Recent lattice results on QCD thermodynamics

(a short and personally biased overview)

Christian Schmidt

FIAS Frankfurt Institute for Advanced Studies

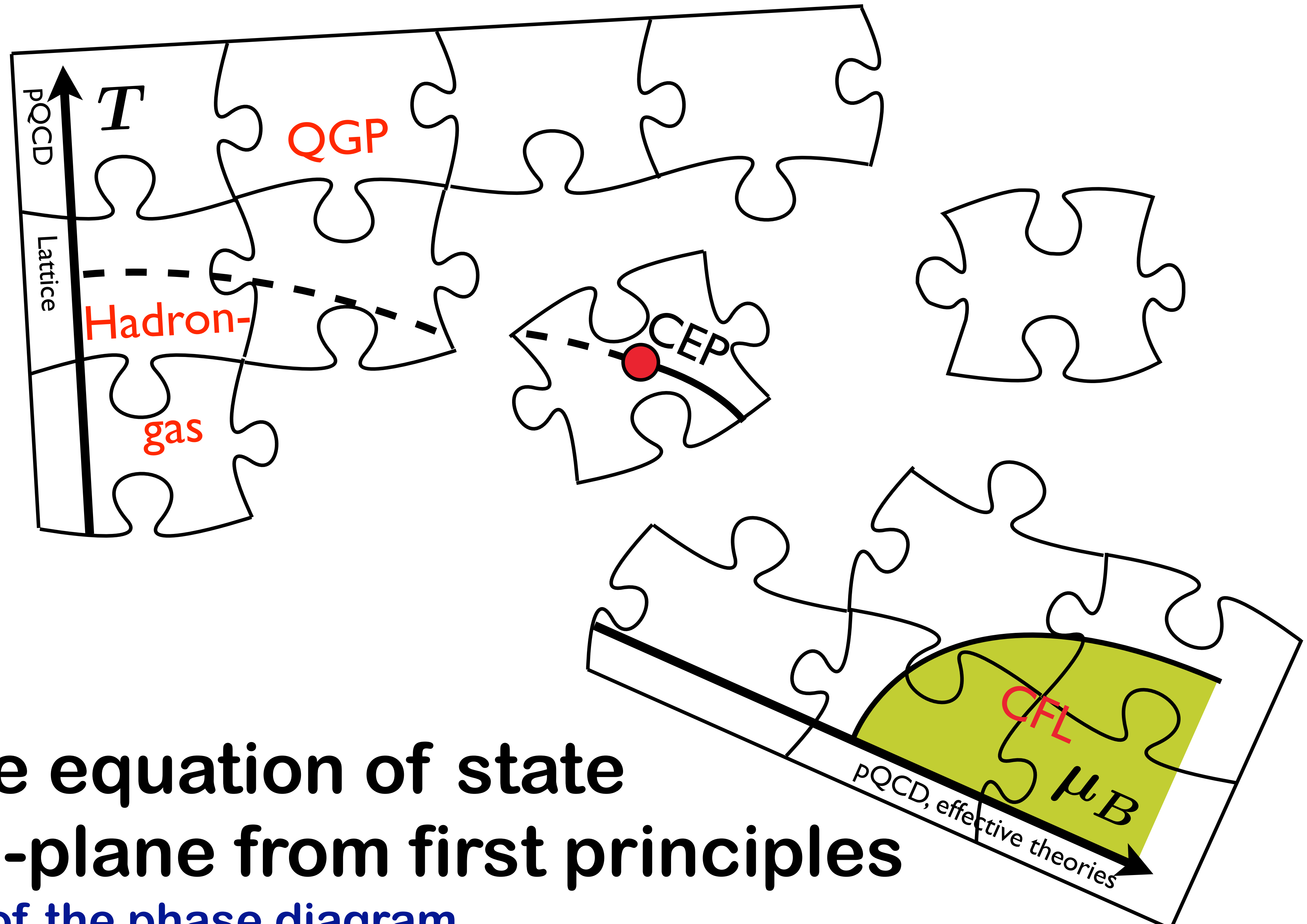
and

GSI Helmholtzzentrum für Schwerionenforschung

LOEWE – Landes-Offensive zur Entwicklung Wissenschaftlich-ökonomischer Exzellenz

HIC for FAIR
Helmholtz International Center

The lattice tasks

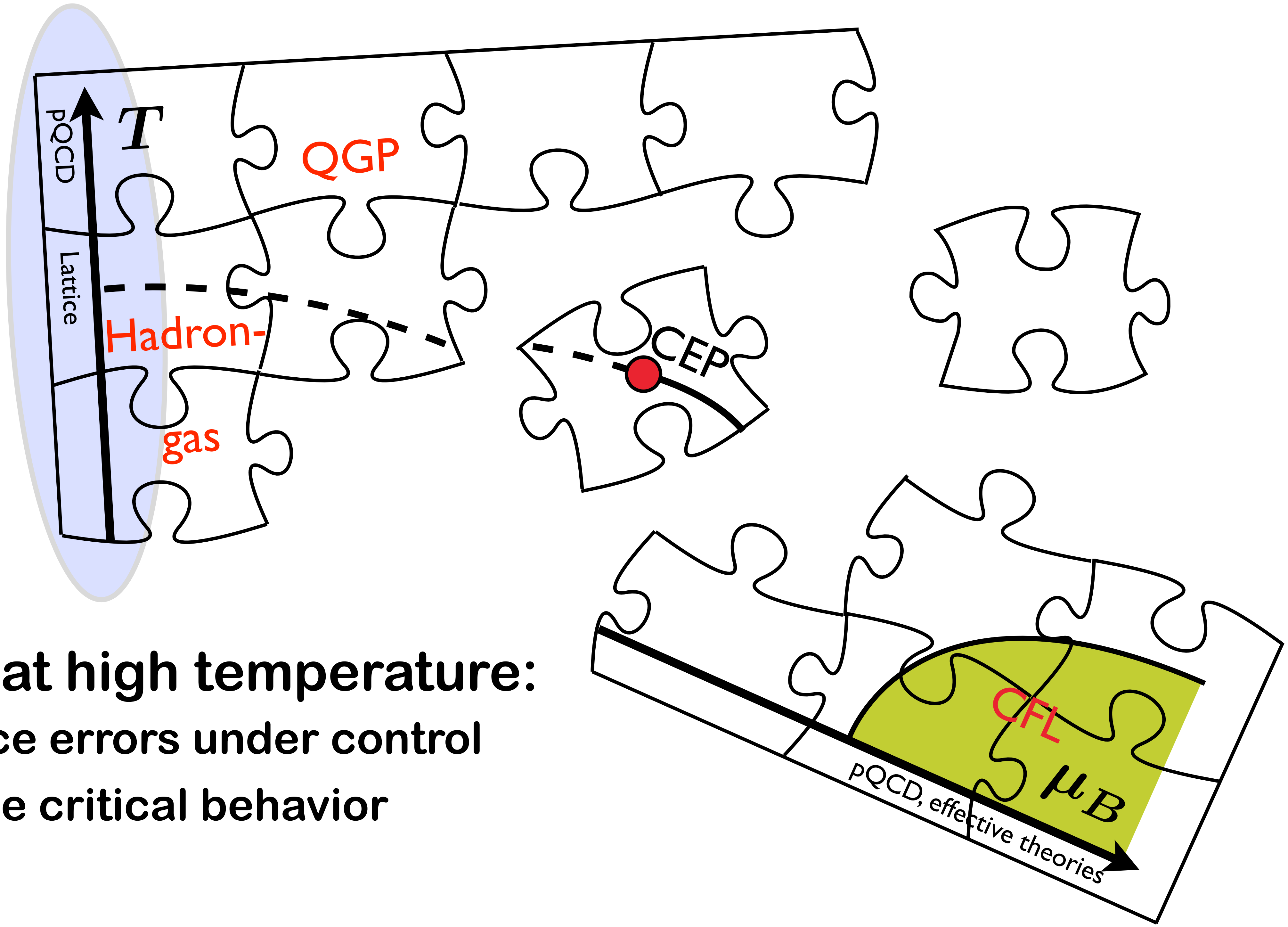


Analyzing the equation of state in the (T, μ_B)-plane from first principles

$\longrightarrow$ determination of the phase diagram

$\longrightarrow$ understanding underling mechanism of the transition

The lattice tasks

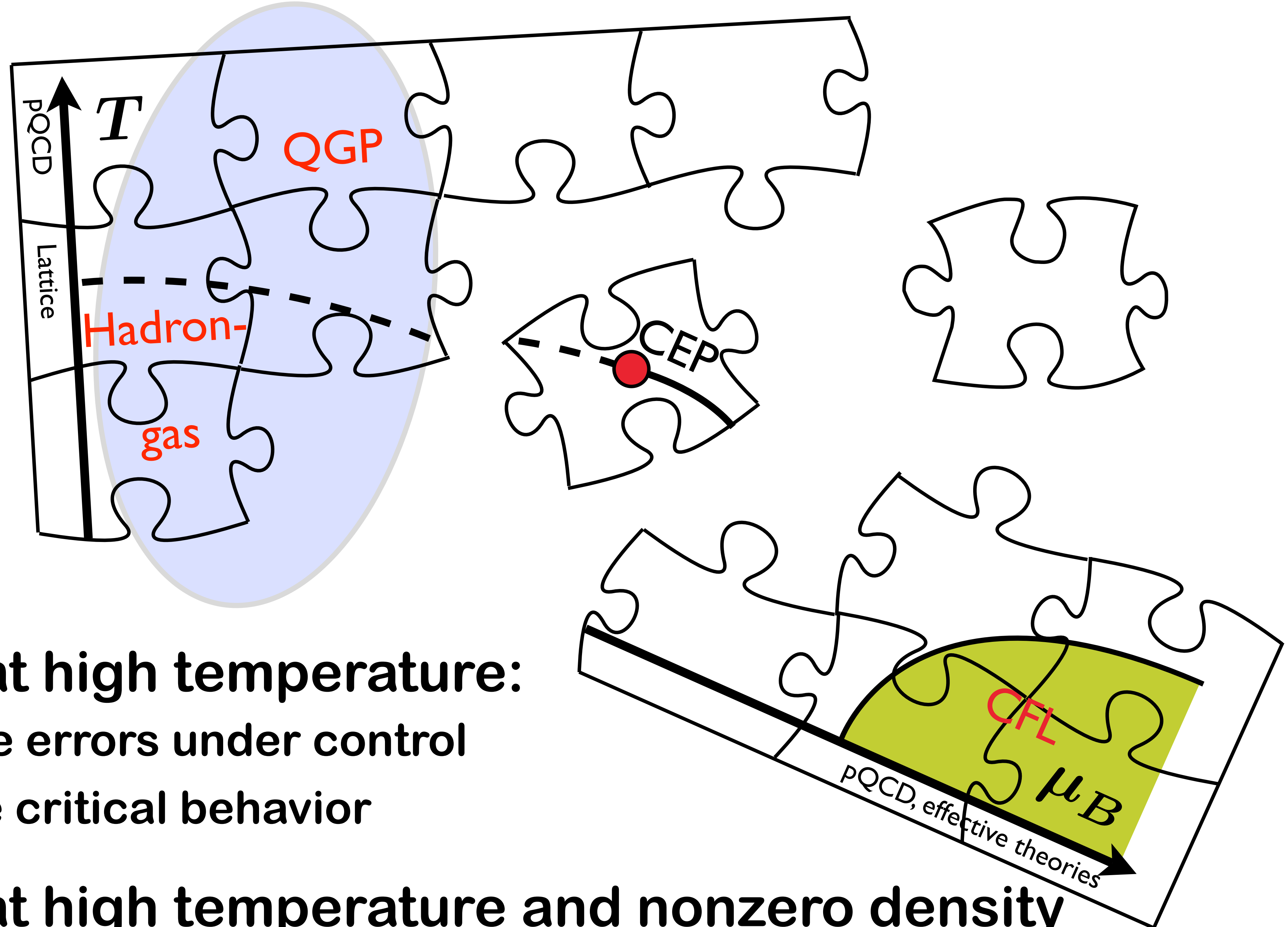


Overview:

★ **Lattice QCD at high temperature:**

- getting lattice errors under control
- analyzing the critical behavior

The lattice tasks



Overview:

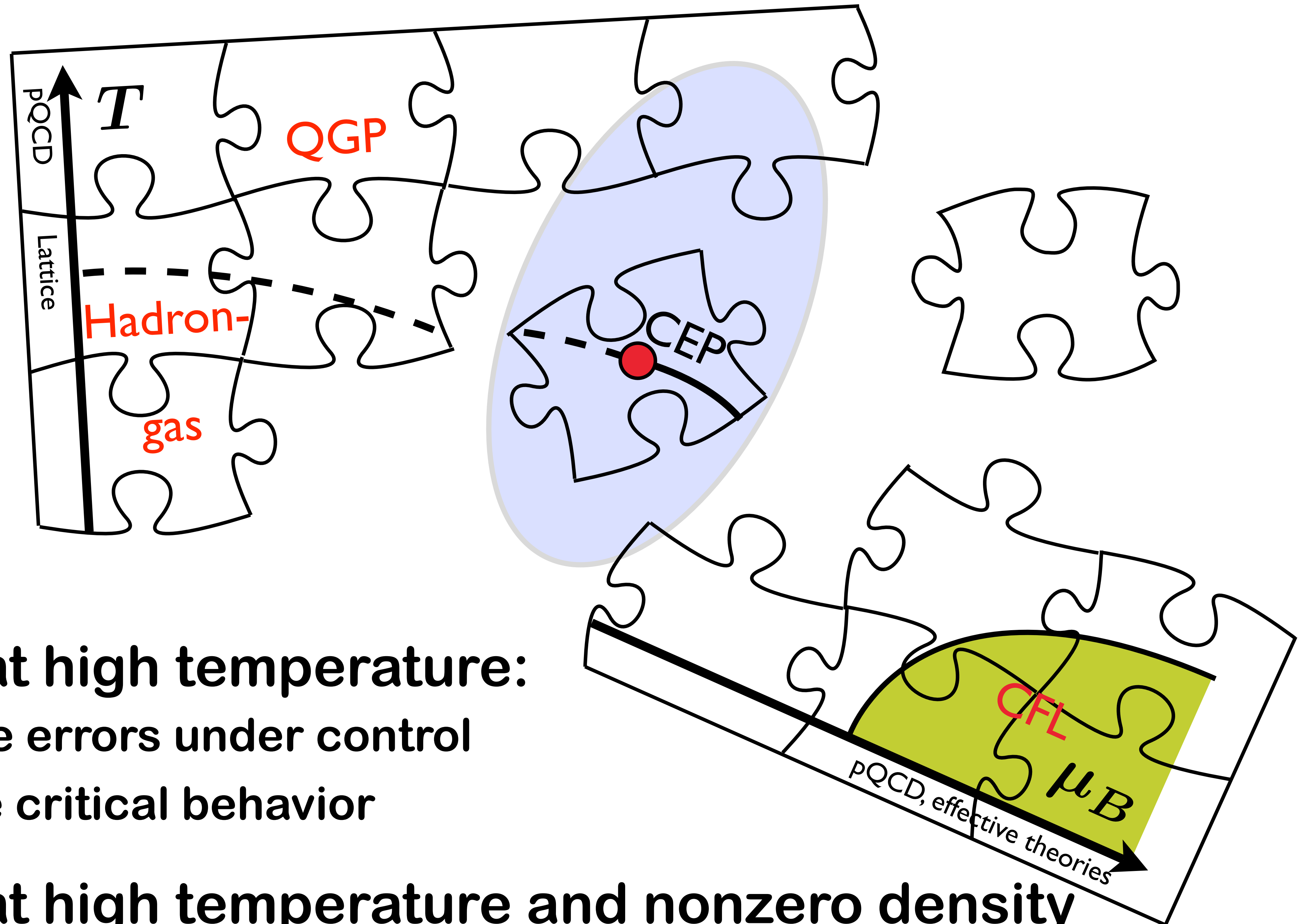
★ **Lattice QCD at high temperature:**

- **getting lattice errors under control**
- **analyzing the critical behavior**

★ **Lattice QCD at high temperature <u>and nonzero density</u>**

- **isentropic-EoS**
- **hadronic fluctuations**

The lattice tasks



Overview:

★ **Lattice QCD at high temperature:**

- getting lattice errors under control
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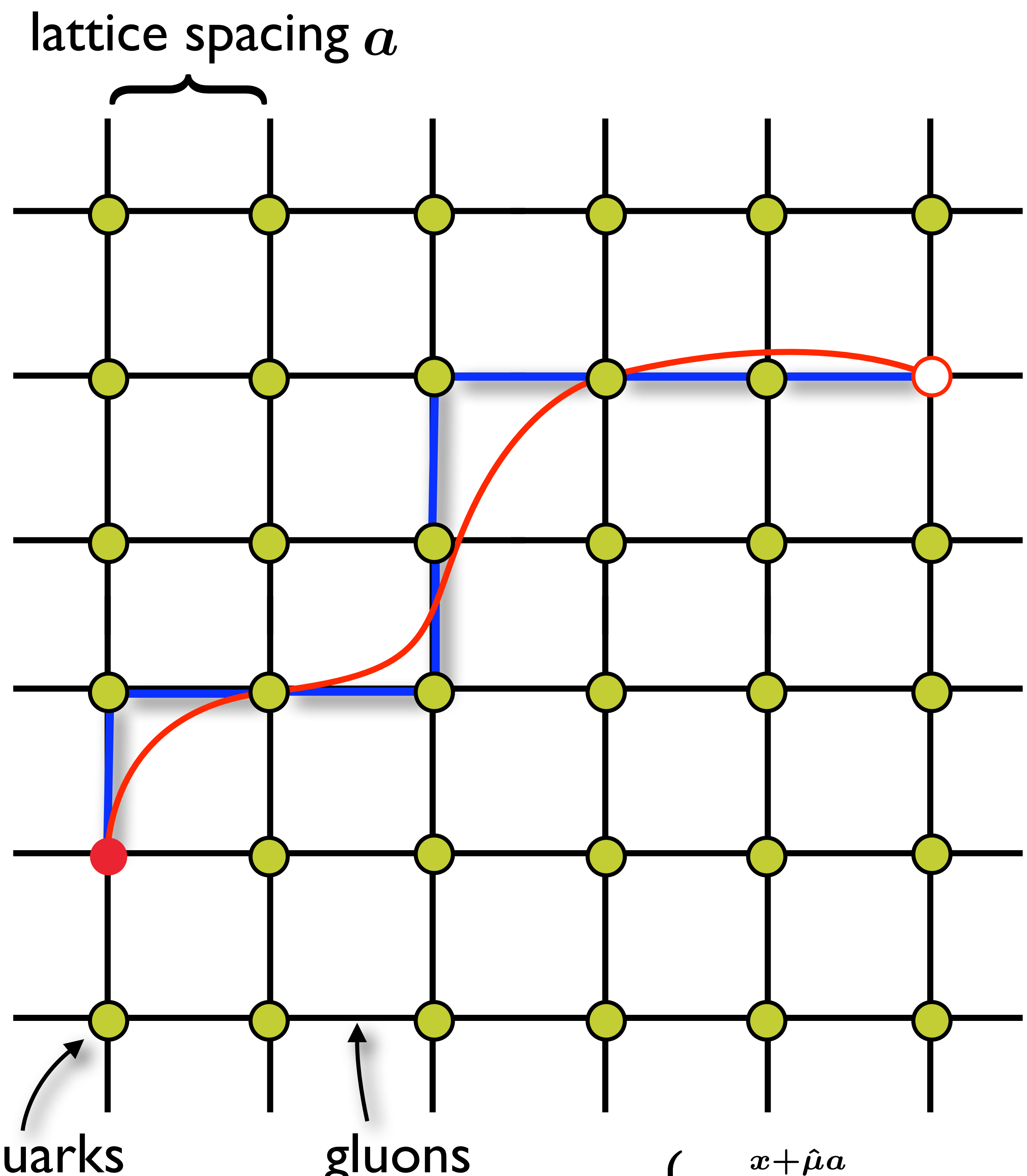
★ **Lattice QCD at high temperature and nonzero density**

