The order of the finite temperature QCD transition physical quark masses in the continuum

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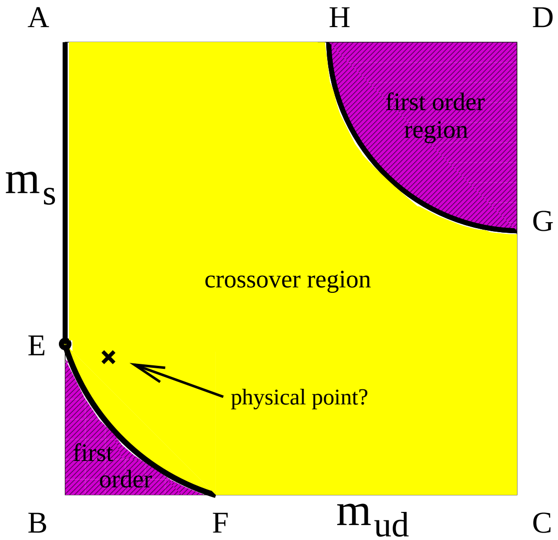
1. Standard picture of the phase diagram and its uncertainties

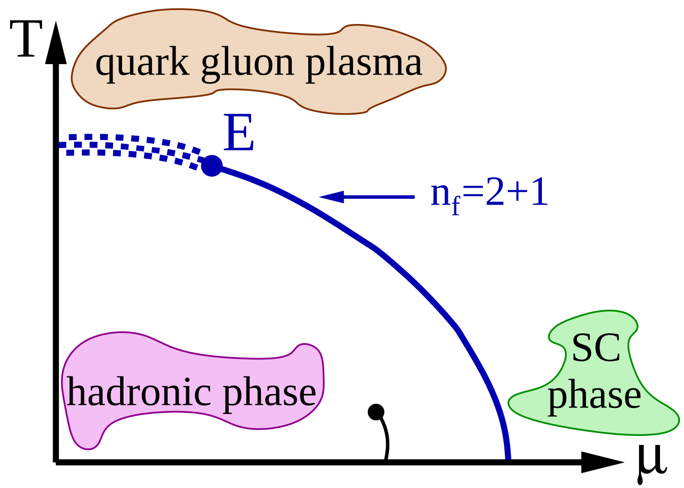
2. Lattice results (staggered fermionic action with fourth root)

   a. Order of a transition: physical quark masses, continuum
   b. Results at N_t=4,6
   c. Finite size scaling in the continuum: N_t=4,6,8,10

3. Conclusions

Standard picture of the phase diagram and its uncertainties





- Chiral phase transition (PT)

$n_f = 2$ with $m_q = 0$ at $\mu = 0$ $\Rightarrow$ 2^{nd} order phase transition

$n_f = 3$ with $m_q = 0$ at $\mu = 0$ $\Rightarrow$ 1^{st} order phase transition

$n_f = 2+1$ with physical m_q at $\mu = 0$ $\Rightarrow$ cross-over
$n_f = 2+1$ with physical m_q at $T = 0$ $\Rightarrow$ 1^{st} order PT
$\Longrightarrow$ results in the standard picture of the QCD phase diagram

$n_f = 2+1$ with physical m_q $\Rightarrow$ critical point (E) at μ,T$\neq$0

- Is our picture correct? (cross over $\longrightarrow$ critical point)

What can we really say about the physical point (cross-over)?

