



3FD

CBM/Trento,
01.06.06

Physical
Input

Dir. Flow

Elliptic Flow

Excitation
Functions

1st order tr.

Summary



Transverse Flow in 3-Fluid Dynamics (AGS to SPS)

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CBM Workshop, Trento, May 29 - June 2, 2006



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Energy density:
$$\varepsilon(n_B, T) = \underbrace{\varepsilon_{\text{gas}}(n_B, T)}_{\text{hadron gas in mean field}} + \underbrace{W(n_B)}_{\text{mean field}}$$

Pressure:
$$P(n_B, T) = \underbrace{P_{\text{gas}}(n_B, T)}_{\text{hadron gas in mean field}} + \underbrace{n_B \frac{dW(n_B)}{dn_B} - W}_{\text{mean field}}$$

$$\frac{\varepsilon(n_B, 0)}{m_N n_0} = a \left(\frac{n_B}{n_0}\right)^{5/3} - b \left(\frac{n_B}{n_0}\right)^2 + c \left(\frac{n_B}{n_0}\right)^{7/3}$$

$\varepsilon(n_B, 0)$ saturates the cold matter at

$n_0 = 0.15 \text{ fm}^{-3}$ and

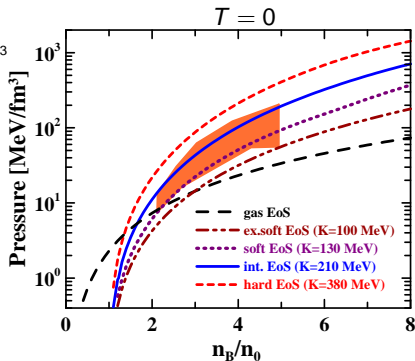
$\varepsilon(n_0, T=0)/n_0 - m_N = -16 \text{ MeV}$,

and provides incompressibility K .

Astrophysical constraints on EoS

T. Klahn et al., nucl-th/0602038:

hard EoS's are preferable at $T = 0$.



Danielewicz, Lacey, Lynch (2002)



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- Hadronic **EoS**

- Enhanced Friction

fitted to observed stopping power (i.e. proton rapidity distributions)

Function of 3 variables, $dN/dy(y, b, E_{lab})$, is fitted by function of 1 variable, $\xi_h^2(s)$!

- Freeze-out: $\varepsilon_{frz} \approx 0.2 \text{ GeV/fm}^3$ ($\approx 0.1 \text{ GeV/fm}^3$ at $E_{lab} < 3 \text{ A}\cdot\text{GeV}$)

fitted to observed proton p_T spectra

Funct. of 4 variables, $(1/2\pi m_T)d^2N/dm_T dy(m_T, y, b, E_{lab})$, is fitted by 1 parameter ε_{frz} !

- Formation Time $\tau = 2 \text{ fm/c} \Rightarrow \tau_{particle} \approx 1 \text{ fm/c}$

fitted to observed pion production at $E_{lab} > 30 \text{ A}\cdot\text{GeV}$

○ $\tau_{particle} < \tau = \int d^3p \tau_{particle} \gamma_{particle} f(p) / \int d^3p f(p)$ If $T \approx 100 \text{ MeV}$, $\tau_{particle} \approx 1 \text{ fm/c}$.



How it works

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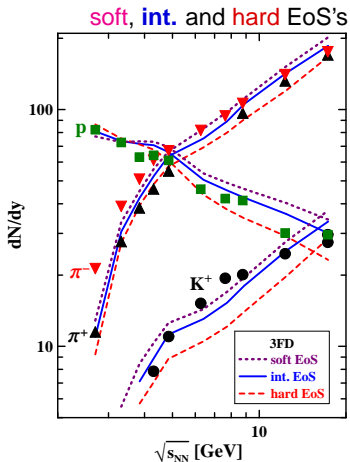
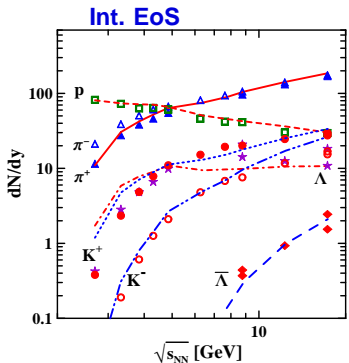
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Summary

dN/dy at midrapidity in central Au+Au and Pb+Pb coll.



