Phase diagram of QCD: the critical point

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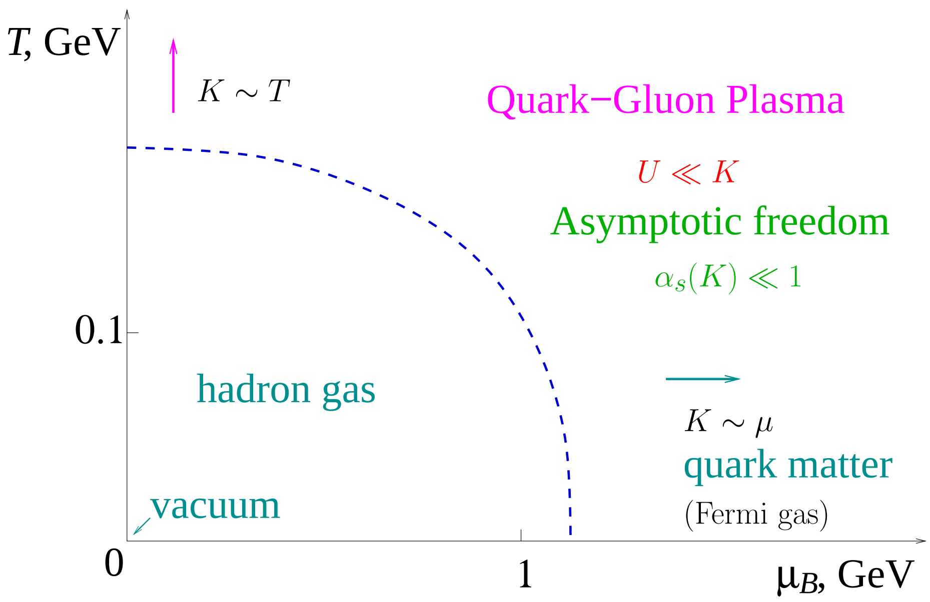
Phase Diagram of QCD

- Basic arguments, quark confinement and asymptotic freedom, predict a transition at $T \sim \Lambda_{\rm QCD}$, $\mu_B \sim N_{\rm color}\Lambda_{\rm QCD}$:

Hadron/resonance gas (π,N, resonances) becomes a (color) plasma of quarks and gluons ($\Lambda_{\rm QCD} \sim$ (hadron size)$^{-1}$).

Simple arguments lead to the sketch:



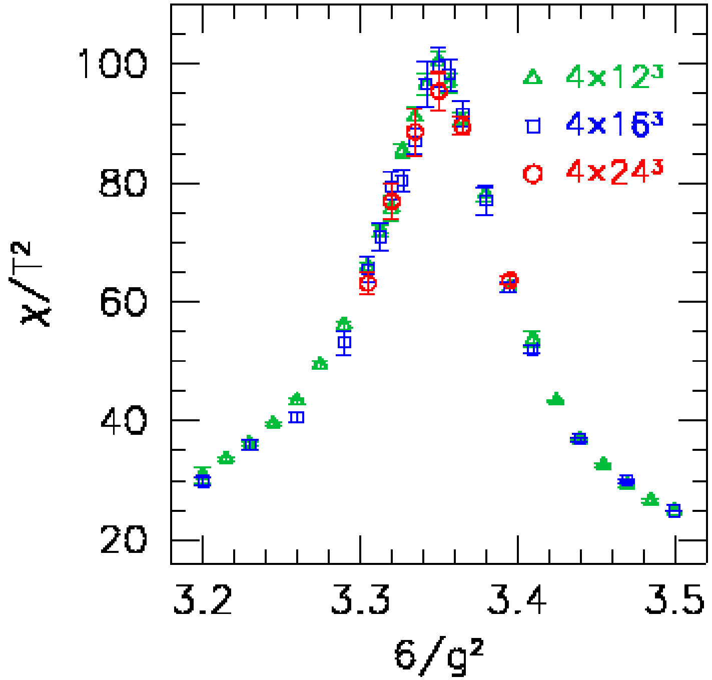
- Order of transition?

Originally, arguments suggested 1st order (discontinuous):
e.g., $S_{\rm QGP} \sim N_{\rm color}^2$, while $S_{\rm HG} \sim N_{\rm color}^0$.

Lattice says: crossover (at $\mu = 0$)

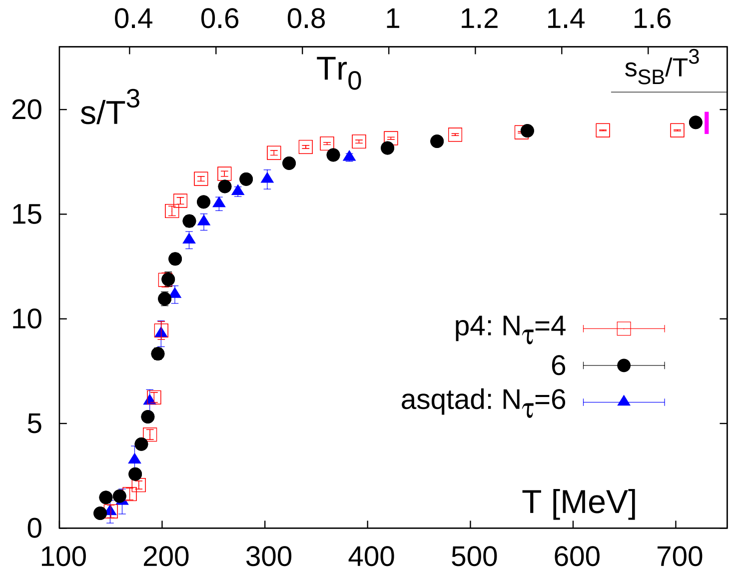
- Earliest: Columbia group, PRL 65(1990)2491
- Recent: Wuppertal-Budapest group, Nature 443(2006)675.

Wuppertal-Budapest:



