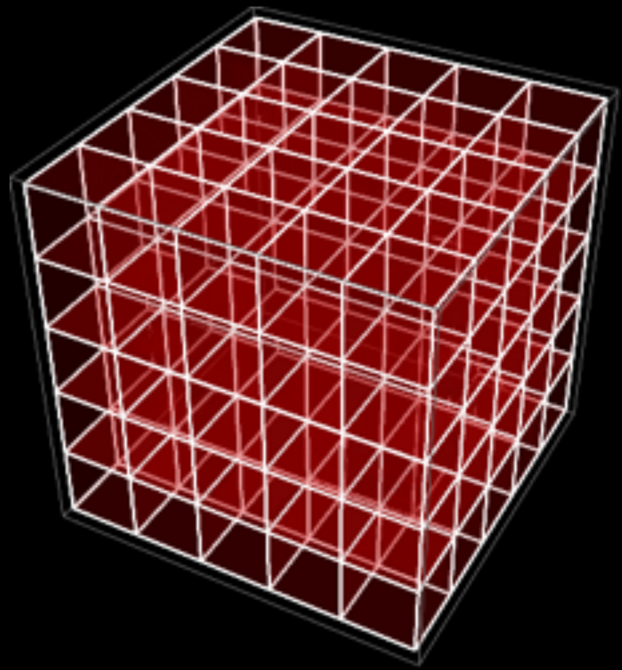
Resonances from lattice QCD



Christian B. Lang
University of Graz, Austria

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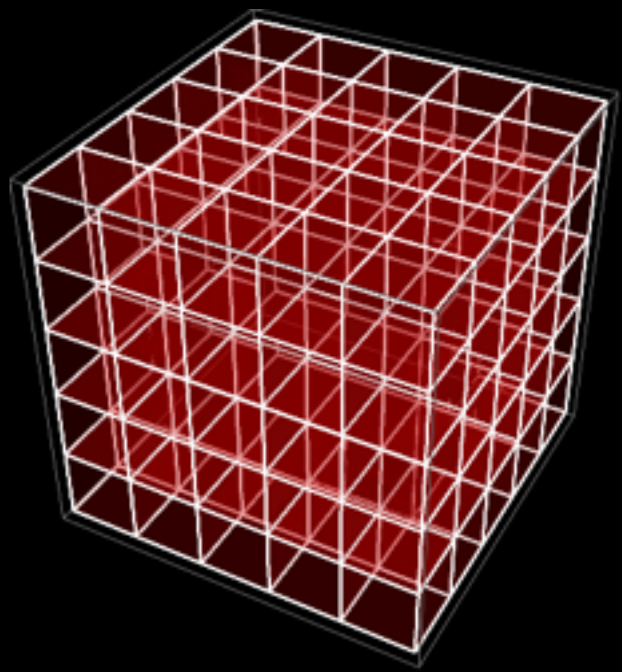

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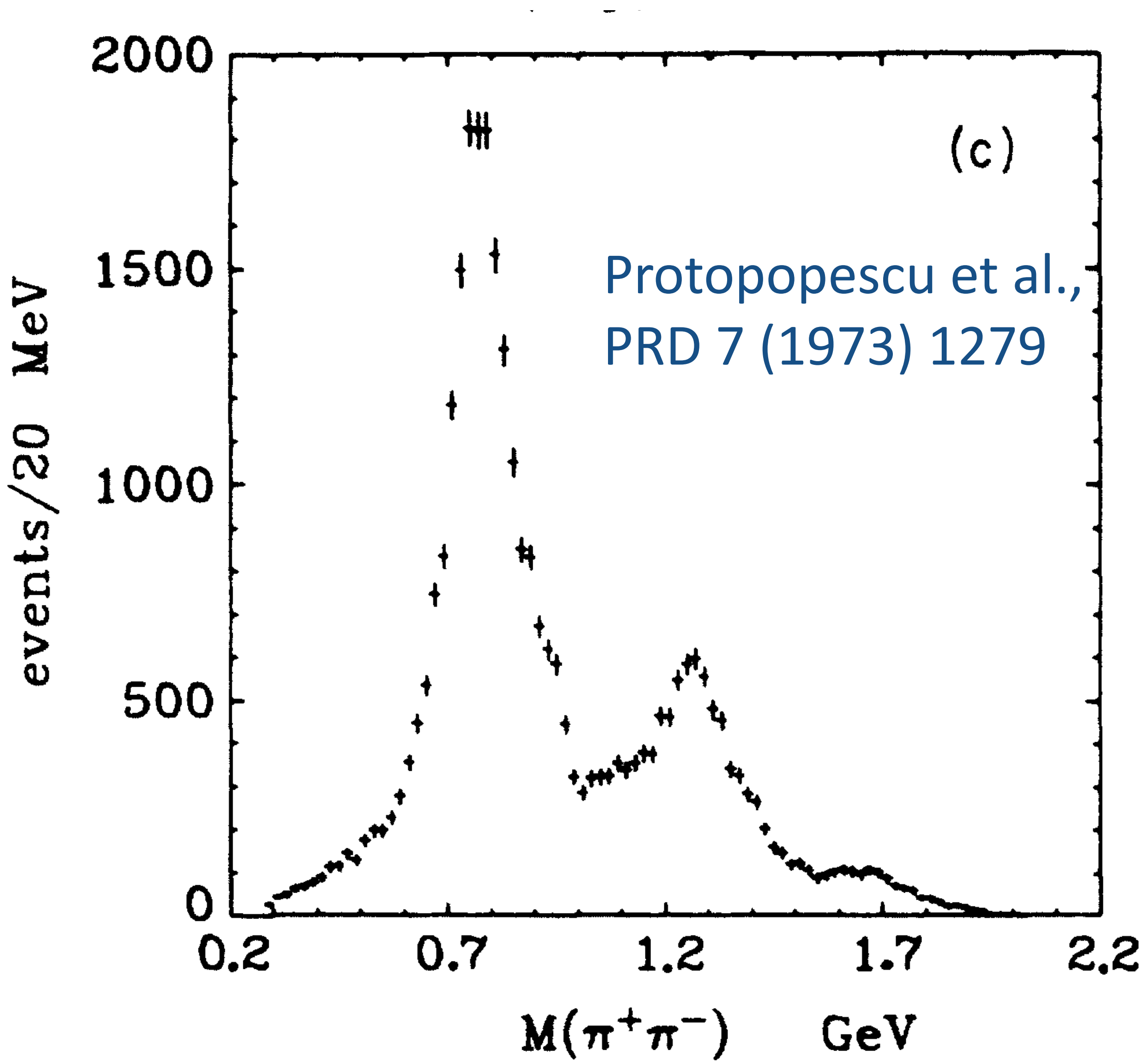
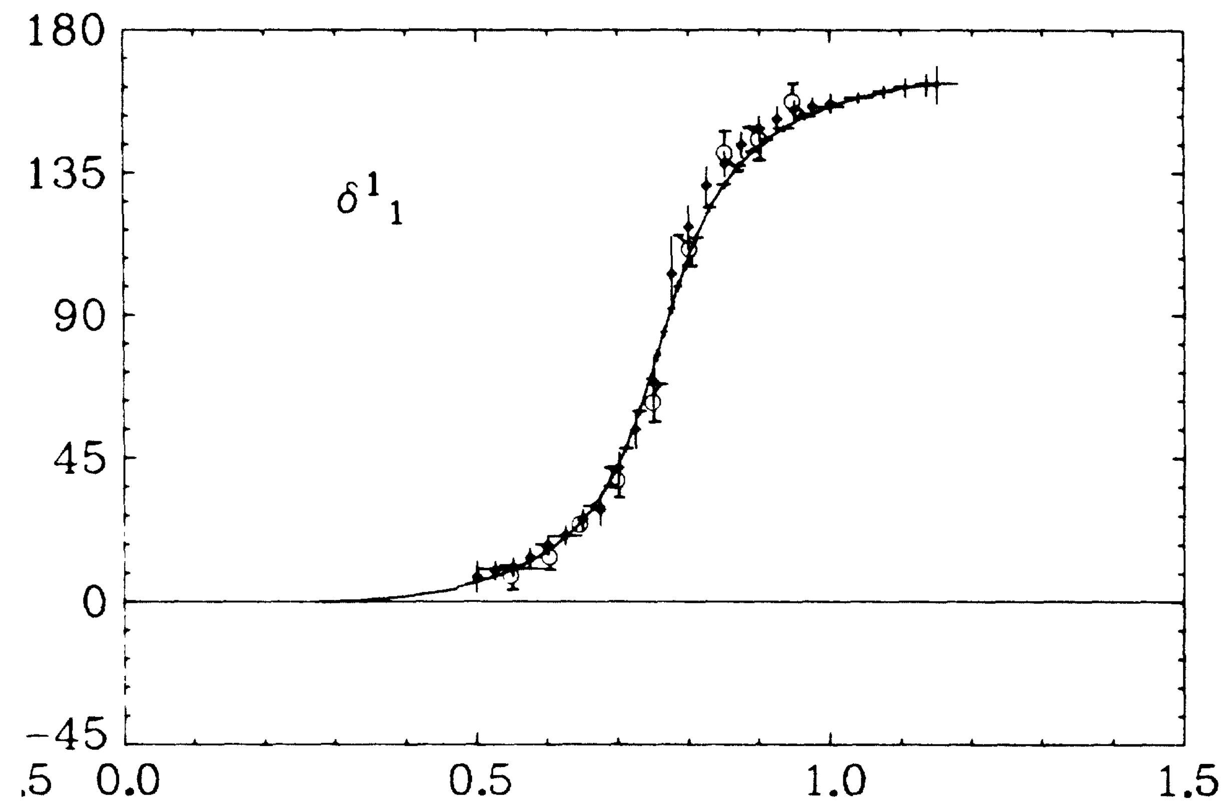
Experiment ➛ Analysis



Protopopescu et al., PRD 7 (1973) 1279

Partial wave scattering amplitudes

Phase shifts



Partial wave amplitude f_ℓ

Unitarity:

$$|S_\ell| = 1$$

$$S_\ell = 1 + 2\mathrm{i}\,\rho\, f_\ell = \mathrm{e}^{2\mathrm{i}\,\delta_\ell}$$

$$f_\ell^{-1}(s) = \rho(s)\ \cot\delta_\ell(s) - \mathrm{i}\,\rho(s)$$

$s > s_{threshold}$

$(s = E^2_{cms})$

$\rho(s)$ phase space factor

Analyticity: Partial wave amplitude f_ℓ

f_ℓ is analytic: that would be boring!

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f_ℓ is analytic: that would be boring!

f_ℓ is analytic up to cuts and poles

Cuts:

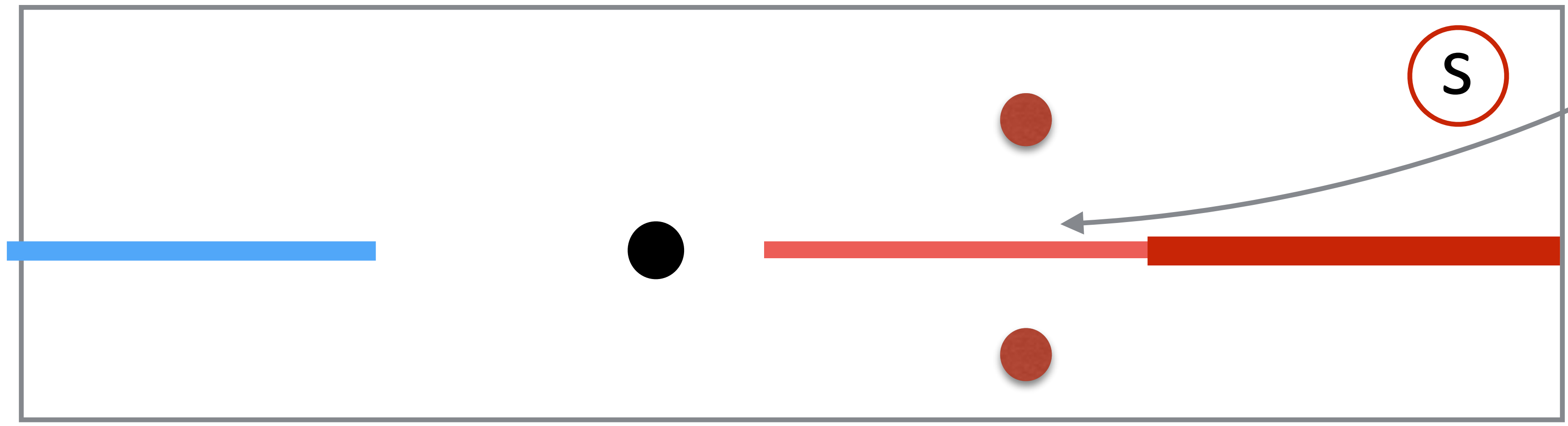
right hand cuts due to unitarity

left hand cuts due to exchange processes (crossing)