Int. EoS is also good for:

- multiplicities
- m_T spectra

Flow?



Directed Flow at AGS

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Physical
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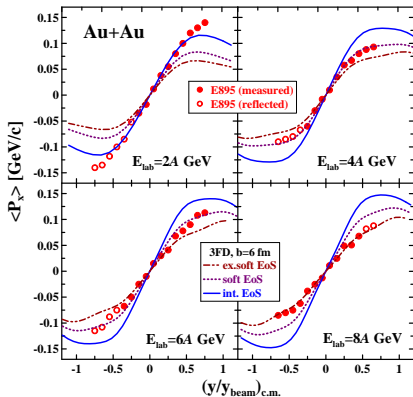
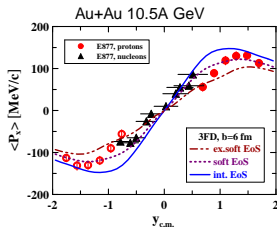
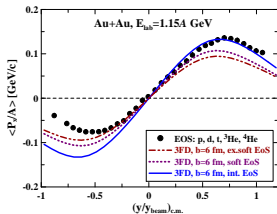
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Data favor: **int. EoS (1A GeV)** \Rightarrow **soft EoS (4A GeV)** \Rightarrow **extra-soft EoS (8A GeV)** \Rightarrow **?(SPS)**

HSD, UrQMD, JAM: **no softening required**

HSD, UrQMD, JAM: **“transition from hadronic to string matter”**[1] \Rightarrow **effective softening**

[1] P.K. Sahu, W. Cassing, U. Mosel and A. Ohnishi, Nucl. Phys. **A672**, 376 (2000)



Directed Flow at SPS

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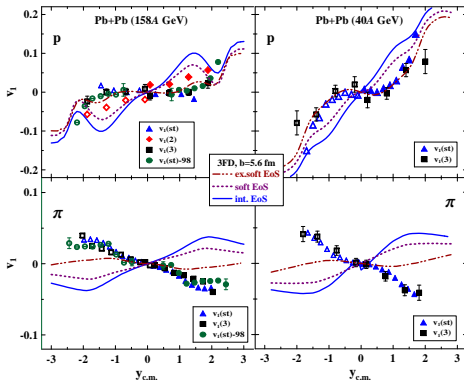
Elliptic Flow

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Summary

HSD: Cassing and Bratkovskaya, PR **308**, 65 (1999); **UrQMD:** Bass *et al.*, PPNP **41**, 225 (1998);
JAM: Isse *et al.*, PRC **72**, 064908 (2005).



Pions: no shadowing
⇒ no anticorrelation

Softening saturates: int. EoS (1A GeV) ⇒ soft EoS (4A GeV) ⇒ extra-soft EoS ($E > 8A$ GeV)

HSD: “transition from hadronic to string matter” saturates at top AGS energies

Initial stage is of prime importance for flow!

Int. EoS is good for “late-stage” and “stopping-power” observables.



Elliptic Flow at AGS

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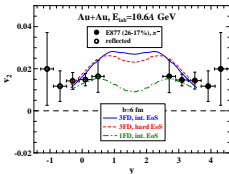
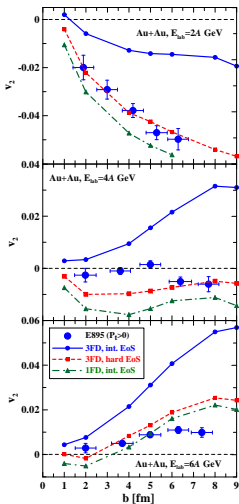
Dir. Flow

Elliptic Flow

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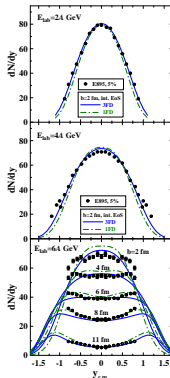
Summary



**Dir. flow requires softer EoS
Elliptic flow — harder EoS**

Similar to Danielewicz *et al.*,
Science **298**, 1592 (2002)

