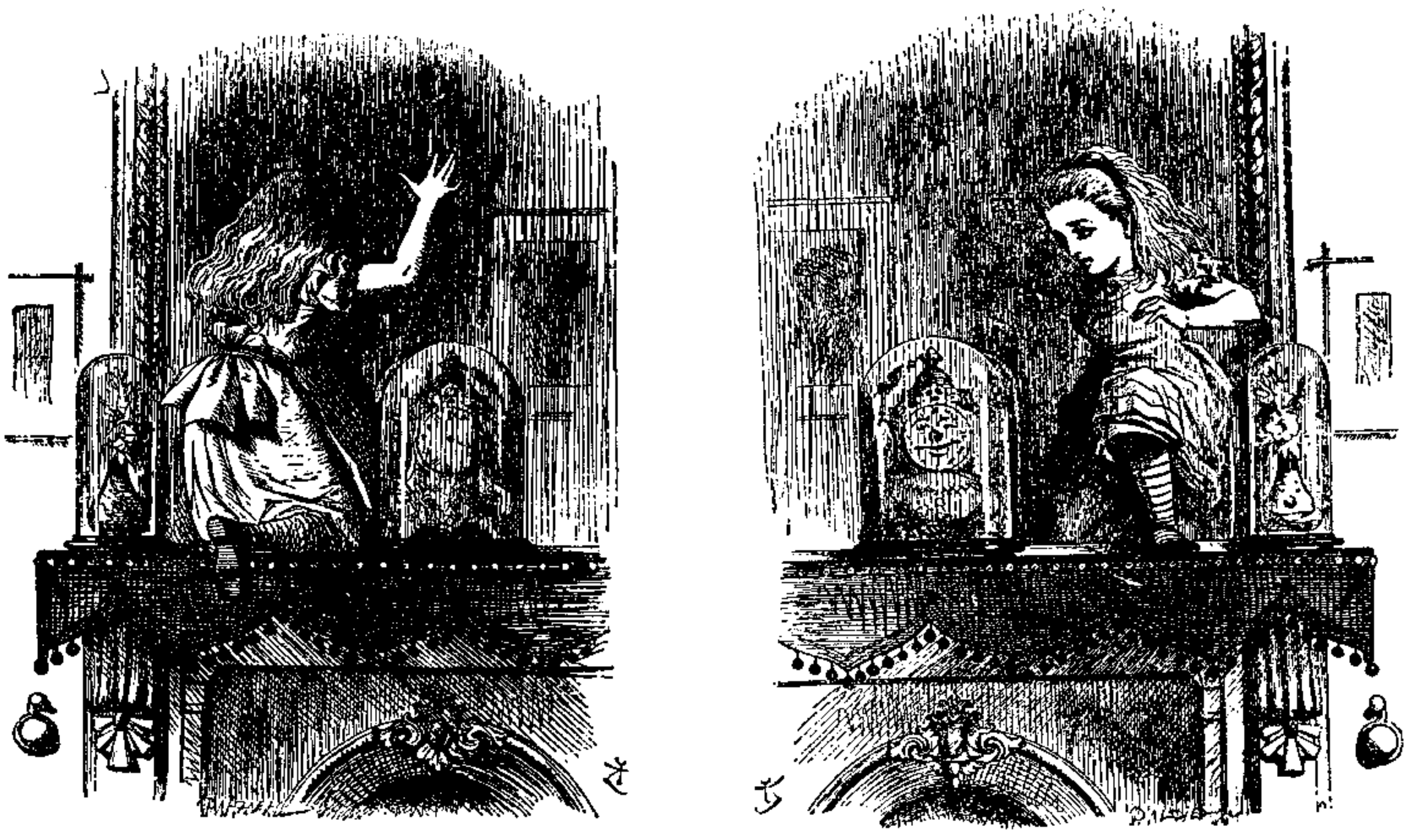
Symmetries, Dark Matter, Axion

- The Fundamental Symmetries of Parity (mirror reflection) and Charge conjugation (matter-antimatter)
- P and C in Electromagnetism: why are they conserved?
- P and CP in **QCD**: Why aren't they violated?
- The Axion: a possible explanation
- The Axion, Cosmology, and the puzzle of Dark Matter
- Relating the Dark Matter abundance to the Axion particle's mass (Still Work in Progress!)

Parity



Physics as seen in a looking glass (mirror) (Spiegel)