the peak should have grown $8\times$ for 1st order transition

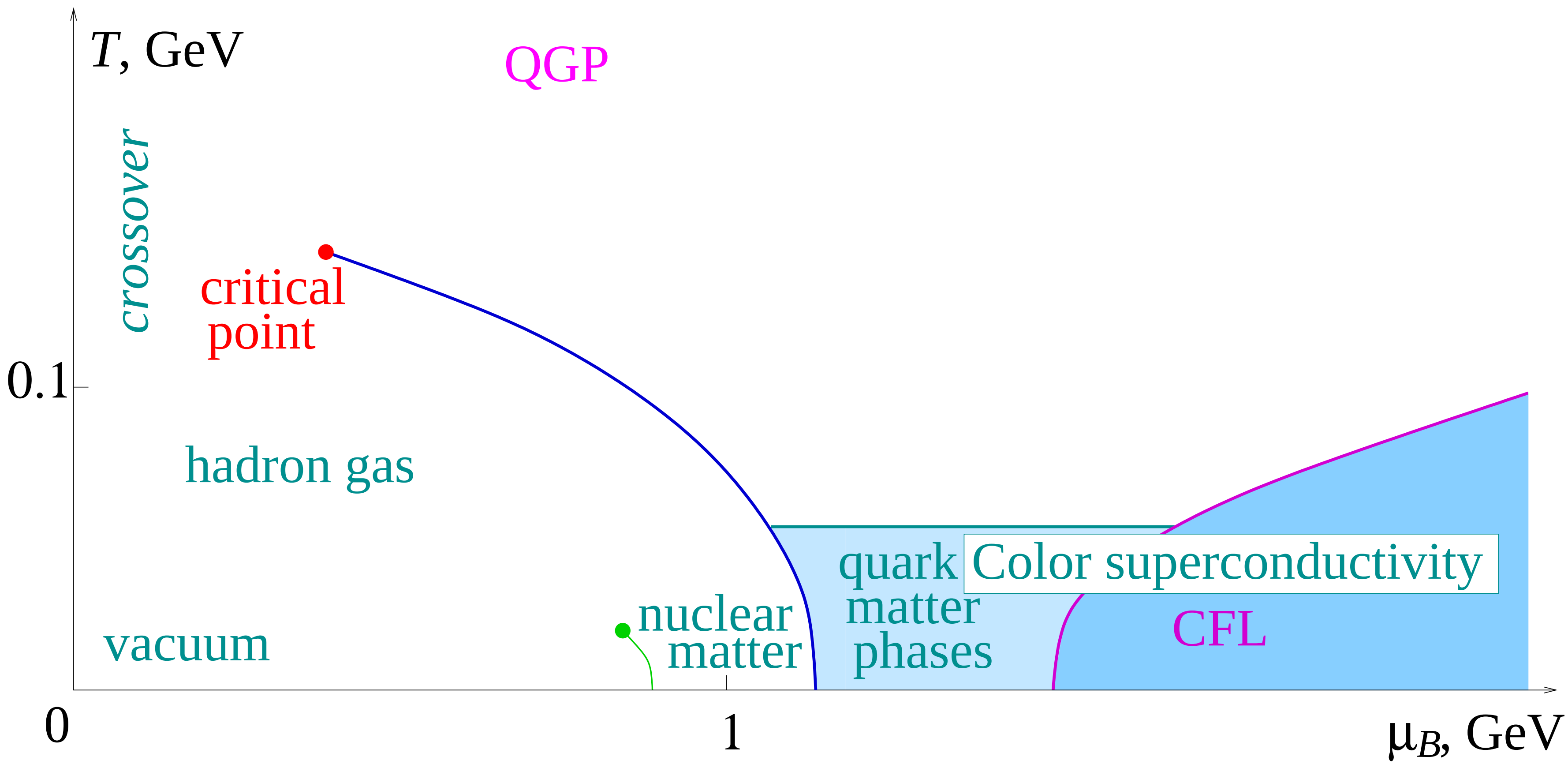
RBC-BI:



Entropy/$T^3 \sim \#$ of d.o.f. grows (color is liberated) but no discontinuity

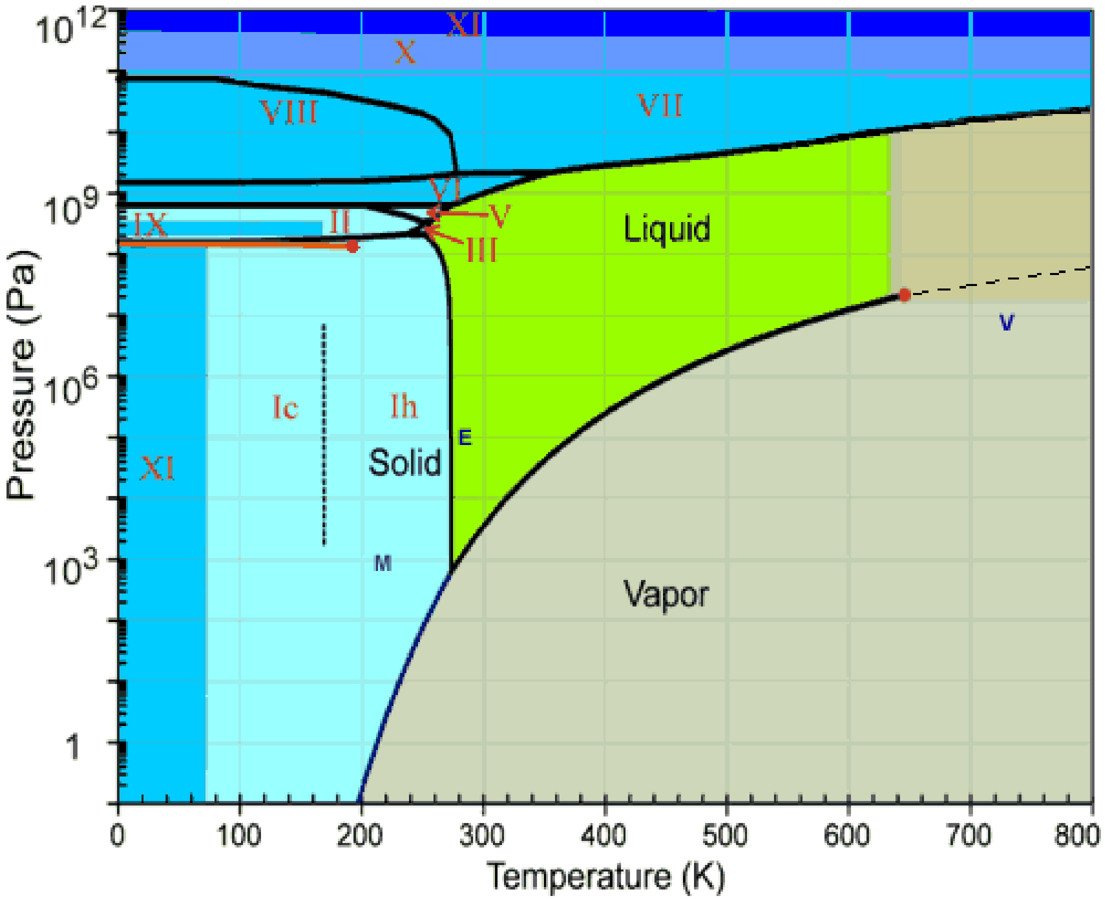
- Quarks are important: w.o. them the transition would be 1st order. Too many quarks – also 1st order.

QCD phase diagram (contemporary view)



- Models (and lattice) suggest crossover turns into 1st order at some μ_B.
- Large μ_B – Color Flavor Locked phase ("QCD ice"). Challenge - translate theoretical understanding into predictions (e.g., for neutron stars): mechanical (rigidity), transport (viscosity), magnetic properties.
- Crossover – "supercritical" fluid. Almost perfect. Strongly coupled. New methods needed.

Water



- Critical point is a common feature of liquids

1822

THE

ANNALS

OF

PHILOSOPHY.

NEW SERIES.

JANUARY TO JUNE, 1823.

ARTICLE IX.

An Account of some Results obtained by the combined Action of Heat and Compression upon certain Fluids, such as Water, Alcohol, Sulphuric Ether, and the rectified Oil of Petroleum. By M. le Baron Cagniard de la Tour.*

The last experiment was made with a glass tube about one-third full of water; this tube lost its transparency, and broke a few seconds afterwards. It appears that at a high temperature water is capable of decomposing glass by combining with its alkali; this suggests the idea that some other result interesting to chemistry may, perhaps, be obtained by increasing the applications of this process of decomposition.