- isentropic-EoS
- hadronic fluctuations
- update on the critical point determination

QCD on the lattice



$\psi(x), \bar{\psi}(x)$

$$U_\mu(x) = P \exp\left\{ ig \int_x^{x+\hat{\mu}a} dx_\mu A_\mu(x) \right\}$$

discretize space time and hence all „paths“ of quarks and gluons

lattice spacing: a

- continuum limit: $a \to 0$
- momentum cutoff $\mathcal{O}(1/a)$
- observables in units of a

$\longrightarrow$ freedom of choosing the lattice action (QCD has to be recovered in the continuum limit)

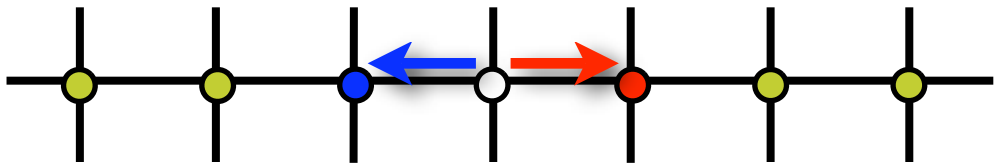
$\longrightarrow$ different lattice groups mainly differ by their choice of the lattice action

Two different improvements

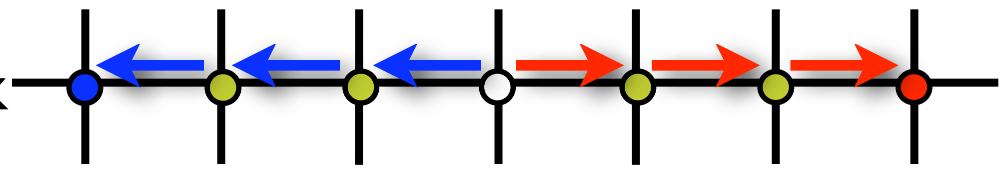
improving the dispersion relation ← **remove O(a²)-effects** → improving the flavor symmetry breaking

⟶ discretization of covariant derivative

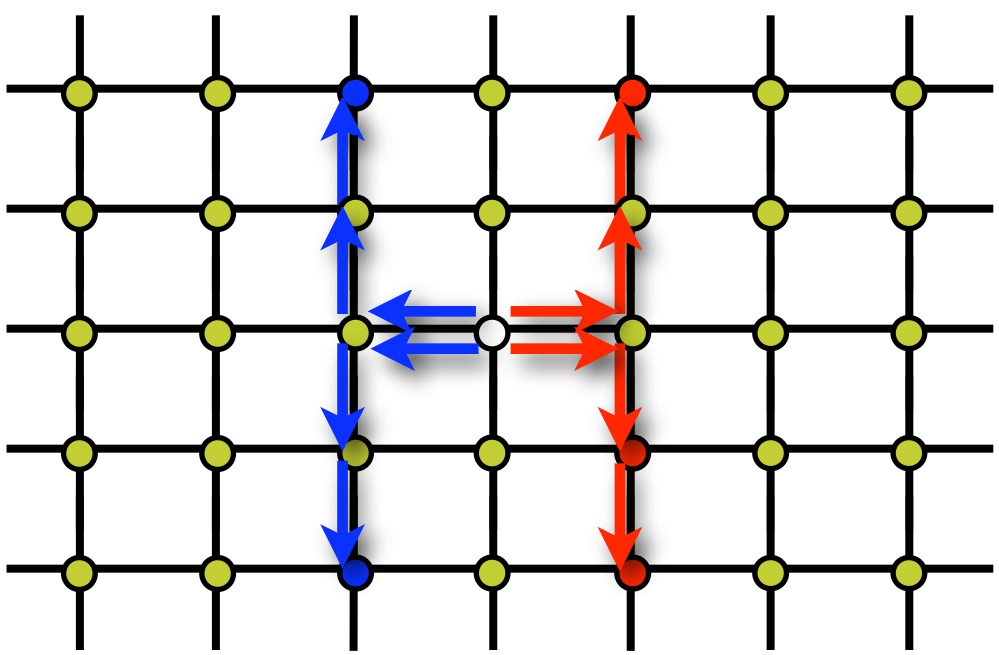
•std



•Naik



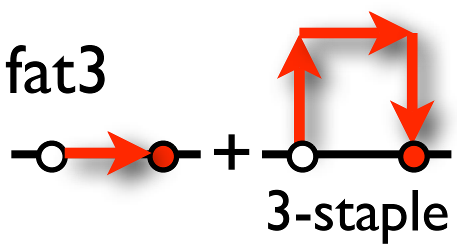
•p4



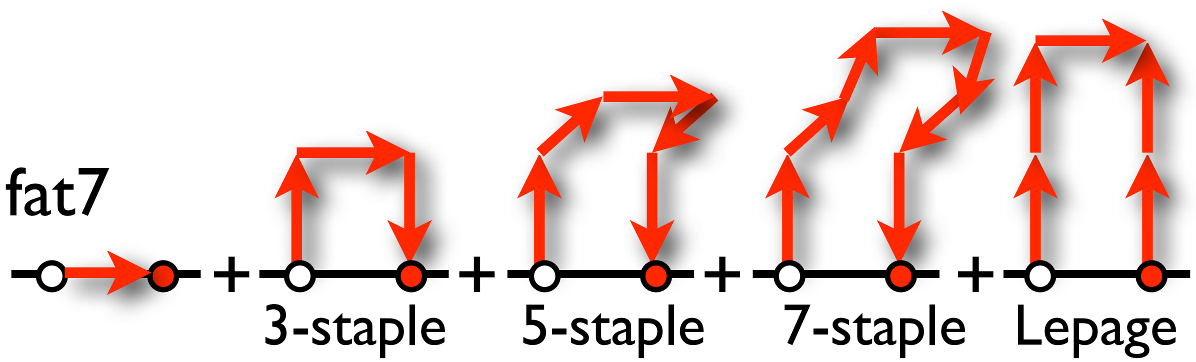
⟶ important to obtain the correct Stefan-Boltzmann limit

⟶ remove the high frequency modes

•fat3



•fat7



•multi-level smearing, where links remain in SU(3)

⟶ important to obtain the correct hadronic spectrum

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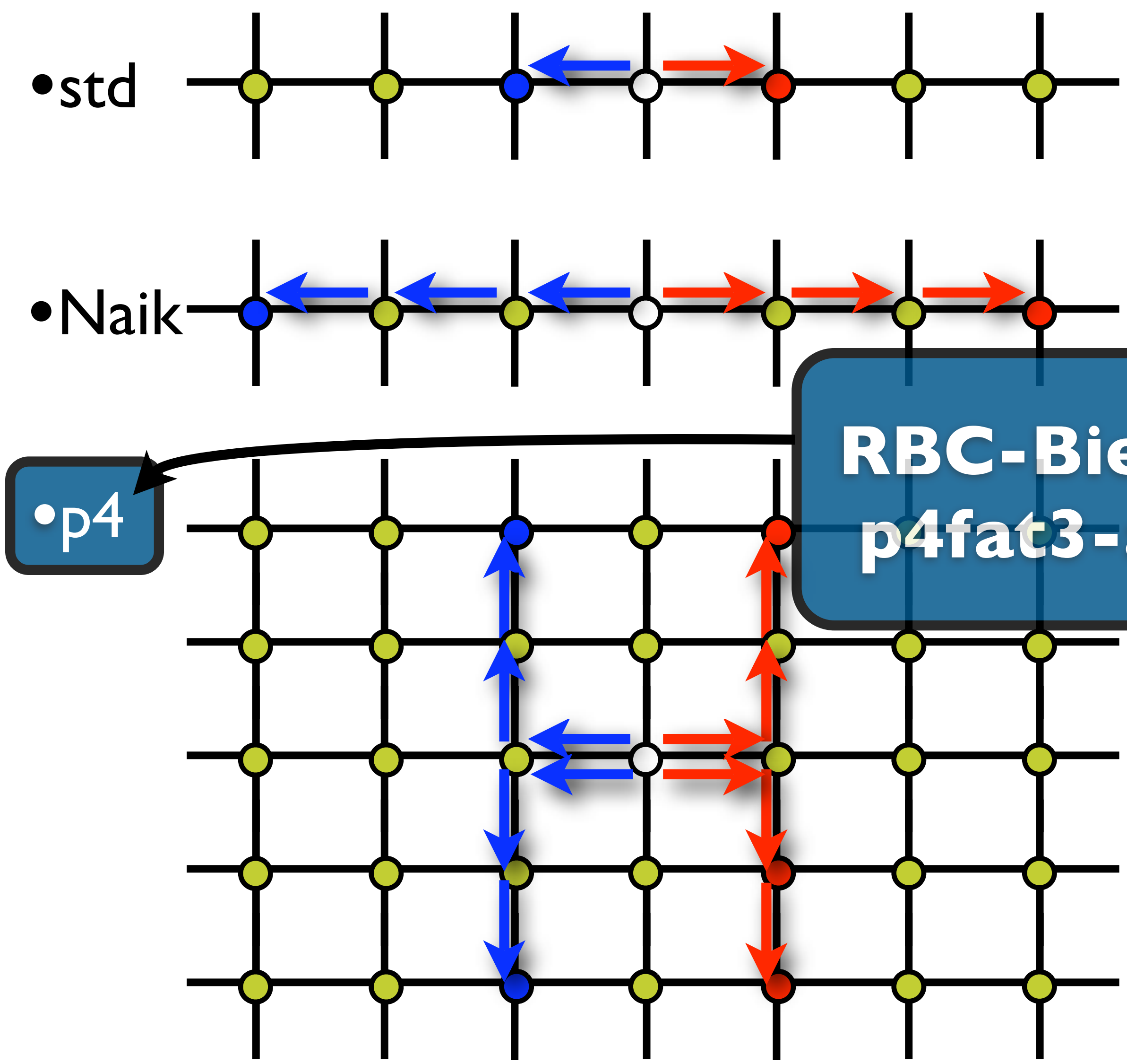
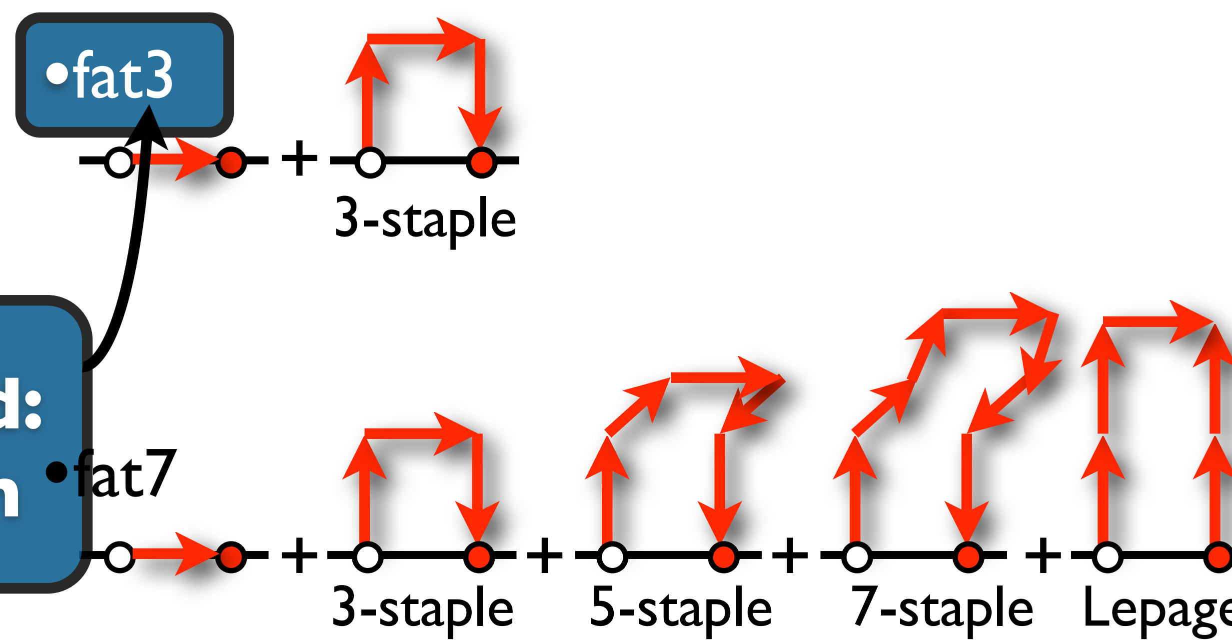


RBC-Bielefeld: p4fat3-action



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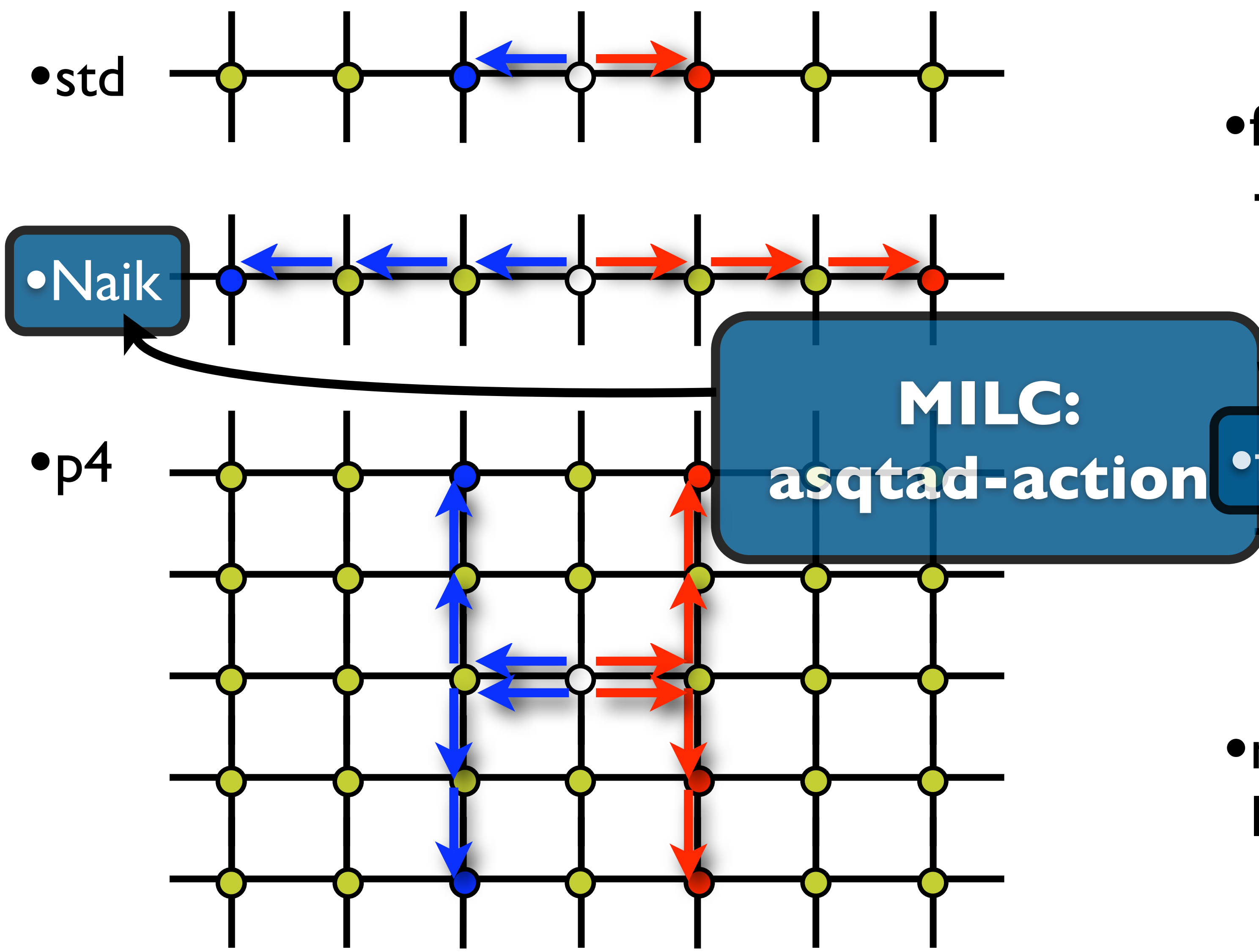
improving the dispersion relation