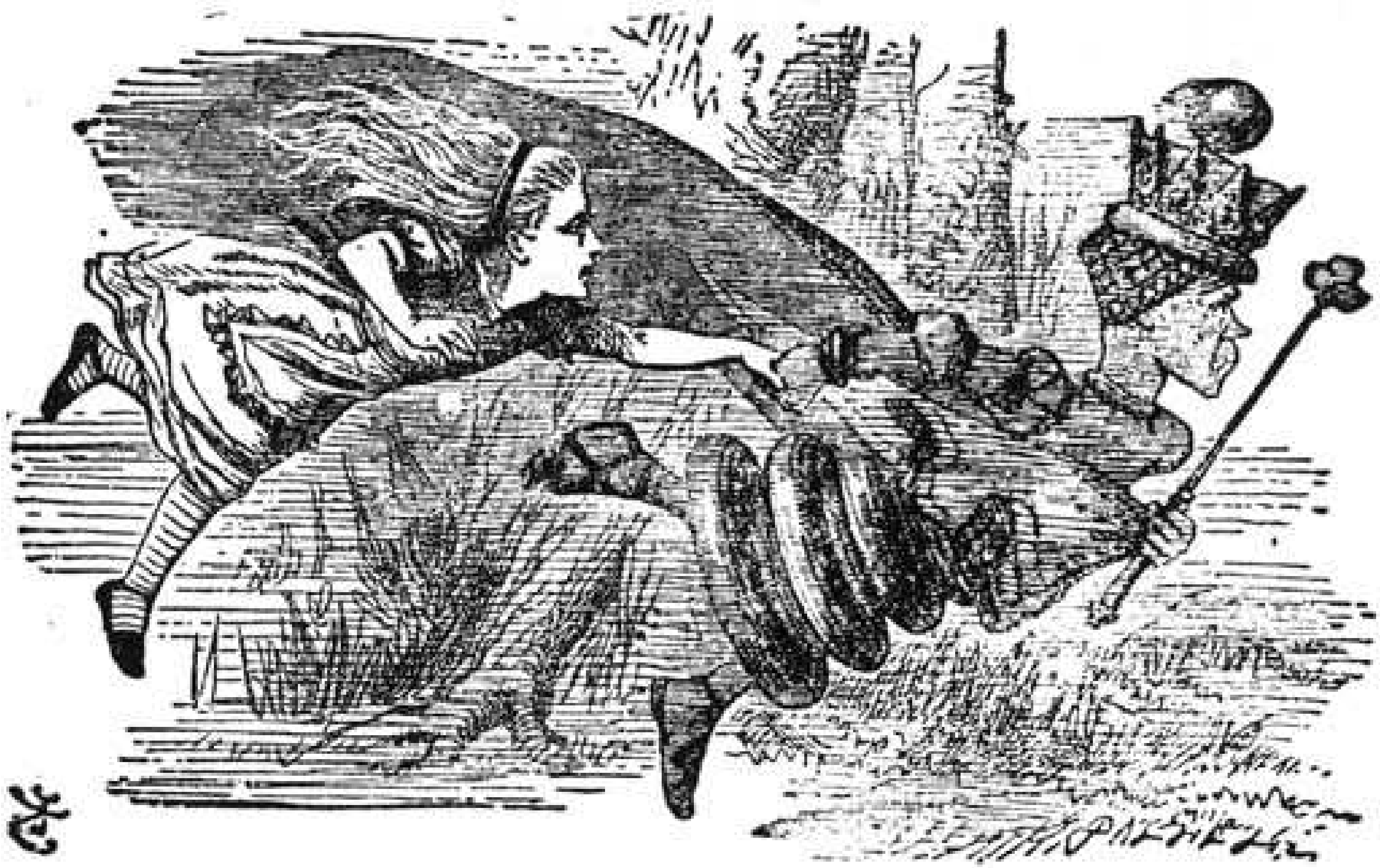
Spontaneous Parity Violation

Circumstances choose to make it look different: laws same

Spontaneous Parity Violation

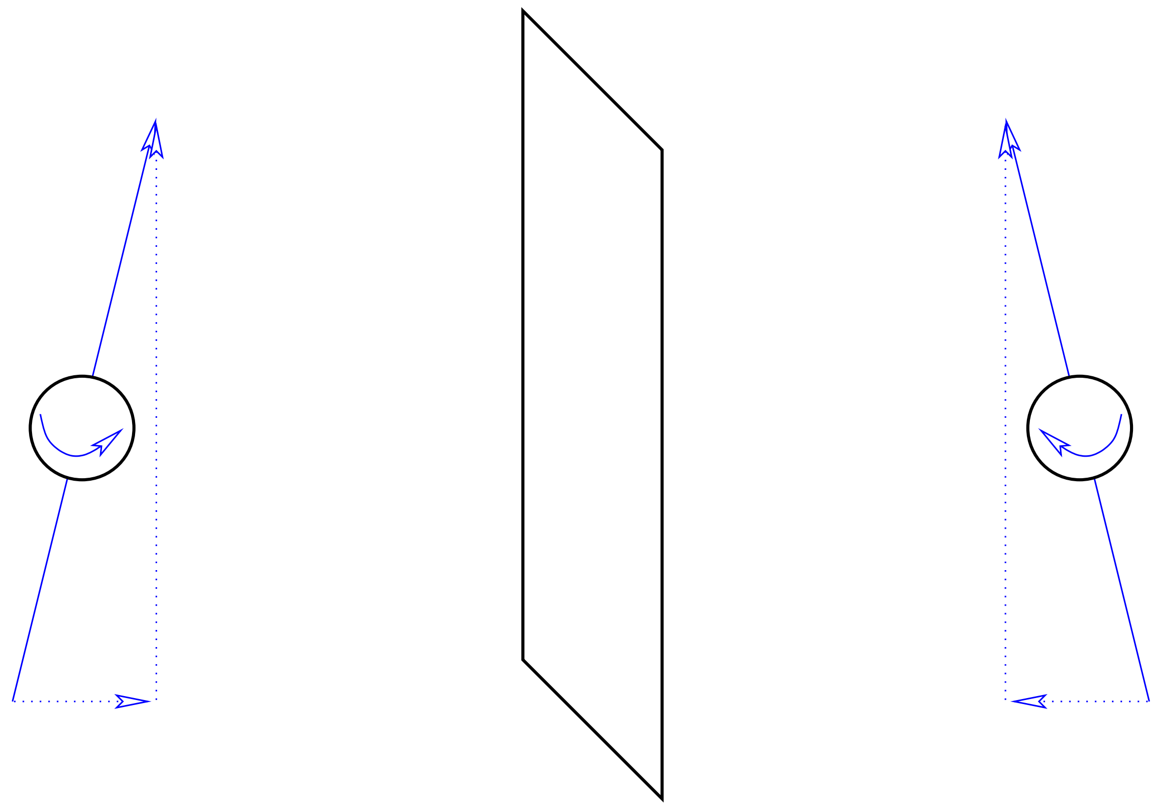
Could have occurred differently different places!

Explicit Parity Violation



Physical laws are different on the other side!

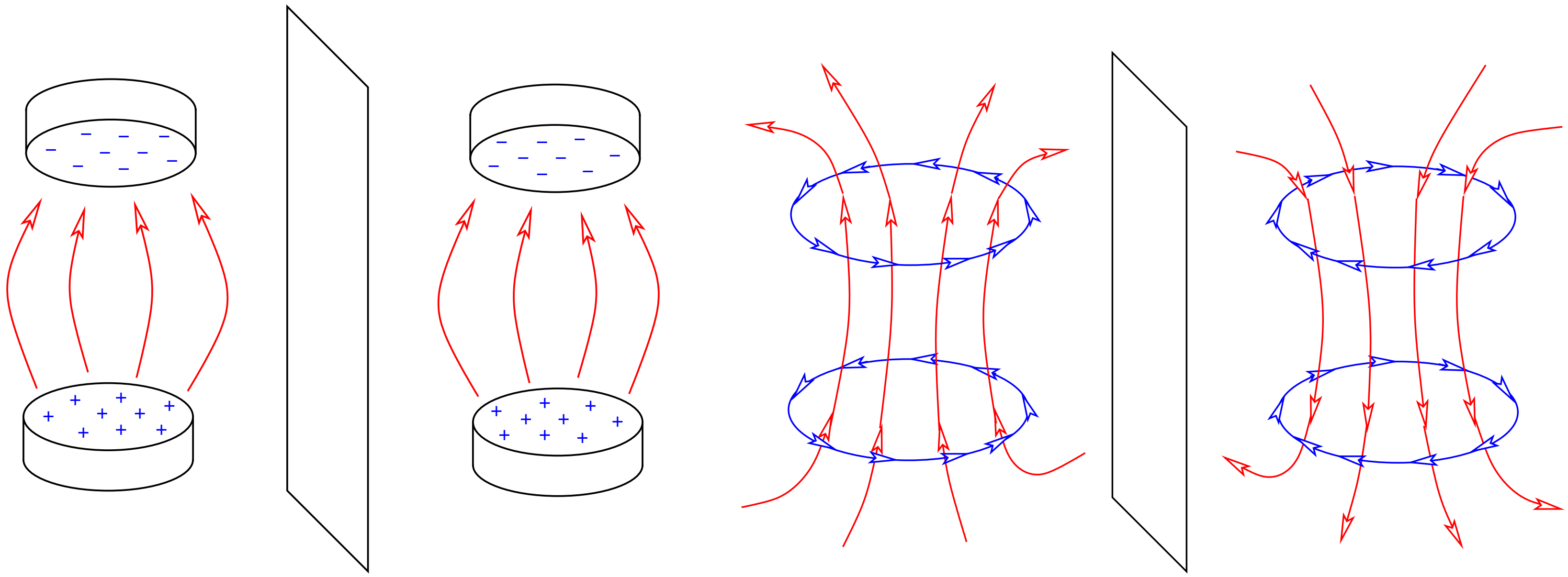
Parity–More Examples



Direction of motion reversed only orthogonal to mirror.