It took a century to explain the phenomenon of critical opalescence – divergent ξ of density fluctuations (Smoluchowski, Einstein). And another 1/2 century to describe critical phenomena quantitatively – scaling, universality, RG (Landau, Kadanoff, Wilson).

What would it take to discover QCD critical point?

- Experiment: heavy ion collision energy scan
- Theory: locate the critical point in a lattice QCD calculation

Sign Problem

- Thermodynamics follows from partition function

$$Z = \sum_{\text{quantum states}} \exp\{-\beta E\} = \int \mathcal{D}(\text{paths}) \, \exp\{-S_E\}$$

  - S_E - action on a path in imaginary time τ from 0 to β.

- Usually S_E - real. So $\int \mathcal{D}(\text{paths})\, e^{-S_E}$ - itself is a partition function for *classical* statistical system in $3+1$ dimensions. Monte Carlo methods work.

- Not so for $\mu \neq 0$.

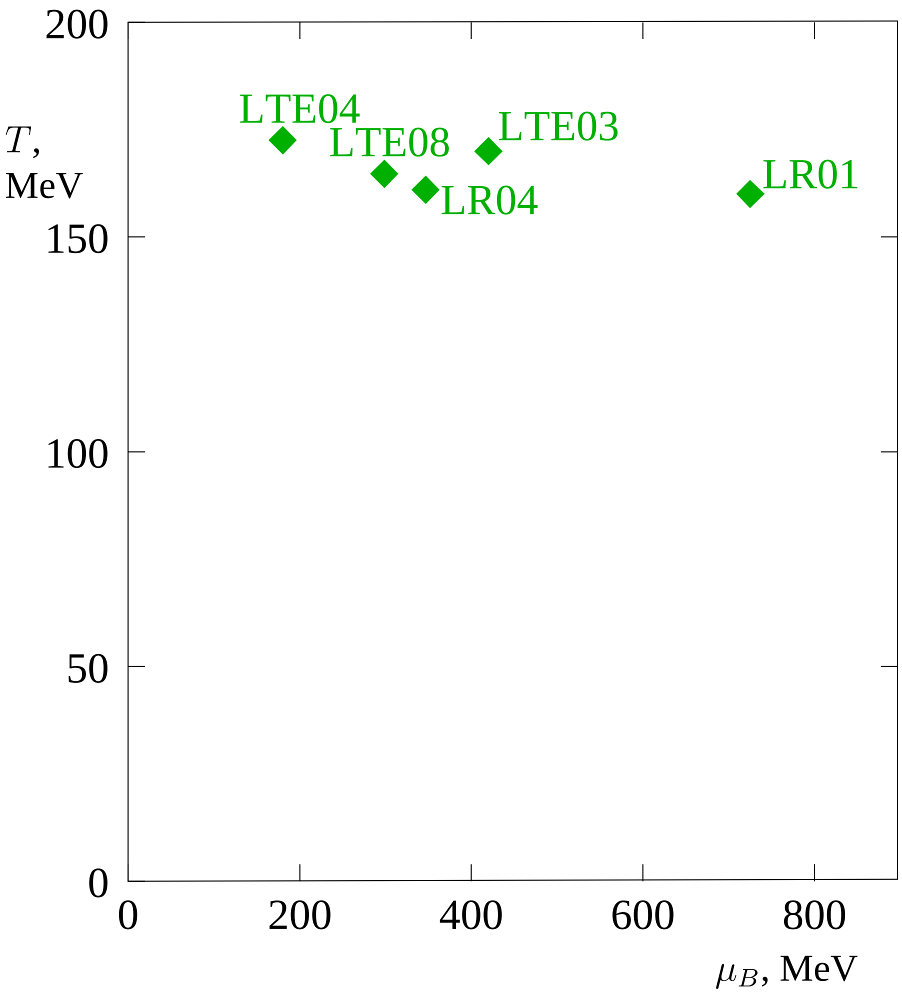
$$e^{-S_E} = e^{-S_{\text{gluons}}} \, \det D_{\text{quarks}}.$$

and $\det D_{\text{quarks}}$ is complex for $\mu \neq 0$.

Monte Carlo translates weight e^{-S_E} into probability and fails if S_E is not real.

- Recent progress based on various techniques of circumventing the problem:
  - Reweighting (use weight at $\mu = 0$);
  - Taylor expansion;
  - Imaginary μ;
  - ...

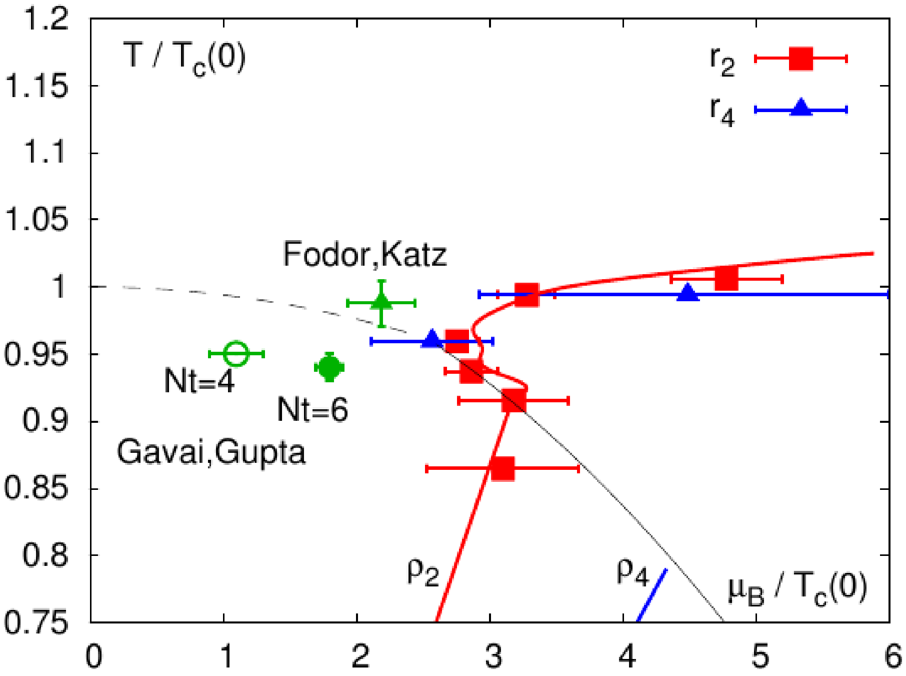
Location of the critical point from the Lattice



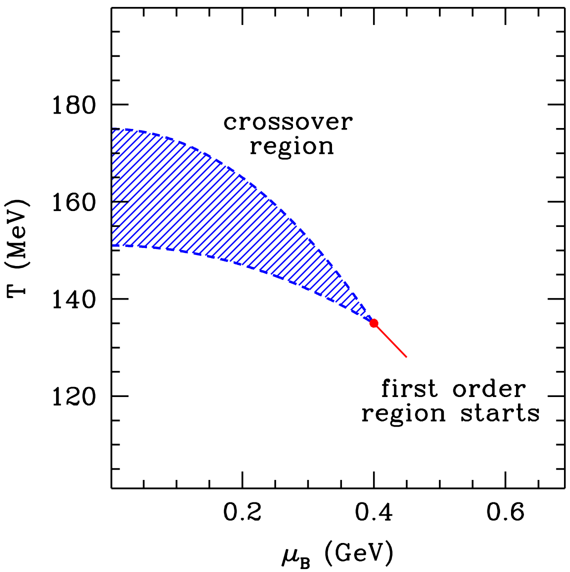
- Systematic errors are not shown.
- So far lattice results disfavor $\mu_B < 200$ MeV.
- de Forcrand-Philipsen: maybe $\mu_B > 500$ MeV?
- Strong N_t dependence:
  - continuum limit is still far?
  - role of anomaly and “rooting”?