Poles:

below threshold on the real axis: bound states

in the 2nd Riemann sheet: resonances



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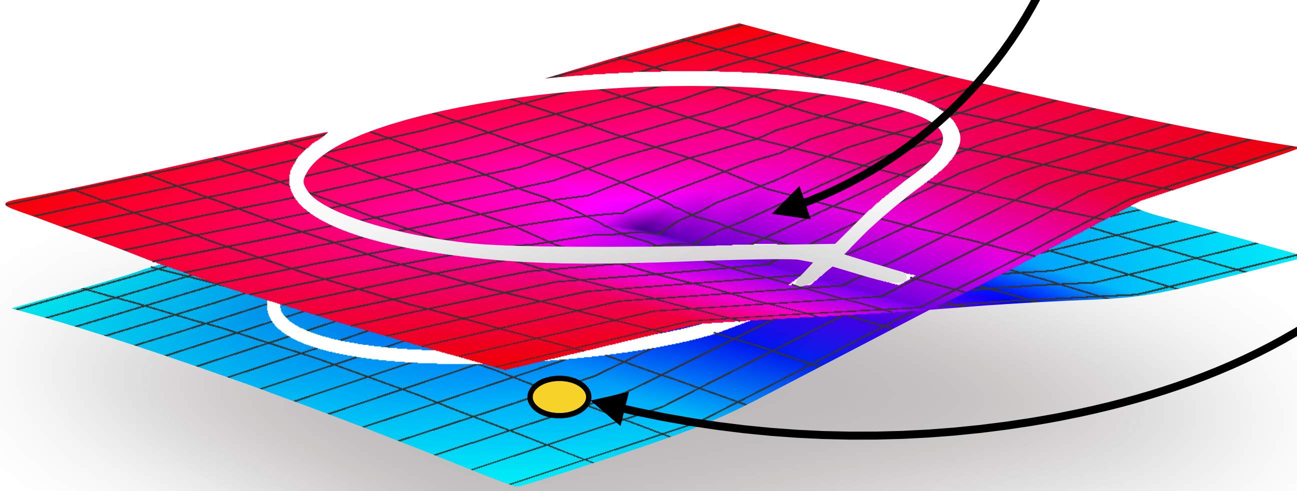
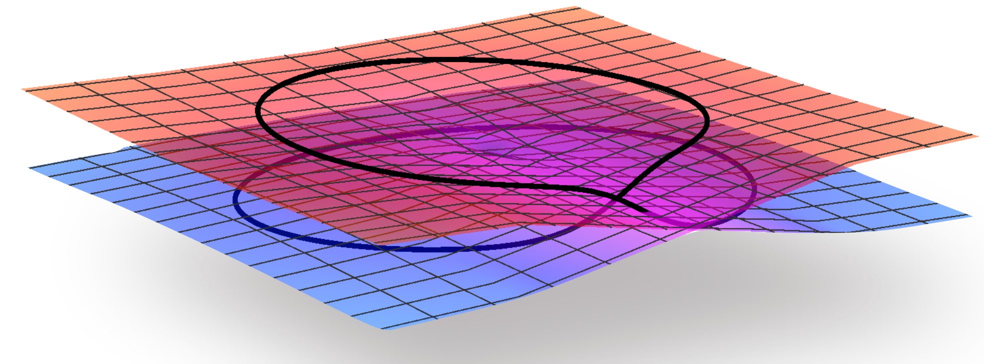
Analyticity: f_ℓ is analytic up to cuts and poles

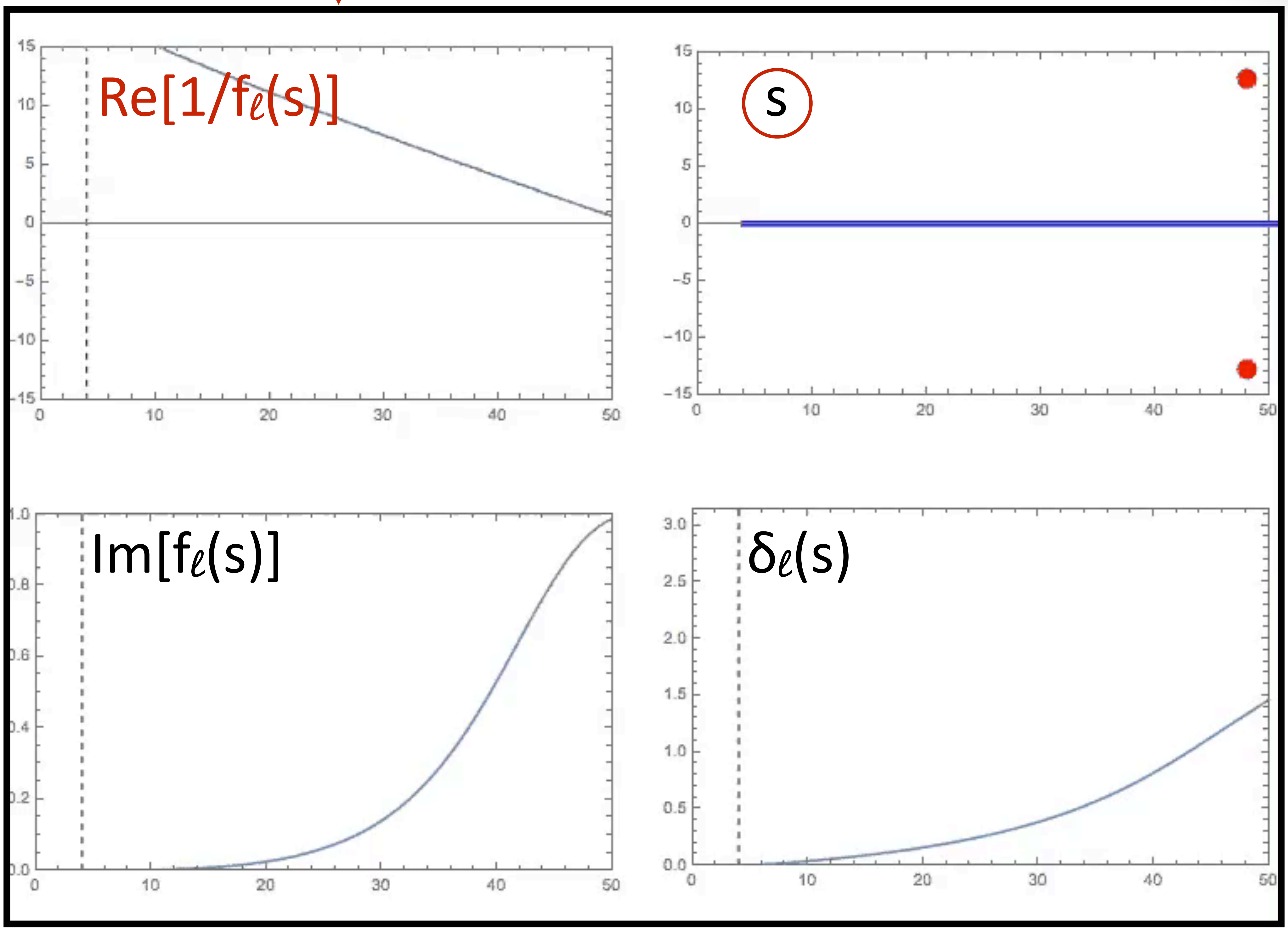
$f_\ell^{-1}(s) = 0$ ➛ pole in the complex plane



Resonance or bound state

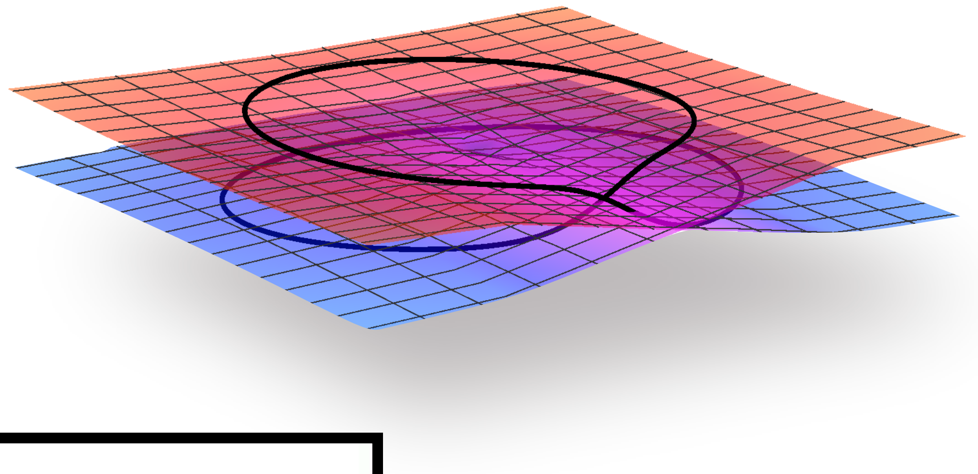


We will need this later



- pole 2nd sheet
- pole 1st sheet

Resonance or bound state



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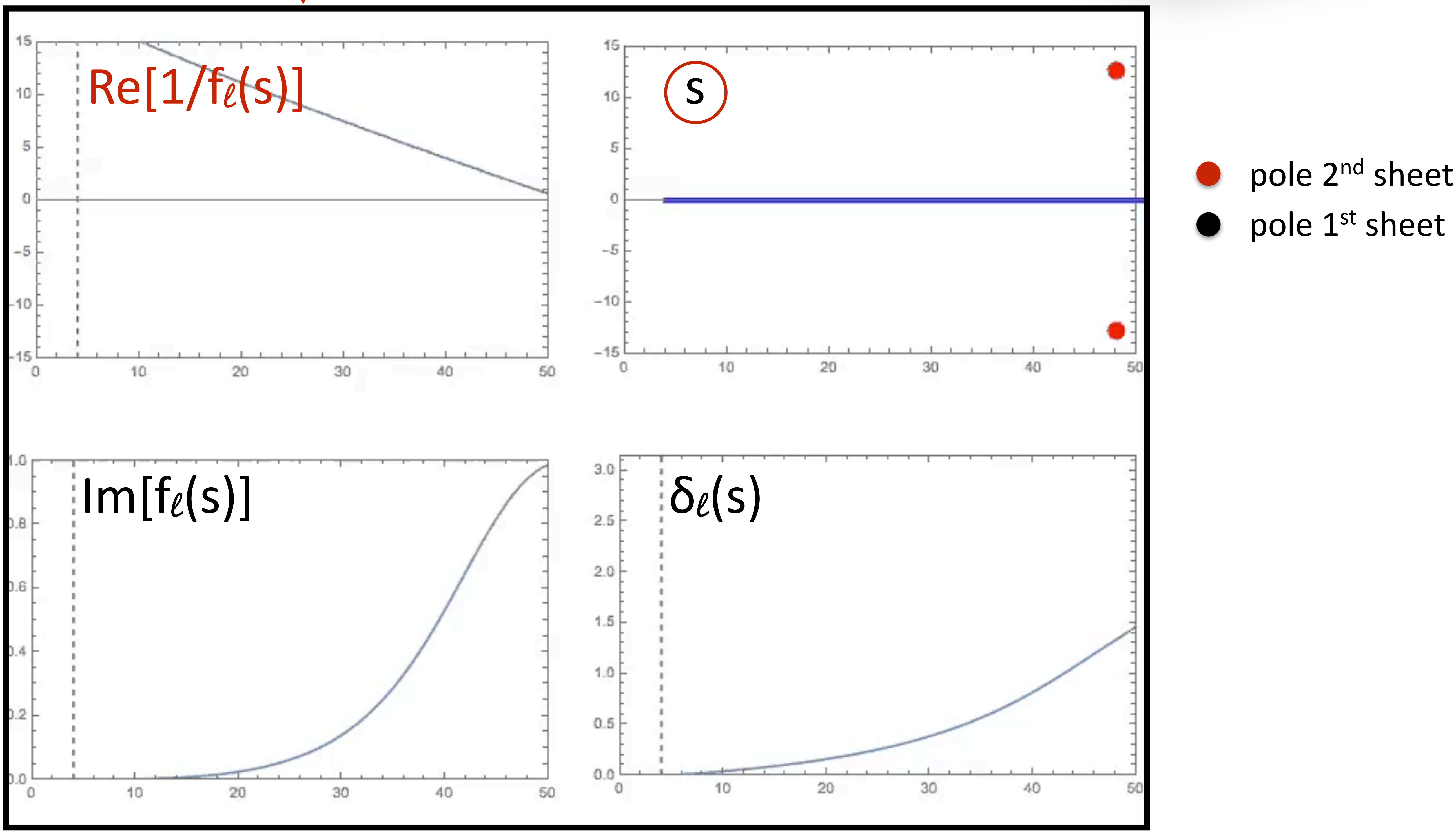
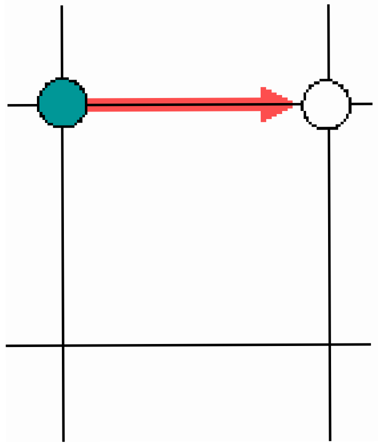


The lattice approach

Regularization: Lattice QCD *(1974)*

Ken Wilson

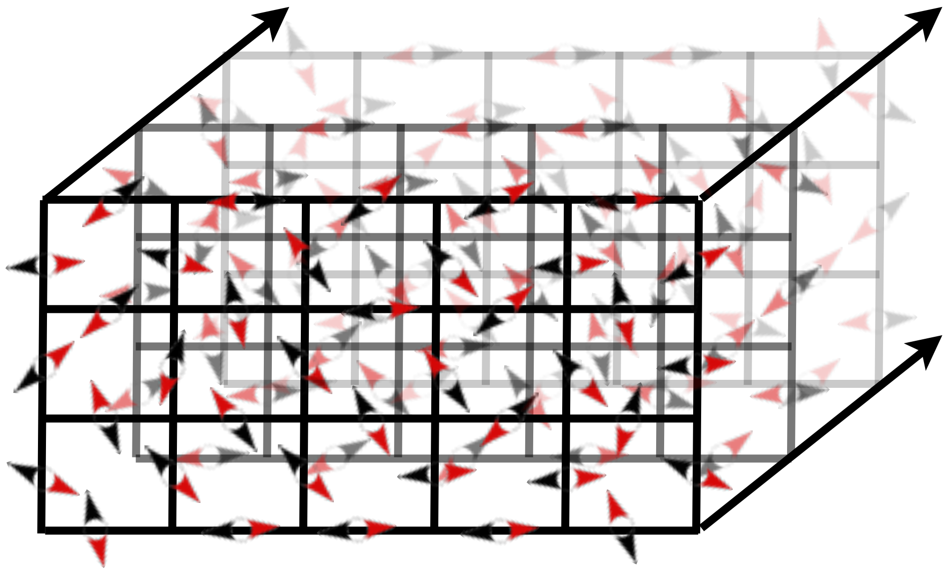


lattice spacing a

Gluon $U_\mu = e^{i\,A_\mu} \in \mathrm{SU}(3)$

Quark ψ

Antiquark $\overline{\psi}$



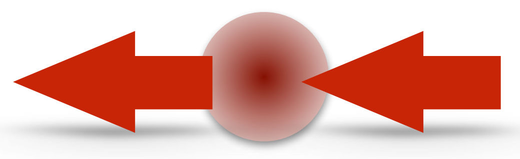
$U_\mu(x, y, z, \tau)$

Quantization:

$$\int [dU][d\psi][d\bar\psi] \to \sum_{\{U.\psi,\bar\psi\}}$$

The path integral becomes a well-defined (very large) sum over field configurations

