Phases of QCD

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Can results of

Lattice QCD Thermodynamics

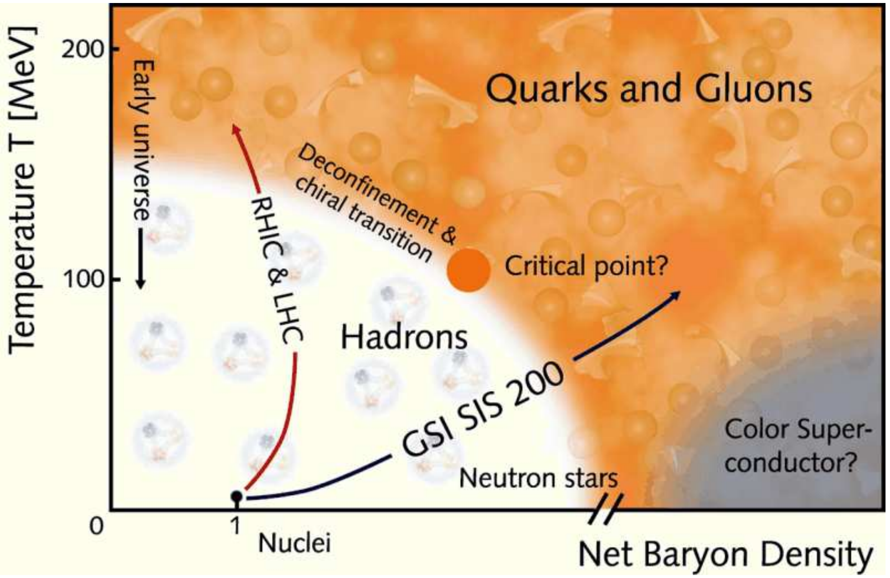
be understood in terms of

QUASIPARTICLE

degrees of freedom?

$N_f = 2$ $N_f = 2+1$ $N_f = 3$

Introduction



- QCD has a rich phase structure
- Many challenging items:
  - order of the phase transition
  - critical point
  - deconfinement and chiral symmetry
  - colour superconductivity at high μ

- Status of lattice QCD thermodynamics:
  - precise data available in the pure gauge sector
  - quarks easily introduced at $\mu = 0$
  - first lattice data at finite (small) μ (F. Karsch, Z. Fodor, S. Katz, P. de Forcrand, O. Philipsen, M. D'Elia, M. P. Lombardo).

PNJL (Polyakov loop extended NJL) model

Starting point: three-flavour NJL model in temporal background gauge field

NJL four-fermion interaction

$$\mathcal{L}_{PNJL} = \bar{\psi}\left(i\gamma_\mu D^\mu - \hat{m}_0\right)\psi + \frac{G}{2}\sum_{f=u,d,s}\left[\left(\bar{\psi}_f\psi_f\right)^2 + \left(\bar{\psi}_f i\gamma_5\vec{\lambda}\psi_f\right)^2\right]$$
$$-\frac{K}{2}\left[\det_{i,j}\left(\bar{\psi}_i(1+\gamma_5)\psi_j\right) + \det_{i,j}\left(\bar{\psi}_i(1-\gamma_5)\psi_j\right)\right] - V\left(\Phi, T\right),$$

NJL six-fermion interaction

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$\hat{m}_0 = diag\,\{m_u, m_u, m_s\}$

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$$-\frac{K}{2}\left[\det_{i,j}\left(\bar{\psi}_i(1+\gamma_5)\psi_j\right) + \det_{i,j}\left(\bar{\psi}_i(1-\gamma_5)\psi_j\right)\right] - V\left(\Phi, T\right),$$

NJL six-fermion interaction

Polyakov loop phenomenological potential

Coupling between Polyakov loop and quarks uniquely determined by covariant derivative D_μ.
We recall that:

$$\Phi = \frac{1}{N_c}\mathrm{Tr}\left[\mathcal{P}\exp\left(i\int_0^\beta A_4 d\tau\right)\right], \qquad A^0 = -iA_4.$$

Parameters: m_u, m_s, G, K, Λ fixed in the hadronic sector.

$$N_f = 2$$

Parameters

Λ [GeV]	0.651
G [GeV^{-2}]	10.078
m_u [MeV]	5.5

Physical quantities

f_π [MeV]	92.4
$\lvert\langle\bar{\psi}\psi\rangle\rvert^{1/3}$ [MeV]	247
m_π [MeV]	139.3

$$N_f = 2 + 1$$

Parameters

Λ [GeV]	0.6023
$G\Lambda^2$	3.67
$K\Lambda^5$	24.72
m_u [MeV]	5.5
m_s [MeV]	140.7

Physical quantities

f_π [MeV]	92.4
$\lvert\langle\bar{\psi}\psi\rangle_{u,d}\rvert^{1/3}$ [MeV]	241.9
$\lvert\langle\bar{\psi}\psi\rangle_s\rvert^{1/3}$ [MeV]	257.7
m_π [MeV]	139.3
m_K [MeV]	497.7

R. Pisarski (2000)

Polyakov loop potential

- The Polyakov loop is the order parameter related to the $Z(N_c)$ symmetry

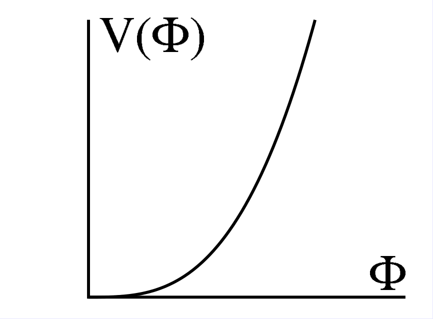
$$\frac{V(\Phi,T)}{T^4} = -\frac{b_2(T)}{2}\Phi^*\Phi - \frac{b_3}{6}\left(\Phi^3 + (\Phi^*)^3\right) + \frac{b_4}{4}(\Phi^*\Phi)^2$$

with

$$b_2(T) = a_0 + a_1\left(\frac{T_0}{T}\right) + a_2\left(\frac{T_0}{T}\right)^2 + a_3\left(\frac{T_0}{T}\right)^3$$

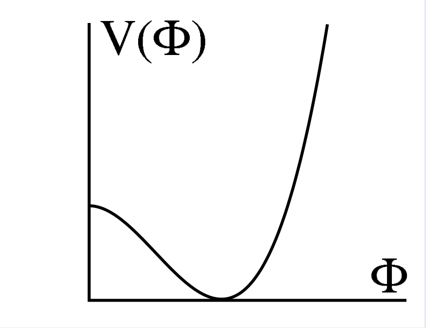
$T < T_c$

- color confinement
- $\langle\Phi\rangle = 0 \longrightarrow Z(3)$ unbroken



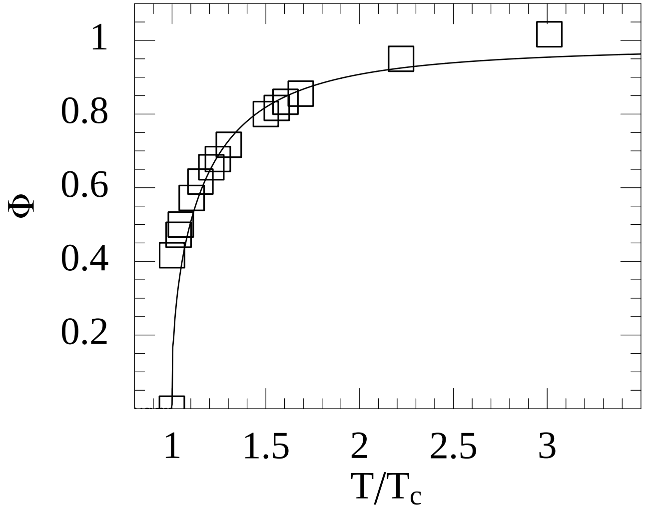
$T > T_c$

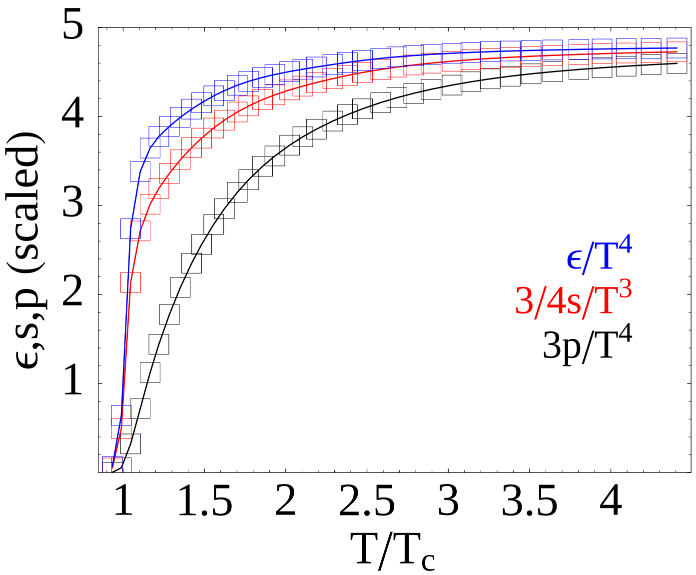
- color deconfinement
- $\langle\Phi\rangle \neq 0 \longrightarrow Z(3)$ broken



