In-Medium Studies of Omega, Nucleon and Open Charm with QCD Sum Rules

Ronny Thomas
Forschungszentrum Dresden-Rossendorf / TU Dresden

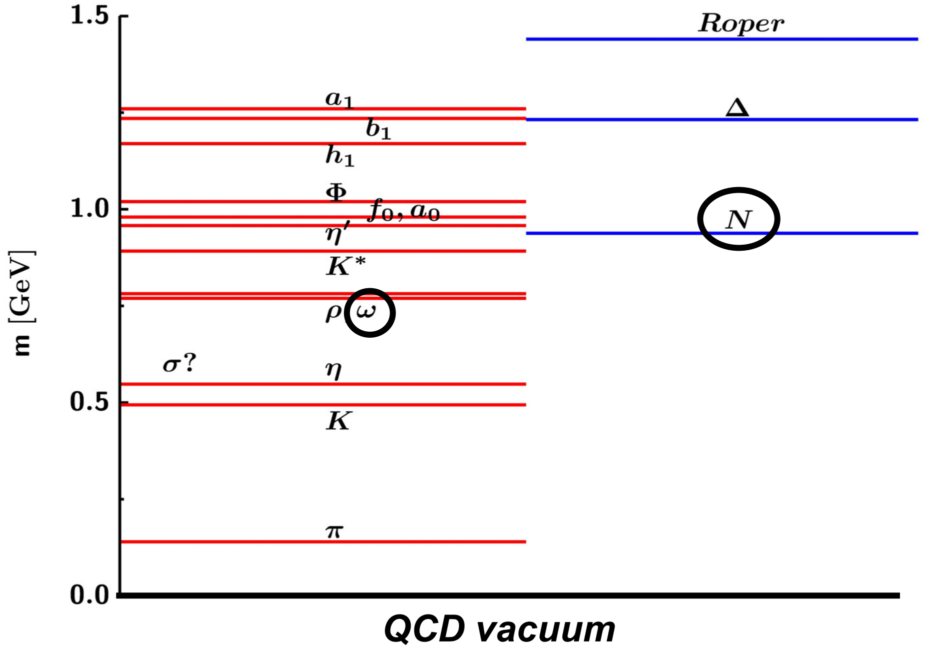
Dresden, September 2007

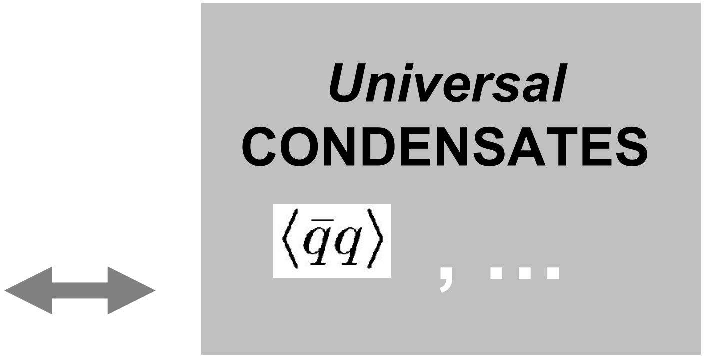
- In-Medium Modification of Hadrons: ω, ***N*, *D***
- QCD Sum Rules
- Four-Quark Condensates

work with T. Hilger, S. Zschocke and B. Kämpfer

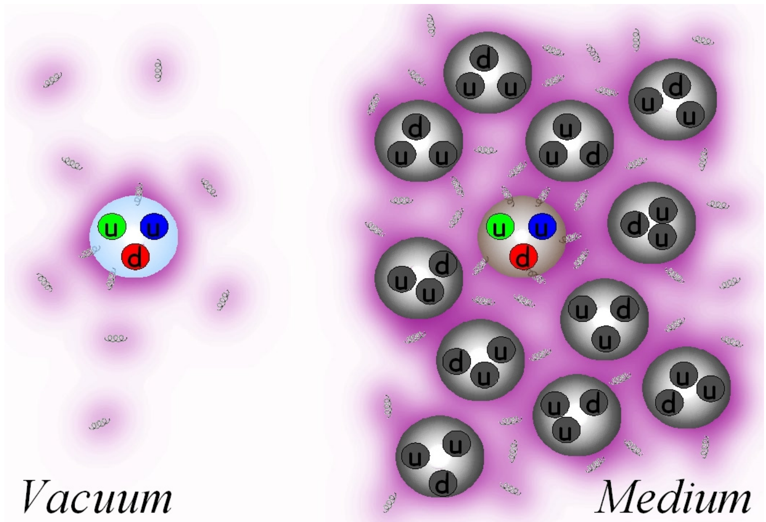
supported by BMBF, GSI, EU, Helmholtz

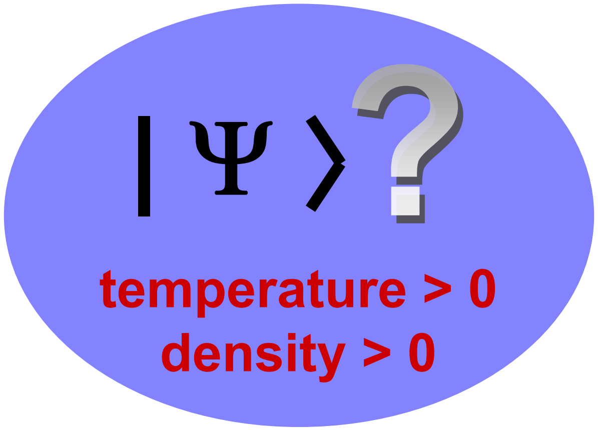
In-Medium Modifications





QCD Sum Rules

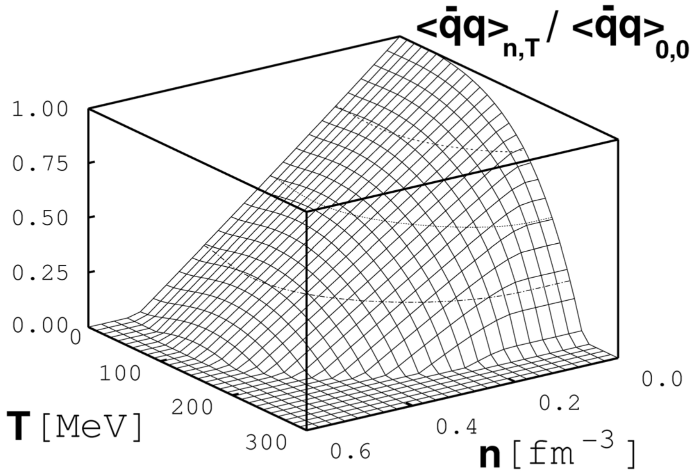




Analogy:
Stark & Zeemann effects

$$\langle \mathcal{O} \rangle = \langle \mathcal{O} \rangle_0 + \frac{n}{2M_N} \langle N | \mathcal{O} | N \rangle + \frac{T^2}{8} \langle \pi | \mathcal{O} | \pi \rangle + \dots .$$

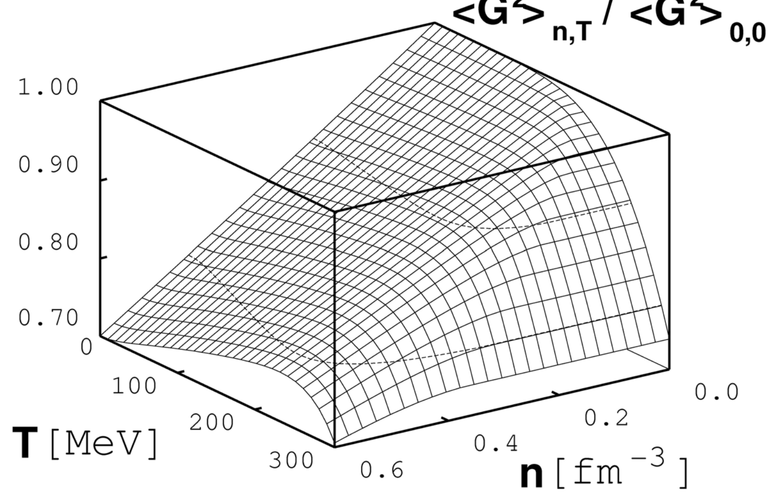
Chiral condensate



Zschocke et al (2002)

$m_\pi^2 f_\pi^2 = -m_q \langle \bar{\psi}\psi \rangle$ (PCAC)