1FD is quite good at lower AGS energies \Rightarrow



HSD, JAM: **proper momentum dependence is of prime importance!**



Elliptic Flow at SPS

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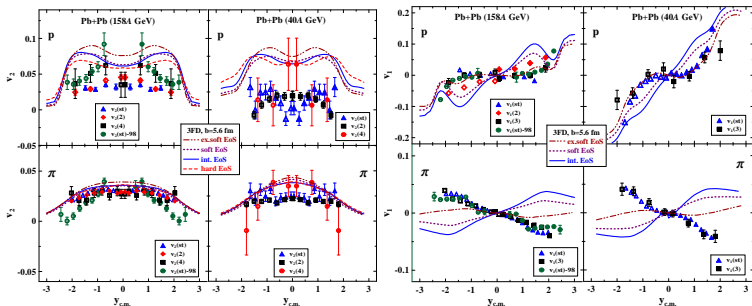
Dir. Flow

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Summary



HSD, JAM: proper momentum dependence is of prime importance!



Early-stage nonequilibrium is important for $v_2 = \langle (p_x^2 - p_y^2) / p_T^2 \rangle$

because otherwise this momentum dependence is integrated over in EoS



Excitation Functions of Flow

3FD

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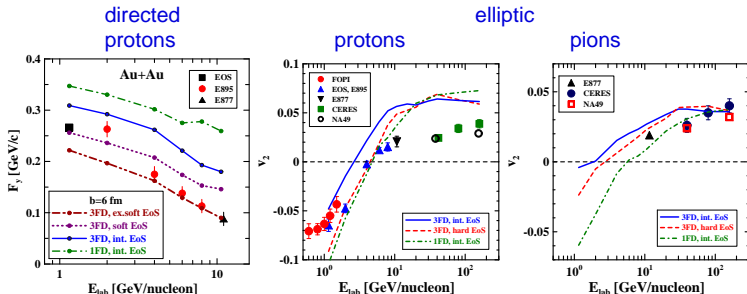
Dir. Flow

Elliptic Flow

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Functions

1st order tr.

Summary



Summary:

- **Effective softening at initial stage is required** by directed flow
“transition from hadronic to string matter” Sahu et al., Nucl. Phys. A672, 376 (2000)
Initial stage is of prime importance for flow!
- Proper momentum dependence is of prime importance for elliptic flow which means that
Early-stage nonequilibrium is important for v_2
- **3FD with int. EoS is good for “late-stage” and “stopping-power” observables.**



EoS with 1st order phase transition

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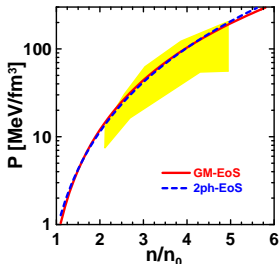
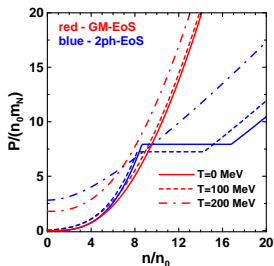
Elliptic Flow

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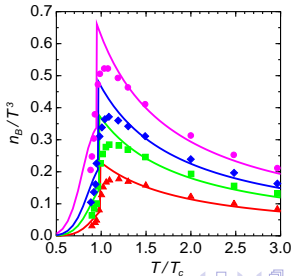
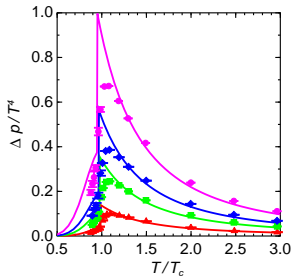
Summary

A.S.Khvorostukhin, V.V.Skovov, K.Redlich, V.D.Toneev, nucl-th/0605069



Danielewicz,
Lacey, Lynch
(2002)

Bag with heavy quarks ($m_{u/d} = 70\text{MeV}$, $m_s = 145\text{MeV}$) and gluons (700 MeV),
to fit lattice data (Fodor et al.)



$\mu_b = 210,$
 $330, 410,$
 530 MeV



2ph-EoS Preliminary!!!