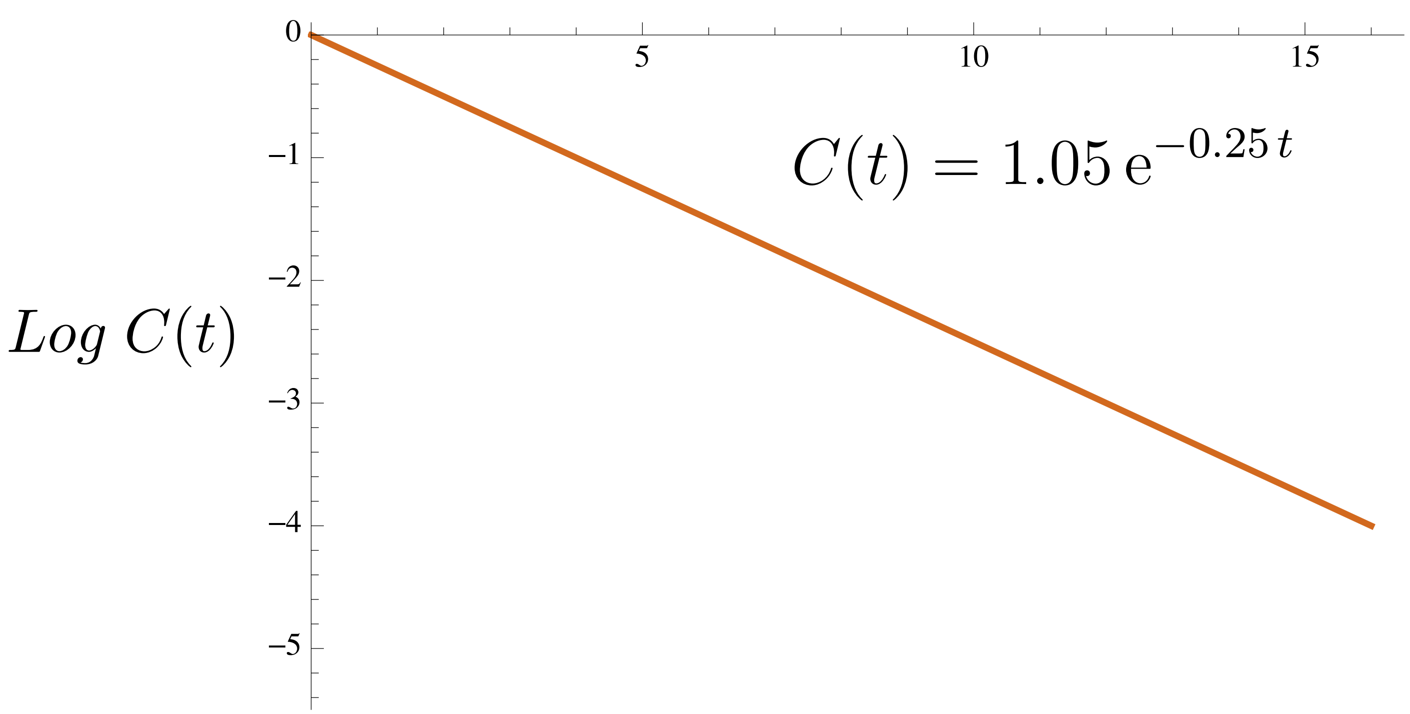
Lattice tools: correlation functions

$X_i(t)$  $\overline{X}_j(0)$

$$C_{ij}(t) \equiv \langle X_i(t)\overline{X}_j(0)\rangle = \sum_n \langle X_i|n\rangle\, \mathrm{e}^{-t\,E_n}\, \langle n|\overline{X}_j\rangle$$

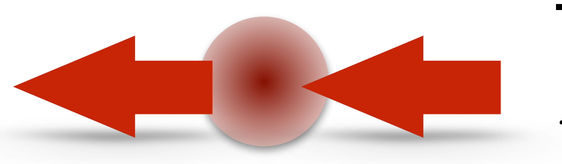
X_i lattice operator

n “physical” eigenstate



slope=
energy of
ground state

Lattice tools: correlation functions

$X_i(t)$  $\overline{X}_j(0)$

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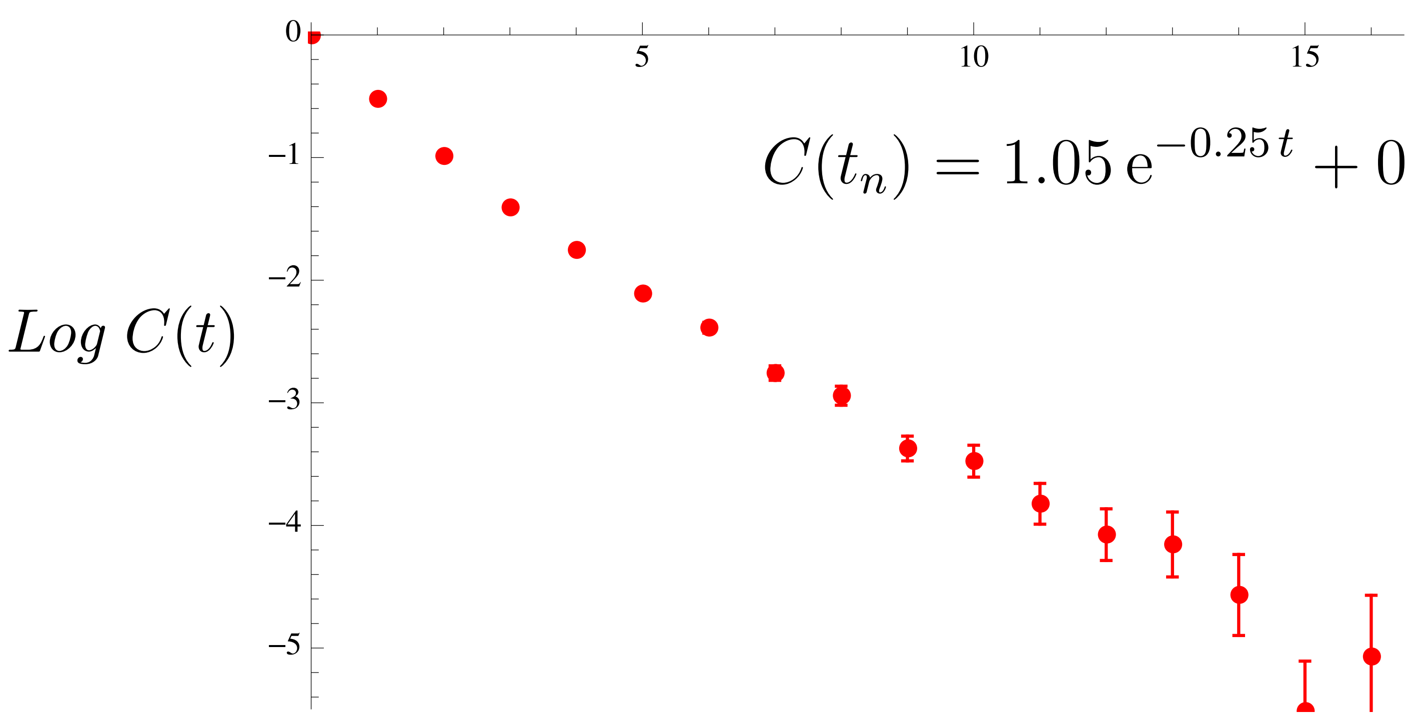


$$C(t_n) = 1.05\,\mathrm{e}^{-0.25\,t} + 0.78\,\mathrm{e}^{-0.55\,t} + 0.54\,\mathrm{e}^{-0.85\,t} + \ldots + \text{noise}$$

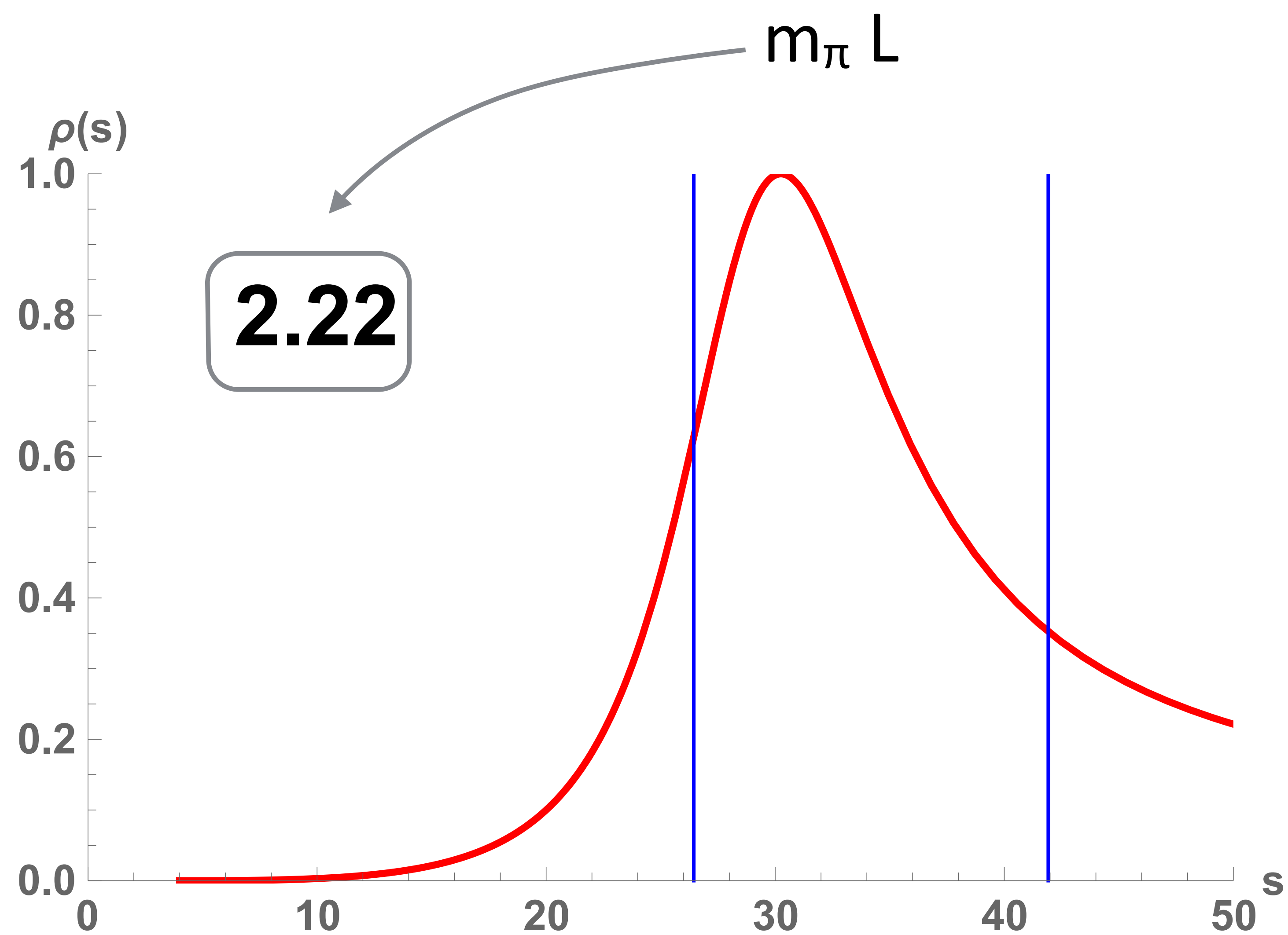
slope=
energy of
ground state

Continuum vs. lattice

Correlation functions have discrete energy levels!

Example:

Spectral density of a simple resonance in continuum and the discrete energies for a lattice volume



One cannot arbitrarily fix the energies: they are eigenvalues depending on the control parameters (volume, couplings,...).