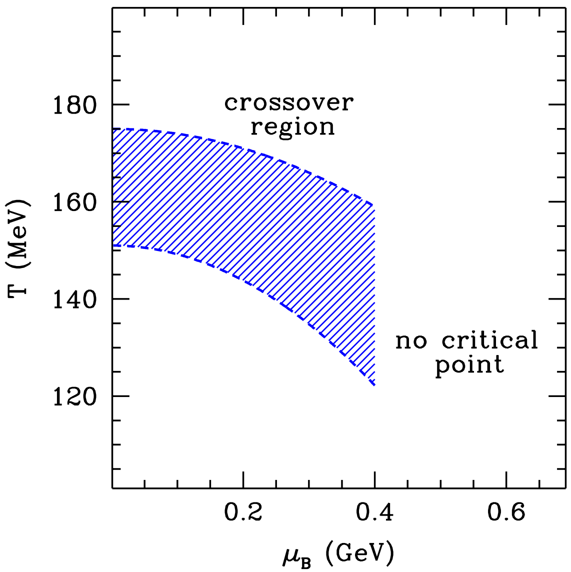
  Wilson fermions might help.

Recent developments



- RBBC, Gavai-Gupta: Radius of convergence of Taylor expansion

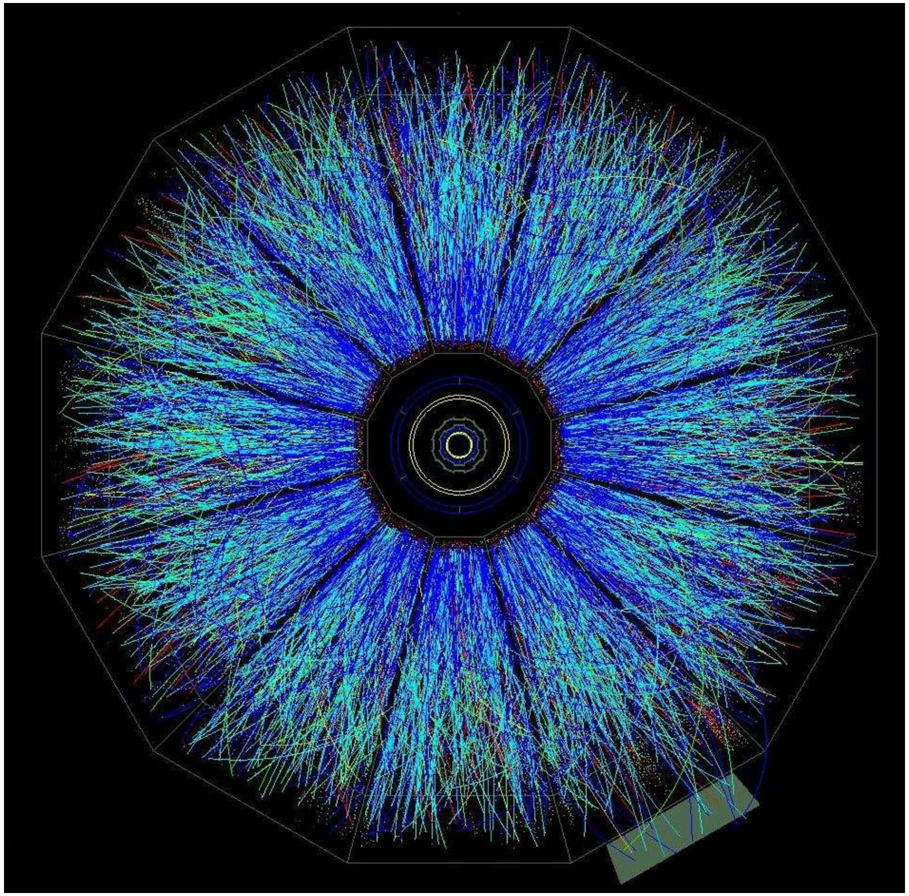




- Fodor et al: Compare crossover curvatures for 2 observables
- Strong N_t dependence

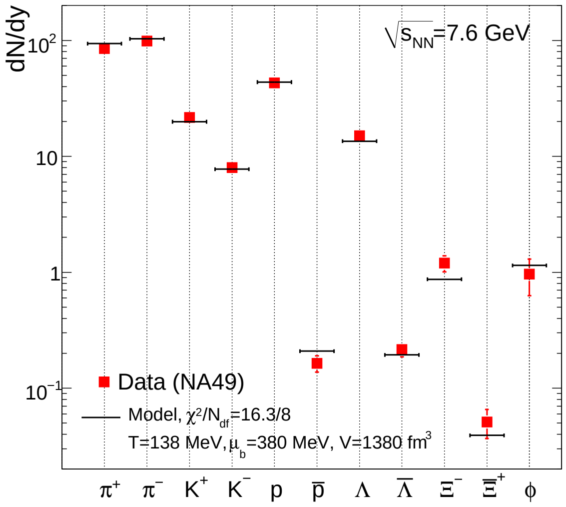
Heavy-ion collisions and the phase diagram

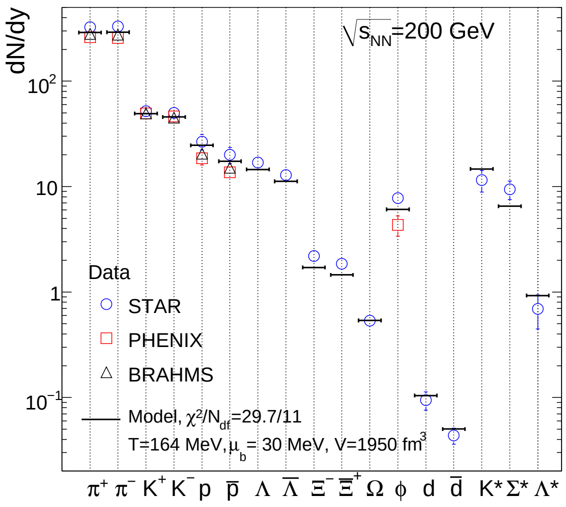
STAR@RHIC



an event
“Little Bang”

