Baryon Resonances with Strangeness

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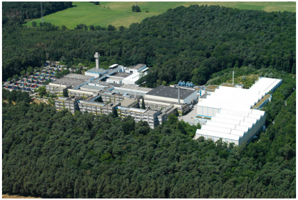
Resonances in QCD



GSI Darmstadt, Germany

10/13/2015

Outline

1. Introduction
   - The Spectrum of Baryons
   - Lattice Calculations
2. Properties of Ξ Resonances
   - Experimental Situation
   - Recent Efforts in Photoproduction
3. Cascades at GlueX
   - Opportunities with Secondary K_L^0 Beams
4. Summary and Outlook

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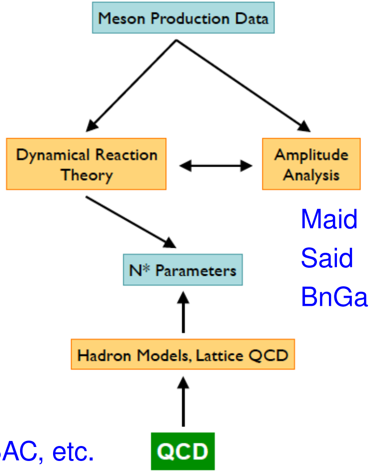
Extraction of Resonance Parameters in N^* Physics

- Double-polarization measurements
- Measurements off neutron and proton to resolve isospin contributions:
  1. $\mathcal{A}(\gamma N \to \pi,\ \eta,\ K)^{I=3/2} \quad \Longleftrightarrow \quad \Delta^*$
  2. $\mathcal{A}(\gamma N \to \pi,\ \eta,\ K)^{I=1/2} \quad \Longleftrightarrow \quad N^*$
- Re-scattering effects: Large number of measurements (and reaction channels) needed to extract full scattering amplitude.

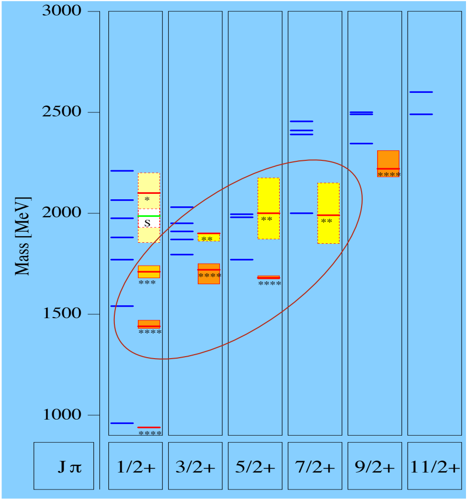
Coupled Channels



Maid
Said
BnGa

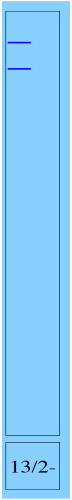
Jülich, Gießen, EBAC, etc.

Spectrum of N^* Resonances



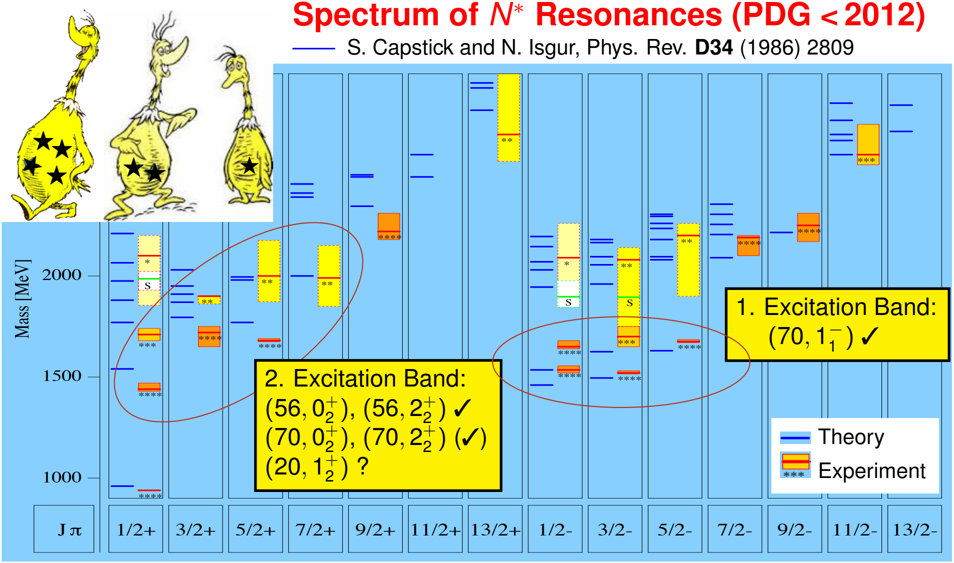
V. C. & W. Roberts, Rep. Prog. Phys. **76** (2013)