Fit of Pure Gauge QCD lattice data

- Minimization of $V(\Phi, T)$: Polyakov loop behaviour as a function of T
- Comparison with lattice data from Kaczmarek *et al.* PLB 543 (2002)





- $p(T) = -V(\Phi(T), T)$
- $s(T) = \frac{dp}{dT} = -\frac{dV(\Phi(T),T)}{dT}$
- $\epsilon(T) = T\frac{dp}{dT} - p = Ts(T) - p(T)$
- Comparison with lattice data from Boyd *et al.* NPB 469 (1996)

PNJL model at finite temperature and chemical potential

The thermodynamic potential of the system is:

$$\Omega\left(T,\mu\right)=V\left(\Phi,T\right)+\frac{\sigma_{u,d}^2}{2G}+\frac{\sigma_s^2}{4G}-\frac{K}{4G^3}\sigma_{u,d}^2\sigma_s-2\sum_f T\int\frac{\mathrm{d}^3p}{(2\pi)^3}\left\{\mathrm{Tr}_c\ln\left[1+L\,\mathrm{e}^{-\left(E_{p,f}-\mu_f\right)/T}\right]\right.$$

$$\left.+\,\mathrm{Tr}_c\ln\left[1+L^\dagger\,\mathrm{e}^{-\left(E_{p,f}+\mu_f\right)/T}\right]+3\frac{E_{p,f}}{T}\theta\left(\Lambda^2-\vec{p}^{\;2}\right)\right\}.$$

with $E_{p,f}=\sqrt{p^2+M_f^2}$ and $\mathrm{Tr}_c L=\Phi$, $\mathrm{Tr}_c L^\dagger=\Phi^*$.

Interaction with chiral condensate: quarks develop a constituent mass:

$$M_i=m_i-\langle\sigma_i\rangle-\frac{K}{4G^2}\langle\sigma_j\rangle\langle\sigma_k\rangle=m_i-2G\langle\bar{\psi}_i\psi_i\rangle+K\langle\bar{\psi}_j\psi_j\rangle\langle\bar{\psi}_k\psi_k\rangle\qquad i\neq j\neq k$$

Minimization of Ω: behaviour of

- Quark masses (chiral condensates)
- Polyakov loop

as functions of

- temperature
- quark chemical potential.

Final form for Ω

1-quark (antiquark) states, suppressed below T_c

2-quark (antiquark) states, suppressed below T_c

$$\Omega\left(T,\mu,\sigma,\Phi\right) = V\left(\Phi,T\right) + \frac{\sigma_{u,d}^2}{2G} + \frac{\sigma_s^2}{4G} - \frac{K}{4G^3}\sigma_{u,d}^2\sigma_s$$

$$-2\sum_f \int \frac{\mathrm{d}^3p}{(2\pi)^3}\left\{3E_p + T\left[\ln\left[1+3\Phi \mathrm{e}^{-(E_p-\mu)/T} + 3\Phi^* \mathrm{e}^{-2(E_p-\mu)/T} + \mathrm{e}^{-3(E_p-\mu)/T}\right]\right.\right.$$

$$\left.\left. + \ln\left[1+3\Phi^* \mathrm{e}^{-(E_p+\mu)/T} + 3\Phi \mathrm{e}^{-2(E_p+\mu)/T} + \mathrm{e}^{-3(E_p+\mu)/T}\right]\right]\right\}$$

3-quark (antiquark) states, not suppressed even below T_c

High temperature limit: $\Phi \longrightarrow 1$, $\Phi^* \longrightarrow 1$
We re-obtain the standard NJL formula:

$$\ln\left[1+3\Phi \mathrm{e}^{-(E_p-\mu)/T} + 3\Phi^* \mathrm{e}^{-2(E_p-\mu)/T} + \mathrm{e}^{-3(E_p-\mu)/T}\right]$$

$$\downarrow \quad T \longrightarrow \infty$$

$$\ln\left[1+\mathrm{e}^{-(E_p-\mu)/T}\right]^3 = 3\ln\left[1+\mathrm{e}^{-(E_p-\mu)/T}\right]$$

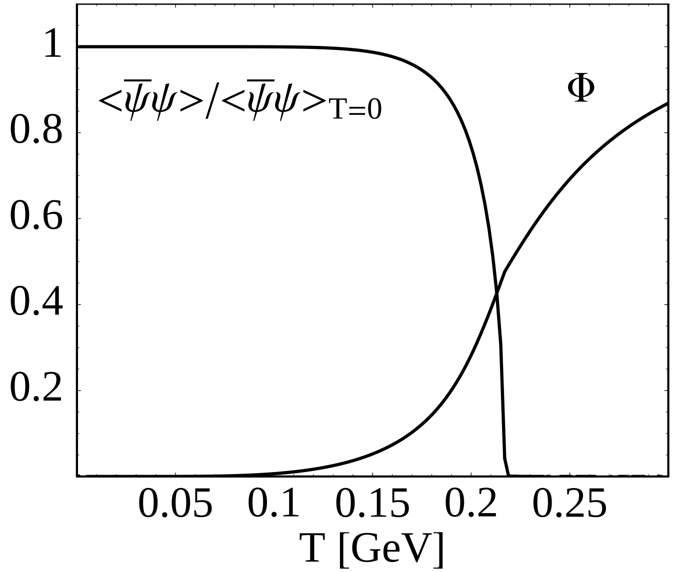
$$N_f = 2 \Leftrightarrow m_s = \infty$$

Lattice vs. PNJL model

Chiral and Deconfinement transitions

Zero and finite chemical potential

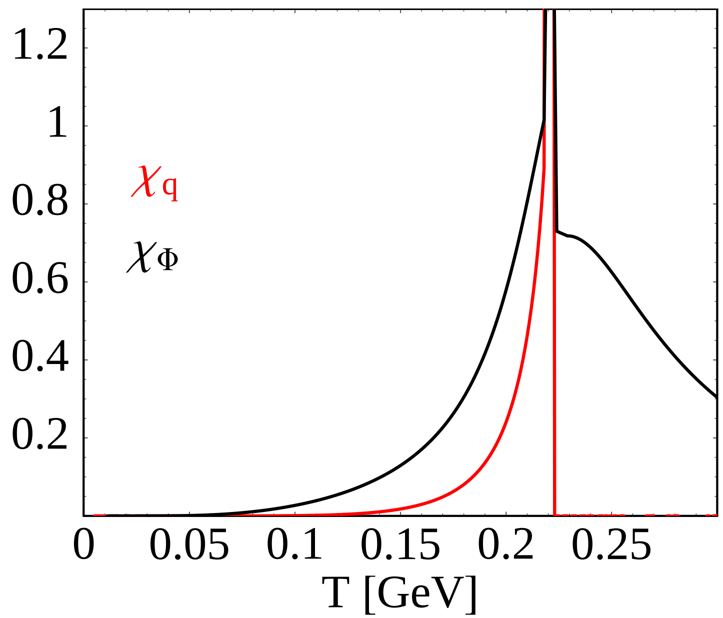
Confinement and chiral symmetry breaking: chiral limit ($m_u = 0$)



$T_c \simeq 270$ MeV in pure gauge

$\Downarrow$

$T_c \simeq 210$ MeV with quarks

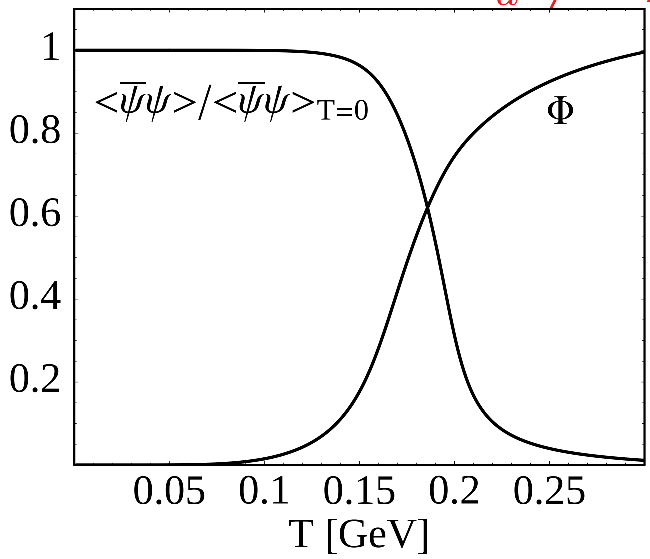


CHIRAL and DECONFINEMENT

transitions coincide in the

CHIRAL LIMIT!