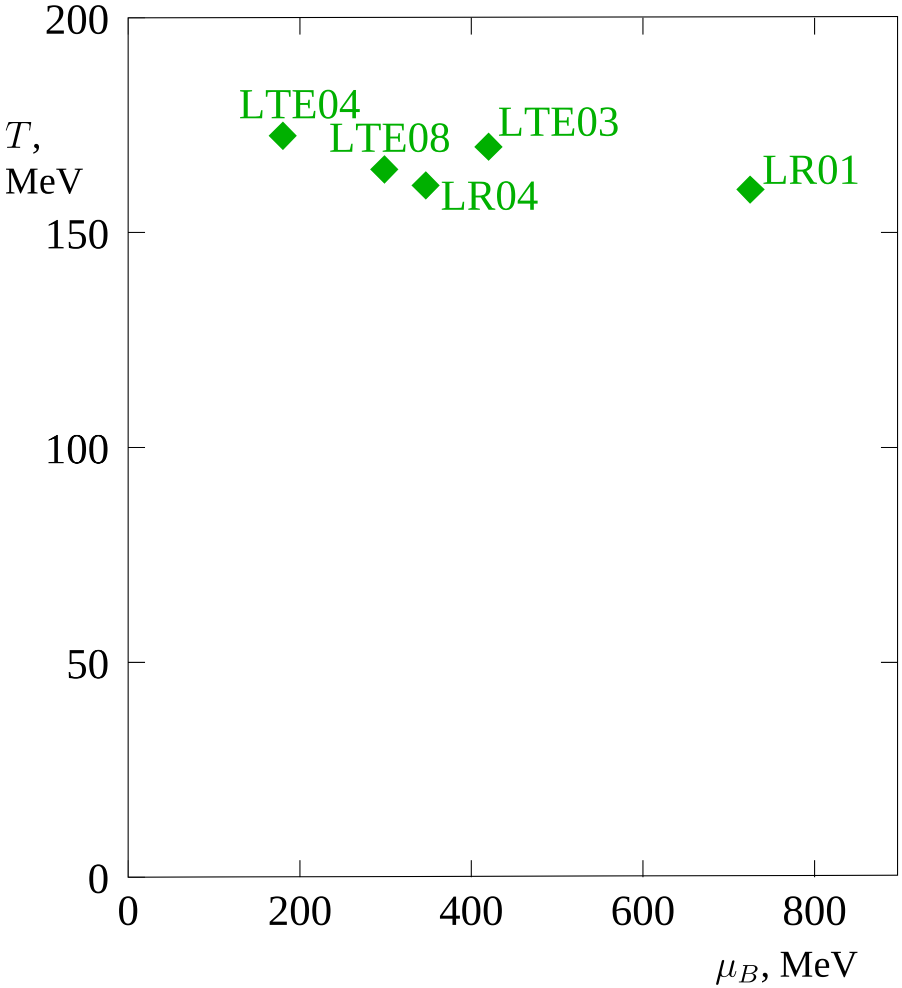
- Final state is thermal





Thermal model 2008
(Andronic-PBM-Stachel)

Location of the critical point vs freeze-out



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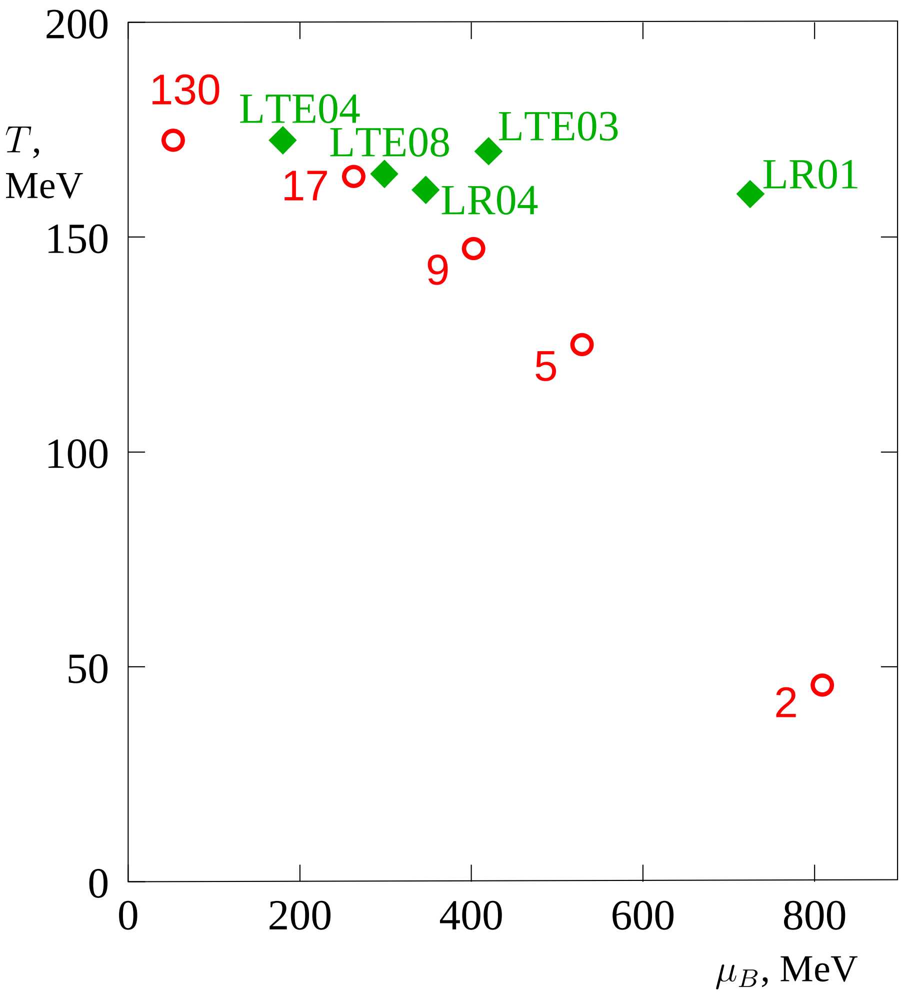