Gluon condensate



Trace anomaly

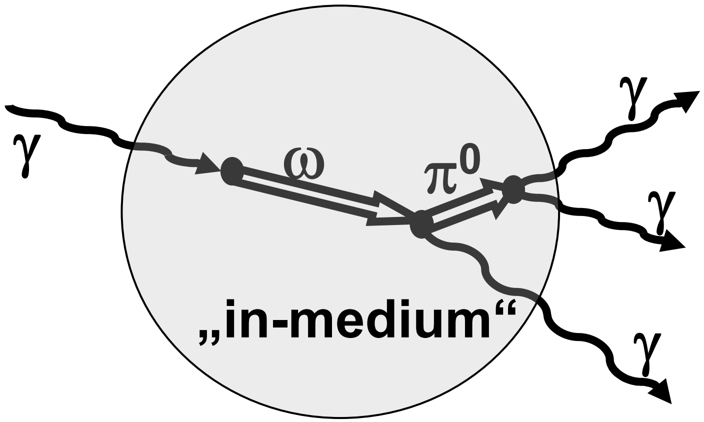
Mixed quark-gluon condensate $\langle \bar{q} g \sigma_{\mu\nu} G^{\mu\nu} q \rangle = m_0^2 \langle \bar{q} q \rangle$

Catalog of four-quark condensates …

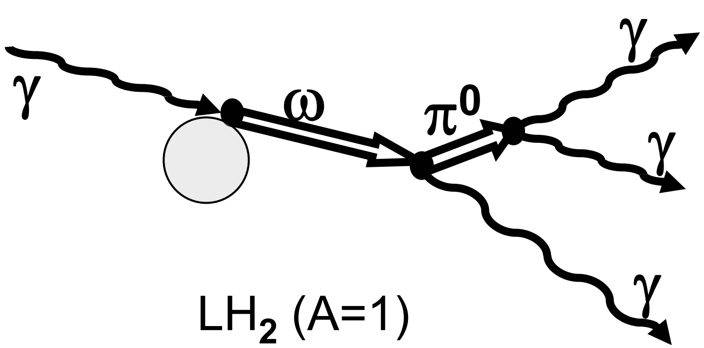
ω in medium

CB-TAPS

$$\gamma + A \rightarrow A' + \omega\ (\rightarrow \pi^0 + \gamma\ )$$

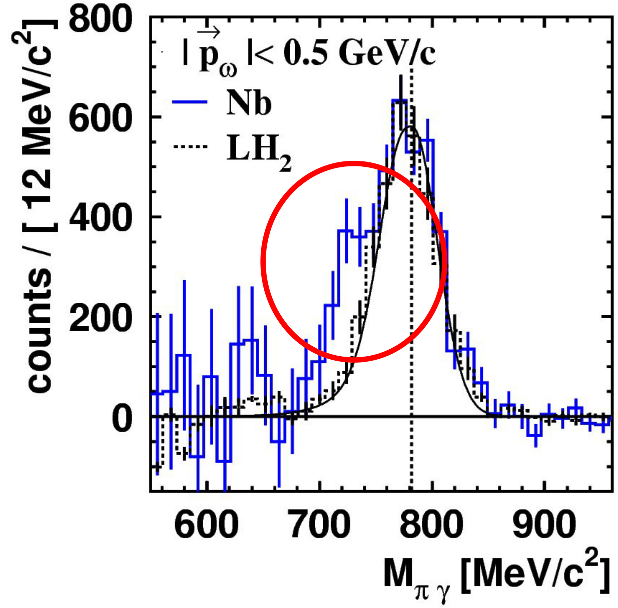


Nb (A=93)



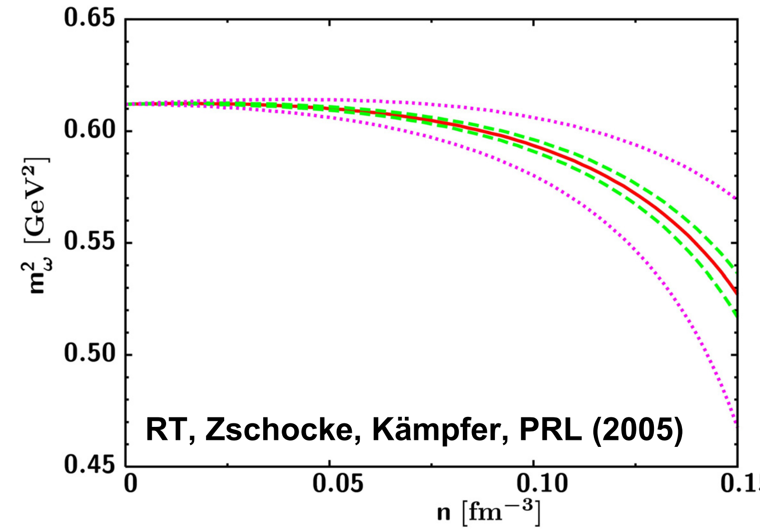
LH_2 (A=1)

Trnka, Metag et al, PRL (2005)



First moment:

$$m_V^2(n, M^2, s_V) \equiv \frac{\int_0^{s_V} ds\ \mathrm{Im}\Pi^{(V)}(s,n)\ e^{-s/M^2}}{\int_0^{s_V} ds\ \mathrm{Im}\Pi^{(V)}(s,n)\ s^{-1} e^{-s/M^2}}$$



⇒ **implies strong density dependence of specific four-quark condensate combinations**

⇒ **change of QCD vacuum** … Brown-Rho scaling (→ In-medium shift → HADES)

QCD Sum Rules

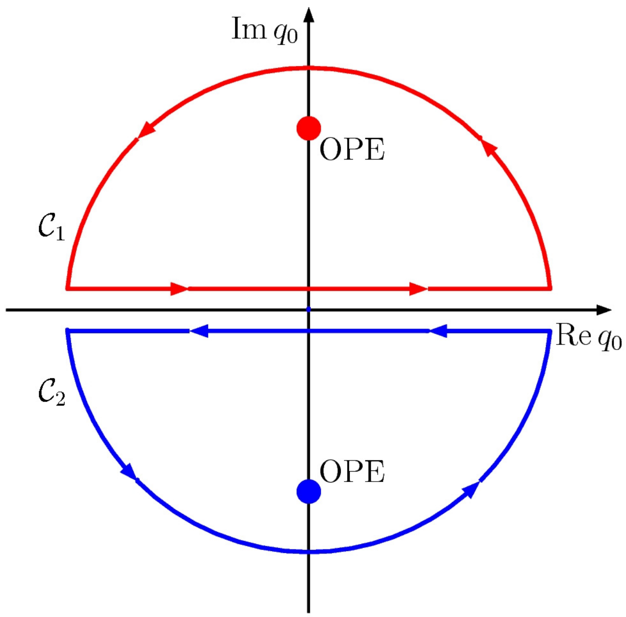
Shifman, Vainshtein, Zakharov (1979)

Current-current correlator

$$\boxed{\Pi(q,n) = i\int d^4x\, e^{iqx}\, \langle\Psi\, |T[j_\mu(x) j^\mu(0)]|\, \Psi\rangle}$$

ω meson:

$$j^\omega_\mu = \frac{1}{2}\left(\bar{u}\gamma_\mu u + \bar{d}\gamma_\mu d\right)$$



Dispersion Relation

$$\frac{1}{\pi}\int_{-\infty}^{+\infty} d\omega \frac{\Delta\Pi(\omega)}{\omega - q_0} = \Pi(q_0)$$

Operator Product Expansion (OPE)

Hadronic side

Next talk: T. Hilger
Aspects of the OPE

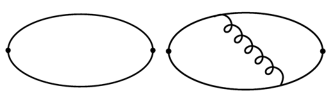
$$\int_{-\infty}^{+\infty} d\omega \mathcal{F}(\omega) = \int_{-\infty}^{\omega_-} d\omega \mathcal{F}(\omega) + \int_{\omega_-}^{0} d\omega \mathcal{F}(\omega) + \int_{0}^{\omega_+} d\omega \mathcal{F}(\omega) + \int_{\omega_+}^{+\infty} d\omega \mathcal{F}(\omega)$$

Twice subtracted, Borel transformed dispersion relation

$$\boxed{\Pi^{(V)}(0,n) - \frac{1}{\pi}\int_0^\infty ds\,\frac{\mathrm{Im}\Pi^{(V)}(s,n)}{s}e^{-s/M^2} = c_0 M^2 + \sum_{i=1}^{\infty}\frac{c_i}{(i-1)!M^{2(i-1)}}}$$

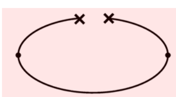
$dR_{\omega\to\pi^0\gamma}/d^4q \propto \Phi(q^2)\,\mathrm{Im}\Pi^\omega(q^2) \longleftrightarrow$ Operator product expansion

$$c_0 = \frac{1}{8\pi^2}\left(1+\frac{\alpha_s}{\pi}\right) \qquad c_1 = -\frac{3}{8\pi^2}(m_u^2+m_d^2)$$