$m_u \neq 0$, $\mu = 0$ predictions



$T_c \simeq 210$ MeV

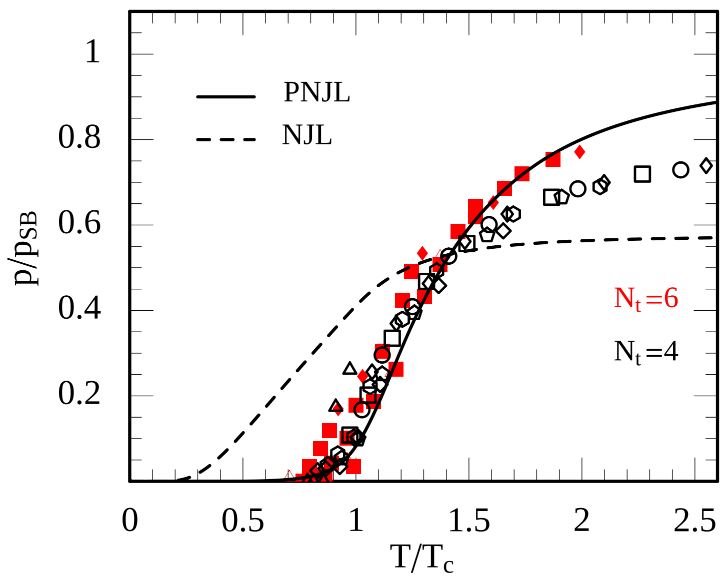
$\Downarrow$

$T_c \simeq 170$ MeV

- Scaled pressure as a function of T/T_c

$$\frac{p\,(T, \mu = 0)}{T^4} = -\frac{\Omega\,(T, \mu = 0)}{T^4}$$

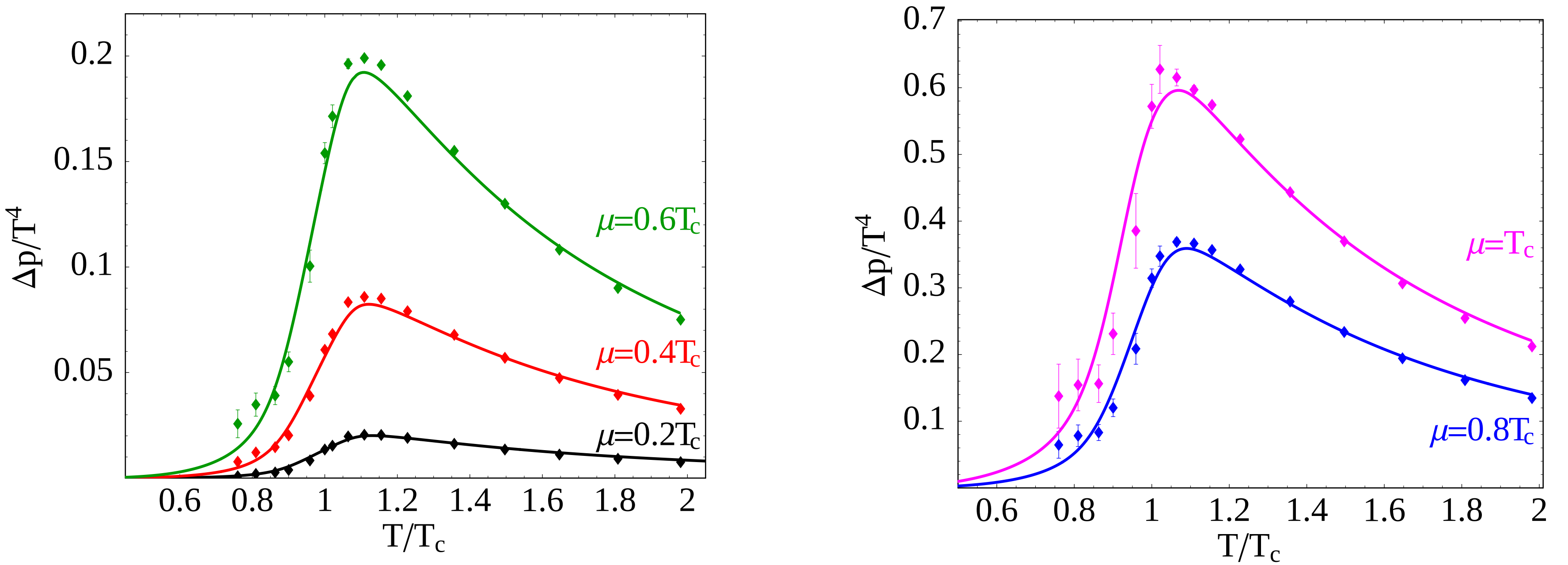
- Comparison with lattice data from CP-PACS collaboration (2001)



Finite μ predictions: pressure difference

- Scaled pressure difference as a function of T/T_c

$$\frac{\Delta p\,(T,\mu)}{T^4} = \frac{p\,(T,\mu) - p\,(T,\mu=0)}{T^4}$$

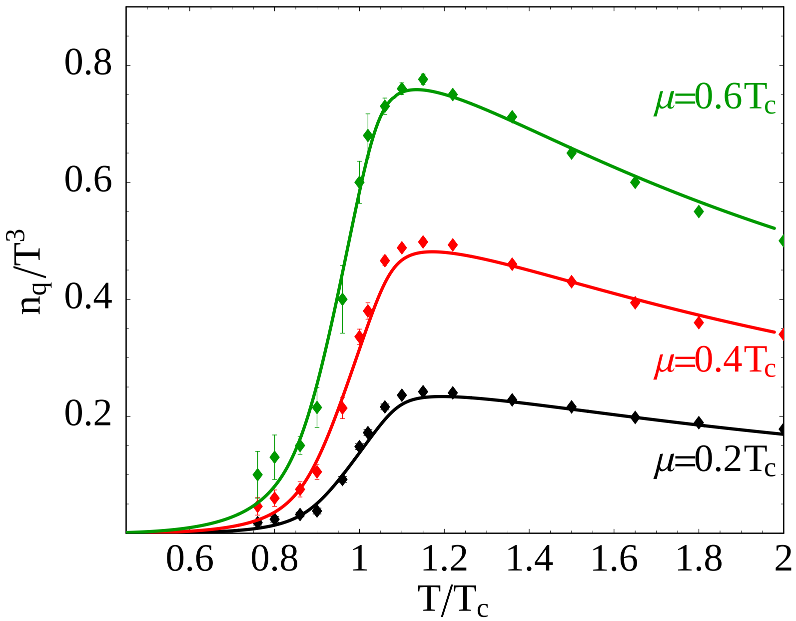


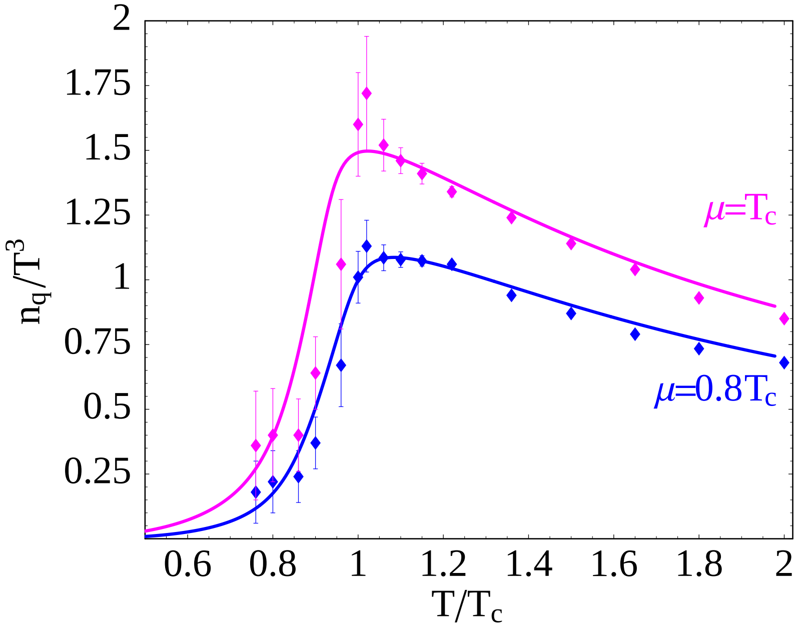
Scaled pressure difference as a function of the temperature at different values of the chemical potential, compared to the corresponding lattice data taken from Allton *et al.*, PRD 68 (2003). PNJL model results from: C.R., M. A. Thaler and W. Weise, PRD 73 (2006).