N^*	J^P ($L_{2I,2J}$)	2010	2014
$N(1440)$	$1/2^+$ (P_{11})	****	****
$N(1520)$	$3/2^-$ (D_{13})	****	****
$N(1535)$	$1/2^-$ (S_{11})	****	****
$N(1650)$	$1/2^-$ (S_{11})	****	****
$N(1675)$	$5/2^-$ (D_{15})	****	****
$N(1680)$	$5/2^+$ (F_{15})	****	****
$N(1685)$			*
$N(1700)$	$3/2^-$ (D_{13})	***	***
$N(1710)$	$1/2^+$ (P_{11})	***	***
$N(1720)$	$3/2^+$ (P_{13})	****	****
$N(1860)$	$5/2^+$		**
$N(1875)$	$3/2^-$		***
$N(1880)$	$1/2^+$		**
$N(1895)$	$1/2^-$		**
$N(1900)$	$3/2^+$ (P_{13})	**	***
$N(1990)$	$7/2^+$ (F_{17})	**	**
$N(2000)$	$5/2^+$ (F_{15})	**	**
~~N(2080)~~	D_{13}	**	
~~N(2090)~~	S_{11}	*	
$N(2040)$	$3/2^+$		*
$N(2060)$	$5/2^-$		**
$N(2100)$	$1/2^+$ (P_{11})	*	*
$N(2120)$	$3/2^-$		**
$N(2190)$	$7/2^-$ (G_{17})	****	****
~~N(2200)~~	D_{15}	**	
$N(2220)$	$9/2^+$ (H_{19})	****	****

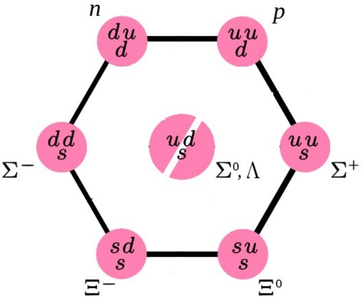


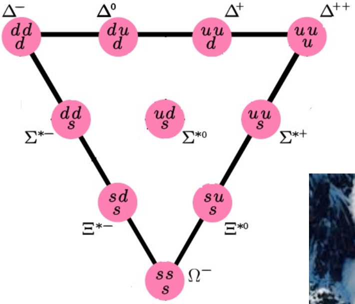
Spectrum of N^* Resonances (PDG < 2012)

S. Capstick and N. Isgur, Phys. Rev. **D34** (1986) 2809



Cascade Spectrum and Multiplets

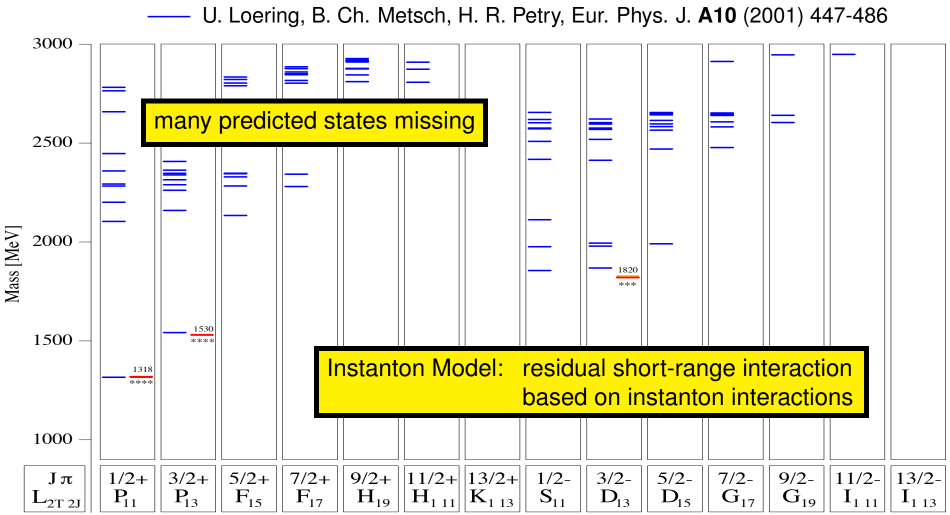




The decuplets consist of Δ^*, Σ^*, Ξ^*, and Ω^* resonances, but also the octets consist of an Ξ^* state.

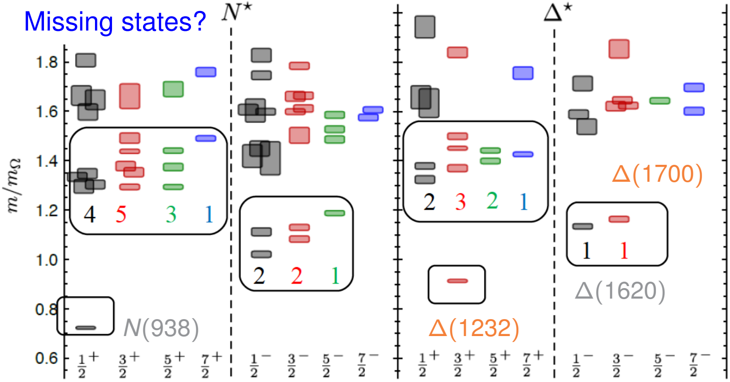
➜ We expect as many Ξ's as N^* & Δ^* states together. Moreover, their properties should be related.