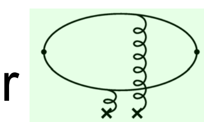


Wilson coefficients **x** **CONDENSATES** → $\mathbf{C_i}$

small

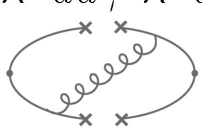


larger



$$c_2 = \frac{1}{2}\left(1+\frac{\alpha_s}{3\pi}\right)\left(m_u\langle\Psi|\bar{u}u|\Psi\rangle + m_d\langle\Psi|\bar{d}d|\Psi\rangle\right) + \frac{1}{24}\left\langle\Psi\left|\frac{\alpha_s}{\pi}G^2\right|\Psi\right\rangle + \ldots$$

$$\begin{aligned} c_3 = &-\frac{\pi}{2}\alpha_s\langle\Psi|\left(\bar{u}\gamma_\mu\gamma_5\lambda^A u\bar{u}\gamma^\mu\gamma_5\lambda^A u + \bar{d}\gamma_\mu\gamma_5\lambda^A d\bar{d}\gamma^\mu\gamma_5\lambda^A d\right)|\Psi\rangle - \pi\alpha_s\langle\Psi|\left(\bar{u}\gamma_\mu\gamma_5\lambda^A u\bar{d}\gamma^\mu\gamma_5\lambda^A d\right)|\Psi\rangle \\ &-\frac{\pi}{9}\alpha_s\langle\Psi|\left(\bar{u}\gamma_\mu\lambda^A u\bar{u}\gamma^\mu\lambda^A u + \bar{d}\gamma_\mu\lambda^A d\bar{d}\gamma^\mu\lambda^A d\right)|\Psi\rangle - \frac{2\pi}{9}\alpha_s\langle\Psi|\left(\bar{u}\gamma_\mu\lambda^A u\bar{d}\gamma^\mu\lambda^A d\right)|\Psi\rangle + \ldots \end{aligned}$$



FOUR-QUARK CONDENSATES

Catalog of Four-Quark Condensates

One flavor: $\langle \bar{q}\Gamma q \bar{q}\Gamma' q\rangle$ time, parity reversal invariance, …

$$\vec{y} = \begin{pmatrix} \langle \bar{q}\lambda^A q\bar{q}\lambda^A q\rangle \\ \langle \bar{q}\gamma_\alpha\lambda^A q\bar{q}\gamma^\alpha\lambda^A q\rangle \\ \langle \bar{q}\not{v}\lambda^A q\bar{q}\not{v}\lambda^A q\rangle / v^2 \\ \langle \bar{q}\sigma_{\alpha\beta}\lambda^A q\bar{q}\sigma^{\alpha\beta}\lambda^A q\rangle \\ \langle \bar{q}\sigma_{\alpha\beta}\lambda^A q\bar{q}\sigma^{\gamma\delta}\lambda^A q\rangle g^\alpha_\gamma v^\beta v_\delta / v^2 \\ \langle \bar{q}\gamma_5\gamma_\alpha\lambda^A q\bar{q}\gamma_5\gamma^\alpha\lambda^A q\rangle \\ \langle \bar{q}\gamma_5\not{v}\lambda^A q\bar{q}\gamma_5\not{v}\lambda^A q\rangle / v^2 \\ \langle \bar{q}\gamma_5\lambda^A q\bar{q}\gamma_5\lambda^A q\rangle \\ \langle \bar{q}\not{v}\lambda^A q\bar{q}\lambda^A q\rangle \\ \langle \bar{q}\gamma_5\gamma^\alpha\lambda^A q\bar{q}\gamma_5\sigma_{\alpha\beta}\lambda^A q\rangle v^\beta \end{pmatrix} \quad \xleftrightarrow{\text{FIERZ}} \quad \vec{x} = \begin{pmatrix} \langle \bar{q}q\bar{q}q\rangle \\ \langle \bar{q}\gamma_\alpha q\bar{q}\gamma^\alpha q\rangle \\ \langle \bar{q}\not{v}q\bar{q}\not{v}q\rangle / v^2 \\ \langle \bar{q}\sigma_{\alpha\beta}q\bar{q}\sigma^{\alpha\beta}q\rangle \\ \langle \bar{q}\sigma_{\alpha\beta}q\bar{q}\sigma^{\gamma\delta}q\rangle g^\alpha_\gamma v^\beta v_\delta / v^2 \\ \langle \bar{q}\gamma_5\gamma_\alpha q\bar{q}\gamma_5\gamma^\alpha q\rangle \\ \langle \bar{q}\gamma_5\not{v}q\bar{q}\gamma_5\not{v}q\rangle / v^2 \\ \langle \bar{q}\gamma_5 q\bar{q}\gamma_5 q\rangle \\ \langle \bar{q}\not{v}q\bar{q}q\rangle \\ \langle \bar{q}\gamma_5\gamma^\alpha q\bar{q}\gamma_5\sigma_{\alpha\beta}q\rangle v^\beta \end{pmatrix}$$

$$\vec{y} = \hat{A}\vec{x}$$

Vacuum: 5
Medium: 5+5

$$\vec{z} = \frac{2}{3}\left(\vec{x} - \frac{3}{4}\vec{y}\right) = \frac{2}{3}\underline{\left(\mathbb{1} - \frac{3}{4}\hat{A}\right)}\vec{x} = \frac{2}{3}\left(\hat{A}^{-1} - \frac{3}{4}\mathbb{1}\right)\vec{y}$$

no inverse:

nucleon $\not\leftrightarrow$ ω

$$\hat{A} = \begin{pmatrix} -7/6 & -1/2 & 0 & -1/4 & 0 & 1/2 & 0 & -1/2 & 0 & 0 \\ -2 & 1/3 & 0 & 0 & 0 & 1 & 0 & 2 & 0 & 0 \\ -1/2 & 1/2 & -5/3 & -1/4 & 1 & 1/2 & -1 & 1/2 & 0 & 0 \\ -6 & 0 & 0 & 1/3 & 0 & 0 & 0 & -6 & 0 & 0 \\ -3/2 & -1/2 & 2 & 1/4 & -2/3 & 1/2 & -2 & -3/2 & 0 & 0 \\ 2 & 1 & 0 & 0 & 0 & 1/3 & 0 & -2 & 0 & 0 \\ 1/2 & 1/2 & -1 & 1/4 & -1 & 1/2 & -5/3 & -1/2 & 0 & 0 \\ -1/2 & 1/2 & 0 & -1/4 & 0 & -1/2 & 0 & -7/6 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -5/3 & -i \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 3i & 1/3 \end{pmatrix}$$

but implies relations:

$$z_2 + z_6 = 0$$
$$4z_1 - 2z_2 - z_4 = 0$$
$$2z_1 - z_4 + 2z_8 = 0$$
$$z_1 - z_3 - z_5 + z_7 = 0$$
$$z_9 - iz_{10} = 0$$

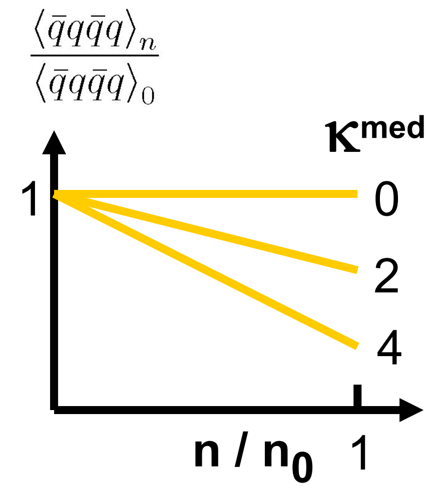
Two flavors: $\langle \bar{u}\Gamma u\bar{d}\Gamma' d\rangle$

Vacuum: 2•5 +10 = 20
Medium: 2•10+24 = 44

→ *flavor symmetry: 10 (20)*

Four-Quark Condensates

Factorization and beyond:



$$\boxed{\langle \bar{q}\Gamma q \bar{q}\Gamma q\rangle = f_\Gamma \langle \bar{q}q\rangle^2_{vac}\, \kappa^{vac}_\Gamma \left[1 + \frac{\kappa^{med}_\Gamma}{\kappa^{vac}_\Gamma} \frac{\sigma_N}{m_q \langle \bar{q}q\rangle_{vac}} n\right]}$$

$$\langle \bar{q}q\rangle = \langle \bar{q}q\rangle_{\mathrm{vac}} + \frac{\sigma_N}{2m_q} n$$

Chiral Symmetry: $U(1)_V \quad \times \quad SU(n_f)_V \quad \times \quad U(1)_A \quad \times \quad SU(n_f)_A$

$\langle \bar{\psi}\psi\rangle$ ***not*** $SU(n_f)_A$ invariant ⟶ order parameter $\quad \Psi' = \exp(i\beta_a T_a \gamma_5)\Psi$

V-A $\quad \langle \mathcal{O}_{4q}\rangle_{V-A} = \langle (\bar{u}\gamma_\mu\gamma_5\lambda_A u - \bar{d}\gamma_\mu\gamma_5\lambda_A d)^2 - (\bar{u}\gamma_\mu\lambda_A u - \bar{d}\gamma_\mu\lambda_A d)^2 \rangle$ not invariant