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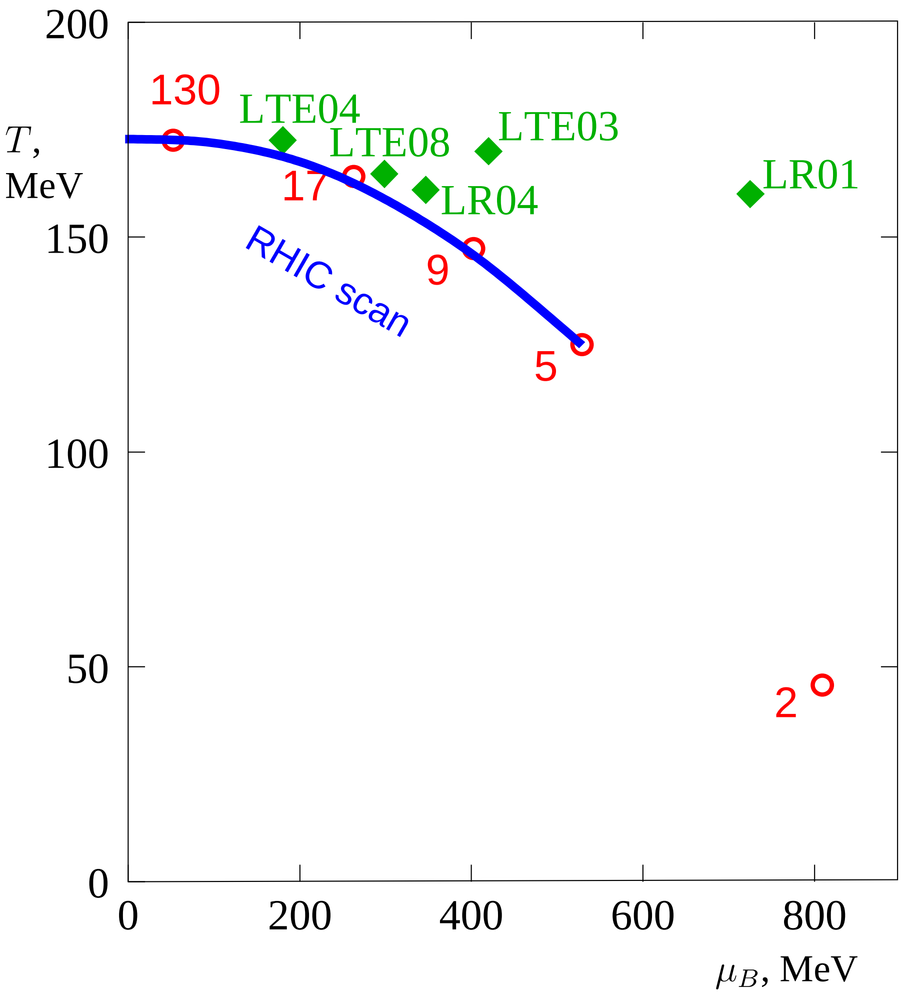


Needed:

- Experiments:
  - RHIC,
  - NA61(SHINE) @ SPS,
  - CBM @ FAIR/GSI
- Improve lattice predictions, understand systematic errors.
- Understand critical phenomena in the dynamical environment of a h.i.c. (understand background) – develop optimal signatures

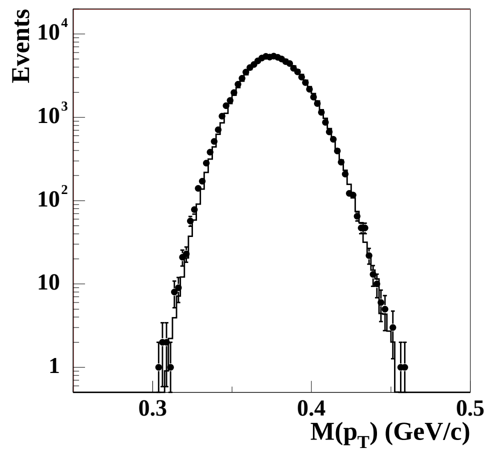
Fluctuation signatures

- Experiments measure for each event: multiplicities N_π, N_p, ..., momenta p, etc.

These quantities fluctuate event-by-event.

  - Typical measure is stdev, e.g., $\langle(\delta N)^2\rangle$.

- What is the magnitude of these fluctuations near the c.p.? (Rajagopal, Shuryak, M.S.)



- *Universality* tells how it grows at the critical point: $\langle(\delta N)^2\rangle \sim \xi^2$.
  - Correlation length is a universal measure of the "distance" from the c.p. It diverges as $\xi \sim (\Delta\mu$ or $\Delta T)^{-2/5}$ as the c.p. is approached.
- Magnitude of ξ is limited by finite time/size effs: $\mathcal{O}($3 fm$)$ - (Berdnikov, Rajagopal).

- "Shape" of the fluctuations can be also measured: non-Gaussian moments.

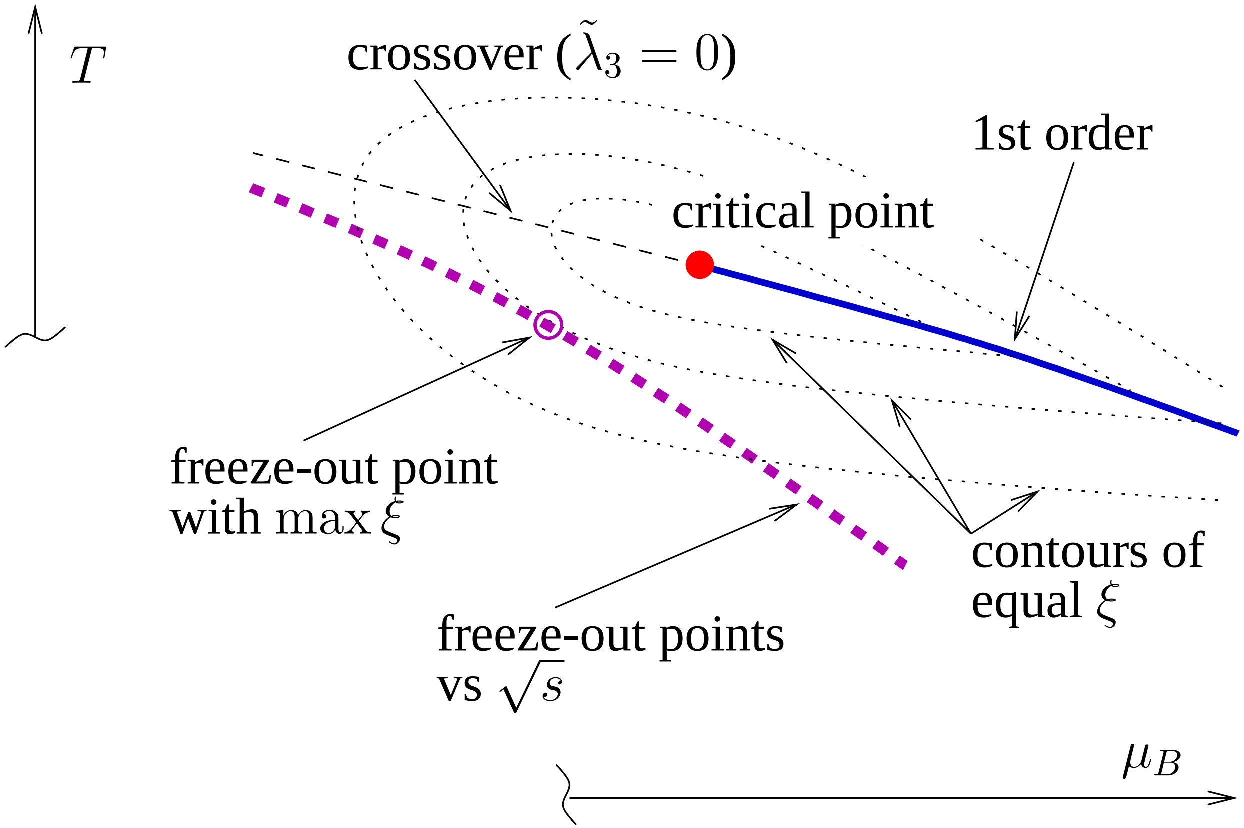
As $\xi \to \infty$ fluctuations become more non-Gaussian.

- Higher cumulants show even stronger dependence on ξ (arxiv:0809.3450):

$$\langle(\delta N)^3\rangle \sim \xi^{4.5}, \qquad \langle(\delta N)^4\rangle - 3\langle(\delta N)^2\rangle^2 \sim \xi^7$$

which makes them more sensitive signatures of the critical point.

