The QCD Phase Diagram and EOS

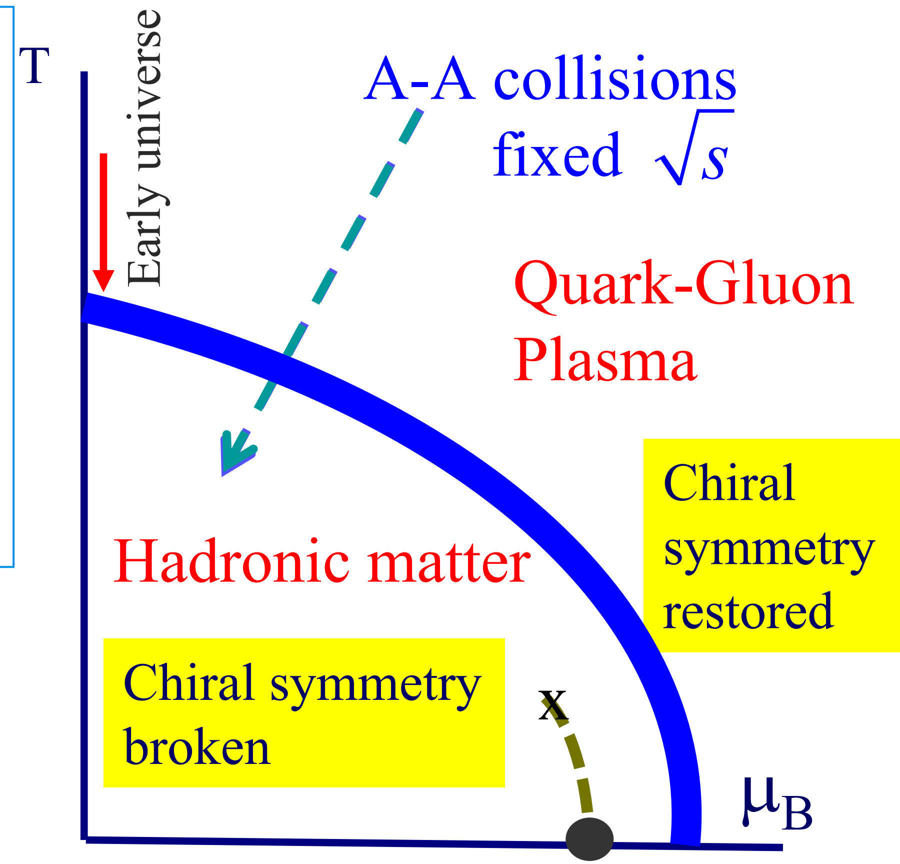
- From Chiral Models

to

- Lattice Gauge Theory

and

- Heavy Ion Collisions



Work done with: B. Friman, E. Nakano, C. Sasaki, V. Skokov, B. Stokic & B.-J. Schaefer

1st principle calculations:

$\mu, T << \Lambda_{QCD}$: $\chi-$perturbation theory

$\mu, T >> \Lambda_{QCD}$: pQCD

$\mu_q < T$: LGT

Krzysztof Redlich University of Wroclaw & CERN

QCD thermodynamics in effective models

- Models based on same symmetries as QCD :

$SU(N_f)_L \times SU(N_f)_R$ chiral with $<\bar{q}q>$ order parameter

$Z(N_c)$ center with $<L>$ order parameter

⇨ due to universality one expects similar critical properties

- Approach based on the Schwinger-Dyson Equation to study critical structure of QCD
- Use AdsCFT/QCD correspondence to gain information on QCD medium properties?

Effective QCD-like models

$$L_{PNJL} = \bar{q}(iD_\mu - m)q + G_S[(\bar{q}q)^2 + (\bar{q}i\vec{\tau}\gamma_5 q)^2] - G_V^{(S)}(\bar{q}\gamma_\mu q)^2$$
$$-G_V^{(V)}(\bar{q}\vec{\tau}\gamma_\mu q)^2 + \mu_q q^+ q + \mu_I q^+ \tau_3 q - U(\Phi[A], \bar{\Phi}[A], T)$$

K. Fukushima, C. Ratti & W. Weise, B. Friman & C. Sasaki , ., ...

B.-J. Schaefer, J.M. Pawlowski & J. Wambach; B. Friman et al.

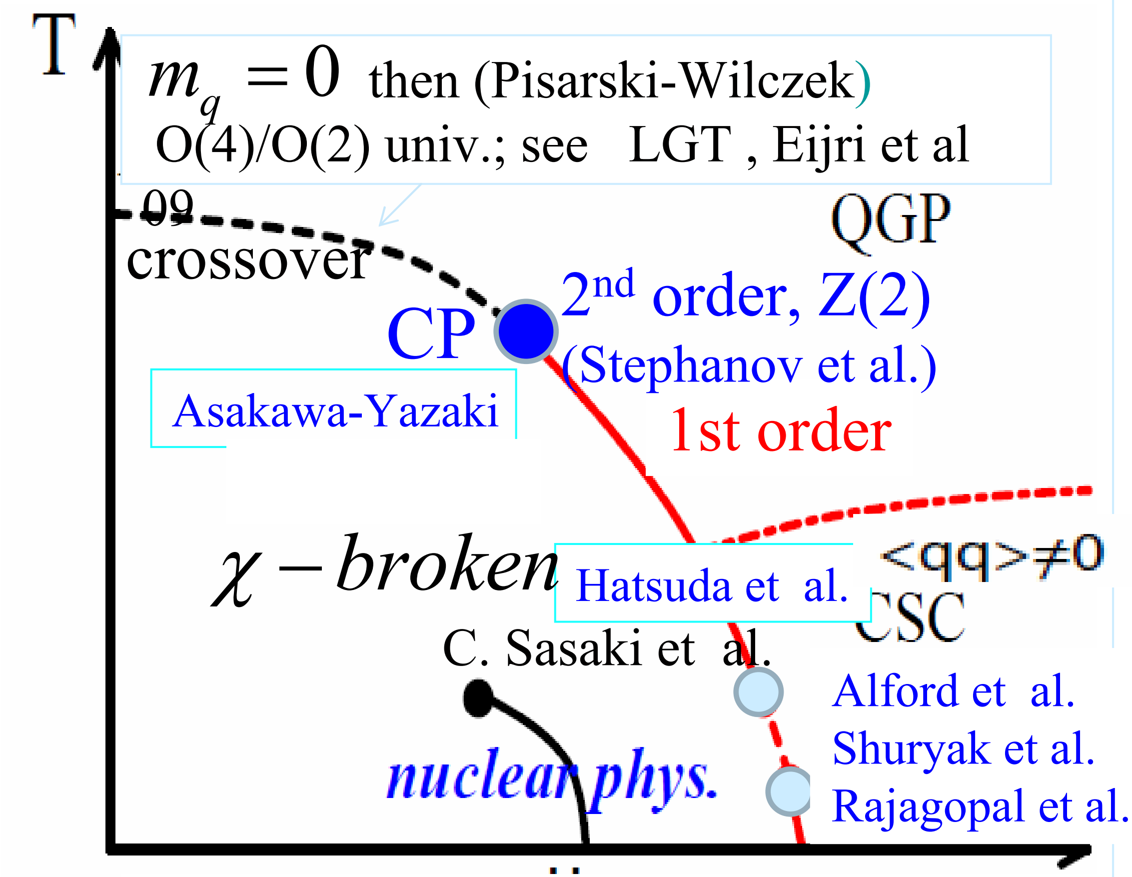
$$L_{PQM} = \bar{q}(iD_\mu - g[\sigma + i\gamma_5\vec{\tau}\vec{\pi}])q + \frac{1}{2}(\partial_\mu\sigma)^2 + \frac{1}{2}(\partial_\mu\vec{\pi})^2$$
$$-U(\Phi[A], \bar{\Phi}[A], T) - U(\sigma, \vec{\pi}^2)$$

$$D_\mu = \partial_\mu - i\delta_{\mu 0}A_0 \qquad \Phi = \frac{1}{N_c} Tr(P\exp[i\int d\tau A_4(\vec{x}, \tau)])$$

Polyakov loop

$$U(\sigma, \vec{\pi}) = \frac{1}{2}m^2\phi^2 + \frac{\lambda}{4}(\phi^2)^2 - h\sigma$$

Generic Phase diagram from effective chiral Lagrangians



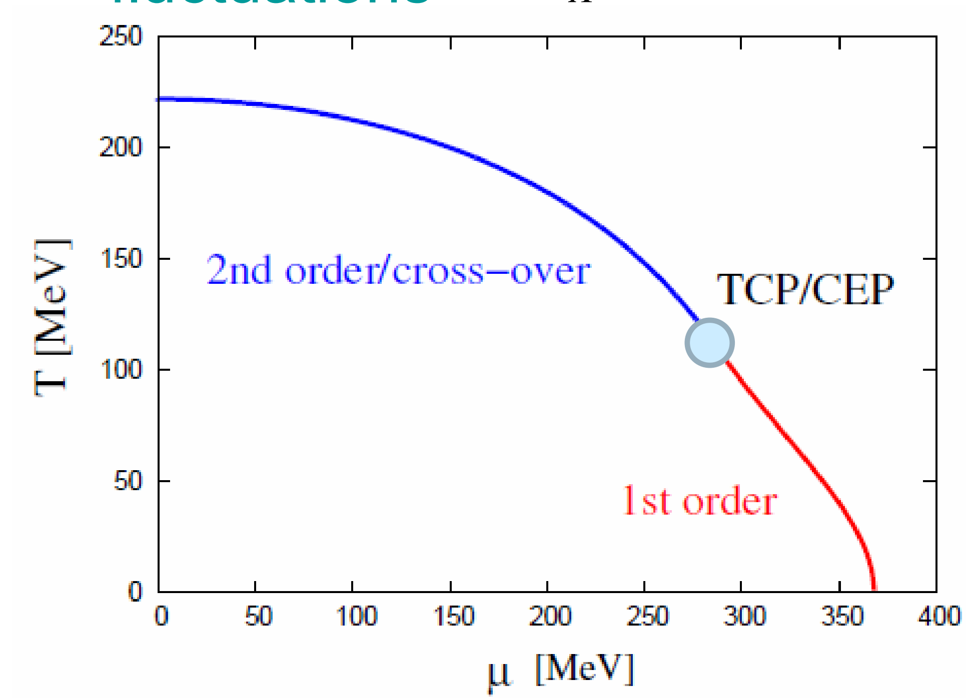
Zhang et al, Kitazawa et al., Hatta, Ikeda; Fukushima et al., Ratti et al., Sasaki et al., Blaschke et al., Hell et al., Roessner et al., ..

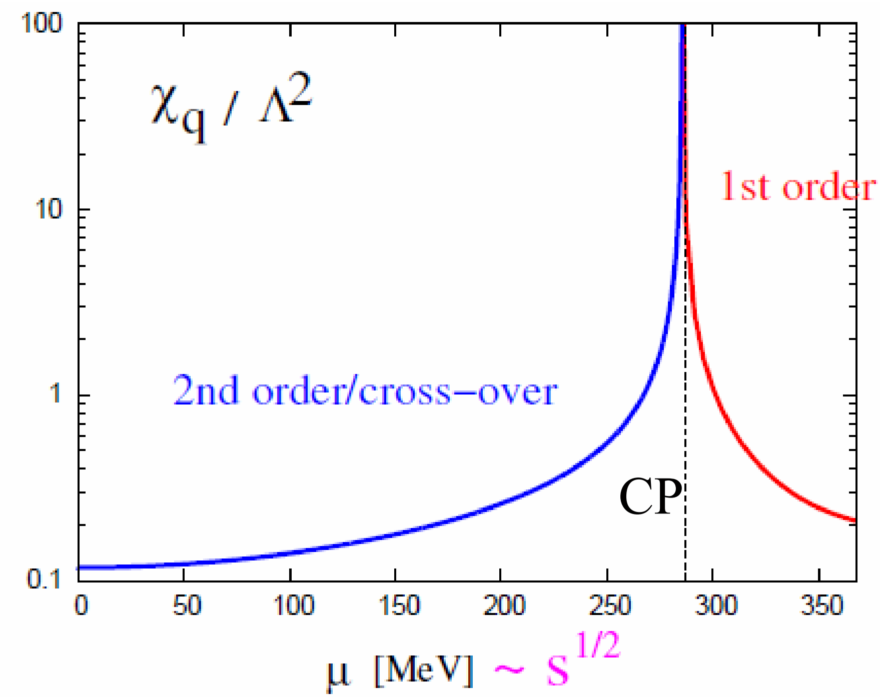
- The existence and position of CP and transition is model and parameter dependent !!
- Introducing di-quarks and their interactions with quark condensate results in CSC phase and dependently on the strength of interactions to new CP's

