Advertising title:

Isentropes in a 'realistic' Phase Diagram

or more descriptive:

Isentropes in the T–μ– and ϵ–ρ–Phase Diagram

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CBM-Workshop: The Physics of High Baryon Density,
ECT* Trento, 01.06.2006

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Isentropes in the T–μ– and ϵ–ρ–Phase Diagram

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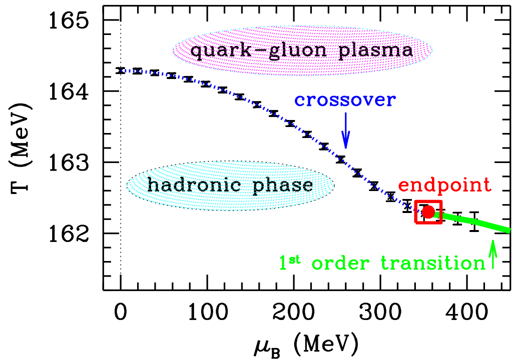
with D. Zschiesche, S. Schramm and H. Stöcker

Outline

1. Motivation: Importance of heavy hadronic states
2. The Chiral Hadronic $\mathrm{SU}(3)$ Model: ... a brief overview
   $+$ introducing a heavy baryon state to the model
3. Results: Phase diagrams with isentropes in T–μ and ϵ–ρ
   D.Zschiesche, G.Z. & S.Schramm, nucl-th/0602073
4. Conclusion and Outlook

The Physics of High Baryon Density, ECT* Trento, 01.06.2006

1. Motivation



- Lattice QCD (lQCD) predicts
  T–μ–phase diagram with critical endpoint
  Z.Fodor & S.D.Katz, JHEP **0404** (2004) 050
  $\rightsquigarrow$ lQCD alone can not disentangle
  the physics of the phase transition (yet?)

- Effective models are a complementary approach to lQCD
  $\rightsquigarrow$ Investigate what drives the phase transition (mechanisms ...)

+ Equation of state (EoS: $p(T,\mu)$, $\epsilon(T,\mu)$) is necessary to describe heavy-ion collisions, neutron stars, ...

1. Motivation ($\mu = 0$)

- Weak dependence of phase transition (p.t.) temperature $T_{\rm p.t.}$ on m_π-changes in lQCD

  F.Karsch & E.Laermann, hep-lat/0305025

  $\Longleftrightarrow$ Strong m_π dependence of $T_{\rm p.t.}$ in models w/o heavy states

  A.Dumitru, D.Röder & J.Ruppert, Phys.Rev. D **70** (2004) 074001

- Scaled hadronic resonance gas gives reasonable description for thermodynamic behavior of lQCD in the *hadronic phase*

  F.Karsch, K.Redlich & A.Tawfik, *e.g.* Eur.Phys.J. C **29** (2003) 549

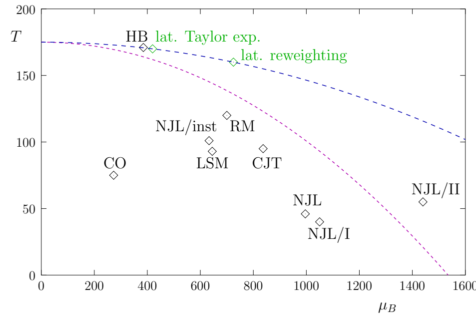
$\Rightarrow$ Contributions from heavy states are important (below $T_{\rm p.t.}$)

$\rightsquigarrow$ Investigate the role of heavy hadronic states / 'resonances'

1. Motivation ($\mu > 0$)

- Effective models with pure order parameter dynamics (LSM, NJL) but w/o heavy states typically yield too low T_c of endpoints, e.g.:

  M.Stephanov, Prog.Theor.Phys.Suppl. **153** (2004) 139



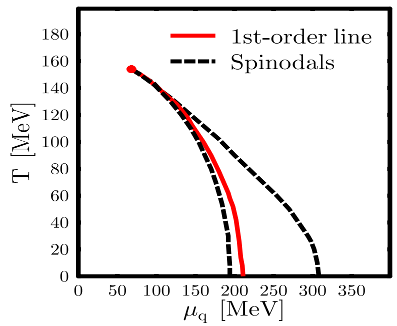
$\Rightarrow$ Contributions from heavy states also important for endpoint!?!