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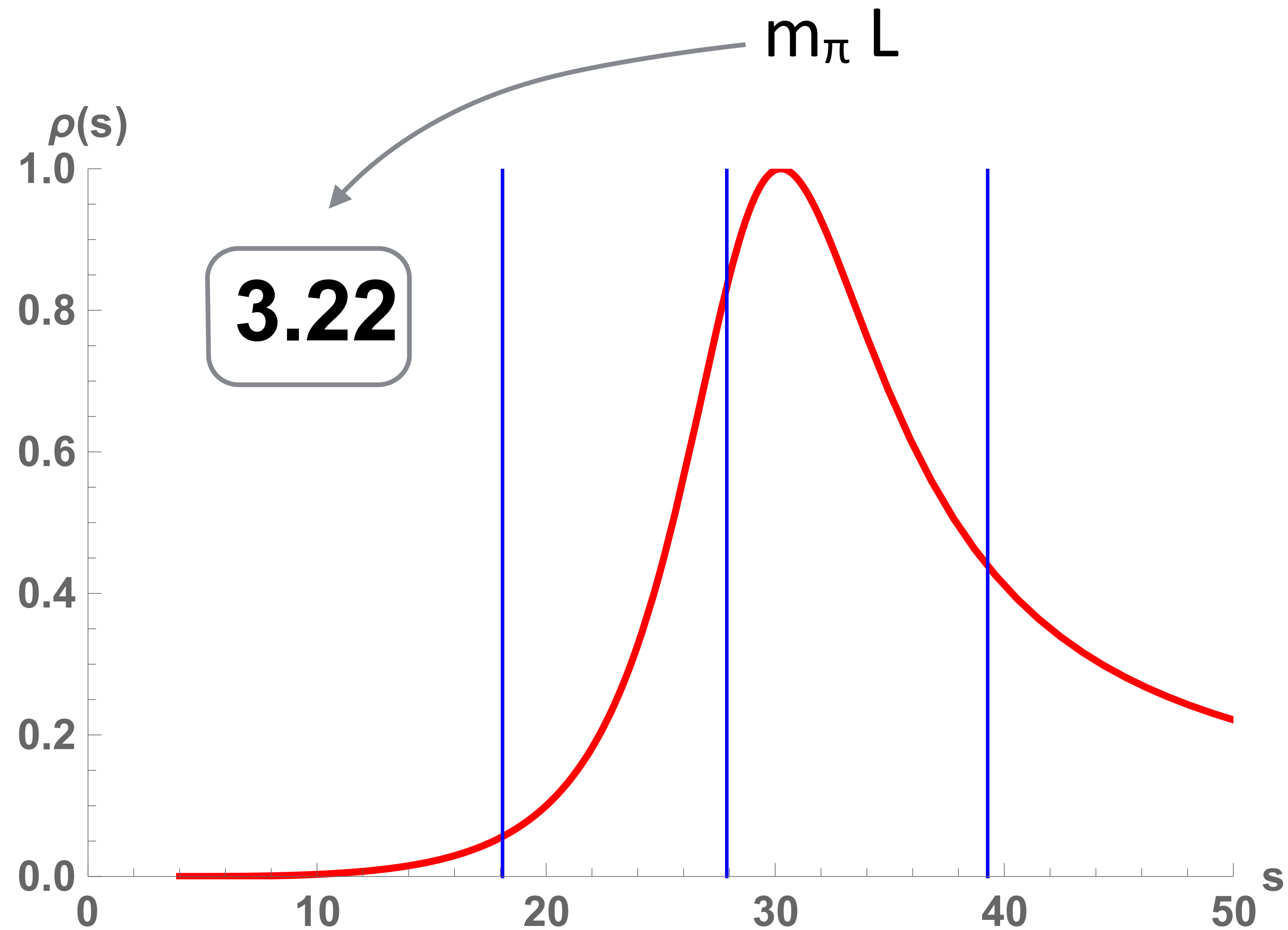

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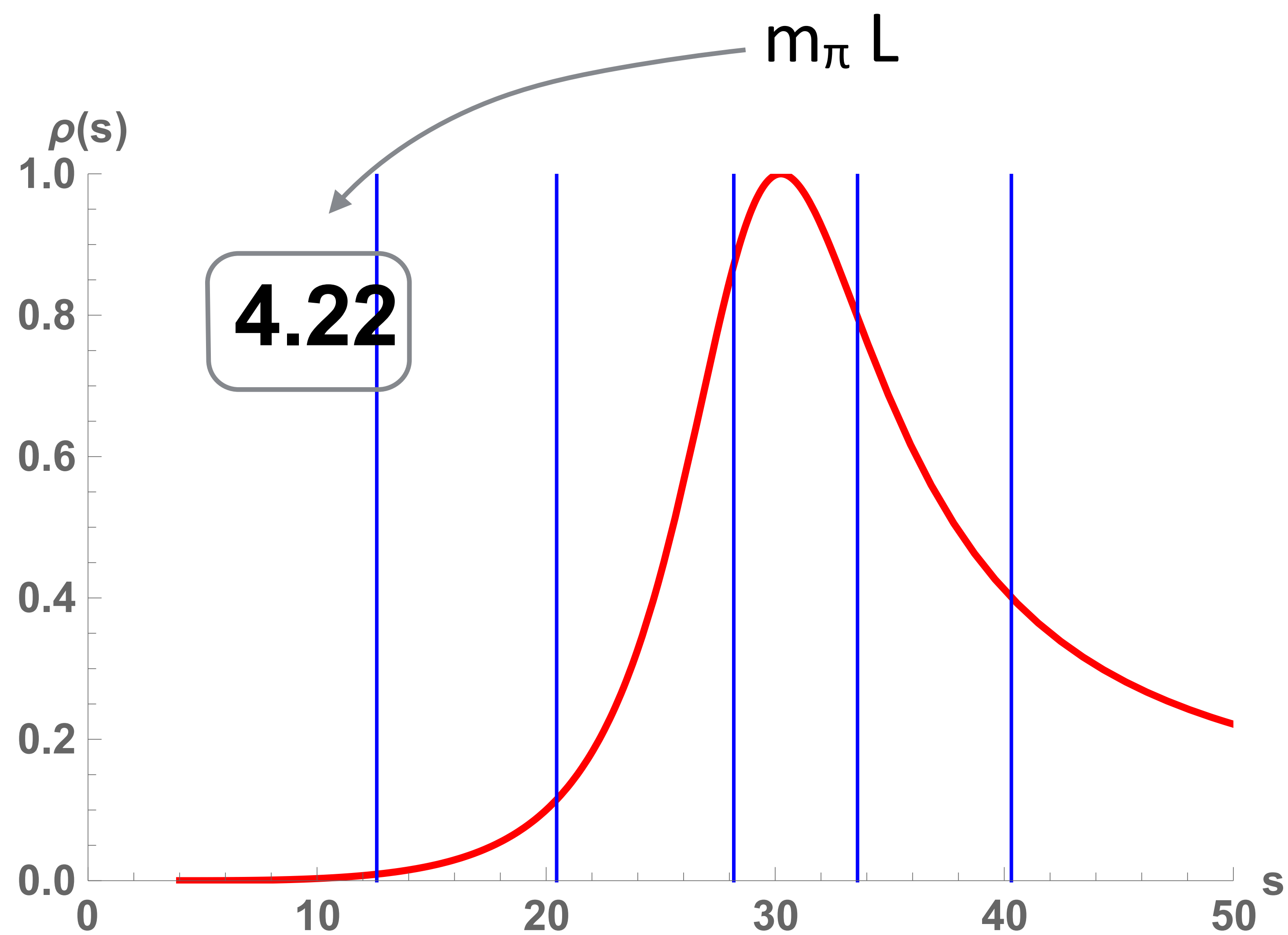

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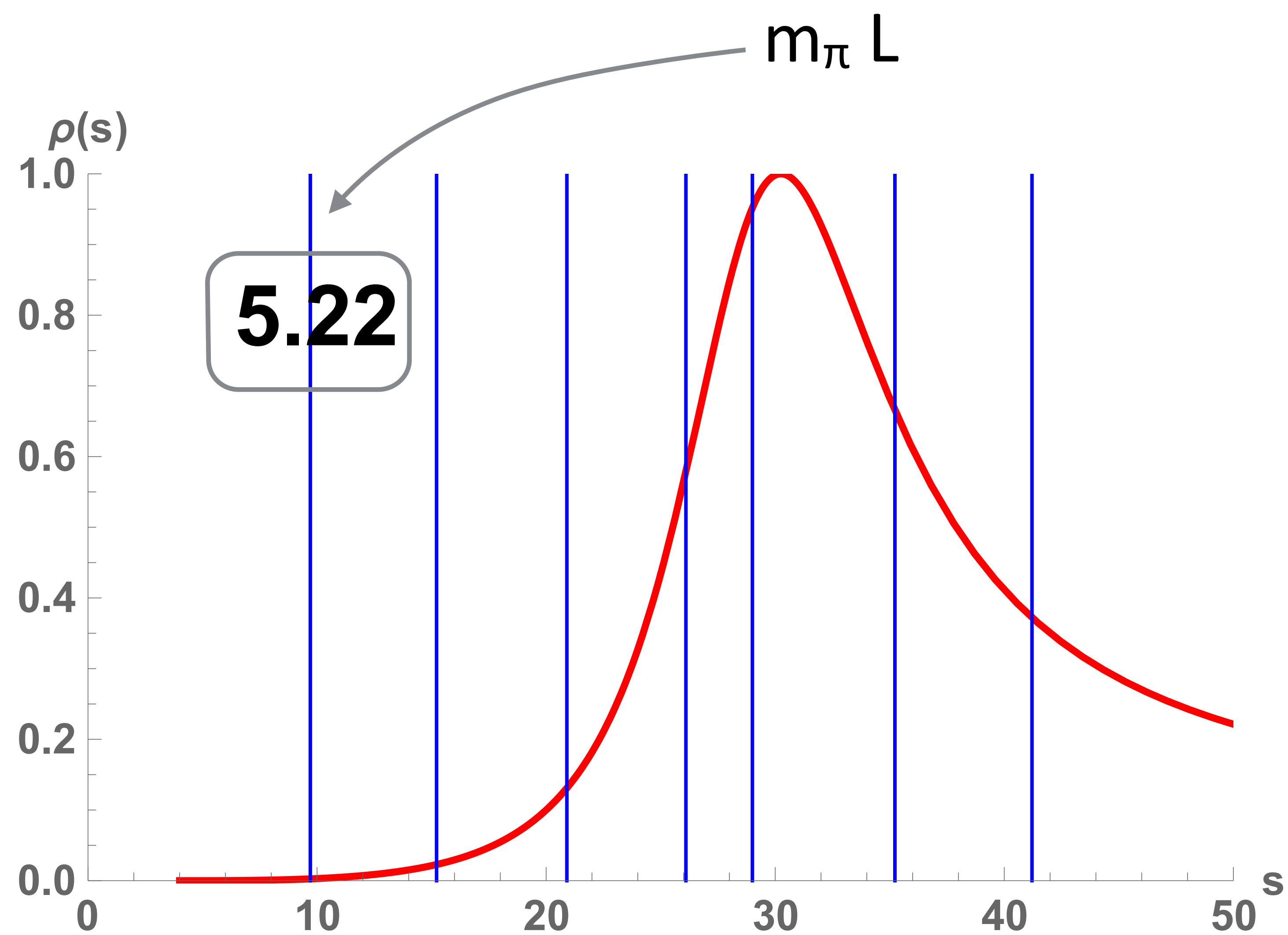

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Spectroscopy

Ground state spectroscopy

Is correct only for stable particles.
Most hadrons are resonances:
We need to study excited states!

Excited states spectroscopy

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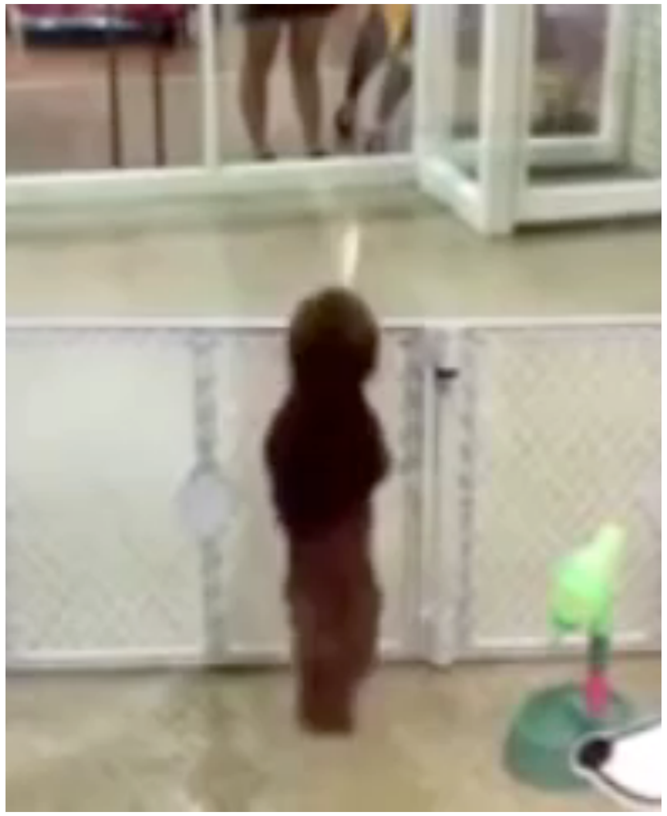
Excited states spectroscopy



Example for an excited state

How to get the energy levels?

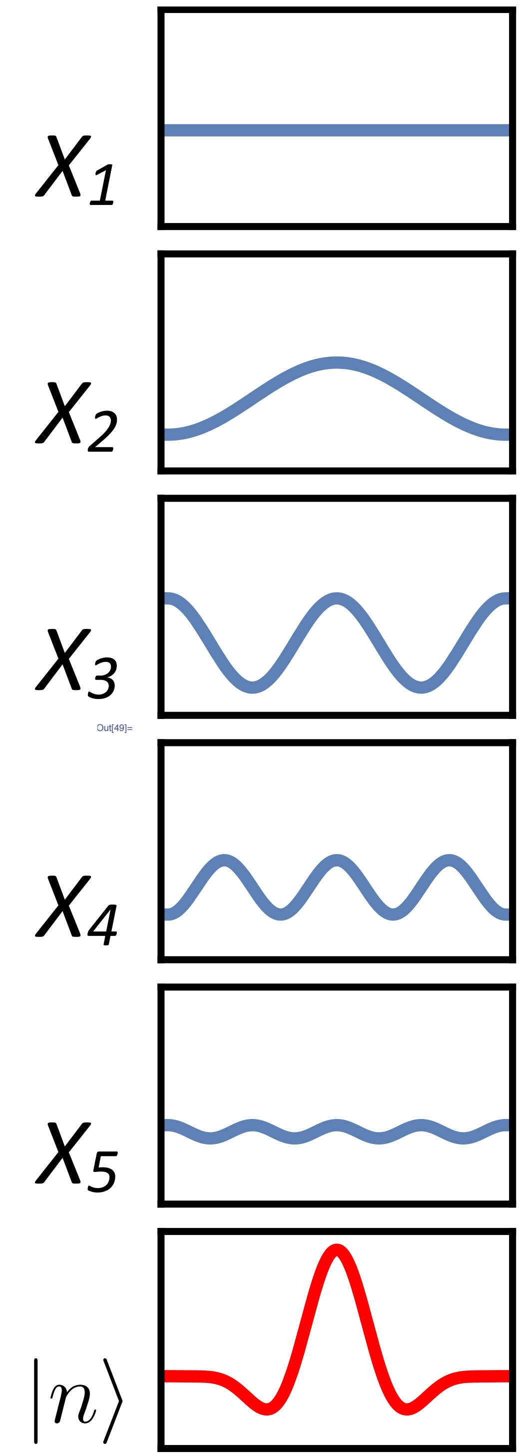
Lüscher, Wolff: NPB339(90)222
Michael, NPB259(85)58
See also Blossier et al., JHEP0904(09)094