$\Longrightarrow$ physical quark masses: m_π=140 MeV, m_K= 500 MeV

$\Longrightarrow$ continuum limit: extrapolation to vanishing lattice spacings

in "usual" lattice simulations these two limits are missing

quark masses are important for the order of the transition:

n_f=2+1 theory with m_q=0 or ∞ gives a first order transition

for intermediate quark masses we have an analytic cross over

F. Karsch et al., Nucl. Phys. Proc. 129 (2004) 614

continuum limit is important for the order of the transition:

n_f=3 case (standard action, μ=0, N_t=4): critical $m_\pi \approx$300 MeV

with different discretization error (p4 action): critical $m_\pi \approx$70 MeV

the physical pion mass is just between these two values

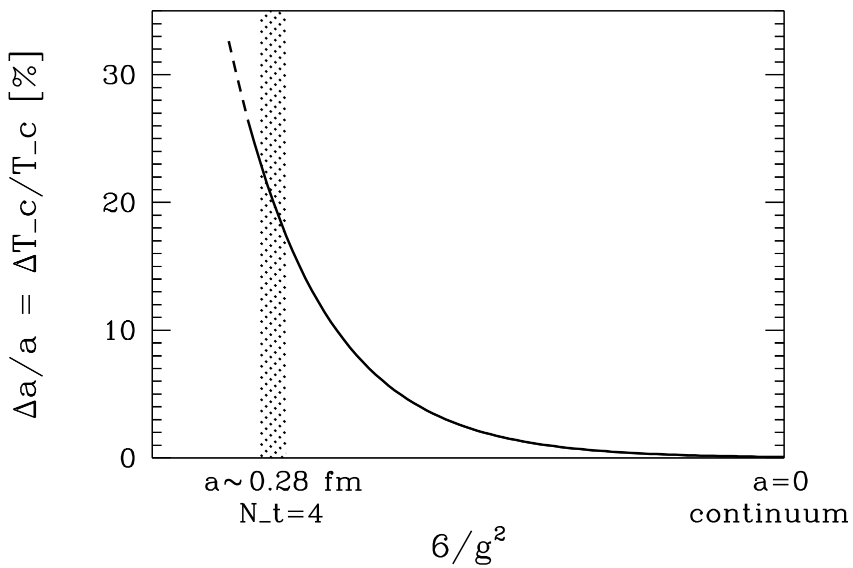
what happens with physical quark masses, in the continuum?

Order of the QCD transition: physical quark masses, continuum

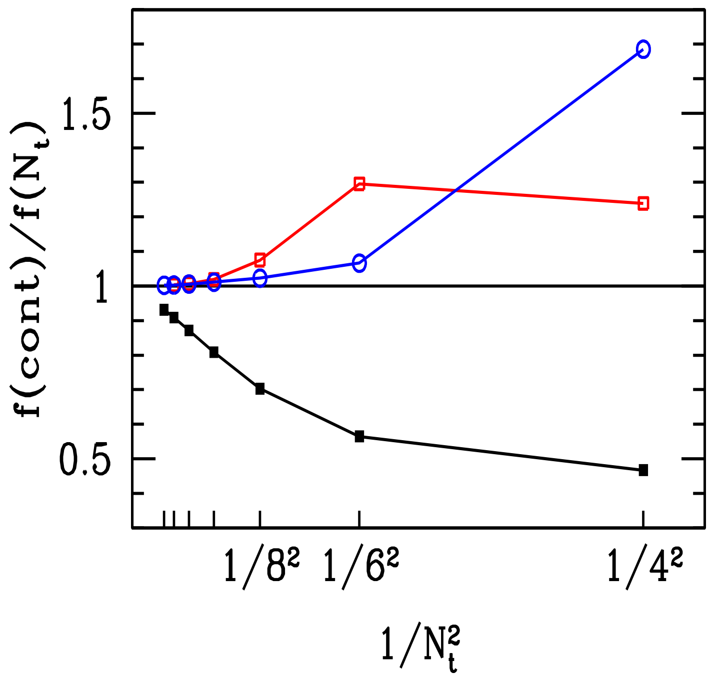
Y. Aoki, G. Endrodi, Z. Fodor, S.D. Katz, K.K. Szabo