Sense of rotation and/or spin reversed ($\epsilon_{ijk} \rightarrow -\epsilon_{ijk}$)

Parity and Fields

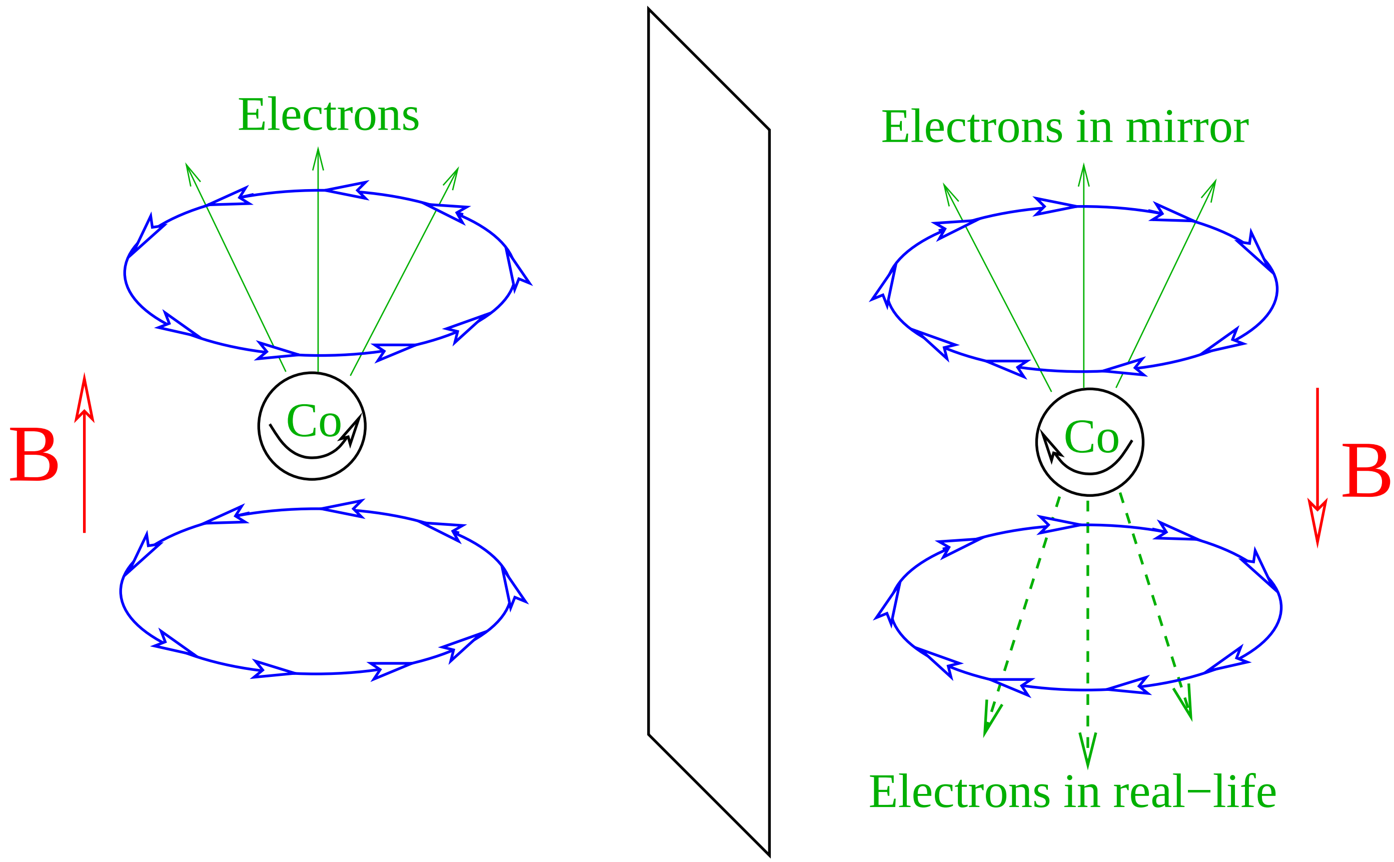


E fields: vector. Flips *only* along mirrored axis

B fields: axial-vector. Flips *except* along mirrored axis

Charges and currents act normally: B involves $\nabla\times$, or ϵ_{ijk}

Parity Violation in Cobalt (Wu 1956)



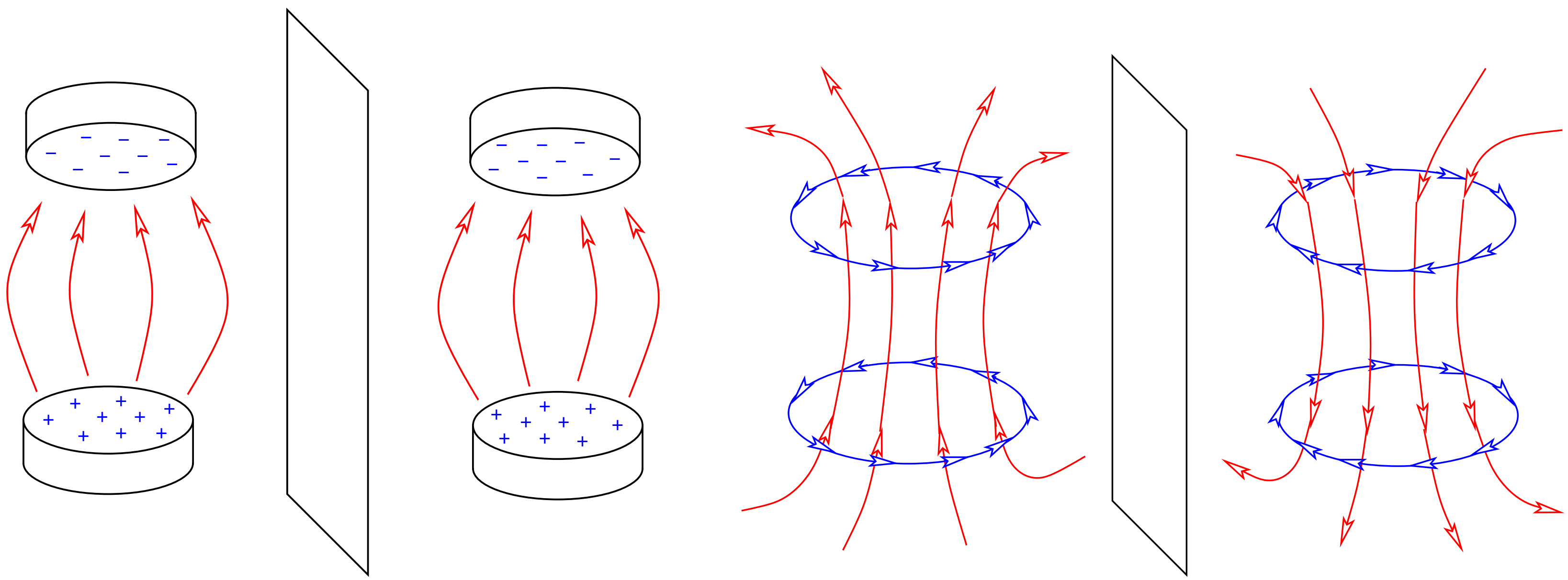
B field lines up nucleus. e^- go with B field.