- Compute all cross-correlations for <u>several</u> lattice operators

  $$C_{ij}(t) \equiv \langle X_i(t) \overline{X}_j(0) \rangle$$

- Solve the eigenvalue problem. The eigenvalues give the energy levels:

$$\lambda^{(n)}(t) \propto e^{-t\,E_n}\left(1 + \mathcal{O}(e^{-t\Delta E_n})\right)$$



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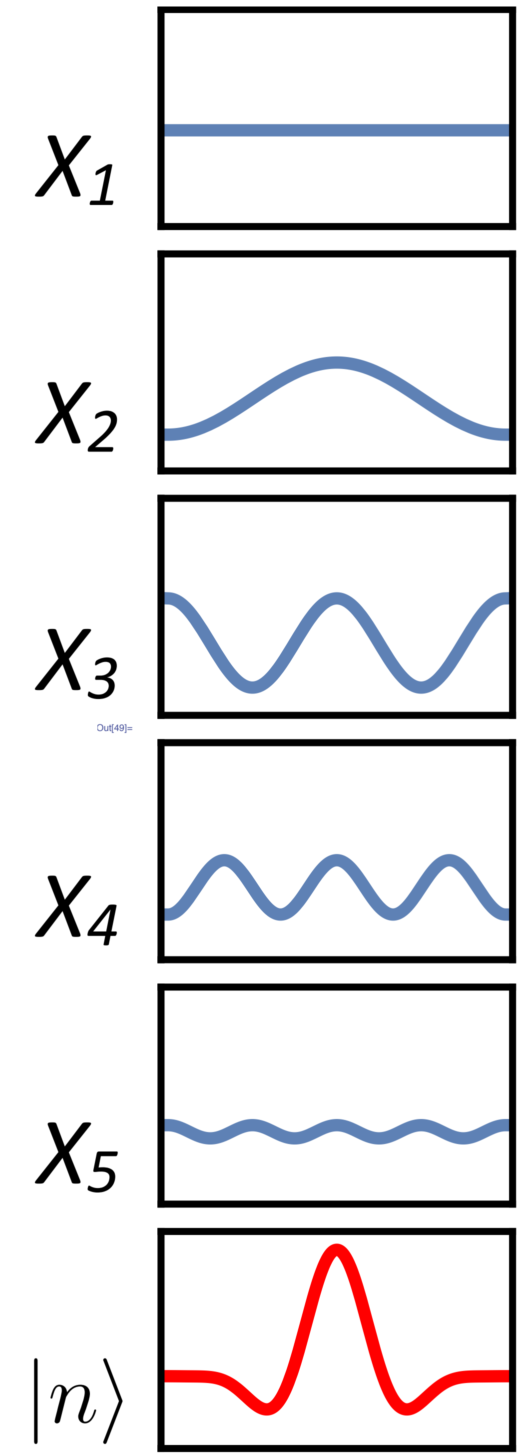

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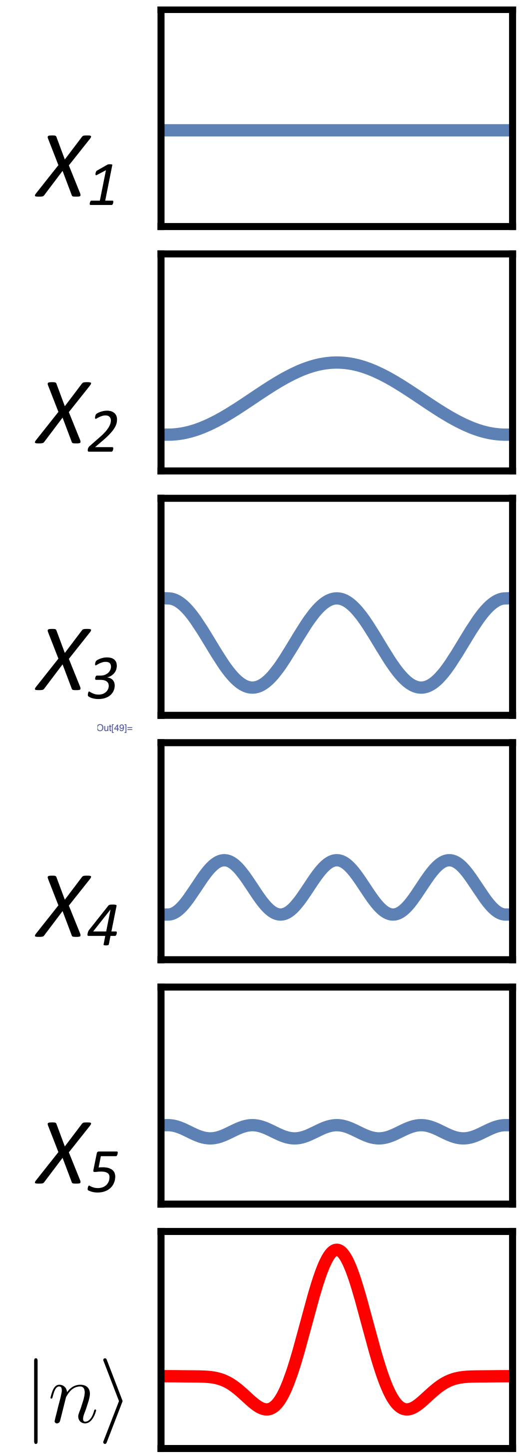
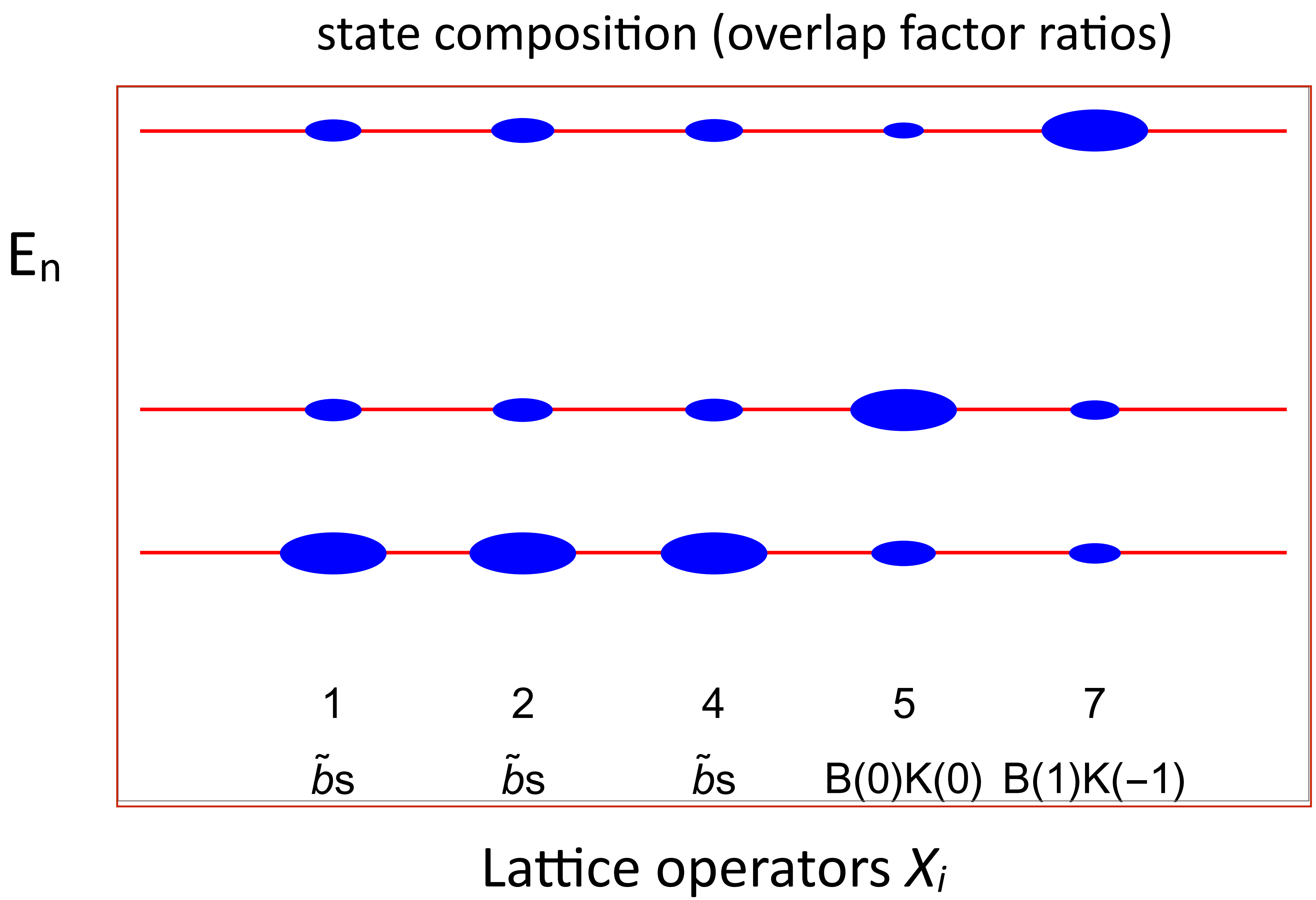
- Solve the eigenvalue problem. The eigenvalues give the energy levels:
  $$\lambda^{(n)}(t) \propto e^{-t\boxed{E_n}}\left(1+\mathcal{O}(e^{-t\Delta E_n})\right)$$
- The eigenvectors are "fingerprints" of the state and allow to identify the "composition" of the state:

  overlap factors $\langle X_i|n\rangle$



Example: Ratios of overlap factors



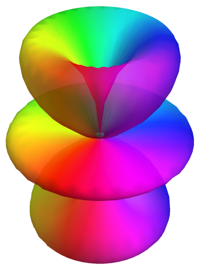
BK scattering in $J^{PC}=0^{++}$ near threshold

CBL, Mohler et al., Physics Letters B 750 (2015) 17

Lattice operators (interpolators) X_i

Irreps of cubic group and its little groups contribute to different angular momenta in continuum

Moore & Fleming , Phys. Rev. D 73, 014504 (2006)
Leskovec, & Prelovsek, PR D85 (2012) 114507
Göckeler et al., PR D 86, 094513 (2012)



J	irreps
0	$A_1(1)$
1	$T_1(3)$
2	$T_2(3) \oplus E(2)$
3	$T_1(3) \oplus T_2(3) \oplus A_2(1)$
4	$A_1(1) \oplus T_1(3) \oplus T_2(3) \oplus E(2)$

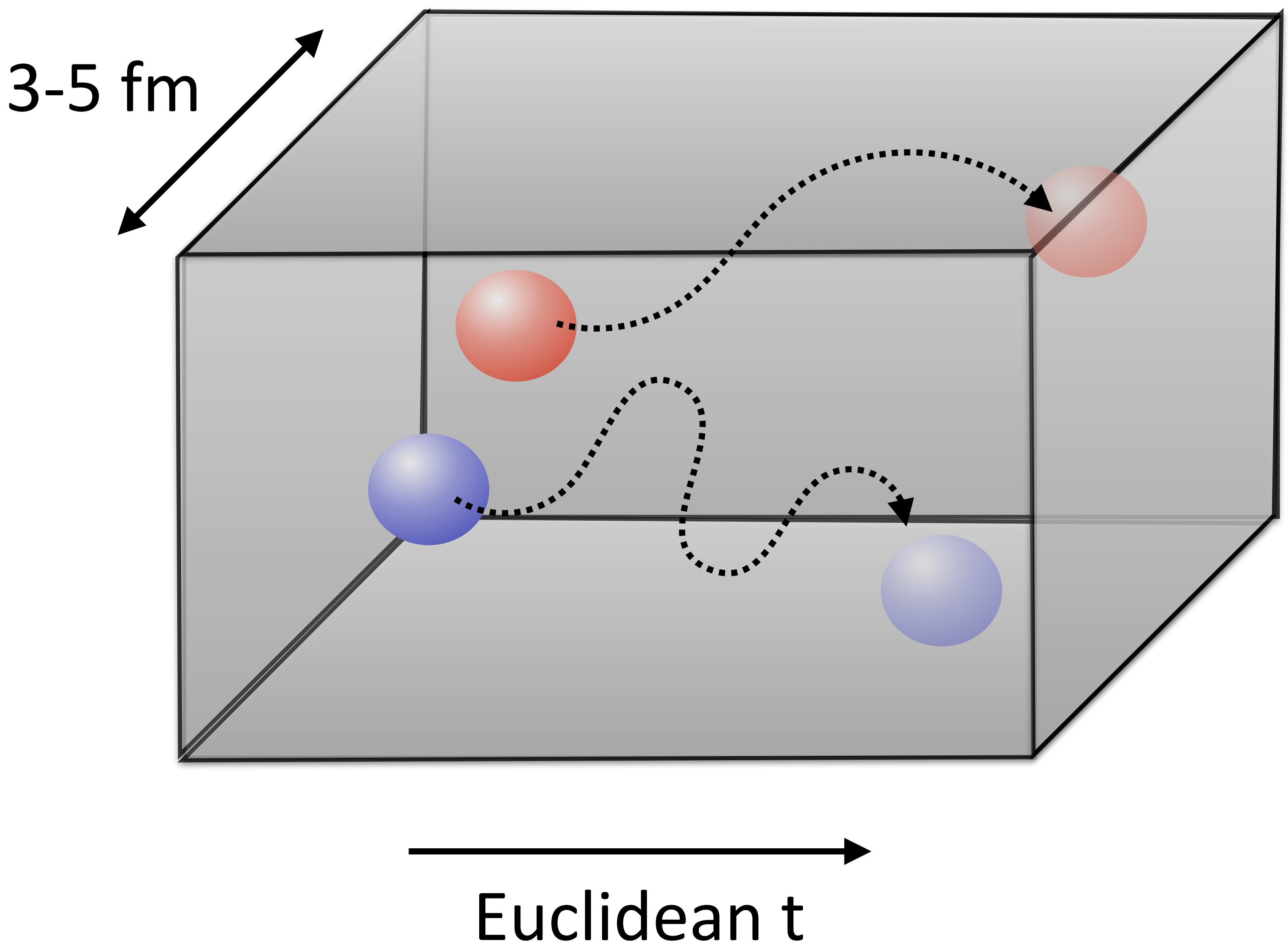
Construction of lattice operators by projection from continuum (subduction)