Cascade Resonances: Status of 2015



The N^* and Δ^* Spectrum from Lattice QCD

R. Edwards *et al.*, Phys. Rev. D **84**, 074508 (2011); Phys. Rev. D **87**, 054506 (2013)



$m_\pi = 396$ MeV

Exhibits broad features expected of $SU(6) \otimes O(3)$ symmetry

➜ Counting of levels consistent with non-rel. quark model, no parity doubling.

The Ξ^* and Ω^* Spectrum from Lattice QCD

R. Edwards *et al.*, Phys. Rev. D **87**, no. 5, 054506 (2013)

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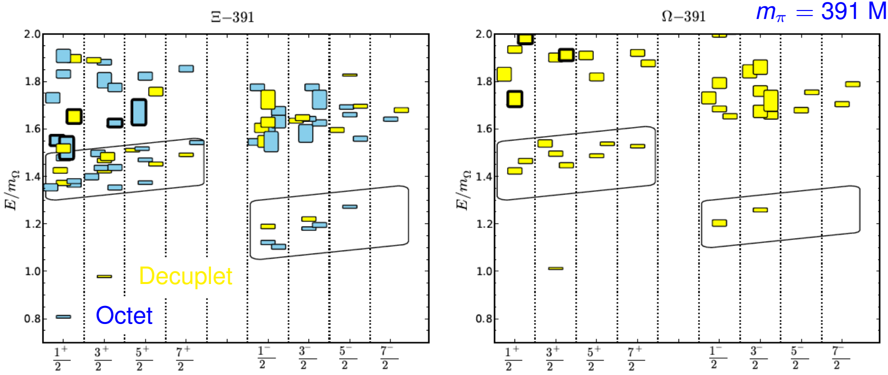

Exhibits broad features expected of $SU(6) \otimes O(3)$ symmetry

➜ Counting of states of each flavor and spin consistent with QM for the lowest negative- and positive-parity bands.

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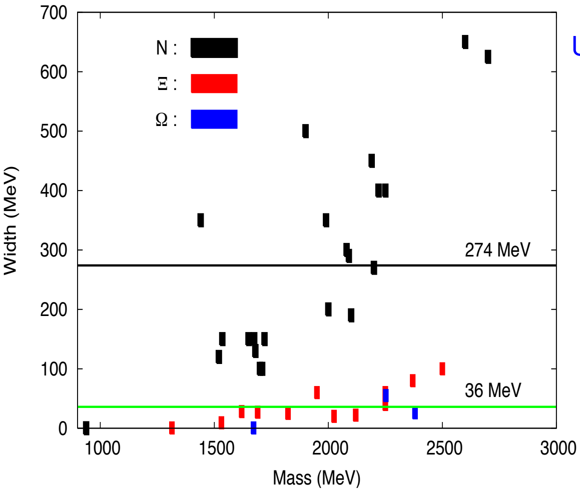
Baryon Widths



Unique features of Ξ^* and Ω^*:

- Strangeness $S > 1$
- Narrow widths:

  $\Gamma_{\Xi^*} \approx 10\,\text{-}\,20$ MeV?

Why are Cascade Resonances Narrow?

Possible explanations:

1. Narrowness related to the number of light quarks
   ➜ Specifically, resonance width proportional to square of the number of light quarks: $\Gamma_{N^*,\Delta^*} : \Gamma_{\Lambda^*,\Sigma^*} : \Gamma_{\Xi^*} \approx 9 : 4 : 1$
   (D. O. Riska, Eur. Phys. J. **A17**, 297 (2003))
2. Alternative explanation based on Cascade structure and decay modes (S. Capstick)
   - Decays $\Xi^* \to \Xi\pi$ are suppressed relative to $N^*, \Delta^* \to N\pi$.
     Non-rel. one-gluon exchange model: Chao, Isgur, Karl, PR **D23**, 155 (1981)
   - Flavor symmetry requires s-quarks at ends of ρ oscillator; excitation energy is given by the square root of K/M where K will be equal for all quark pairs if the confinement potential is flavor independent.

     ➜ Excitation of ρ costs less energy than the excitation of λ.
     (Argument only valid for lowest states.)

Measurements at BNL in $K^- p \to K^+_{\rm slow} + X^-$

“Existence of Ξ Resonances above 2 GeV”
(C.M. Jenkins *et al.*, Phys. Rev. Lett. **51**, 951 (1983))

Observed Ξ States:

$\Xi(1320)$ **** $I(J^P) = \frac{1}{2}(\frac{1}{2}^+)$

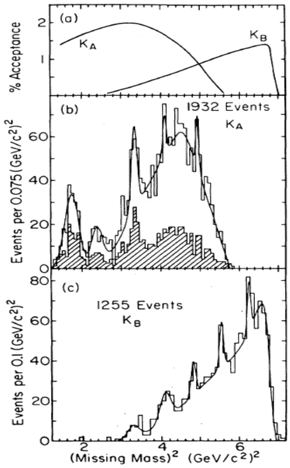
$\Xi(1530)$ **** $I(J^P) = \frac{1}{2}(\frac{3}{2}^+)$