In mirror–B field reversed, but e^- are same.

Occurs due to weak interactions – NOT **QCD** or E&M

Another Possible Symmetry: T

Forget mirrors. "Run the movie backwards"



Charges stay same. But currents are reversed. B flips.

Spontaneous T breaking all around. But explicit breaking?

Electromagnetism, P, T

E&M: a theory of fields. A (the first!) relativistic theory.
Nature is quantum mechanical $\Rightarrow$

Need Relativistic Quantum Field Theory

Start with action, $S = \int L\, dt$.
Field Theory: $L = \int \mathcal{L}\, d^3x$, $\mathcal{L}$ the "Lagrangian Density"

$$S = \int dt\, L = \int \mathcal{L}(E, B)\, d^4x$$

Relativity: $\mathcal{L}$ must be a (Lorentz) scalar.

Finding Lorentz scalars in E&M

Transformation rules: E, B fit in an antisymmetric tensor

$$F^{\mu\nu} = \begin{bmatrix} 0 & E_x & E_y & E_z \\ -E_x & 0 & -B_z & B_y \\ -E_y & B_z & 0 & -B_x \\ -E_z & -B_y & B_x & 0 \end{bmatrix} = \partial^\mu A^\nu - \partial^\nu A^\mu \,, \quad A^\mu = \begin{bmatrix} \Phi \\ A_x \\ A_y \\ A_z \end{bmatrix}$$

Lorentz transform: multiply left-and-right by same matrix used to transform (ct, x, y, z).

Long story short: two ways to form a scalar:

$$\frac{1}{2} F^{\mu\nu} F_{\mu\nu} = \vec{E}^2 - \vec{B}^2 \,, \qquad \frac{1}{8} \epsilon_{\mu\nu\alpha\beta} F^{\mu\nu} F^{\alpha\beta} = \vec{E} \cdot \vec{B}$$

Most general version of Electromagnetism

The Action can be

$$S = \int dt \int d^3x \left(C_1(\vec{E}^2 - \vec{B}^2) + \frac{C_2}{8\pi^2} \vec{E} \cdot \vec{B} \right)$$

(Plus stuff for electrons, invariant under P and T)

First term: B flips sign under P or T.
But term *as a whole* stays same. P, T invariant.

Second term: B flips sign unter P or T, E doesn't.
Under P or T, $\vec{E} \cdot \vec{B} \to -\vec{E} \cdot \vec{B}$.

If $C_2 \neq 0$, it appears E&M violates P and T.

P, T violation is Illusory!

The $C_2\vec{E}\cdot\vec{B}$ term has no *consequences* because it turns out that $\vec{E}\cdot\vec{B}$ is a *total derivative*

$$\frac{1}{4}\epsilon_{\mu\nu\alpha\beta}F^{\mu\nu}F^{\alpha\beta} = \partial^\mu K_\mu\,, \qquad K^\mu \equiv \frac{1}{2}\epsilon_{\mu\nu\alpha\beta}A^\nu F^{\alpha\beta}$$

I can integrate $C_2\vec{E}\cdot\vec{B}$ to a boundary term. For any local charge/current configuration, $F^{\alpha\beta}$ and therefore K_μ vanishes at the boundary.

So the term has no physical consequences.

But it would in **QCD** (the theory of the strong [nuclear] interactions) !!!

QCD and its Lagrangian

QCD is like 8 copies of E&M, but with non-linearities:

$$\text{Field strength}: \quad F_a^{\mu\nu} = \partial^\mu A_a^\nu - \partial^\nu A_a^\mu + g f_{abc} A_b^\mu A_c^\nu \,,$$

g: coupling. F_a, A_a: E&M-like fields $(a = 1, 2, \ldots 8)$
f_{abc} the constants which describe nonlinearity

$$S = \int dt \int d^3x \left(C_1 (\vec{E}_a^2 - \vec{B}_a^2) + \tfrac{C_2}{8\pi^2} \vec{E}_a \cdot \vec{B}_a \right)$$

where $\vec{E}_a \cdot \vec{B}_a$ still a total derivative:

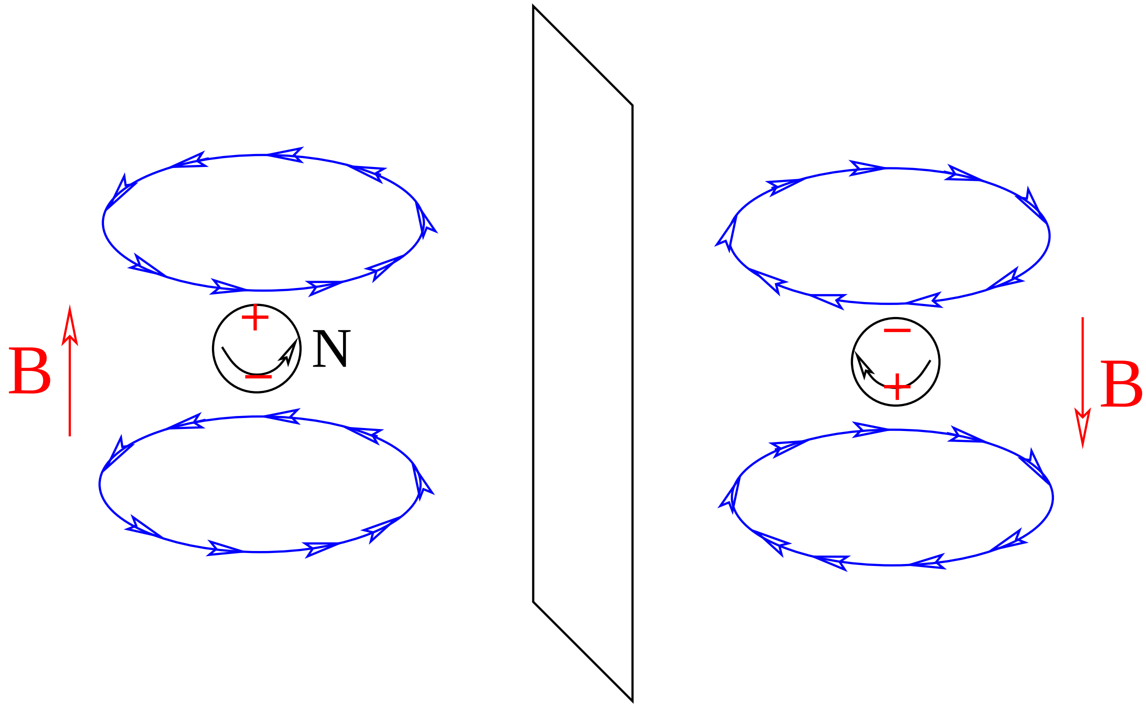
$$\vec{E}_a \cdot \vec{B}_a = \partial^\mu K_\mu \,, \qquad 2K_\mu = \epsilon_{\mu\nu\alpha\beta} \left(A_a^\nu F_a^{\alpha\beta} + \frac{g f_{abc}}{3} A_a^\nu A_b^\alpha A_c^\beta \right)$$

Last term *need not* vanish on boundary even if $\vec{E}_a = 0 = \vec{B}_a$ there!

