Latest lattice QCD thermodynamics results

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RBC-Bielefeld:
M. Cheng, N.H. Christ, S.Datta, J. van der Heide, C. Jung, F. Karsch, O. Kaczmarek, E. Laermann, R.D. Mawhinney, C. Miao, P. Petreczky, K. Petrov, CS, W. Söldner and T. Umeda.

HotQCD:
A. Bazavov, T. Battacharya, M. Cheng, N.H. Christ, C. De Tar, S. Ejiri, S. Gottlieb, R. Gupta, U. Heller, K. Hübner, C. Jung, F. Karsch, E. Laermann, L. Levkova, T. Luu, R.D. Mawhinney, C. Miao, P. Petreczky, D. Renfrew, CS, W. Söldner, R. Soltz, R. Sugar, D. Toussaint, O. Vranas.

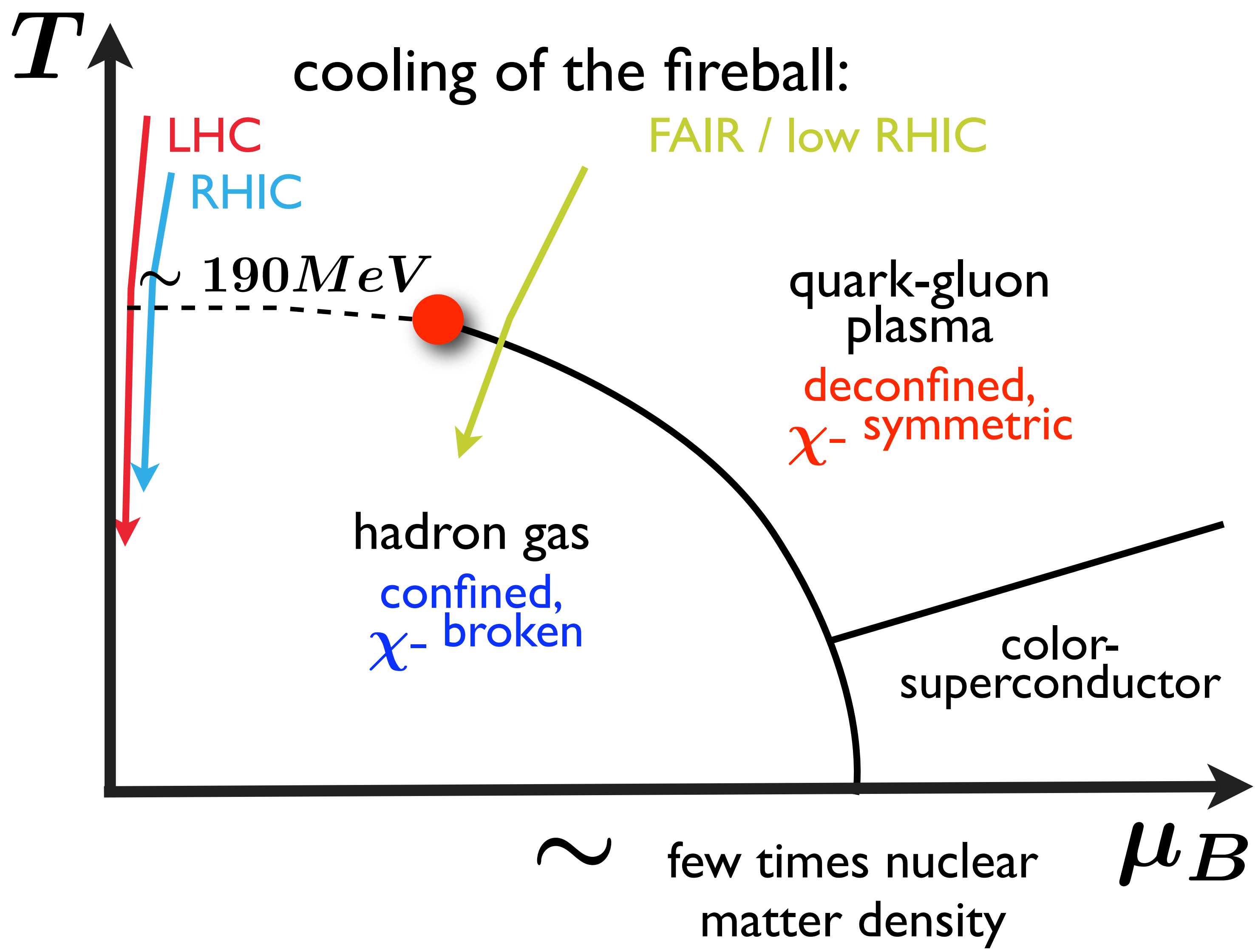
Outline

- Introduction
- QCD Thermodynamics at zero baryon number density (EoS)
- Hadronic fluctuations and the QCD critical point
- EoS at non-zero density
- Summery

The QCD phase diagram

- LGT* at $\mu = 0$
  RHIC, LHC
- LGT at $\mu > 0$
  RHIC at low energies, FAIR@GSI
- Observable that connects experiment and LGT:

  B,S,Q fluctuations ?

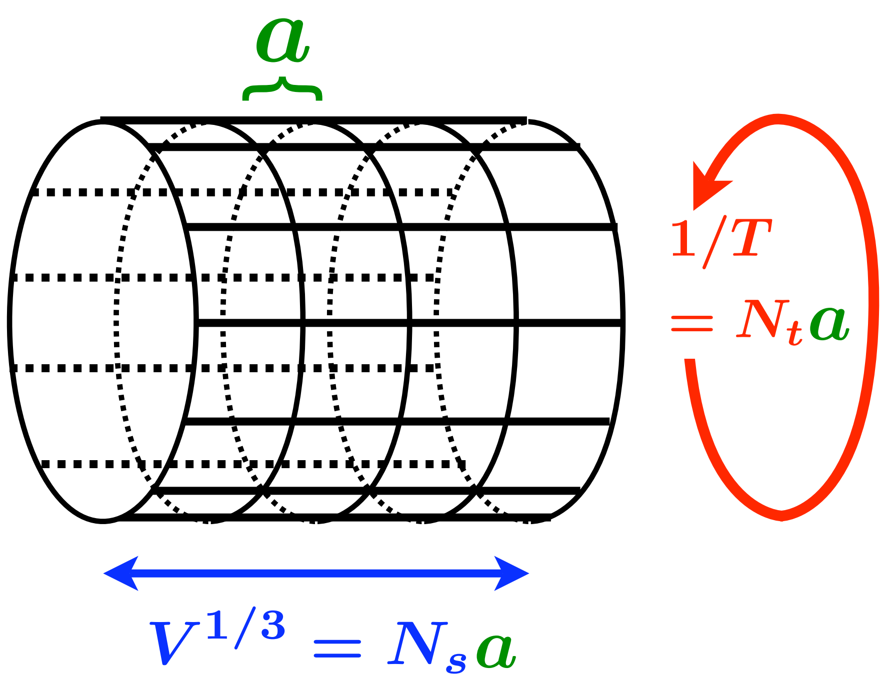


$$\mu_S = \mu_Q = 0$$

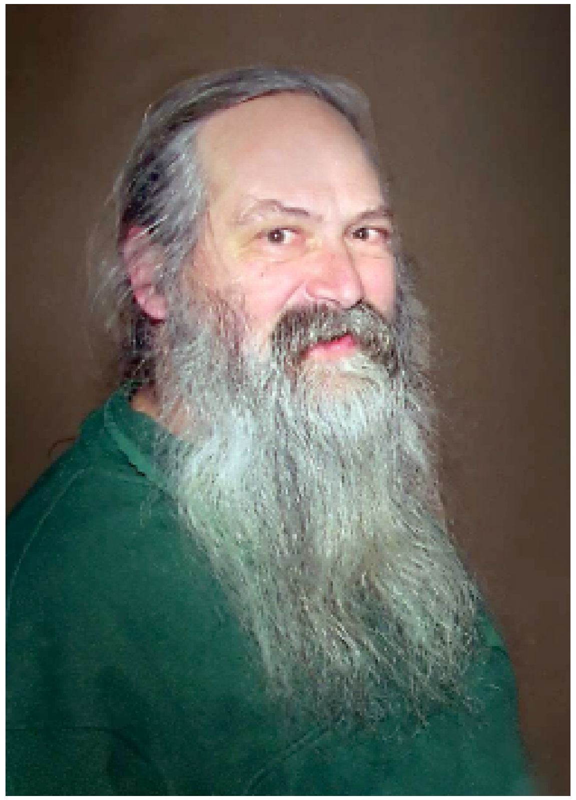
*LGT = Lattice gauge theory

Lattice QCD

- Analyzing hot and dense matter on the lattice: $N_s^3 \times N_t$



Michael Creutz,
PRD 21 (1980) 2306



- Using only the QCD partition function:

$$Z(V,T,\mu) = \int \mathcal{D}A\mathcal{D}\psi\mathcal{D}\bar{\psi}\ \exp\{-S_E\}$$

$$S_E = \int_0^{1/T} dx_0 \int_V d^3x \mathcal{L}_E(A,\psi,\bar{\psi},\mu)$$

$\longrightarrow$ $\approx 10^6$ grid points, $\approx 10^8$ d.o.f. integrate eq. of motion $\longrightarrow$ need fast computers!

Cut-off effects and the choice of action

- **free staggered fermions (infinite temperature)**
  - $\mathcal{O}(a^2)$ discretization errors

    $\longrightarrow$ add irrelevant operators to the action

$$\frac{p}{p_{SB}} = 1 + \frac{248}{147}\left(\frac{\pi}{N_\tau}\right)^2 + \frac{635}{147}\left(\frac{\pi}{N_\tau}\right)^4 + \ldots \quad (standard)$$

$$\frac{p}{p_{SB}} = 1 + 0 - \frac{1143}{980}\left(\frac{\pi}{N_\tau}\right)^4 + \frac{73}{2079}\left(\frac{\pi}{N_\tau}\right)^6 + \ldots \quad (p4)$$

$$\frac{p}{p_{SB}} = 1 + 0 - \frac{1143}{980}\left(\frac{\pi}{N_\tau}\right)^4 - \frac{365}{77}\left(\frac{\pi}{N_\tau}\right)^6 + \ldots \quad (Naik)$$

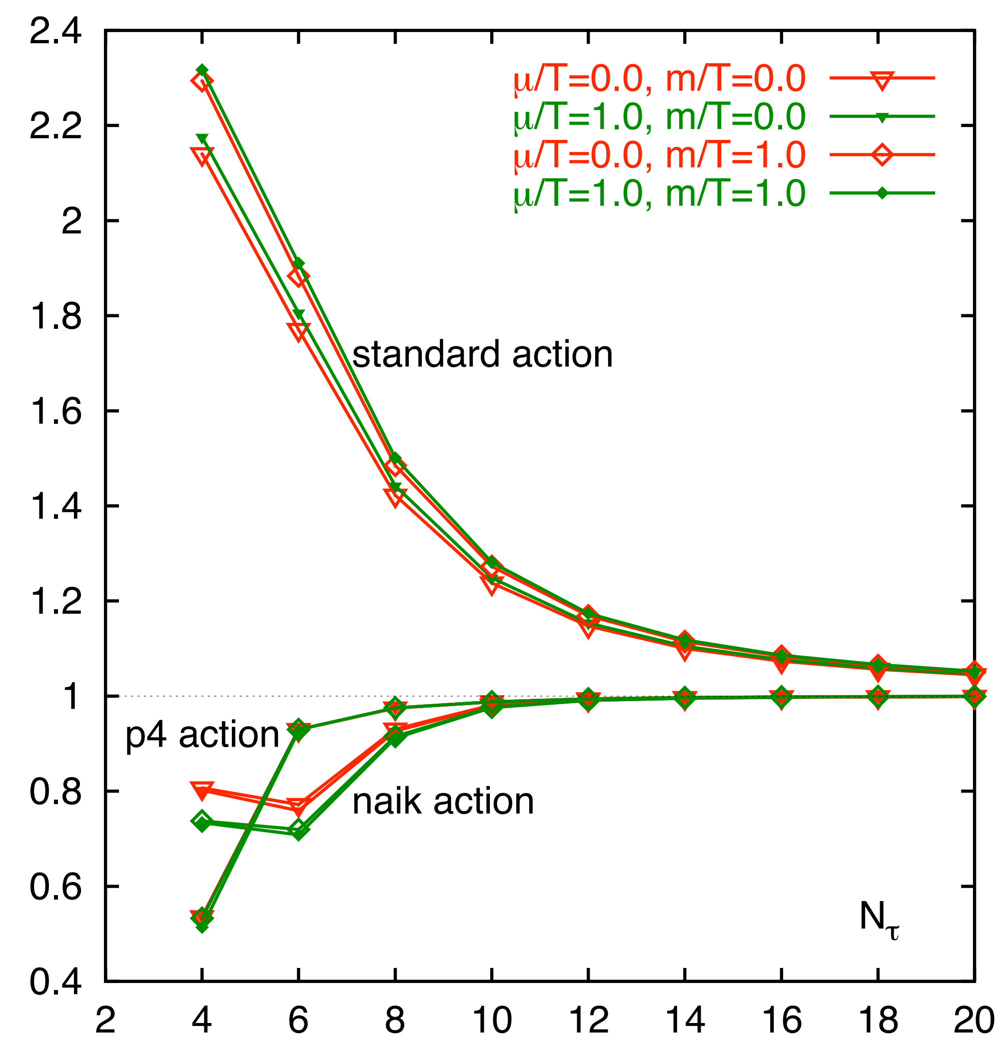
P. Hegde et al., arXiv:0801.4883.

  - $\mathcal{O}(a^2)$ flavor mixing effects

    $\longrightarrow$ use various smearing techniques: 3-link staple, 7-link staple (asqtad)

  - cut-off dependence on μ is very small

    $\longrightarrow$ quark number susceptibilities show similar cut-off effects as pressure



Bulk thermodynamics at $\mu = 0$

- **The interaction measure**

$$\frac{\epsilon - 3p}{T^4} = T\frac{d}{dT}\left(\frac{p}{T^4}\right) = \left(a\frac{d\beta}{da}\right)_{LCP} \frac{dp/T^4}{d\beta}$$

$$\text{(gluon)} = R_\beta \left(\langle S_G\rangle_0 - \langle S_G\rangle_T\right) N_\tau^4$$

$$\text{(fermion)} - R_\beta R_m \left[2\hat{m}_l \left(\left\langle\bar\psi\psi\right\rangle_{l,0} - \left\langle\bar\psi\psi\right\rangle_{l,T}\right) + \hat{m}_s \left(\left\langle\bar\psi\psi\right\rangle_{s,0} - \left\langle\bar\psi\psi\right\rangle_{s,T}\right)\right] N_\tau^4$$

$$\text{(mass ratio)} - R_\beta R_h \hat{m}_s \left(\left\langle\bar\psi\psi\right\rangle_{s,0} - \left\langle\bar\psi\psi\right\rangle_{s,T}\right) N_\tau^4$$

$$R_\beta = -a\frac{d\beta}{da} \qquad R_m = \frac{1}{\hat{m}_l}\frac{d\hat{m}_l}{d\beta} \qquad R_h = \frac{\hat{m}_l}{\hat{m}_s}\frac{d(\hat{m}_s/\hat{m}_l)}{d\beta}$$

$\longrightarrow$ need T-scale and various β-functions to quite some accuracy

- **The pressure**

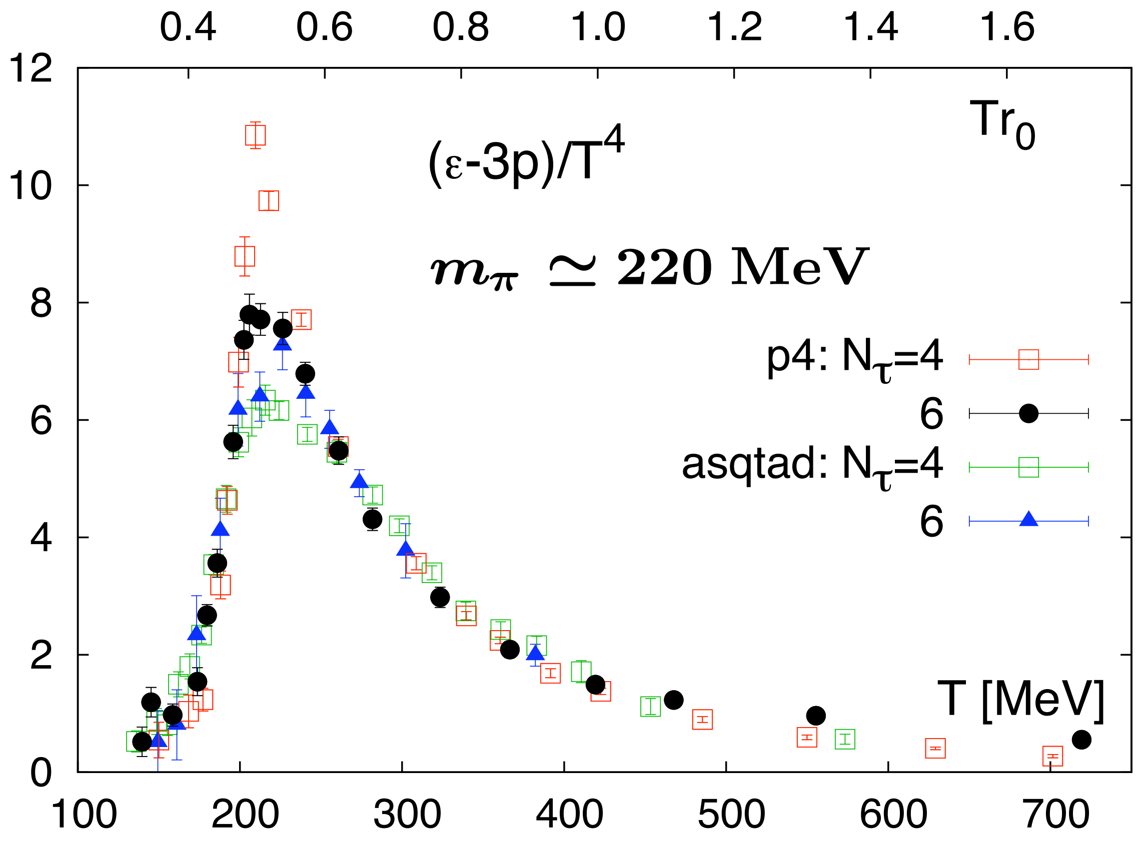
$$\frac{p(T)}{T^4} - \frac{p(T_0)}{T_0^4} = \int_{T_0}^{T} dT' \, \frac{1}{T'} \, \frac{\epsilon - 3p}{T'^4}$$

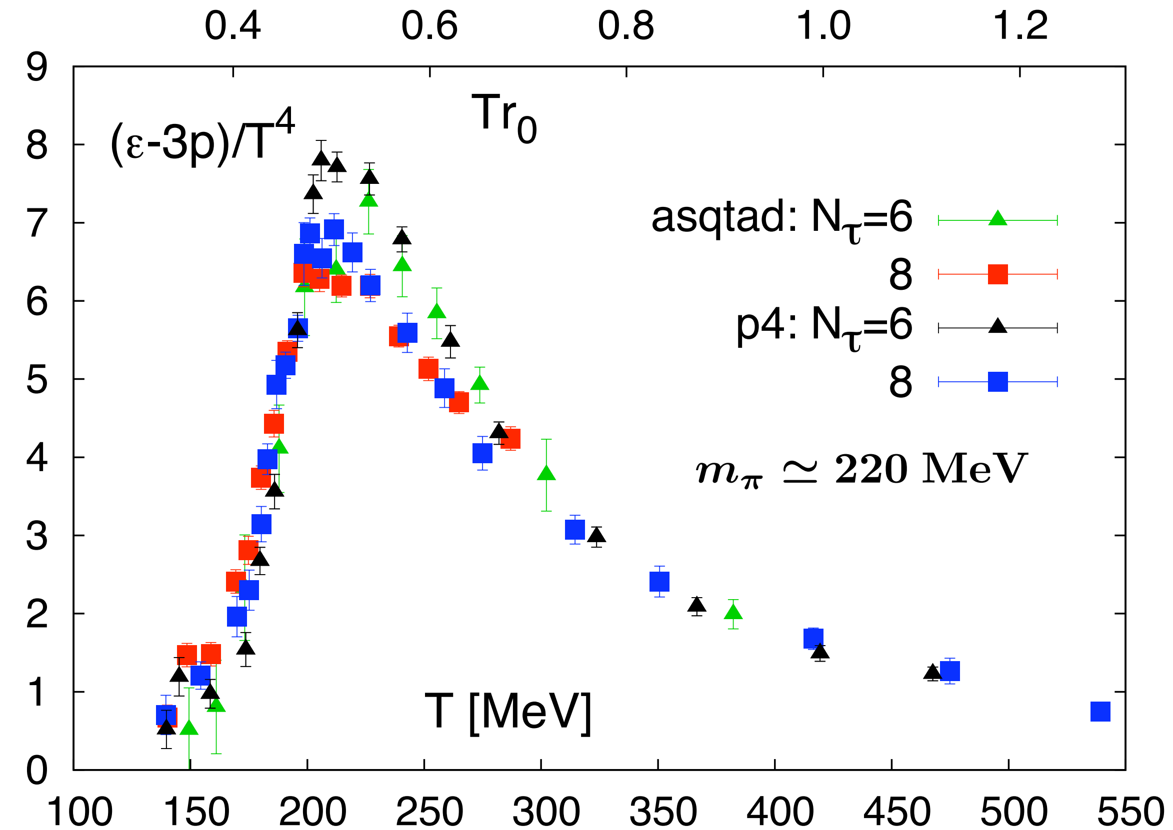
Integral-Method

$\longrightarrow$ unknown integration constant

Bulk thermodynamics at $\mu = 0$

- **The interaction measure**





LCP: $m_q = 0.1 m_s$

$\longrightarrow$ $m_\pi \approx 220 MeV$

$\longrightarrow$ tune m_s to physical strange quark mass, using $m_K, m_{\bar{s}s}$

p4 vs asqtad:

$\longrightarrow$ overall good agreement

- $T\left[(\epsilon - 3p)/T^4|_{max}\right] \approx 200 MeV$ (~ softest point of EoS)
- cut-off effects persist in peak region

$N_\tau = 4, 6$

- p4-data: **RBC-Bielefeld**, M. Cheng et al., PRD 77, 014511 (2008)
- asqtad-data: **MILC**, C. Bernard et al., PRD 75, 094505 (2007)

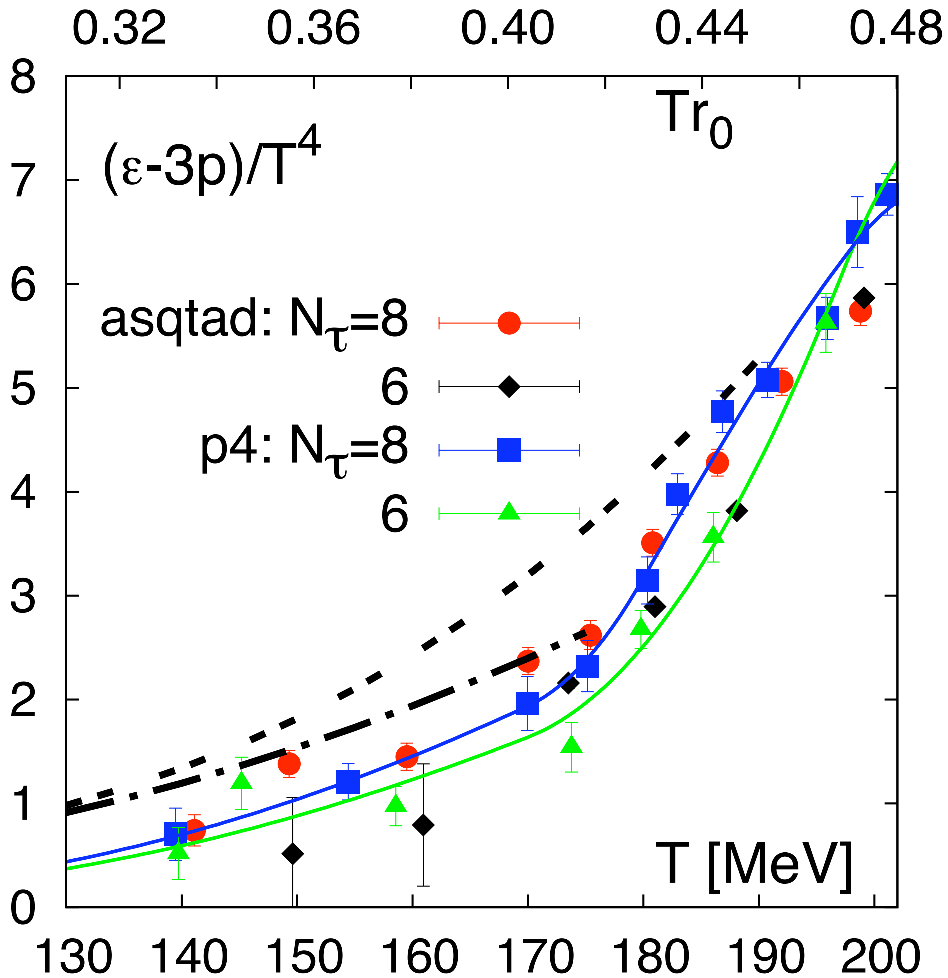
$N_\tau = 8$

- p4-, asqtad-data: **HotQCD preliminary**

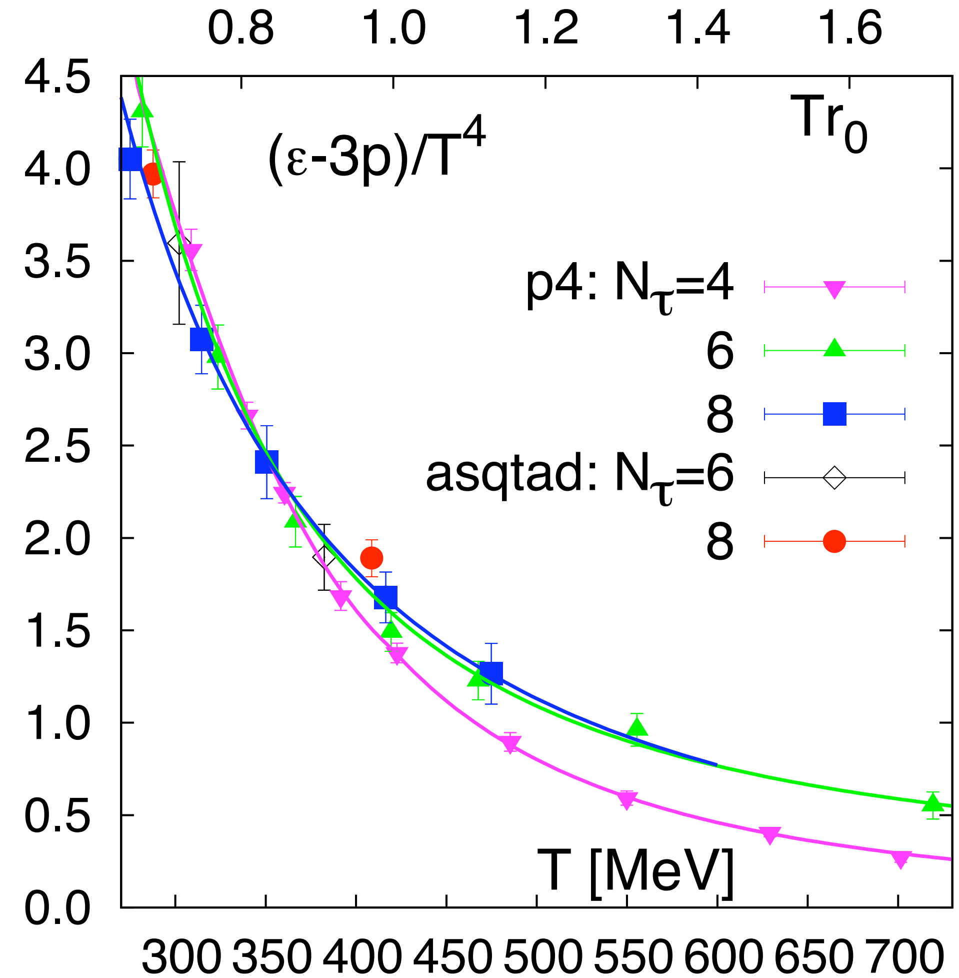
Bulk thermodynamics at $\mu = 0$

- **The interaction measure ...towards the continuum limit**

low T regime



high T regime

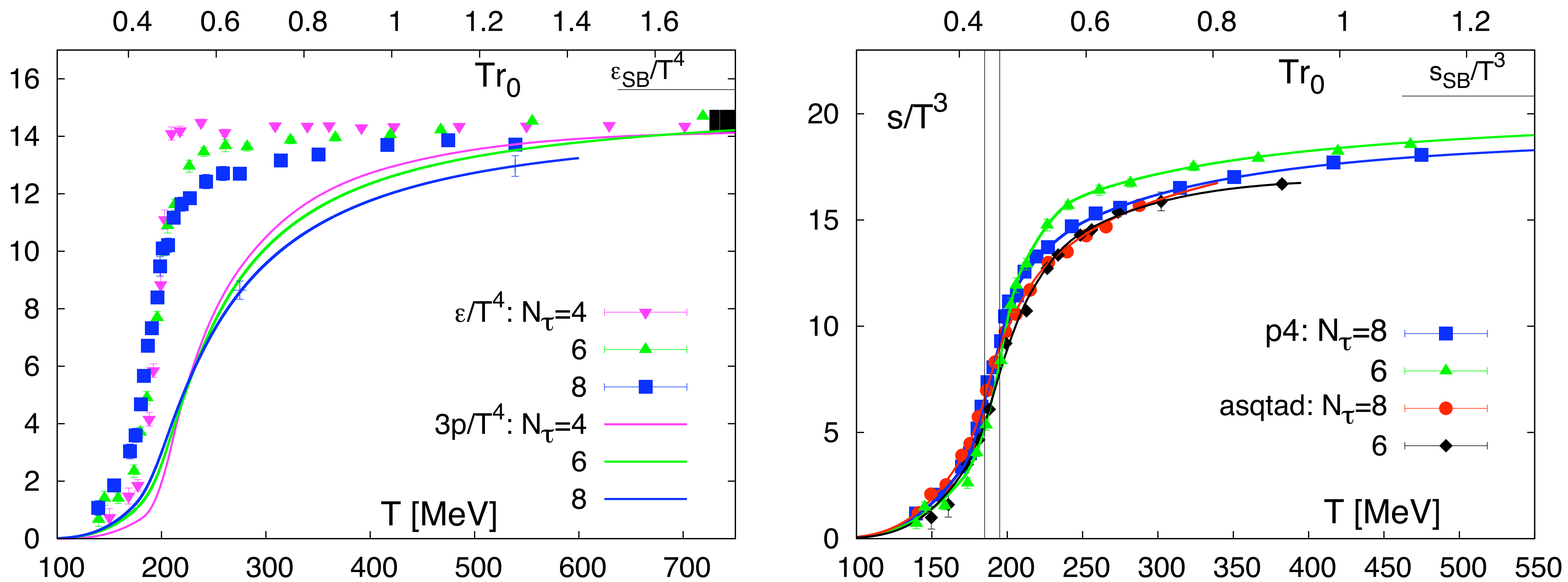


- $N_\tau = 6 \rightarrow 8$: small shift of transition region
  ⟶ better agreement with HRG
- approach to physical quark masses
  ⟶ further shift of T-scale $\mathcal{O}(5MeV)$

- good agreement between Nt=8 and 6 results for $T > 300MeV$
  ⟶ more data needed to make contact with (resummed) perturbative QCD
- strong deviations from conformal limit: find $(\epsilon - 3p)/T^4 \sim a/T^2 + b/T^4$

Bulk thermodynamics at $\mu = 0$

- **The Pressure, Energy and Entropy**



HotQCD preliminary

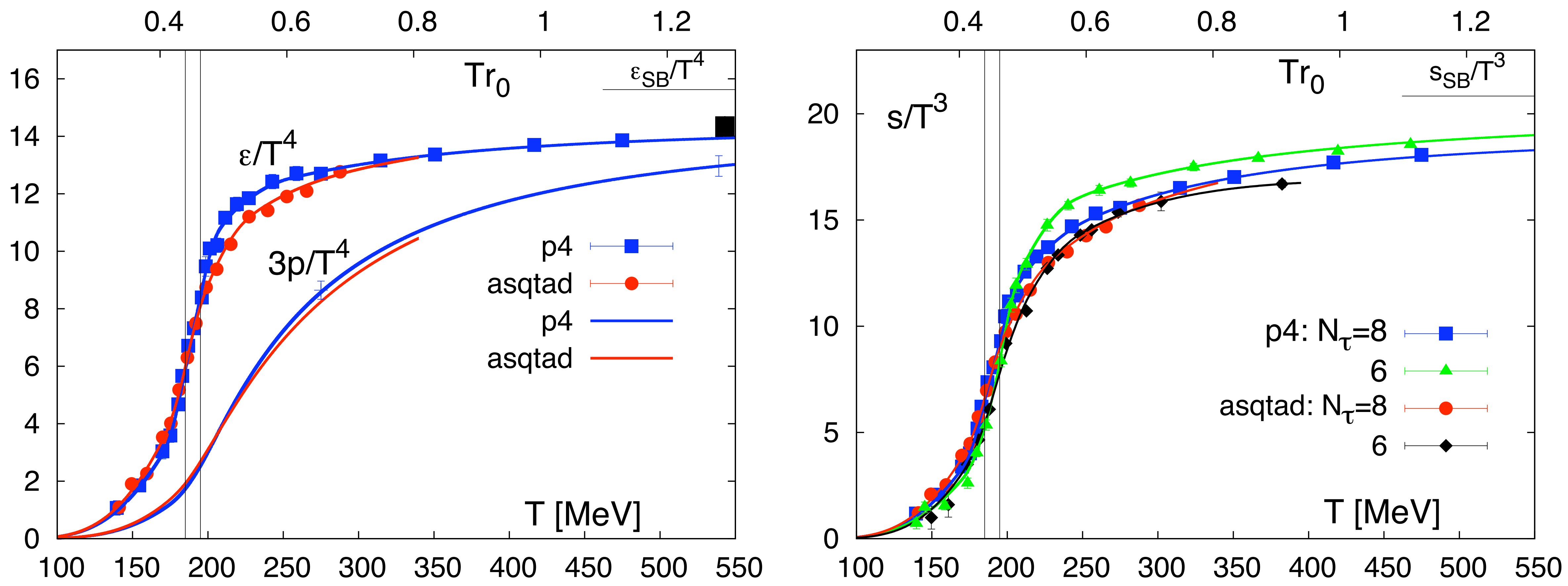
- p/T^4 from integrating over $(\epsilon - 3p)/T^5$

$\longrightarrow$ systematic error from starting the integration at $T_0 = 100 MeV$ with $p(T_0) = 0$

$\longrightarrow$ use HRG to estimate systematic error: $\left[p(T_0)/T_0^4\right]_{HRG} \approx 0.265$

Bulk thermodynamics at $\mu = 0$

- **The Pressure, Energy and Entropy**



HotQCD preliminary

- p/T^4 from integrating over $(\epsilon - 3p)/T^5$
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Bulk thermodynamics at $\mu = 0$

- **EoS and velocity of sound**

$$c_s^2 = \frac{dp}{d\epsilon} = \epsilon \frac{d(p/\epsilon)}{d\epsilon} + \frac{p}{\epsilon}$$

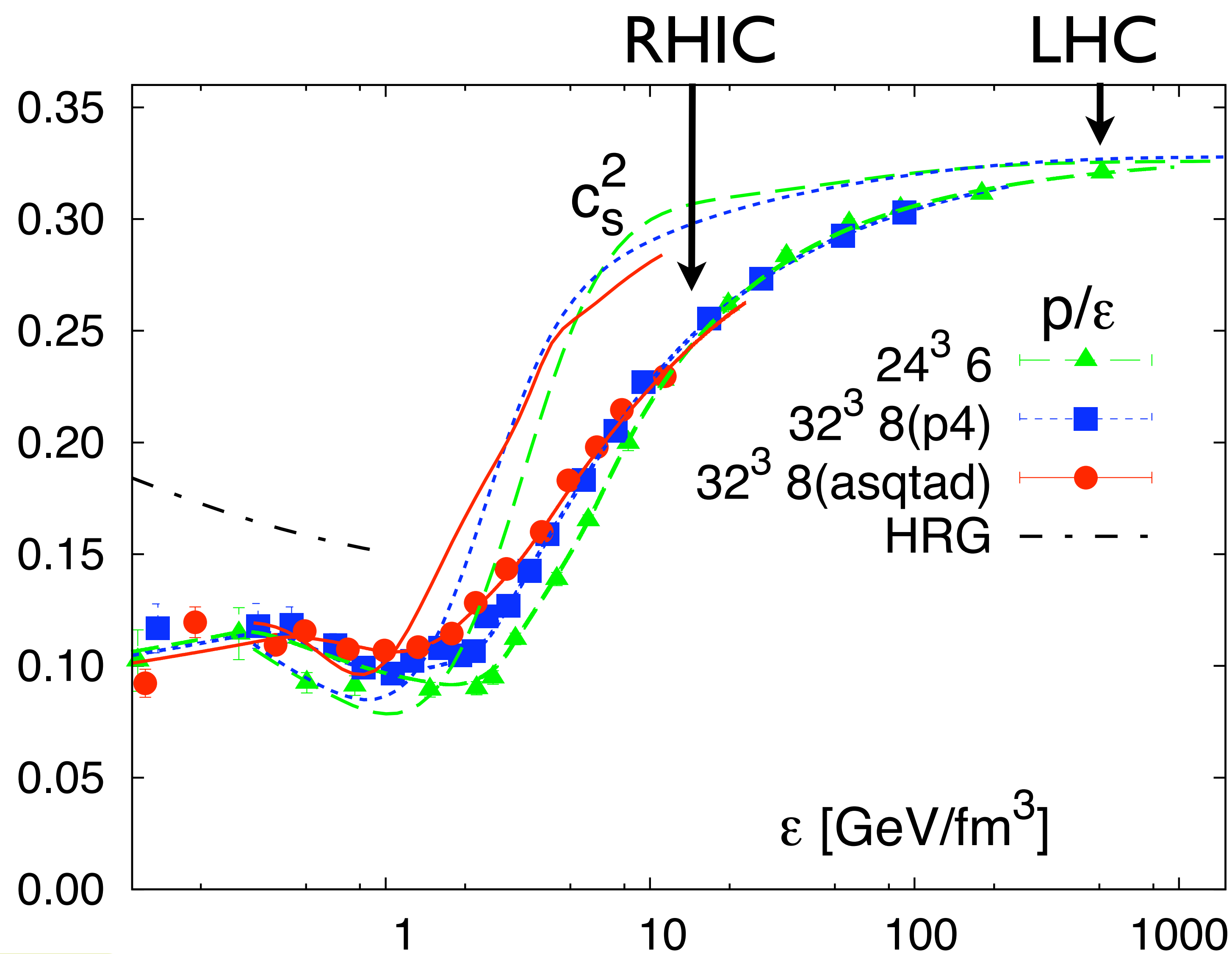
- fit: $p/\epsilon = c - a/(1 + b\epsilon)$
  for $\epsilon > 4 GeV/fm^3$
  5-th order polynomial
  for $\epsilon < 4 GeV/fm^3$

- evaluate velocity of sound from fit:

$\longrightarrow$ $c_s^2 \approx 1/3$
for $\epsilon > 100 GeV/fm^3$

$\longrightarrow$ $c_s^2 \approx 0.09$
for $\epsilon \approx (1-2) GeV/fm^3$

slows down hydrodynamic expansion



HotQCD preliminary

Lattice QCD at nonzero density

- The „sign problem“

$$Z(V,T,\mu) = \int \mathcal{D}A\mathcal{D}\psi\mathcal{D}\bar{\psi}\ \exp\{S_F(A,\psi,\bar{\psi}) - \beta S_G(A)\}$$
$$= \int \mathcal{D}A\ \det[M](A,\mu)\ \exp\{-\beta S_G(A)\}$$

complex for $\mu > 0$

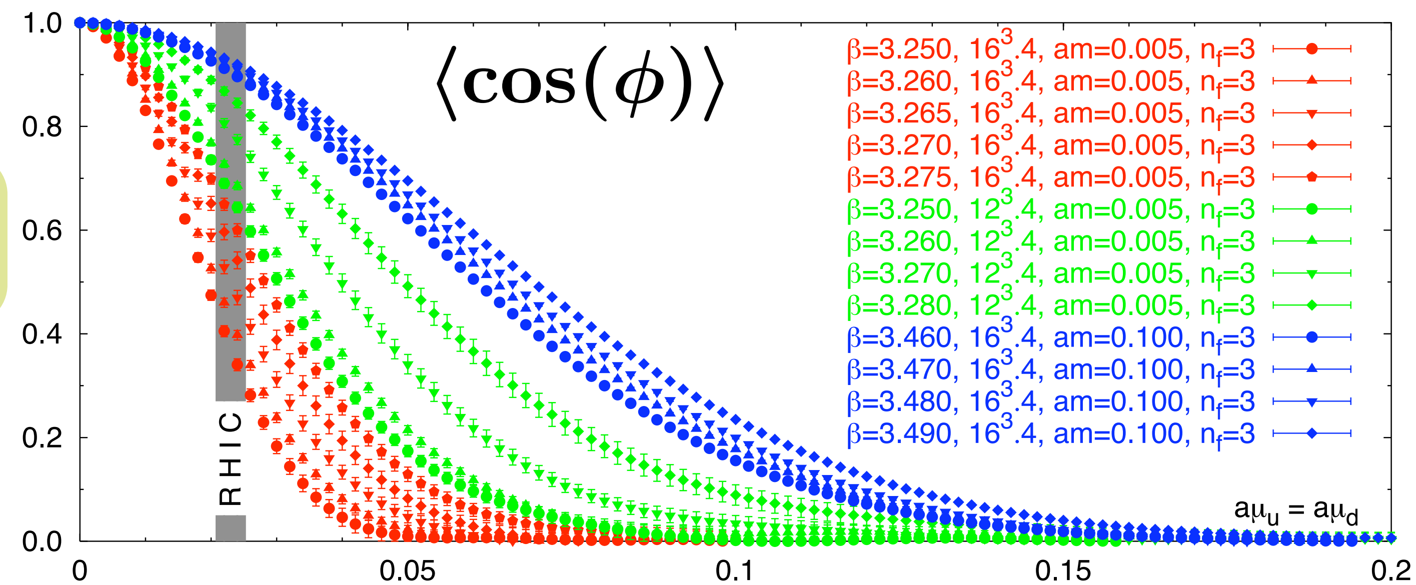
propabilistic interpretation necessary for Monte Carlo