Probing CP with charge fluctuations

- Net quark-number χ_q,isovector χ_I and electric charge χ_Q fluctuations $\chi_A = <A^2> - <A>^2$

$$\chi_Q = \frac{1}{36}\chi_q + \frac{1}{4}\chi_I + \frac{1}{6}\frac{\partial^2 P}{\partial\mu_q \partial\mu_I}$$





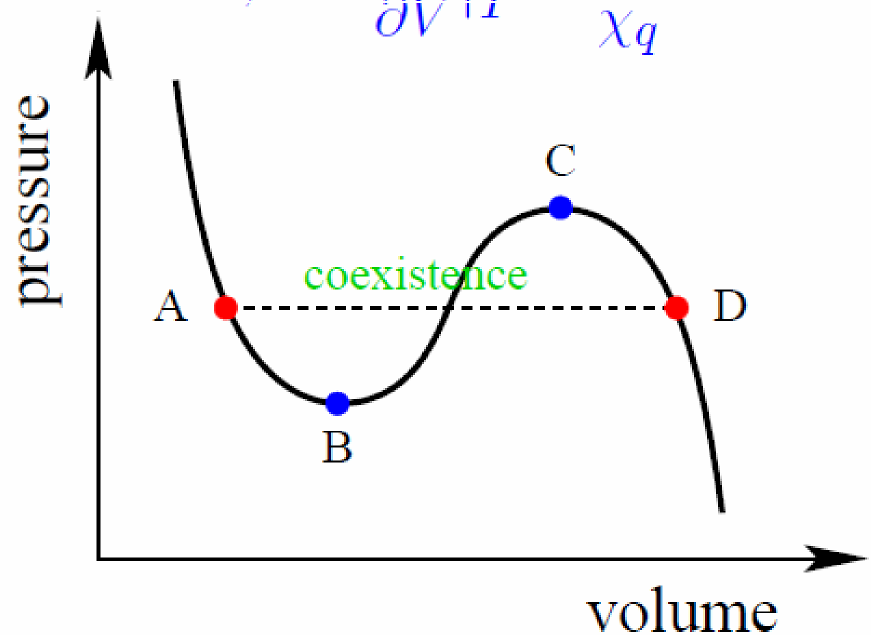
The CP ($m_{u,d} \neq 0$) and TCP ($m_{u,d} = 0$) are the only points where the baryon number and electric charge densities diverged !

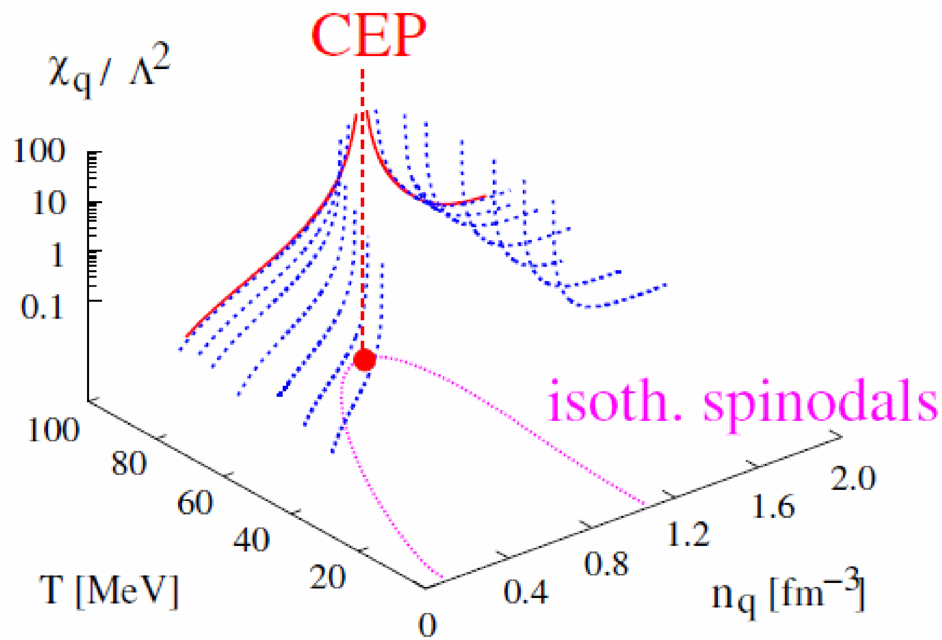
A non-monotonic behaviour of charge Density fluctuations (χ_q, χ_Q) is an excellent probe of the CP

- **heavy-ion collisions: deviation from equilibrium**
  - nature of first order phase transitions: spinodal instabilities
    $\Rightarrow$ enhancement of baryon and strangeness fluctuations

    [Heiselberg et al. (88), Bower, Gavin (01), Chomaz et al. (04), V. Koch et al. (05)]
  - model-indep. signature of 1st-order tr. $\chi_q \to \infty$ Sasaki, Friman et al. 07

$$B, C : \frac{\partial P}{\partial V}\Big|_T \propto \frac{1}{\chi_q} = 0$$



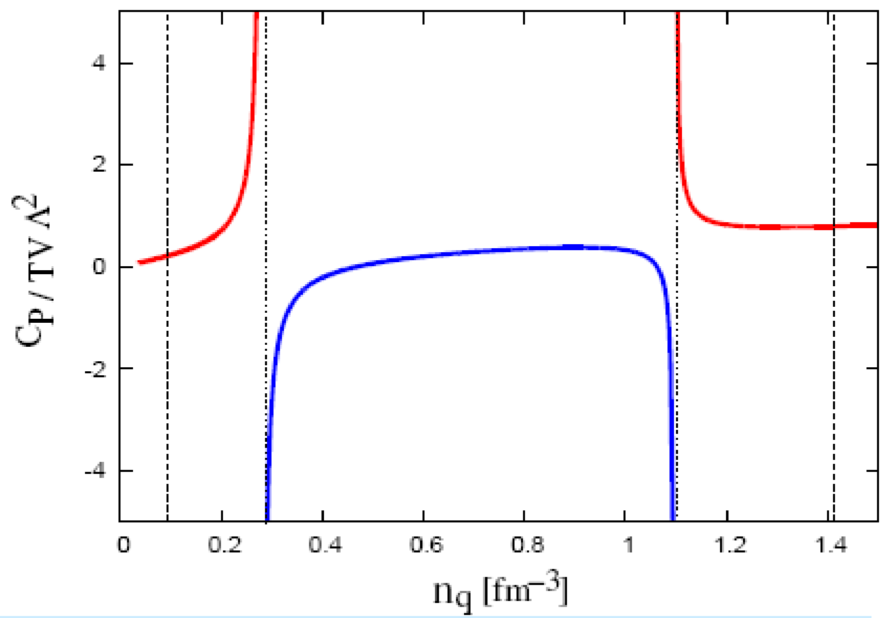


2 divergent branches approaching toward a CP

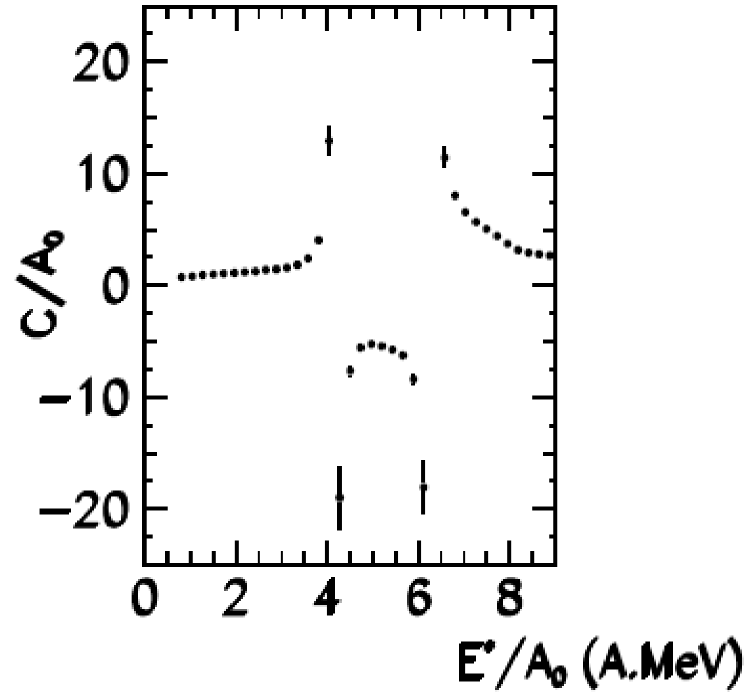
$\Rightarrow$ large baryon number fluctuations in a wider range of phase diagram

Experimental Evidence for 1st order transition

Specific heat for constant pressure:



Low energy nuclear collisions



$$C_P = T\left(\frac{\partial S}{\partial T}\right)_P = TV\left[\chi_{TT} - \frac{2s}{n_q}\chi_{\mu T} + \frac{s^2}{n_q^{\ 2}}\chi_q\right]$$

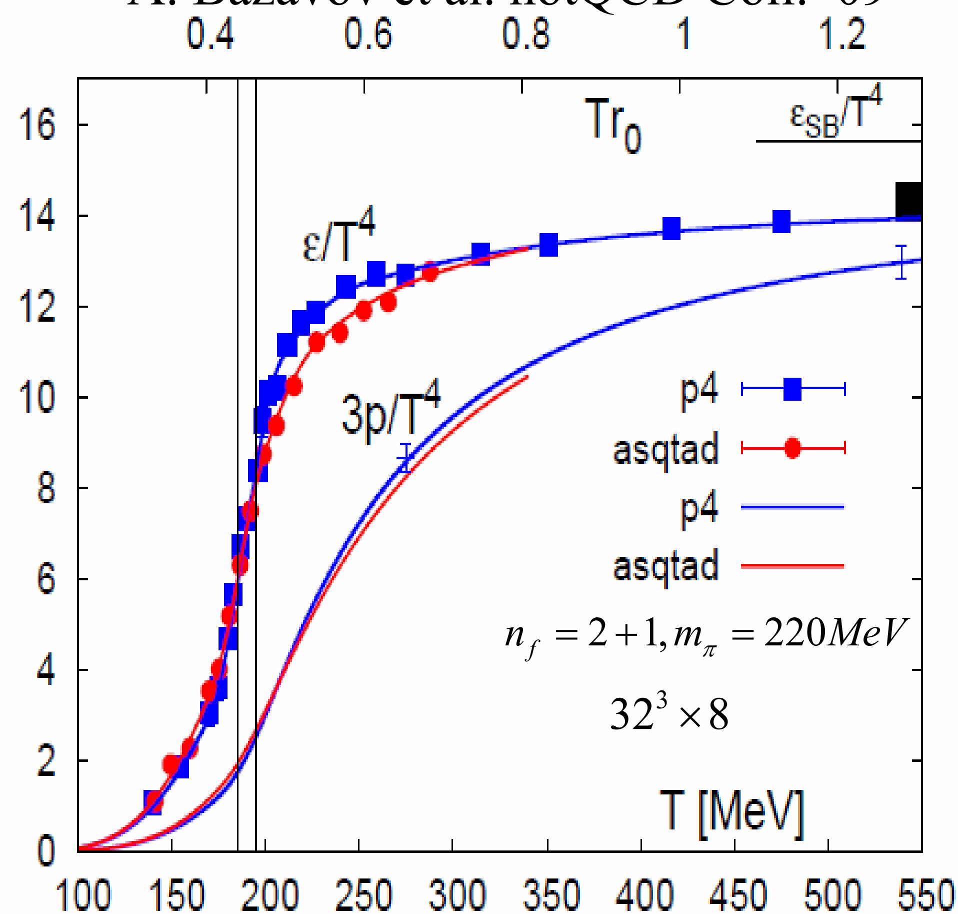
M. D'Agostino *et al.*, Phys. Lett. B 473, 219 (2000)

negative heat capacity : anomalously large fluctuations

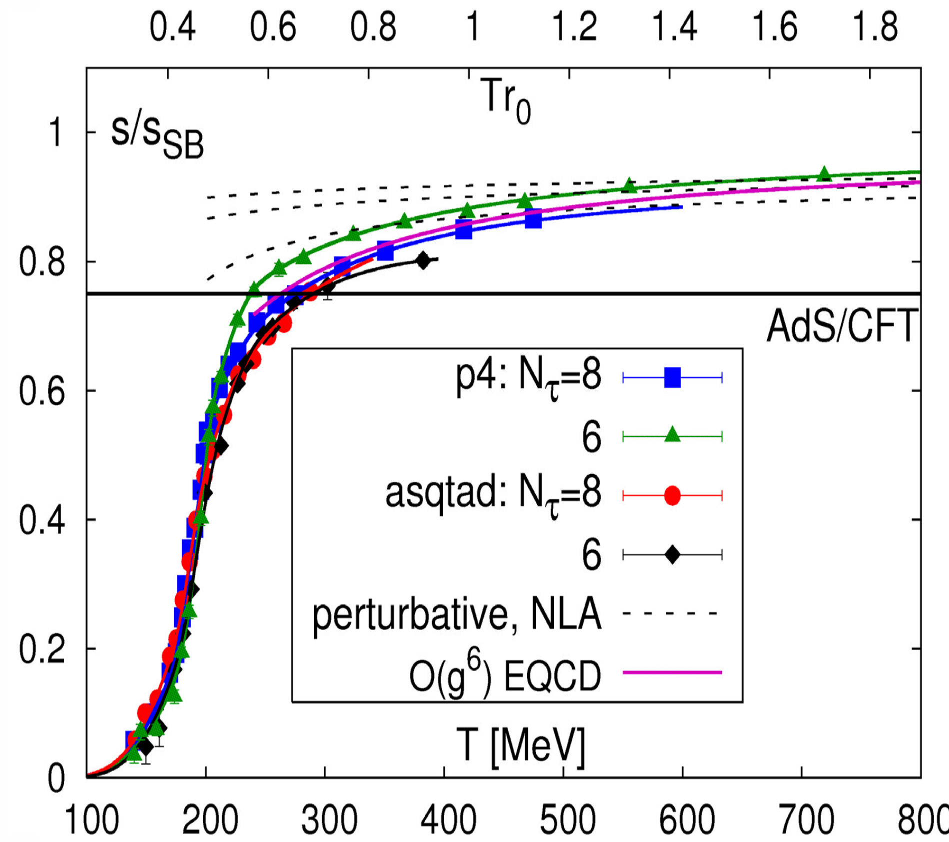
$\Rightarrow$ an evidence of the 1st order liquid-gas phase transition

