG: TCP (as expected) and CEP found
- Size of critical region via $\chi_q(T,\mu)$ and $\chi_\sigma(T,\mu)$

  $\longrightarrow$ “compressed” w/ fluctuations (cf. Figs)
- RG calc.: critical exponents consistent w/ *3d* Ising universality class @ CEP
- novel crossover phenomenon

Polyakov Quark Meson Model (preliminary)

J. Pawlowski, BJS, J. Wambach