- Factorization of the fermion determinant

$$\det[M] \equiv |\det[M]| \exp\{i\phi\}$$

consider the phase quenched ensemble
$$\langle \mathcal{O} \rangle\ (\mu) = \langle \mathcal{O} \cos(\phi) \rangle_{|\det[M](\mu)|}$$

the sign problem
$$\langle \cos(\phi) \rangle_{|\det[M](\mu)|} \underset{T<T_c}{\longrightarrow} 0$$



$$\propto \left(1 - \frac{4\mu^2}{m_\pi^2}\right)^{N_f+1}$$

from mean field treatment of a chiral Lagrangian
K. Splittorff, J.J.M. Verbaarschot, PRL98:031601,2007

Lattice QCD at nonzero density

- start from Taylor expansion of the pressure,

$$\frac{p}{T^4} = \frac{1}{VT^3} \ln Z(V,T,\mu_u,\mu_d,\mu_s) = \sum_{i,j,k} c_{i,j,k}^{u,d,s} \left(\frac{\mu_u}{T}\right)^i \left(\frac{\mu_d}{T}\right)^j \left(\frac{\mu_s}{T}\right)^k$$

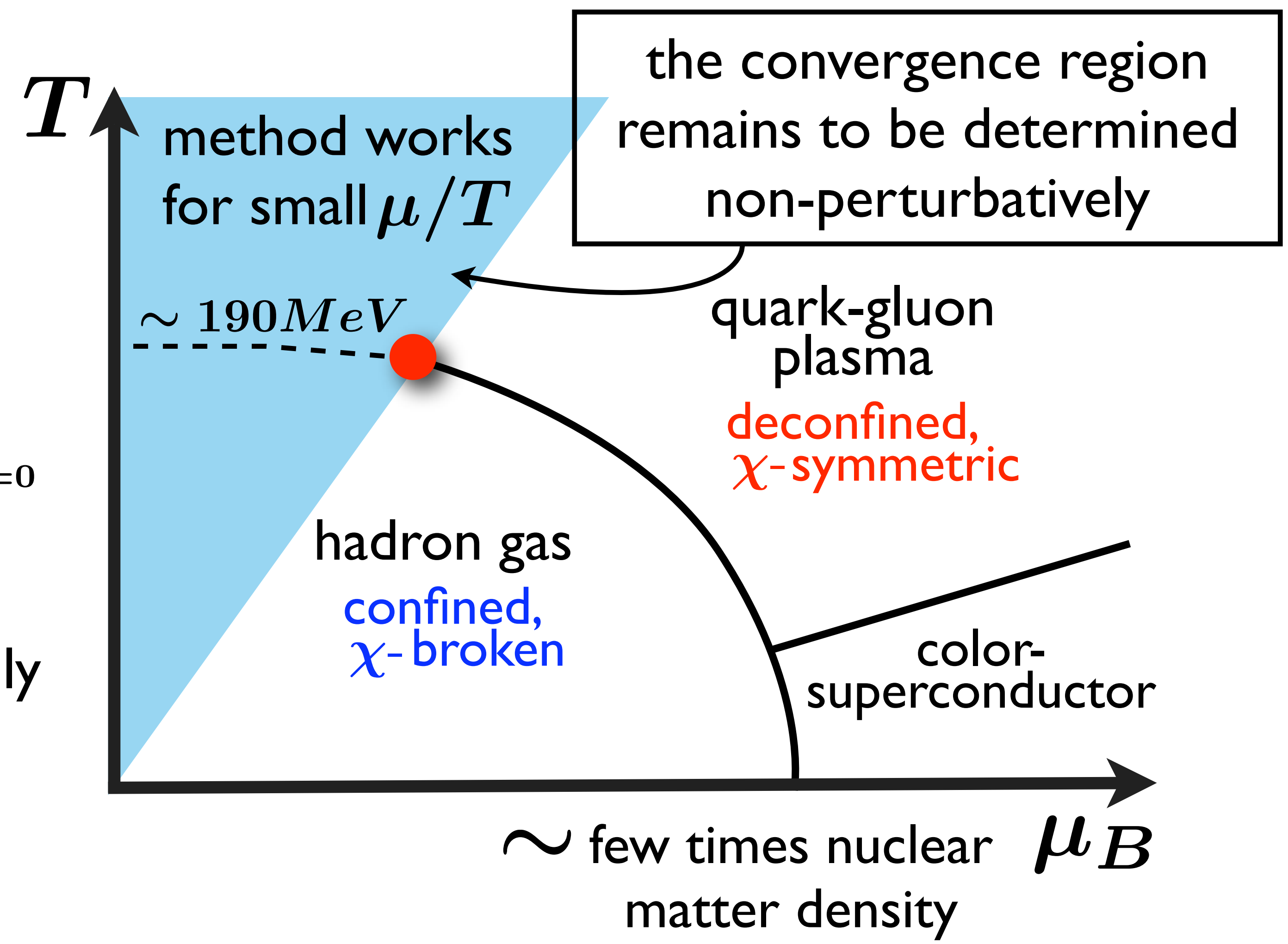
- calculate expansion coefficients for fixed temperature

- no sign problem:
  all simulations are done at $\mu = 0$

$$c_{i,j,k}^{u,d,s} \equiv \frac{1}{i!j!k!} \frac{1}{VT^3} \cdot \left. \frac{\partial^i \partial^j \partial^k \ln Z}{\partial(\frac{\mu_u}{T})^i \partial(\frac{\mu_d}{T})^j \partial(\frac{\mu_s}{T})^k} \right|_{\mu_{u,d,s}=0}$$

- method is straight forward:
  all terms can be generated automatically

Allton *et al.*, PRD66:074507,2002;
Allton *et al.*, PRD68:014507,2003;
Allton *et al.*, PRD71:054508,2005.



- use unbiased, noisy estimators to calculate $c_{i,j,k}^{u,d,s}$
  $\longrightarrow$ see C. Miao, CS, PoS (Lattice 2007) 175.

- line of constant physics: $m_q = m_s/10$
  (physical strange quark mass)

- measure currently up to
$$\mathcal{O}(\mu^8) \quad \longleftrightarrow \quad (N_t = 4)$$
$$\mathcal{O}(\mu^4) \quad \longleftrightarrow \quad (N_t = 6)$$

- expansion coefficients $c_{i,j,k}^{u,d,s}$ are related to B,S,Q-fluctuations

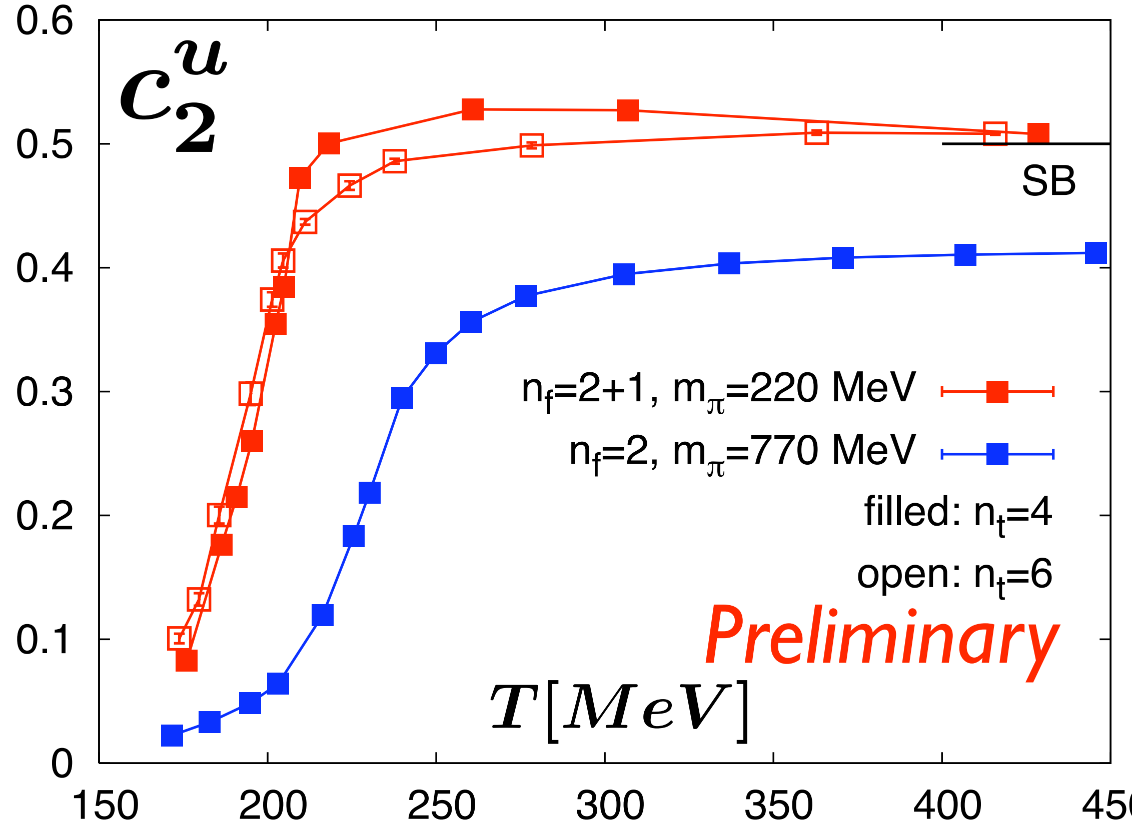
$$\begin{aligned} n_B &= \frac{\partial(p/T^4)}{\partial(\mu_B/T)} = \frac{1}{3}(n_u + n_d + n_s) \\ n_S &= \frac{\partial(p/T^4)}{\partial(\mu_S/T)} = -n_s \\ n_Q &= \frac{\partial(p/T^4)}{\partial(\mu_Q/T)} = \frac{1}{3}(2n_u - n_d - n_s) \end{aligned}$$

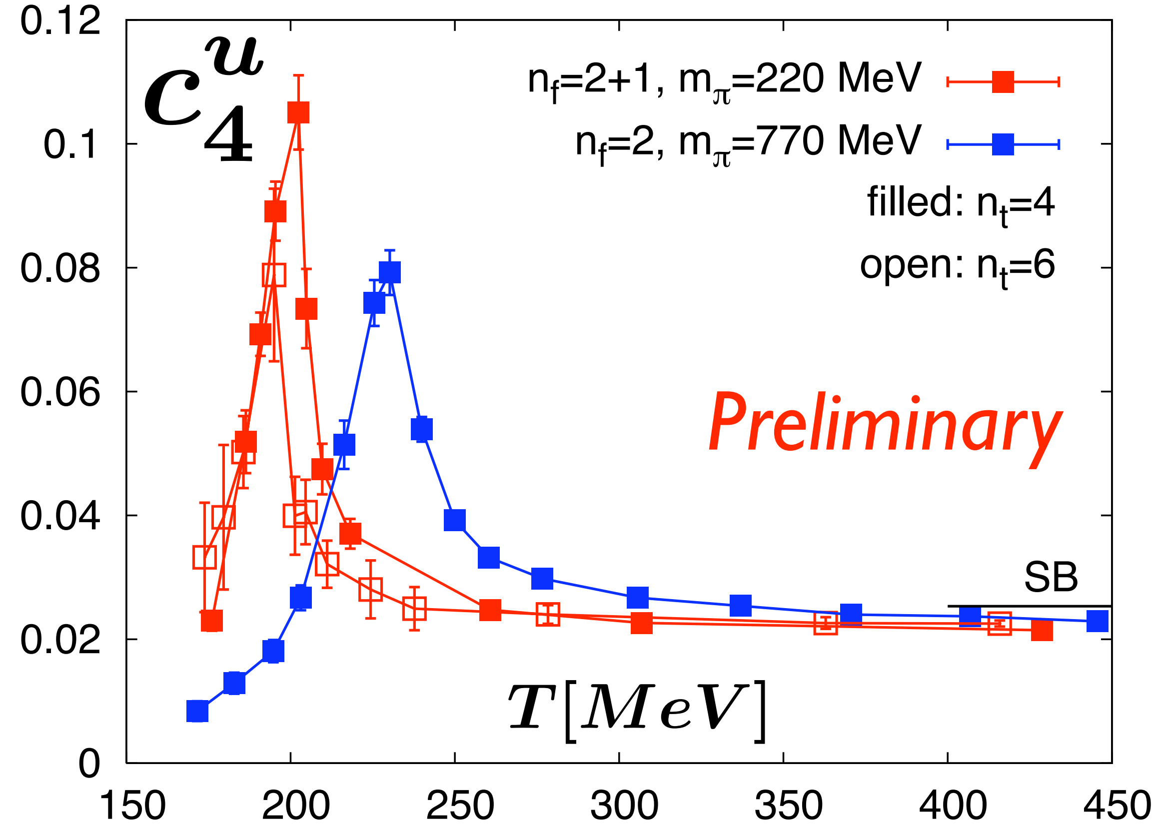
$$\begin{aligned} \mu_u &= \frac{1}{3}\mu_B + \frac{2}{3}\mu_Q \\ \mu_d &= \frac{1}{3}\mu_B - \frac{1}{3}\mu_Q \\ \mu_s &= \frac{1}{3}\mu_B - \frac{1}{3}\mu_Q - \mu_S \end{aligned}$$

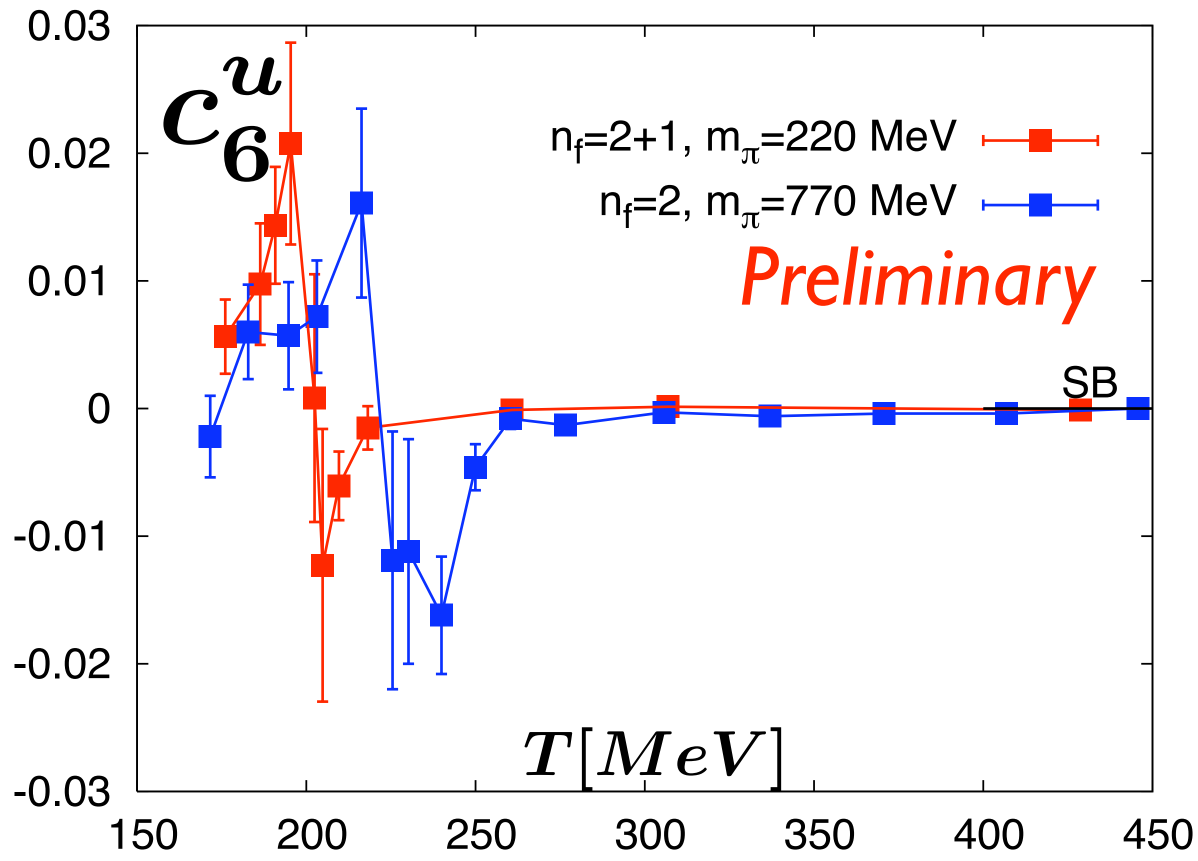
- choice of $\mu_u \equiv \mu_d$ is equivalent to $\mu_Q \equiv 0$

Taylor expansion of the pressure

- Results for expansion coefficients $c_{i,j,k}^{u,d,s}$







Cut-off dependance:

⟶ **Small cut-off effects in the transition region (similar to p, e-3p, ...)**

Mass dependance:

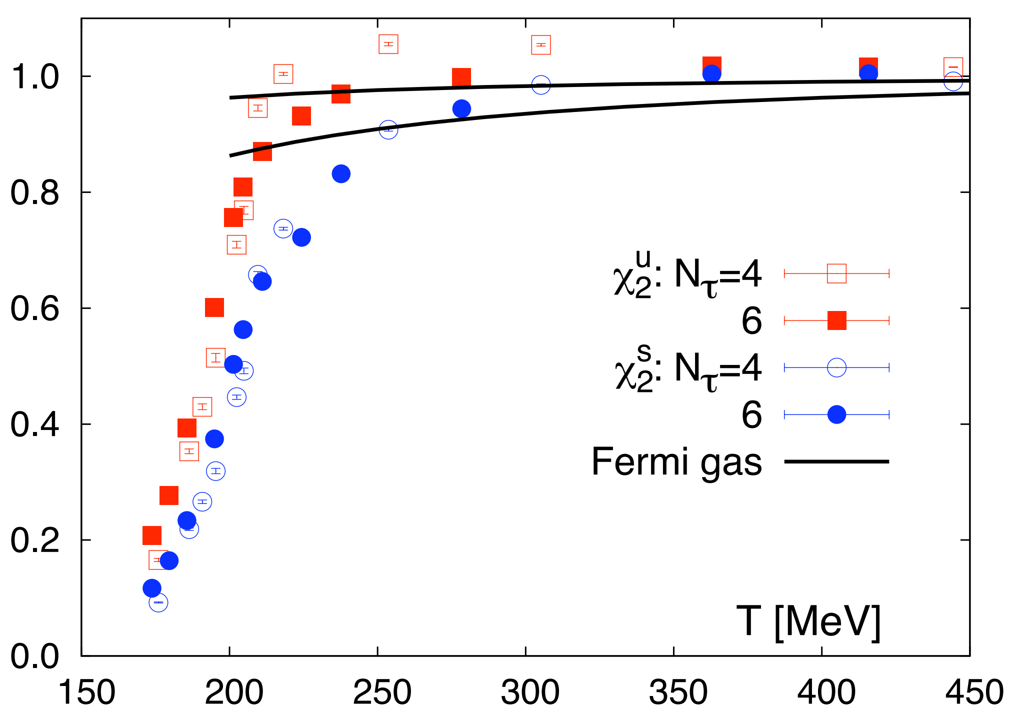
⟶ **Tc decreases with decreasing mass**

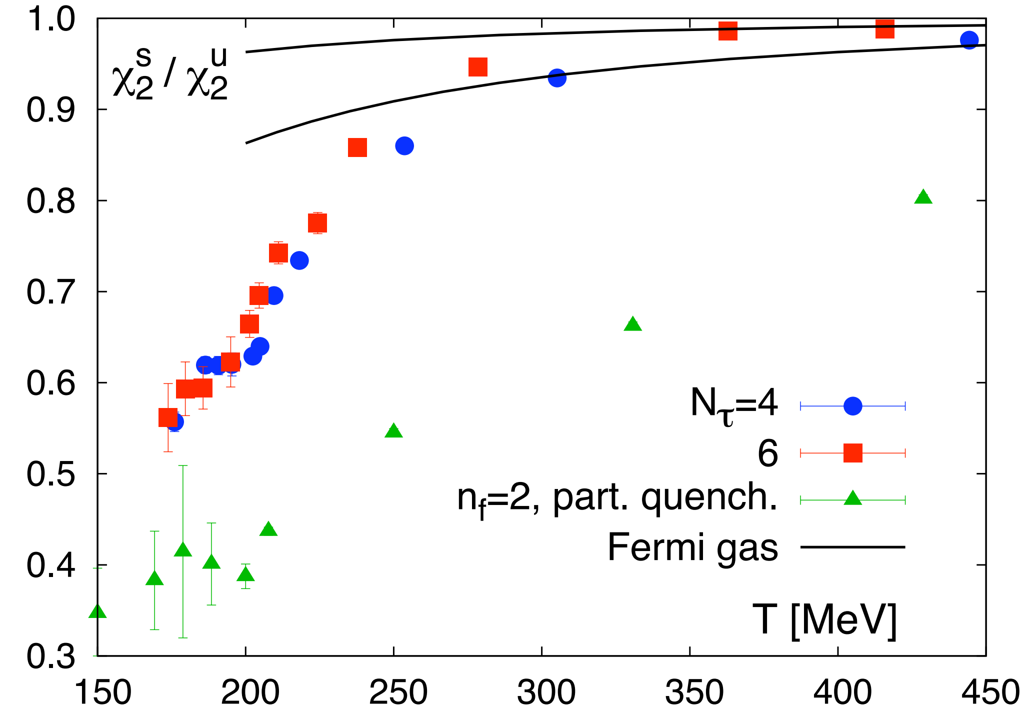
⟶ **Fluctuations increase with decreasing mass**

red: RBC-Bielefeld, preliminary

blue: PRD71:054508,2005.