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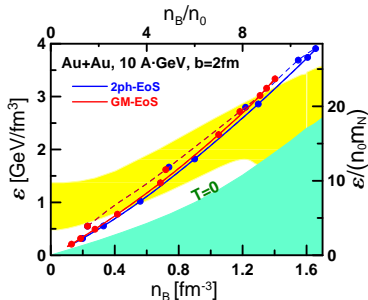
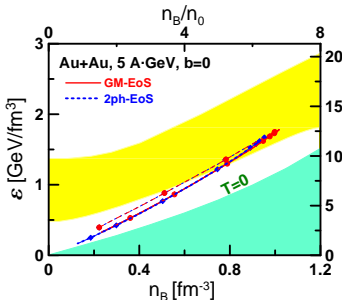
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Summary

Dynamic Trajectories for central box $4 \text{ fm} \times 4 \text{ fm} \times 4 \text{ fm}/\gamma_{cm}$
(Preliminary!)



Phase transition starts at 5A GeV

At 10A GeV the matter mostly lives in the phase-coexistence region



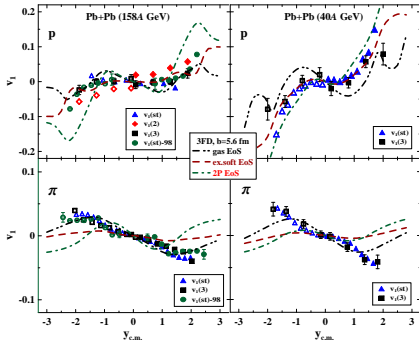
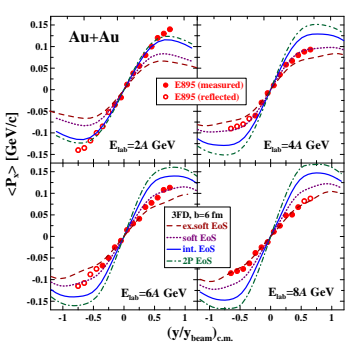
2ph-EoS Preliminary!!!

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Directed flow (Preliminary!)



2P-EoS:

too hard in hadronic phase

too soft in QGP phase



2ph-EoS Preliminary!!!

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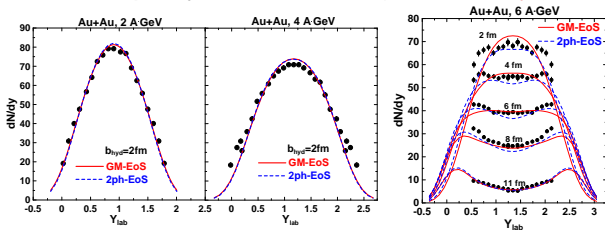
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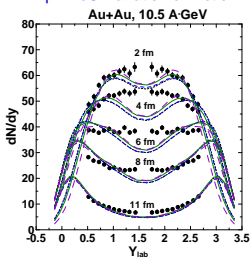
1st order tr.

Summary

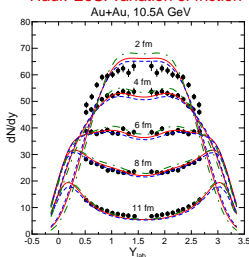
Proton Rapidity Distributions (Preliminary!)



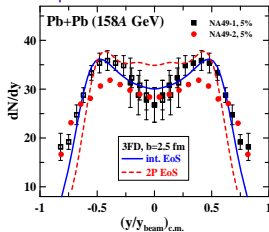
2ph-EoS: variation of friction



Hadr. EoS: variation of friction



2ph-EoS: variation of friction



Phase transition happens too early



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Summary

Hydrodynamics addresses **Equation of State!**
Simple hadronic EoS \implies **Natural reference point**

- Simple *hadronic EoS* reasonably reproduces observables (except flows).
 - Flows require a softer EoS and hence leave room for **phase transition to quark-gluon phase** or **“transition from hadronic to string matter”** (HSD, UrQMD, JAM)
- EoS with 1st order phase transition:
 - **Phase transition starts too early!** (Au+Au at 5A GeV)
 - **2ph-EoS is too hard in hadronic phase**
 - **2ph-EoS is too soft in QGP phase**