It's always $8\pi^2 N$ with N integer. So C_2 mod 2π has *physical consequences*

Neutron Electric Dipole Moment

A Test of whether **QCD** obeys T symmetry:



Put neutron in $\vec{B}$ field – spin lines up with $\vec{B}$.

Does it have an electric dipole moment aligned with spin?

If so: physics when you run movie backwards is different! T violating!

Neutron Electric Dipole Moment

Theory: Neutron electric dipole moment should exist,

$$d_n = -3.8 \times 10^{-16}\, e\,\mathrm{cm} \times C_2$$

so long as C_2 is not zero! Guo *et al*, arXiv:1502.02295, assumes C_2, modulo 2π, is small

Experiment: Consistent with zero! Baker *et al* (Grenoble), arXiv:hep-ex/0602020

$$|d_n| < 2.9 \times 10^{-26}\, e\,\mathrm{cm}$$

Either $\left|\frac{C_2}{8\pi^2}\right| < 10^{-10}$ by (coincidence? accident?) or there is something deep going on here.

Why an accident seems unlikely

- T is not a fundamental symmetry. We have observed its violation (Kaon, B-meson, possibly D-meson physics)
- More T violation almost surely out there – otherwise, tough to explain why Universe is filled with matter!
- Renormalization: T viol. one place finds its way into other places, including C_2. C_2 does *not get smaller* as you go from high to low scales R.G. marginal – feels T viol from all scales.

Axion: an explanation for $C_2 = 0$

Hypothesize an extra *complex scalar* field $\varphi = \phi e^{i\theta_A}$:

- Field: takes a value at each point in space Think $\vec{E}$
- Scalar: value is a number, without direction but with units ...
- Complex scalar: value is a real and imaginary part

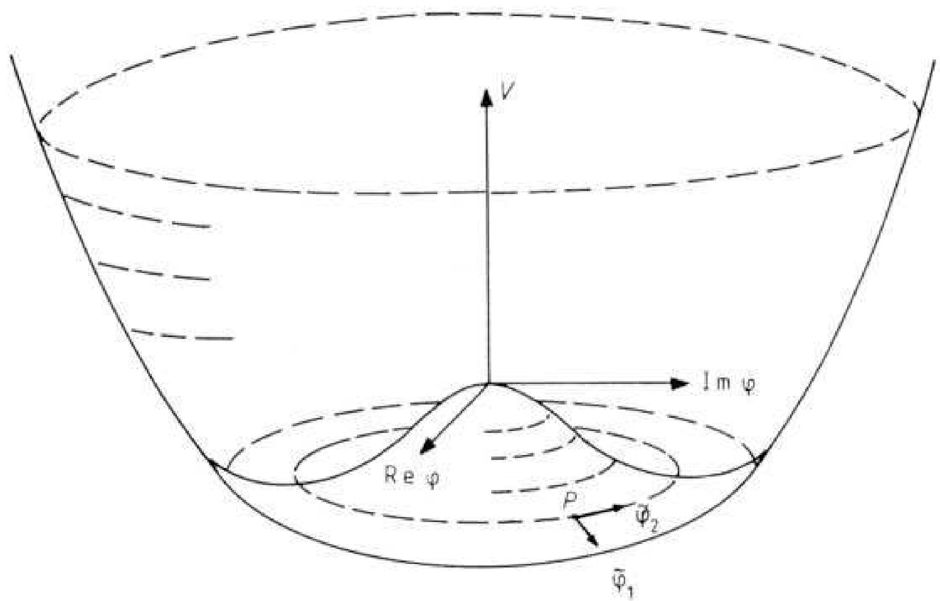
Assume a *symmetry*: $\varphi \to e^{i\theta}\varphi$

Energy should be

$$\text{Energy} = \int d^4x \, |\dot{\varphi}|^2 + |\vec{\nabla}\varphi|^2 + V(|\varphi|)$$

Axion: spontaneous symmetry breaking

Potential function $V(|\varphi|)$ can look like this:



Min of V

at $\varphi \neq 0$!

Lowest energy state has $\varphi \neq 0$.

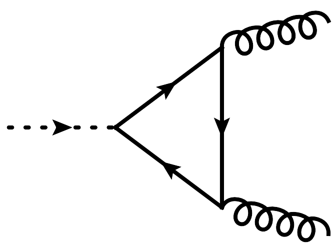
Minimum not unique; all values of θ_A equally good.

Or are they??

Indirect Interactions with QCD

Suppose Axions ⇔ New Heavy quarks ⇔ QCD

Introduces axion-QCD interactions at low energy:



$$\mathcal{L}_{\mathbf{QCD}} + = k_1 \ln(\phi)(\vec{E}_a^2 - \vec{B}_a^2) + \frac{n}{8\pi^2}\theta_A \vec{E}_a \cdot \vec{B}_a$$

k_1: (heavy) radial ϕ excitations interact with QCD not important.

Last term looks like the mysterious $C_2 \vec{E} \cdot \vec{B}$ interaction!

n is integer and can be 1. I will assume $n = 1$

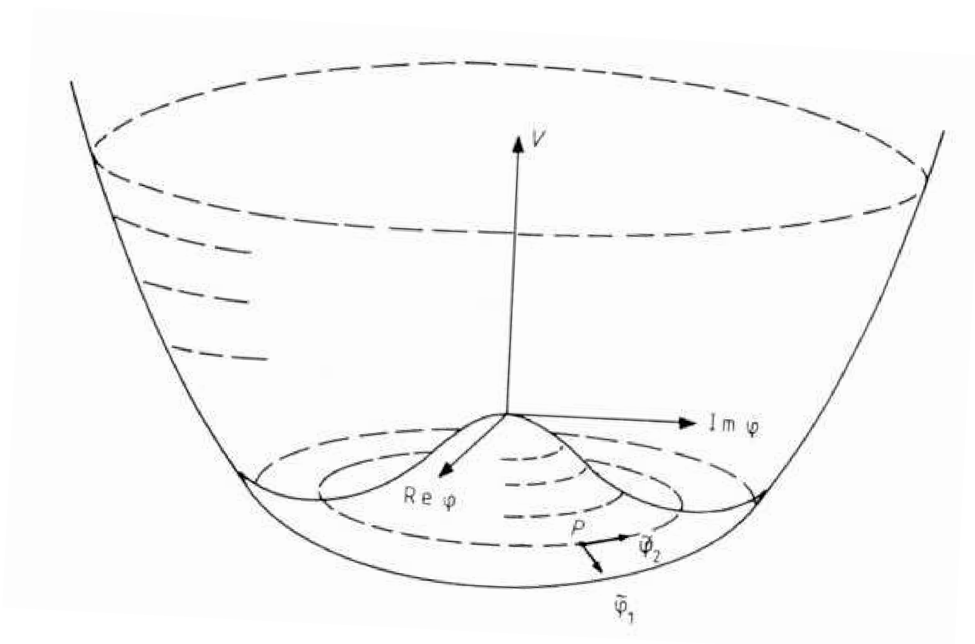
T violation determined by $C_2 + \theta_A$ (modulo 2π)

No T violation if θ_A happens to take value $-C_2$.