Finite μ predictions: quark number density

- Scaled quark number density as a function of T/T_c

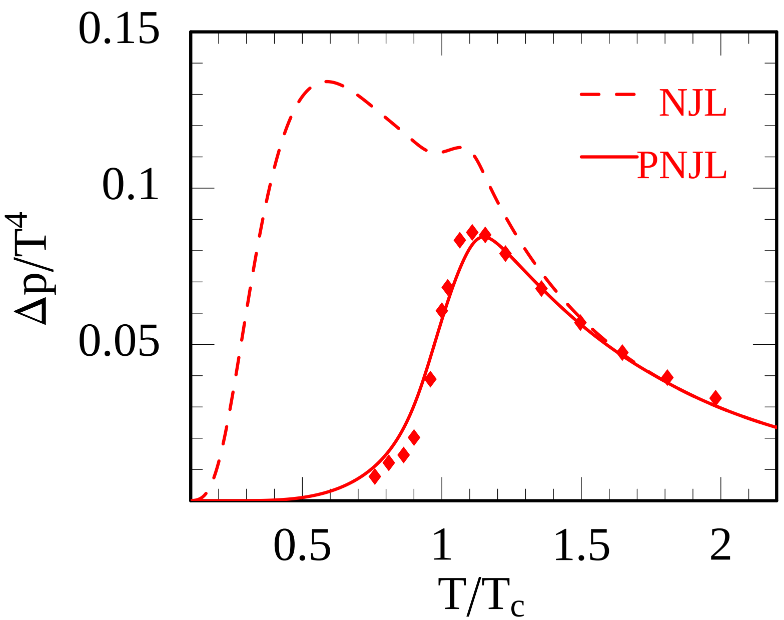
$$\frac{n_q(T,\mu)}{T^3} = -\frac{1}{T^3}\frac{\partial\Omega(T,\mu)}{\partial\mu}$$

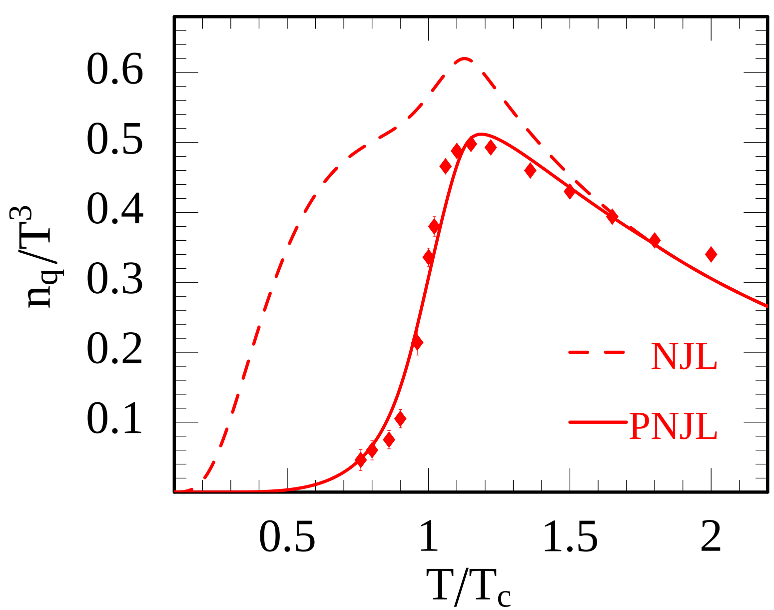




Scaled quark number density as a function of the temperature at different values of the chemical potential, compared to the corresponding lattice data taken from Allton *et al.*, PRD 68 (2003). PNJL model results from: C.R., M. A. Thaler and W. Weise, PRD 73 (2006).

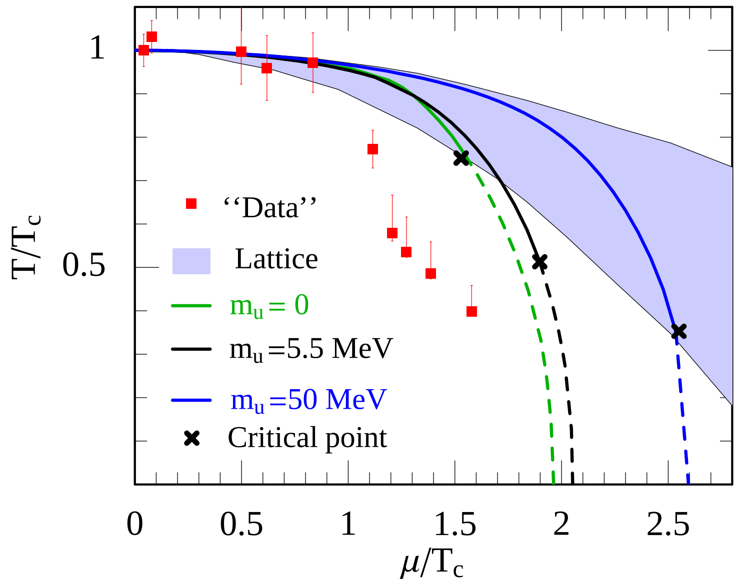
Finite μ results in the standard NJL model





- Comparison between PNJL and standard NJL model results at finite μ
- NJL model alone fails in reproducing lattice data
- Incorporation of confinement crucial to reproduce lattice results

Towards the PHASE DIAGRAM ($N_f = 2$)



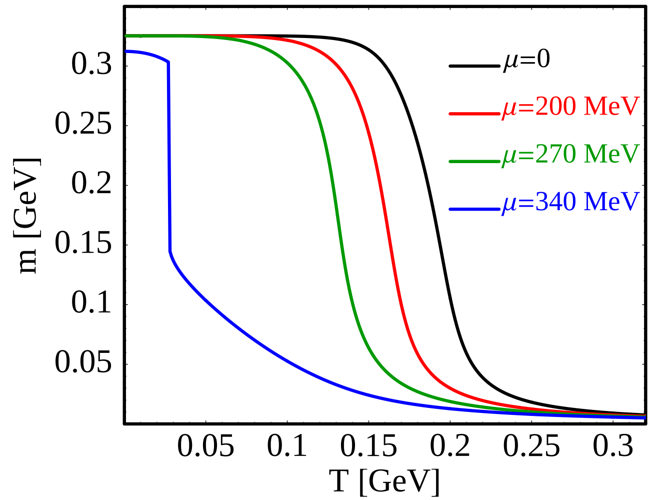
Therm. fit data from Andronic *et al.* (2005)
Lattice data from Allton *et al.* (2002)

First order transition

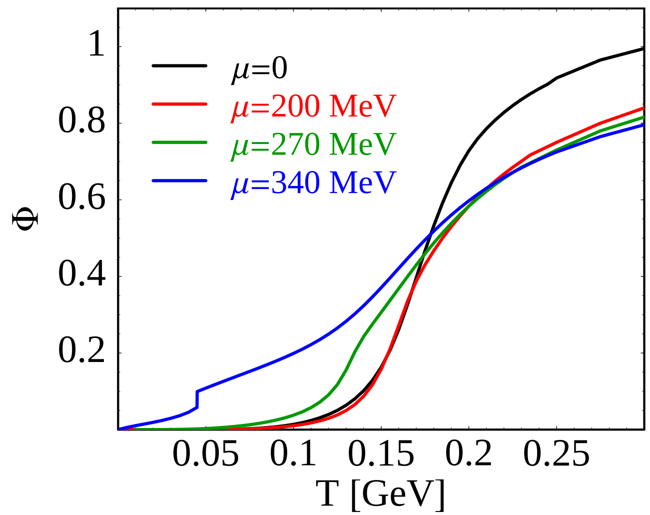
at large chemical potential!