LGT EOS at finite T and vanishing baryon density

A. Bazavov et al. hotQCD Coll. 09



P.P., NPA 830 (2009) 11

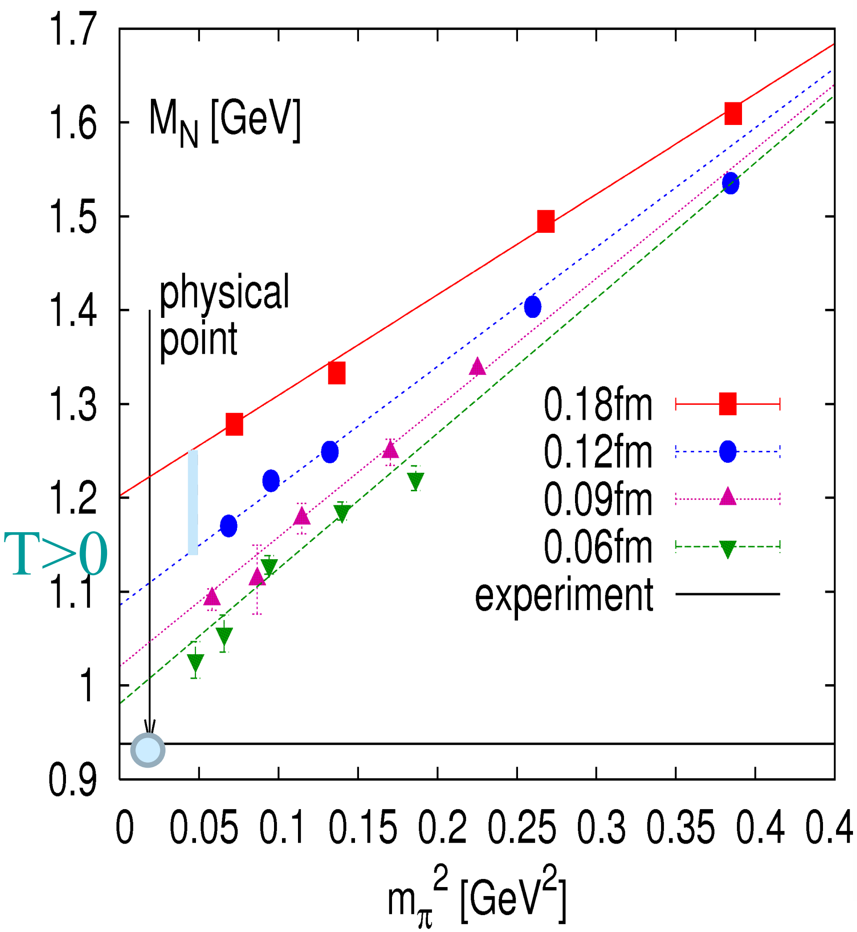


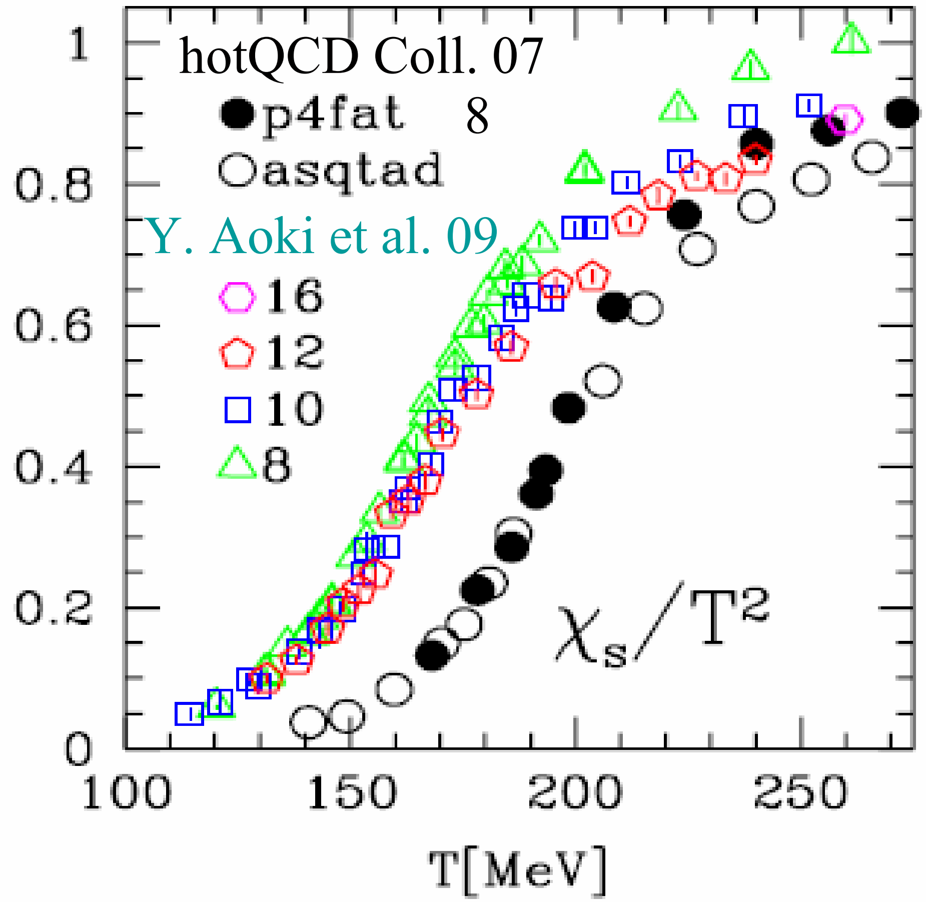
$\frac{1}{9}$

Abrupt but smooth change of energy density indicates crossover transition :
Results still not free from finite size effects!

Entropy consistent with PQCD at large T
At low T LGT essentially below the hadron resonance gas results

Finite Size Effects in LGT Thermodynamics





Thermodynamics from p4fat action with $N_\tau = 8$ is still calculated with non-physical mass spectrum; $M_N > 1.1\ [GeV]$

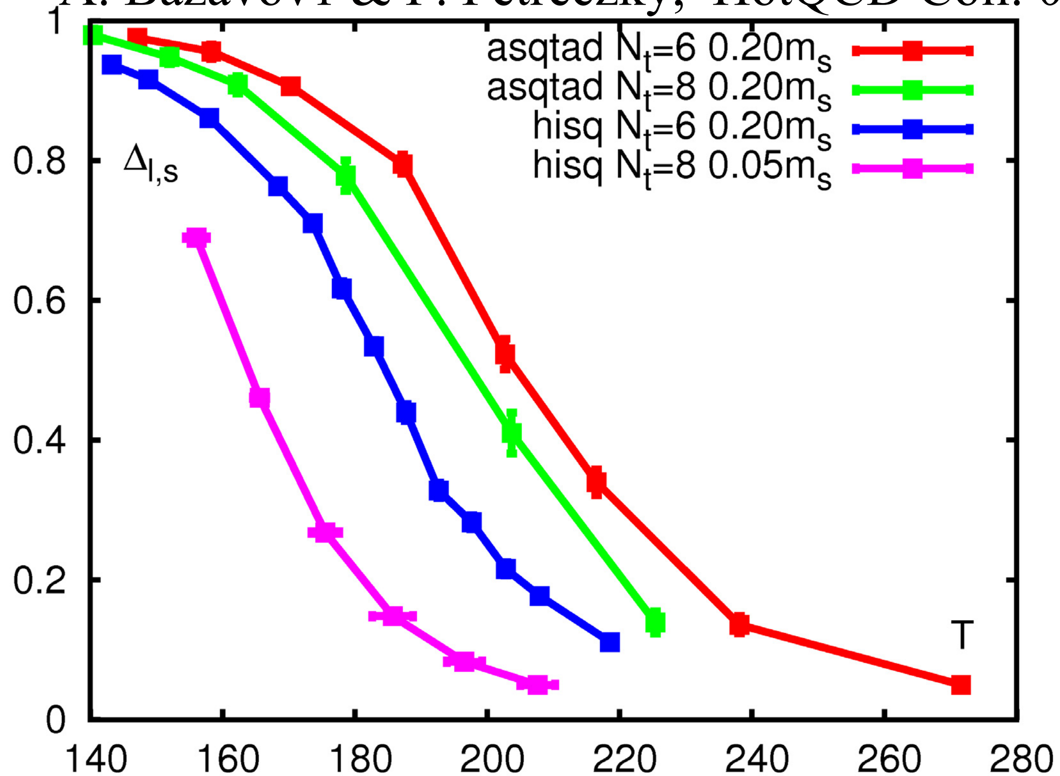
Different critical temperatures:

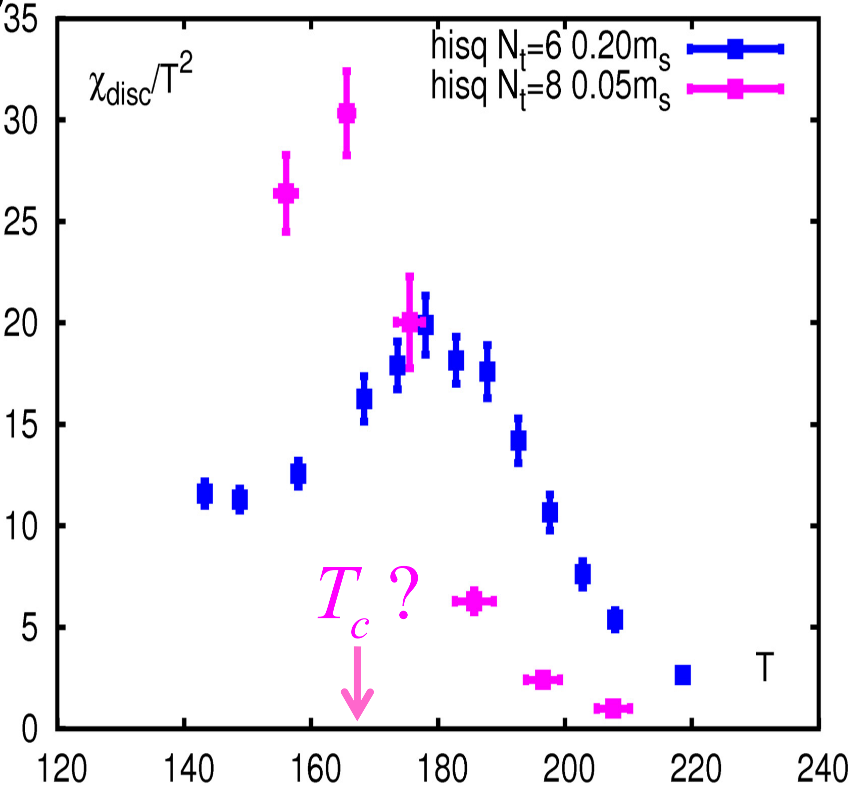
$T_c = 169 \pm 3 \pm 3$ Y. Aoki et al 09