- relative suppression of strange quark to light quark fluctuations





⟶ find suppression of 0.6 at Tc, reach unity at T>1.7 Tc

⟶ agreement with free massive Fermi gas for T>1.5 Tc

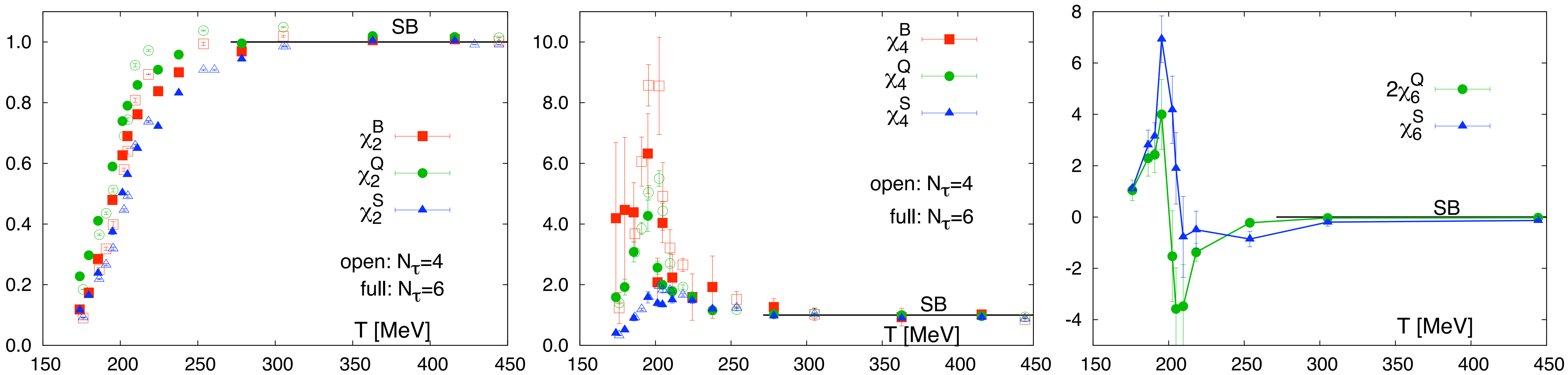
red/blue: RBC-Bielefeld, arXiv:0811.1006

green: Gavai, Gupta, PRD 73, 014004 (2006)

- we define fluctuations of charge X as

$$
\begin{aligned}
\chi_2^X &= \frac{1}{VT^3}\left\langle N_X^2\right\rangle = 2!\, c_2^X \\
\chi_4^X &= \frac{1}{VT^3}\left(\left\langle N_X^4\right\rangle - \left\langle N_X^2\right\rangle^2\right) = 4!\, c_4^X \\
\chi_6^X &= \frac{1}{VT^3}\left(\left\langle N_X^6\right\rangle - 15\left\langle N_X^4\right\rangle\left\langle N_X^2\right\rangle + 30\left\langle N_X^2\right\rangle^3\right) = 6!\, c_6^X
\end{aligned}
$$

- **B,Q,S-fluctuations**



⟶ small cut off effects in the transition region (similar to e-3p, p, ...)

⟶ general pattern can be understood by the singular behavior of the free energy

$$\chi_{2n}^B \sim \left|\frac{T-T_c}{T_c}\right|^{2-n-\alpha}, \qquad \alpha \approx -0.25$$

χ_2^B dominated by the regular part, χ_4^B develops a cusp.

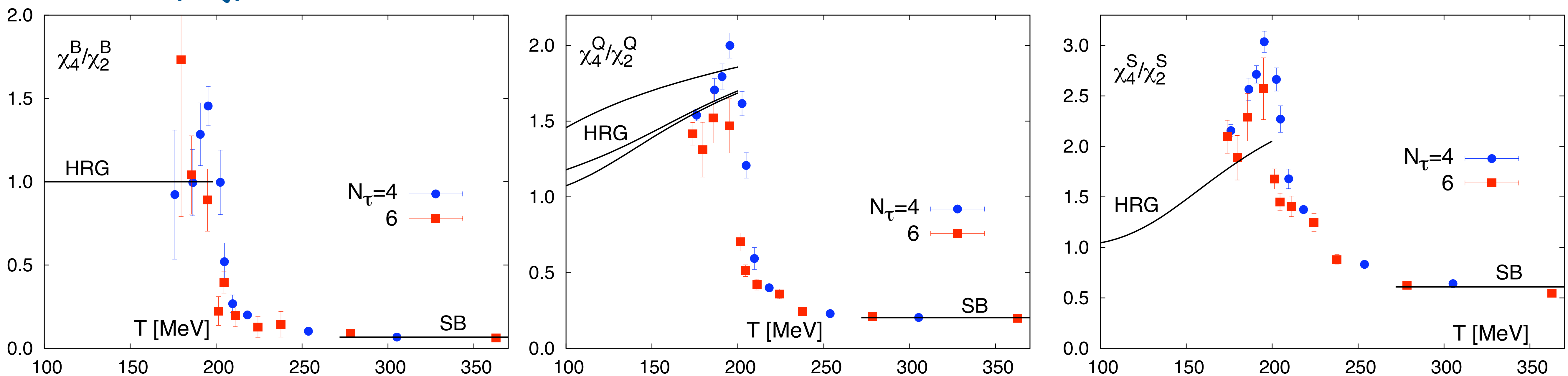
- We define fluctuations of charge X as

$$\chi_2^X = \frac{1}{VT^3}\langle N_X^2\rangle = 2!\ c_2^X$$

$$\chi_4^X = \frac{1}{VT^3}\left(\langle N_X^4\rangle - \langle N_X^2\rangle^2\right) = 4!\ c_4^X$$

$$\chi_6^X = \frac{1}{VT^3}\left(\langle N_X^6\rangle - 15\langle N_X^4\rangle\langle N_X^2\rangle + 30\langle N_X^2\rangle^3\right) = 6!\ c_6^X$$

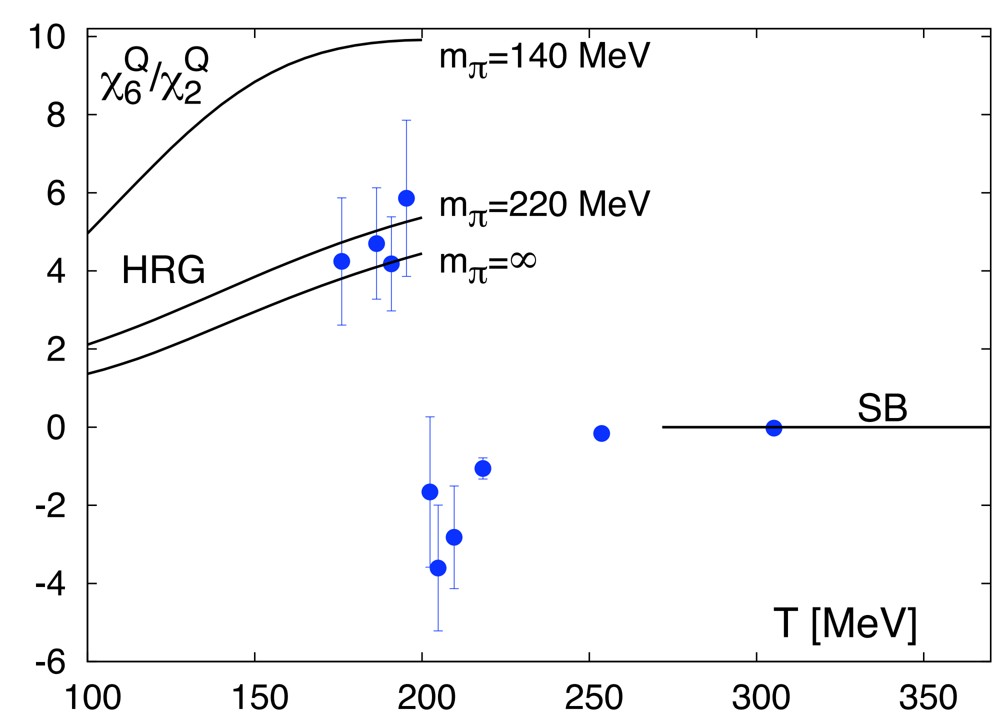
- **B,Q,S-kurtosis**



⟶ agreement with free gas results for T>1.5 Tc

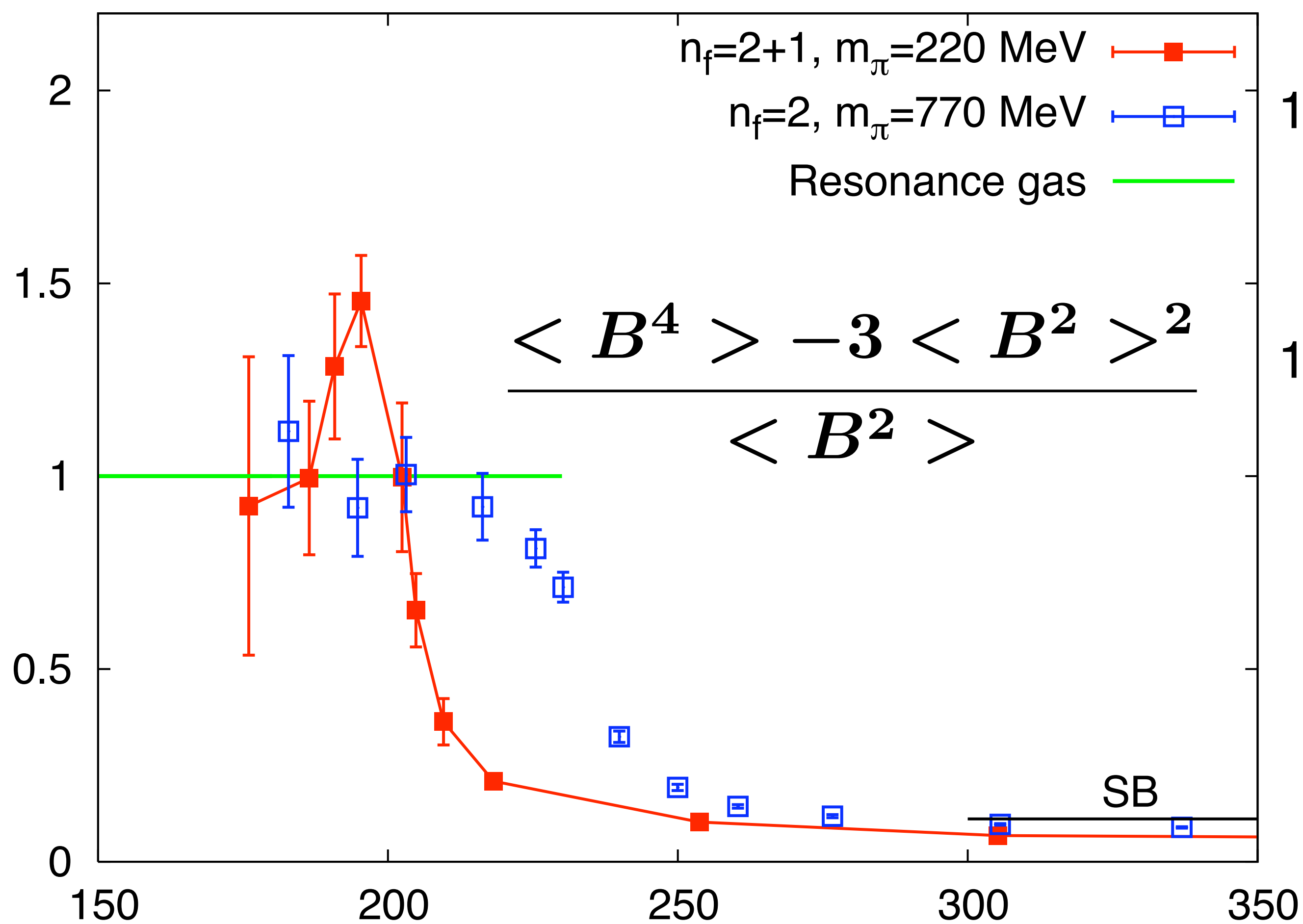
⟶ qualitative agreement with the resonance gas below Tc

⟶ for electric charge fluctuations: increasingly strong sensitivity to the mass of the charged pions

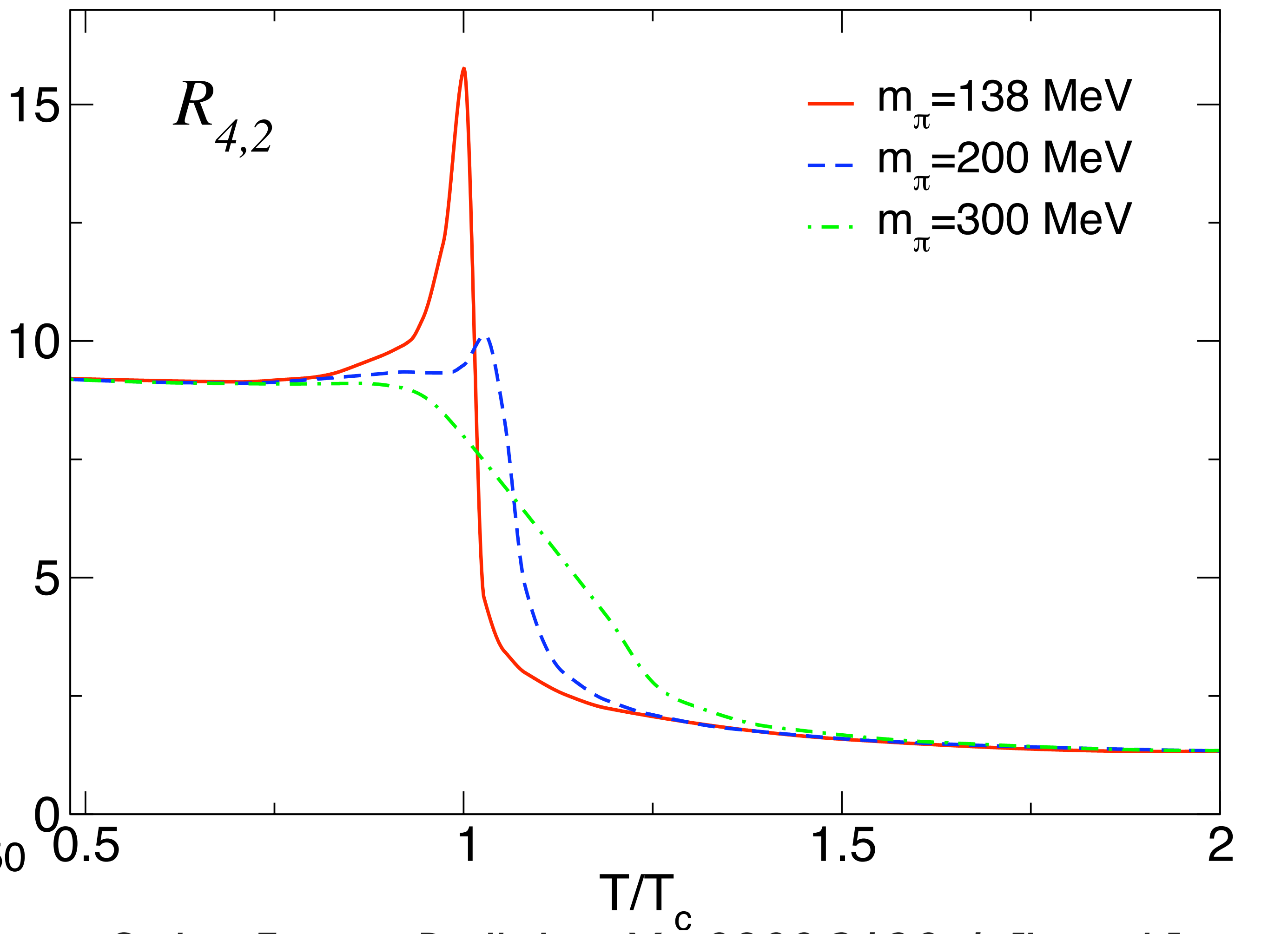


Do fluctuations increase over the resonance gas value?
(expected from chiral models)
⟶ need more detailed studies and a better control over the continuum limit

- **B-kurtosis (mass dependence)**



red: RBC-Bielefeld, preliminary
blue: Allton et all., Phys. Rev. D71 (2005) 054508.



Stokic, Friman, Redlich, arXiv:0809.3129v1 [hep-ph]

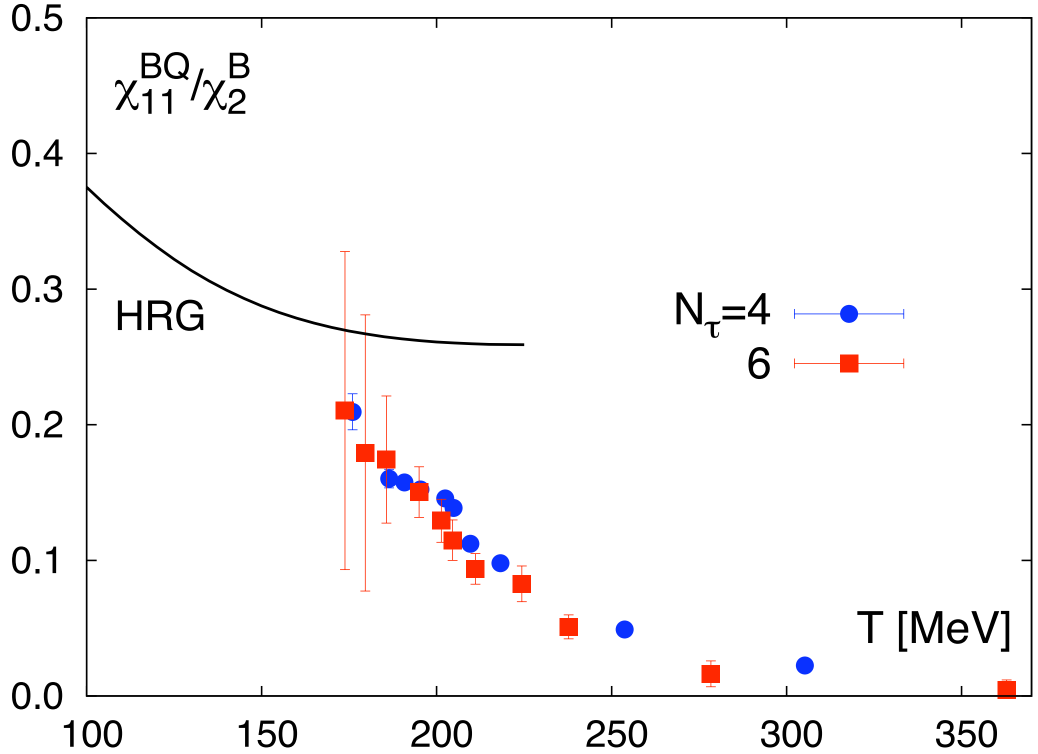
⟶ enhancement over resonance gas value at physical masses ?