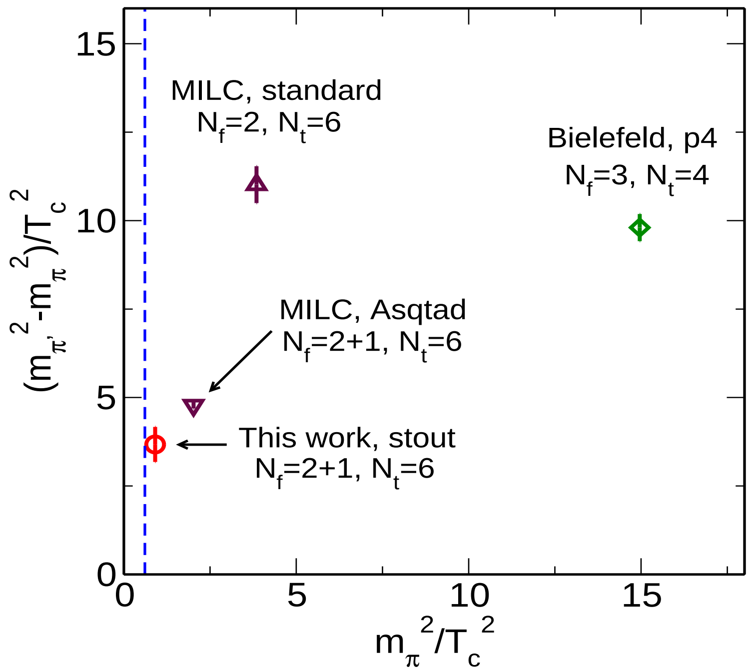
exact RHMC simulation algorithm on N_t=4,6,8,10 lattices
correspond to $a\approx$ 0.28 fm, 0.19 fm, 0.14 fm, 0.11 fm

lattice spacing or transition temperature in physical units is
sensitive to the quantity we use to set the scale (m_ρ, m_N or σ)
Δa/a=ΔT/T $\sim$ 20% at N_t=4: inconvenient mismatch
in the strong coupling limit Δa/a$\longrightarrow$-∞
most probably the force at intermediate distance is a good choice



Symanzik improved gauge, stout improved fermionic action
simulations along the line of constant physics (LCP):
masses are fixed to their physical value: m_π=140 MeV, m_K=500 MeV