ω $\quad \left[\langle \bar{\psi}\gamma_5\gamma_\mu\lambda^A\psi\bar{\psi}\gamma_5\gamma^\mu\lambda^A\psi\rangle + \frac{2}{9}\langle \bar{\psi}\gamma_\mu\lambda^A\psi\bar{\psi}\gamma^\mu\lambda^A\psi\rangle\right]$ invariant

N (vacuum)

$$[\langle \bar{\psi}\gamma_5\gamma_\mu\psi\bar{\psi}\gamma_5\gamma^\mu\psi\rangle + 3\langle \bar{\psi}\gamma_\mu\psi\bar{\psi}\gamma^\mu\psi\rangle]$$
$$-\frac{3}{4}[\langle \bar{\psi}\gamma_5\gamma_\mu\lambda^A\psi\bar{\psi}\gamma_5\gamma^\mu\lambda^A\psi\rangle + 3\langle \bar{\psi}\gamma_\mu\lambda^A\psi\bar{\psi}\gamma^\mu\lambda^A\psi\rangle]$$

invariant

Nucleon in medium

$$\eta(x) = \epsilon_{abc}[u_a^T(x) C\gamma_\mu u_b(x)]\gamma_5\gamma^\mu d_c(x)$$

RT, Hilger, Kämpfer, NPA (2007)

Weighted dispersion relation

condensates

$$\boxed{\begin{aligned}(E-\bar{E})\frac{1}{\pi}\int_0^{\omega_+} d\omega \Delta\Pi(\omega) e^{-\omega^2/\mathcal{M}^2} &= \tilde{\Pi}^{\rm e}(\mathcal{M}^2) - \frac{1}{\pi}\int_{\omega_+}^{\infty} d\omega\,\omega \Pi^{\rm e}_{\rm per}(\omega) e^{-\omega^2/\mathcal{M}^2}\\ -\bar{E}\left\{\tilde{\Pi}^{\rm o}(\mathcal{M}^2) - \frac{1}{\pi}\int_{\omega_+}^{\infty} d\omega \Pi^{\rm o}_{\rm per}(\omega) e^{-\omega^2/\mathcal{M}^2}\right\} &+ \frac{1}{\pi}\int_{\omega_-}^{-\omega_+} d\omega \Delta\Pi(\omega)[\omega - \bar{E}] e^{-\omega^2/\mathcal{M}^2}\end{aligned}}$$

condensates

moments: $$E = \frac{\int_0^{\omega_+} d\omega \Delta\Pi(\omega)\omega e^{-\omega^2/\mathcal{M}^2}}{\int_0^{\omega_+} d\omega \Delta\Pi(\omega) e^{-\omega^2/\mathcal{M}^2}} \qquad \bar{E} = \frac{\int_{\omega_-}^{0} d\omega \Delta\Pi(\omega)\omega e^{-\omega^2/\mathcal{M}^2}}{\int_{\omega_-}^{0} d\omega \Delta\Pi(\omega) e^{-\omega^2/\mathcal{M}^2}}$$

Parametrization of hadronic side:

$$(E_+ - E_-)\frac{1}{\pi}\int_0^{\omega_+} d\omega \Delta\Pi(\omega) e^{-\omega^2/\mathcal{M}^2} = -\frac{\lambda_N^2}{1-\Sigma_q}(\not{q} + M_N^* - \not{v}\Sigma_v) e^{-E_+^2/\mathcal{M}^2}$$

$\rightarrow$ *Sum rules in pole ansatz:* **Furnstahl, Griegel, Cohen, PRC (1992)**

$$\Delta G(q_0) = \frac{\pi}{1-\Sigma_q}\frac{\not{q} + M_N^* - \not{v}\Sigma_v}{E_+ - E_-}\left(\delta(q_0 - E_-) - \delta(q_0 - E_+)\right)$$

Analysis: Nucleon in medium

$$\Pi(q) = \Pi_s(q^2, qv)\mathbb{1} + \Pi_q(q^2, qv)\not{q} + \Pi_v(q^2, qv)\not{v}$$

four-quark condensates

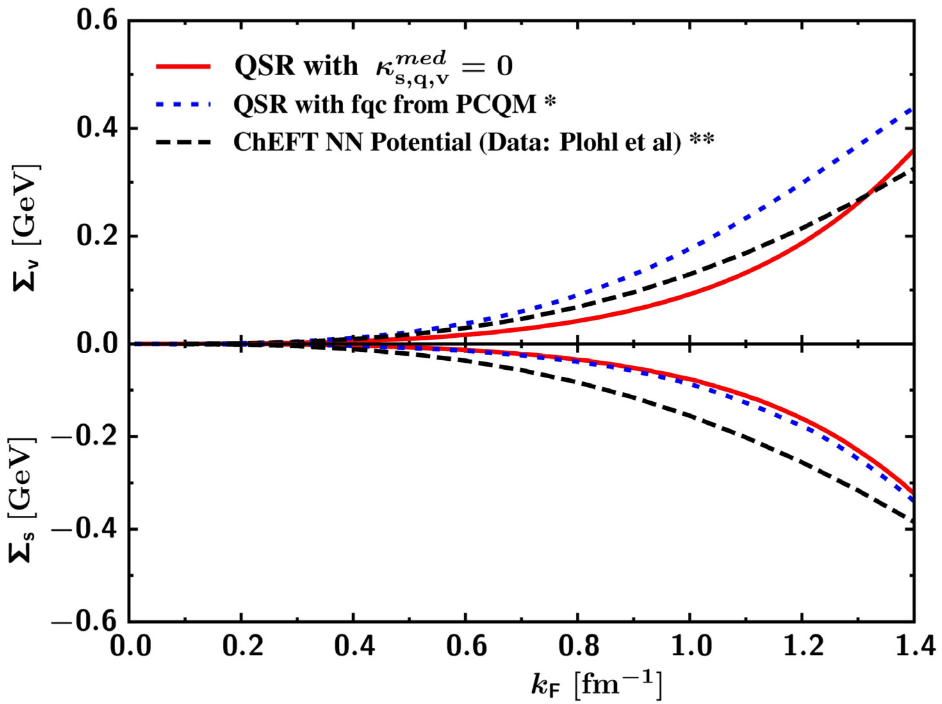
$\kappa_{\mathrm{s}}^{\mathrm{med}}$ $\kappa_{\mathrm{q}}^{\mathrm{med}}$ $\tilde{\kappa}_{\mathrm{v}}^{\mathrm{med}}$

3 coupled sum rule equations:

$$-\lambda_N^{*2} M_N^* e^{-E_+^2/\mathcal{M}^2} = (E_+ - E_-)\frac{1}{\pi}\int_0^{\omega_+} d\omega \Delta\Pi_s(\omega) e^{-\omega^2/\mathcal{M}^2}$$

$$-\lambda_N^{*2} e^{-E_+^2/\mathcal{M}^2} = (E_+ - E_-)\frac{1}{\pi}\int_0^{\omega_+} d\omega \Delta\Pi_q(\omega) e^{-\omega^2/\mathcal{M}^2}$$

$$\lambda_N^{*2}\Sigma_v e^{-E_+^2/\mathcal{M}^2} = (E_+ - E_-)\frac{1}{\pi}\int_0^{\omega_+} d\omega \Delta\Pi_v(\omega) e^{-\omega^2/\mathcal{M}^2}$$