→ discretization of covariant derivative



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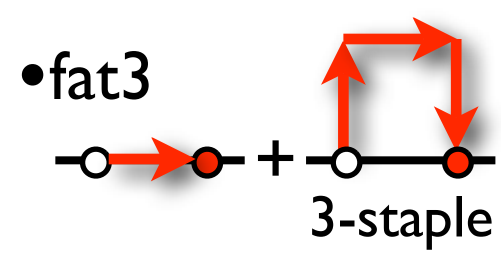

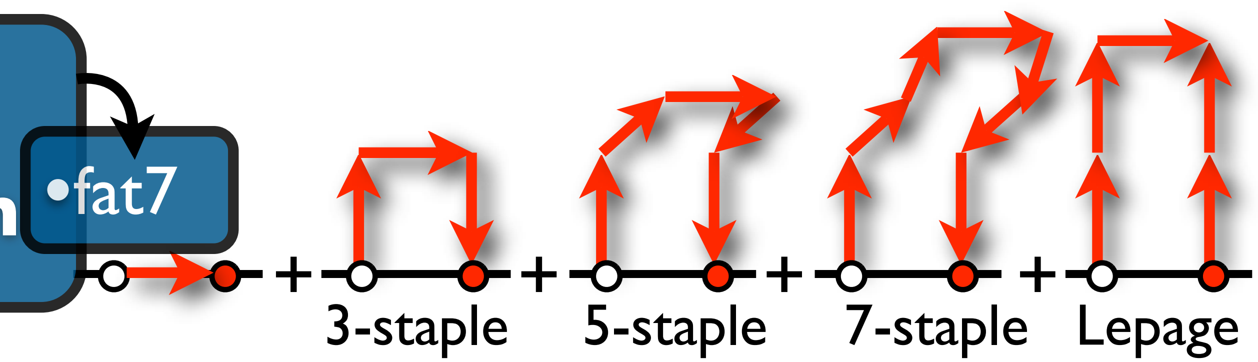

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Two different improvements

remove $O(a^2)$-effects

improving the dispersion relation

improving the flavor symmetry breaking

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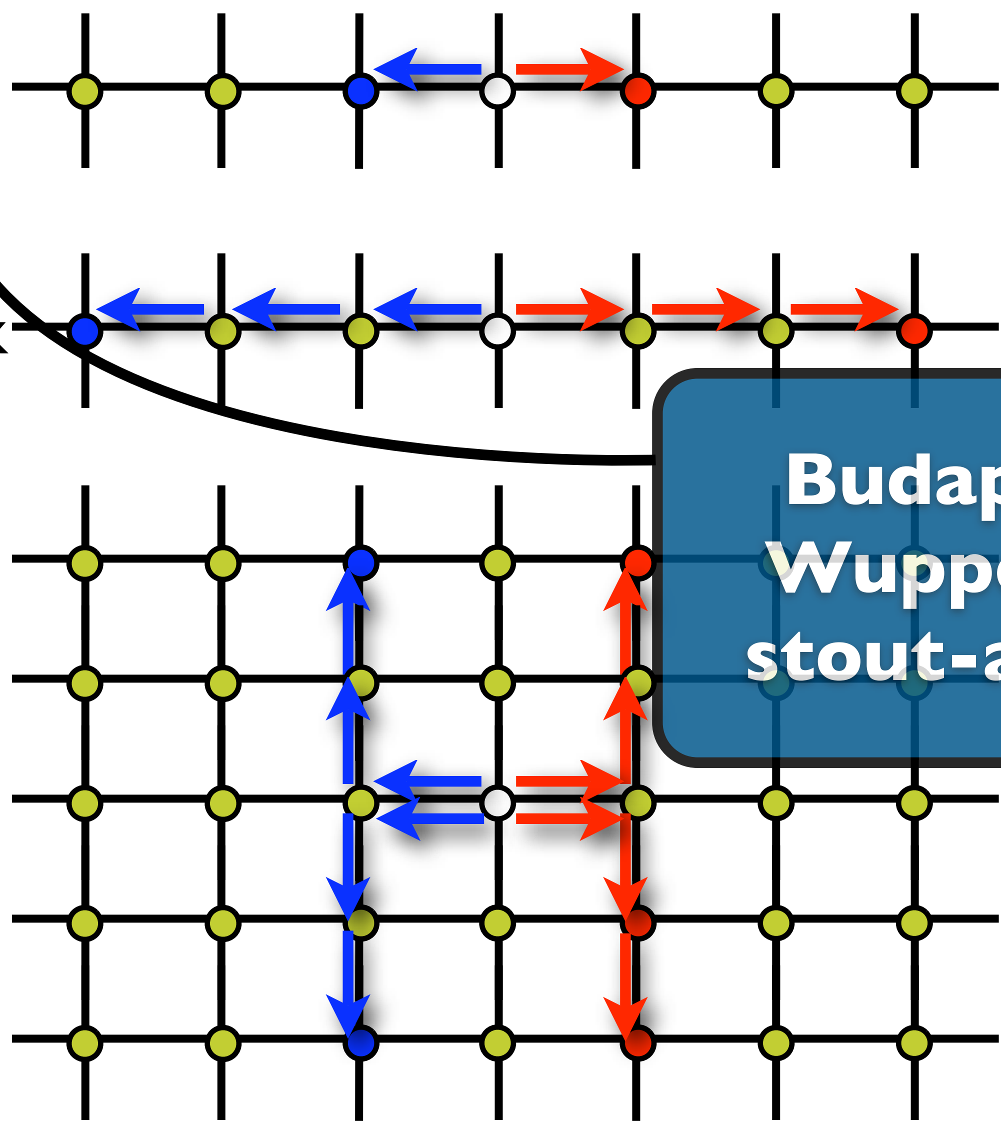

Budapest-Wuppertal: stout-action

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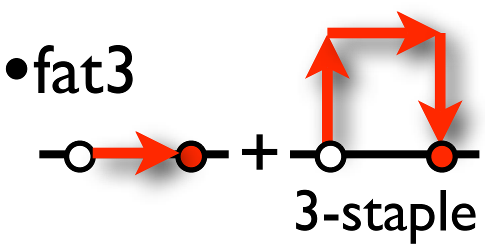

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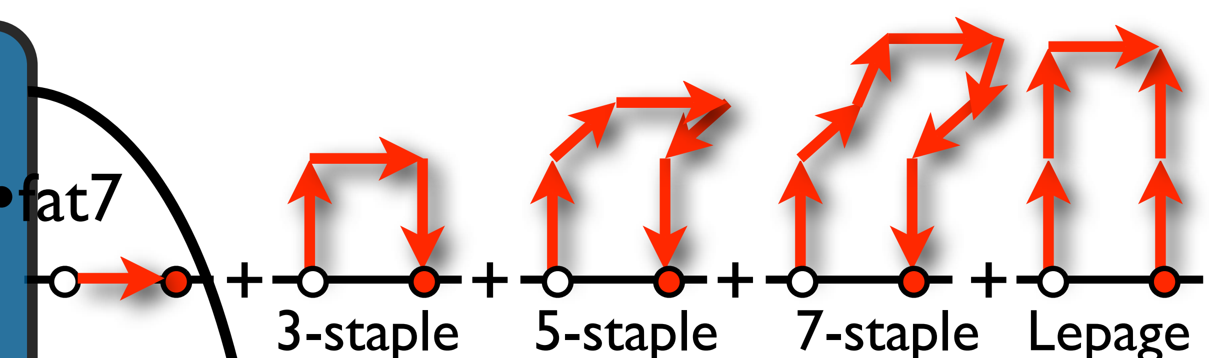

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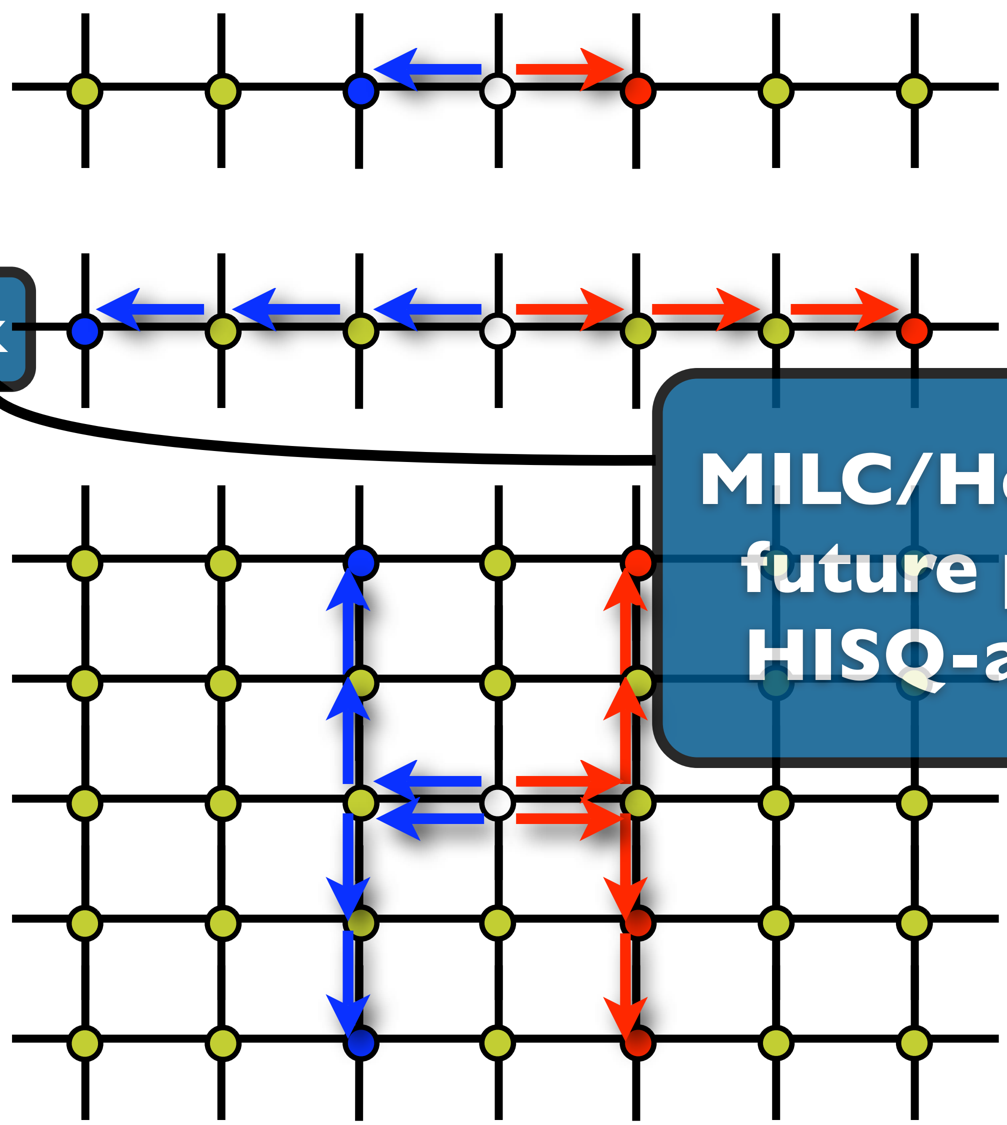

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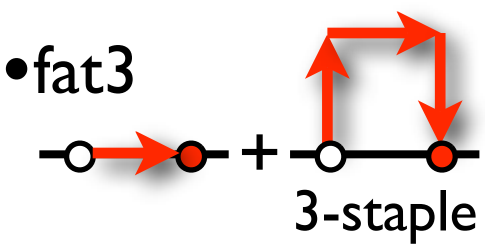

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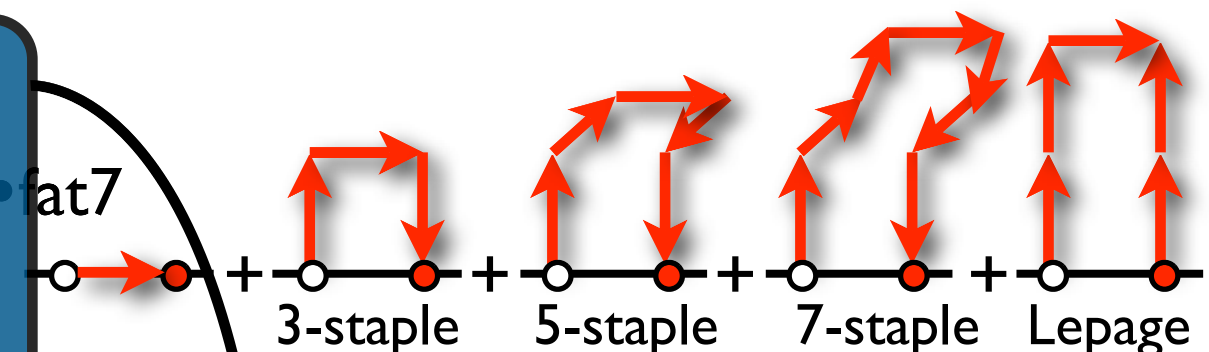


MILC/HotQCD future plans: HISQ-action

•multi-level smearing, where links remain in SU(3)

⟶ important to obtain the correct Stefan-Boltzmann limit

⟶ important to obtain the correct hadronic spectrum

Simulation parameters

- N_f=2+1: two degenerate u/d quarks + strange quark
- RHMC algorithm
- two lines of constant physics: $m_l/m_s = 0.1$, $m_l/m_s = 0.05$

$\downarrow$ $m_\pi \approx 220$ MeV, $\downarrow$ $m_\pi \approx 150$ MeV

- lattice size: $N_\sigma/N_\tau = 4$, $N_\tau = 4, 6, 8, 12$★

$$T = \frac{1}{N_\tau a}$$

$\downarrow$ $a = 0.25, 0.17, 0.13, 0.08$ fm

(at $T = 200$ MeV)

★$N_\tau = 6, 8$: HotQCD (asqtad, p4), Phys.Rev.D80:014504,2009.
BW (stout), Phys.Lett.B643:46-54,2006

$N_\tau = 12$: HotQCD (asqtad) ***preliminary***
BW (stout), JHEP 0906:088,2009.

The Collaborations

MILC + RBC-Bielefeld $\gtrsim$

HotQCD Collaboration:

A. Bazavov, T. Bhattacharya, M. Cheng, N. Christ, C. DeTar, S. Gottlieb, R. Gupta, P. Hegde, U. Heller, C. Jung, F. Karsch, E. Laermann, L. Levkova, C. Miao, R. Mawhinney, S. Mukherjee, P. Petreczky, D. Renfrew, C. Schmidt, R. Soltz, W. Söldner, R. Sugar, D. Toussaint, W. Unger, P. Vranas

BW Collaboration:

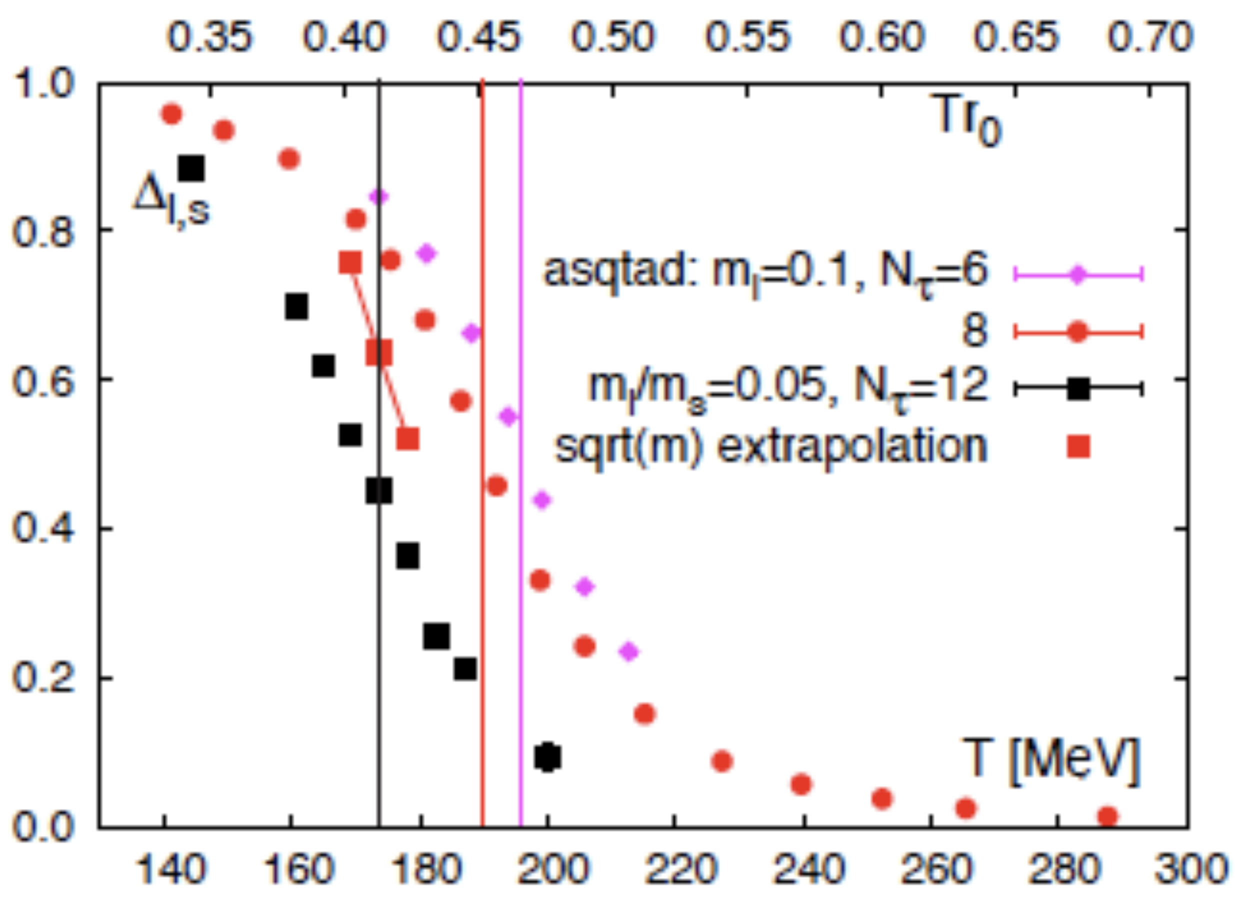
Y. Aoki, S. Borsanyi, S.Durr, Z. Fodor, S. Katz, S.Krieg, K.Szabo