$\Xi(1820)$ *** $I(J^P) = \frac{1}{2}(\frac{3}{2}^-)$

$\Xi(2030)$ *** $I(J^P) = \frac{1}{2}(\geq \frac{5}{2}^?)$

$\Xi(2370)$ *** $I(J^P) = \frac{1}{2}(?^?)$

$\Xi(2500)$ *** $I(J^P) = \frac{1}{2}(?^?)$



Excited Cascade Resonances at BaBar

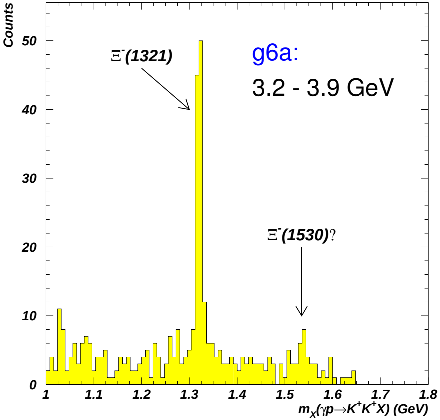
BaBar collaboration has recently studied the $\Xi(1530)$ resonance in the decay $\Lambda_c^+ \to K^+\Xi(1530)^0 \to K^+(\Xi^-\pi^+)$ (V. Ziegler, MENU 2007)

- Complementary to the search in the reaction $\gamma p \to K^+(\Xi^-\pi^+)$
- Dalitz plot shows only one dominant structure: $\Xi(1530)^0 \to \Xi^-\pi^+$
  ➜ Spin-parity analysis favors a $J^P = 3/2^+$ assignment
- Evidence for *S*-*P*-wave interference in the $\Xi^-\pi^+$ system provides hints for possible $\Xi(1690)$ production
- Further indications for the $\Xi(1690)$ state have been found in the study of $\Lambda_c^+ \to \Lambda K_S K^+$
  ➜ BaBar data favor a spin-1/2 assignment at the 56.4 % C.L.

CLAS g6 Runs

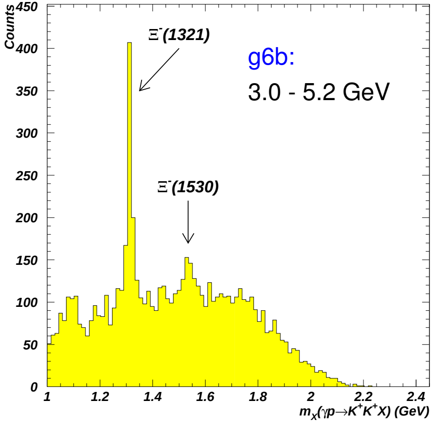
First exclusive photoproduction of Ξ^-

J. Price *et al.*, PRC **71**, 058201 (2005)



Look for $\gamma p \to K^+K^+\,(X)$

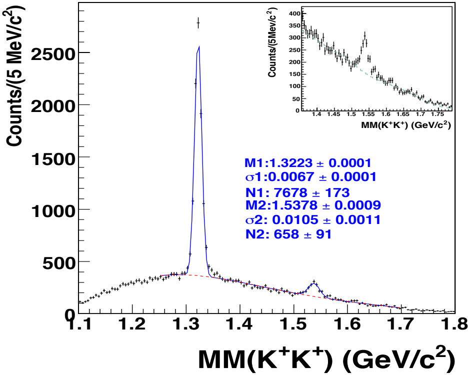
with $B = +1, \; Q = -1, \; S = -2$



CLAS g11a Run

More data mining based on much higher statistics

Guo *et al.*, PRC **76**, 025208 (2007)

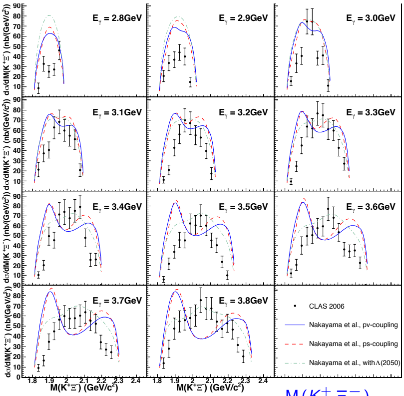


Reaction $\gamma p \rightarrow K^+ K^+ (X)$

- Very strong Ξ(1320) signal
- Clear signal for Ξ(1530)
- Ratio $N_{\Xi} \,/\, N_{\Xi(1530)} \approx 10 : 1$
  - ➜ About the same as for the CLAS-g6 data.