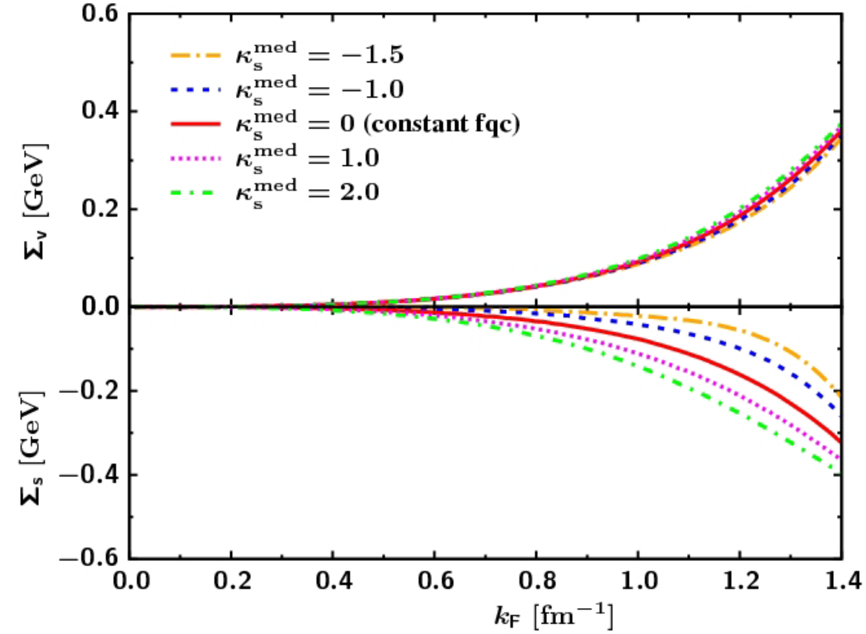


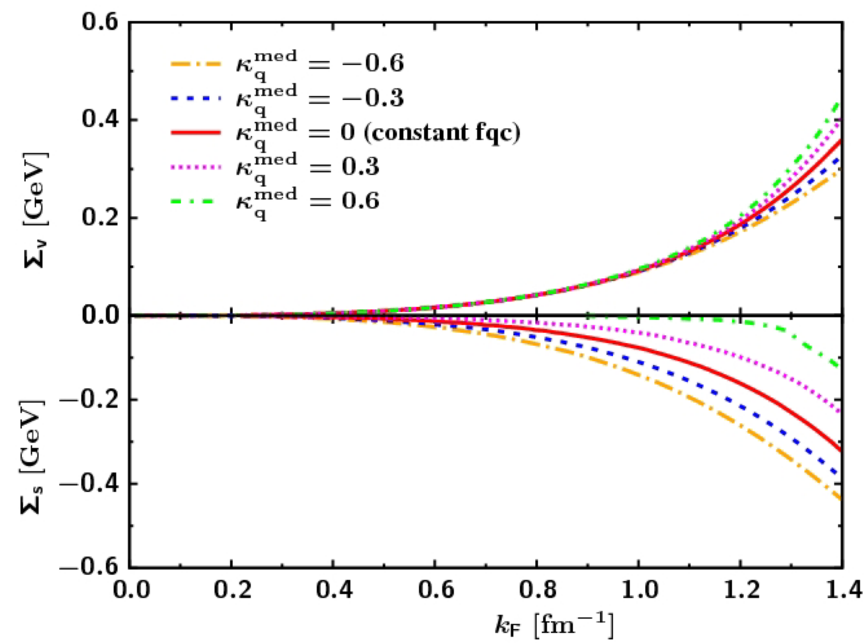
* Drukarev et al, PRD (2003)

** Plohl, Fuchs, PRC (2006)

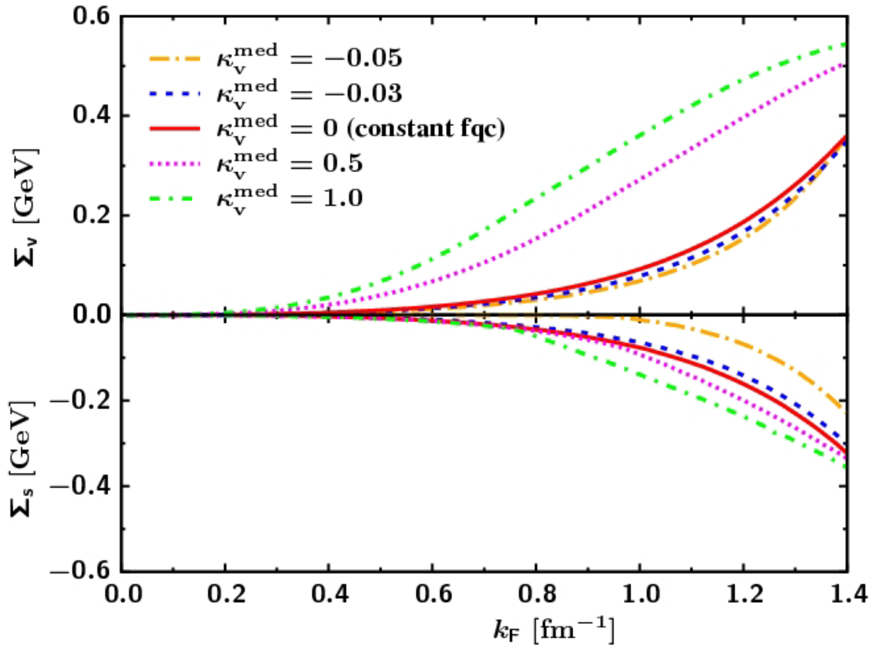
Numerical impact of density dependence of combined four-quark condensates:

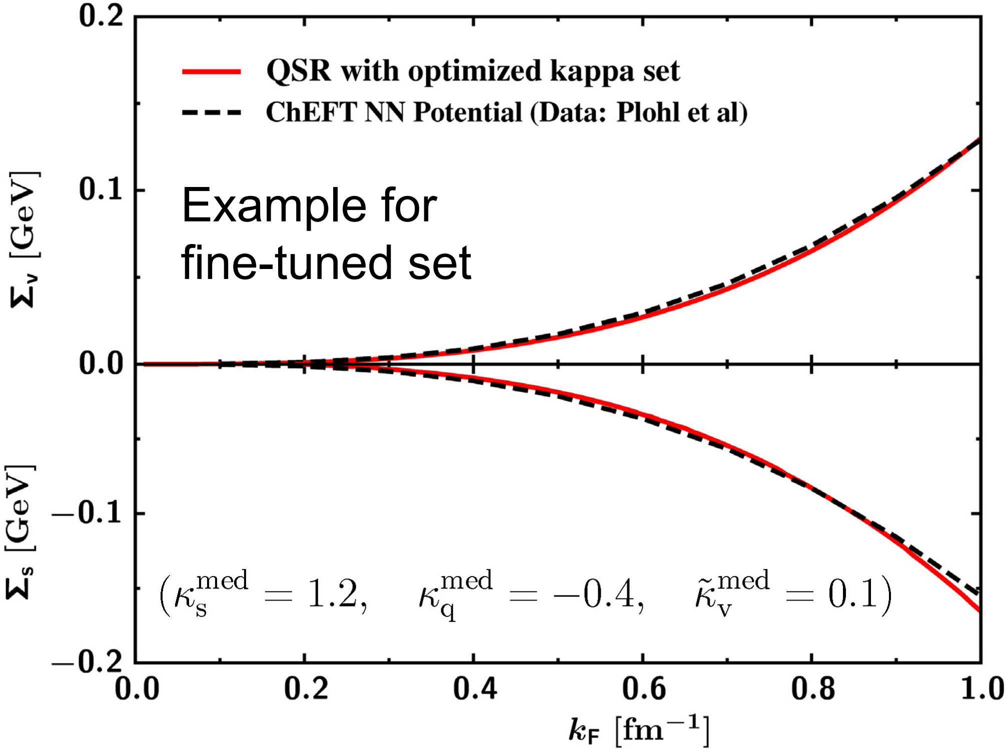
Scalar self-energy





Vector self-energy





Vector, scalar self-energies

Analytical estimates:

$$\Sigma_v \sim (0.16 + 1.22\tilde{\kappa}_{\mathrm{v}}^{\mathrm{med}})\ \mathrm{GeV}\frac{n}{n_0}$$

opposite effects

$$\Sigma_s \sim -(0.32 + 0.11\kappa_{\mathrm{s}}^{\mathrm{med}} - 0.31\kappa_{\mathrm{q}}^{\mathrm{med}})\ \mathrm{GeV}\frac{n}{n_0}$$

In-medium Ioffe approximation:

$$\Sigma_v \sim 0.36\ \mathrm{GeV}\frac{n}{n_0}$$

$$\Sigma_s \sim -0.37\ \mathrm{GeV}\frac{n}{n_0}$$

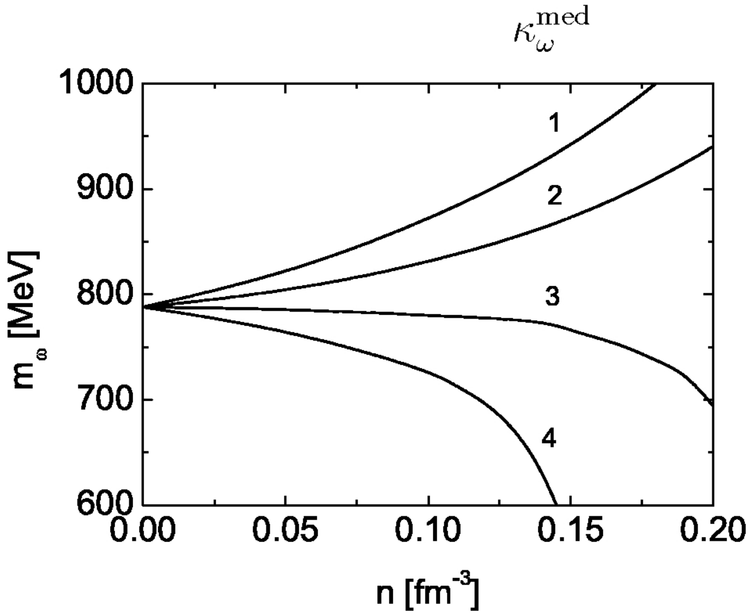
$$\kappa_{\mathrm{s}}^{\mathrm{med}} \langle \bar{q}q \rangle_{\mathrm{vac}} \frac{3}{2} n = \langle \bar{u}\not{v}u\bar{d}d \rangle + \frac{1}{2} \langle \bar{u}\gamma_5\gamma_\kappa u \bar{d}\sigma_{\lambda\pi} d \epsilon^{\kappa\lambda\pi\xi} v_\xi \rangle - \frac{3}{4}\left[\text{color structures with } \lambda^A\right]$$