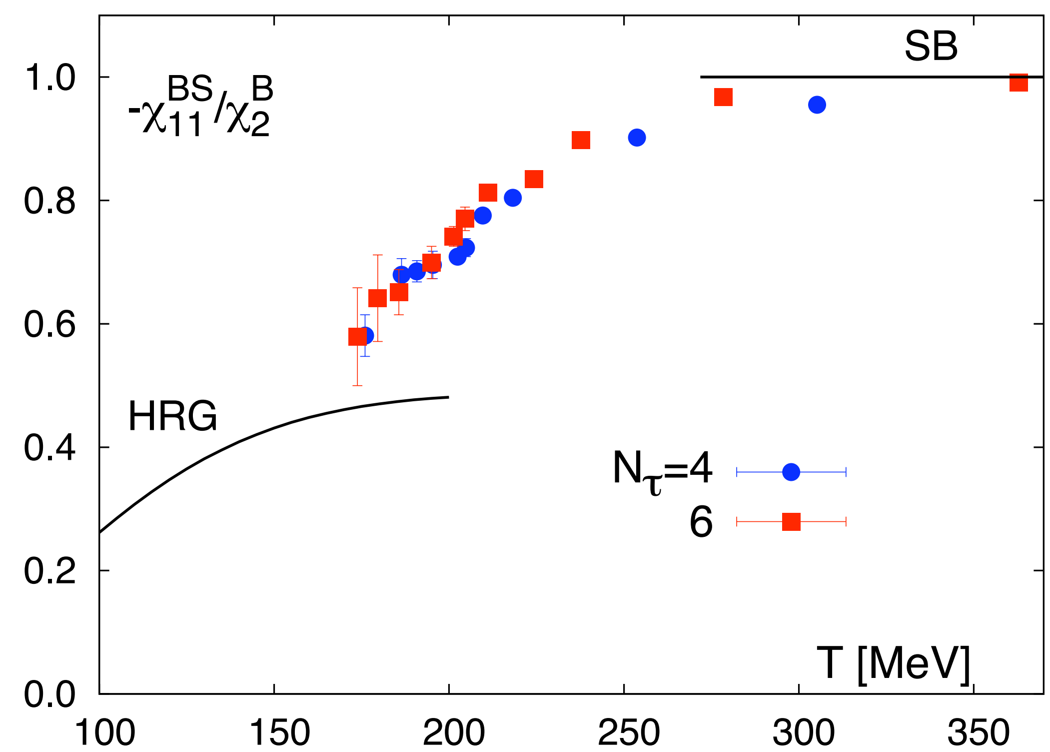
⟶ **may be a good experimental observable**

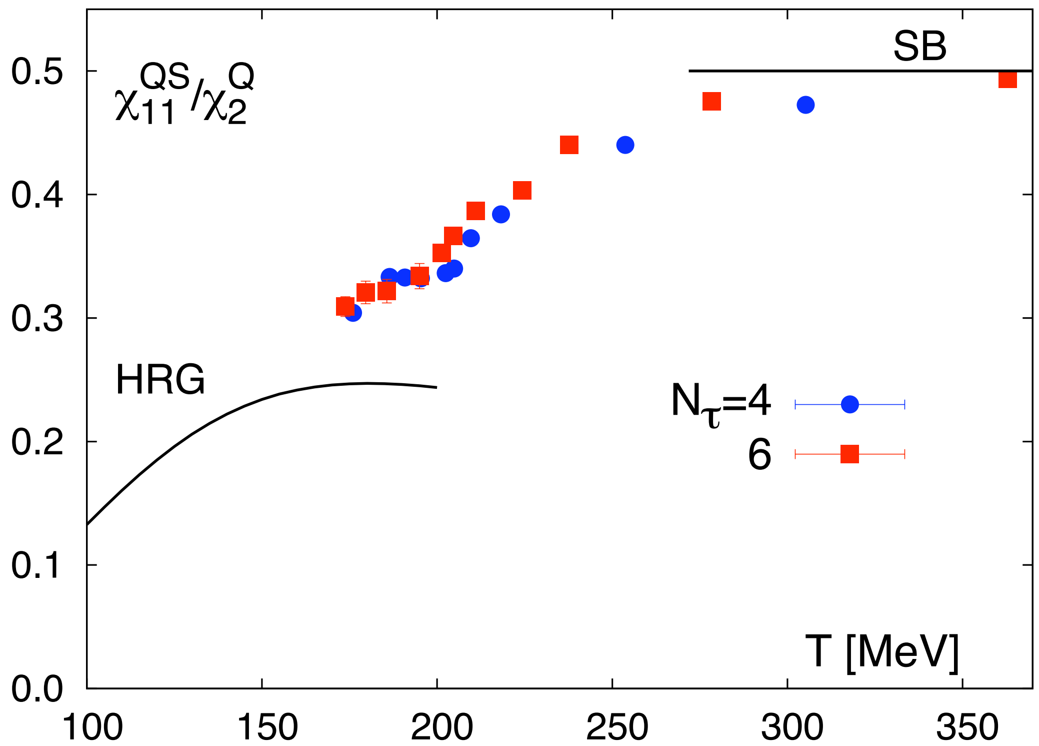
chiral limit:

$$\chi_4^B, \chi_4^Q \propto |T - T_c|^{-\alpha} + \text{regular}$$

- **Correlations among charges**







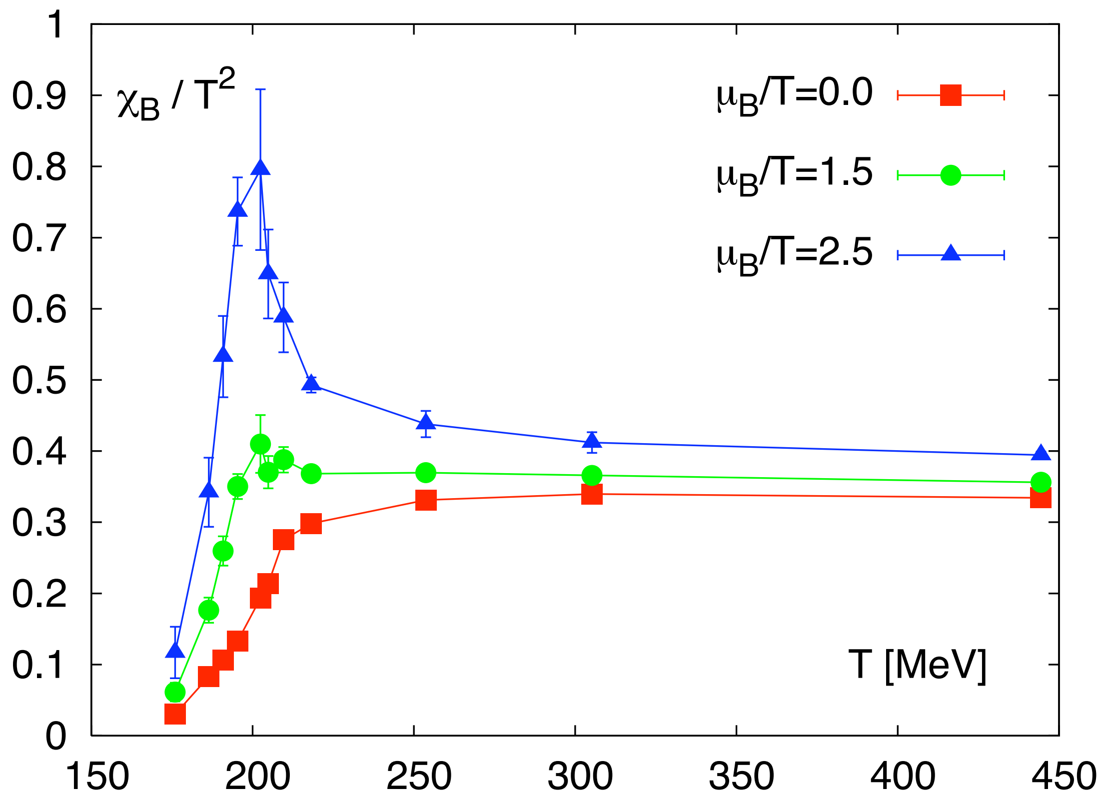
⟶ agreement with free gas results for T>1.5 Tc

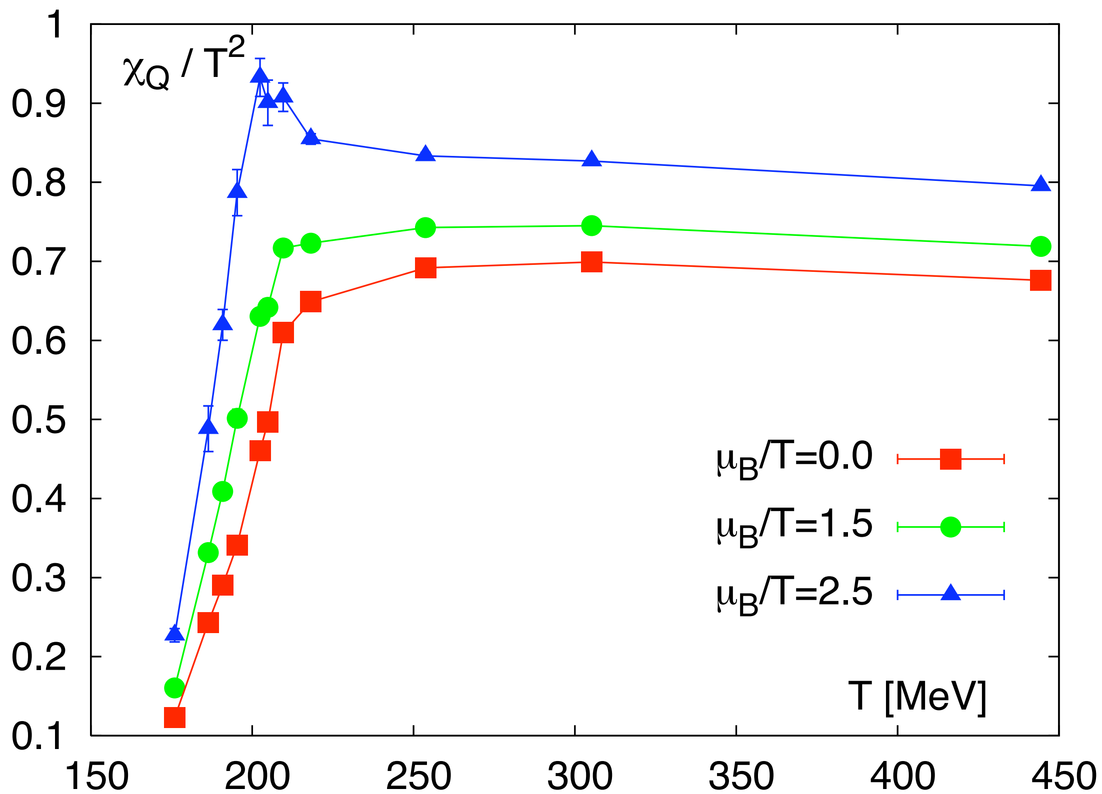
⟶ qualitative agreement with the resonance gas below Tc

Hadronic fluctuations and the QCD critical point

- **Hadronic fluctuations** $(\mu_B > 0)$ $\quad (\mu_S = \mu_Q = 0)$

$$\frac{\chi_B}{T^2} = 2c_2^B + 12c_4^B \left(\frac{\mu_B}{T}\right)^2 \qquad \frac{\chi_Q}{T^2} = 2c_2^Q + 2c_{22}^{BQ} \left(\frac{\mu_B}{T}\right)^2$$





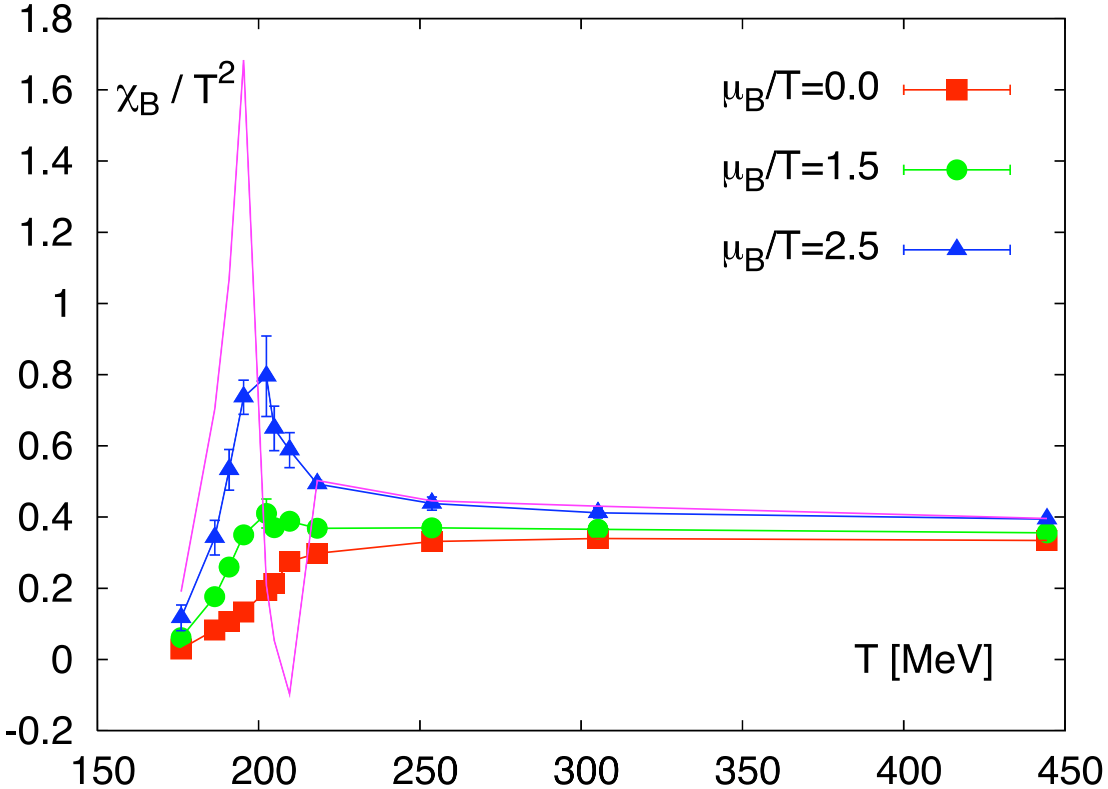
⟶ to be studied in event-by-event fluctuations

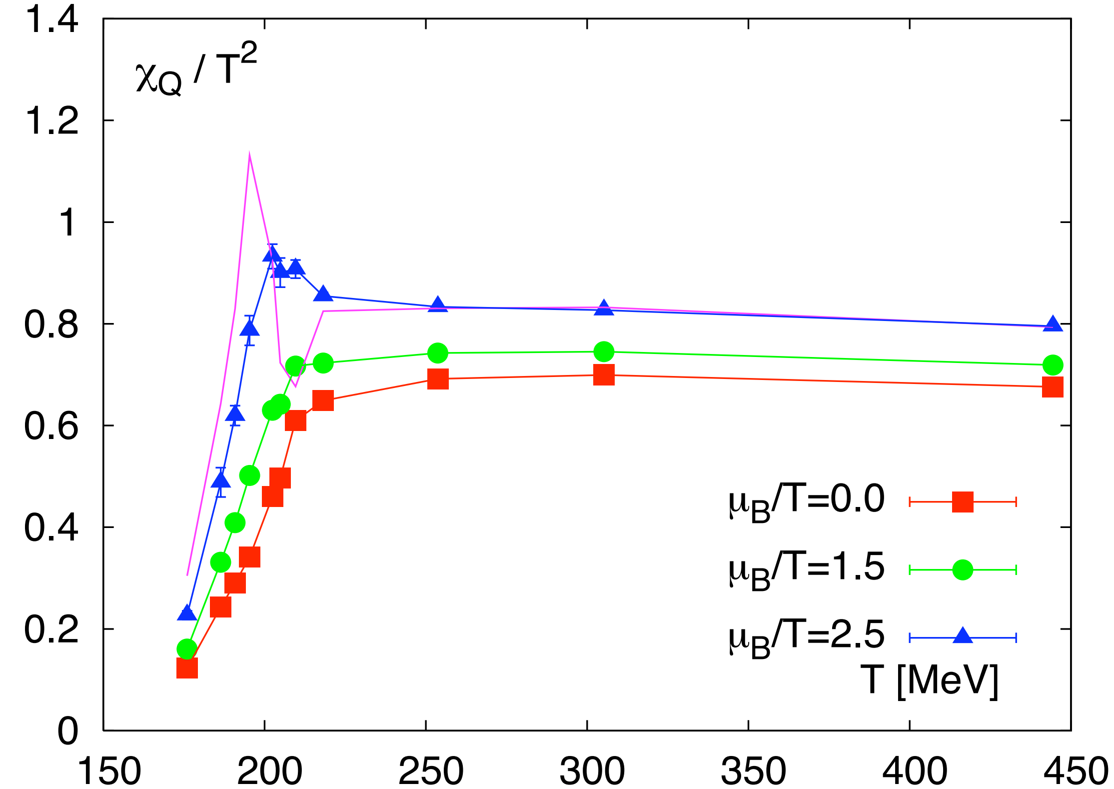
⟶ **evidence for a critical point ?**

Seeing „true“ singular behavior as a signal for a critical point requires large volumes and high order Taylor expansions

- Hadronic fluctuations $(\mu_B > 0)$ $\quad (\mu_S = \mu_Q = 0)$

$$\frac{\chi_B}{T^2} = 2c_2^B + 12c_4^B \left(\frac{\mu_B}{T}\right)^2 + 30c_6^B \left(\frac{\mu_B}{T}\right)^4 \qquad \frac{\chi_Q}{T^2} = 2c_2^B + 2c_{22}^{BQ} \left(\frac{\mu_B}{T}\right)^2 + 2c_{42}^{BQ} \left(\frac{\mu_B}{T}\right)^4$$





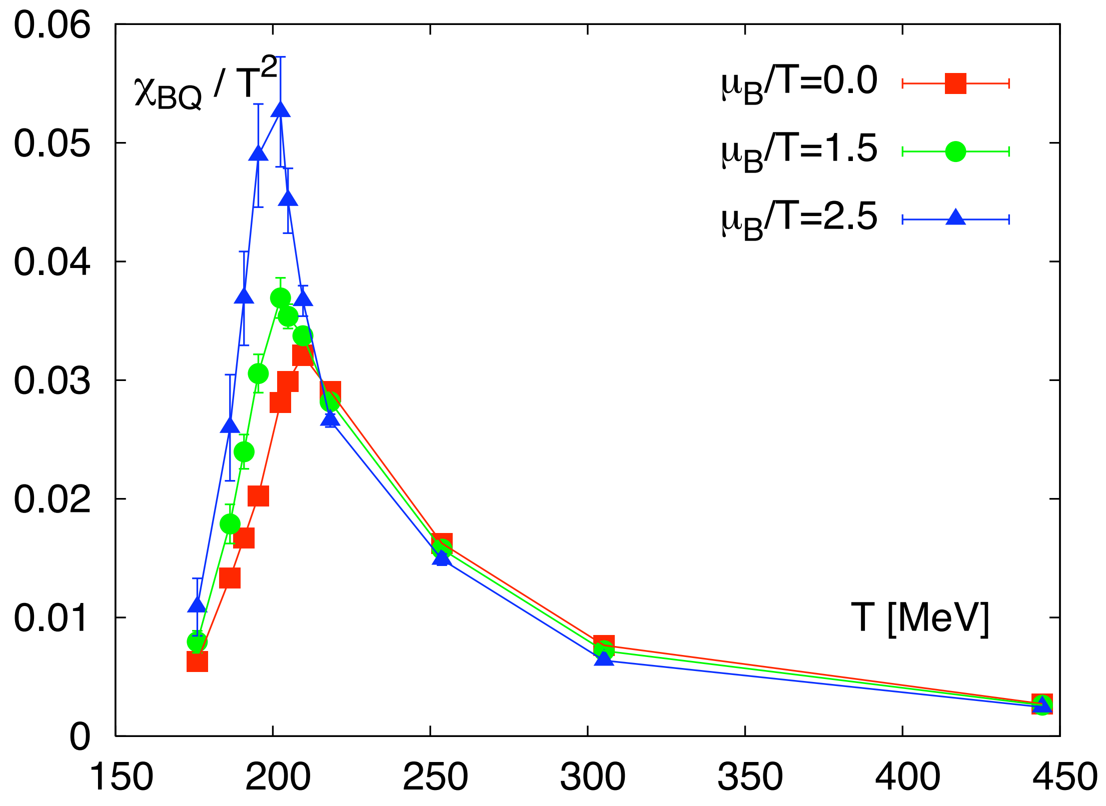
⟶ to be studied in event-by-event fluctuations