Chiral Hadronic SU(3) Model

- Applied to: Nuclear matter PRC57&63, finite nuclei PRC59&66, compact stars JPG29, HIC PRC65, PLB547, JPG30, . . .

- Baryon decuplet allows for higher endpoints *but:* problems at nuclear matter ($\mu_B = \mu_0$)

  D.Zschiesche, G.Z., S.Schramm & H.Stöcker, J.Phys. G **31** (2005) 935



$\rightsquigarrow$ Use this chiral model as basis and extend it further / differently

2. The Chiral Hadronic SU(3) Model

- Degrees of freedom: Baryons und Mesons ($\mathrm{SU}(3)$-Multiplets)
- A σ-ω-Model: Interactions induced via scalar and vector fields
- Chiral symmetry
  - Non-linear realization $\rightsquigarrow$ more natural baryon coupling schemes
  - Spontaneous symmetry breaking (SSB)

    $\Rightarrow$ dynamical baryon mass generation, in-medium masses
  - Explicite symmetry breaking (ESB) $\Rightarrow$ PCAC
- Scale breaking potential $\Rightarrow$ mimics trace anomaly of QCD
- Mean-field approximation: mesonic fields treated classically

$$\sigma, \zeta, \chi, \omega, \phi \; : \quad \hat{\varphi} \; \longrightarrow \; \varphi = \langle \hat{\varphi} \rangle$$

- Model shows phase transition to chirally 'restored' phase for large T and μ, if heavy degrees of freedom are included (besides baryon octet)

General Lagrangian — *2. The Model (2a)*

$$\mathcal{L}^{\rm chiral} = \mathcal{L}_0 + \mathcal{L}_{\rm BM} + \mathcal{L}_{\rm BV} + \mathcal{L}_{\rm Vec} + \mathcal{L}_{\rm ESB}$$

- Scalar Potential

$\sigma \sim \langle \bar{q} q \rangle$	non-strange chiral condensate
$\zeta \sim \langle \bar{s} s \rangle$	strange chiral condensate
$\chi \sim \langle G_{\mu\nu} G^{\mu\nu} \rangle$	dilaton field (gluon condensate)

$\Rightarrow$ *SSB*: finite vaccuum expectation values $\sigma_0,\ \zeta_0,\ \chi_0$

$\Rightarrow$ Breaking of scale invariance ($\rightsquigarrow$ trace anomaly)

$$\begin{aligned} \mathcal{L}_0 \;=\; & -\frac{1}{2} k_0 \chi^2 \left(\sigma^2 + \zeta^2\right) + k_1 \left(\sigma^2 + \zeta^2\right)^2 + k_2 \left(\frac{\sigma^4}{2} + \zeta^4\right) + k_3 \chi \sigma^2 \zeta \\ & - k_4 \chi^4 - \frac{1}{4} \chi^4 \ln \frac{\chi^4}{\chi_0^4} + \frac{\delta}{3} \chi^4 \ln \frac{\sigma^2 \zeta}{\sigma_0^2 \zeta_0} \end{aligned}$$

General Lagrangian — 2. The Model (2b)

$$\mathcal{L}^{\text{chiral}} = \mathcal{L}_0 + \mathcal{L}_{\text{BM}} + \mathcal{L}_{\text{BV}} + \mathcal{L}_{\text{Vec}} + \mathcal{L}_{\text{ESB}}$$

- Baryon – scalar meson interaction ⇒ *Attraction*

  ⇒ Dynamical mass generation (of baryon octet) by scalar fields

$$\mathcal{L}_{\text{BM}} = -\sum_i \bar{\Psi}_i \, m_i^* \, \Psi_i$$

$$m_i^* = g_{i\sigma}\sigma + g_{i\zeta}\zeta + \delta m_i^{\text{const.}} \; ; \quad i = N, \Lambda, \Sigma, \Xi$$

- Baryon – vector meson interaction ⇒ *Repulsion*

  ⇒ Effective chemical potentials μ_i^*

$$\mathcal{L}_{\text{BV}} = -\sum_i \bar{\Psi}_i \gamma^0 \left[g_{i\omega}\omega_0 + g_{i\phi}\phi_0 \right] \Psi_i$$

General Lagrangian — 2. The Model (2c)

$$\boxed{\mathcal{L}^{\text{chiral}} = \mathcal{L}_0 + \mathcal{L}_{\text{BM}} + \mathcal{L}_{\text{BV}} + \mathcal{L}_{\text{Vec}} + \mathcal{L}_{\text{ESB}}}$$