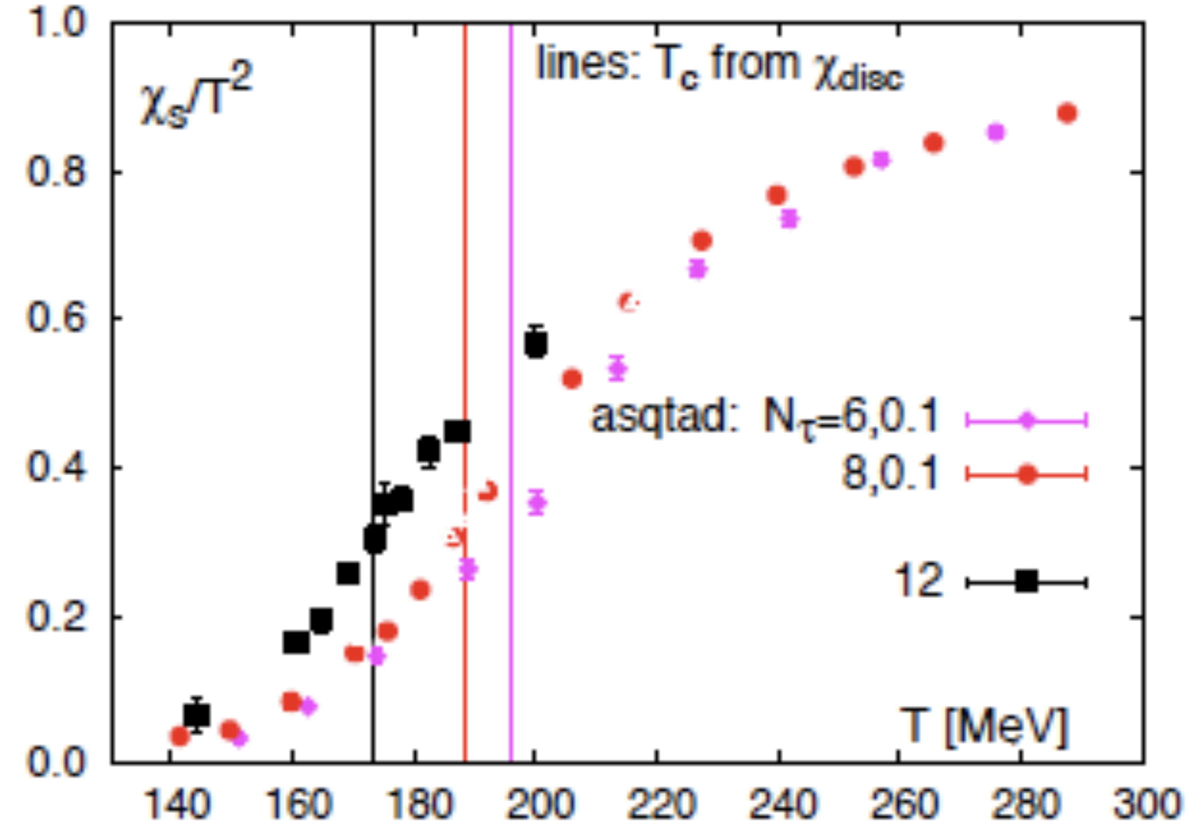
The transition at $\mu_B = 0$

χ-symmetry vs. deconfinement transition

$$\Delta_{l,s}(T) = \frac{\langle\bar\psi\psi\rangle_{l,T} - \frac{\hat m_l}{\hat m_s}\langle\bar\psi\psi\rangle_{s,T}}{\langle\bar\psi\psi\rangle_{l,0} - \frac{\hat m_l}{\hat m_s}\langle\bar\psi\psi\rangle_{s,0}}$$

$$\chi_s = \frac{1}{VT^3}\frac{\partial^2 \ln Z}{\partial \mu_s^2} = \frac{1}{VT^3}\left(\langle S^2\rangle - \langle S\rangle^2\right)$$





red boxes: $N_\tau = 12,\ m_l = 0.1 m_s$ through $\sqrt{m_l}$ extrapolation

at low T: $\sim \exp(-m_K/T)$

significant changes in the same temperature range

The transition at $\mu_B = 0$

asqtad (HotQCD) vs. stout (BW)

$$\Delta_{l,s}(T) = \frac{\langle\bar\psi\psi\rangle_{l,T} - \frac{\hat m_l}{\hat m_s}\langle\bar\psi\psi\rangle_{s,T}}{\langle\bar\psi\psi\rangle_{l,0} - \frac{\hat m_l}{\hat m_s}\langle\bar\psi\psi\rangle_{s,0}}$$

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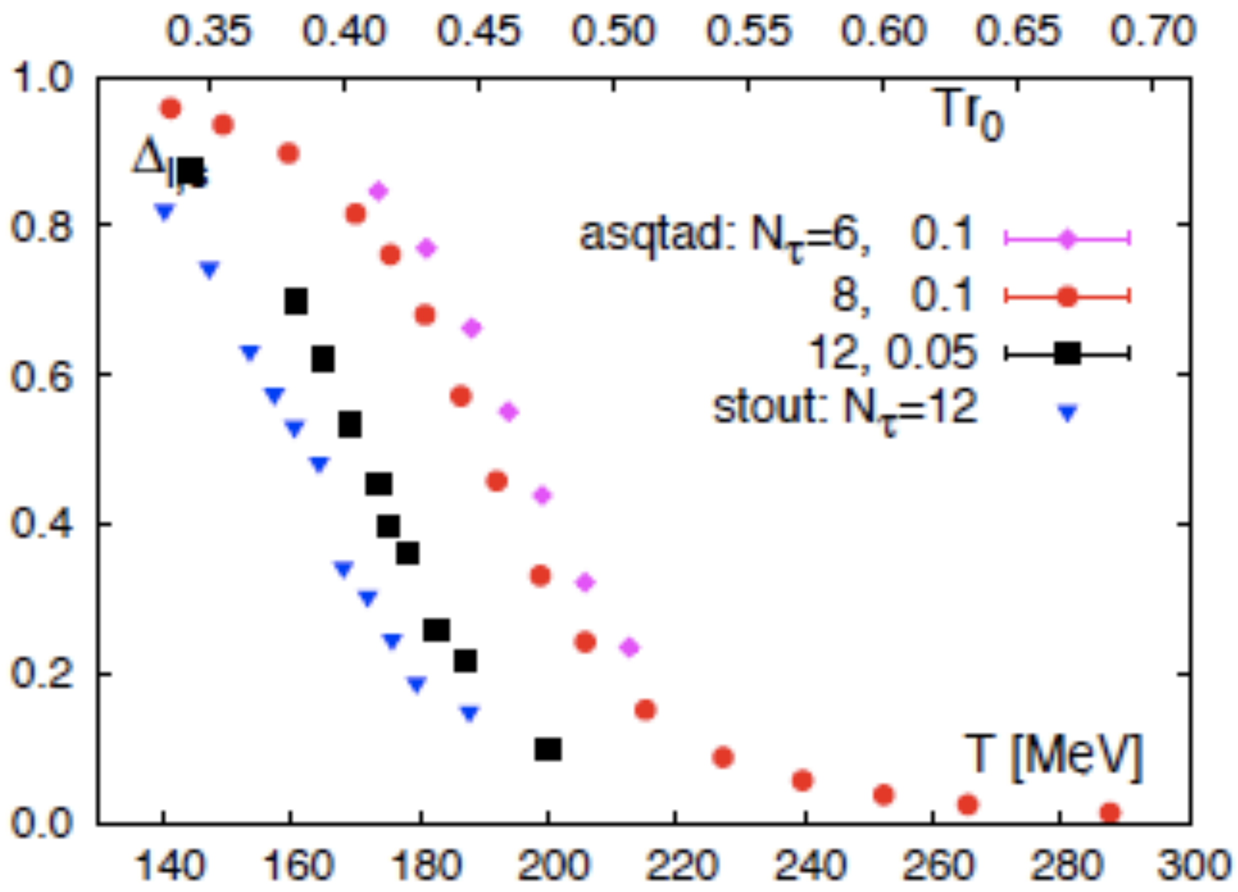
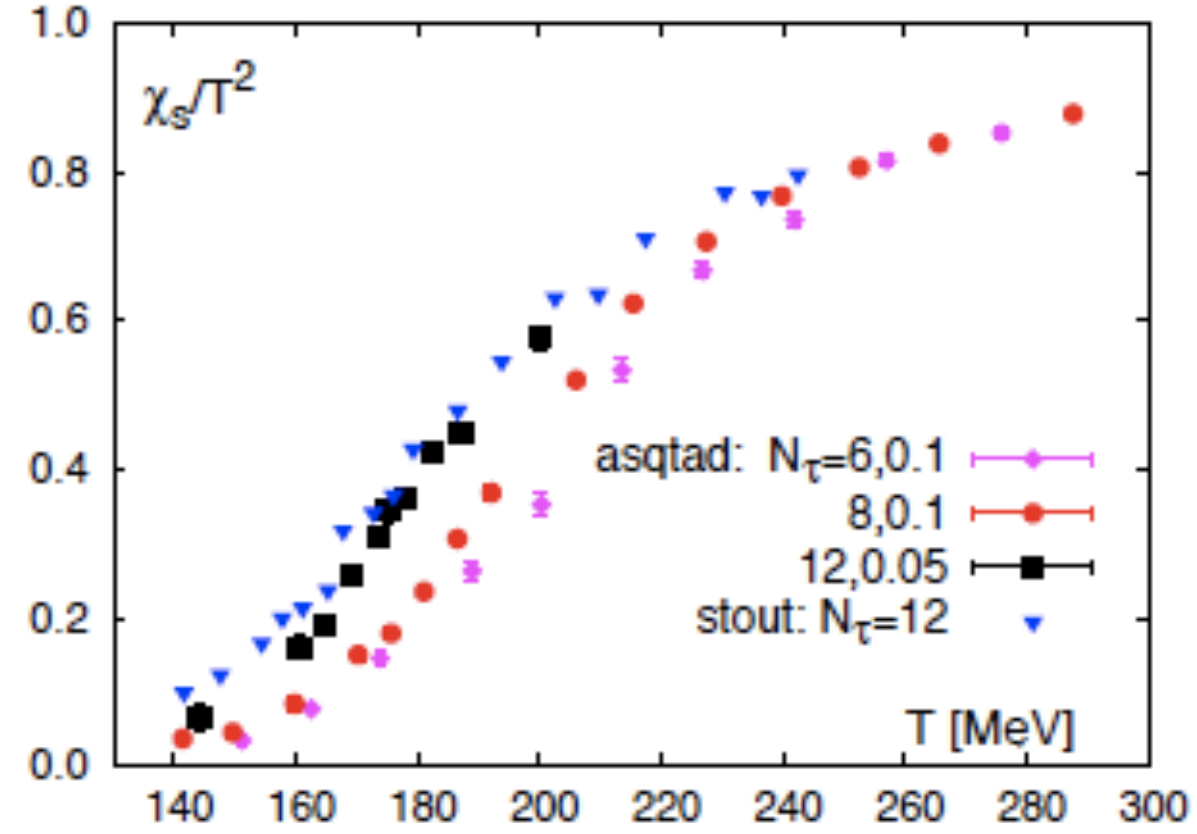
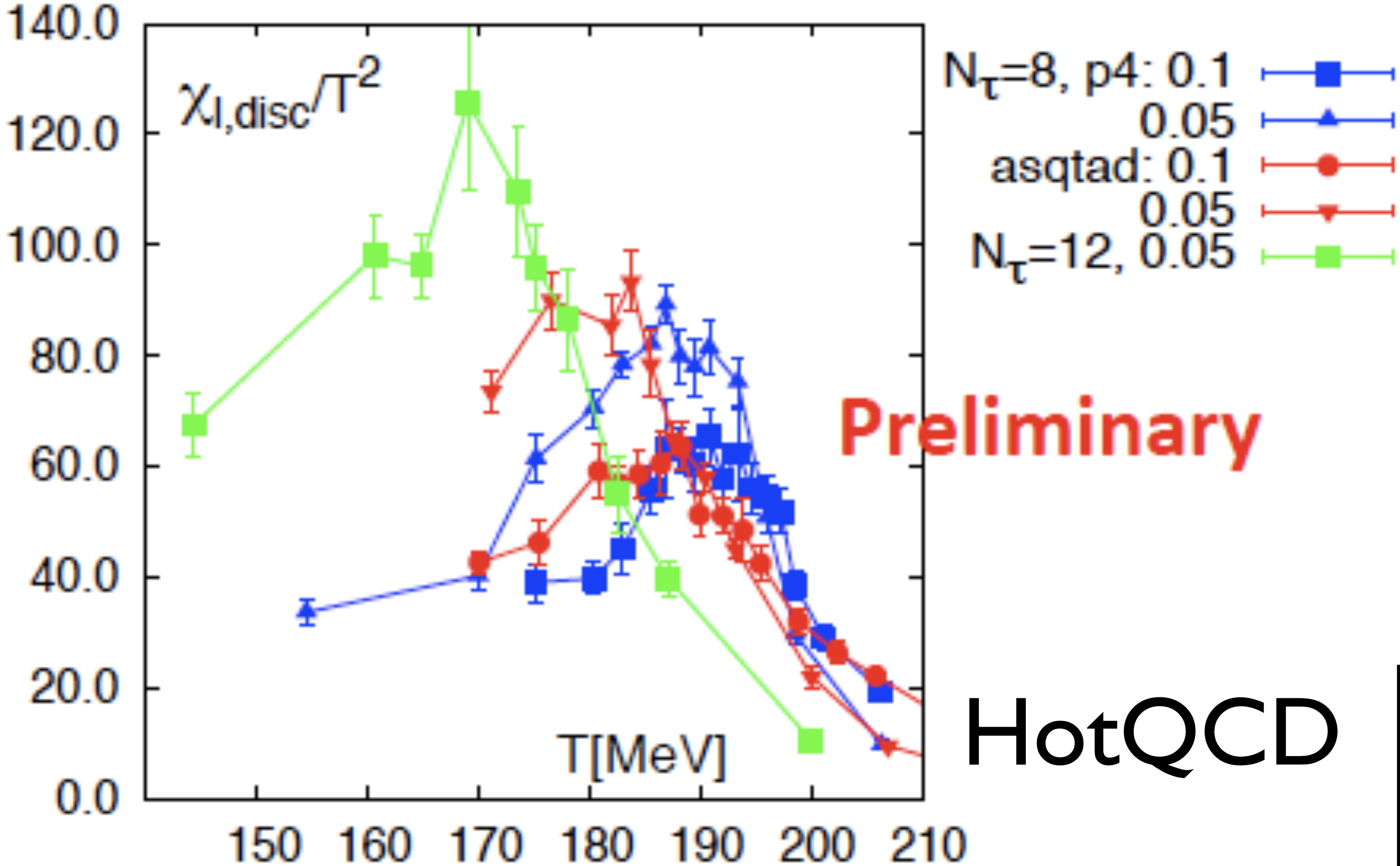




differences decrease with decreasing a

disconnected chiral susceptibility



HotQCD

$N_\tau = 8, m_l/m_s = 1/20$:
$T_c \approx (180 - 190)$ MeV

$N_\tau = 12$: suggests continuum extrapolated value < 170 MeV

work in progress

BW

continuum extrapolated values:

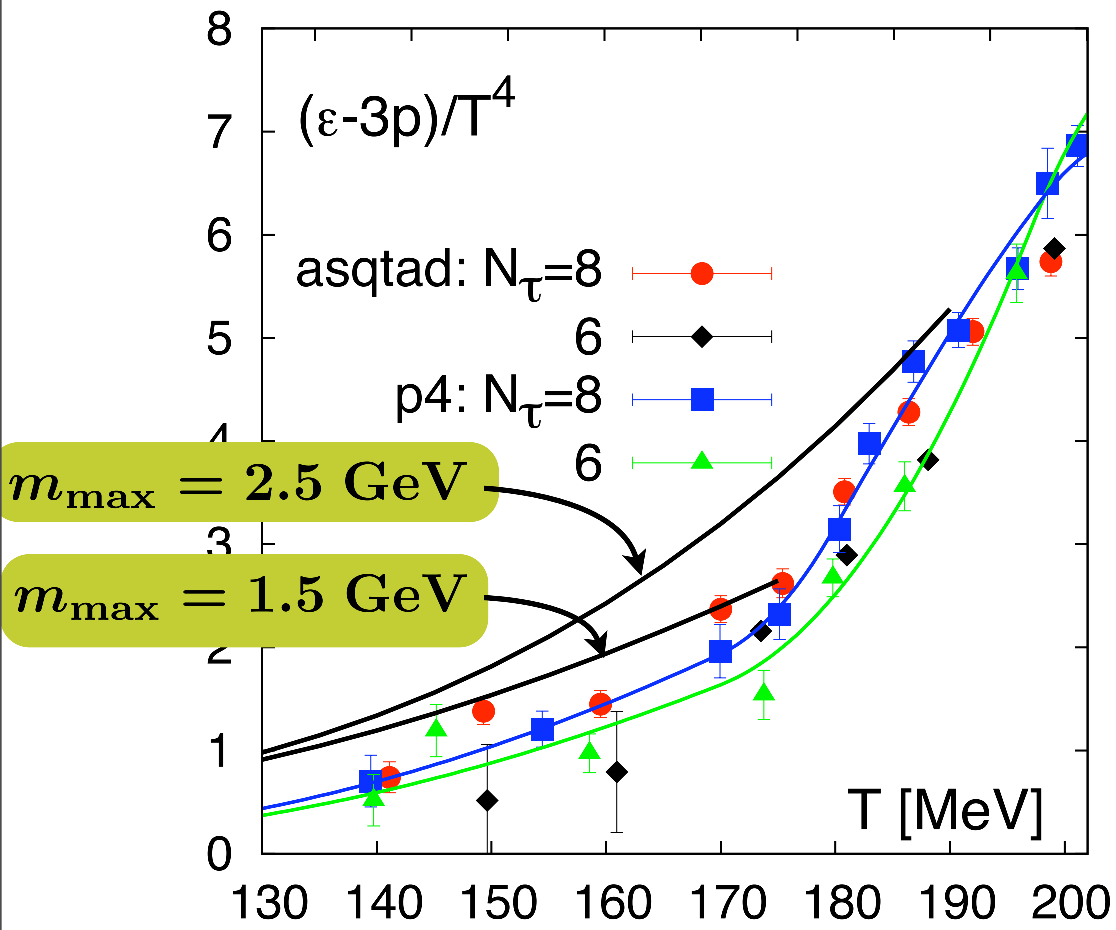
$T_c(\chi_{\bar\psi\psi}/T^2) = 152(3)(3)$