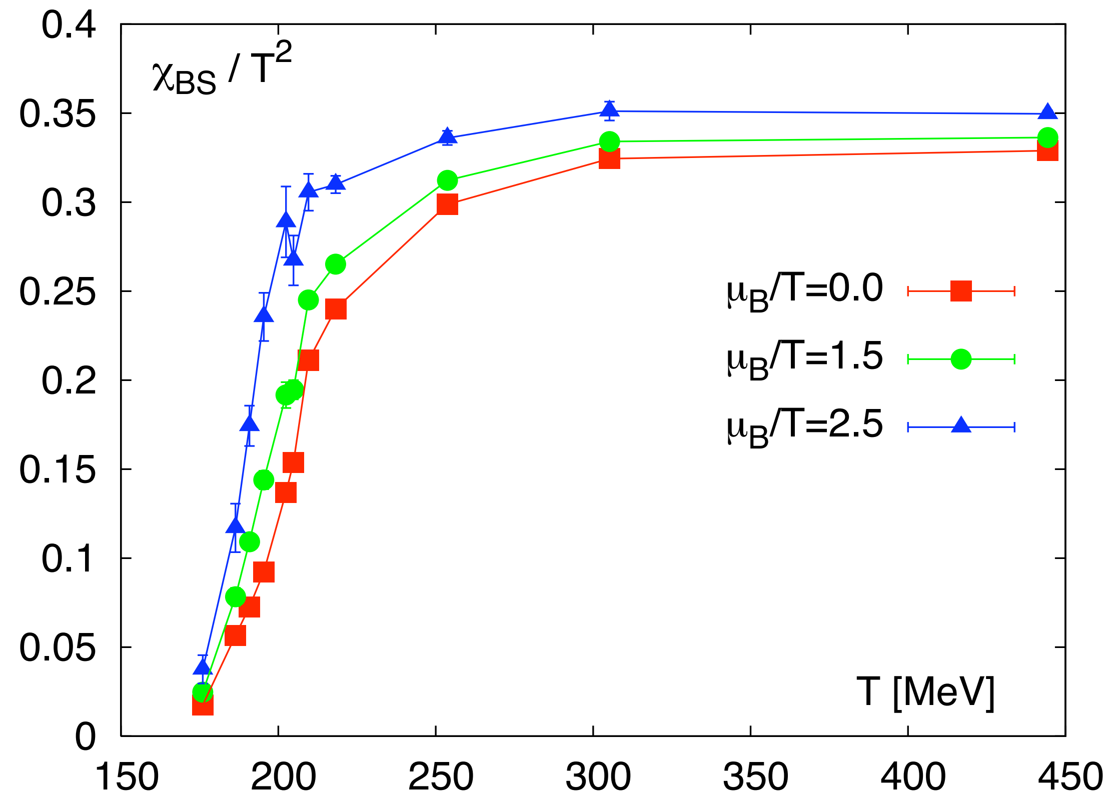
⟶ **evidence for a critical point ?**

Seeing „true“ singular behavior as a signal for a critical point requires large volumes and high order Taylor expansions

- **Hadronic correlations** $(\mu_B > 0)$ $(\mu_S = \mu_Q = 0)$

$$\frac{\chi_{11}^{BQ}}{T^2} = c_{11}^{BQ} + c_{31}^{BQ}\left(\frac{\mu_B}{T}\right)^2 \qquad \frac{\chi_{11}^{BS}}{T^2} = c_{11}^{BS} + c_{31}^{BS}\left(\frac{\mu_B}{T}\right)^2$$





⟶ to be studied in event-by-event fluctuations

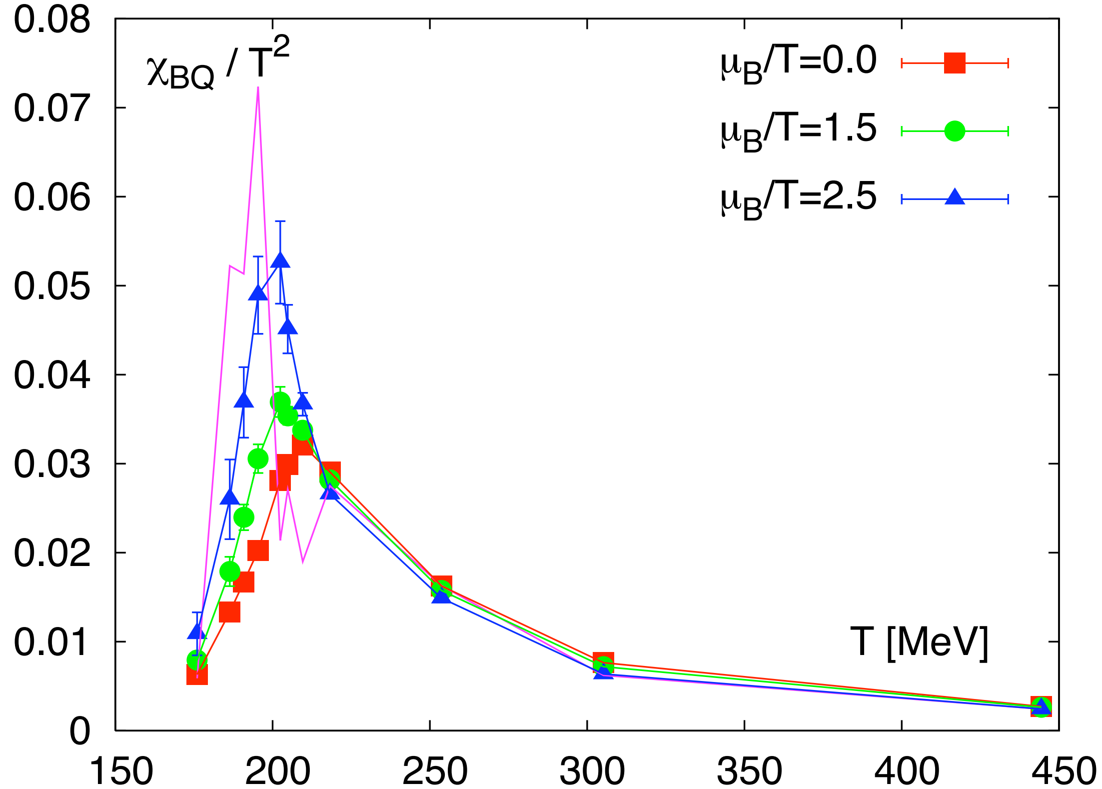
⟶ **evidence for a critical point ?**

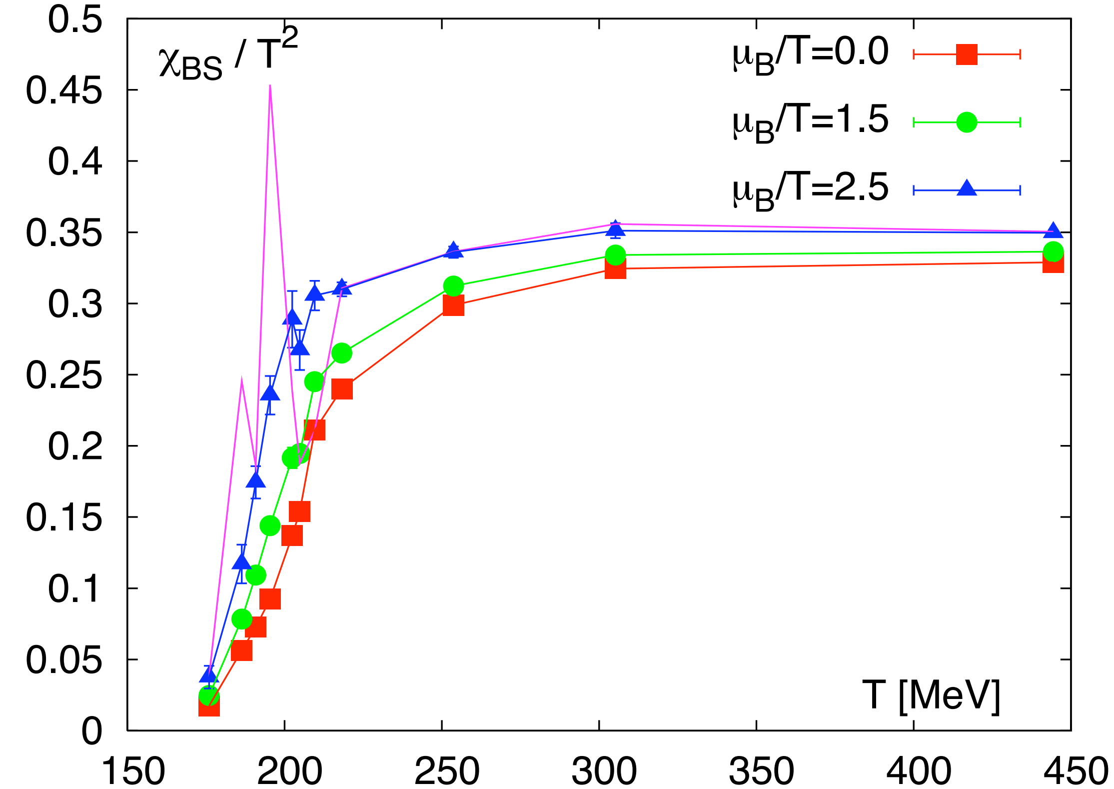
Seeing „true" singular behavior as a signal for a critical point requires large volumes and high order Taylor expansions

- **Hadronic correlations** $(\mu_B > 0)$ $\quad (\mu_S = \mu_Q = 0)$

$$\frac{\chi_{11}^{BQ}}{T^2} = c_{11}^{BQ} + c_{31}^{BQ}\left(\frac{\mu_B}{T}\right)^2 + 2c_{51}^{BQ}\left(\frac{\mu_B}{T}\right)^4$$

$$\frac{\chi_{11}^{BS}}{T^2} = c_{11}^{BQ} + c_{31}^{BS}\left(\frac{\mu_B}{T}\right)^2 + 2c_{51}^{BS}\left(\frac{\mu_B}{T}\right)^4$$





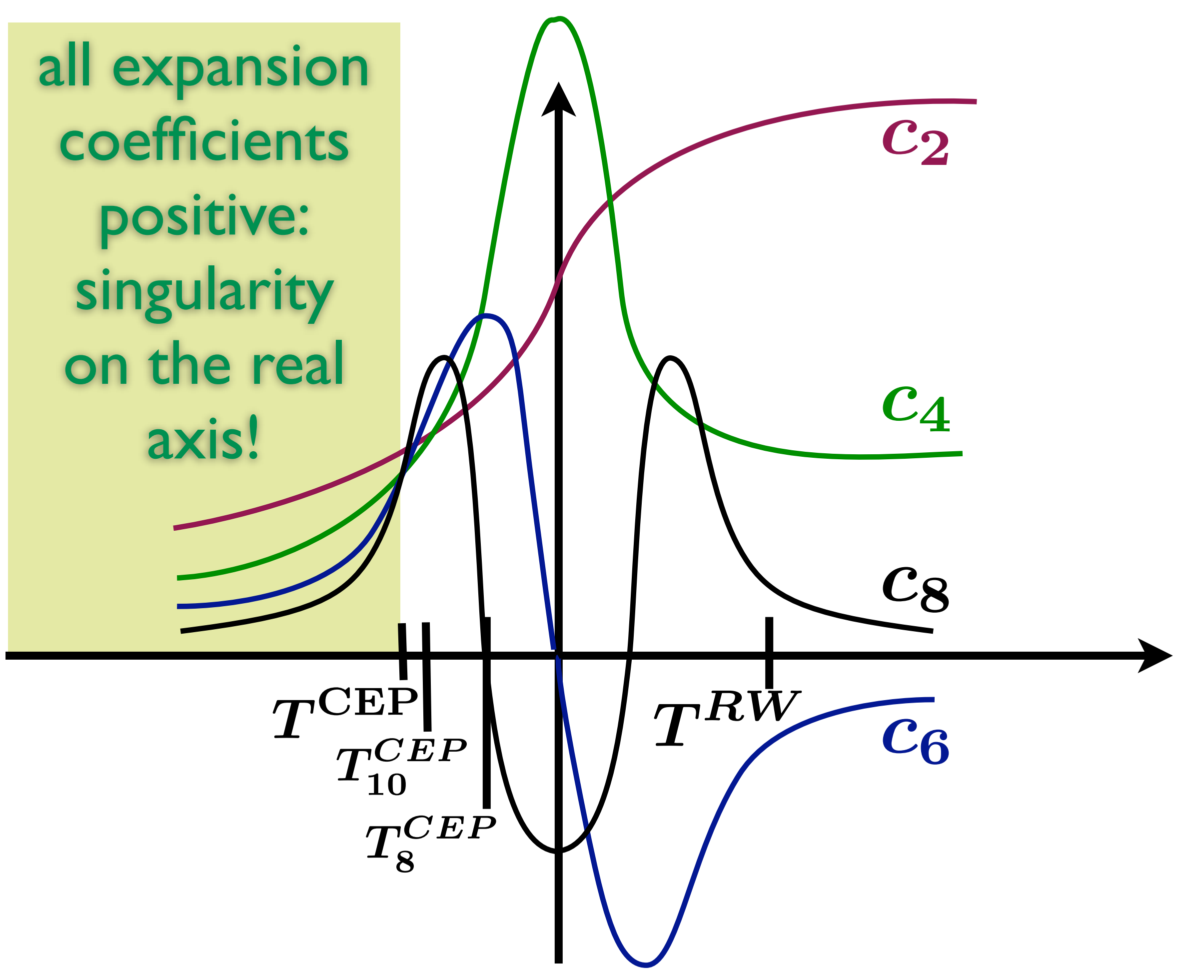
⟶ to be studied in event-by-event fluctuations

⟶ **evidence for a critical point ?**

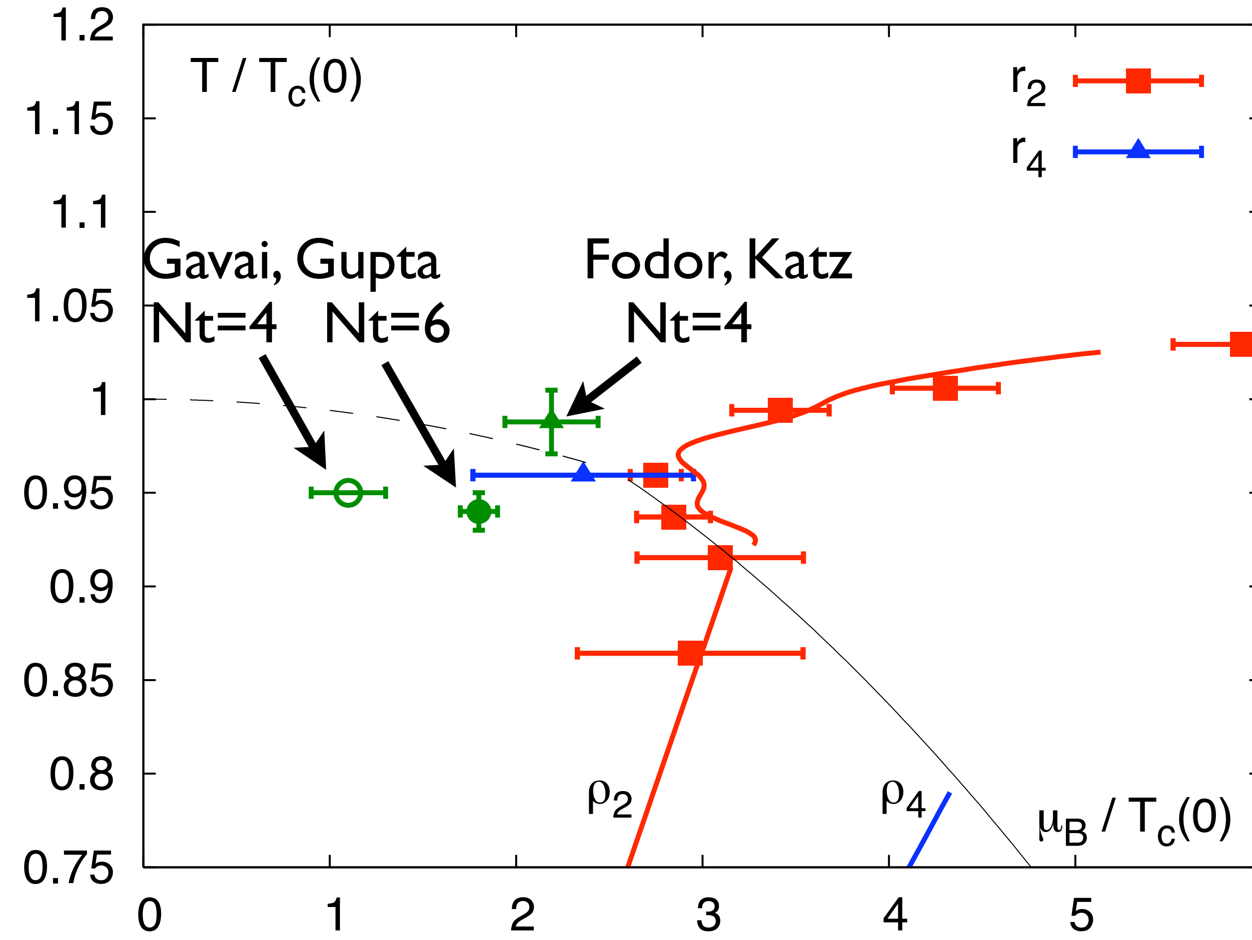
Seeing „true“ singular behavior as a signal for a critical point requires large volumes and high order Taylor expansions

Method to determine the CEP:

- find largest temperature were all expansion coefficients are positive → $T^{\rm CEP}$
- determine the radius of convergence at that temperature → $\mu^{\rm CEP}$



→ first non-trivial estimate of $T^{\rm CEP}$ from c_8
second non-trivial estimate of $T^{\rm CEP}$ from c_{10}



$$\mu^{\rm CEP} = \lim_{n\to\infty} \mu_n^{\rm CEP}$$

$$\mu_n^{\rm CEP} = T^{\rm CEP}\sqrt{c_n/c_{n+2}}$$

The Resonance gas limit:

$$\frac{p}{T^4} = G(T) + F(T)\cosh\left(\frac{\mu_B}{T}\right)$$

$$\longrightarrow \rho_n = \sqrt{(n+2)(n+1)}$$

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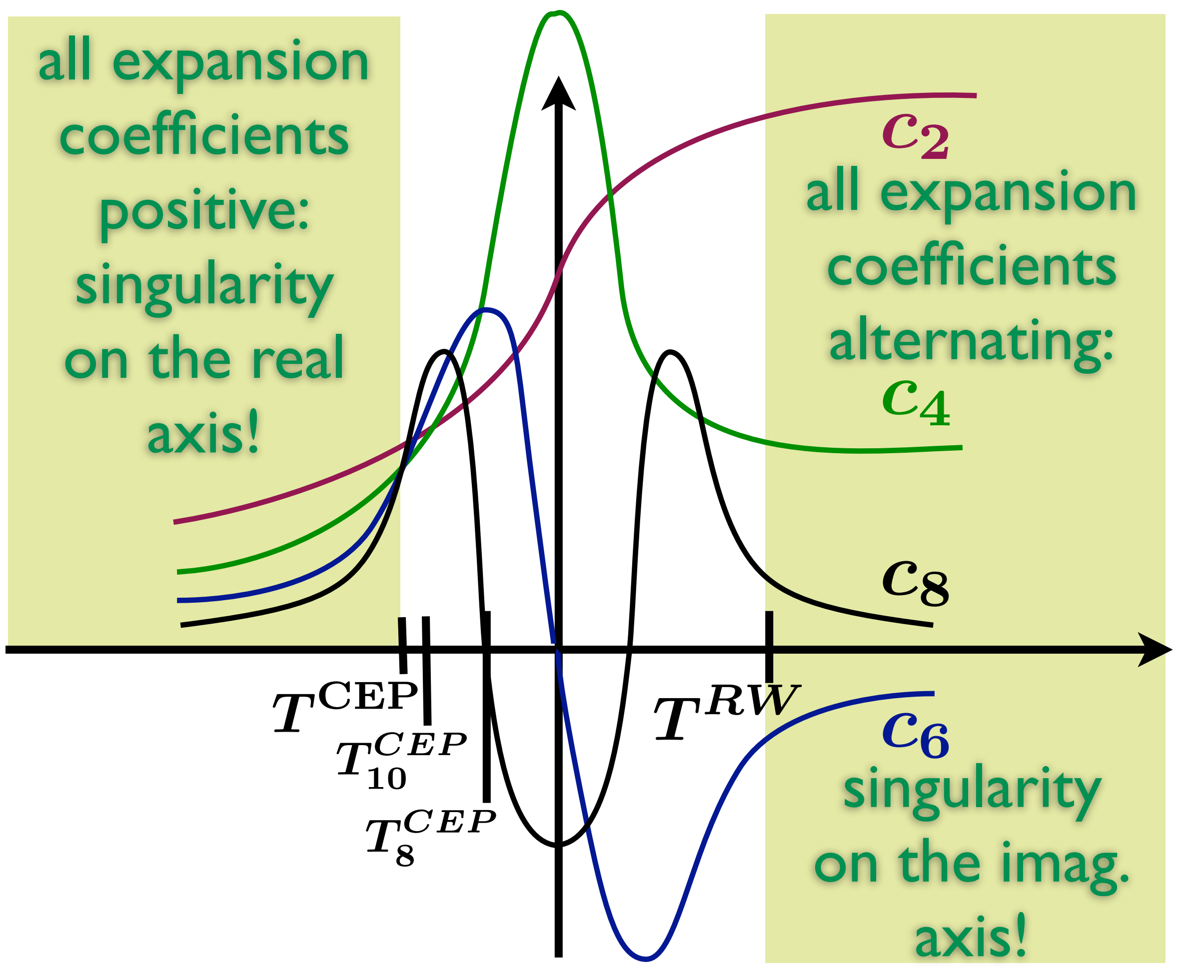