- Vector meson potential

$$\mathcal{L}_{\text{Vec}} = \frac{1}{2} m_\omega^2 \frac{\chi^2}{\chi_0^2} \omega^2 + \frac{1}{2} m_\phi^2 \frac{\chi^2}{\chi_0^2} \phi^2 + g_4^4 (\omega^4 + 2\phi^4)$$

- Explicit symmetry breaking
  $\Rightarrow$ PCAC, finite π and K mass

$$\mathcal{L}_{\text{ESB}} = -\left(\frac{\chi}{\chi_0}\right)^2 \left[m_\pi^2 f_\pi \, \sigma + \left(\sqrt{2} m_K^2 f_K - \frac{1}{\sqrt{2}} m_\pi^2 f_\pi \right) \zeta \right]$$

Grand Canonical Potential — *2. The Model (3)*

$$\frac{\Omega}{V}(T,\mu) = -\mathcal{L}_{\mathrm{Vec}} - \mathcal{L}_0 - \mathcal{L}_{\mathrm{ESB}} - \mathcal{V}_{\mathrm{vac}}\big|_{T=0,\mu=0}$$
$$\mp T \sum_{i\in\left\{\substack{\text{Baryons}\\ \text{Mesons}}\right.} \frac{d_i}{(2\pi)^3}\int \mathrm{d}^3k \left[\ln\left(1 \pm e^{-\frac{1}{T}[E_i^*(k)-\mu_i^*]}\right)\right]$$

where $E_i^*(k) = \sqrt{\vec{k}_i^{\,2} + m_i^{*\,2}}$ and $\mu_i^* = \mu_i - g_{i\omega}\omega - g_{i\phi}\phi$

Minimizing the potential with respect to the fields

$\rightsquigarrow$ Coupled equations of motions for σ, ζ, ω and ϕ (and take $\chi \equiv \chi_0$)

solved numerically such that $\rho_{\mathrm{strange}} = 0$ (vanishing net strangeness)

$\Rightarrow$ Thermodynamic quantities p, ρ_i, s, ϵ can be calculated

Adding Heavy Hadronic States — *2. The Model (4)*

- Chiral restoration properties depend on baryon couplings

  D.Zschiesche, G.Z., S.Schramm & H.Stöcker, J.Phys. G **31** (2005) 935

- Introduce a heavy baryon ('resonance') state with

$$\mathcal{L}_{\mathrm{R}} = -d_{\mathrm{R}}\, \bar{\Psi}_{\mathrm{R}} \left(m_{\mathrm{R}}^{*} + \gamma^{0} g_{\mathrm{R}\omega} \omega_{0} \right) \Psi_{\mathrm{R}}$$

  to model the higher mass resonance spectrum *effectively*

  ⇒ Effective mass $\boxed{m_{\mathrm{R}}^{*} = m_{0} + g_{\mathrm{R}\sigma}\, \sigma}$ choose degeneracy d_{R} as of Δ-s $(d_{\mathrm{R}} = 16)$

  ⇒ Vector coupling

  $g_{\mathrm{R}\omega} = r_{\mathrm{V}}\ g_{\mathrm{N}\omega}$ use relative coupling w.r.t. octet $\boxed{r_{\mathrm{V}} = g_{\mathrm{R}\omega}/g_{\mathrm{N}\omega}}$

⇝ Investigate scalar *and* vector couplings: explicit constant mass term m_0 as well as 'vacuum mass' $m_{\mathrm{R}} \equiv m_{\mathrm{R}}^{*}(\sigma_0\,;\ g_{\mathrm{R}\sigma}\,,\ m_0)$ (with $g_{\mathrm{R}\sigma} = \frac{m_{\mathrm{R}} - m_0}{\sigma_0}$) *and* relative vector coupling r_{V}

3. Results — Endpoint in the T–μ–Plane

Choice of parameters:

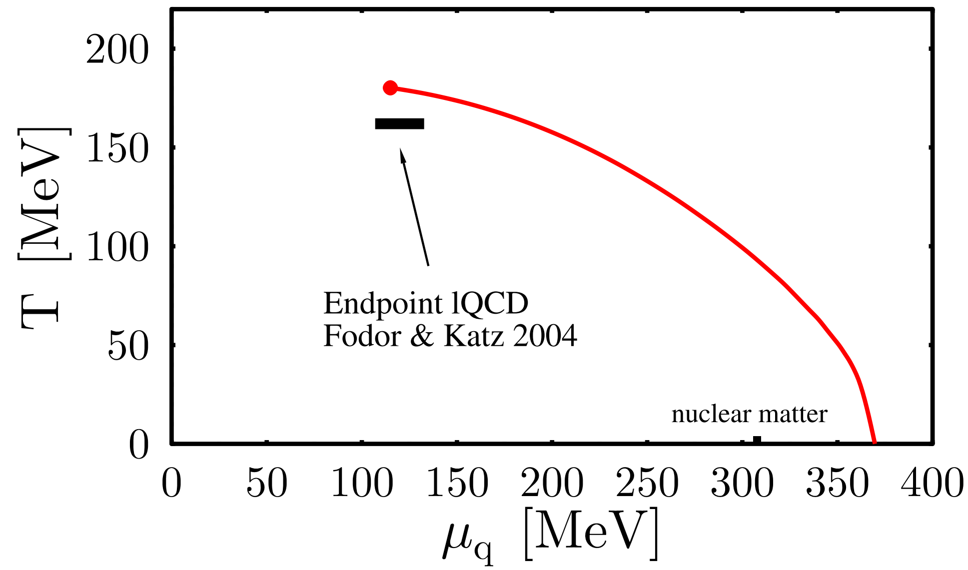
$m_{\rm R} \equiv m^*_{\rm R}(\sigma_0) = 2$ GeV with $m_0 = 570$ MeV , and $r_{\rm V} = 0.4$

$\rightsquigarrow$ $T_{\rm c}(\mu = 0) \approx 185$ MeV

$\rightsquigarrow$ $T_{\rm End} \approx 180$ MeV

$\rightsquigarrow$ $\mu_{q,\rm End} \approx 125$ MeV

$\hat{=}\ \mu_{\rm B,End} \approx 375$ MeV



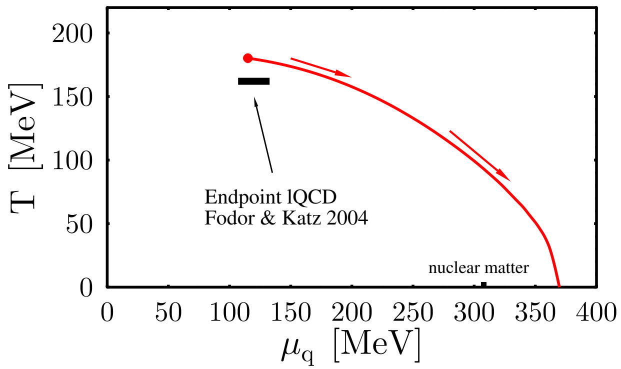
- Reasonable agreement with endpoint from 3-flavor lQCD calculations

  Z.Fodor & S.D.Katz, JHEP **0404** (2004) 050 : $T_{\rm End} = 162 \pm 2$ MeV , $\mu_{\rm B,End} = 360 \pm 40$ MeV

- Small slope of critical line $\rightsquigarrow$ *Model can describe nuclear matter, too!*