Dudek et al. (HSC), Phys. Rev. D 82, 034508 (2010)

Construction of multi-particles states

Moore et al., Phys. Rev. D 74, 054504 (2006)
Thomas et al. (HSC),Phys. Rev. D 85, 014507 (2012)
Wallace [arXiv:1506.05492]

Femto universe



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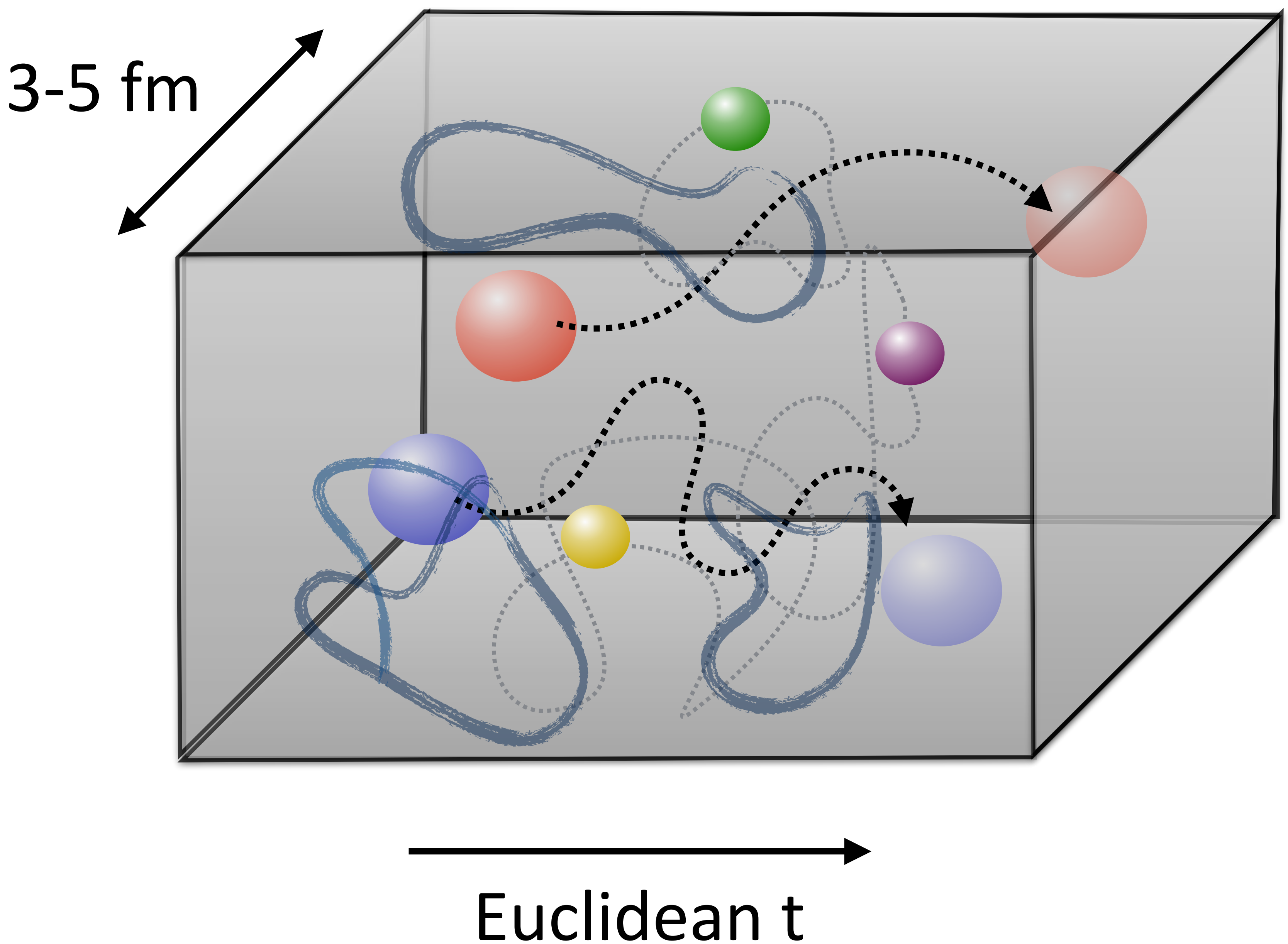
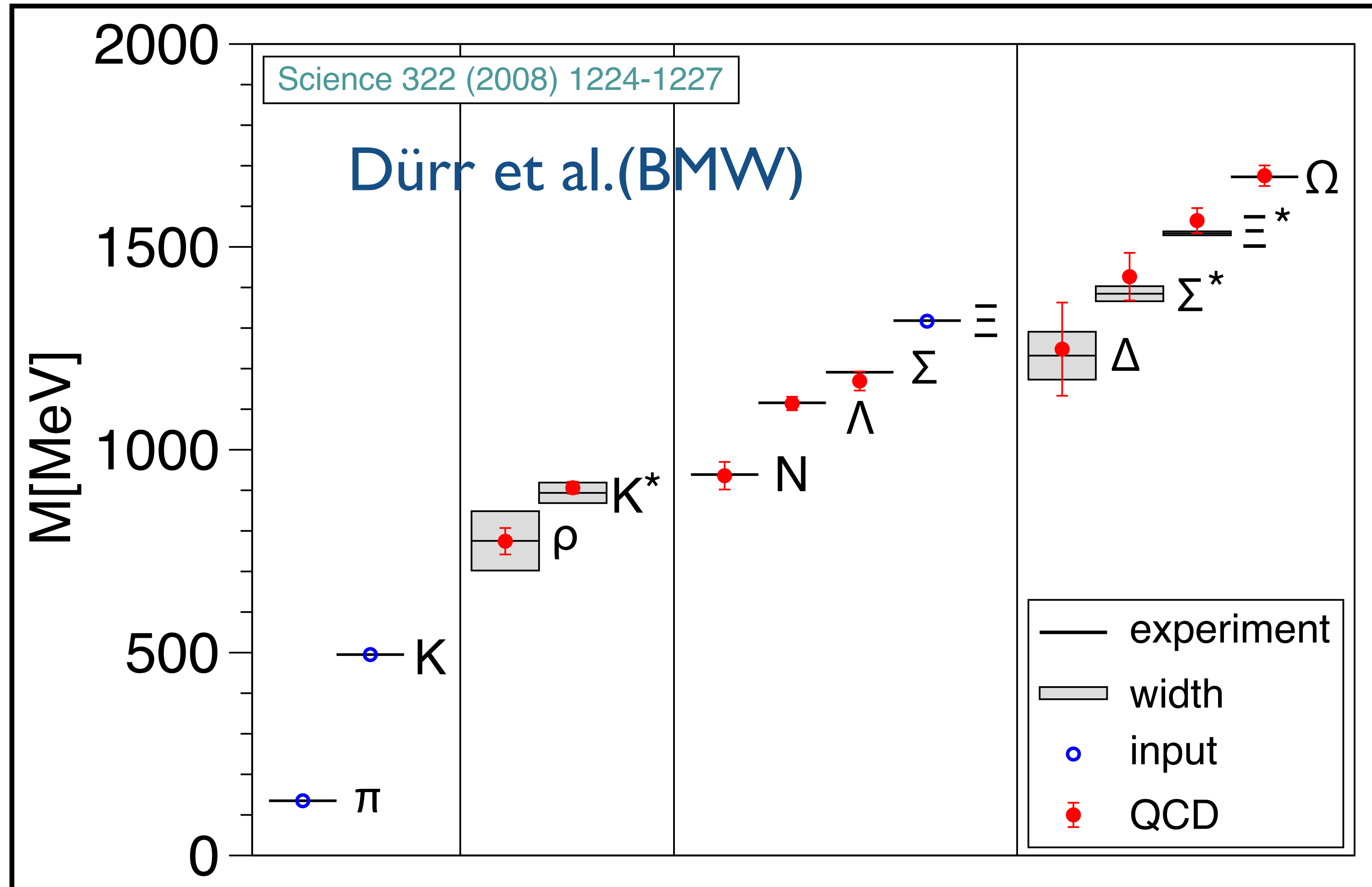


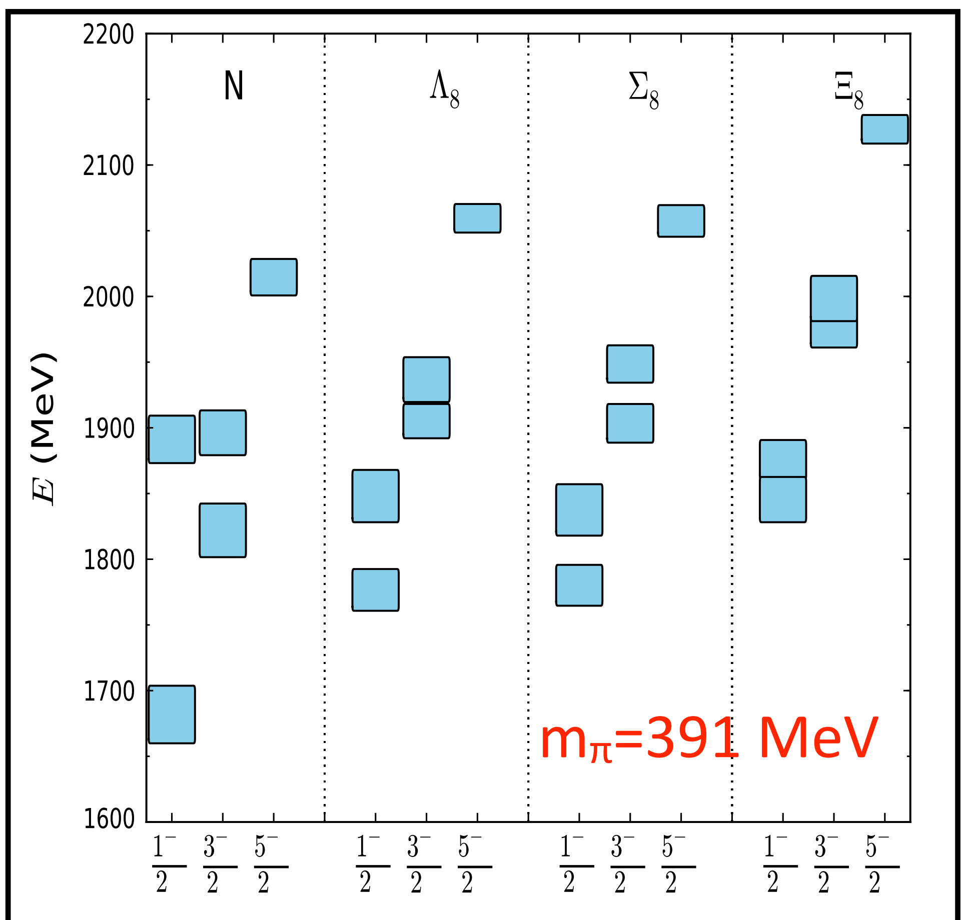
Milestones

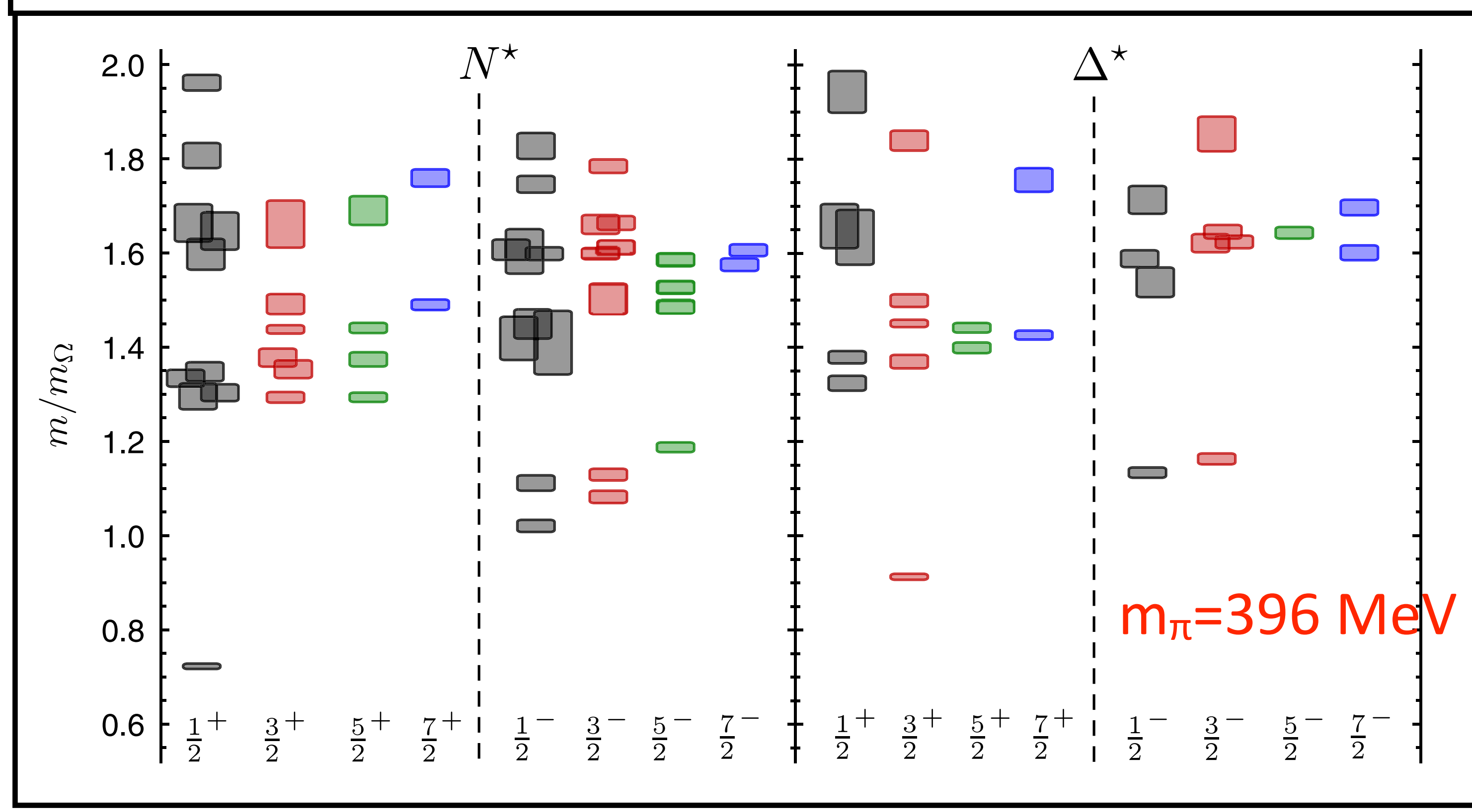
Single hadron approximation

BMW(2008)