$T_c = 190 \pm 5$ hotQCD Coll. 07

Thermodynamics in LGT with Highly Improved Staggered Quarks (HISQ)

A. Bazavov1 & P. Petreczky, HotQCD Coll. 09



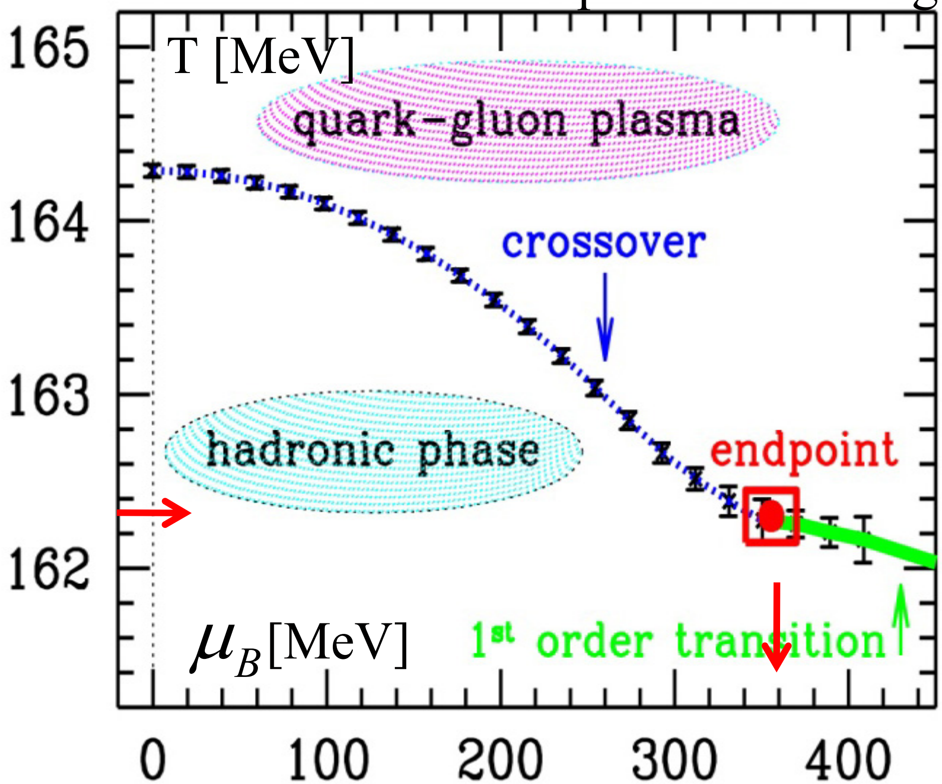


Tendency of shifting the transition region to lower temperatures than in previous asqtad and p4 studies on $N_\tau = 8$ lattice !

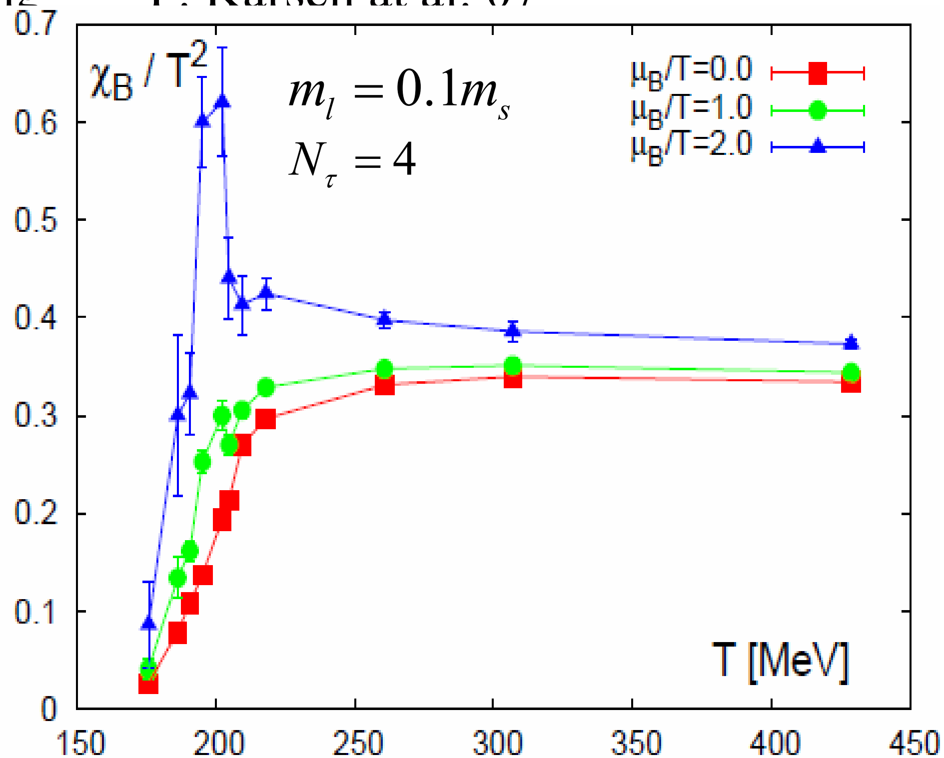
Finite baryon density LGT in 2+1 f and CEP

$$Z(\mu,\beta)=\int DU\exp(-S_g(\beta,U))\det M(\mu=0,U)\times\frac{\det M(\mu,U)}{\det M(\mu=0,U)}$$

Fodor & Katz 04: multi-parameter rewighting



F. Karsch at al. 07



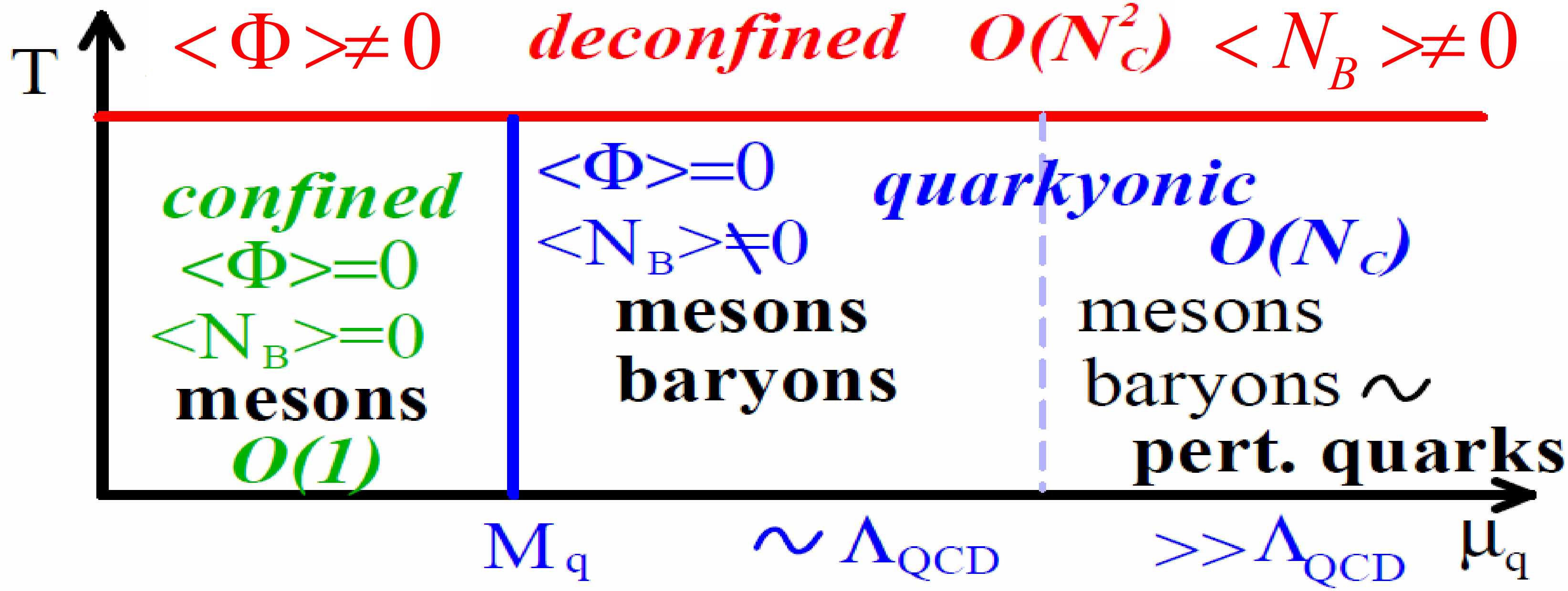
Det M exactly included in calculations
Use the Lee-Yung theorem to identify CP
See also: S. Ejiri 08

Det M through Taylor expansion method:
Study radius of convergence and fluctuations
to identify CP; No strong evidence of CP till now

Novel Phase of QCD in the limit $N_c \to \infty$

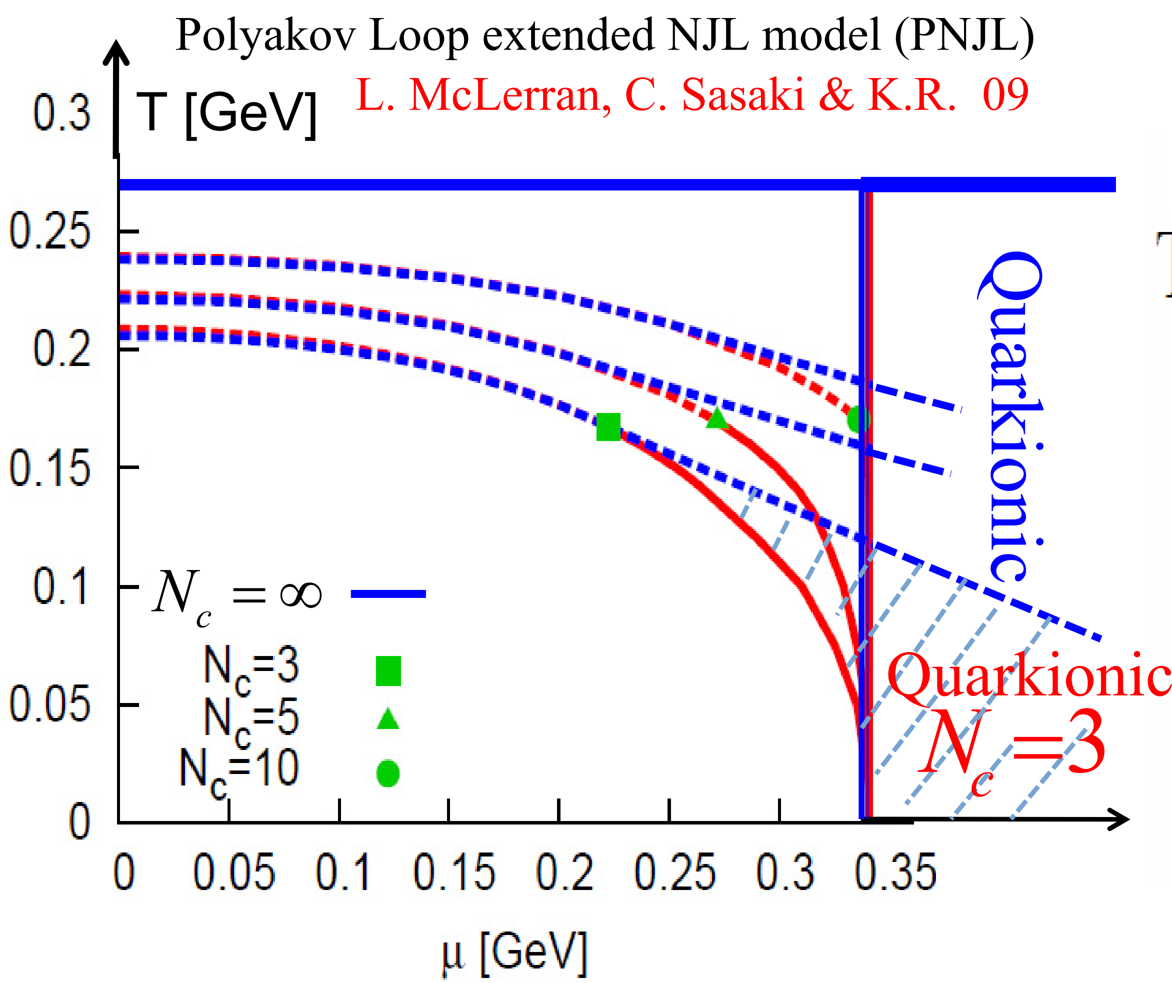
“ Quarkyonic Phase “ L. McLerran & R. Pisarski 07