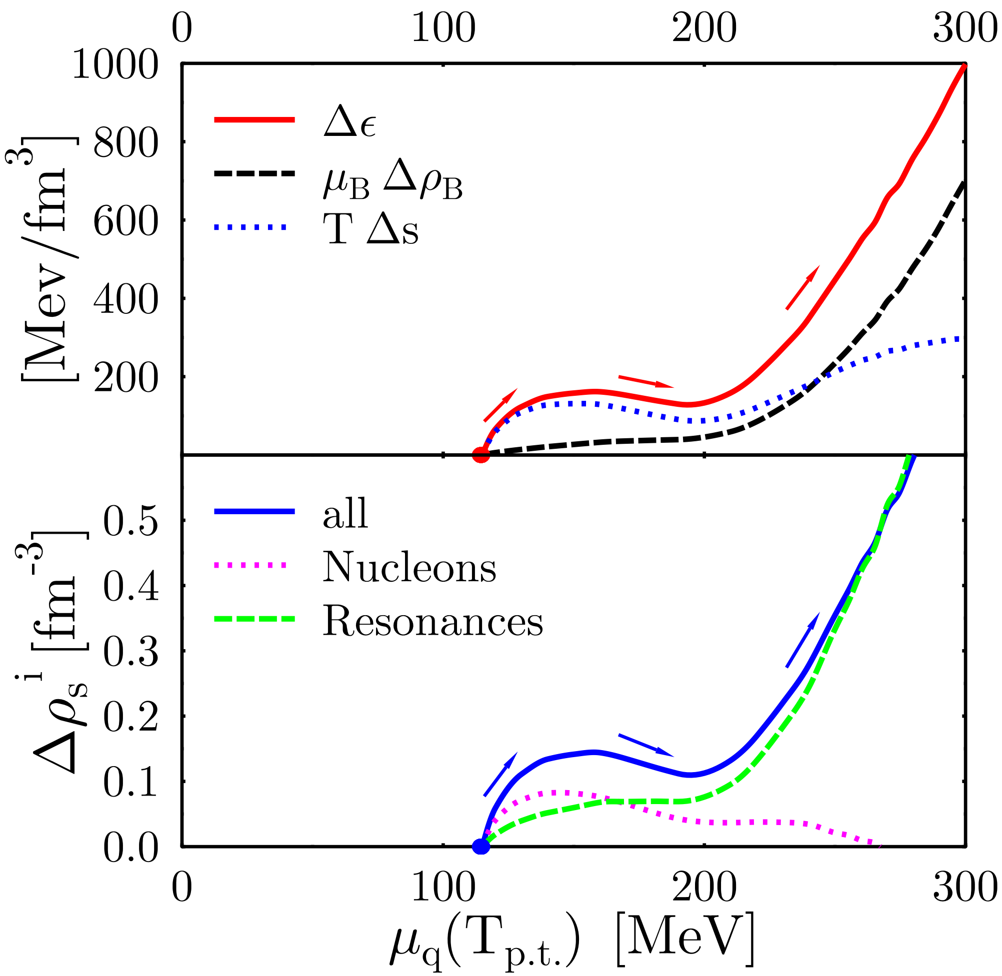
Contributions to Latent Heat — *3. Results (2)*

Along the line
of first-order phase transitions . . .



- *Non-monotonic* latent heat due to entropie part $\Delta\epsilon_s \equiv T\Delta s$
- Change of *leading* contributions to the jump in scalar densities $\Delta\rho_{s,i}$:

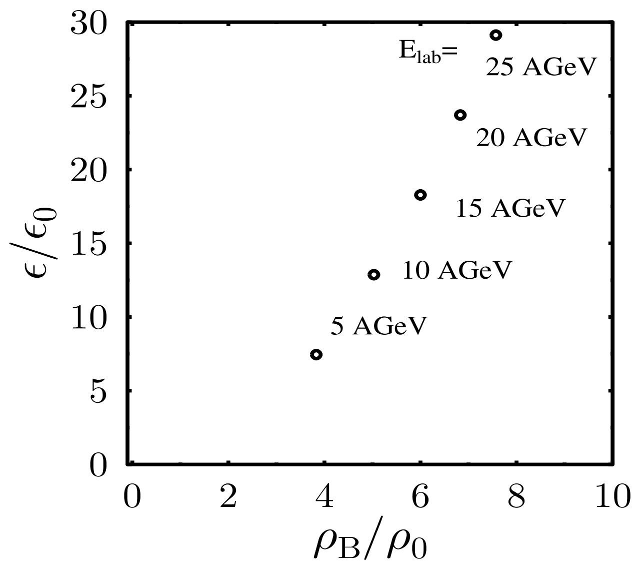
  Nucleons $\leftrightarrow$ Resonances

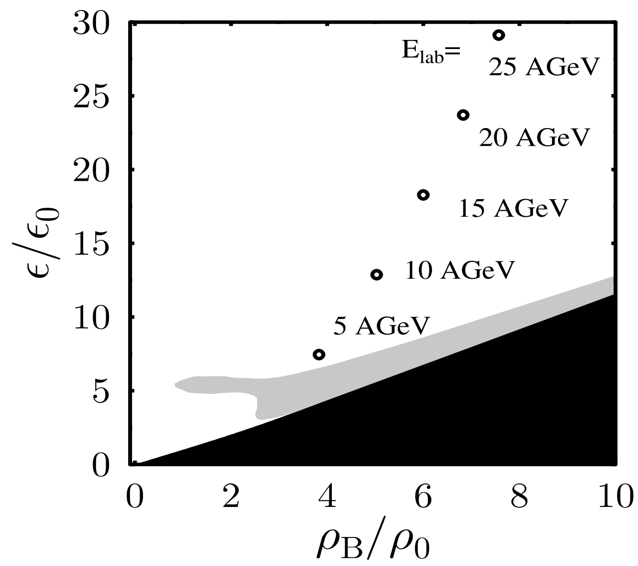


Initial Conditions and ϵ–ρ–Phase Diagram — *3. Results (3)*

Initial conditions: $\epsilon, \rho_{\mathrm{B}}$ from simple overlap model

$$\rho_{\mathrm{B}}^{\mathrm{initial}} = 2\,\gamma_{\mathrm{c.m.}}\,\rho_0 \quad \text{and} \quad \epsilon^{\,\mathrm{initial}} = \sqrt{s}\;\rho_{\mathrm{B}}^{\mathrm{initial}} = \sqrt{s}\;\gamma_{\mathrm{c.m.}}\,\rho_0$$





No EoS-dependence *of initial conditions*

$\leftrightarrow$ only T, μ and S/A connected via EoS

Here together with EoS :

– Grey area is mixed phase of EoS

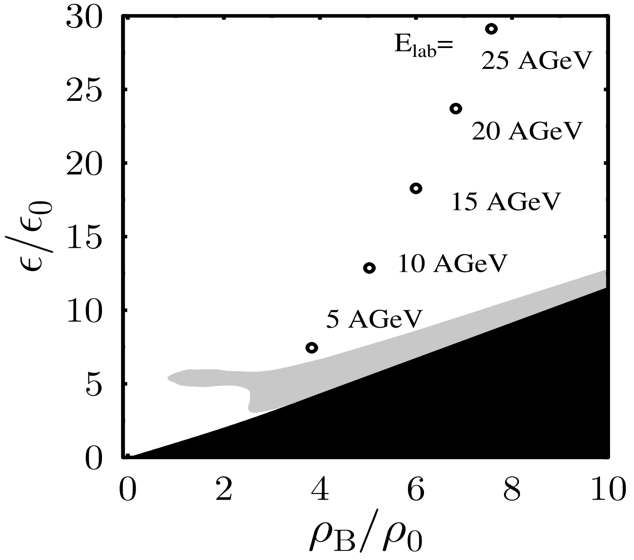
– Black area is *forbidden region*

(below the $\epsilon(\rho_{\mathrm{B}})$-isoline for $T = 0$)

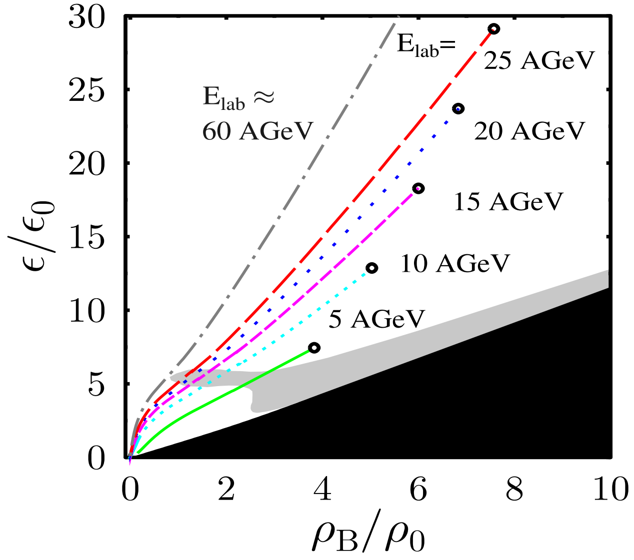
Initial Conditions and Isentropes — *3. Results (4)*

Ansatz: Ideal expansion along paths of constant entropie per baryon ($S/A =$ const.)