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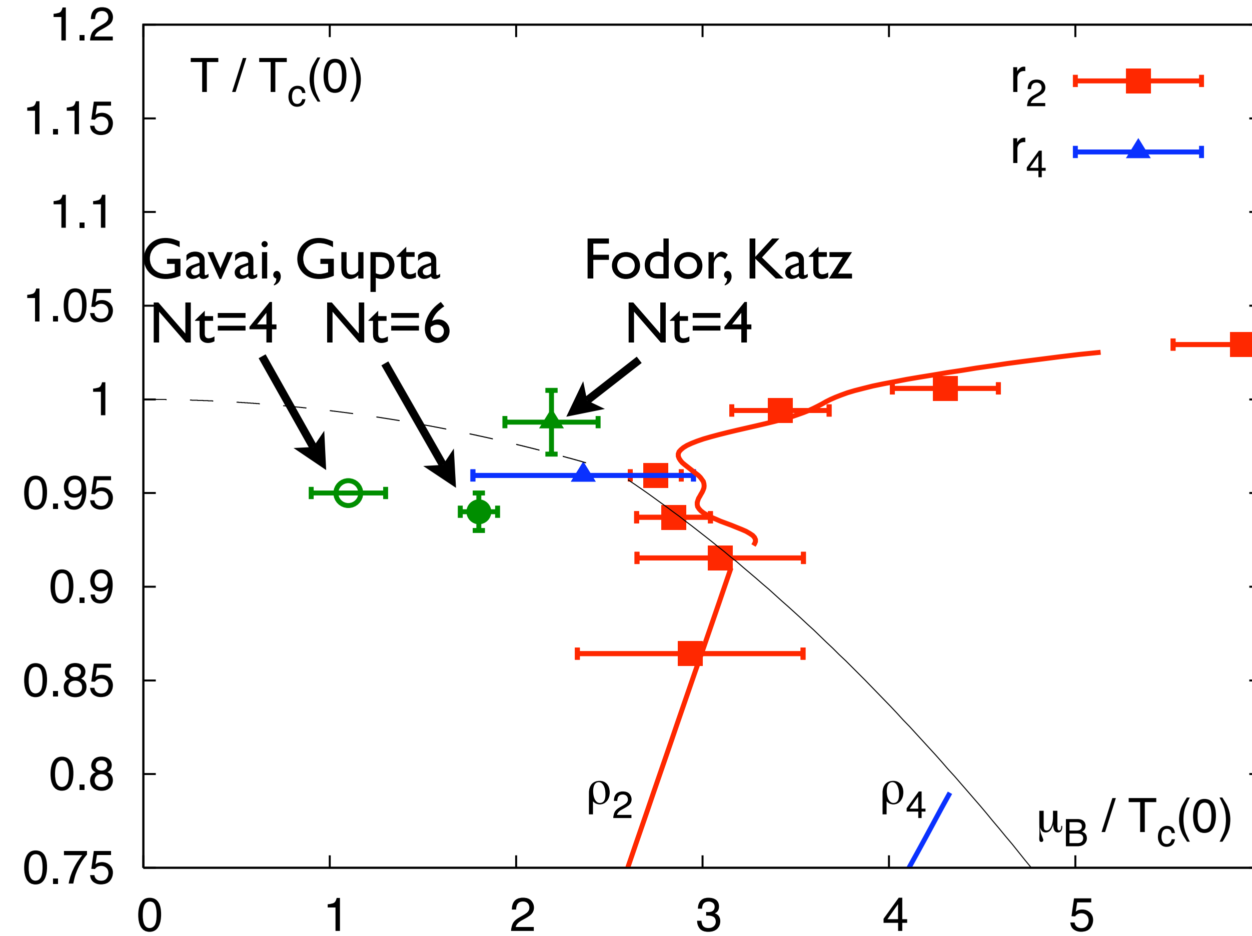

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$$\frac{p}{T^4} = G(T) + F(T)\cosh\left(\frac{\mu_B}{T}\right)$$

$$\longrightarrow \quad \rho_n = \sqrt{1/(n+2)(n+1)}$$

The EoS at non zero density

- **Taylor expansion of the trace anomaly**

$$\frac{\epsilon - 3p}{T^4} = \sum_{n=0}^{\infty} {c'}_n^B(T, m_l, m_s) \left(\frac{\mu_B}{T}\right)^n$$

⟶ Coefficients are defined by

$${c'}_n^B(T, m_l, m_s) = T \frac{dc_n^B(T, m_l, m_s)}{dT}$$

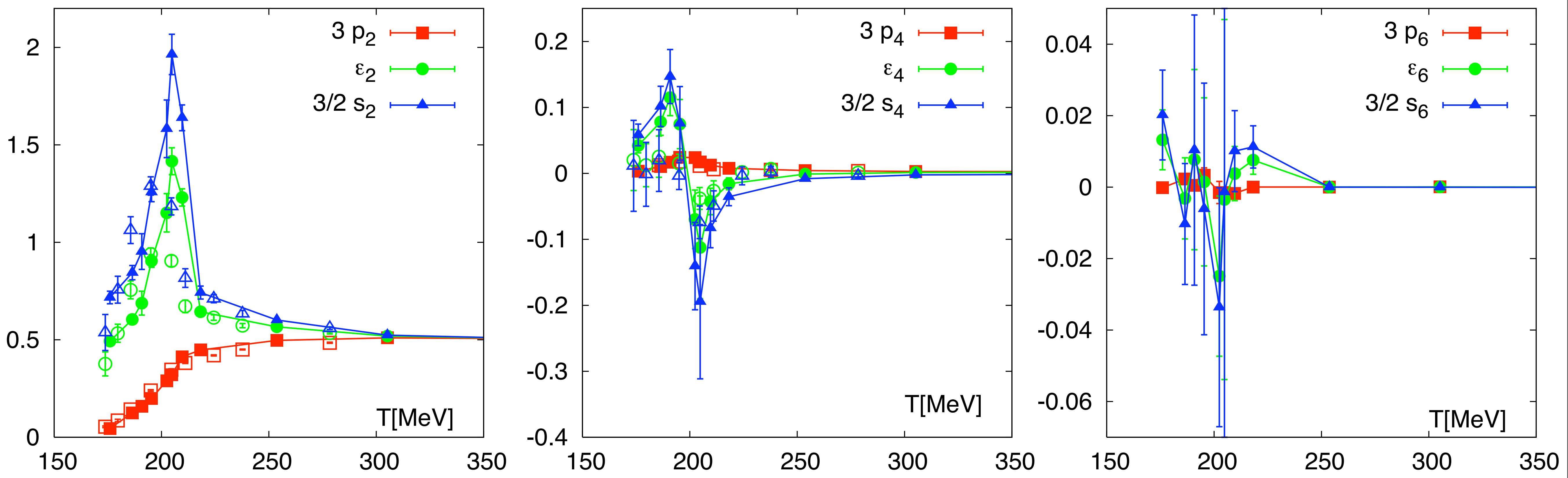
- **Taylor expansion of energy and entropy densities**

$$\frac{\epsilon}{T^4} = \sum_{n=0}^{\infty} \left(3c_n^B(T, m_l, m_s) + {c'}_n^B(T, m_l, m_s)\right) \left(\frac{\mu_B}{T}\right)^n \equiv \sum_{n=0}^{\infty} \epsilon_n^B \left(\frac{\mu_B}{T}\right)^n$$

$$\frac{s}{T^3} = \sum_{n=0}^{\infty} \left((4-n)c_n^B(T, m_l, m_s) + {c'}_n^B(T, m_l, m_s)\right) \left(\frac{\mu_B}{T}\right)^n \equiv \sum_{n=0}^{\infty} s_n^B \left(\frac{\mu_B}{T}\right)^n$$

The EoS at non zero density

- **Coefficients of the μ_B-expansion**

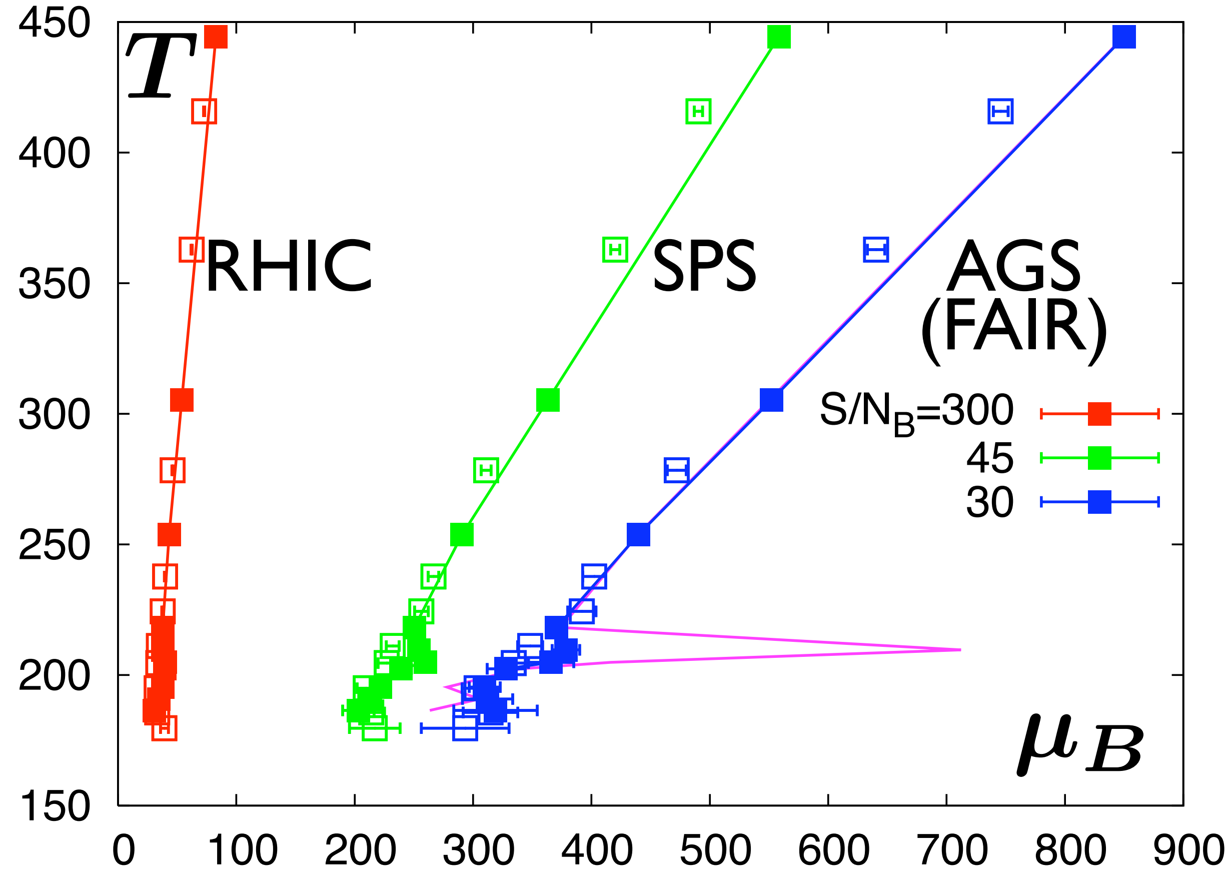


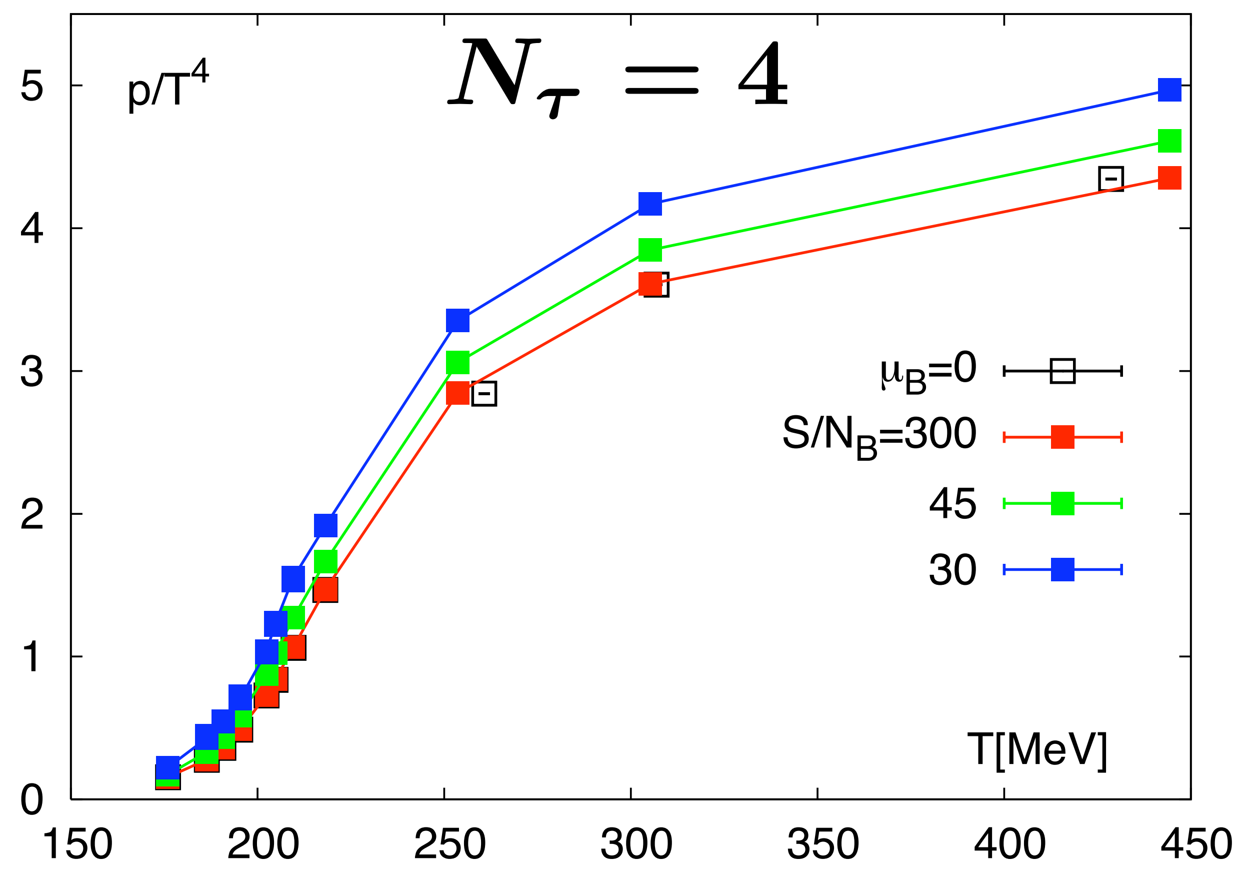
⟶ corrections to the leading terms are small: $\approx 10\%$

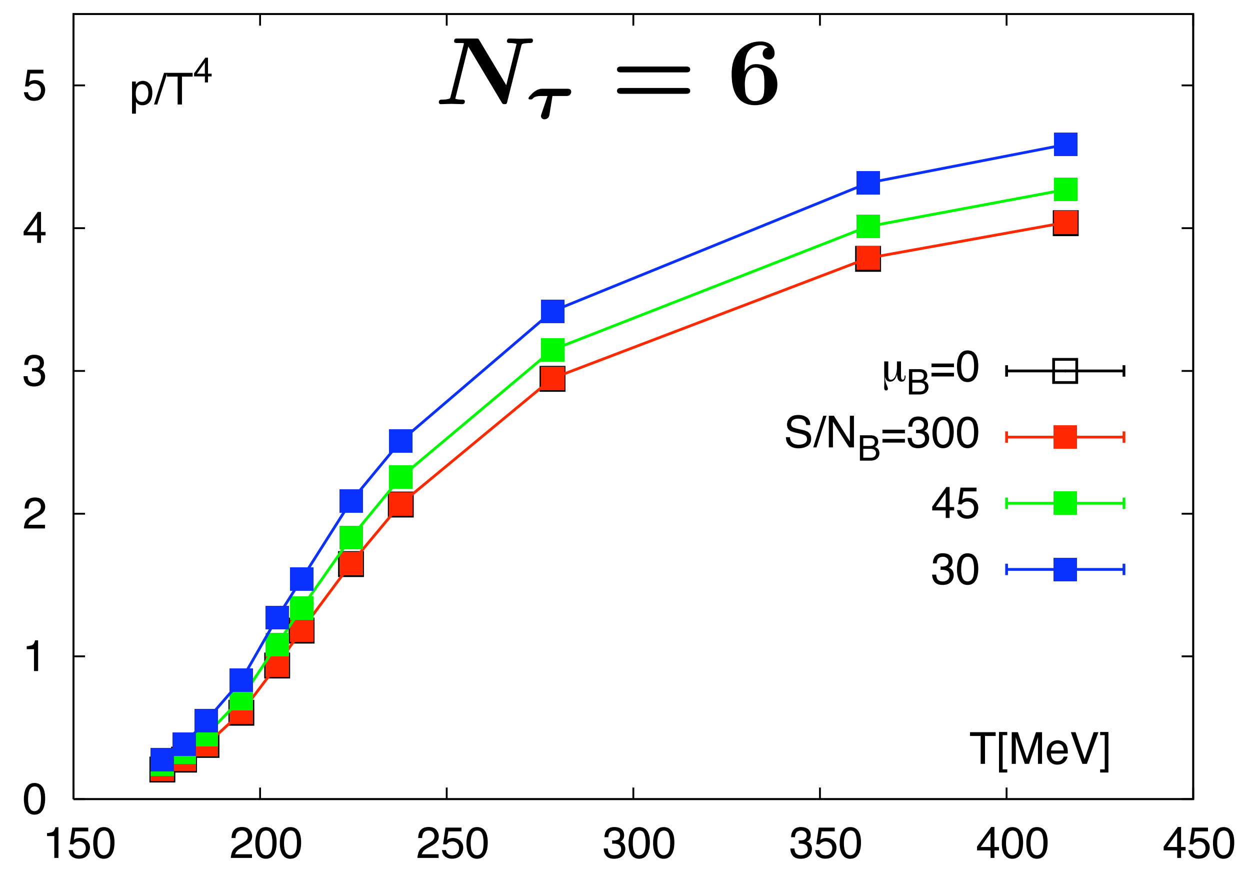
⟶ pattern of ϵ_n and s_n is that of c_{n+2}

The isentropic EoS

- **Isentropic trajectories**
  - ⟶ solve numerically for $S(T,\mu_B)/N_B(T,\mu_B) = \text{const.}$
  - ⟶ non-monotonic trajectories ?
  - ⟶ calculate pressure and energy density along isentropic trajectories
  - ⟶ pressure and energy density increase by $\approx 10\%$ for S/N=30.







The isentropic EoS

- **Isentropic trajectories**

  ⟶ solve numerically for

  $$S(T,\mu_B)/N_B(T,\mu_B) = \text{const.}$$

  ⟶ non-monotonic trajectories ?