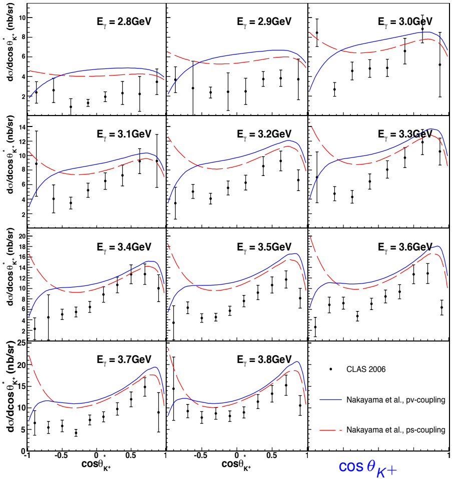
CLAS g11a Run: Differential Cross Sections

E_γ range of 2.75 – 3.85 GeV



M$(K^+ \Xi^-)$

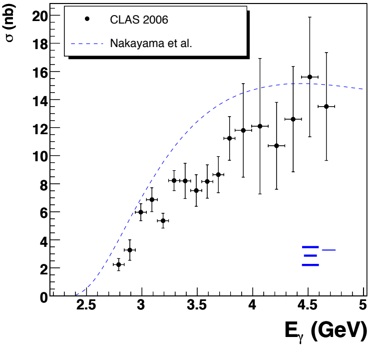
$\cos \mathrm{d}\sigma/\mathrm{d}\cos\theta_{K+}$ [nb/sr]

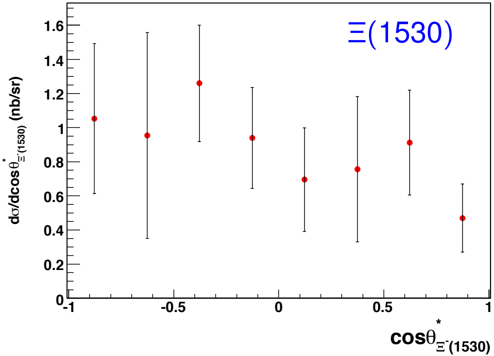


$\cos\theta_{K+}$

CLAS g11a: Cross Sections of Ξ^- and $\Xi(1530)$

E_γ range of 2.75 – 4.75 GeV

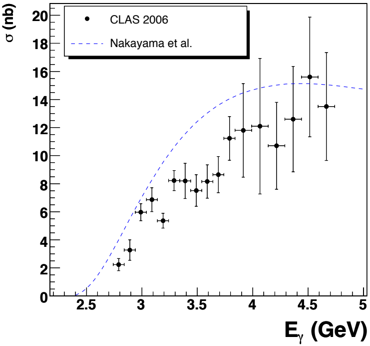


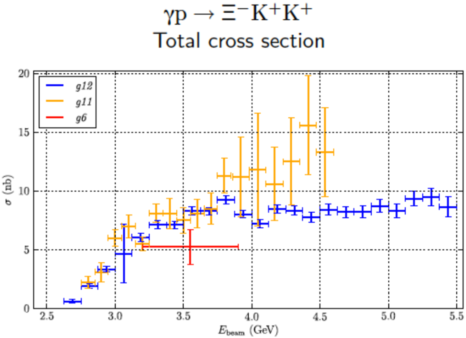


K. Nakayama, Y. Oh, and H. Haberzettl, Phys. Rev. **C74**, 035205 (2006)

CLAS g12: Total Cross Section of Ξ^- (preliminary)

E_γ range of 2.75 – 4.75 GeV

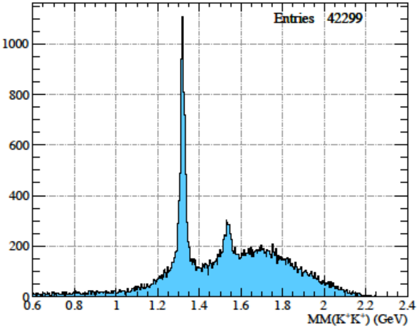


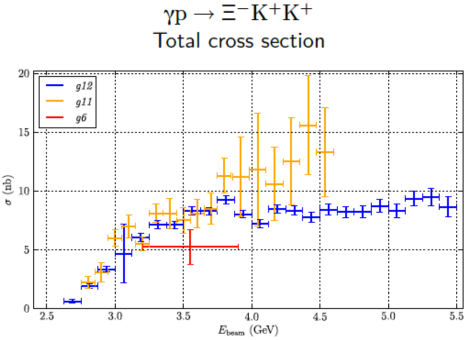


K. Nakayama, Y. Oh, and H. Haberzettl, Phys. Rev. **C74**, 035205 (2006)

CLAS g12: Total Cross Section of Ξ^- (preliminary)

Johann Goetz (CLAS Collaboration), UCLA, Ph.D. Thesis

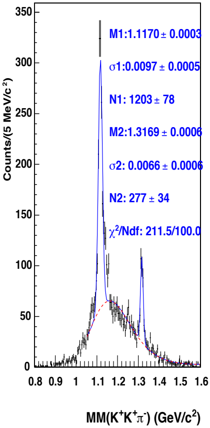


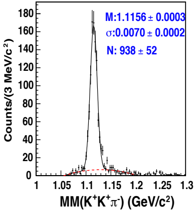


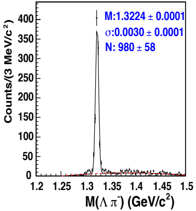
Upper Limits (integrated over 3.5-5.4 GeV):