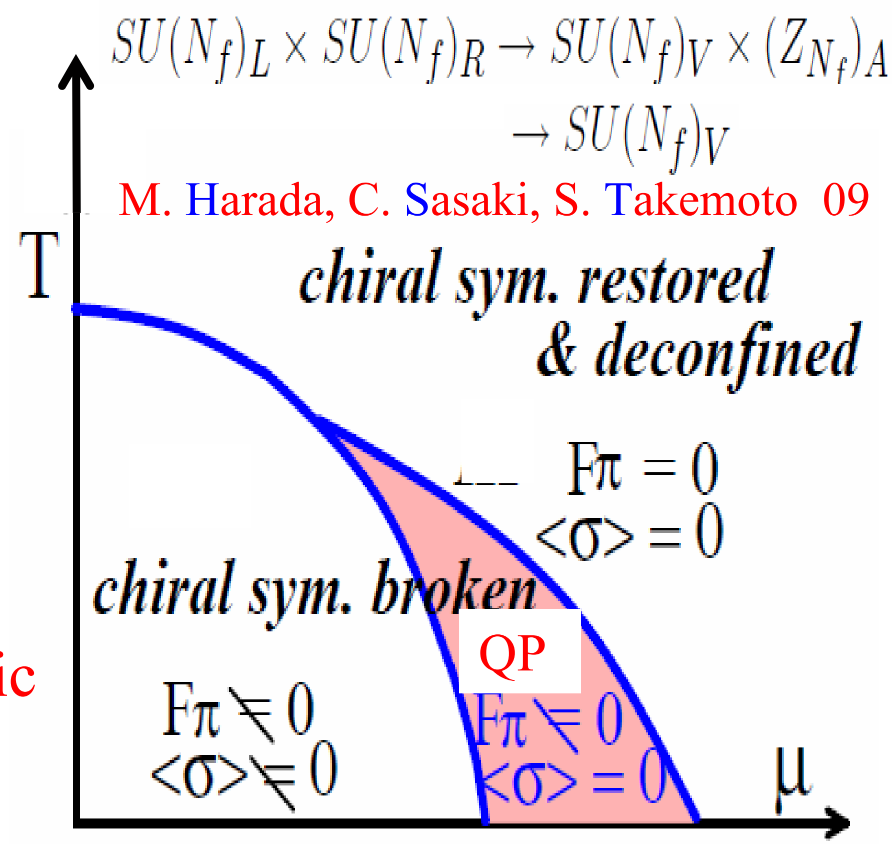
$$N_c \to \infty \Rightarrow < N_B > \approx e^{-N_c (M_q - \mu_q)/T} \qquad \text{Quarks } O(N_c) \quad \text{Gluons } O(N_c^2)$$



- 3 distinct phases in large N_c: deconf. $(\langle \Phi \rangle = 1, \langle N_B \rangle \neq 0)$, mesonic $(\langle \Phi \rangle = 0, \langle N_B \rangle = 0)$, quarkyonic $(\langle \Phi \rangle = 0, \langle N_B \rangle \neq 0)$
- “quark-yonic” : elementary fermionic excitations at very large μ (quarks) and at moderate μ (baryons)
- Quarkyonic phase can persists for $N_c = 3$ { L. McLerran, C. Sasaki and K.R. 09; M. Harada, C. Sasaki, S. Takemoto 09

Quarkionic Phase (QP) in models with $N_c = 3$





$< \sigma > = < \bar{q}q >$ Order parameter of $(Z_{N_f})_A$ sym.

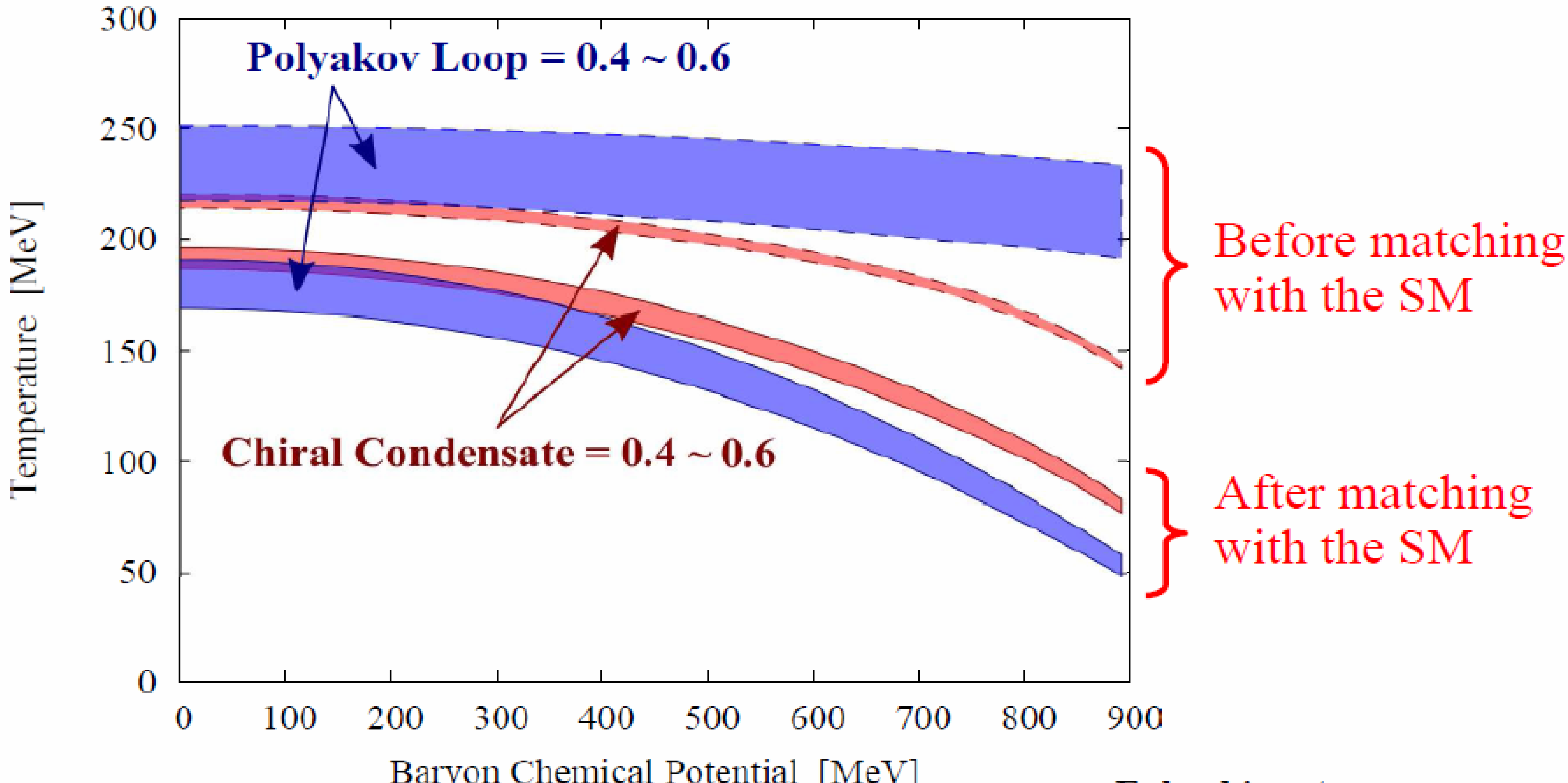
$F_\pi = a < \sigma > + b < \sigma^2 > + ..$

Order parameter of chiral symmetry restoration

The Quarkionic Phase can persists for $N_c = 3$, thus could be also there in QCD?

Matching Gluon Potential with SM (K. Fukushima)

Exotic scenario may be disfavored...?

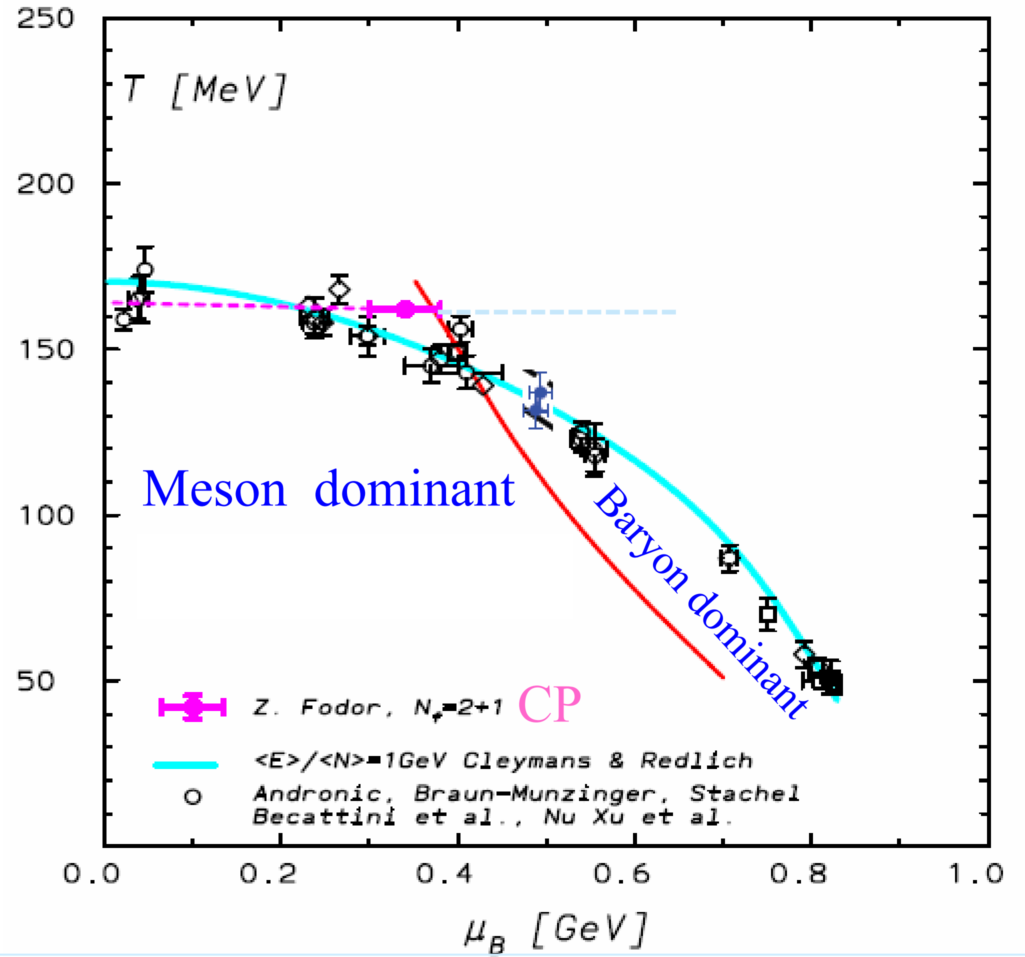


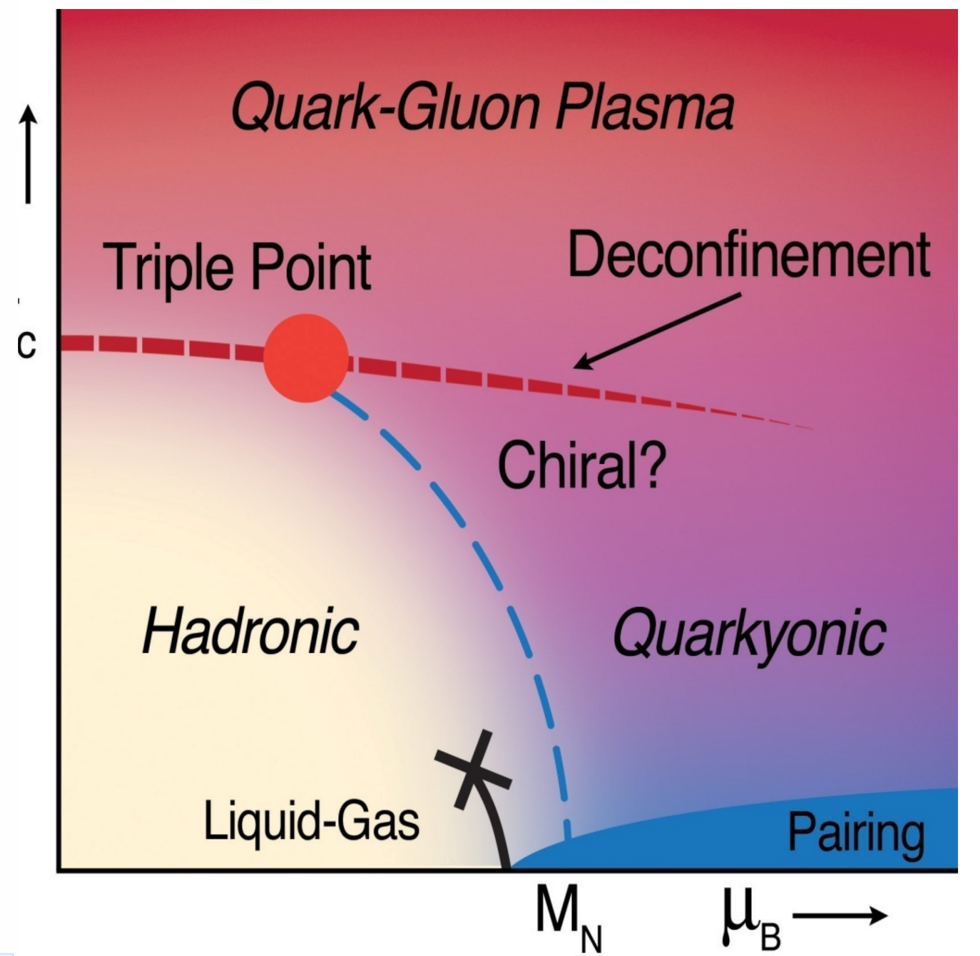
Fukushima to appear soon

April 6 2010 at DM2010

Probing the QCD Phase Boundary in HIC

Andronic,Braun-Munzinger,Blaschke,Cleymans,McLerran,Oeschler,Pisarski,Sasaki,Satz,Stachel &K.R.