  ⟶ calculate pressure and energy density along isentropic trajectories

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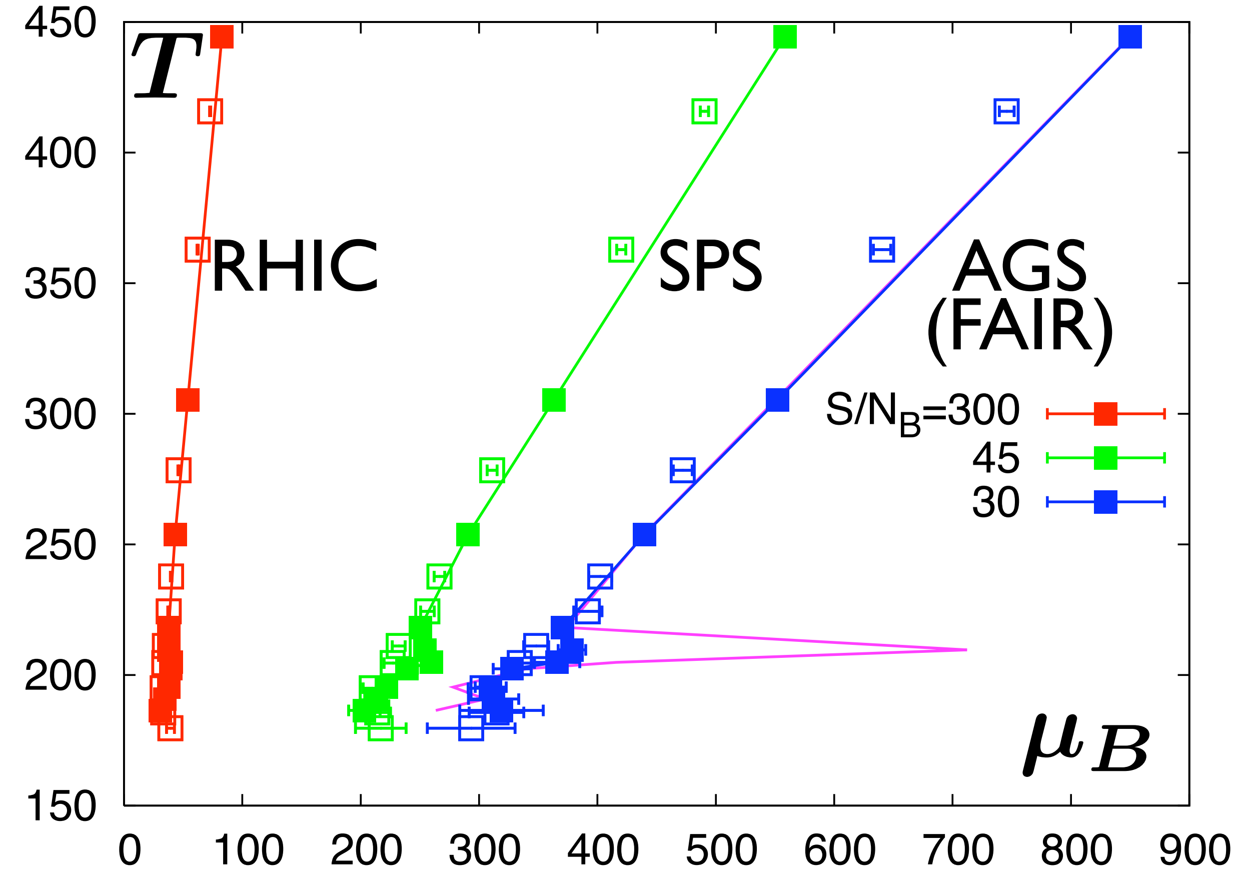
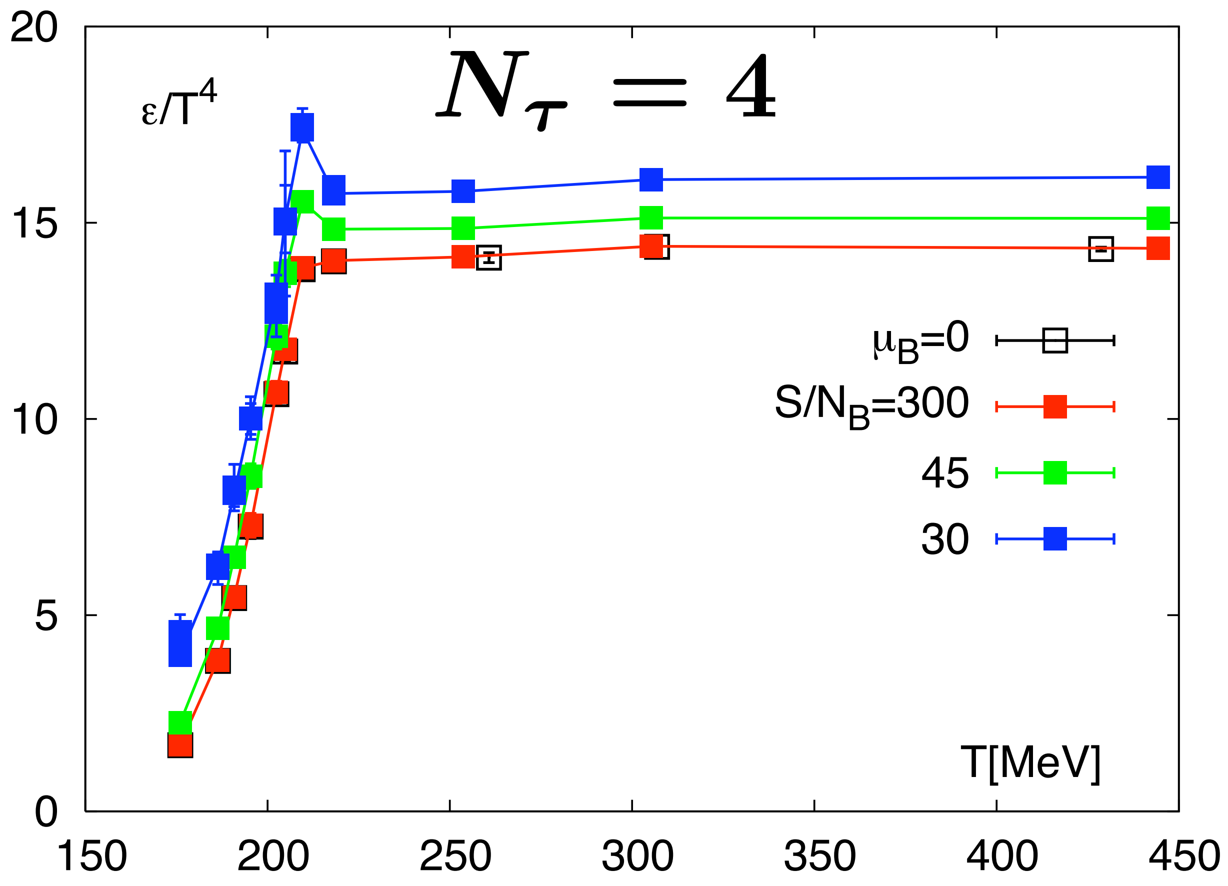
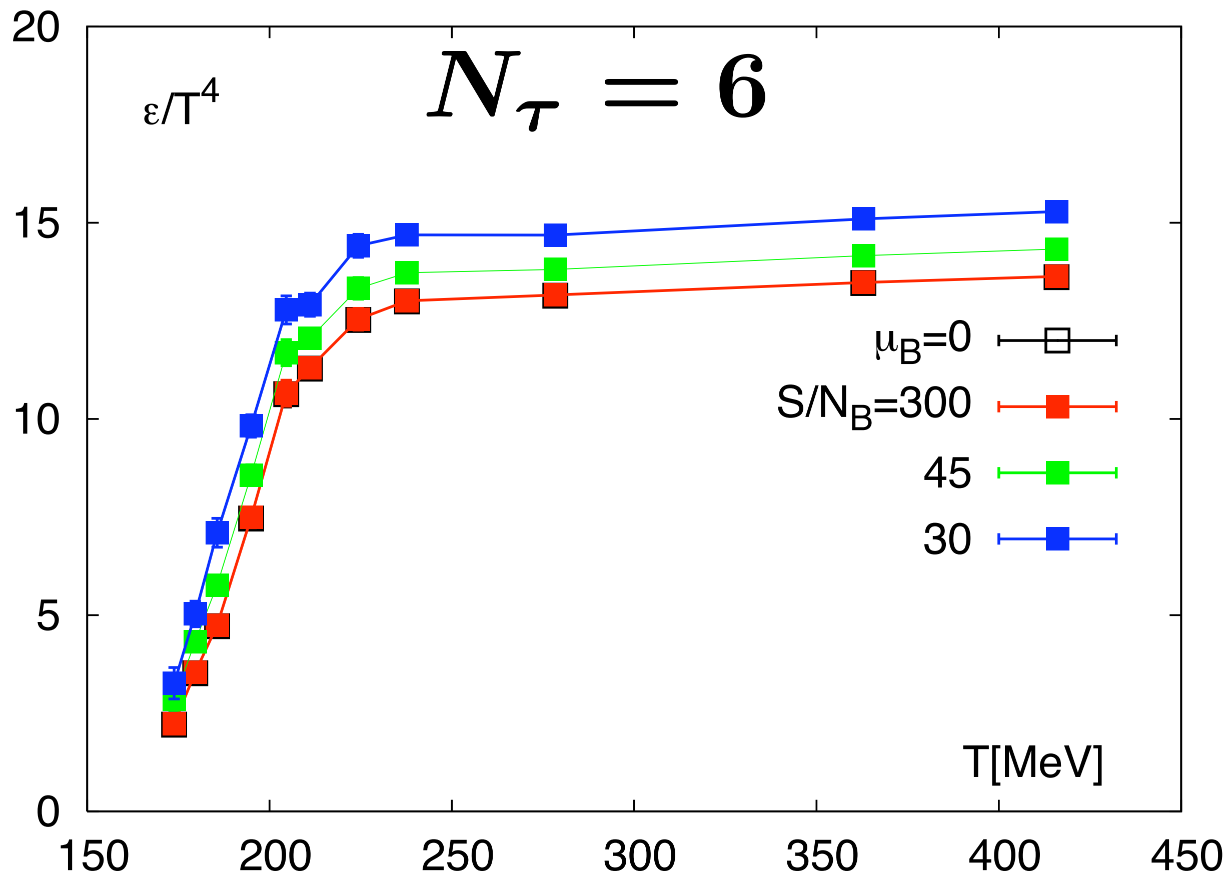






Summary

- Improved actions drastically reduce cut-off effects: **p4** and **asqtad** actions lead to consistent thermodynamics on lattices of Nt=6 and 8.
- Cut-off effect for Taylor expansion coefficients are small and sizable only in the transition region (similar to the interaction measure e-3p).
- Fluctuations and correlations are well described by a free gas of quarks above T>(1.5-1.7)Tc and by a resonance gas for T<Tc.
- Higher order cumulants signal the break down of the resonance gas at temperatures close but below Tc.
- We find non-monotonic behavior in the radius of convergence for $N_\tau = 4$ which could be a first hint for a critical region in the T, μ_B - plane. This needs to be confirmed by $N_\tau = 6$.
- Finite density correction for EoS are small, pressure and energy density increase by $\approx 10\%$ for S/N=30 (AGS/FAIR), corrections cancel to large extent in p/ϵ .
- Taylor expansion method will provide valuable input for HIC phenomenology.

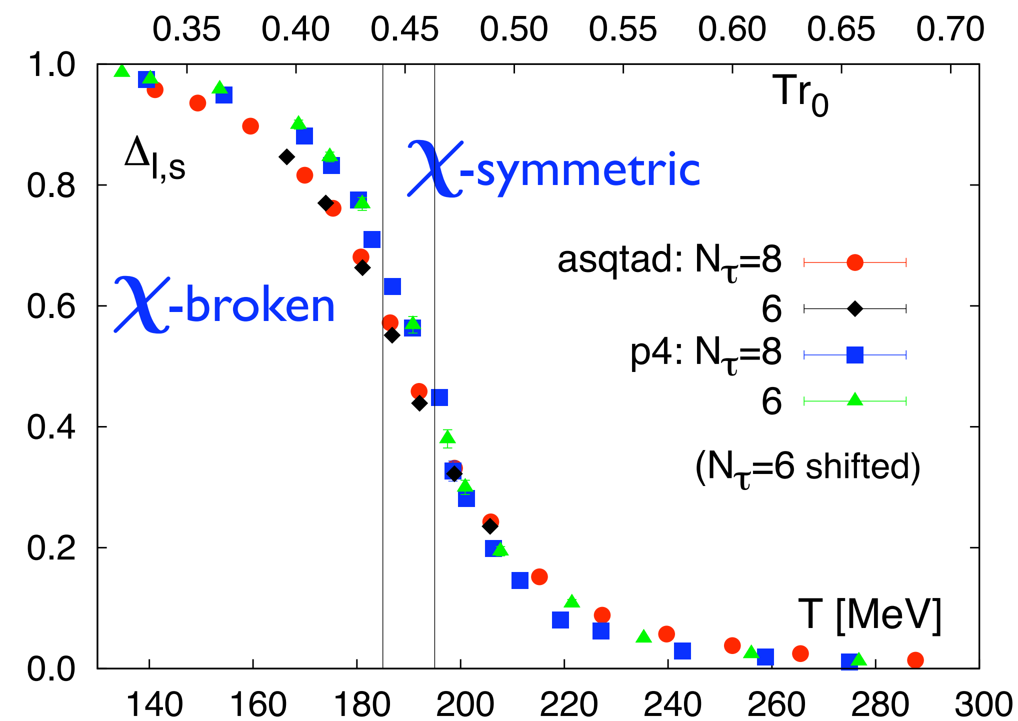
De-confinement and χ-symmetry

- **Order parameter in the chiral limit: the chiral condensate**

  (sensitive to chiral symmetry restoration)

$$\Delta_{l,s}(T) = \frac{\langle\bar\psi\psi\rangle_{l,T} - \frac{\hat m_l}{\hat m_s}\langle\bar\psi\psi\rangle_{s,T}}{\langle\bar\psi\psi\rangle_{l,0} - \frac{\hat m_l}{\hat m_s}\langle\bar\psi\psi\rangle_{s,0}}$$

(additive and multiplicative re-normalization factors are removed by this combination of light and strange chiral condensate at zero and finite temperature)



p4, asqtad 2+1 flavor

$m_\pi \approx 220 MeV$

(line of const. physics)

HotQCD preliminary

- **Order parameter in the pure gauge limit: the Polyakov loop**

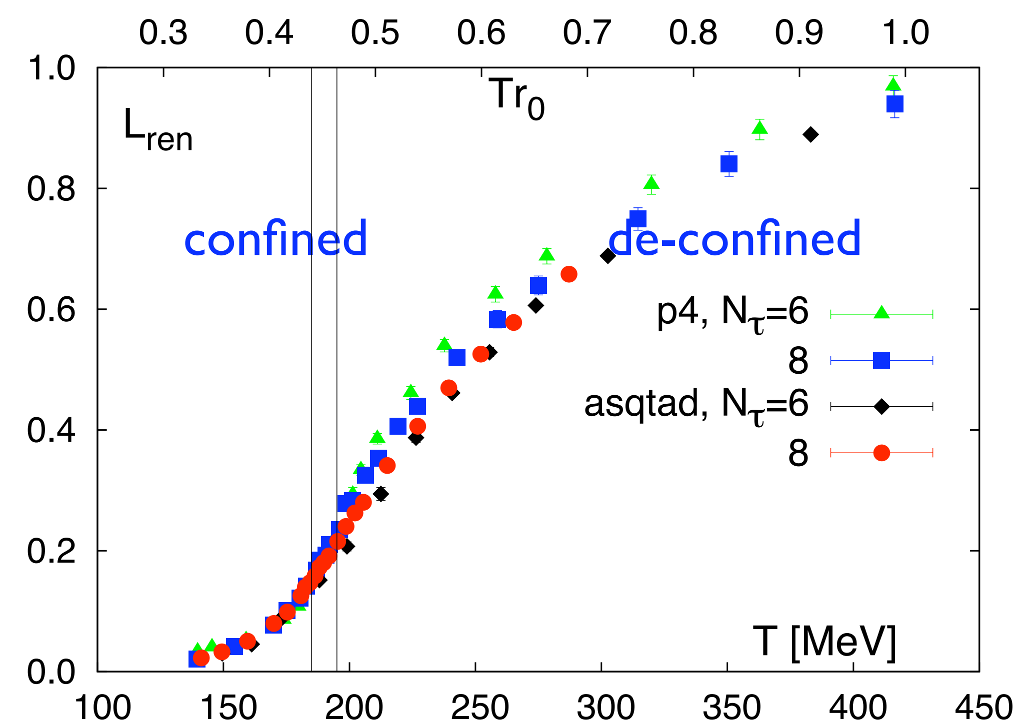
  (sensitive to the de-confinement transition)

$$\langle L\rangle = \left\langle \frac{1}{N_\sigma^3}\sum_{\vec x} L_{\vec x}\right\rangle \quad \text{with}$$

$$L_{\vec x} = \frac{1}{3}\mathrm{Tr}\prod_{x_0=1}^{N_\tau} U_{(x_0,\vec x),\hat 0}$$

$$L_{ren}(T) = Z_{ren}^{N_\tau}(\beta)\langle L\rangle$$

(re-normalization factors are obtained by matching the static quark potential to the string potential) $\longrightarrow$

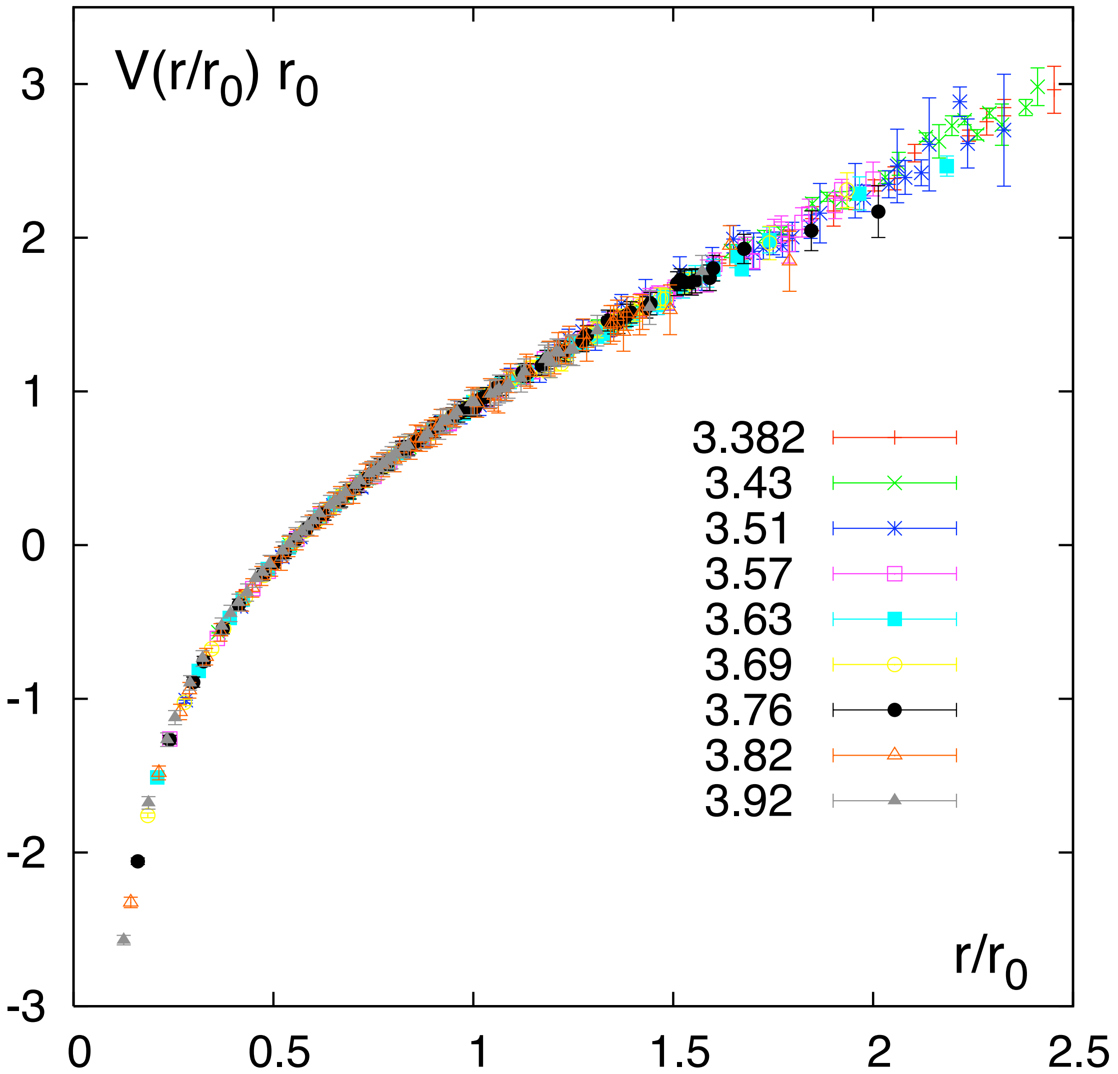


small cut-off effects

Scale setting at T=0

use r_0 or string tension to set the scale for $T = 1/N_\tau a(\beta)$

$$V(r) = -\frac{\alpha}{r} + \sigma r \quad , \quad r^2 \frac{dV(r)}{dr}\Big|_{r=r_0} = 1.65$$



no significant cut-off dependence when cut-off varies by a factor 5

i.e. from the transition region on $N_\tau = 4$ lattices ($a \simeq 0.25$ fm) to that on $N_\tau = 20$ lattices ($a \simeq 0.05$ fm) !!

Scale setting at T=0

- high precision studies of several experimentally well known observables in lattice calculations with staggered (asqtad) fermions led to convincing agreement $\Rightarrow$ gold plated observables
- simultaneous determination of r_0/a in these calculations determines the scale r_0 in MeV

knowing any of these experimentally accessible quantities accurately from a lattice calculation is equivalent to knowing r_0, which is a fundamental parameter of QCD

C.T.H. Davies et al., PRL 92 (2004) 022001

A. Gray et al., PRD72 (2005) 094507