(1) Ξ(1620): 0.78 nb (2) Ξ(1690): 0.97 nb (3) Ξ(1820): 1.09 nb

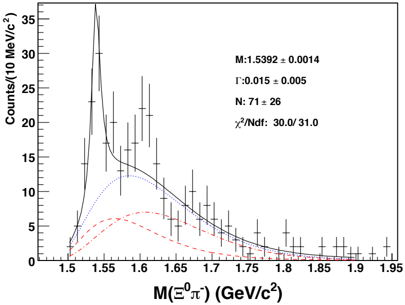
CLAS g11a: Excited States in $\gamma p \to K^+ K^+ \pi^- \, (X)$







1. Only Ξ(1530) statistically significant
2. Ξ(1620) signal "plausible", but simulated K^{*0} events also peak in 1600 MeV/c^2 region ($\gamma p \to K^+ K^{*0} \Xi^0, \; K^{*0} \to K^+ \pi^-$)

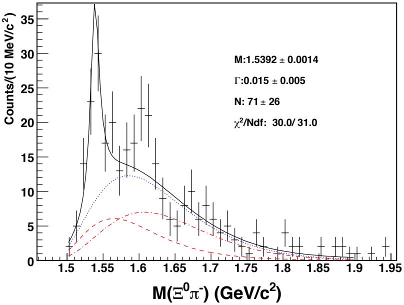


CLAS g11a: Excited States in $\gamma p \to K^+K^+\pi^-\,(X)$

From the paper: *Although a small enhancement is observed in the $\Xi^0\pi^-$ invariant mass spectrum near the controversial 1-star $\Xi^-(1620)$ resonance, it is not possible to determine its exact nature without a full partial wave analysis.*

Need high-statistics, high-energy data from an experiment designed to see Ξ states:

- 3- or 4-track trigger
- Reconstruction of full decay chain
- Higher photon energy
- Improved detectors



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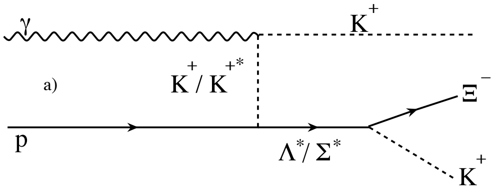
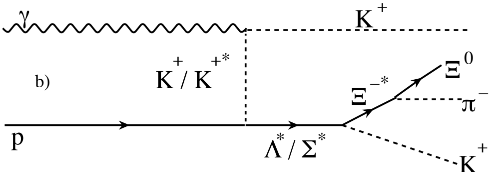
Possible Production Mechanisms



$K^+(\Xi^- K^+)$, $K^+(\Xi^0 K^0)$, $K^0(\Xi^0 K^+)$

➜ Cross sections, beam asymmetries (similar to $p\pi\pi$ & pKK^*)



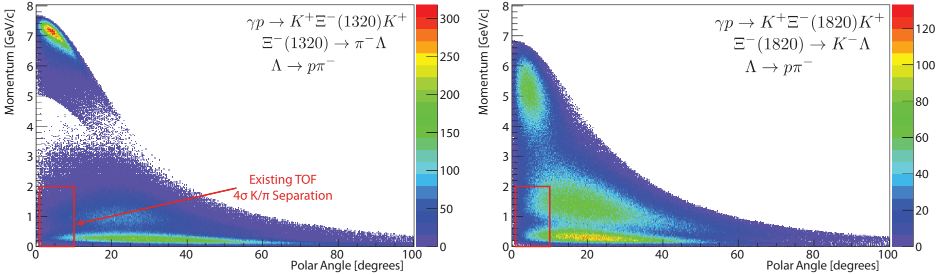
Production of excited states via a

1. forward-going K^0 meson
   ➜ $K^0\,(\,\Xi^-\,\pi^+\,)\,K^+$, etc.
2. forward-going K^+ meson
   ➜ $K^+\,(\,\Xi^-\,\pi^+\,)\,K^0$,
   $K^+\,(\,\Xi^0\,\pi^-\,)\,K^+$, etc.

* W. Roberts *et al.*, Phys. Rev. C **71**, 055201 (2005)

Ξ Spectroscopy with the GlueX Detector

GlueX Proposal on "Decays to Strange Final States" (JLab PAC 39, 40 & 42)



Efficiency should be adequate for conducting a study of excited Ξ states with the baseline detector:

- Detailed studies of the production, especially of the ground state Ξ's, and a parity measurement * will likely require enhanced kaon identification in the forward direction ➜ Components of the BaBar DIRC for GlueX.

* e.g. Nakayama *et al.*, Phys. Rev. C **85**, 042201 (2012)

Ξ Spectroscopy with the GlueX Detector

The Ξ octet ground states (Ξ^0, Ξ^-) will be challenging to study via exclusive *t*-channel (meson exchange) production. The typical final states have kinematics for which the baseline GlueX detector has very low acceptance due to:

- the high-momentum forward-going kaon and
- the relatively low-momentum pions produced in the Ξ decay.