$T_c(\chi_S) = 169(3)(3)$

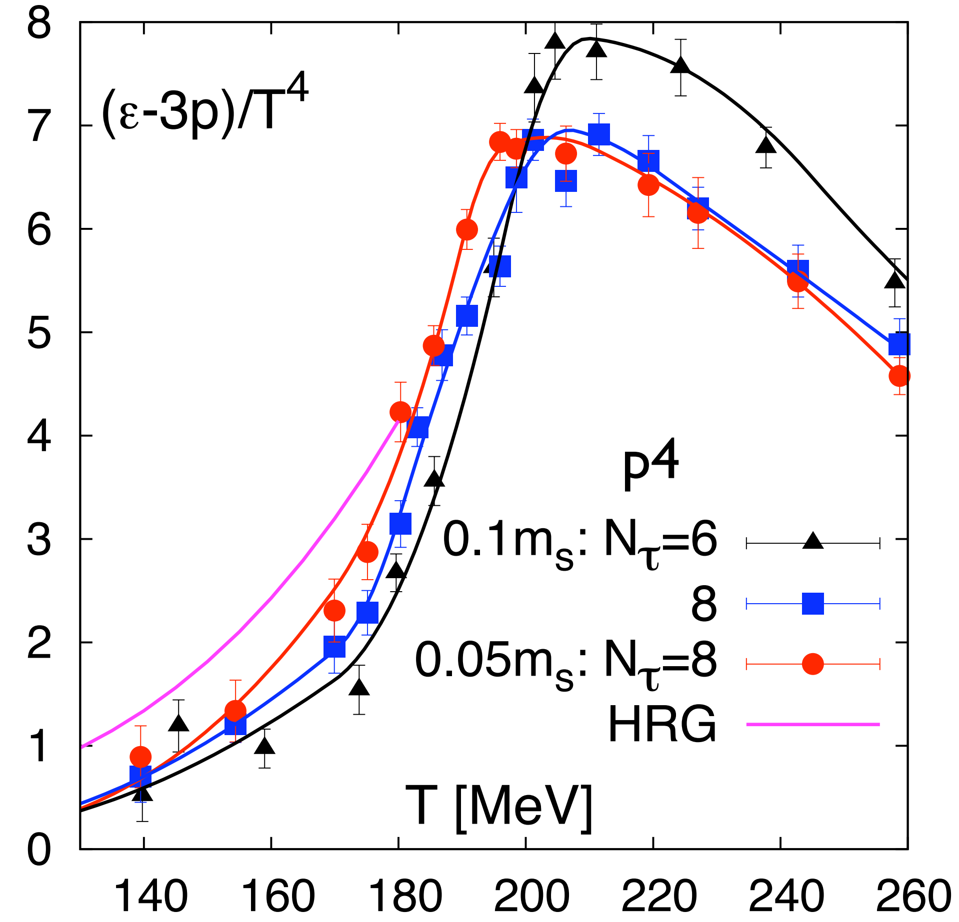
EoS at $\mu_B = 0$

(e-3p): lattice vs. the Hadron Resonance Gas

$$\left(\frac{\epsilon - 3p}{T^4}\right)_{low\,T} = \sum_{m_i \le m_{max}} \frac{d_i}{2\pi^2} \sum_{k=1}^{\infty} (\pm)^{k+1} \frac{1}{k} \left(\frac{m_i}{T}\right)^3 K_1(km_i/T)$$



HotQCD (asqtad, p4), Phys.Rev.D80:014504,2009.



RBC-Bielefeld (p4), Phys.Rev.D81:054504,2010.

non-negligible contribution of heavy resonances in HRG

reducing discretization effects or quark mass lowers crossover termerature

EoS at $\mu_B = 0$

(e-3p): lattice vs. the Hadron Resonance Gas

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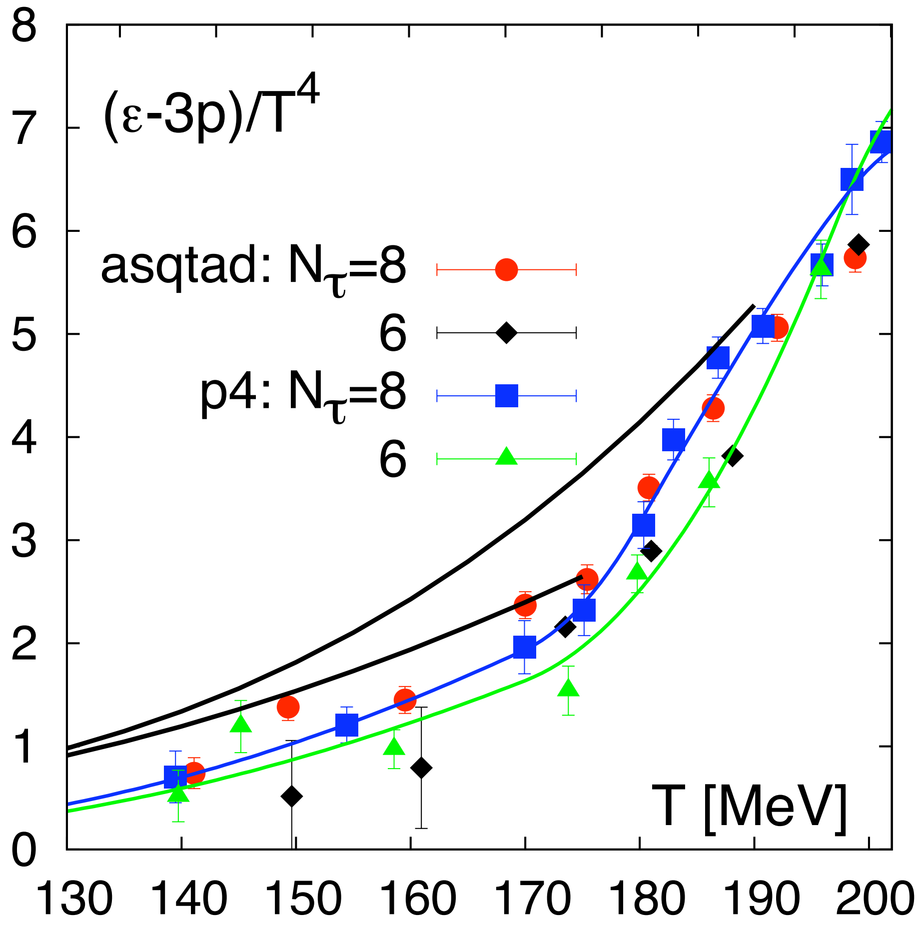
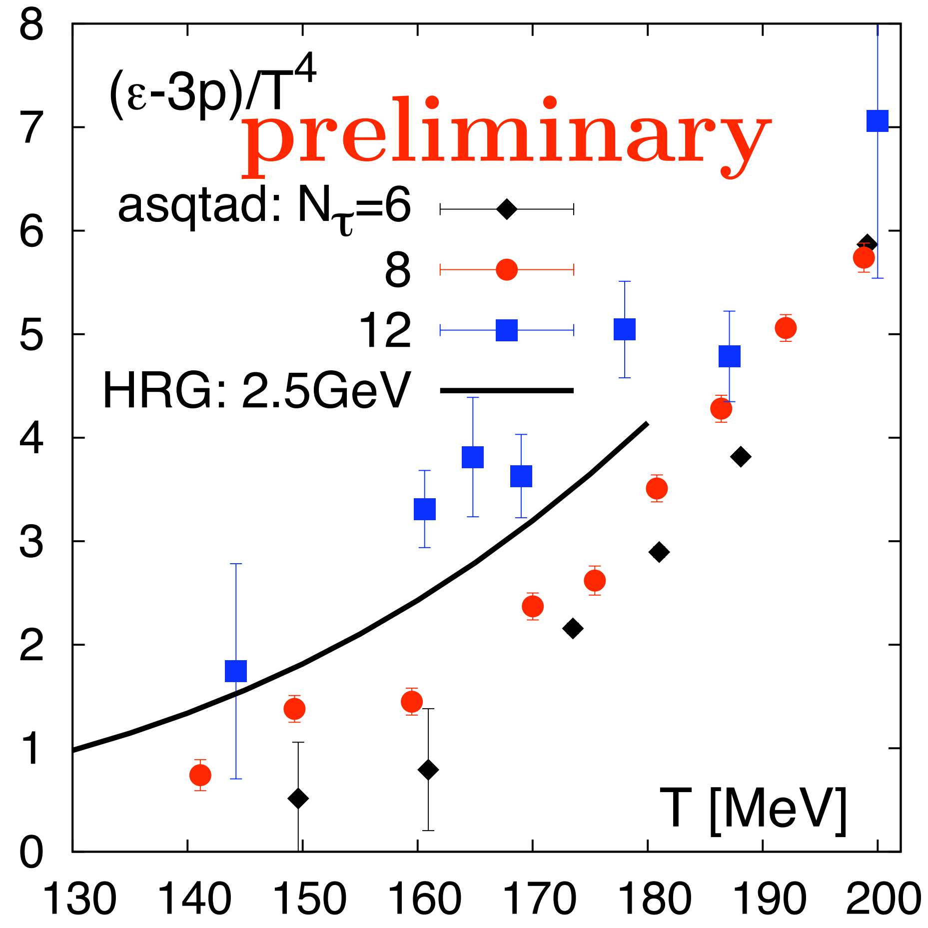


HotQCD (asqtad, p4), Phys.Rev.D80:014504,2009.



HotQCD (asqtad), preliminary

non-negligible contribution of heavy resonances in HRG

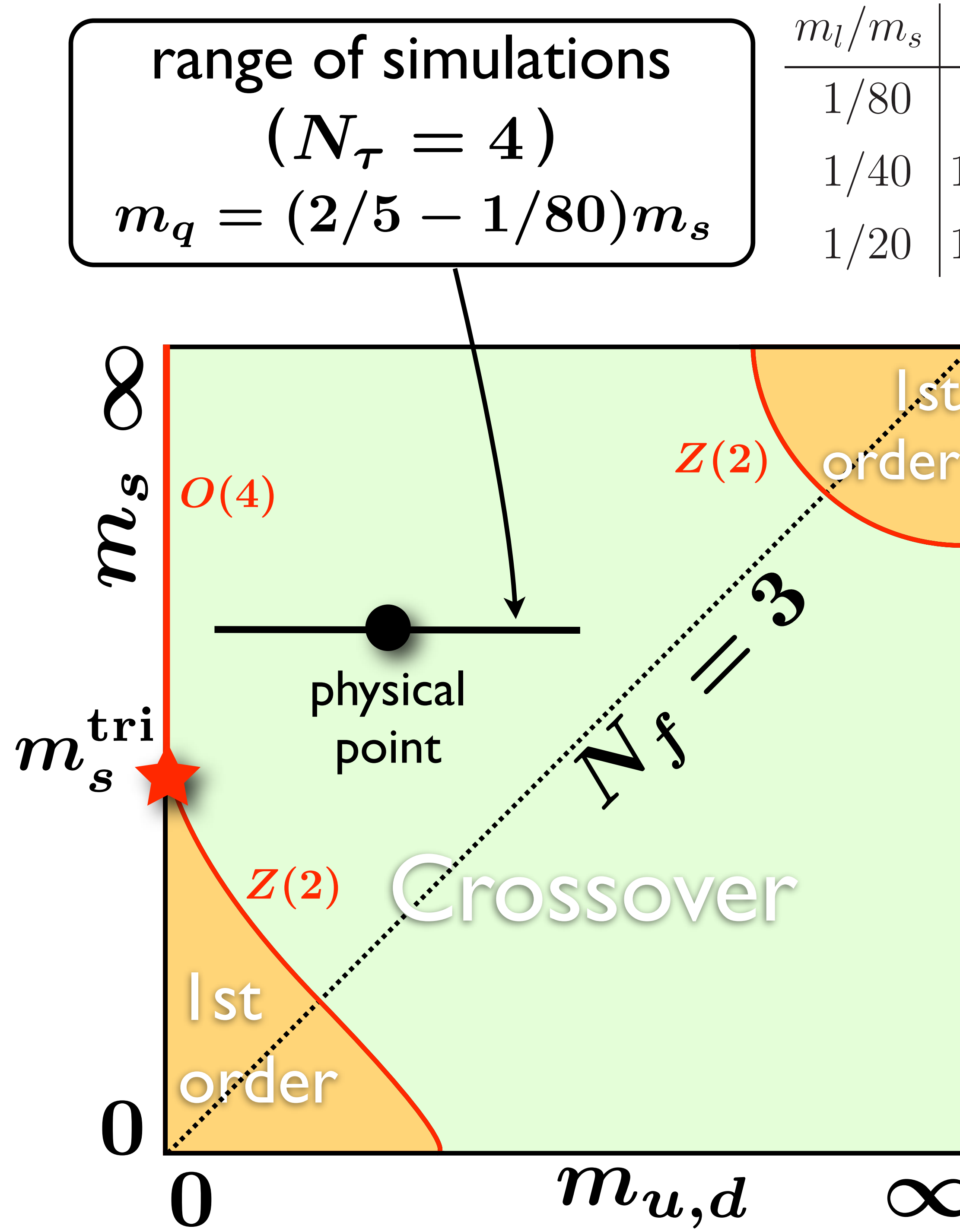
reducing discretization effects from $N_\tau = 8 \rightarrow 12$ seems to raise e-3p

Critical Behavior of QCD

Simulations with improved staggered fermions (p4fat3)

- chiral symmetry of 2-flavor QCD
  $SU_L(2) \times SU_R(2) \simeq O(4)$
- hence, if expect m_s is large in (2+1)-flavor QCD:
  expect universal behavior as of 3d-$O(4)$ spins in the vicinity of T_c and the chiral limit
- so far no clear evidence from simulations
- staggered fermions preserve a flavor non-diagonal $U(1)$-part of chiral symmetry even at $a > 0$
  $\longrightarrow$ look for $O(2)$-critical behavior

m_l/m_s	m_π
1/80	75 MeV
1/40	105 MeV
1/20	150 MeV



Magnetic EoS in $O(N)$-spin models

- order parameter:

  magnetization $M = h^{1/\delta} f_G(z)$

  universal scaling function

- scaling variable:

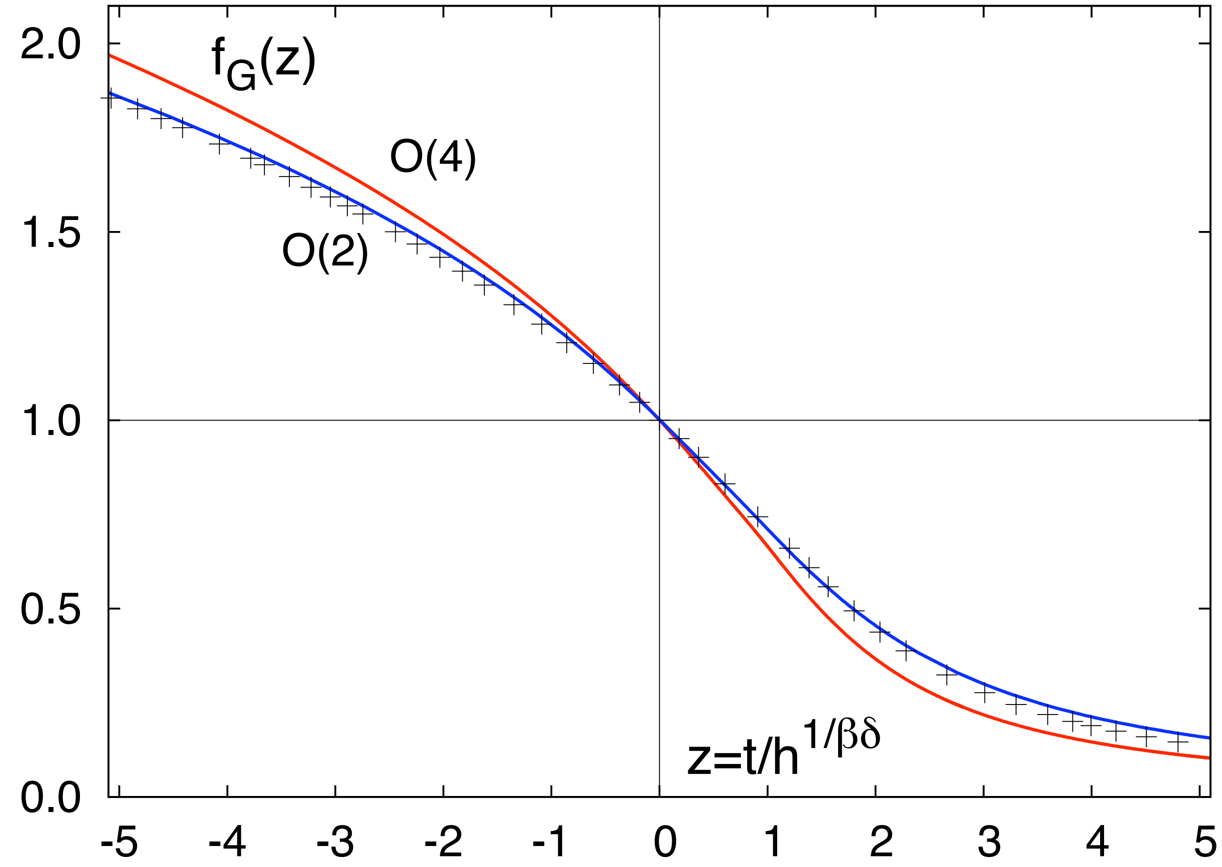
  $z = t/h^{1/\beta\delta}$

  where $t = \dfrac{1}{t_0}\dfrac{T - T_c}{T_c}$

  (reduced temperature)