Now the fun part

θ_A is a *field* and can change value.

QCD *cares about* value of $C_2 + \theta_A$. Detailed **QCD** calculation *Topological Susceptibility of* **QCD**: Potential tilted



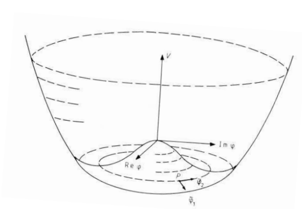
Energy lowest where $C_2 + \theta_A = 0$, where T conserved.

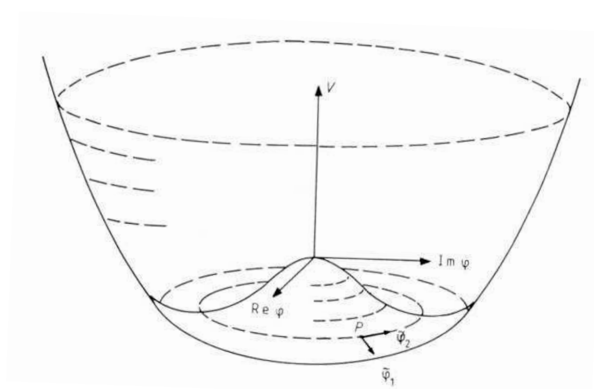
Size of tilt: nonperturbative **QCD**. *Solved.*

What is predicted:

- Neutron EDM: 10^{-5} smaller than current limit
- New particle, Axion, exists, with specific interactions
- Strength of axion interactions scale with its mass (Pseudo-Nambu-Goldstone Boson)

What is not predicted: Axion Mass!





The larger ϕ, the flatter the potential, and lighter the axion. ϕ not predicted by model.

One more prediction: Dark Matter!!

Recall: Cold Dark Matter makes up 25% of the density of the Universe. *Mystery*. We know only 3 things about it:

- It is matter: feels gravitational attraction, which makes it clump (makes up 80% of galactic haloes, galaxy clusters)
- It is dark: electric charge $\simeq 0$, interactions too feeble for us to have detected it except gravitationally
- It is cold: almost pressureless. When Universe was 50,000yr old and 3000$\times$ hotter, it was non-relativistic enough to gravitationally clump $(|v| < 10^{-5}c)$

Could the CDM be axions?

Axion in the Early Universe

Suppose axion field starts at some value $\varphi = \phi e^{i\theta_A}$.

Around when $tm_A = 1$, field starts oscillating

Oscillations damp as

- mass rises with time
- Hubble drag acts on field

Oscillating field a good CDM candidate!

Large ϕ $\Rightarrow$ oscillations start later, keep larger amplitude.

Also, large ϕ $\Rightarrow$ small m_A! *CDM density, m_A correlated!*

Best Axion search experiments can only look in narrow mass range. Need approx. mass *before* you do experiment.

We theorists should give them clean prediction! Problems:

- Axion mass arises from **QCD** effects. Different for hot **QCD** (early Universe) than vacuum **QCD** (Universe today).

  Effect should be calculable. But *accurate* calculation will need new techniques...

- Axion field likely started out *inhomogeneous*. Dynamics complicated by *topological* structures.

Let's concentrate on the second problem!

Initial state of φ field?

Maybe it's the same everywhere in space.

But most likely, it's randomly different in different places!

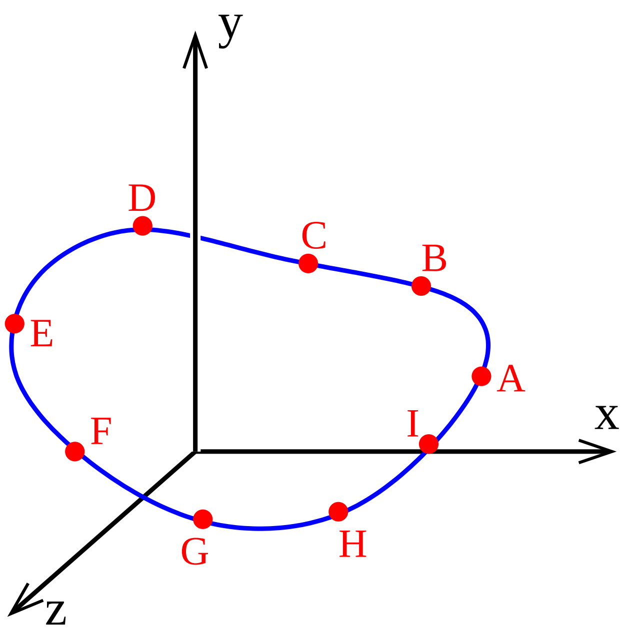
- *Inflation* seems likely. Can be "high-scale" or "low-scale". High: axion starts out random. Low: axion theory has problems!

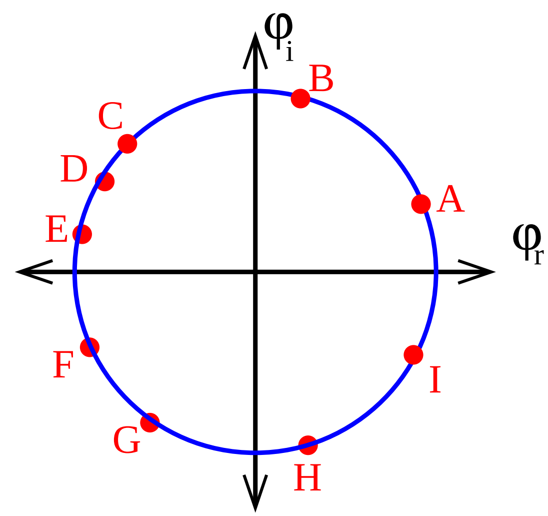
- Gets scrambled *after* inflation if Universe was ever really hot $T > 10^{11}$ GeV.

Random case more complicated – also more predictive.

Axions and Topology I

Consider the value of φ around a circle in *space*:





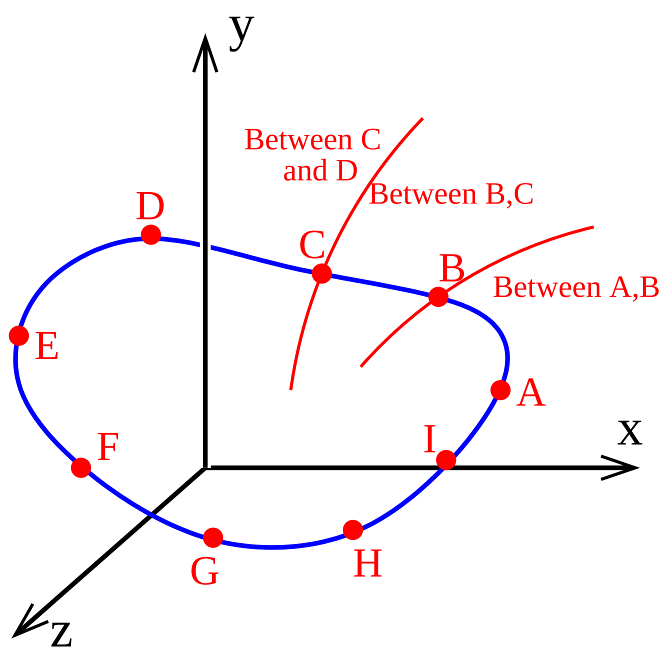
Each point maps to a point in field-space.