C. R., S. Rößner, W. Weise, in preparation

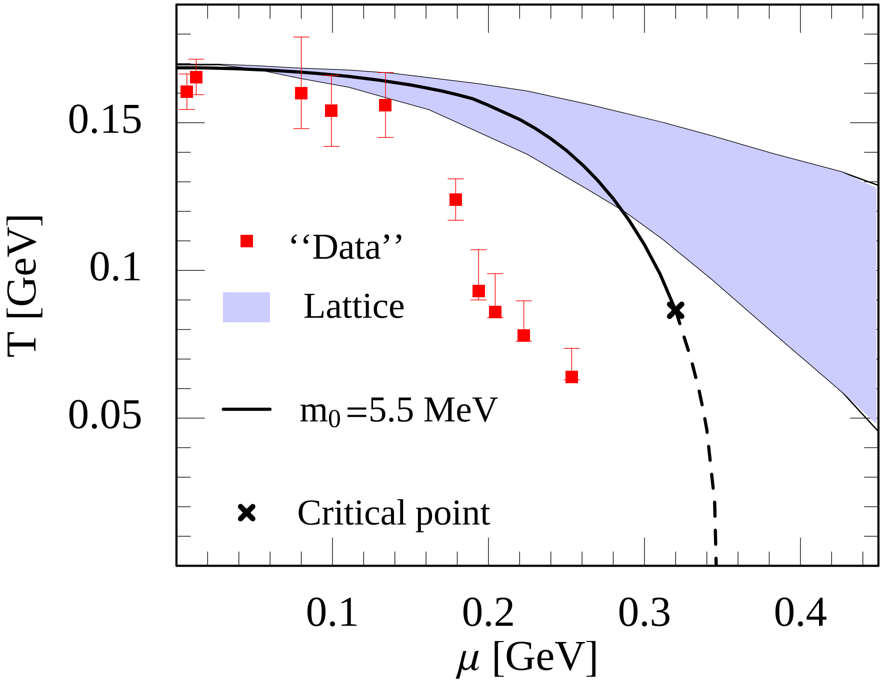
Constituent quark mass

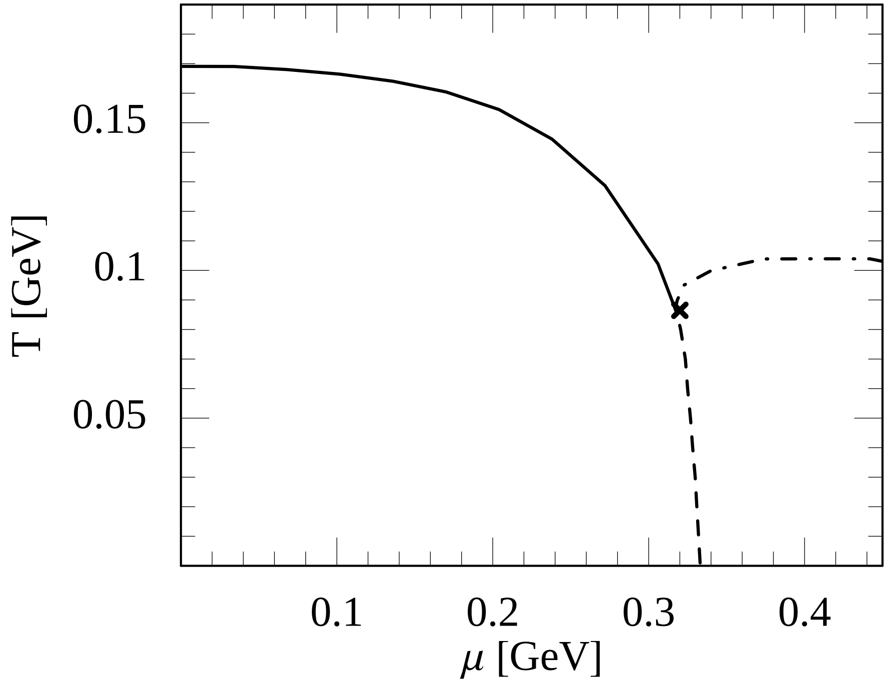


Polyakov loop



Phase diagram at large μ (preliminary)





- Inclusion of diquark condensation does not seem to affect the critical point position

$$N_f = 2 + 1 \Leftrightarrow m_s \neq m_u$$

Lattice vs. PNJL model

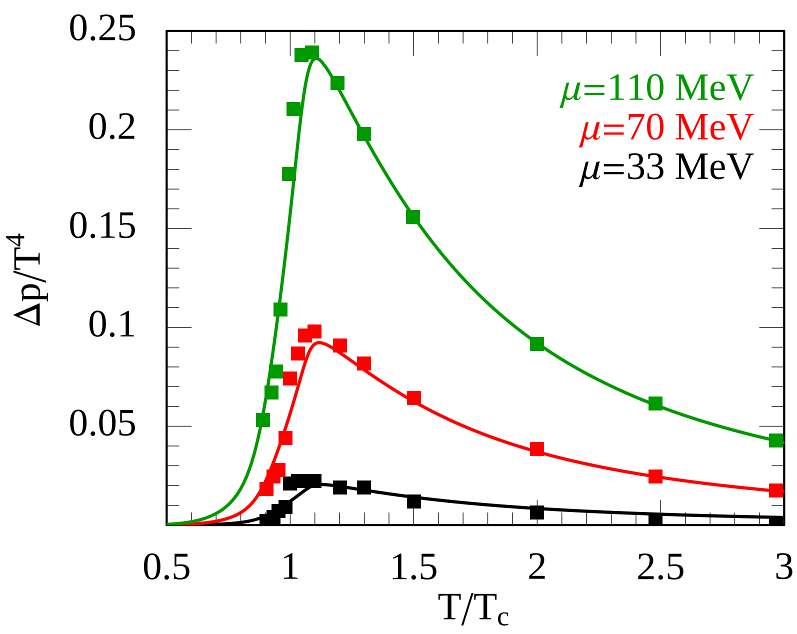
Finite chemical potential

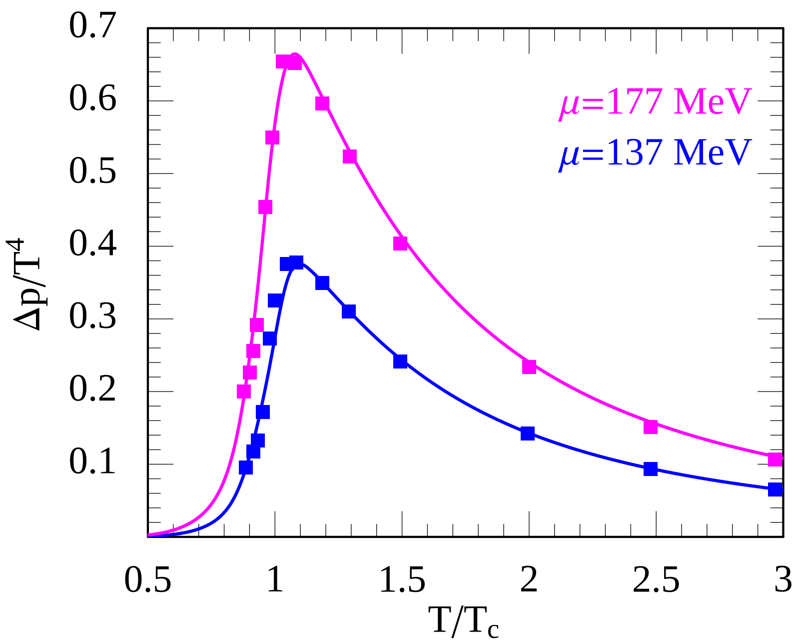
C. R., M. A. Thaler and W. Weise, in preparation

Finite μ predictions: pressure difference ($\mu_s = 0$)

- Scaled pressure difference as a function of T/T_c

$$\frac{\Delta p\,(T,\mu)}{T^4} = \frac{p\,(T,\mu) - p\,(T,\mu=0)}{T^4}$$



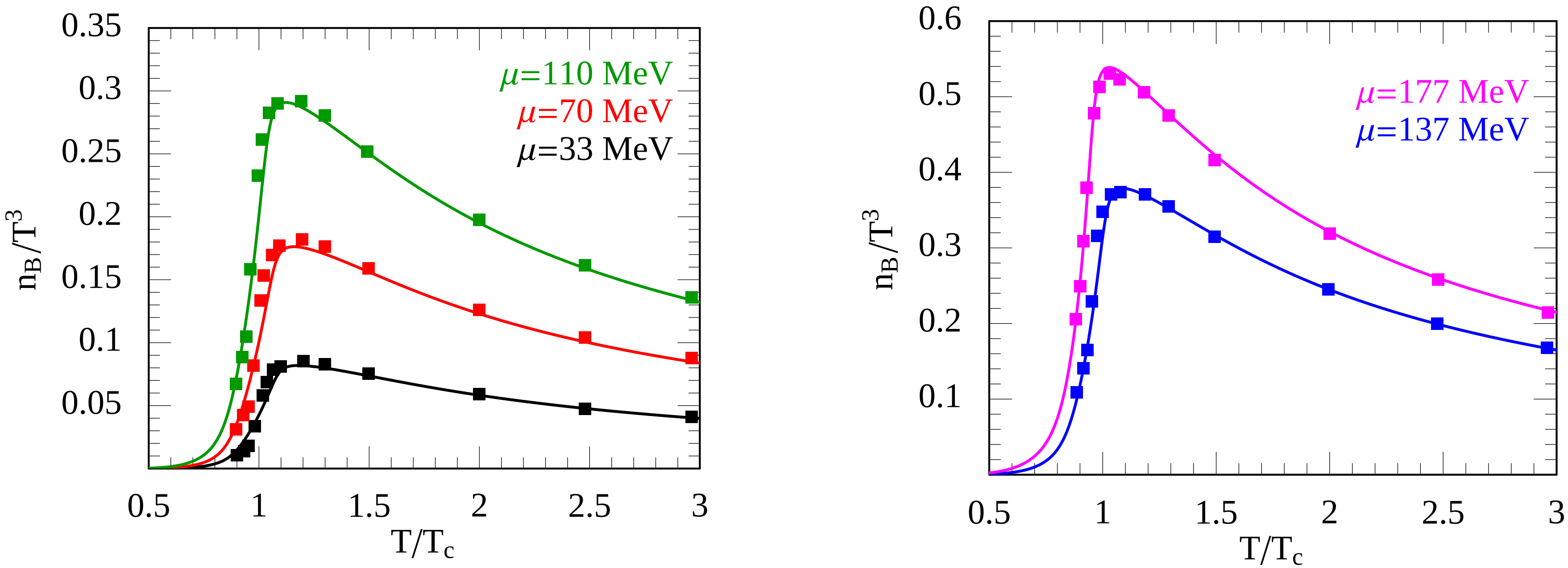


Scaled pressure difference as a function of the temperature at different values of the chemical potential, compared to the corresponding lattice data taken from Z. Fodor *et al.*, PLB 568 (2003).