— starting at initial ϵ–ρ conditions from overlap model



Isentropes ⟹



Specific model predictions:

- First-order phase transition: $E_{\rm Lab} \approx 5 - 10\,A$ GeV
- Second-order endpoint: $E_{\rm Lab} \approx 25 - 60\,A$ GeV

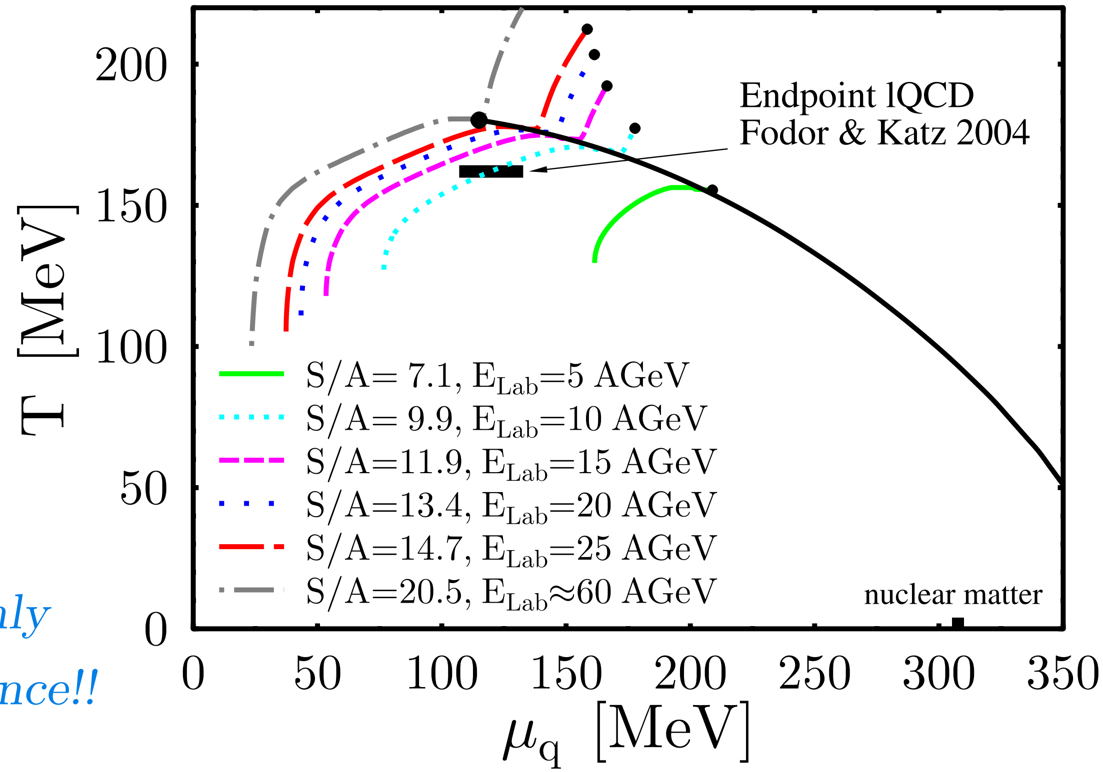
Isentropes in T–μ–Phase Diagram — *3. Results (5)*

. . . now in the T–μ–plane: Ideal expansion along pathes of $S/A =$ const.
(starting from initial ϵ–ρ conditions of overlap model connected to T–μ via EoS)

Specific model predictions:

- Focussing / Bending visible
- 1st-order: $5 - 10\,A$ GeV
- Endpoint: $25 - 60\,A$ GeV

Caution: Do not concentrate on isentrope through lQCD endpoint only
$\rightsquigarrow$ *strong model/parameter dependence!!*



CBM: Lowest E_{Lab} sufficient to overshoot p.t. line
$+$ Endpoint might well be in range! (Here also: *model/parameter dependence*)

4. Conclusions

- An example was presented, that
  a phase diagram in (semi-)quantitative agreement with lQCD
  can be obtained from an effective model which includes heavy states
- Isentropes in the corresponding phase diagram give
  an orientation on the experimental window $(T–\mu$ or $\epsilon–\rho)$ of experiments
- Specific model predictions:
  - Non-monotonic latent heat $\Delta\epsilon$ due to change of contributions to Δs
  - **CBM** :
    1st-order regime (and even 2nd-order endpoint) seems in reach!

4. Conclusions – Outlook

Include full resonance spectrum (*work in progress*)

$\leftrightarrow$ also allow for re-scaling of masses according to lQCD calculations

$\rightsquigarrow$ Calculate corresponding observables for heavy-ion collisons, e.g.:

- Use EoS in dynamical hydro simulations
- Extract chemical freeze-out parameters

* What about effects due to finite system ($\rightarrow$fireball) size ???

$\Rightarrow$ Whishes (from *or* for **CBM**, from audience) ?!?

Thanks for your attention

Thanks to Detlef Zschiesche and Stefan Schramm, to Horst Stöcker,
and especially to Adrian Dumitru and Jørgen Randrup/**CBM**