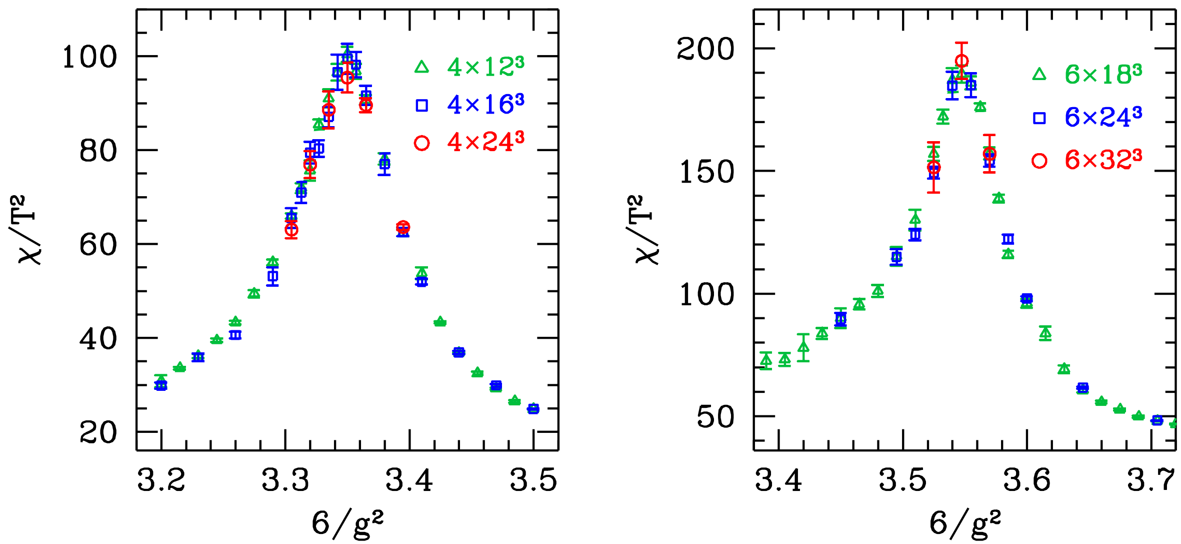


N_t=4,6 don't scale reduce unphysical nondegeneracy of π's $\approx T_c$

- Finite size scaling of the transition

Chiral susceptibilities: χ=(T/V)∂^2logZ/∂m^2

first order transition $\Longrightarrow$ peak width $\propto$ 1/V, peak height $\propto$ V
cross over $\Longrightarrow$ peak width $\approx$ constant, peak height $\approx$ constant



for aspect ratios 3–6 (an order of magnitude larger volumes)
volume independent scaling $\Longrightarrow$ cross-over

do we get the same result (cross-over) in the continuum limit?
one might have the unlucky case as we had in n_f=3 QCD:
for physical m_π discretization errors changed the order

• How to get rid of the discretization errors?

a. susceptibility for fixed physical volumes in the continuum
b. finite size analysis of the continuum extrapolated values

renormalize the susceptibility the same way as the pressure

$$p(T) \propto \log Z(T{\neq}0)/V_4 - \log Z(T{=}0)/\bar{V}_4$$

$p(T)$ has a continuum limit and we can use $m_r{=}Z_m{\cdot}m$

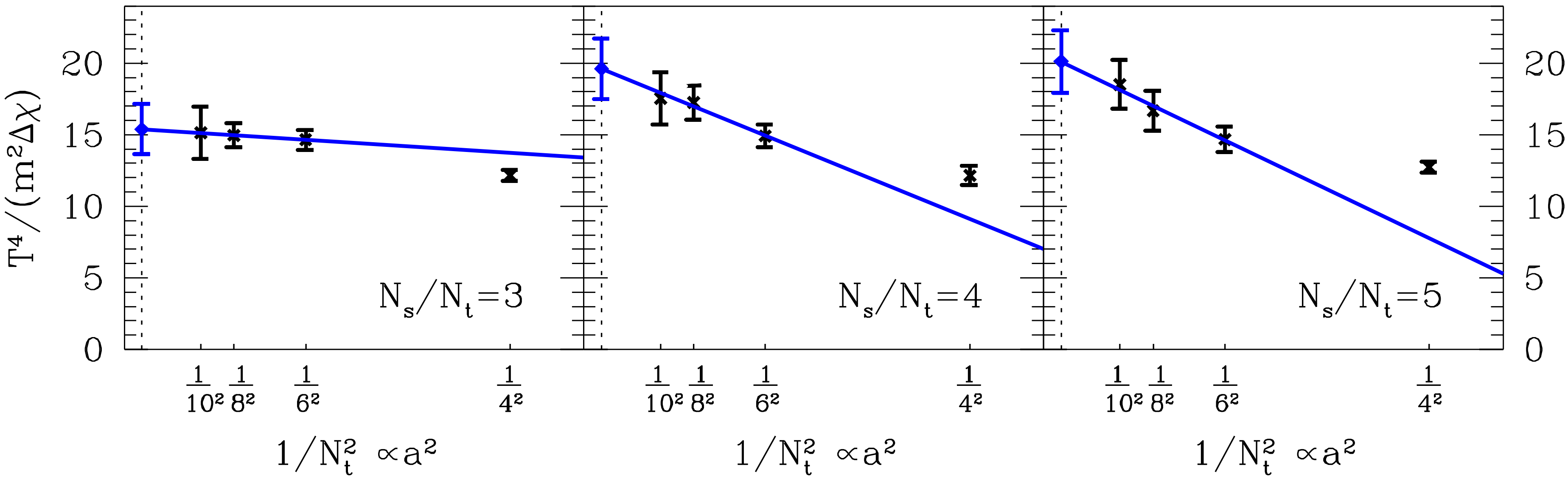
$$\chi_r(T) = \partial^2/(\partial m_r^2)\ [\log Z(T{\neq}0)/V_4 - \log Z(T{=}0)/\bar{V}_4]$$