Finite μ predictions: quark number density ($\mu_s = 0$)

- Scaled quark number density as a function of T/T_c

$$\frac{n_B\left(T,\mu\right)}{T^3} = -\frac{1}{3}\frac{1}{T^3}\sum_f \frac{\partial\Omega\left(T,\mu_f\right)}{\partial\mu_f}$$



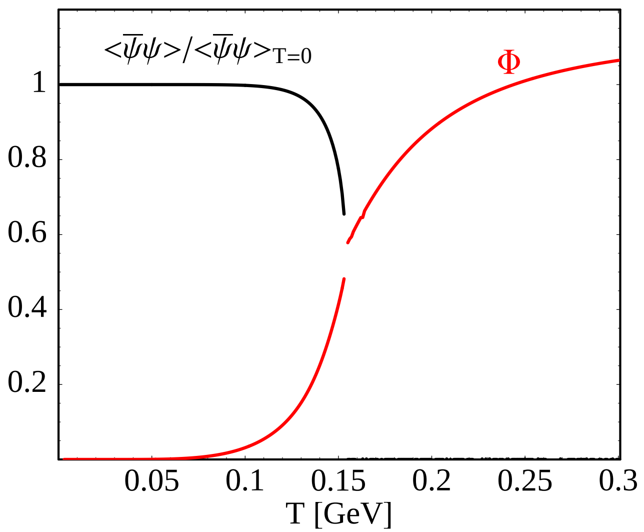
Scaled quark number density as a function of the temperature at different values of the chemical potential, compared to the corresponding lattice data taken from Z. Fodor *et al.*, PLB 568 (2003).

$$N_f = 3 \Leftrightarrow m_s = m_u = 0$$

Phase diagram

Influence of ’t Hooft interaction

C. R., S. Rößner, M. A. Thaler and W. Weise, in preparation

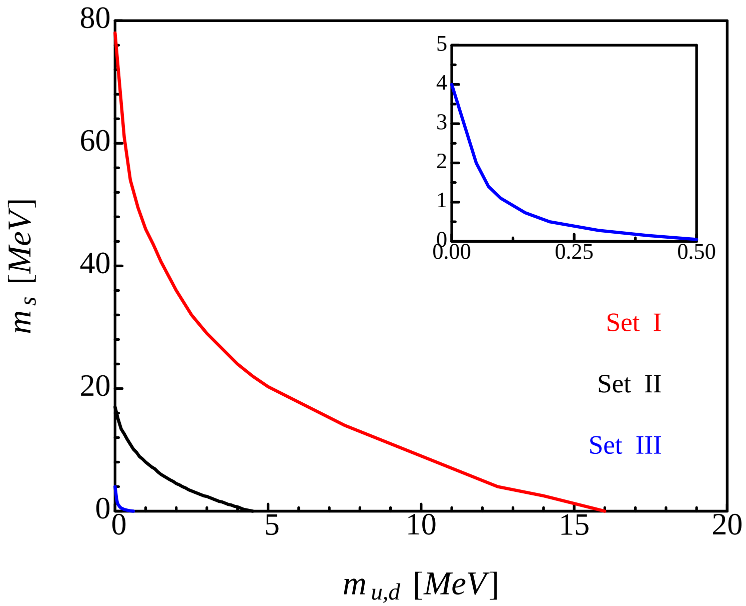


First order phase transition

for three massless flavours

at $\mu = 0$

	Λ[GeV]	$G\Lambda^2$	$K\Lambda^5$
set I	0.6023	3.072	40.00
set II	0.6023	3.67	24.72
set III	0.6023	4.2426	10.00



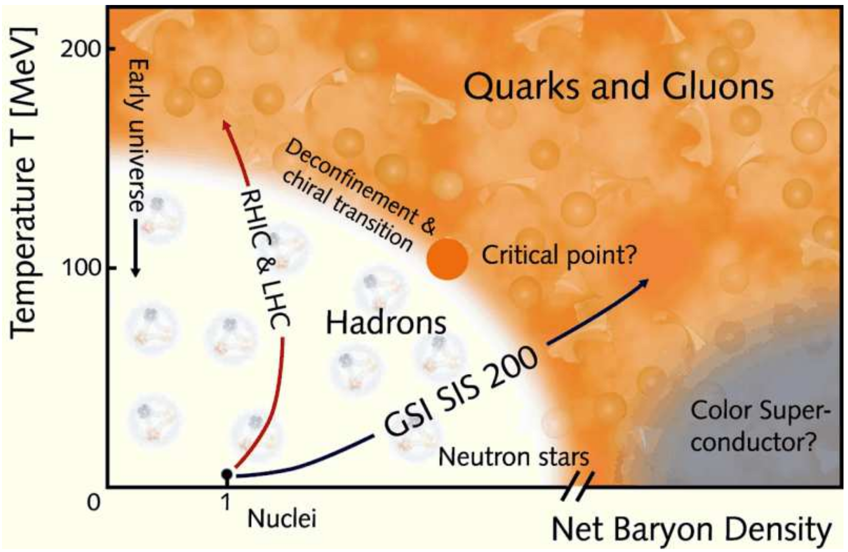
't Hooft six-fermion interaction responsible

for first-order phase transition!

Conclusions

- The standard NJL model fails in reproducing QCD thermodynamics
- PNJL model as a minimal synthesis of confinement and chiral symmetry breaking
- A description of QCD thermodynamics with our simple model works very well
- $N_f = 2$
  - → $\mu = 0$
  - → $\mu \neq 0$
- $N_f = 2 + 1$
- $N_f = 3$

Outlook



- Exploration of the phase diagram and its quark mass dependence for $N_f = 2+1$ and $N_f = 3$
- Improvement of Polyakov loop potential
- Improvement of approximation: going beyond mean field approximation

Improving the Polyakov-loop potential

- Present Polyakov loop potential:

$$\frac{V(\Phi,T)}{T^4} = -\frac{b_2(T)}{2}\Phi^*\Phi - \frac{b_3}{6}\left(\Phi^3+\Phi^{*3}\right) + \frac{b_4}{4}\left(\Phi^*\Phi\right)^2$$

- We IMPOSED (by proper parameter choice) that

$$\Phi \to 1 \qquad \text{for} \qquad T \to \infty \qquad \text{in pure gauge}$$

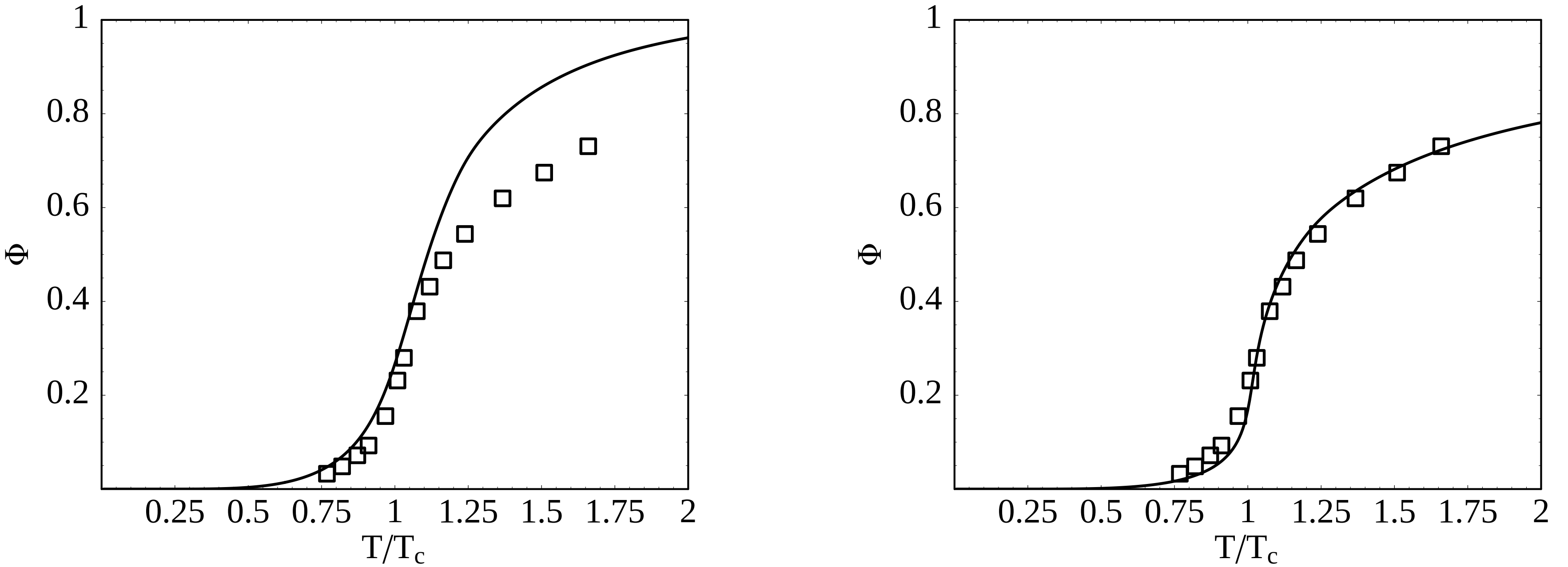
- Nothing prevents Φ from going above 1 in the system of quarks and gluons
- The potential can constraint the Polyakov loop to stay always below 1

$$\frac{V(\Phi,T)}{T^4} = -\frac{b_2(T)}{2}\Phi^2 - b_4\left(\frac{T_0}{T}\right)^3 \ln[(-(-1+\Phi)^3)(1+3\Phi)]$$

Improving the Polyakov-loop potential (II)