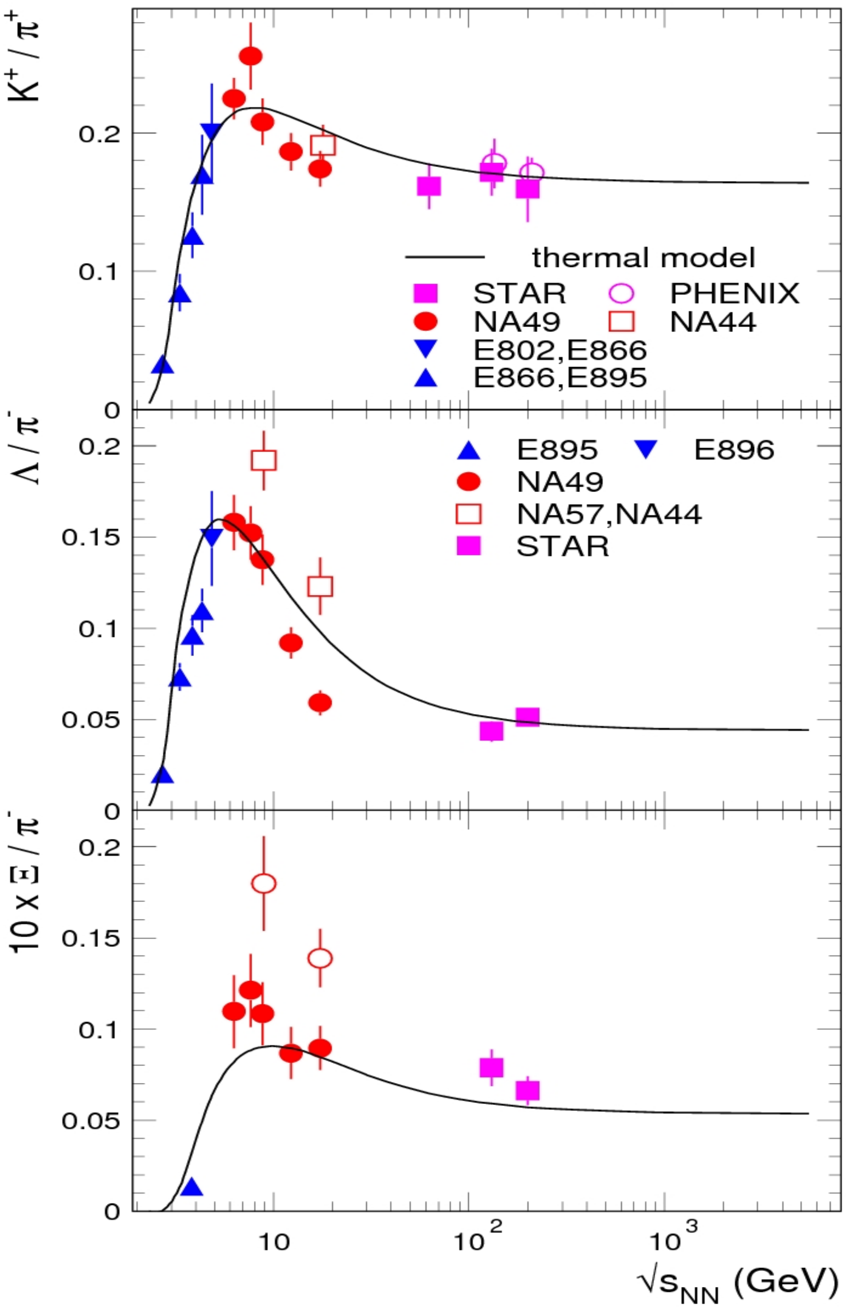




Freezeout line provide lower bound of the QCD phase boundary with clear separation of baryonic and mesonic dominance at $\sqrt{s} \approx 10 GeV$

Possible phase diagram of QCD with "Triple Point" and Quarkyonic Phase

A. Andronic, P. Braun-Munzinger, J. Stachel 09



Particle ratios in HIC from SIS to RHIC

An abrupt change in excitation functions of particle ratios at

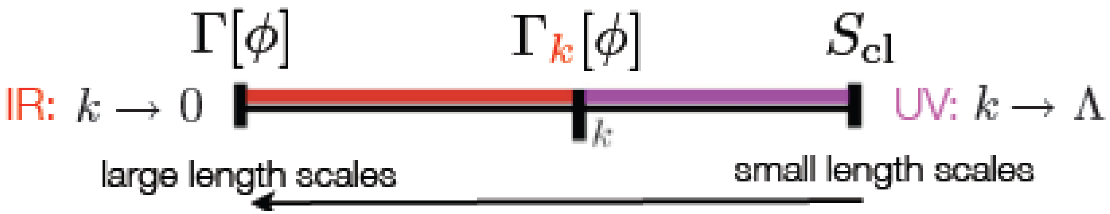
$\sqrt{s} \approx 10$ [GeV] ⇒

change of production dynamics from hadronic

($\pi N \to K\Lambda, \quad K\Lambda \to \Xi\pi,$ etc.)

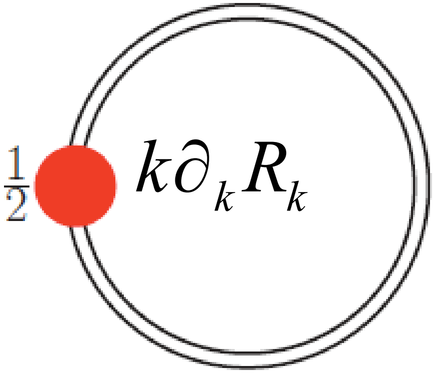
to partonic via hadronization of the QGP?

Including quantum fluctuations: FRG approach



k-dependent full propagator

$$k\partial_k \Gamma_k[\phi] = \frac{1}{2}$$



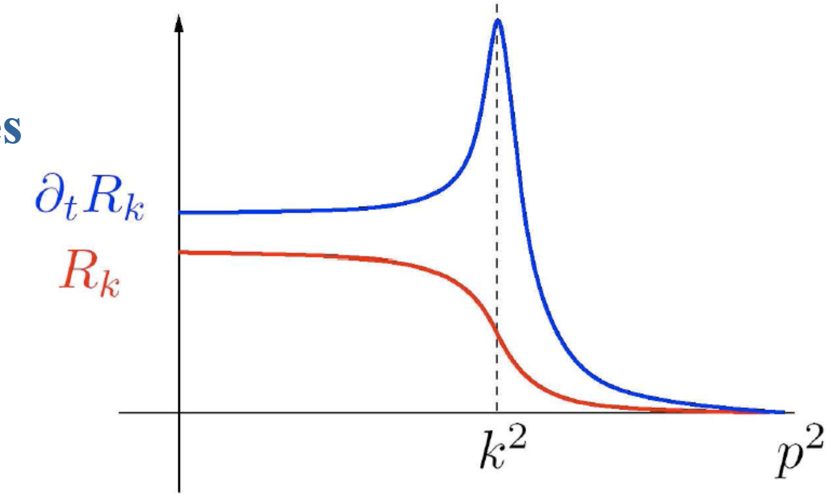
start at classical action and include quantum fluctuations successively by lowering k

FRG flow equation (C. Wetterich 93)

$$k\partial_k \Gamma_k \equiv \partial_t \Gamma_k = \frac{1}{2} \mathrm{Tr} \frac{\partial_t R_k}{\Gamma_k^{(2)} + R_k}$$

$$\Gamma_k^{(2)} = \frac{\delta^2 \Gamma_k}{\delta\phi\delta\phi}$$

Regulator function suppresses particle propagation with momentum Lower than k



$$\Omega(T,V) = \lim_{k\to 0}(\Omega_k = (T/V)\Gamma_k)$$

FRG for quark-meson model

$$\partial_t \Gamma_k = \frac{1}{2}\mathrm{Tr}\left[(\Gamma^{(2)}_{B,k} + R_{B,k})^{-1}\partial_t R_{B,k}\right. - \left.\mathrm{Tr}\left[(\Gamma^{(2)}_{F,k} + R_{F,k})^{-1}\partial_t R_{F,k}\right]\right.$$

- LO derivative expansion (J. Berges, D. Jungnicket, C. Wetterich) (η small)
  - Optimized regulators (D. Litim, J.P. Blaizot et al., B. Stokic, V. Skokov et al.)
    - Thermodynamic potential: B.J. Schaefer, J. Wambach, B. Friman et al.

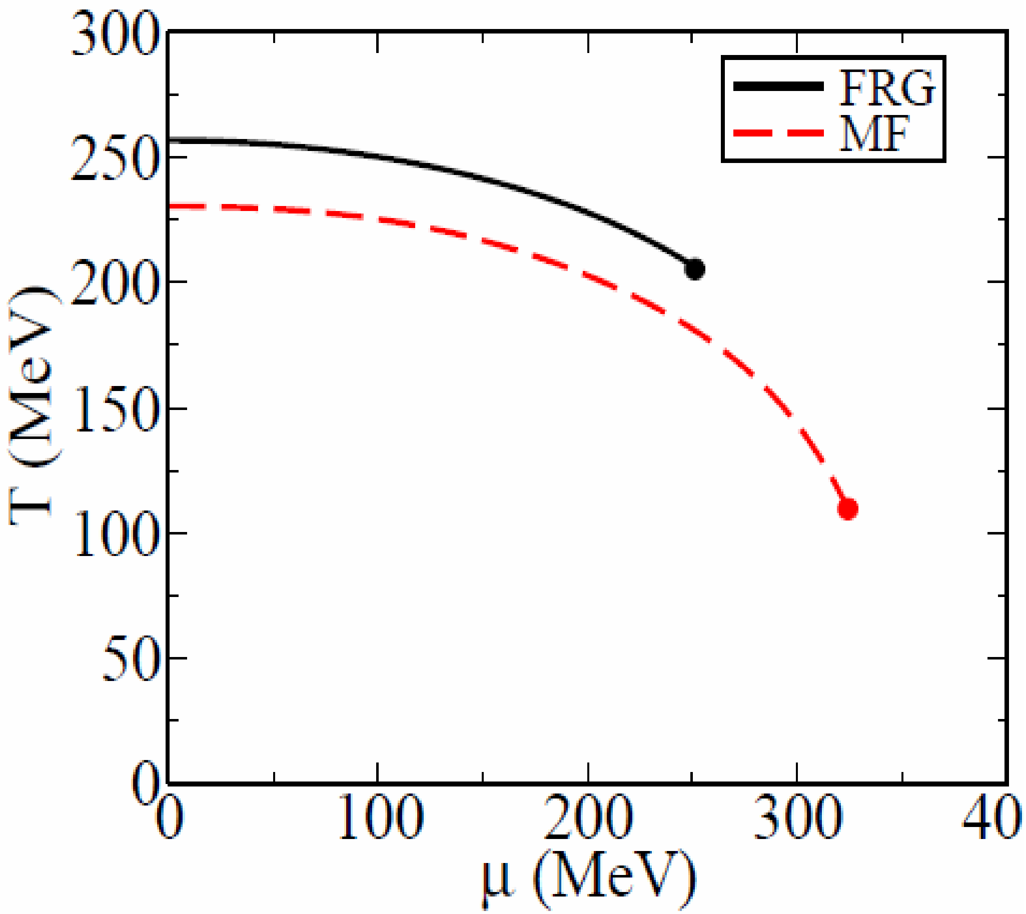
$$\partial_k \Omega_k (T, \mu : \rho_{o,k}) \approx [3\frac{1+2n_\pi}{E_\pi} + \frac{1+2n_\sigma}{E_\sigma} - 4N_f N_c \frac{1-n_q-n_{a-q}}{E_q}]$$

Non-linearity through self-consistent determination of disp. rel.