  $h = \dfrac{H}{h_0}$

  (external field)



- scaling function and critical exponents are known to high precision in condensed matter literature [e.g. Engels *et al.*]

- scaling function includes Goldstone effect in the limit of $z \longrightarrow -\infty$

  $z \rightarrow -\infty: \quad h \rightarrow 0, t < 0 \quad M(t,h) = M(t,0) + c_2(t)\sqrt{h} + \cdots$

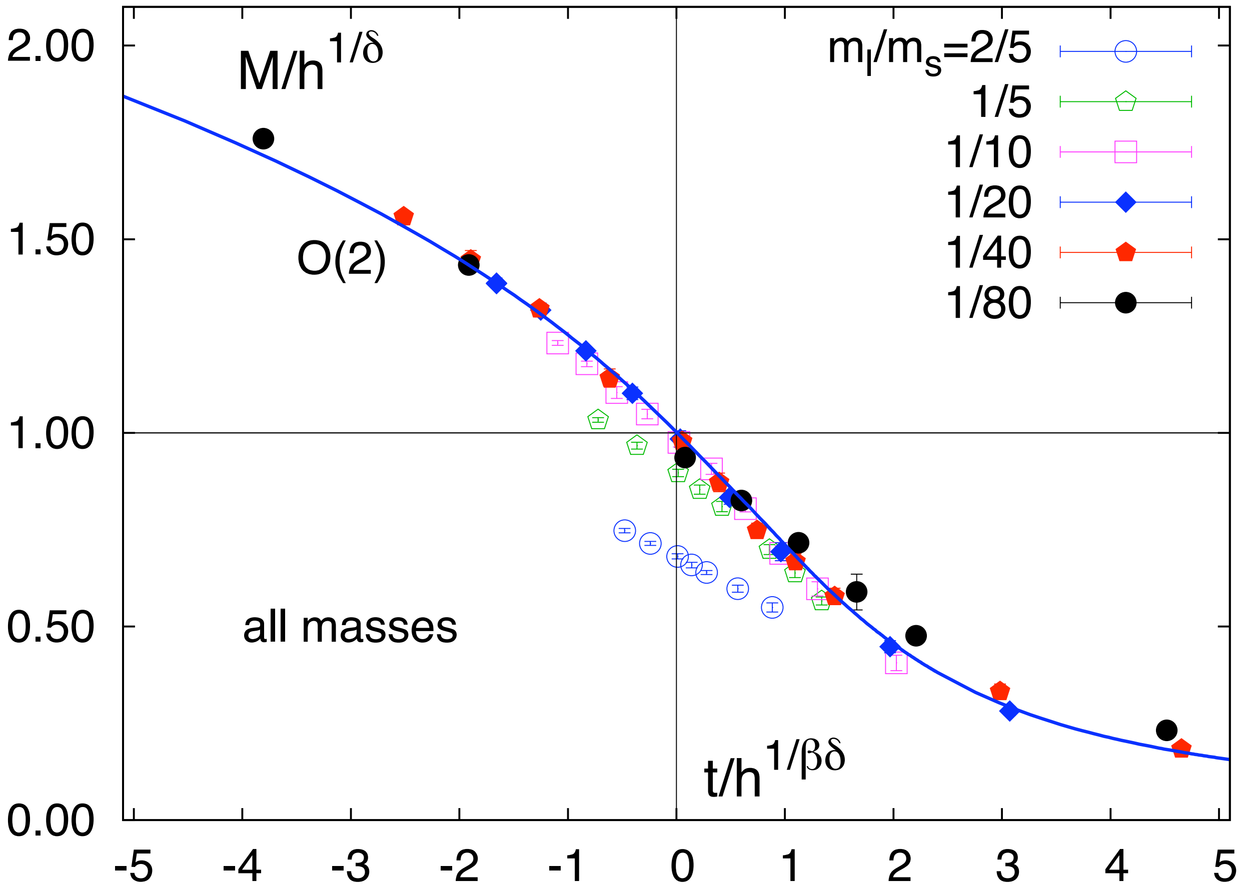
Magnetic EoS in QCD (Nt=4)

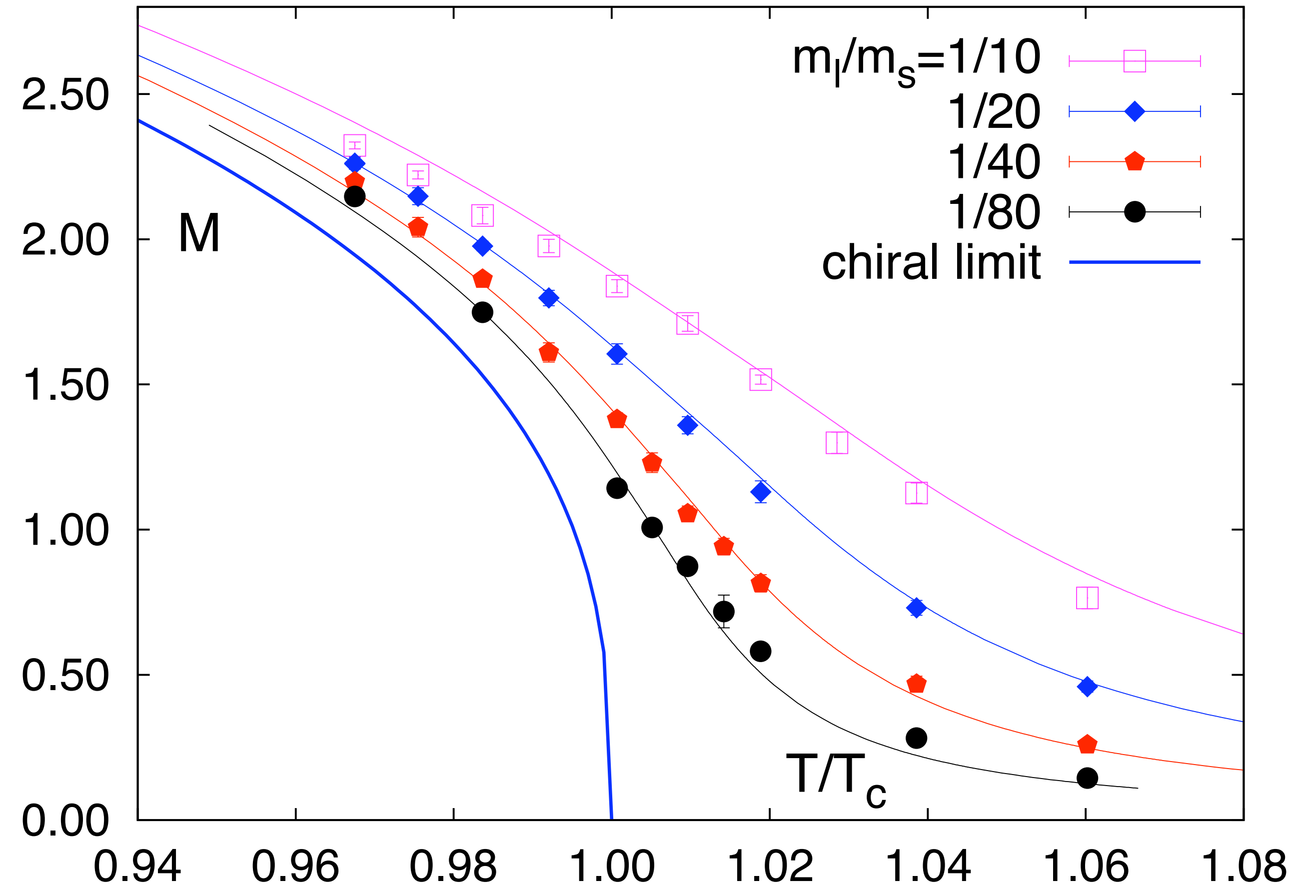
- order parameter: $M = m_s \left(\langle \bar\psi\psi \rangle_l - \frac{m_l}{m_s} \langle \bar\psi\psi \rangle_s \right) N_\tau^4 = h^{1/\delta} f_G(z)$ (chiral condensate)
- scaling variable: $z = t/h^{1/\beta\delta}$

$$t = \frac{1}{t_0} \frac{T - T_c}{T_c} \qquad h = \frac{1}{h_0} \frac{m_l}{m_s}$$

(quark mass)

non-universal constants, determined by fits to the data





→ good agreement with the O(2)-scaling function for $m_l/m_s \leq 1/20$

S. Ejiri *et al.* [RBC-Bielefeld], PRD 80 (2009) 094505.

Summary $\mu_B = 0$

News:

- preliminary N_t=12, asqtad results from HotQCD

The Transition:

- reducing a and m_l effects studies observables in the same way
- N_t=12 suggests a continuum extrapolated value of $T_c \lesssim 170$ MeV

The EoS:

- N_t=12 suggests that e-3p approaches the HRG value from above

The critical behavior:

- finally strong indications for O(N) scaling of the magnetic EoS
- hits that the physical point is in the attraction region of a critical point at m_l=0
- optimistic signal for future studies of the QCD phase diagram

Lattice QCD at nonzero density

- direct MC-simulations for $\mu > 0$ not possible

$$\begin{aligned} Z(V,T,\mu) &= \int \mathcal{D}A\mathcal{D}\psi\mathcal{D}\bar{\psi}\ \exp\{S_F(A,\psi,\bar{\psi}) - \beta S_G(A)\} \\ &= \int \mathcal{D}A\ \det[M](A,\mu)\exp\{-\beta S_G(A)\} \end{aligned}$$

complex for $\mu > 0$

Interpretation as probability is necessary for MC-Integration

$\longrightarrow$ perform a Taylor expansion around $\mu = 0$

Lattice QCD at nonzero density

- Taylor-expansion of the pressure

$$\frac{p}{T^4} = \frac{1}{VT^3} \ln Z(V,T,\mu_u,\mu_d,\mu_s) = \sum_{i,j,k} c_{i,j,k}^{u,d,s} \left(\frac{\mu_u}{T}\right)^i \left(\frac{\mu_d}{T}\right)^j \left(\frac{\mu_s}{T}\right)^k$$

- calculate Taylor coefficients at fixed temperature

- no sign-problem: all simulations at $\mu = 0$

$$c_{i,j,k}^{u,d,s} \equiv \frac{1}{i!j!k!} \frac{1}{VT^3} \cdot \left. \frac{\partial^i \partial^j \partial^k \ln Z}{\partial(\frac{\mu_u}{T})^i \partial(\frac{\mu_d}{T})^j \partial(\frac{\mu_s}{T})^k} \right|_{\mu_{u,d,s}=0}$$

- expansion coefficients reflect fluctuations of various quantum numbers

generalized susceptibilities

$$2! c_2^X = \chi_2^X = \frac{1}{VT^3}\left(\langle X^2\rangle - \langle X\rangle^2\right)$$ quadratic fluctuations

$$4! c_4^X = \chi_4^X = \frac{1}{VT^3}\left(\langle X^4\rangle - 3\langle X^2\rangle^2\right)$$ quartic fluctuations

$$X = u, d, s, B, Q, S, \cdots$$

- Taylor-expansion of the pressure

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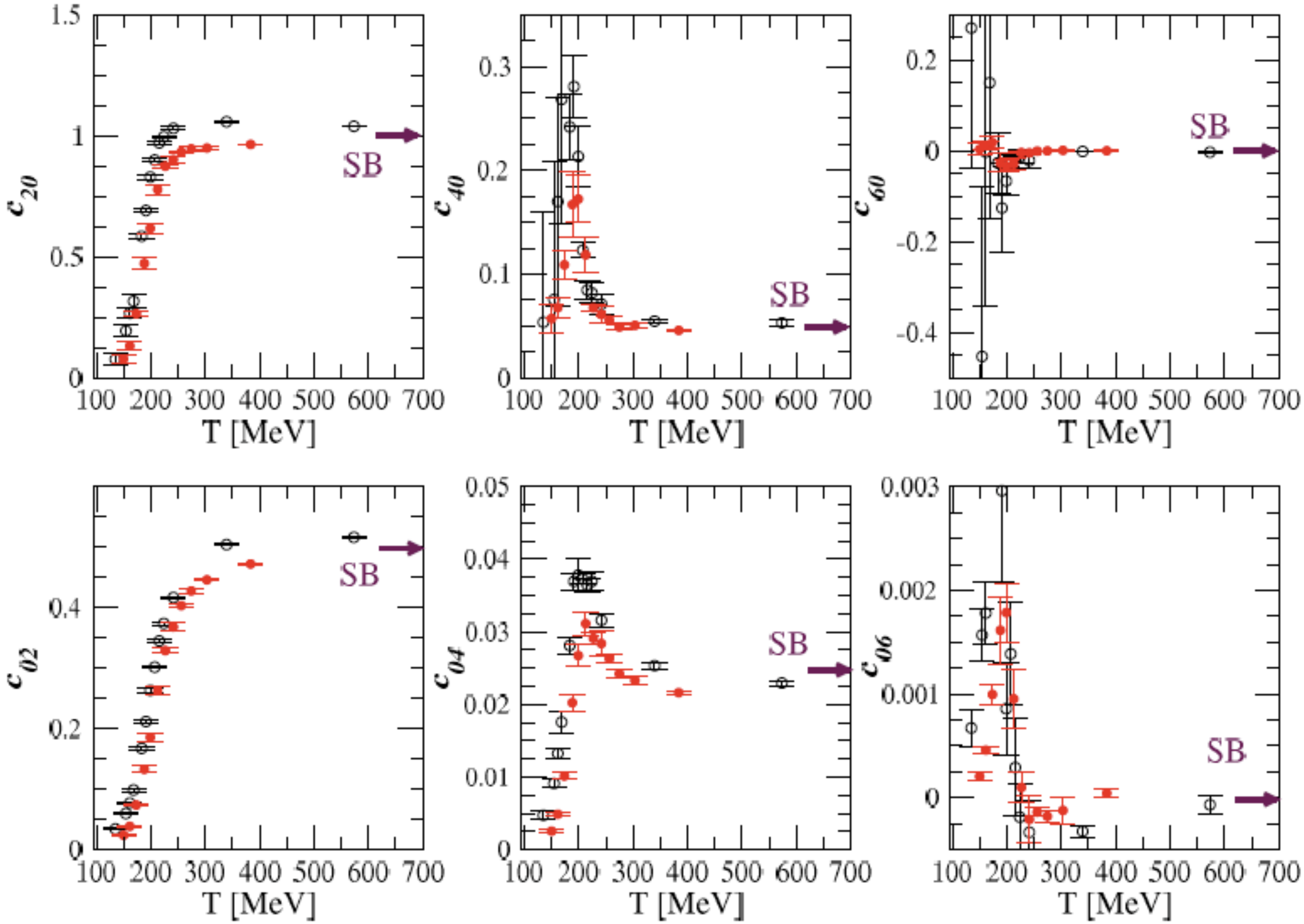
The expansion coefficients of p/T^4