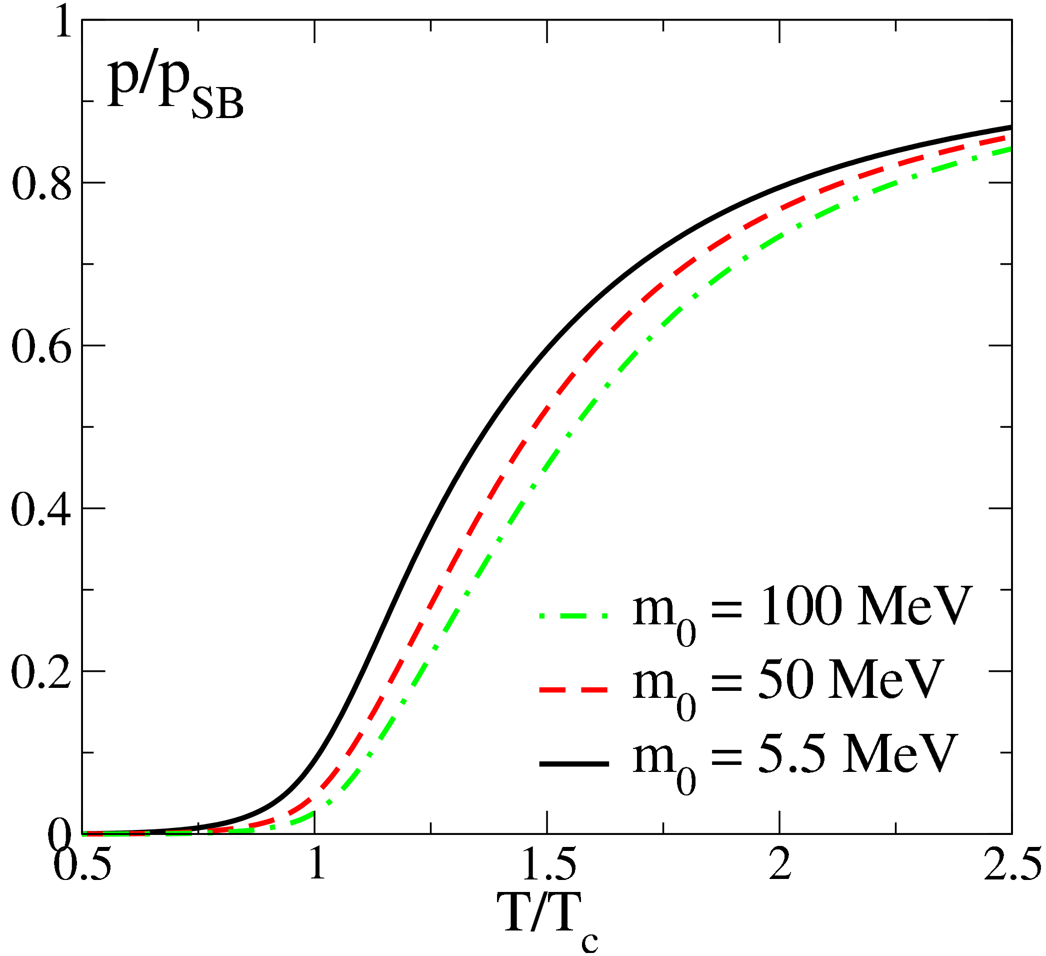
- We can do better than this: the potential can constrain the Polyakov loop to stay always below 1

$$\frac{V(\Phi,T)}{T^4} = -\frac{b_2(T)}{2}\Phi^2 - b_4\left(\frac{T_0}{T}\right)^3 \ln[(-(-1+\Phi)^3)(1+3\Phi)]$$