$E_i = \sqrt{k^2 + M_i^2}$ with $M_\pi^2 = \overline{\Omega}_k'$ $\quad M_\sigma^2 = \overline{\Omega}_k' + 2\rho_{0,k}\overline{\Omega}_k''$ $\quad M_q^2 = 2g^2\rho_{0,k}$

and $\overline{\Omega}_k' = \partial\overline{\Omega}_k / \partial\rho |_{\rho=\rho_{0,k}}$ with $\overline{\Omega}_k = \Omega_k + h\sqrt{2\rho}$

Thermodynamics of PQM model in the presence of mesonic fluctuations within FRG approach



- Quantitative modification of the phase diagram due to quantum and thermal fluctuations.
- Shift of CP to the lower density and higher temperature

FRG at work –O(4) scaling

- Near T_c critical properties obtained from the singular part of the free energy density

$$t = \frac{T - T_c}{T_c}$$

$$F_S(t, h) = b^{-d} F(b^{1/\nu} t, b^{\beta\delta/\nu} h)$$

h : external field

- Phase transition encoded in the "equation of state"

$$< \sigma >= -\frac{\partial F_s}{\partial h} \Rightarrow$$

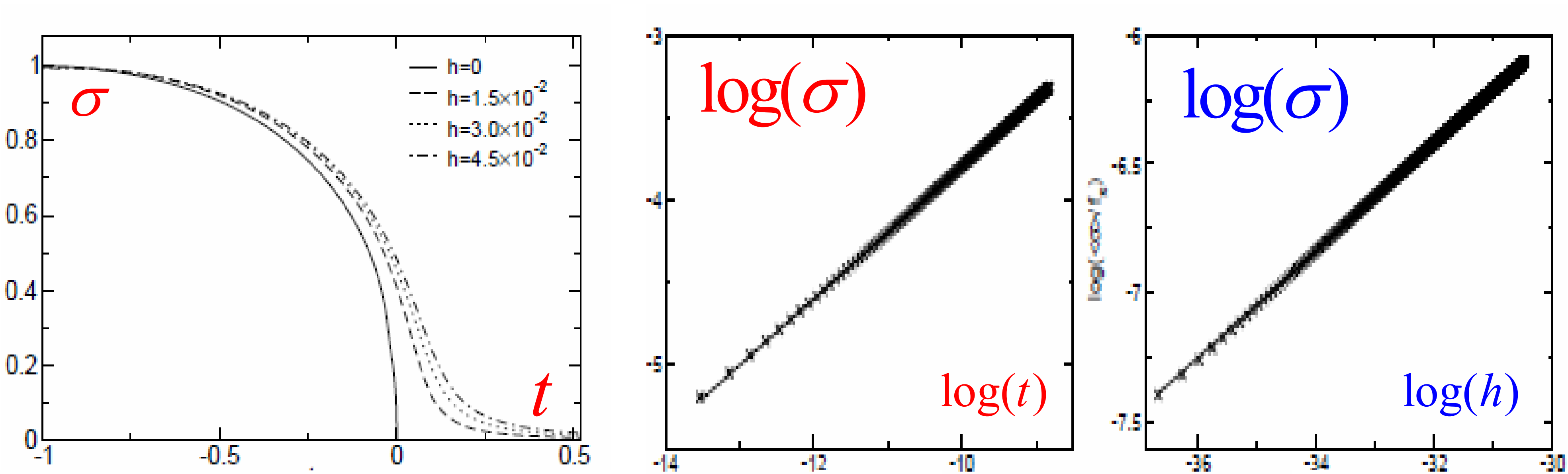
$$< \sigma >= h^{1/\delta} F_h(z) \ , \quad z = t h^{-1/\beta\delta}$$

$$< \sigma >= | t |^{\beta} \ F_s^{'}(h \, | t |^{-\beta\delta})$$

- Resulting in the well known scaling behavior of $< \sigma >$

$$< \sigma >= \begin{cases} B(-t)^{\beta}, h = 0, \ t < 0 & \text{coexistence line} \\ B h^{1/\delta} \quad , \ t = 0, \ h > 0 & \text{pseudo-critical point} \end{cases}$$

FRG-Scaling of an order parameter in QM model



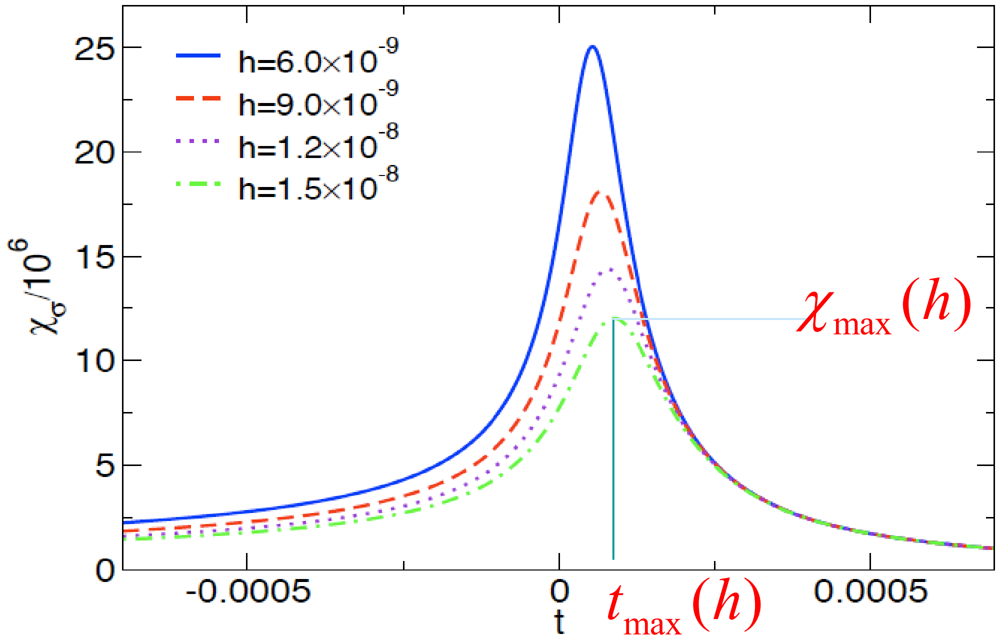
- The order parameter shows scaling. From the one slope one gets

	β	δ
MF	0.5	3
FRG	0.401(1)	4.818(29)
LGT	0.3836(46)	4.851(22)

- However we have neglected field-dependent wave function renormal. Consequently $\eta = 0$ and $\delta = 5$. The 3% difference can be attributed to truncation of the Taylor expansion at 3th order when solving FRG flow equation:

see D. Litim analysis for O(4) field Lagrangian

Fluctuations & susceptibilities



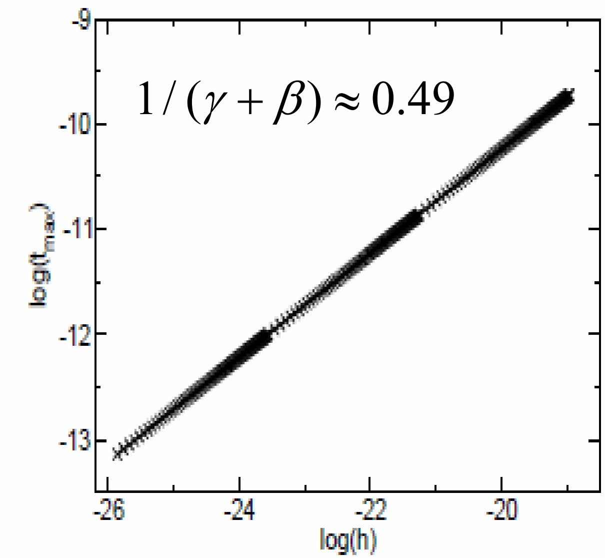
- Two type of susceptibility related with order parameter

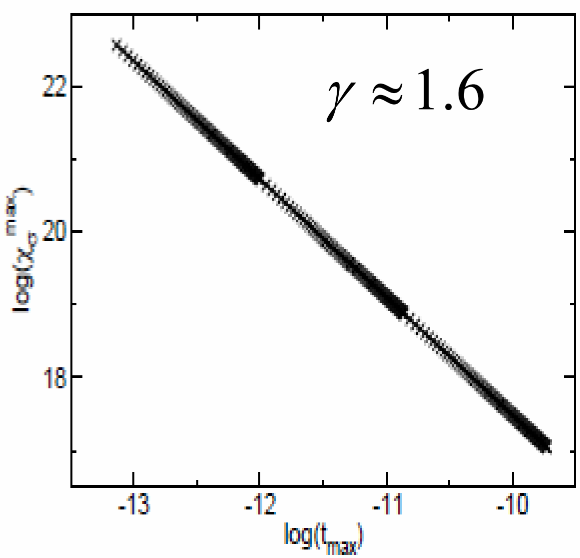
1. longitudinal

$$\chi_l = \chi_\sigma = \partial\sigma / \partial h$$

2. transverse

$$\chi_t = \chi_\pi = \sigma / h$$





- Scaling properties at t=0 and $h \to 0$

$$\chi_\sigma \square \delta = \chi_\pi = B\ h^{1/\delta-1}$$

$$t_{\max} \approx h^{1/(\gamma+\delta)}$$

$$\chi_\sigma(t_{\max}) \approx t_{\max}{}^{-\gamma}$$

Focusing of Isentrops and their signature

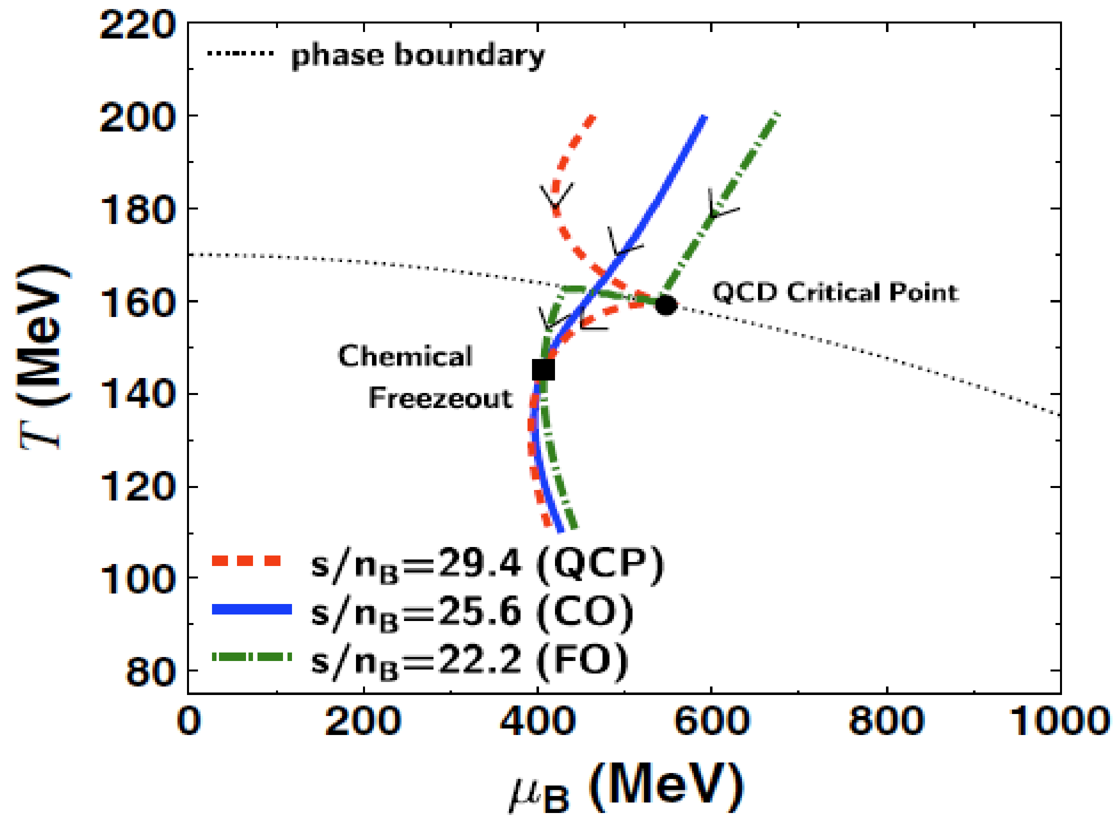
Asakawa, Bass, Müller, Nonaka

- Idea: $\bar{p}/p$ ratio sensitive to μ_B

  large q_t baryons emitted early, small q_t later

Isentropic trajectories dependent on EoS

In Equilibrium:
momentum-dep.
of $\bar{p}/p$ ratio
reflects history



Caveats:

•Critical slowing down

•Focusing non-unsiversal!!