These may make a circuit around field-min. circle

Continuity $\rightarrow$ somewhere in middle, field "goes over top"

Axions and Topology II

What are points *inside* our space-circle with the same field-values? Use continuity . . . .



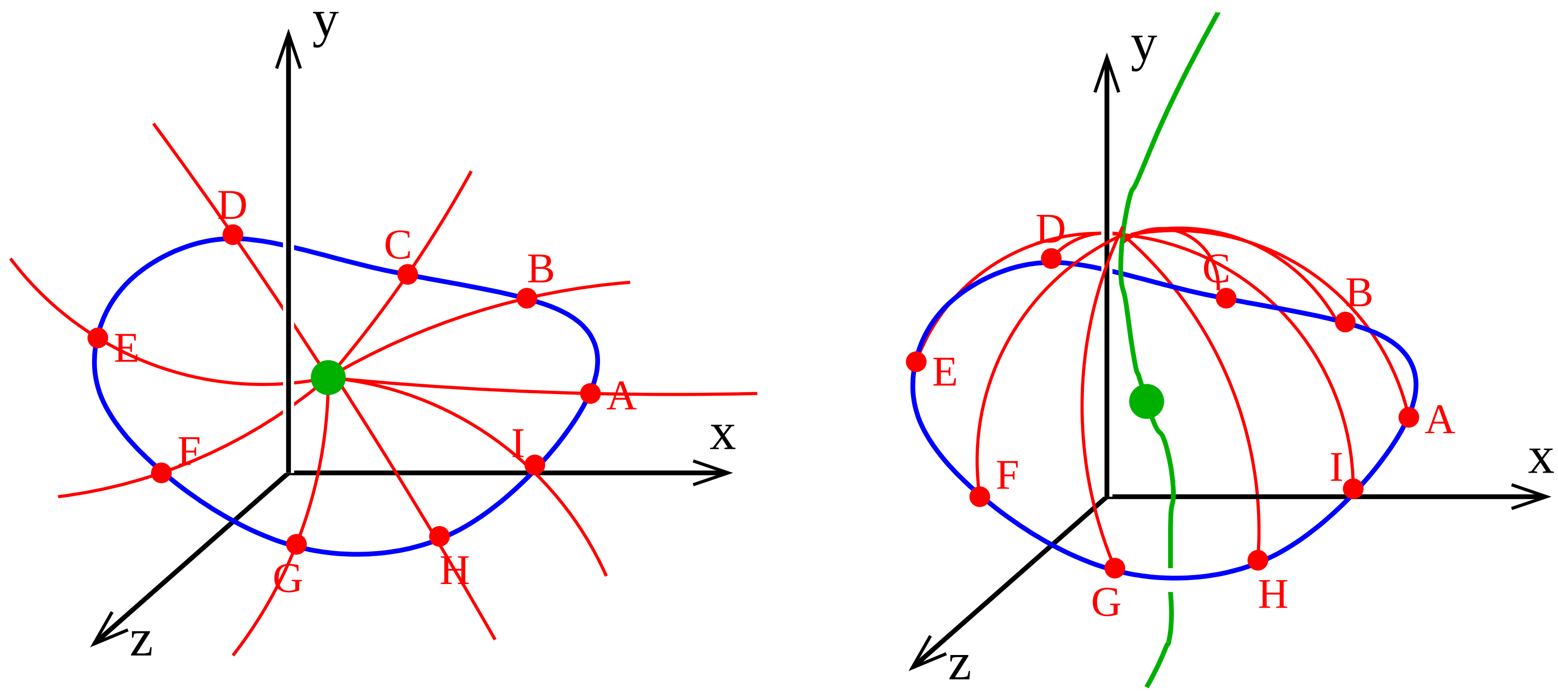
These lines can only cross where field is discontinuous

Or where it “leaves the ring.” Energy cost!

Minimize energy – have all lines meet at one point.

Axions, Topology III

Point where lines meet – field leaves min-energy circle



Think about any other surface bounded by the circle:

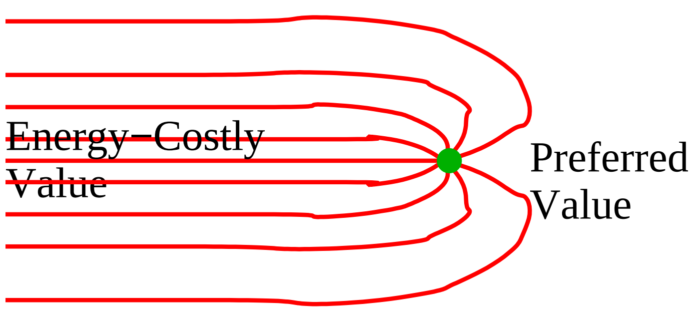
Consider locus of all such points.

φ leaves vacuum circle along a “string” (line)

Strings and Walls

Random starting conditions $\Rightarrow$ network of these "strings"
Network evolves, strings straighten out, find each other.

Potential tilts: $\theta_A \to 0$ where it can. Near string: explores all values. String has "domain wall" attached.



Pulls strings towards each other, speeds breakup of network.
Complex string – domain wall network dynamics.

This dynamics produces Axions!

Big complication

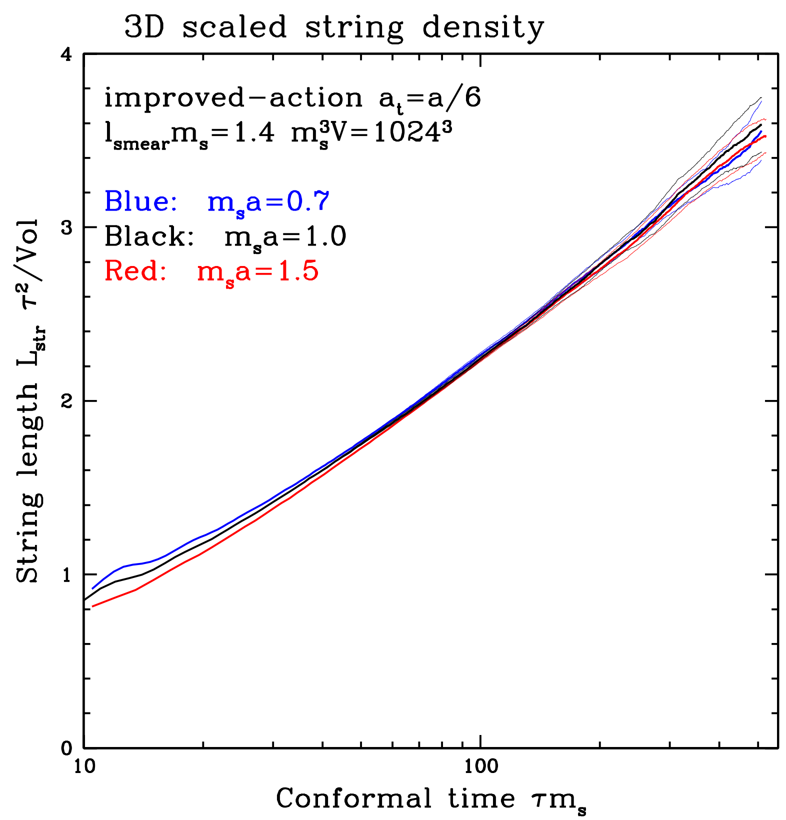
These strings are like onions (or scallions)