The production of the Ξ decuplet ground state, Ξ(1530), and other excited Ξ's decaying to $\Xi\pi$ results in a lower momentum kaon at the upper vertex, and these heavier Ξ states produce higher momentum pions in their decays.

The lightest excited Ξ states are expected to decouple from $\Xi\pi$ and can be searched for and studied also in their decays to $\Lambda\overline{K}$ and $\Sigma\overline{K}$:

$$\gamma p \to K\,Y^* \quad \to \quad K^+\,(\,\overline{K}\Lambda\,)_{\Xi^{-*}}\,K^+, \quad K^+\,(\,\overline{K}\Lambda\,)_{\Xi^{0*}}\,K^0, \quad K^0\,(\,\overline{K}\Lambda\,)_{\Xi^{0*}}\,K^+,$$

$$\gamma p \to K\,Y^* \quad \to \quad K^+\,(\,\overline{K}\Sigma\,)_{\Xi^{-*}}\,K^+, \quad K^+\,(\,\overline{K}\Sigma\,)_{\Xi^{0*}}\,K^0, \quad K^0\,(\,\overline{K}\Sigma\,)_{\Xi^{0*}}\,K^+.$$

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Ξ Spectroscopy with the GlueX Detector

Expected yields of Ξ states using (PAC 40 proposal):

$$N = \epsilon\,\sigma\,n_\gamma\,n_t\,T \quad \text{where}$$

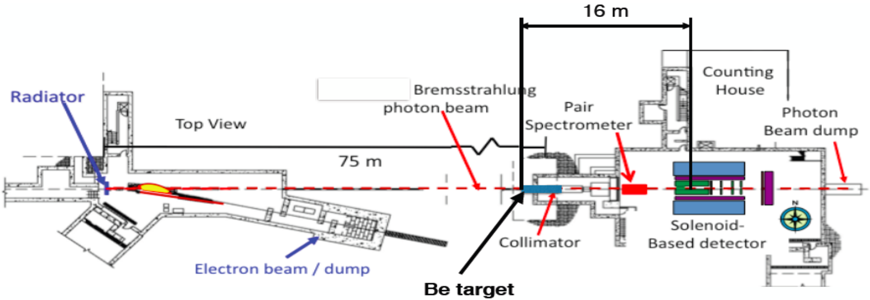
$\sigma_{\Xi(1320)} = 15$ nb and $\sigma_{\Xi(1530)} = 2$ nb at $E_\gamma = 5$ GeV

$\epsilon_{\Xi(1820)} \approx 30\,\%$ (BDT: signal purity 0.9)

➜ $800,000\ \Xi^-(1320)$ events
$100,000\ \Xi^-(1530)$ events
$90,000\ K^+K^+K^-\Lambda$ events (based on PYTHIA)

➜ At least x10 more statistics than previous CLAS result.

Opportunities with Secondary K_L^0 Beams in Hall D



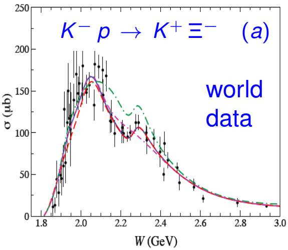
K_L^0 mesons from collimated photon beam on a Be target ($R = 2$ cm, $L = 40$ cm)

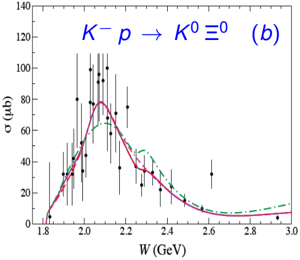
- $\Delta L \approx 16$ m (between Be & LH_2 targets); thick lead absorber to stop photons.
- $\approx 2000\ K_L^0$ / sec (x10 higher than in LASS experiment at SLAC).
- Resolution of $\Delta p/p \approx 0.3\,\%$ for K_L^0 momenta based on TOF.
- Reduced n rate compared to LASS.

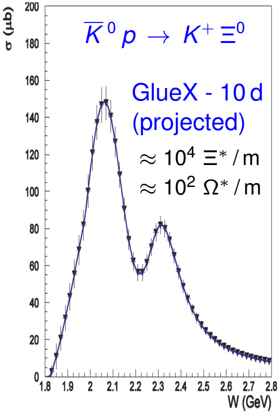
Opportunities with Secondary K_L^0 Beams in Hall D

Possible reactions to be studied (elastic and charge-exchange reactions):

- 2- & 3-body reactions producing $S = -1$ hyperons
- 2-body reactions producing $S = -2$ hyperons
  ➜ $K_L^0\, p \to K^+\, \Xi^0;\ \pi^+ K^+\, \Xi^-;\ K^+\, \Xi^{0*};\ \pi^+ K^+\, \Xi^{-*}$
- 3-body reactions producing $S = -3$ hyperons
  ➜ $K_L^0\, p \to K^+ K^+\, \Omega^-;\ K^+ K^+\, \Omega^{-*}$







Opportunities with Secondary K_L^0 Beams in Hall D

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Physics with Neutral Kaon Beam at JLab Workshop
February 1-3, 2016
Thomas Jefferson National Accelerator Facility
Newport News, VA

Outline

1. Introduction
   - The Spectrum of Baryons
   - Lattice Calculations
2. Properties of Ξ Resonances
   - Experimental Situation
   - Recent Efforts in Photoproduction
3. Cascades at GlueX
   - Opportunities with Secondary K_L^0 Beams
4. Summary and Outlook

Summary and Outlook

Baryon Spectroscopy: Are we there, yet? Certainly not ...