Scan



Skewness $\neq 0$.

Concluding remarks

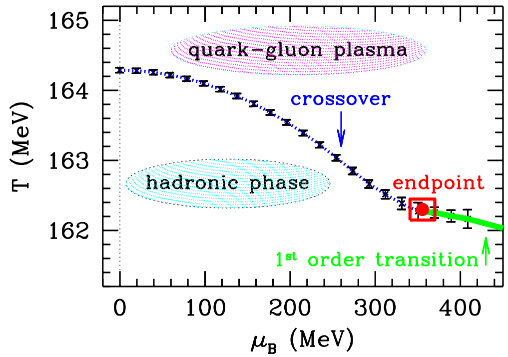
- Phase diagram of QCD still contains many unknowns.

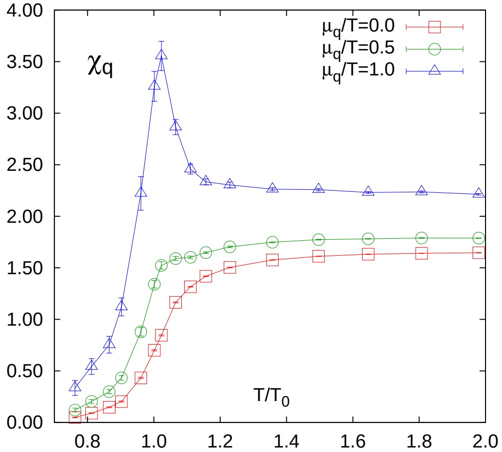
- The QCD critical point is one of the central features of the QCD phase diagram. Its discovery will transform the phase diagram from theoretical speculation to text-book knowledge.

- The theory and experiment each have their own challenges. And each approach needs “data” from the other.

Appendix

Critical point on the lattice





Allton, *et al*: peak in χ_B, but not in χ_I

Several approaches:

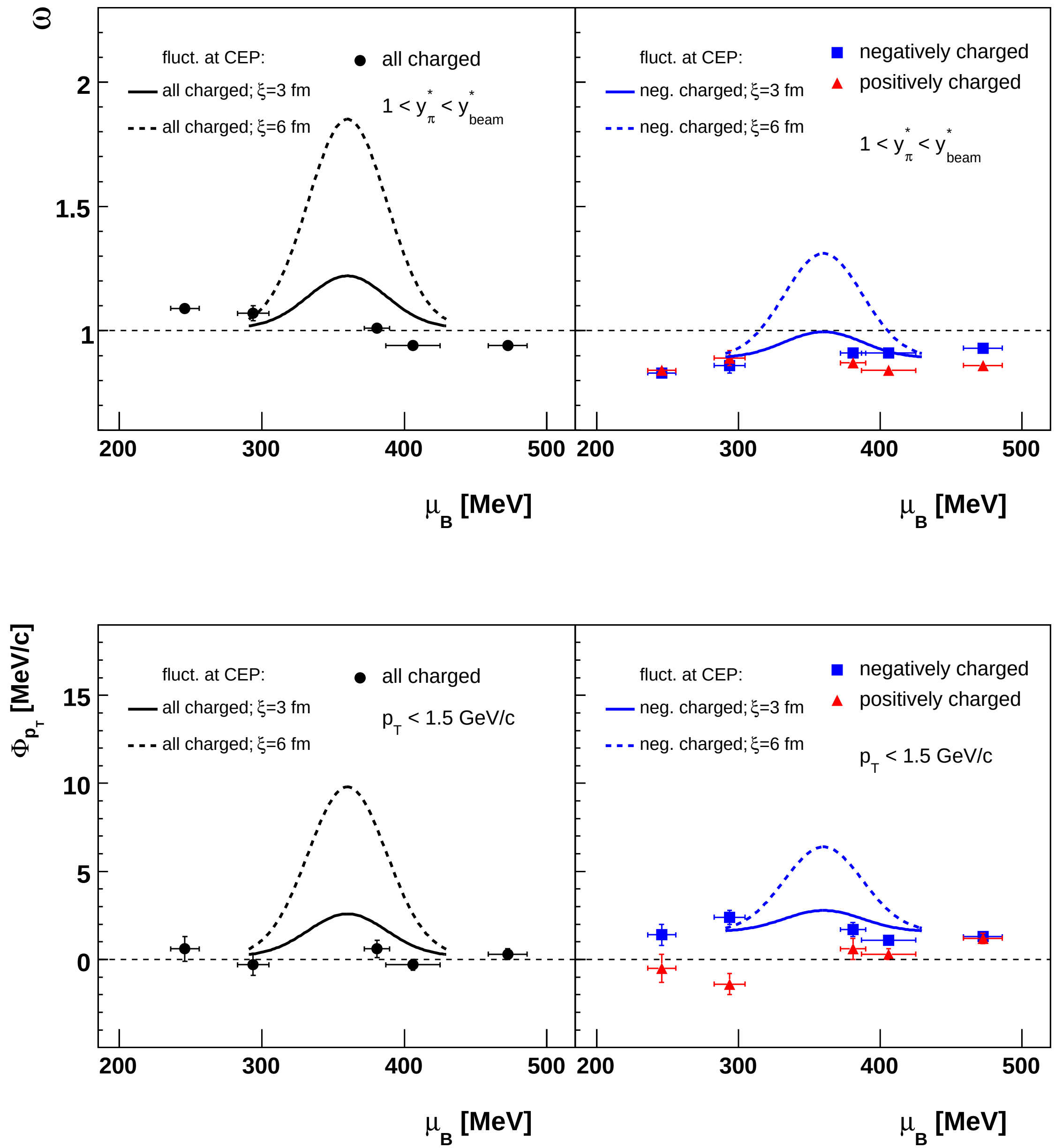
- Reweighting: Fodor-Katz
  - 2001: $\mu_B \sim 725$ MeV
  - 2004: $\mu_B \sim 360$ MeV (smaller m_q and larger V)
- Taylor expansion: Bielefeld-Swansea (to μ^6)
  - 2003: $\mu_B \sim$ 420 MeV
  - 2005: 300 MeV $\lesssim \mu_B \lesssim$ 500 MeV
- Taylor expansion: Gavai-Gupta (to μ^8)
  - From convergence radius: $\mu_B \sim 180$ MeV (more precisely > 180 MeV)
- Imaginary μ: deForcrand-Philipsen, Lombardo, *et al*
  - Sensitive to m_s, perhaps $\mu_B \gg$ 300 MeV
- Fixed density: deForcrand, Kratochvila; Density of states: Fodor, Katz, Schmidt.
  - ? ($N_f = 4$, small volumes)

Observables – theory comments

- Fluctuations:
  - Multiplicities –
    pro: larger signal – especially protons (coupled to critical mode);
    con: larger background (impact param. fluct.)
  - Ratios, mean p_t –
    pro: no impact param. fluct.;
    con: smaller signal.
  - Non-gaussian fluctuations (higher moments: skewness, kurtosis) –
    pro: strong dependence on ξ – large signal;
    con: difficult to estimate either signal or background.
  - Fluctuations from 1st order transition (nonequilibrium)?
    pro: presumably more drammatic;
    con: difficult to predict – requires more detailed dyn. assumptions.
- Non-fluctuation observables:
  - $\bar{p}/p$ – Asakawa-Bass-Müller-Nonaka; based on focusing – trajectories are "pulled" to larger μ_B at earlier times $+$ earlier freezeout of higher p_t.