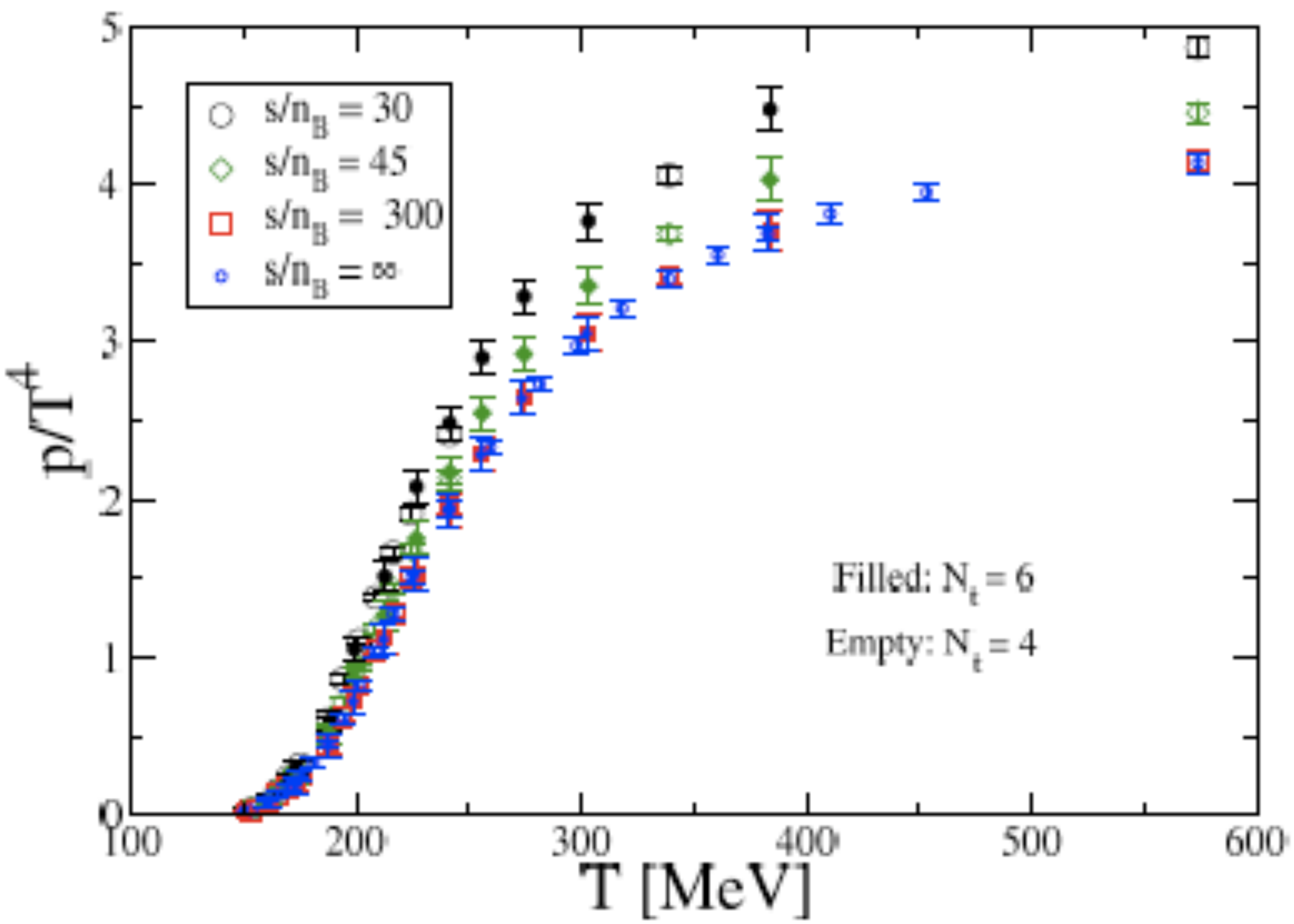


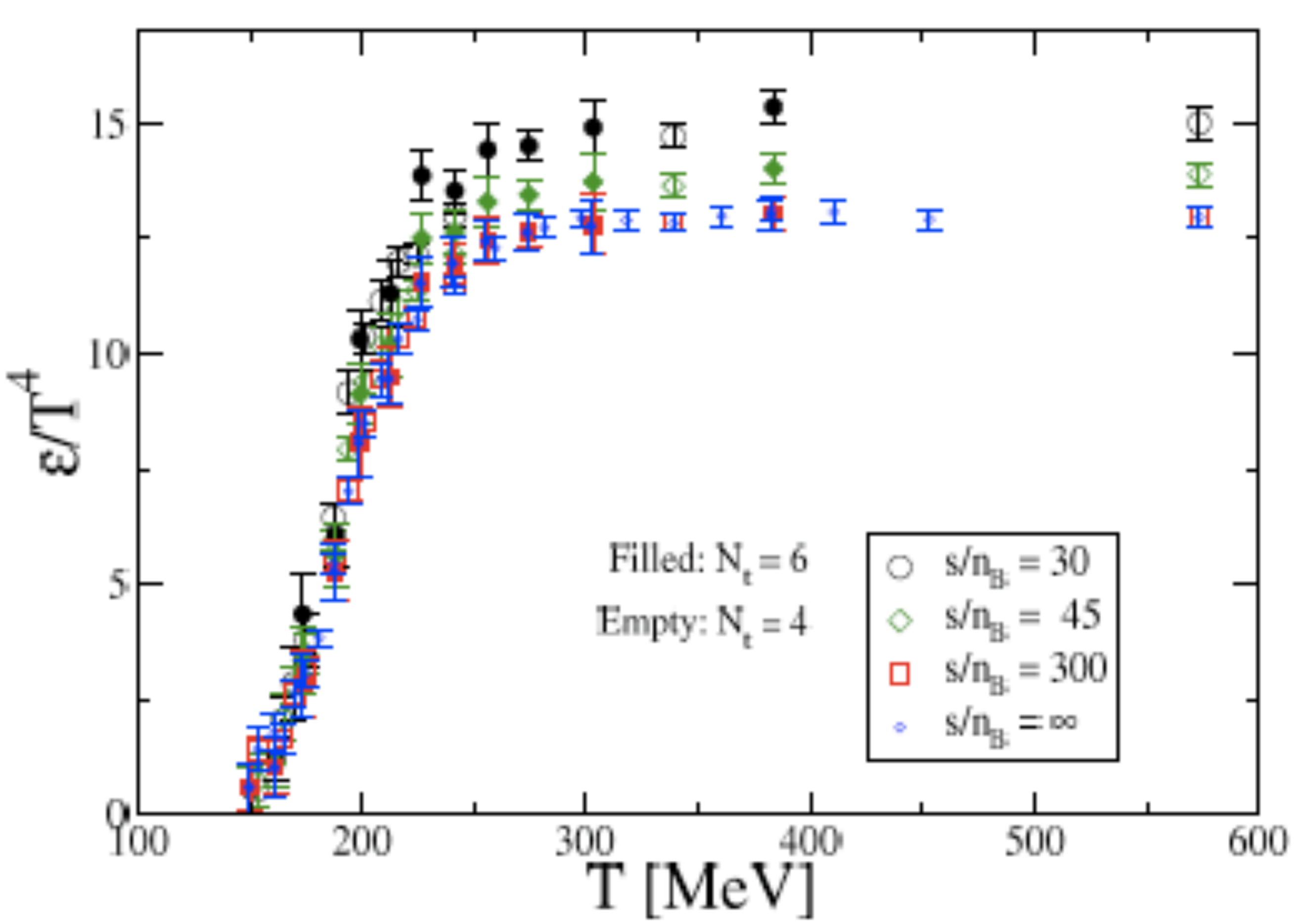
MILC (asqtad),
arXiv:1003.5682[hep-lat].

black: $N_\tau = 4$
red: $N_\tau = 6$

The isentropic EoS

Finding trajectories in the (μ_l, μ_s, T) -plane with $s/n_B = \mathrm{const}, \ n_S = 0$
for AGS, SPS, RHIC we have $s/n_B = 30, 45, 300$, respectively





MILC (asqtad),
arXiv:1003.5682[hep-lat].

dominated by the 0th-order in μ

⟶ corrections to p, e are small

⟶ lattice discretization effects similar in $N_\tau = 4, 6$ lattices

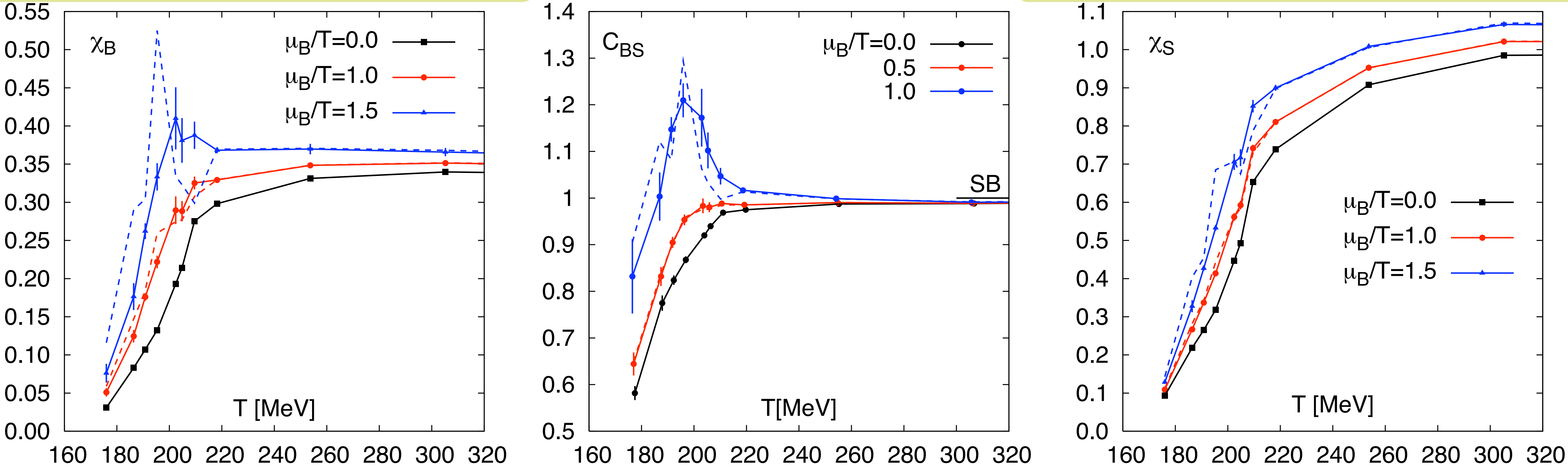
Hadronic fluctuations

at $\mu_B > 0$ $(\mu_S = \mu_Q = 0)$

baryon number fluctuations | **baryon number - strangeness correlations** | **strangeness fluctuations**

$$\chi_B = 2c_2^B + 12c_4^B \left(\frac{\mu_B}{T}\right)^2 + \cdots$$

$$\chi_S = 2c_{0,2}^{B,S} + 2c_{2,2}^{B,S} \left(\frac{\mu_B}{T}\right)^2 + \cdots$$



$$C_{BS} = \frac{c_{1,1}^{B,S} + 3c_{3,1}^{B,S}\left(\frac{\mu_B}{T}\right)^2 + \cdots}{\chi_S\left(\frac{\mu_B}{T}\right)}$$

→ LO introduces a peak in the fluctuations/correlations, NLO shifts the peak towards smaller temperatures

→ truncation errors become large at $\mu_B/T \gtrsim 1.5$

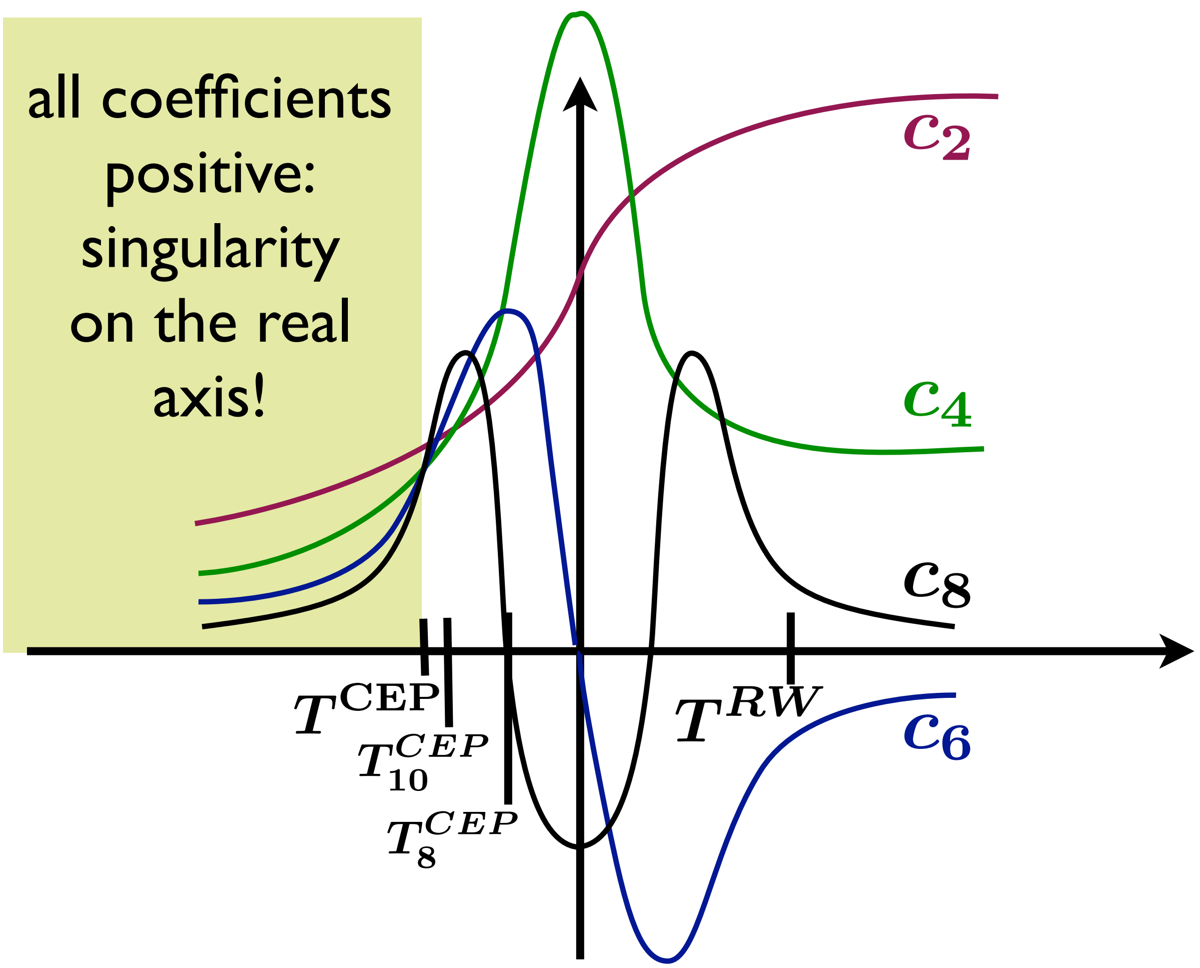
The critical endpoint

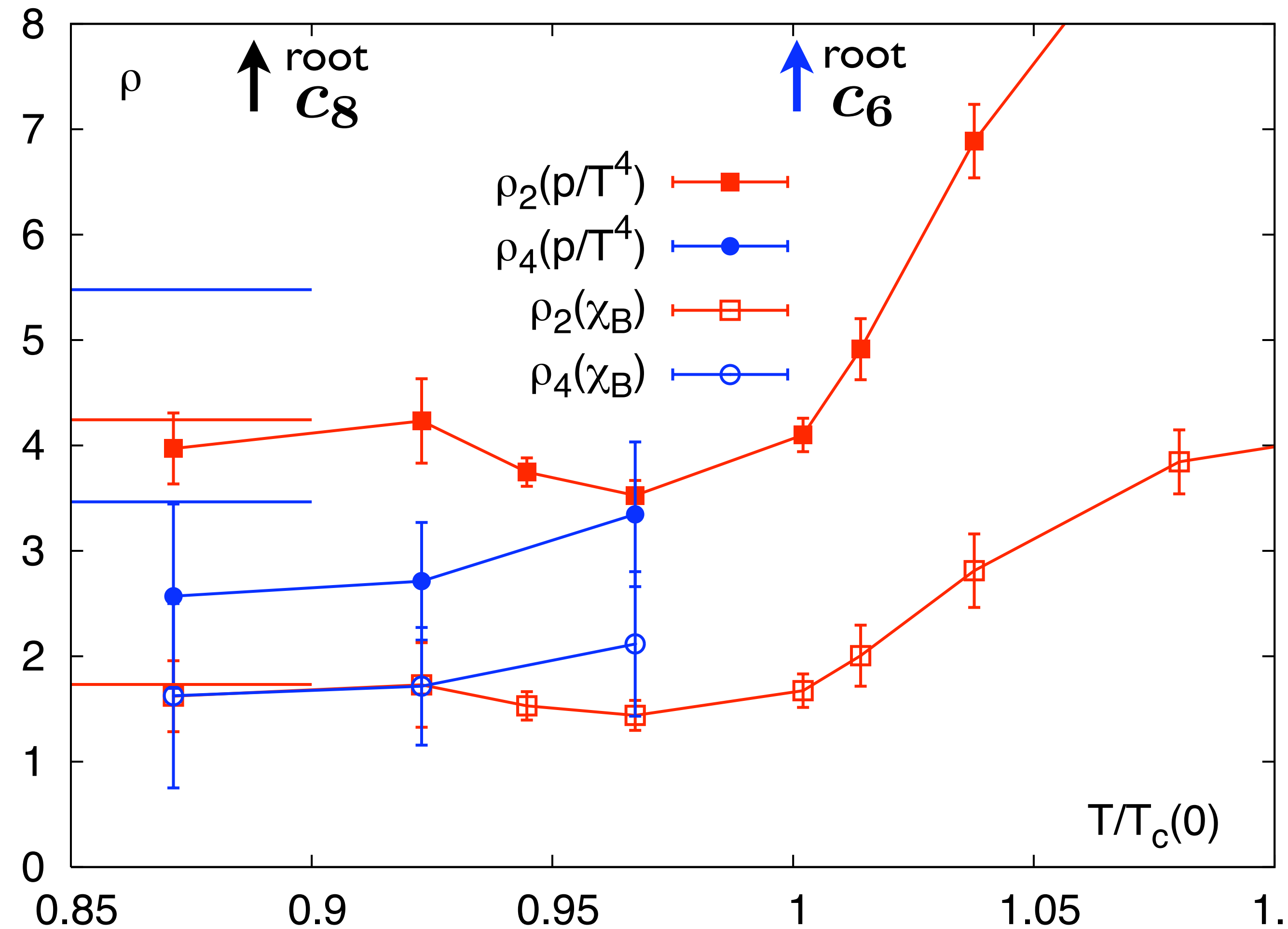
<u>method for locating of the CEP:</u>

- determine largest temperature where all coefficients are positive $\rightarrow T^{\mathrm{CEP}}$
- determine the radius of convergence at this temperature $\rightarrow \mu^{\mathrm{CEP}}$

$$p = c_0 + c_2 \left(\mu_B/T\right)^2 + c_4 \left(\mu_B/T\right)^4 + \cdots$$

$$\chi_B = 2c_2 + 12c_4 \left(\mu_B/T\right)^2 + 30c_6 \left(\mu_B/T\right)^4 + \cdots$$





first non-trivial estimate of T^{CEP} by c_8
second non-trivial estimate of T^{CEP} by c_{10}

$$\rho_n(p) = \sqrt{c_n / c_{n+2}}$$

$$\rho = \lim_{n \to \infty} \rho_n$$

The critical endpoint

method for locating of the CEP:

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- determine the radius of convergence at this temperature → μ^{CEP}