$$\begin{aligned}
\kappa_{\mathrm{q}}^{\mathrm{vac}} \langle \bar{q}q \rangle_{\mathrm{vac}}^2 + \kappa_{\mathrm{q}}^{\mathrm{med}} \langle \bar{q}q \rangle_{\mathrm{vac}} \frac{\sigma_{\mathrm{N}}}{m_{\mathrm{q}}} n = \frac{2}{3} \big(&\langle \bar{u}\gamma_\tau u \bar{u}\gamma^\tau u \rangle - \langle \bar{u}\not{v}u\bar{u}\not{v}u/v^2 \rangle - \langle \bar{u}\gamma_5\gamma_\tau u \bar{u}\gamma_5\gamma^\tau u \rangle + \langle \bar{u}\gamma_5\not{v}u\bar{u}\gamma_5\not{v}u/v^2 \rangle \\
&+ 4 \langle \bar{u}\gamma_\tau u \bar{d}\gamma^\tau d \rangle - \langle \bar{u}\not{v}u\bar{d}\not{v}d/v^2 \rangle + 2 \langle \bar{u}\gamma_5\gamma_\tau u \bar{d}\gamma_5\gamma^\tau d \rangle + \langle \bar{u}\gamma_5\not{v}u\bar{d}\gamma_5\not{v}d/v^2 \rangle \big) \\
&- \frac{3}{4}\left[\text{color structures with } \lambda^A\right]
\end{aligned}$$

$$\begin{aligned}
\tilde{\kappa}_{\mathrm{v}}^{\mathrm{med}} \langle \bar{q}q \rangle_{\mathrm{vac}} \frac{\sigma_{\mathrm{N}}}{m_{\mathrm{q}}} n = \frac{2}{3} \Big(&-\frac{1}{4} \langle \bar{u}\gamma_\tau u \bar{u}\gamma^\tau u \rangle + \langle \bar{u}\not{v}u\bar{u}\not{v}u/v^2 \rangle + \frac{1}{4} \langle \bar{u}\gamma_5\gamma_\tau u \bar{u}\gamma_5\gamma^\tau u \rangle - \langle \bar{u}\gamma_5\not{v}u\bar{u}\gamma_5\not{v}u/v^2 \rangle \\
&- \frac{1}{4} \langle \bar{u}\gamma_\tau u \bar{d}\gamma^\tau d \rangle + \langle \bar{u}\not{v}u\bar{d}\not{v}d/v^2 \rangle + \frac{1}{4} \langle \bar{u}\gamma_5\gamma_\tau u \bar{d}\gamma_5\gamma^\tau d \rangle - \langle \bar{u}\gamma_5\not{v}u\bar{d}\gamma_5\not{v}d/v^2 \rangle \Big) \\
&- \frac{3}{4}\left[\text{color structures with } \lambda^A\right]
\end{aligned}$$

N $\not\Leftrightarrow$ **ω**

Comparison: ω in medium



Zschocke, Pavlenko, Kämpfer, PLB (2003)

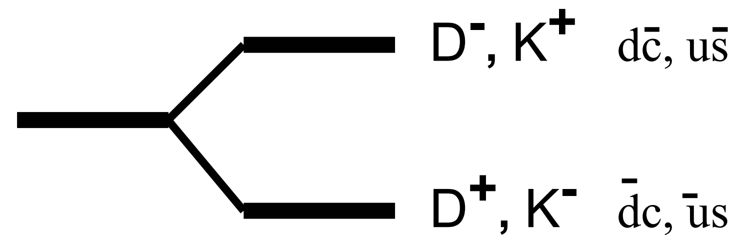
$$\kappa_\omega^{\mathrm{vac}} \langle \bar{q}q \rangle_{\mathrm{vac}}^2 + \kappa_\omega^{\mathrm{med}} \langle \bar{q}q \rangle_{\mathrm{vac}} \frac{\sigma_{\mathrm{N}}}{m_{\mathrm{q}}} n = \frac{81}{112} \Bigg(\frac{1}{2} \langle \bar{u}\gamma_5\gamma_\mu\lambda^A u \bar{u}\gamma_5\gamma^\mu\lambda^A u \rangle + \frac{1}{2} \langle \bar{d}\gamma_5\gamma_\mu\lambda^A d \bar{d}\gamma_5\gamma^\mu\lambda^A d \rangle$$
$$+ \langle \bar{u}\gamma_5\gamma_\mu\lambda^A u \bar{d}\gamma_5\gamma^\mu\lambda^A d \rangle + \frac{2}{9} \langle \bar{u}\gamma_\mu\lambda^A u \bar{d}\gamma^\mu\lambda^A d \rangle$$
$$+ \frac{1}{9} \langle \bar{u}\gamma_\mu\lambda^A u \bar{u}\gamma^\mu\lambda^A u \rangle + \frac{1}{9} \langle \bar{d}\gamma_\mu\lambda^A d \bar{d}\gamma^\mu\lambda^A d \rangle \Bigg)$$

Open Charm: D

CBM and PANDA @ FAIR

D^+ similar to K^- ← **-** 80 MeV at saturation density

Scheinast et al (KaoS), PRL (2006)



D meson in medium:

$$\frac{q_0^4}{\pi}\int_0^\infty \frac{ds}{s^2}\frac{1}{s-q_0^2}\left(\mathrm{Im}\Pi_+(s)+\mathrm{Im}\Pi_-(s)\right)+subtractions=\mathrm{Re}\Pi^e_{\mathrm{OPE}}(q_0)$$

$$\frac{q_0^5}{\pi}\int_0^\infty \frac{ds}{s^{5/2}}\frac{1}{s-q_0^2}\left(\mathrm{Im}\Pi_+(s)-\mathrm{Im}\Pi_-(s)\right)+subtractions=\mathrm{Re}\Pi^o_{\mathrm{OPE}}(q_0)$$

$(m_{D^-}-m_{D^+})$ $\qquad$ $\frac{m_{D^-}+m_{D^+}}{2}$

Morath, Weise (2001) $\qquad$ Hayashigaki (2000)

↓ $\qquad$ ↓

~ 30 … 50 MeV $\qquad$ ~ **-** (0 … 50) MeV $\qquad$ at saturation density

Zschocke (2005) $\qquad$ $\langle q^\dagger g_s \sigma G q\rangle$ ***?***

Example of Operator Product Expansion for D-meson:

amplifier

$$\Pi_{D^+}^{(2d)e}(\omega^2) = \langle \bar{d}d \rangle \frac{m_c}{\omega^2 - m_c^2} + \langle \frac{\alpha_s}{\pi} G^2 \rangle \frac{1}{12} \left\{ \frac{\frac{m_c}{m_d} - 1}{\omega^2 - m_c^2} + \frac{1}{2} \frac{\omega^2}{(\omega^2 - m_c^2)^2} \right\}$$
$$+ \langle \bar{d}\gamma_0 i D_0 d \rangle 2 \frac{\omega^2}{(\omega^2 - m_c^2)^2} - \langle \bar{d} D_0 D_0 d \rangle 4 \frac{m_c \omega^2}{(\omega^2 - m_c^2)^3}$$
$$+ \langle \bar{d} g \sigma \mathscr{G} d \rangle \frac{m_c}{2} \frac{1}{(\omega^2 - m_c^2)^2}$$
$$+ \langle \frac{\alpha_s}{\pi} \left[(vG)^2 - \frac{G^2}{4} \right] \rangle \frac{1}{3} \left(\ln \frac{\mu^2}{m_d^2} - \frac{1}{3} \right) \frac{\omega^2}{(\omega^2 - m_c^2)^2}$$

$$\frac{1}{\omega} \Pi_{D^+}^{(2d)o}(\omega) = - \langle \bar{d}\gamma_0 d \rangle \frac{1}{\omega^2 - m_c^2} + \langle \bar{d}\gamma_0 D_0 D_0 d \rangle 4 \frac{\omega^2}{(\omega^2 - m_c^2)^3}$$
$$- \langle \bar{d}\gamma_0 g \sigma \mathscr{G} d \rangle \frac{1}{2} \frac{1}{(\omega^2 - m_c^2)^2}$$

mixing of QCD condensates
under renormalization

(Diploma thesis T. Hilger)

Conclusion

- catalog of four-quark condensates

- combinations of in-medium four-quark condensates:

  ω → strong density dependence
  N → some density dependence

  Modified QCD vacuum

- different sum rules
  → no factorization of combined four-quark condensates
  → combinations like e.g. $V - A \sim \langle \bar{q}q\bar{q}q \rangle$ may serve as further order parameters of chiral symmetry breaking

- further hadrons probe other condensates: D ⟶ CBM, PANDA

qqq: Nucleon

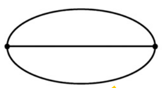
- interpolating field not unique
- inclusion of derivatives

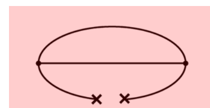
$$\eta'_G = \epsilon_{abc}[u_a^T C \gamma_\mu u_b]\gamma_5 \gamma^\mu \sigma_{\alpha\beta}[G^{\alpha\beta} d]_c$$

Braun, Schäfer et al, PLB (1993)

$\boxed{t = -1}$ Ioffe interpolating field:

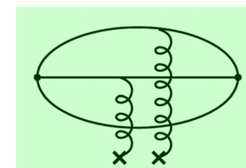
$$\boxed{\eta(x) = \epsilon_{abc}[u_a^T(x) C \gamma_\mu u_b(x)]\gamma_5 \gamma^\mu d_c(x)}$$

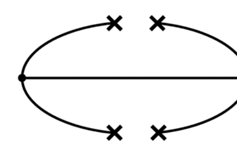




c_i:

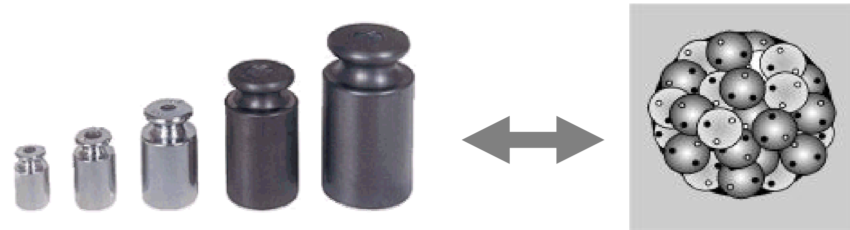
$$\Pi^{(uud)}(q) = \frac{1}{64\pi^4} q^4 \ln(-q^2)\not{q} - \frac{1}{4\pi^2} q^2 \ln(-q^2)\,\langle \bar{q}q\rangle + \frac{1}{32\pi^2}\ln(-q^2)\not{q}\left\langle \frac{\alpha_s}{\pi} G^2 \right\rangle$$
$$+ \frac{2}{3q^2}\not{q}\Bigg(\frac{1}{2}\cdot\left(\langle \bar{u}u\bar{u}u\rangle - \tfrac{3}{4}\langle \bar{u}\lambda^A u\bar{u}\lambda^A u\rangle\right)$$
$$+ \frac{1}{4}\cdot\left(\langle \bar{u}\gamma^\mu u\bar{u}\gamma_\mu u\rangle - \tfrac{3}{4}\langle \bar{u}\gamma^\mu\lambda^A u\bar{u}\gamma_\mu\lambda^A u\rangle\right)$$
$$- \frac{1}{4}\cdot\left(\langle \bar{u}\gamma_5\gamma^\mu u\bar{u}\gamma_5\gamma_\mu u\rangle - \tfrac{3}{4}\langle \bar{u}\gamma_5\gamma^\mu\lambda^A u\bar{u}\gamma_5\gamma_\mu\lambda^A u\rangle\right)$$
$$- \frac{1}{2}\cdot\left(\langle \bar{u}\gamma_5 u\bar{u}\gamma_5 u\rangle - \tfrac{3}{4}\langle \bar{u}\gamma_5\lambda^A u\bar{u}\gamma_5\lambda^A u\rangle\right)$$
$$+ \frac{5}{2}\cdot\left(\langle \bar{u}\gamma^\mu u\bar{d}\gamma_\mu d\rangle - \tfrac{3}{4}\langle \bar{u}\gamma^\mu\lambda^A u\bar{d}\gamma_\mu\lambda^A d\rangle\right)$$
$$+ \frac{3}{2}\cdot\left(\langle \bar{u}\gamma_5\gamma^\mu u\bar{d}\gamma_5\gamma_\mu d\rangle - \tfrac{3}{4}\langle \bar{u}\gamma_5\gamma^\mu\lambda^A u\bar{d}\gamma_5\gamma_\mu\lambda^A d\rangle\right)\Bigg)$$



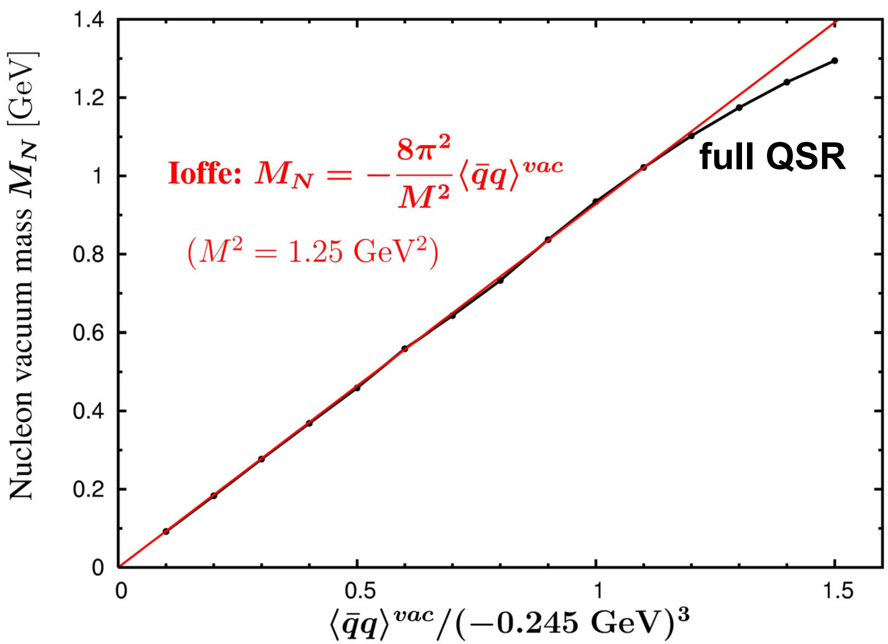


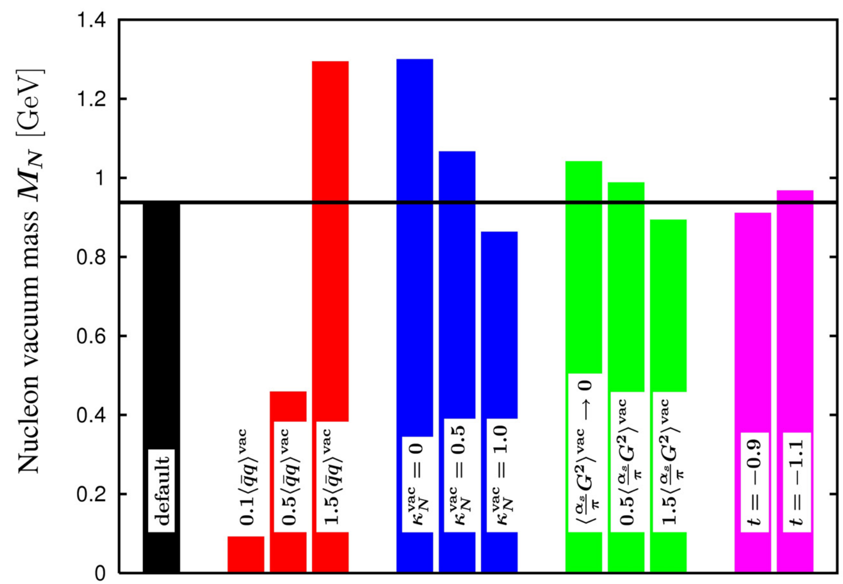
(for vacuum)