True critical point signal should show consistently in several observables.

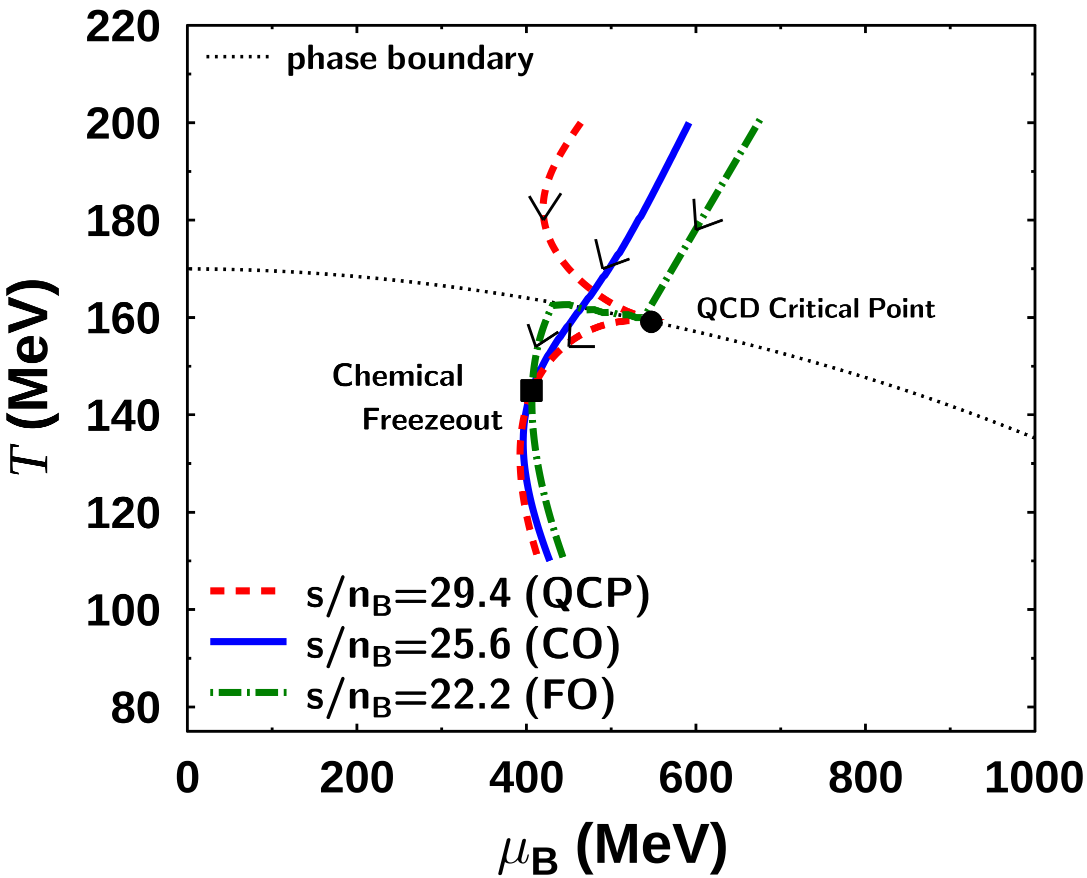
NA49 energy scan (20-30-40-80-160)



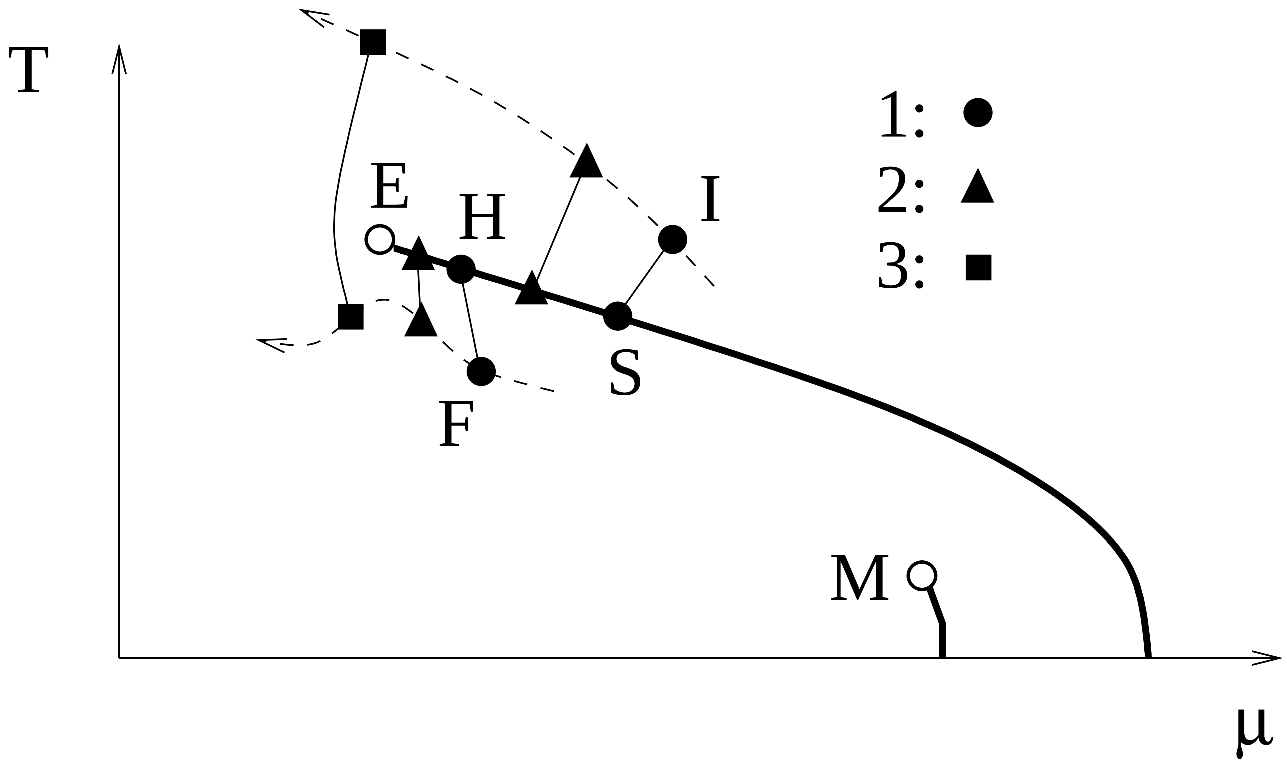
- Critical fluctuations: signal(all) = 2 $\times$ signal(half)
- Not so for B.E.C. signal
- $\omega(N)$ has interesting centrality dependence.

NA61 can study it further also for Φ_{p_T}.

Asakawa-Bass-Müller-Nonaka



Focusing



$$(s/n)_{\mathrm{QGP}} > (s/n)_{\mathrm{HG}}$$