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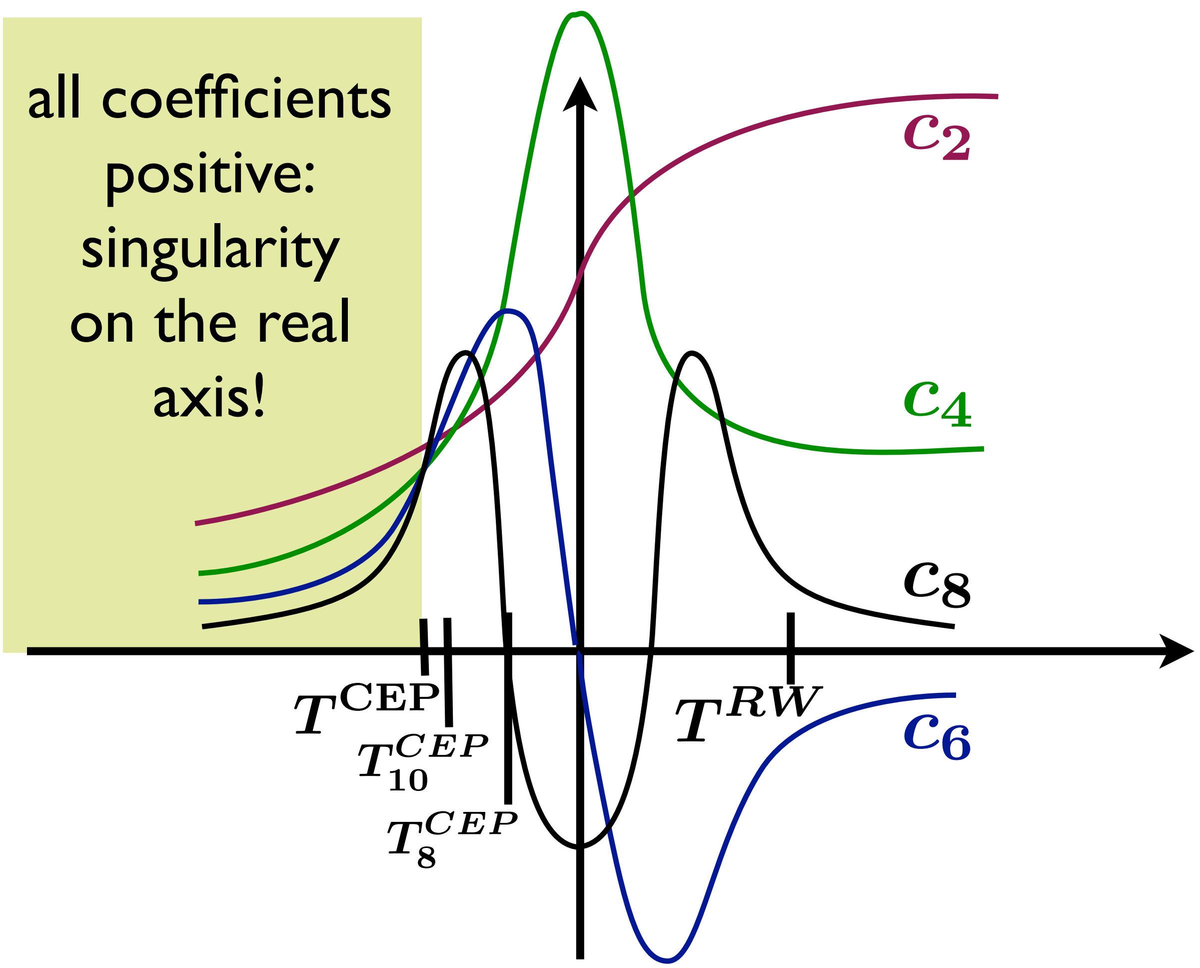

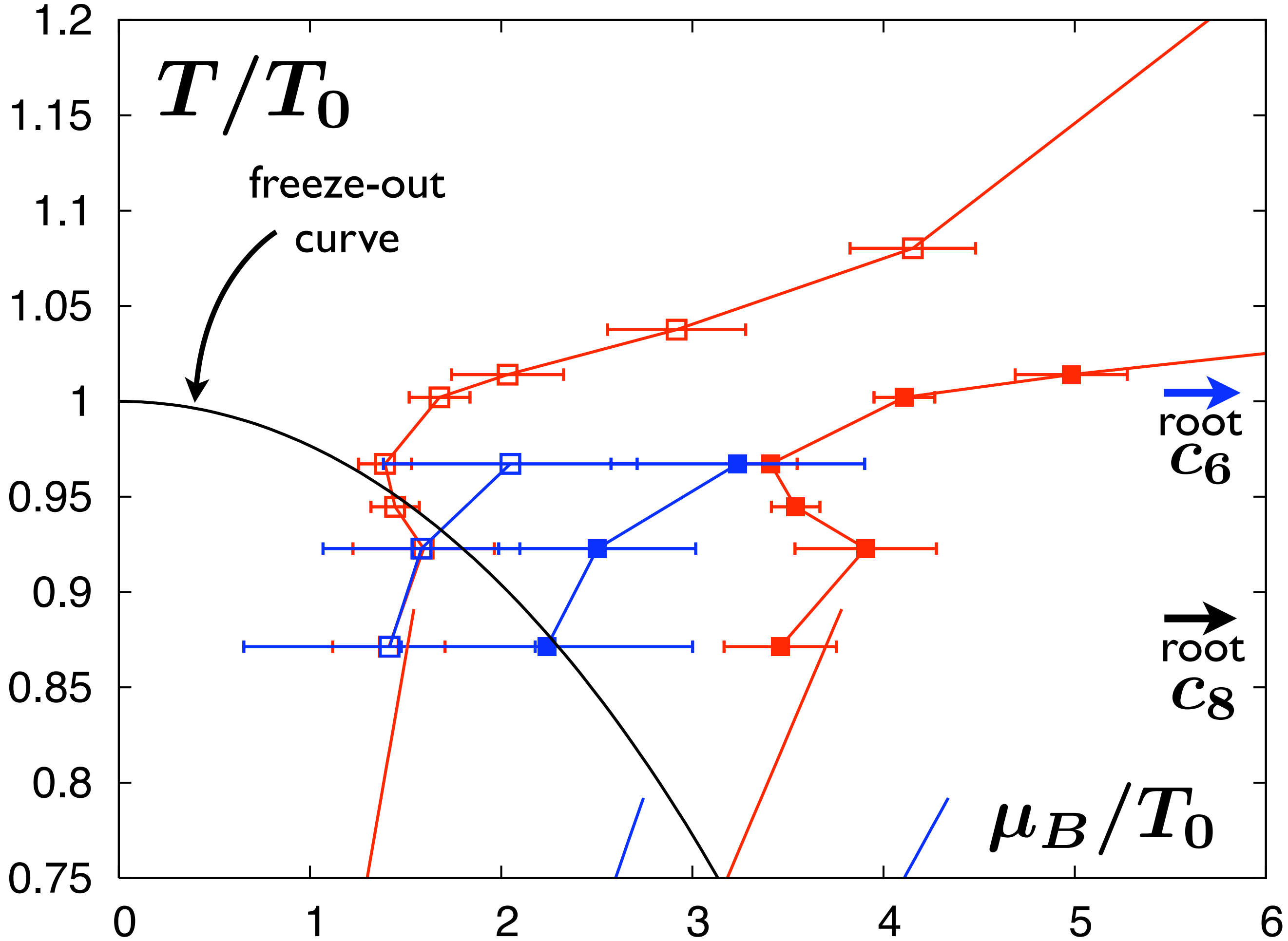

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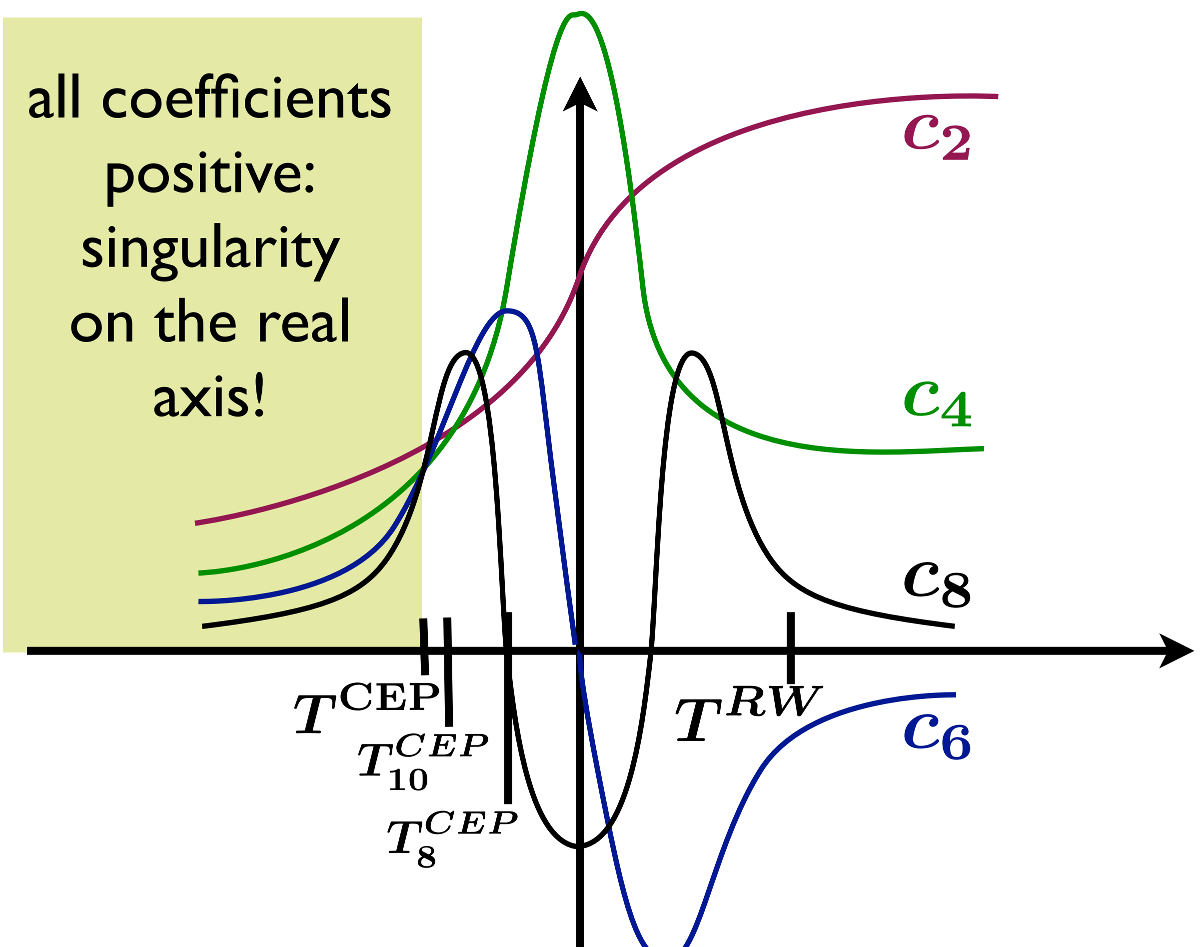

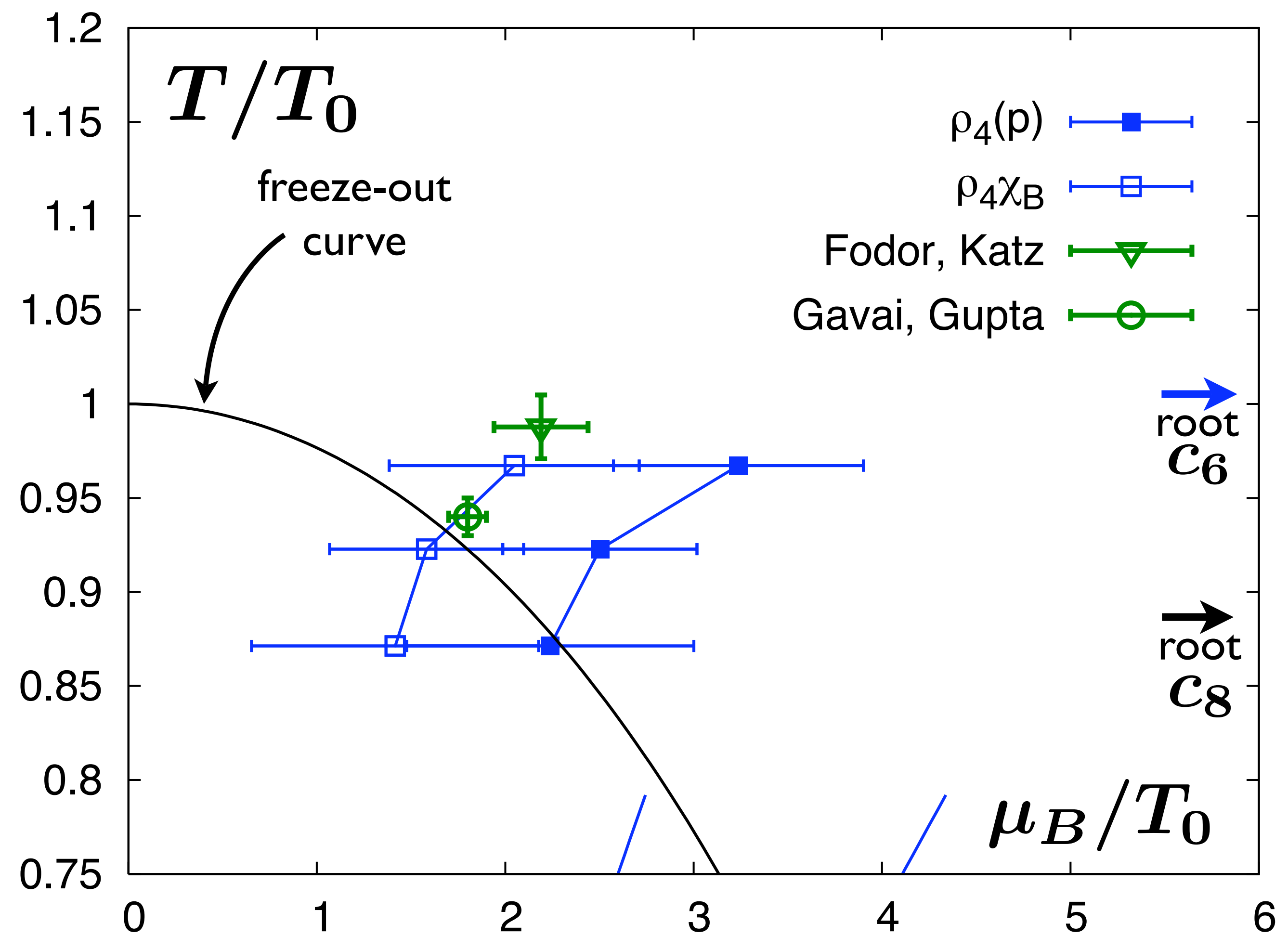

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Summary $\mu_B > 0$

News:

- N_t=6, asqtad results from MILC for Taylor expansion coefficients of the pressure up to the 6th order

The isentropic EoS:

- finite μ corrections to EoS are small for $s/n_B = \mathbf{30, 45, 300}$

Hadronic fluctuations:

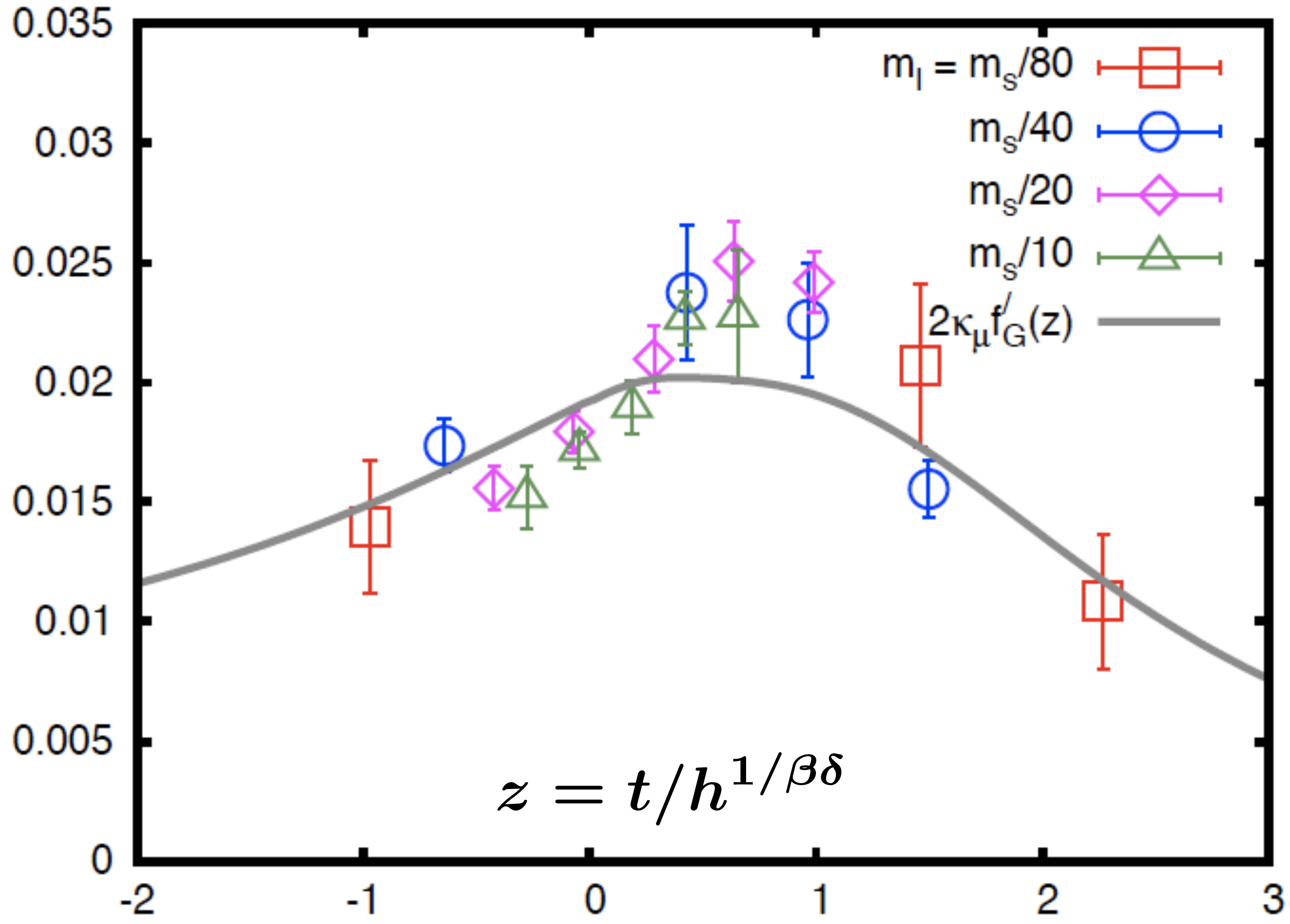
- Fluctuations and correlations are well described by a free gas of quarks above T>(1.5-1.7)Tc and by a resonance gas for $T<T_E$
- Truncation errors of the Taylor series becomes large for $\mu_B/T \gtrsim (1-1.5)$

The critical point:

- The radius of convergence of the Taylor expansion can be used to estimate the critical endpoint

The critical line (chiral limit)

fit to $c_2^{\bar\psi\psi}$



courtesy S.Mukherjee

scaling field (chiral limit):

$$t = \frac{T - T_c}{T_c} + \kappa\mu_B^2$$

magnetic EoS:

$$M = h^{1/\delta} f_G(t/h^{1/\beta\delta})$$

critical line:

$$\frac{T_c(\mu_B)}{T_c(0)} = 1 - \kappa\mu_B^2$$

$$-h^{(1-\beta)/\beta\delta} c_2^{\bar\psi\psi} t_0 m_s T^{-1} = 2\kappa f'_G(z)$$

only one fit-parameter

expected phase diagram