HSC(2011, 2013)







Edwards et al. (HSC) Phys.Rev. D87, 054506 (2013). Edwards et al. (HSC) PR D 84, 074508 (2011)

Beyond the single hadron approximation

Spectroscopy

Ground state spectroscopy

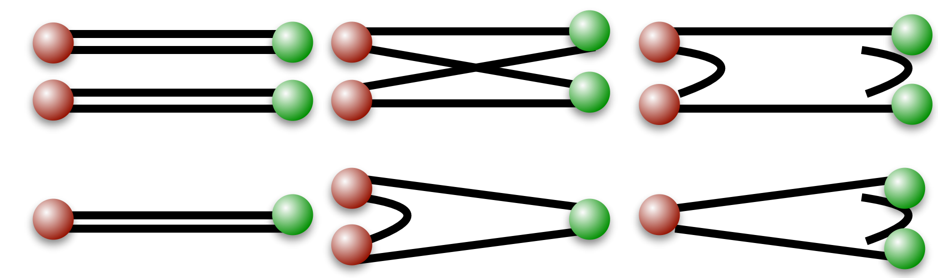
Is correct only for stable particles.
Single hadron approach qqq or qq
is valid only below scattering threshold
("bound states" or "artificial bound states")

Resonances and bound states

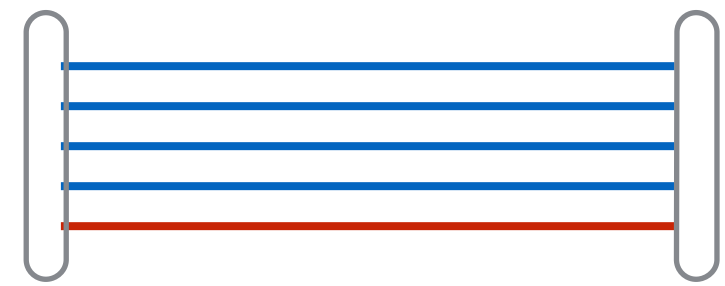
require inclusion of hadron-hadron channels in the calculation.
Multi-hadron approach: we need to extend the space of operators to multi-hadron operators: $(q\underline{q})(q\underline{q}),(qqq)(q\underline{q}),(qqq)(\underline{qqq})$...

What is the challenge?

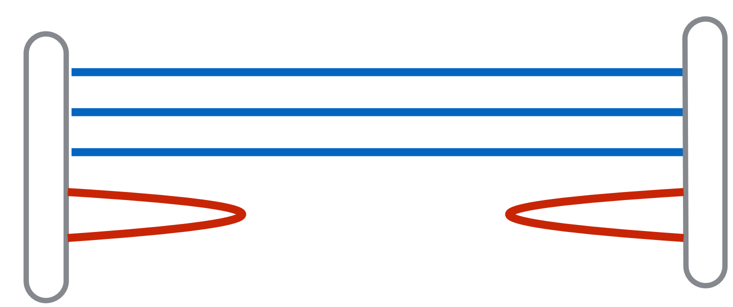
More terms



More quark propagators

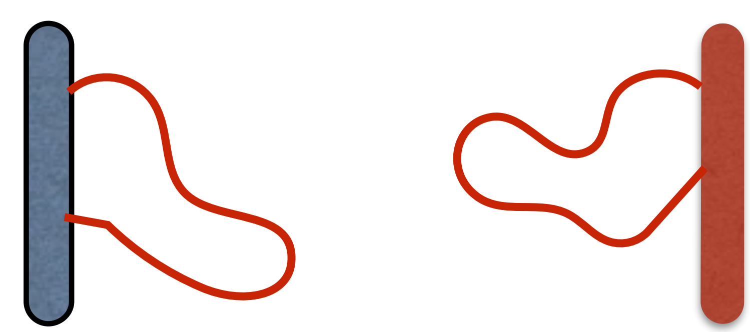


Backtracking loops are expensive!



“All-to-all propagators”:

- Stochastic sources
- Distillation



Peardon et al. (HSC), PR D 80, 054506 (2009).
Morningstar et al., PR D 83, 114505 (2011).

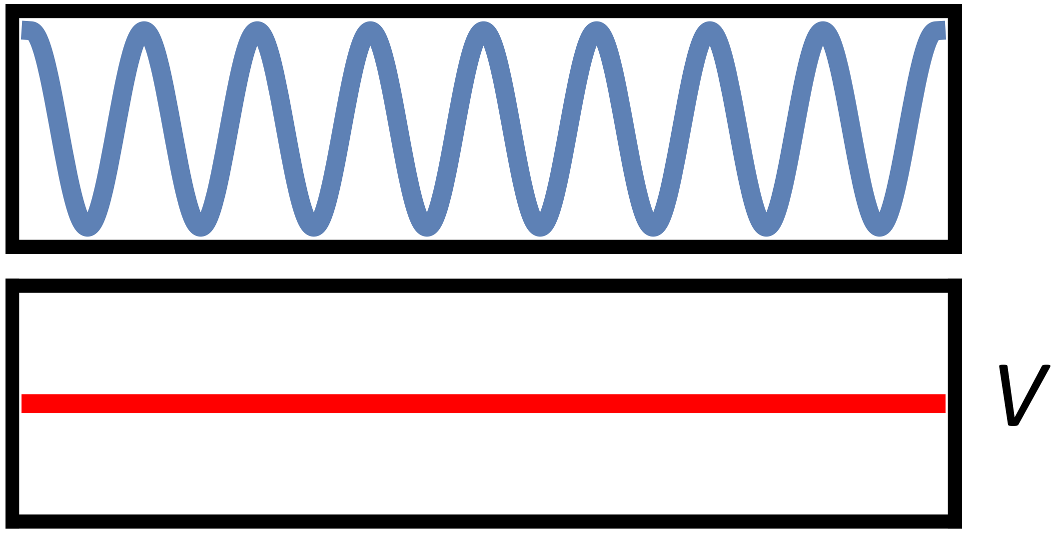
Energy levels ➤ Phase shift points (in the elastic region)

Lüscher, CMP 105(86) 153,
NP B354 (91) 531, NP B 364 (91) 237

$$e^{\mathrm{i}\,k\,L} = 1 \quad \longrightarrow k_n = 2\,n\,\pi/L$$

(e.g. for L=3 fm:
k_1=400 MeV)

periodic b.c.



$V = const.$ $\delta = 0$

Cf., 2d resonance example:
Gattringer & cbl, NPB391 (93) 463

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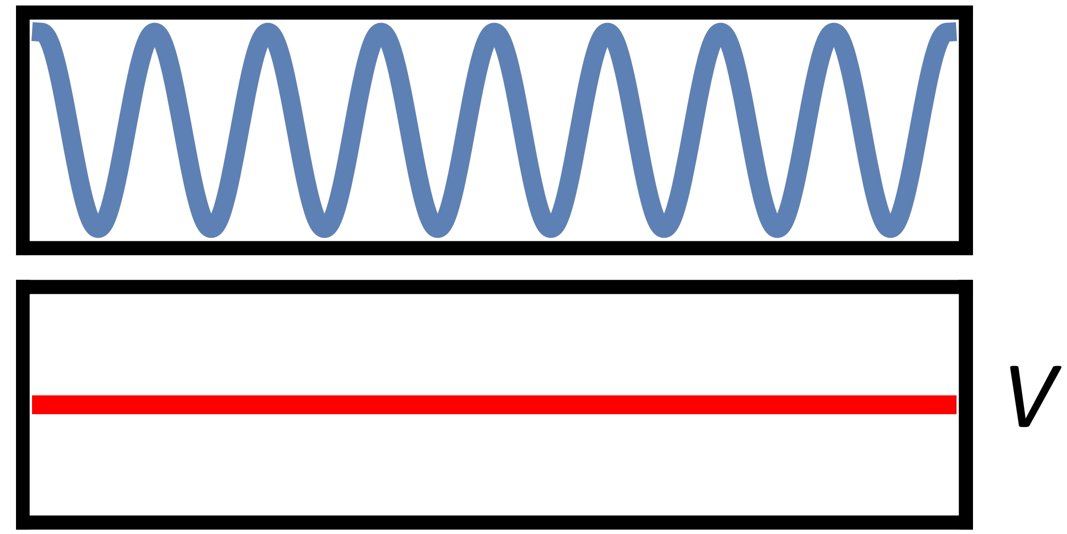
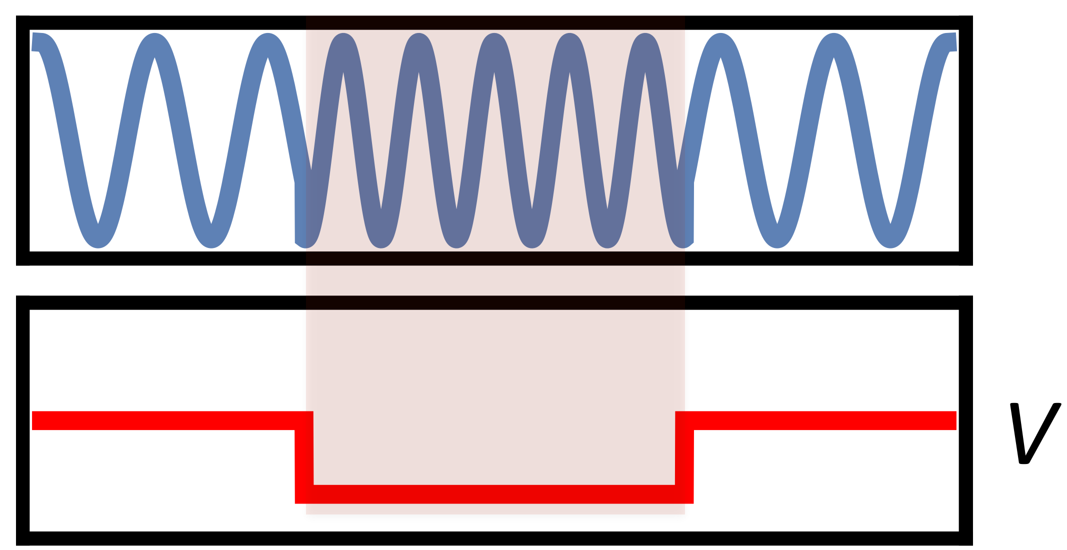


$V = const. \qquad \delta = 0$

$$\mathrm{e}^{\mathrm{i}\,k\,L+2\mathrm{i}\delta(k)} = 1$$
$$\longrightarrow 2\,\delta(k_n) = 2n\pi - k_n L$$



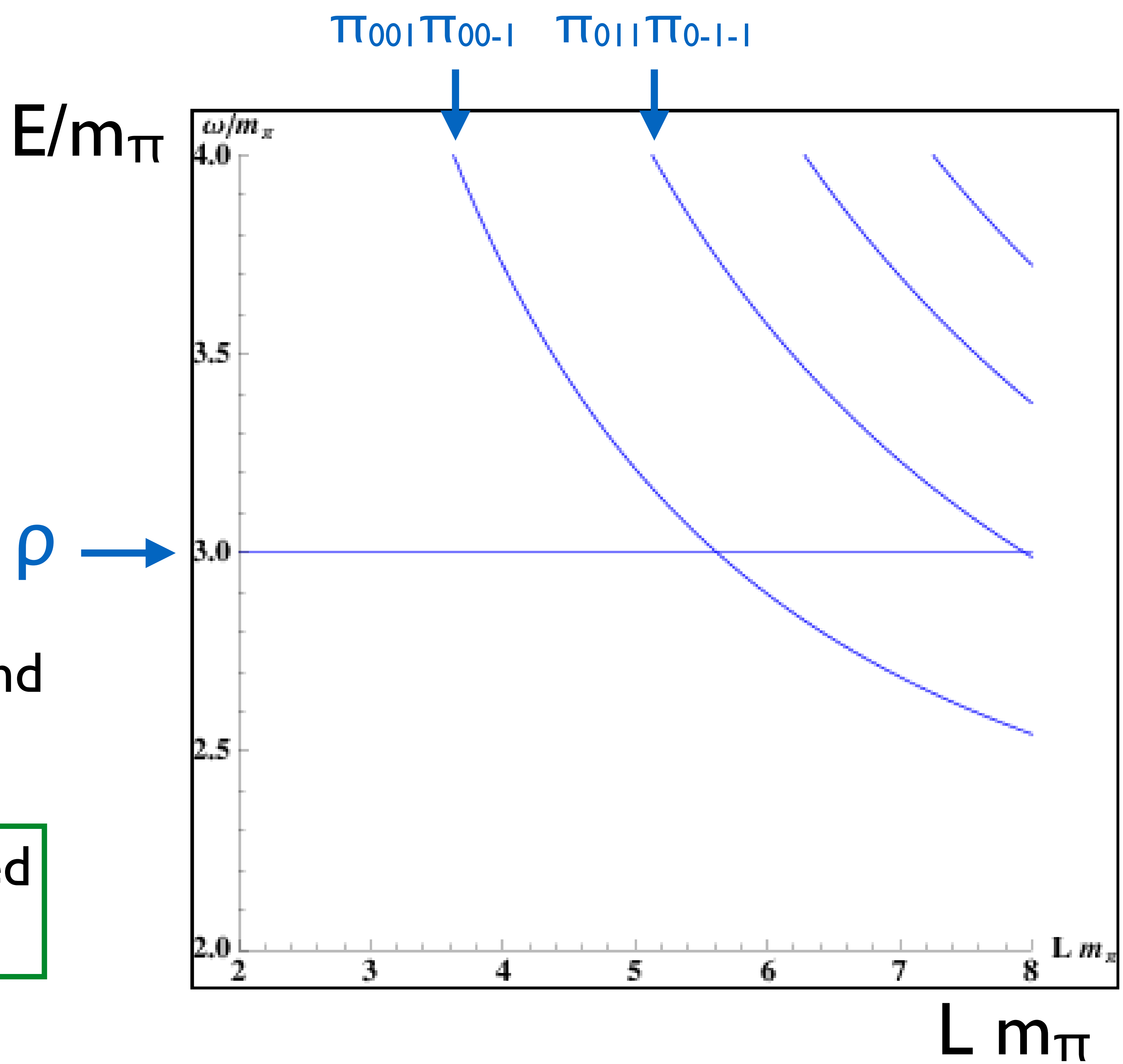
$V = localized \qquad \delta \neq 0$

Cf., 2d resonance example:
Gattringer & cbl, NPB391 (93) 463

Energy levels and phase shifts

without interaction

with interaction



The energy levels depend on the spatial volume.