Layers! Innermost "core" has radius $r \sim 1/|\phi| \sim 10^{-27}$m. Outer size $\sim H^{-1} \sim 10^{2}$m. Energy stored logarithmically with radius $\varepsilon \propto \int \frac{r\,dr}{r^2}$

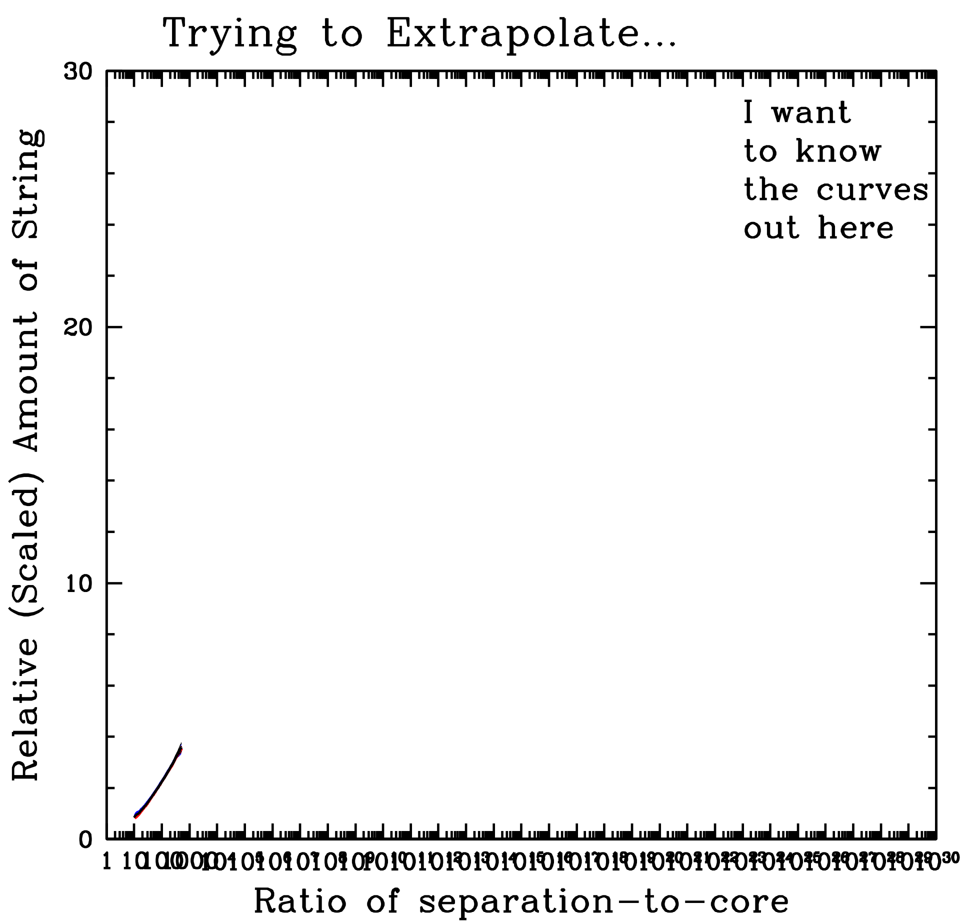
Does this matter?

Yes! As strings get more layers, they get heavier; attractive interactions less important. Density rises ~linearly with # of layers



Numerics: never get log larger than $\sim 10^3$. Need 10^{30}!

Do you trust this extrapolation?



I have to confess some skepticism. Need either model information or new technique

What we have so far

So far, axion production *barely depends* on starting density of strings! **IF** that stays true:

$$m_A = 18\,\mu\text{eV}, \qquad \phi = 3.3 \times 10^{11}\ \text{GeV}$$

Axion starts oscillating when $mt = 1$ at $T \simeq 1.5$ GeV.

Need some technique to treat the string-core explicitly!

Recently developed (but not implemented!) in 2D.

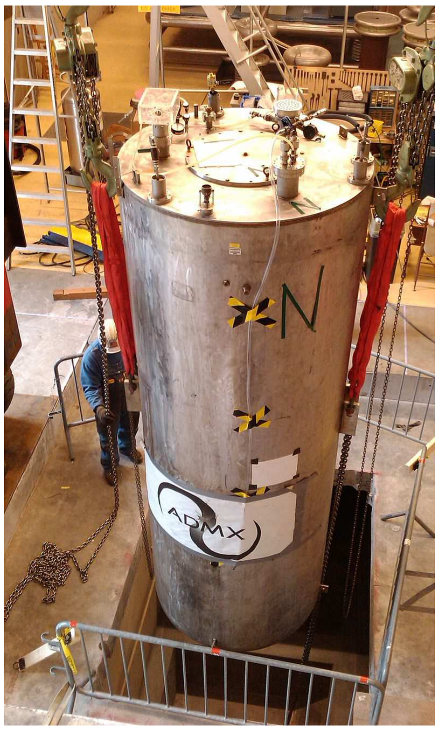
Work in progress

Conclusions

- T, P potential symmetries of nature
- E&M naturally respects both symmetries
- **QCD need not** respect P, T. Experiment: it does!
- Axion could explain T invariance of **QCD**
- Axion role in Dark Matter requires new inputs:
  - ∗ thermal **QCD** inputs
  - ∗ Field-string-network dynamics
- I hope a tight prediction of the axion mass can be made!

How do experimentalists look for Axions?

Most sensitive method: resonant cavity in magnetic field



Microwave cavity inside superconducting solenoid. $\vec{E}$ field of cavity mode aligned with $\vec{B}$ of solenoid. Cavity oscillation: oscillating $\vec{E} \cdot \vec{B}$. If cavity resonance matches $m_A/\hbar$: cavity resonance driven. Squid readout – tuneable cavity ...

What about Anthropic Principle?

Trendy Explanation for “coincidences” or “tunings”

Why is Cosmological Constant so small?

If it were 100 times bigger, matter would fly apart or collapse before life could evolve. Nature plays dice, universes with all values occur, but only universes with life get observed.

Why does **QCD** respect T symmetry?

If **QCD** *violated T, something would go wrong with nuclear physics, which would make life impossible. Nature plays dice, only universes where life is possible get observed.*

Except that life is fine in a world where $C_2 = 10^{-2}$!