construct a quantity in continuum: Z_m drops out from $m^2\partial^2/\partial m^2$

$$\Longrightarrow m_r^2{\cdot}\chi_r(T) = m^2{\cdot}[\chi(T{\neq}0) - \chi(T{=}0)]$$

- we need a continuum extrapolation (width, height)

calculate $m^2\Delta\chi = m^2[\chi(T\neq0)-\chi(T=0)]$ at the transition point

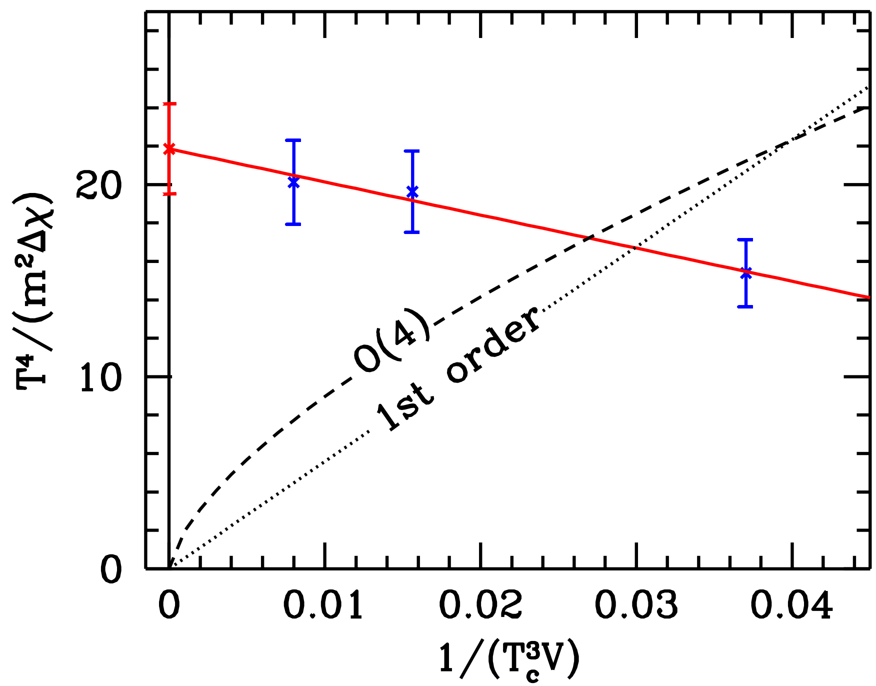


continuum limit values are obtained from N_t=4,6,8,10
N_t=6,8,10 temporal sizes are already in the a^2 scaling region

choice of the action or the line of constant physics is ambiguous
this choice has influence only on the slope, not on the value

the three continuum extrapolated values do not show 1/V scaling

• finite size scaling analysis with continuum extrapolated $m^2\Delta\chi$



the result is consistent with an approximately constant behavior for a factor of 5 difference within the volume range

chance probability for 1/V is 10^{-19} for O(4) is $7 \cdot 10^{-13}$

continuum result with physical quark masses in staggered QCD:

the QCD transition at μ=0 is a cross-over

Conclusions

- order of the nonzero temperature QCD transition was unknown (some conjectures about an analytic cross-over)

- only lattice QCD could give an unambiguous answer

- tell the order of a transition: finite size scaling analysis

- one needs continuum extrapolated results with physical masses

- continuum result with physical quark masses in staggered QCD:

the QCD transition at μ=0 is a cross-over