New era in the spectroscopy of strange baryons (GlueX, LHCb, PANDA, ...)

- Mapping out the spectrum of Ξ baryons is the primary motivation (including parity measurements); some hope for peak hunting.
- Ground-state Ξ in $\gamma p \to KK\,\Xi$ will allow the spectroscopy of Σ^* / Λ^* states.

The multi-strange baryons provide a missing link between the light-flavor and the heavy-flavor baryons. Also:

1. Do the lightest excited Ξ states in certain partial waves decouple from the $\Xi\pi$ channel, confirming the flavor independence of confinement?
2. Ξ baryons as a probe of excited hadron structure?
   ➜ Measurements of the isospin splittings in spatially excited Ξ states appear possible for the first time (similar to $n - p$ or $\Delta^0 - \Delta^{++}$).

Baryons 2016 Conference: May 16 - 20, 2016

➜ baryons2016@hadron.physics.fsu.edu

Baryons 2016
International Conference on the Structure of Baryons
May 16 - 20, 2016

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Welcome!

The BARYONS series of international conferences started in 1970 at Duke University, and since then it has been held roughly every three years in important cities around the world. Following the successful conferences in Osaka, Japan in 2010 and Glasgow, Scotland in 2013, the next in the series will be held in 2016 in Tallahassee, Florida.

The conference will highlight the physics of baryons and related subjects in particle, nuclear, and astrophysics. The main issues to be discussed are our understanding of the structure and reactions of baryons from the fundamental theory of the strong interaction, Quantum Chromodynamics (QCD). In particular, the highly non-perturbative phenomena of quark confinement, mass generation, and spontaneous breaking of chiral symmetry will be addressed. Recent developments of new and forthcoming facilities in the world will also be showcased. We strongly encourage the participation of young scientists, including students and postdocs.

Main Topics Include:

* Spectroscopy of Light / Heavy Flavor Hadrons
* Structure of Hadrons
* Recent Approaches to Non-Perturbative QCD
* Exotic Baryons
* New Facilities and Instrumentation
* ...

Baryons 2016
International Conference on the Structure of Baryons
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Important Dates (Preliminary)

• November 1, 2015:	Start Date for Abstract Submission
• December 14, 2015:	Start Date for Registration
• February 26, 2016:	Deadline for Abstract Submission
• April 1, 2016:	Deadline for Early Registration
• March/April, 2016:	Deadline for Hotel Reservations
• May 16 - 20, 2016:	Conference

Backup Slides

SU(3) Flavor Symmetry

SU(3)$_F$ symmetry is broken by the strange-light quark mass difference
➜ Corrections in quark models are substantial

However:

- Some static properties, such as the masses of the baryons, are found to obey SU(3)$_F$ symmetry:

$$\Delta m\,[N(1440)\frac{1}{2}^+ - N(939)\frac{1}{2}^+] \approx \Delta m\,[\Lambda(1600)\frac{1}{2}^+ - \Lambda(1115)\frac{1}{2}^+]$$

- Also some dynamic properties exhibit SU(3)$_F$ symmetry:
  1. Similarity of near-threshold cross sections of the reactions $\pi^- p \to \eta n$ and $K^- p \to \eta\Lambda$ (S. Prakhov *et al.*, Phys. Rev. **C72**, 015203 (2005))
  2. Similarity in the Dalitz plots for the processes $\pi^- p \to \pi^0\pi^0 n$ and $K^- p \to \pi^0\pi^0\Lambda$ (S. Prakhov *et al.*, Phys. Rev. **C69**, 042202 (2004))

SU(3) Flavor Symmetry: Implications

Conclusion:

Parallel members of different multiplets should exhibit a similar mass difference Δm.

➜ Unfortunately, we cannot compare with any Ξ states because the 2^{nd} $\Xi(?)P_{11}(J^P = \frac{1}{2}^+)$ has not been found, yet.

However, the $\mathrm{SU}(3)_F$ symmetric basis may not be ideal:
(Everything is strongly mixed.)

- Better choice (S. Capstick): *“uds” basis where you (anti-)symmetrize only in u, d quark degrees of freedom.*

  ➜ Isospin much better symmetry than $\mathrm{SU}(3)_F$.

- E.g.: $\phi_\Lambda = (ud - du)s/\sqrt{2}$ $\qquad \phi_\Sigma = uus,\ (ud + du)s/\sqrt{2},\ dds$

  In Ξ states, symmetrize only in ss pair: $\phi_\Xi = ssu,\ ssd$