Nucleon in vacuum



Chiral condensate





QCD Sum Rules: 3 equations

$$\lambda_N^{*2} M_N^* e^{-(E_q^2-\vec{q}^{\,2})/\mathcal{M}^2} = a_1\mathcal{M}^4 + a_2\mathcal{M}^2 + a_3$$

$$\lambda_N^{*2} e^{-(E_q^2-\vec{q}^{\,2})/\mathcal{M}^2} = b_0\mathcal{M}^6 + b_2\mathcal{M}^2 + b_3 + b_4/\mathcal{M}^2$$

$$\lambda_N^{*2} \Sigma_v e^{-(E_q^2-\vec{q}^{\,2})/\mathcal{M}^2} = c_1\mathcal{M}^4 + c_2\mathcal{M}^2 + c_3$$

$$a_1 = -\frac{1}{4\pi^2} E_1 \langle \bar{q}q\rangle_{\rm med}$$

$$a_2 = -\frac{1}{2\pi^2} E_0 \bar{E}_q \langle \bar{q} iD_0 q\rangle_{\rm med}$$

$$a_3 = -\frac{4}{3\pi^2}\vec{q}^{\,2}\left(\langle \bar{q} iD_0 iD_0 q\rangle_{\rm med} + \frac{1}{8}\langle g_s \bar{q}\sigma G q\rangle_{\rm med}\right) - \frac{4}{3}\bar{E}_q \left[\tilde{\kappa}_{\rm s}^{\rm vac}\langle\bar{q}q\rangle_{\rm vac}^2 + \kappa_{\rm s}^{\rm med}\langle\bar{q}q\rangle_{\rm vac} 3n/2\right]$$

$$b_0 = \frac{1}{32\pi^4}E_2$$

$$b_2 = \frac{E_0}{3\pi^2}\bar{E}_q\left\langle q^\dagger q\right\rangle_{\rm med} - \frac{5E_0}{9\pi^2}\left\langle q^\dagger iD_0 q\right\rangle_{\rm med} - \frac{E_0}{16\pi^2}\left\langle \frac{\alpha_s}{\pi}(\vec{E}^2 - \vec{B}^2)\right\rangle_{\rm med} - \frac{E_0}{144\pi^2}\left\langle \frac{\alpha_s}{\pi}(\vec{E}^2 + \vec{B}^2)\right\rangle_{\rm med}$$

$$b_3 = \frac{8\vec{q}^{\,2}}{9\pi^2}\left\langle q^\dagger iD_0 q\right\rangle_{\rm med} + \frac{\vec{q}^{\,2}}{36\pi^2}\left\langle \frac{\alpha_s}{\pi}(\vec{E}^2 + \vec{B}^2)\right\rangle_{\rm med} + \frac{\bar{E}_q}{18\pi^2}\left\langle g_s q^\dagger \sigma G q\right\rangle_{\rm med}$$
$$- 2\bar{E}_q\left(\left\langle q^\dagger iD_0 iD_0 q\right\rangle_{\rm med} + \frac{1}{12}\left\langle g_s q^\dagger\sigma G q\right\rangle_{\rm med}\right) + \frac{2}{3}\left[\kappa_{\rm q}^{\rm vac}\langle\bar{q}q\rangle_{\rm vac}^2 + \kappa_{\rm q}^{\rm med}\langle\bar{q}q\rangle_{\rm vac}\sigma_{\rm N} n/m_{\rm q}\right]$$

$$b_4 = \frac{4}{3\pi^2}\vec{q}^{\,2}\left(\left\langle q^\dagger iD_0 iD_0 q\right\rangle_{\rm med} + \frac{1}{12}\left\langle g_s q^\dagger\sigma G q\right\rangle_{\rm med}\right)$$

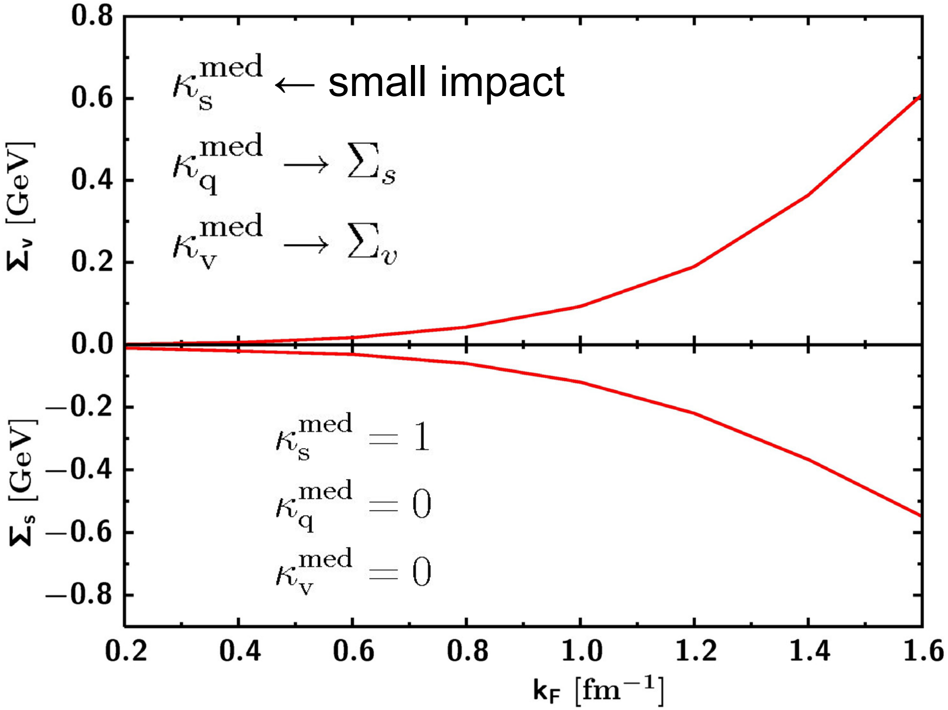
$$c_1 = \frac{2}{3\pi^2}E_1\left\langle q^\dagger q\right\rangle_{\rm med}$$

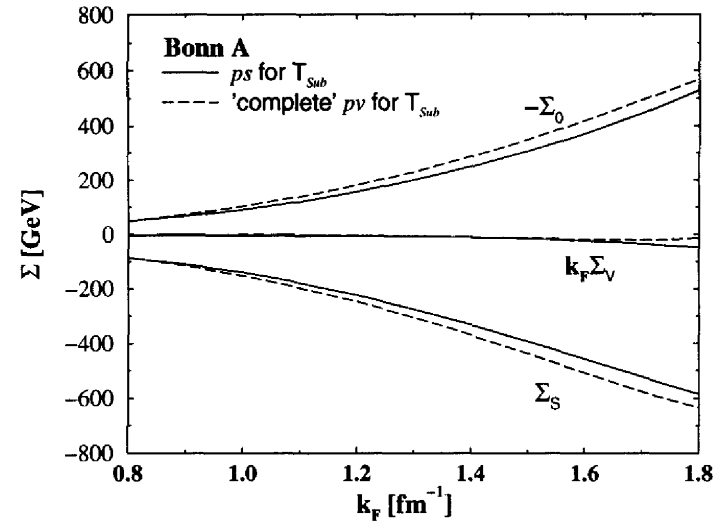
$$c_2 = \frac{20}{9\pi^2}E_0\bar{E}_q\left\langle q^\dagger iD_0 q\right\rangle_{\rm med} + \frac{E_0}{36\pi^2}\bar{E}_q\left\langle \frac{\alpha_s}{\pi}(\vec{E}^2 + \vec{B}^2)\right\rangle_{\rm med} - \frac{E_0}{12\pi^2}\left\langle g_s q^\dagger\sigma G q\right\rangle_{\rm med}$$

$$c_3 = \frac{4}{\pi^2}\vec{q}^{\,2}\left(\left\langle q^\dagger iD_0 iD_0 q\right\rangle_{\rm med} + \frac{1}{12}\left\langle g_s q^\dagger\sigma G q\right\rangle_{\rm med}\right) + \frac{8}{3}\bar{E}_q\left[\tilde{\kappa}_{\rm v}^{\rm vac}\langle\bar{q}q\rangle_{\rm vac}^2 + \tilde{\kappa}_{\rm v}^{\rm med}\langle\bar{q}q\rangle_{\rm vac}\sigma_{\rm N} n/m_{\rm q}\right]$$

$$-\frac{M_N^*}{1-\Sigma_q}e^{-E^2/\mathcal{M}^2} = (E-\bar{E})\frac{1}{\pi}\int_0^{\omega_+} d\omega \Delta\Pi_s(\omega)e^{-\omega^2/\mathcal{M}^2}$$
$$-\frac{1}{1-\Sigma_q}e^{-E^2/\mathcal{M}^2} = (E-\bar{E})\frac{1}{\pi}\int_0^{\omega_+} d\omega \Delta\Pi_q(\omega)e^{-\omega^2/\mathcal{M}^2}$$
$$\frac{\Sigma_v}{1-\Sigma_q}e^{-E^2/\mathcal{M}^2} = (E-\bar{E})\frac{1}{\pi}\int_0^{\omega_+} d\omega \Delta\Pi_v(\omega)e^{-\omega^2/\mathcal{M}^2}$$

3 coupled sum rule equations





Gross-Boelting, Fuchs, Fäßler, NPA (1999)

→ **weak density dependence of specific four-quark condensate combinations**

Drukarev, Fäßler et al, PRC (2004):

- nucleon matrix elements of four-quark operators for the nucleon structures

→ weak density dependence

$$\langle \bar{q}q\bar{q}q\rangle_N \not\leftrightarrow \langle \bar{q}q\bar{q}q\rangle_\omega$$

Lattice: Göckeler, Schäfer et al, NPB (2002)

Analysis: Nucleon in medium

Threshold dependence