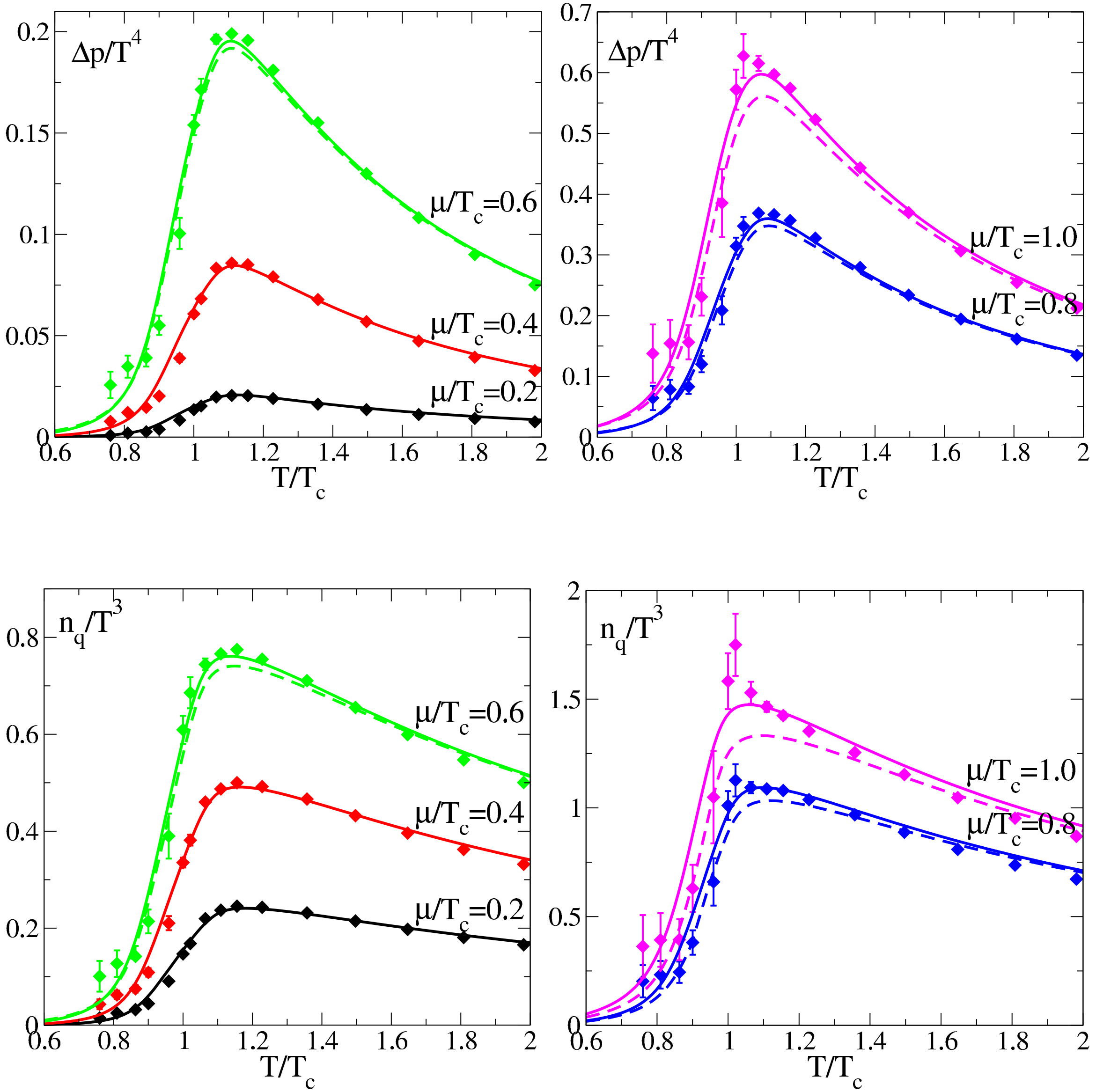


Backup slides

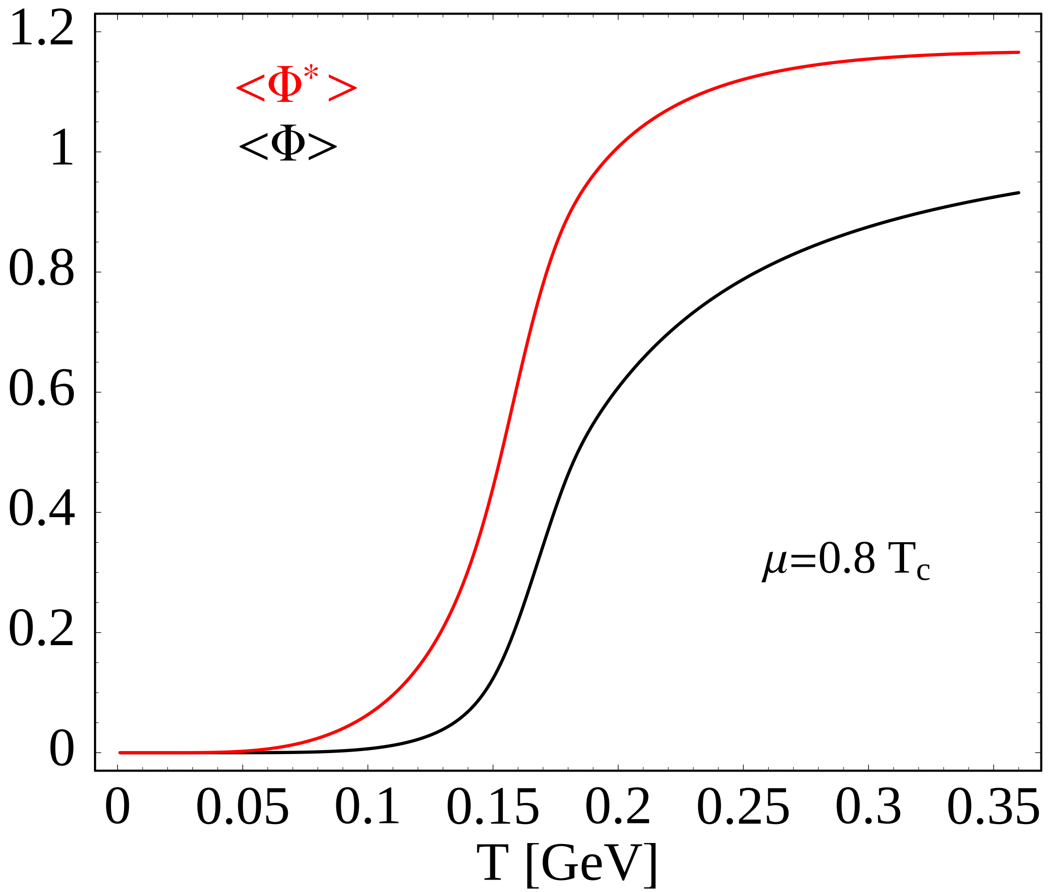
Quark mass dipendence



Taylor expansion



Finite μ results



Fixing the parameters

- In the Polyakov loop potential there are 7 parameters:
  - ➡ $a_0, \quad a_1, \quad a_2, \quad a_3, \quad b_3, \quad b_4, \quad T_0$

  but only 3 are free

- There are 4 constraints:
  - ➡ T_0 is fixed to 270 MeV, the known critical temperature for the pure gauge system
  - ➡ The Polyakov loop must tend to 1 as $T \rightarrow \infty$
  - ➡ $p(T) = -V(\Phi(T), T)$ tends to the ideal gas limit as $T \rightarrow \infty$
  - ➡ At $T = T_0$ the absolute minimum of $V(\Phi, T)$ must jump from $\Phi = 0$ to a finite Φ

- The remaining three parameters are fixed to reproduce the pure gauge lattice data

Parameter fixing

We have three free parameters in the model: $m_0,\ \Lambda,\ G$. They are fixed by:

- The pion decay constant f_π is evaluated in the NJL model through the following relation:

$$f_\pi^2 = 4m^2 I_\Lambda^{(1)}(m) \quad \text{where} \quad I_\Lambda^{(1)}(m) = -iN_c \int \frac{d^4p}{(2\pi)^4} \frac{\theta\left(\Lambda^2 - \vec{p}^{\,2}\right)}{(p^2 - m^2 + i\epsilon)^2}.$$

  The empirical value is $f_\pi = 92.4$ MeV.

- The quark condensate becomes

$$\left\langle \bar{\psi}_u \psi_u \right\rangle = -4m I_\Lambda^{(0)}(m) \qquad \text{with} \qquad I_\Lambda^{(0)}(m) = iN_c \int \frac{d^4p}{(2\pi)^4} \frac{\theta\left(\Lambda^2 - \vec{p}^{\,2}\right)}{p^2 - m^2 + i\epsilon}.$$

  Its “empirical” value derived from QCD sum rules is

$$\langle \bar{\psi}_u \psi_u \rangle^{1/3} \simeq \langle \bar{\psi}_d \psi_d \rangle^{1/3} = -\left(240 \pm 20\right)\ \text{MeV}.$$

- The current quark mass m_0 is fixed from the Gell-Mann, Oakes, Renner (GMOR) relation:

$$m_\pi^2 = \frac{-m_0 \left\langle \bar{\psi}\psi \right\rangle}{f_\pi^2}.$$

Thermodynamic potential

The final form of the thermodynamic potential is

$$\Omega\left(T,\mu,\sigma,\Phi,\Phi^*\right) = V\left(\Phi,\Phi^*,T\right) + \frac{\sigma^2}{2G}$$

$$-2N_cN_f\int\frac{\mathrm{d}^3p}{(2\pi)^3}\Big\{E_p + \frac{T}{3}\left[\ln\left[1+3\left(\Phi+\Phi^*\mathrm{e}^{-(E_p-\mu)/T}\right)\mathrm{e}^{-(E_p-\mu)/T}+\mathrm{e}^{-3(E_p-\mu)/T}\right]\right.$$

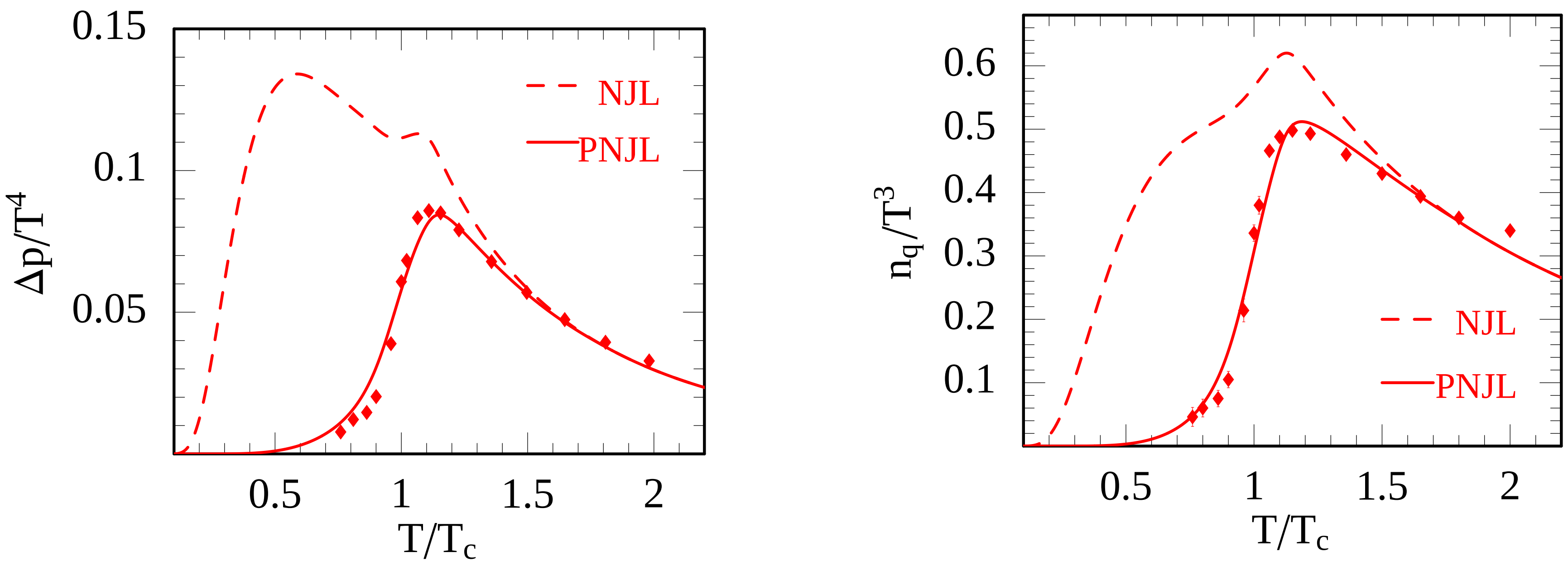
$$\left.+\ \ln\left[1+3\left(\Phi^*+\Phi\mathrm{e}^{-(E_p+\mu)/T}\right)\mathrm{e}^{-(E_p+\mu)/T}+\mathrm{e}^{-3(E_p+\mu)/T}\right]\right]\Big\}$$

with $E_p = \sqrt{p^2+m^2} = \sqrt{p^2+(m_0-\sigma)^2}$.

Field equations:

$$\frac{\partial\Omega\left(T,\mu,\sigma,\Phi,\Phi^*\right)}{\partial\sigma}=0,\qquad \frac{\partial\Omega\left(T,\mu,\sigma,\Phi,\Phi^*\right)}{\partial\Phi}=0,\qquad \frac{\partial\Omega\left(T,\mu,\sigma,\Phi,\Phi^*\right)}{\partial\Phi^*}=0$$

Finite μ results in the standard NJL model



- Comparison between PNJL and standard NJL model results at finite μ
- NJL model alone fails in reproducing lattice data
- Incorporation of confinement crucial to reproduce lattice results

$N_f = 2$

Parameters

Λ [GeV]	0.651
G [GeV^{-2}]	10.078
m_0 [MeV]	5.5

Physical quantities

f_π [MeV]	92.4
$\lvert\langle\bar{\psi}\psi\rangle\rvert^{1/3}$ [MeV]	247
m_π [MeV]	139.3

$N_f = 2 + 1$

Parameters

Λ [GeV]	0.6023
$G\Lambda^2$	3.67
$K\Lambda^5$	24.72
$m_{0u,d}$ [MeV]	5.5
m_{0s} [MeV]	140.7

Physical quantities

f_π [MeV]	92.4
$\lvert\langle\bar{\psi}\psi\rangle_{u,d}\rvert^{1/3}$ [MeV]	241.9
$\lvert\langle\bar{\psi}\psi\rangle_{s}\rvert^{1/3}$ [MeV]	257.7
m_π [MeV]	139.3
m_K [MeV]	497.7