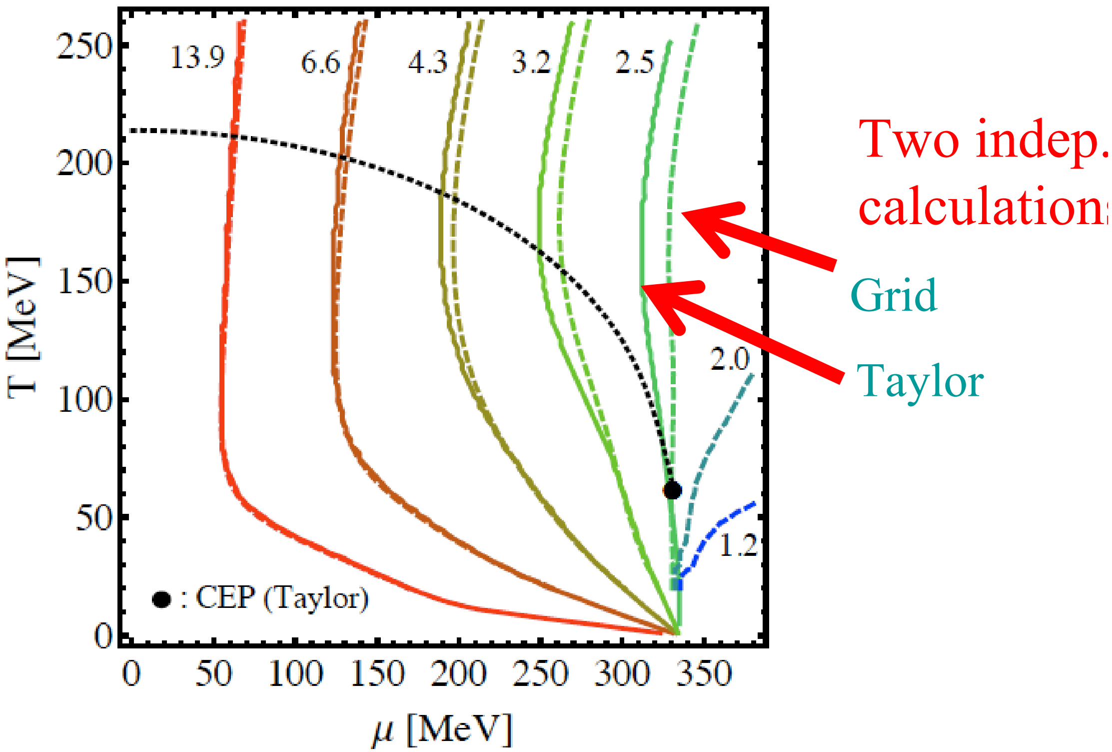
FRG at work –global observables

- Lines of constant s/n

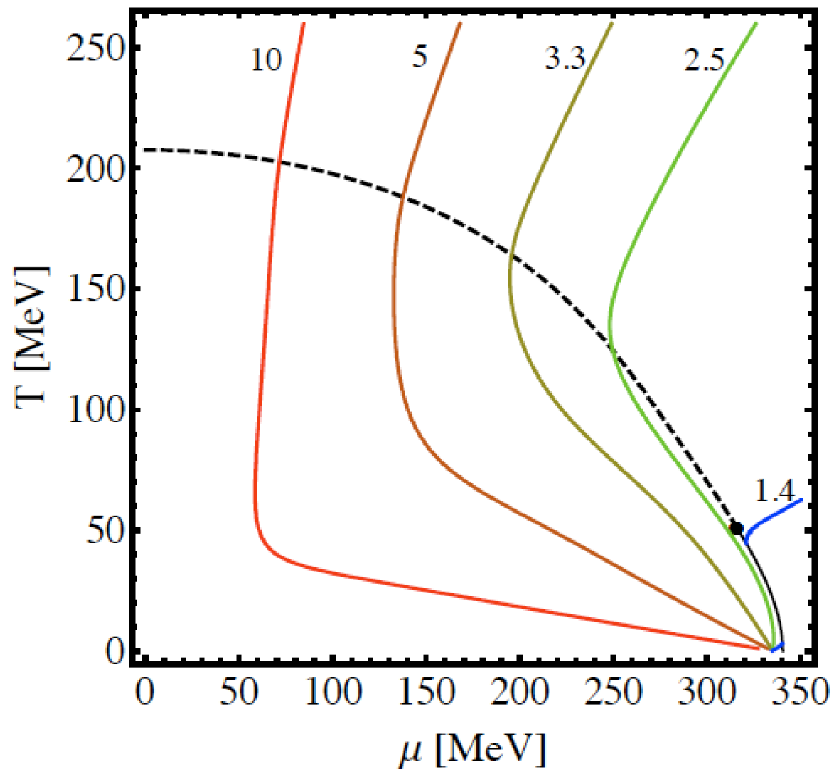
$$s=\frac{\partial p}{\partial T}=-\left.\frac{\partial\left[a_{0,k}-h\sqrt{2\rho_{0,k}}\right]}{\partial T}\right|_{k=0}$$

$$n=\frac{\partial p}{\partial \mu}=-\left.\frac{\partial\left[a_{0,k}-h\sqrt{2\rho_{0,k}}\right]}{\partial \mu}\right|_{k=0}$$

FRG - E. Nakano et al.



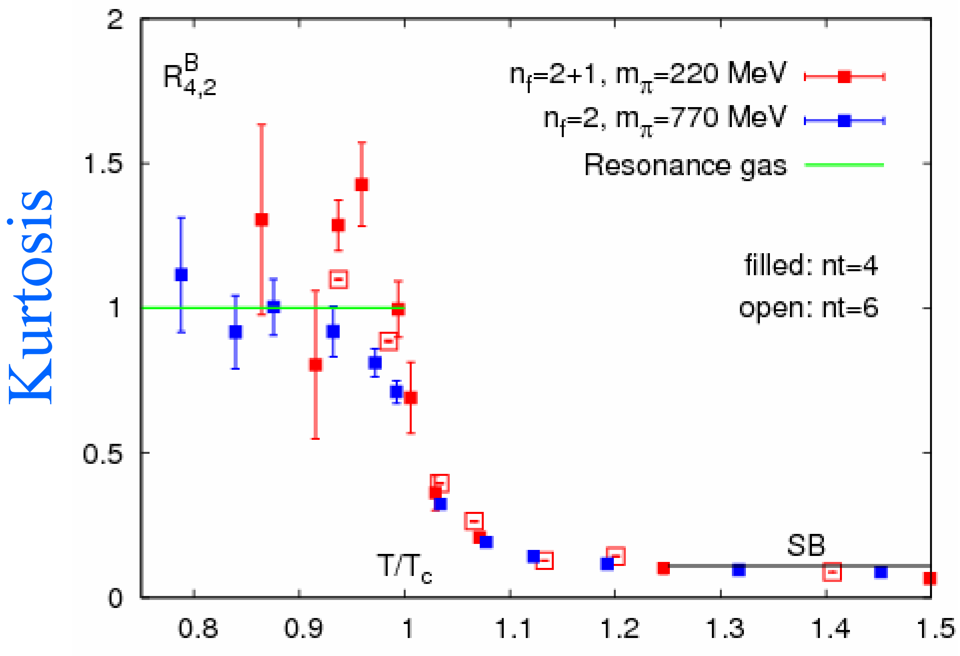
MF results: see also K. Fukushima



Kurtosis as excellent probe of deconfinement

F. Karsch, Ch. Schmidt et al., S. Ejiri et al.

$$R_{4,2}^{\ B} = \frac{1}{9}\frac{c_4}{c_2}$$



Observed quark mass dependence of kurtosis, remnant of chiral O(4) dynamics?

- HRG factorization of pressure:

$$P^B(T,\mu_q) = F(T)\cosh(3\mu_q/T)$$

consequently: $c_4/c_2 = 9$ in HRG

- In QGP, $SB = 6/\pi^2$
- Kurtosis=Ratio of cumulants

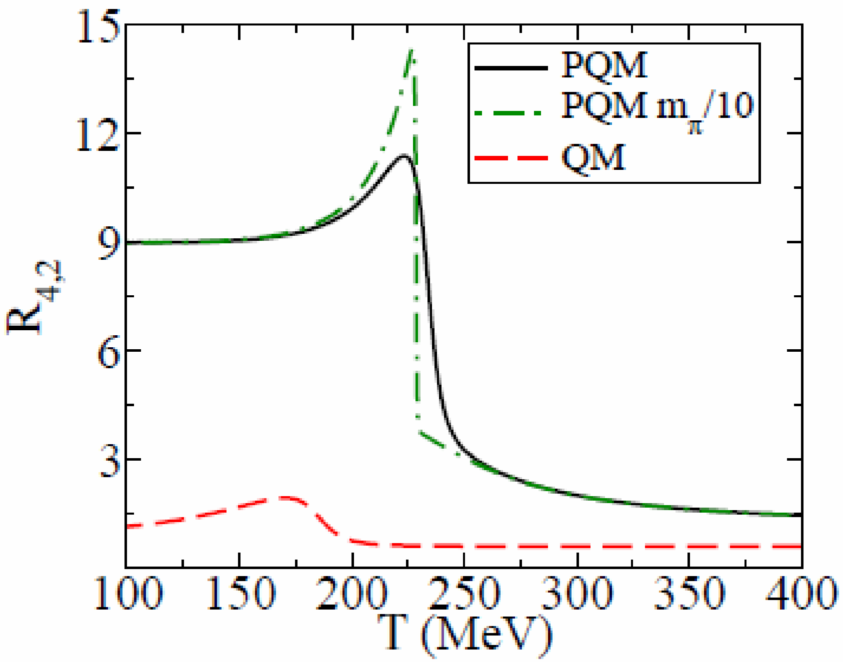
$$c_4^q / c_2^q = \frac{<(\delta N_q)^4>}{<(\delta N_q)^2>} - 3<(\delta N_q)^2>$$

excellent probe of deconfinement

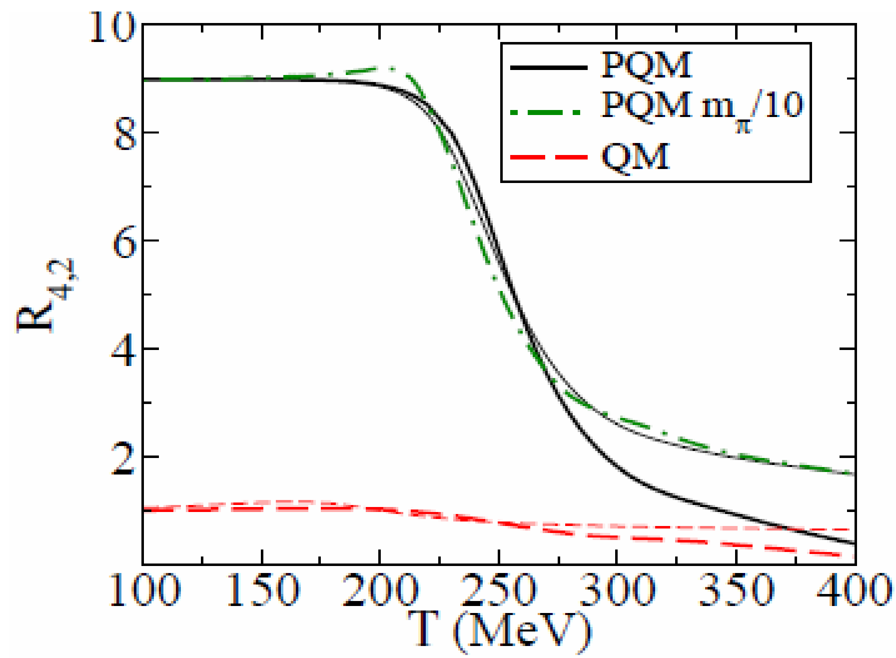
Kurtosis of net quark number density in PQM model

V. Skokov, B. Friman &K.R.

MF results



FRG results



- Strong dependence on pion mass, remnant of O(4) dynamics
- Smooth change with a rather weak dependen-ce on the pion mass

Summary

- Effective chiral models provide a powerful tool to study the critical consequences of the chiral symmetry restoration and deconfinement in QCD, however

- To quantify the QCD phase diagram and thermodynamics require the first principle LGT calculations

- Systematic studies of heavy ion collisions in broad range of collision energies
  SIS → FAIR → SPS → LHC with possibly small energy steps are essential to quantify the QCD phase structure and medium properties