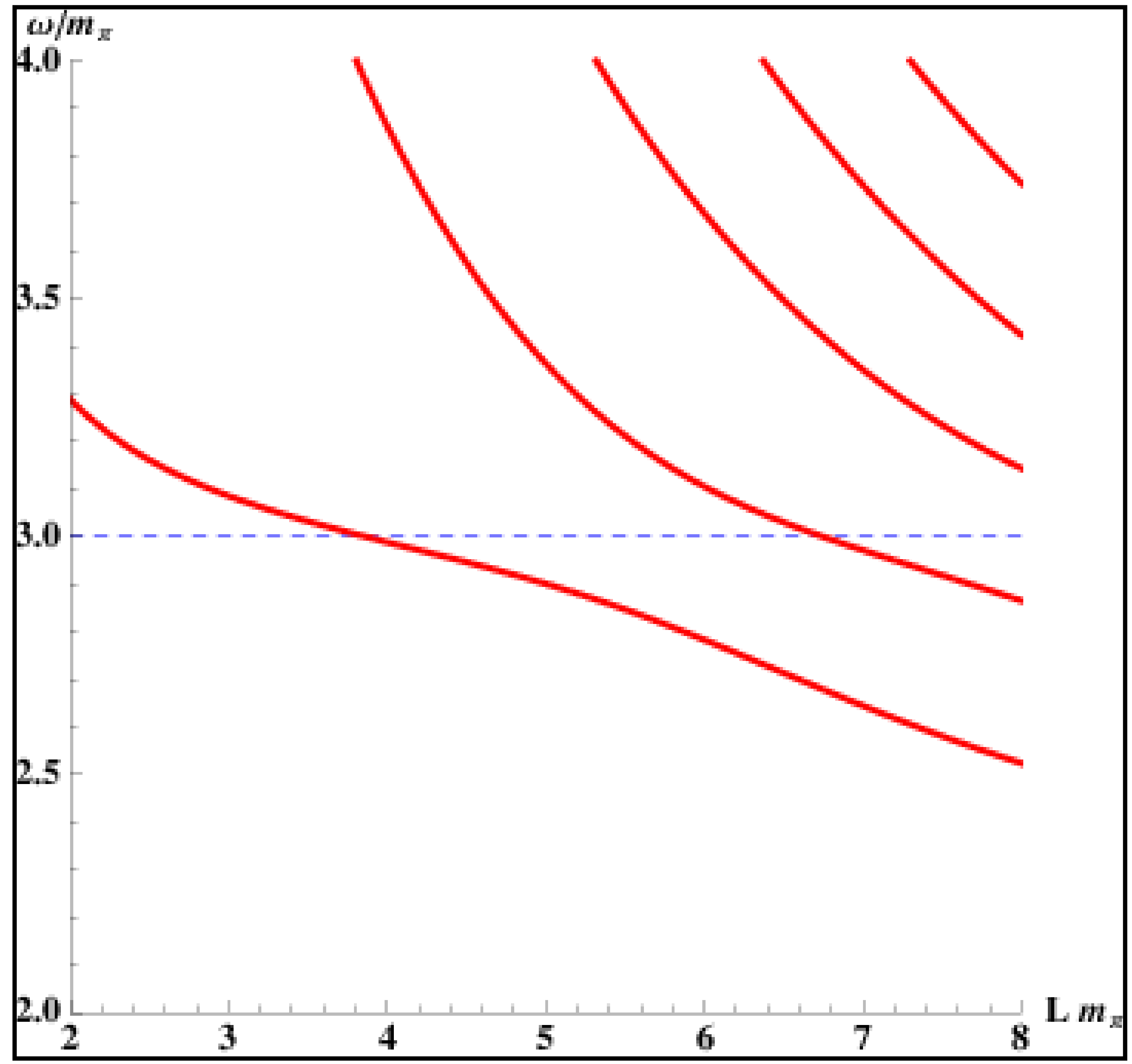
Resonance region: avoided level crossing

Energy levels and phase shifts

without interaction

with interaction

E/m$_\pi$



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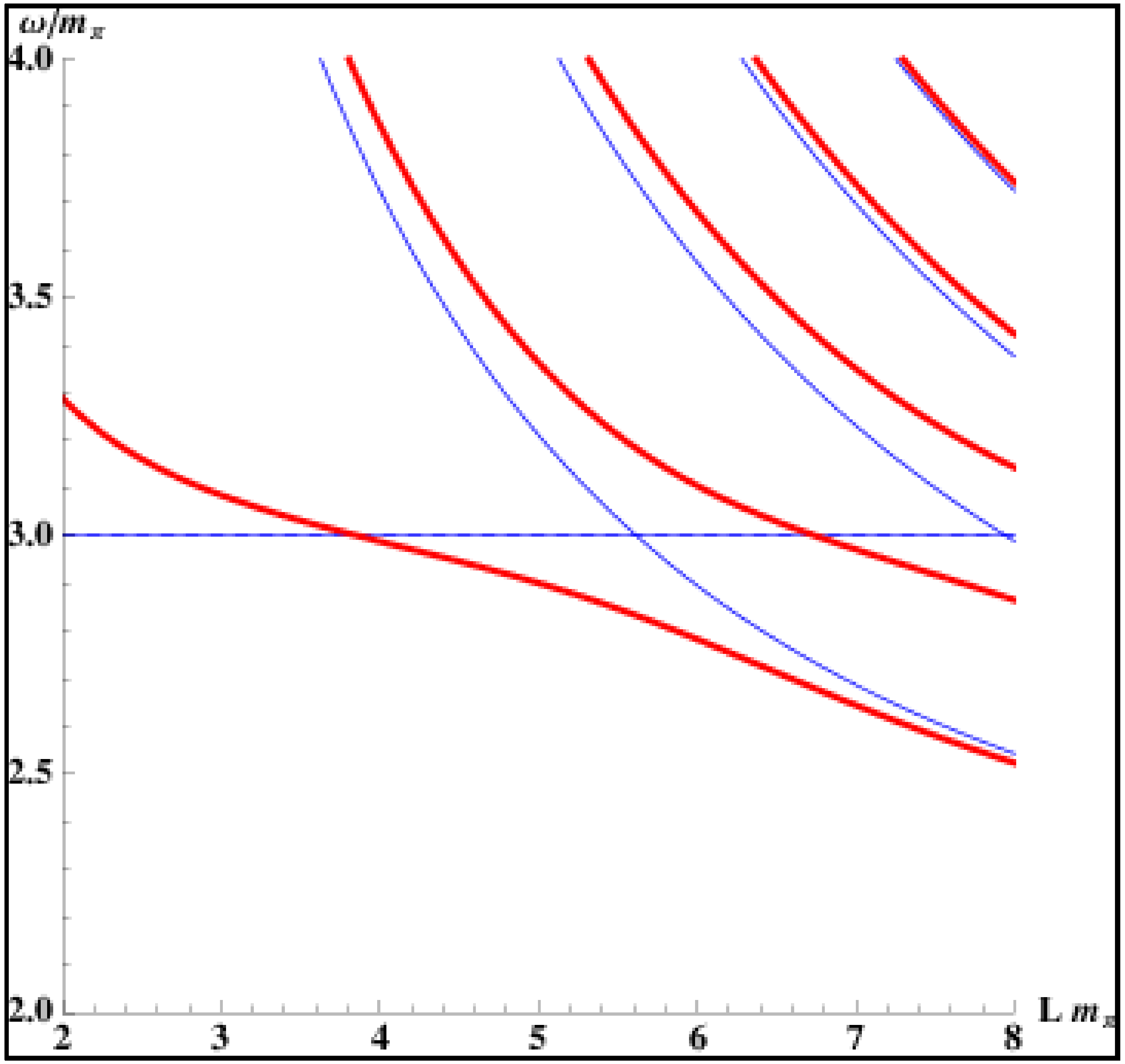

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The energy levels depend on the spatial volume.

Resonance region: avoided level crossing

Energy levels and phase shifts

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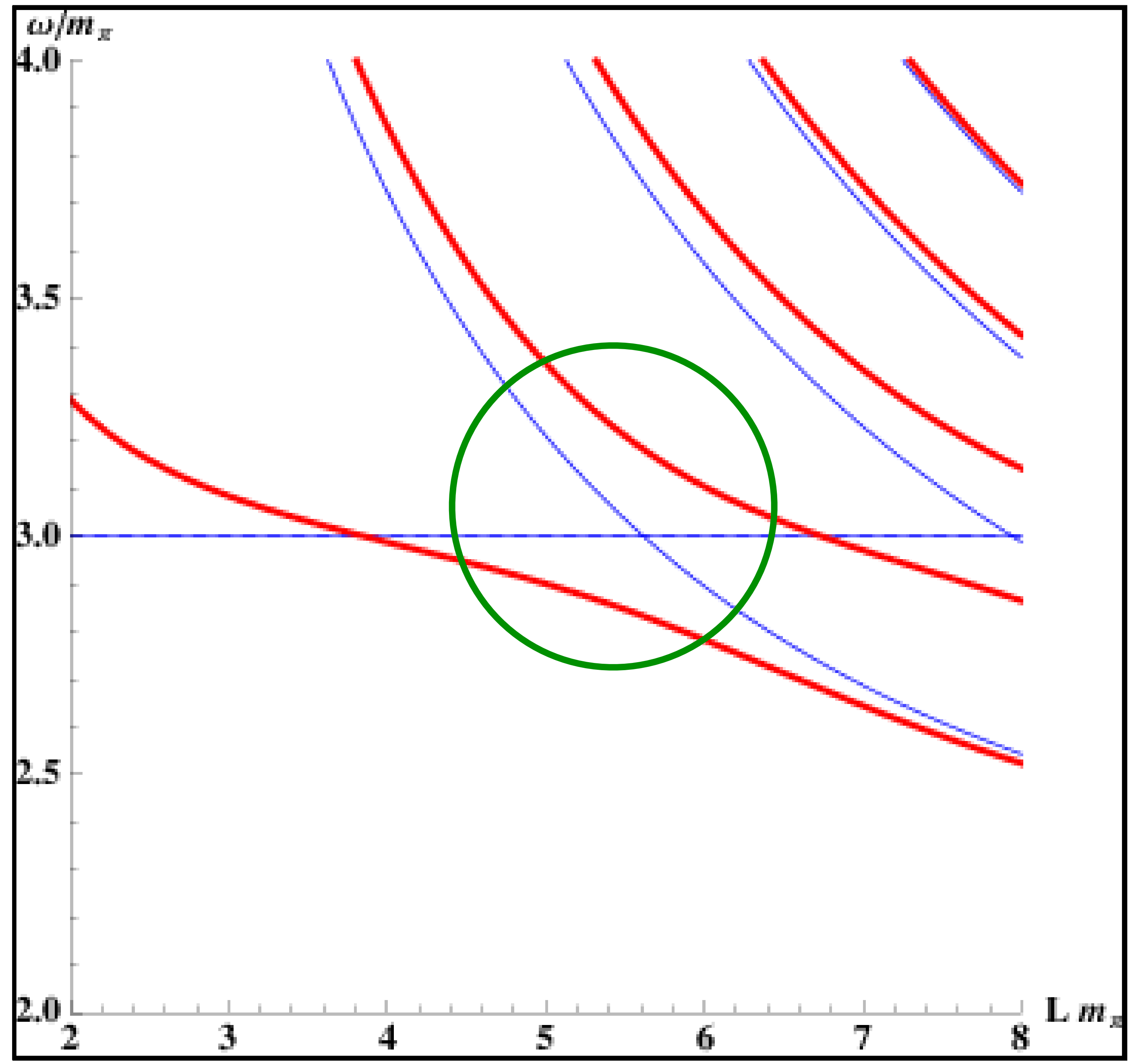

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The energy levels depend on the spatial volume.

Resonance region: avoided level crossing

From energy levels to phase shifts

(in the elastic region)

$$\boxed{\mathrm{Re}(f^{-1}) \;-\; c\,\mathcal{Z}_{00}\left(1;\left(\frac{pL}{2\pi}\right)^2\right) = 0}$$

$$f_\ell^{-1}(s) = \rho(s)\,\cot\delta_\ell(s) - \mathrm{i}\,\rho(s)$$

Energy levels E_n → $\rho \cot \delta$ → $\delta(E_n)$

From energy levels to phase shifts

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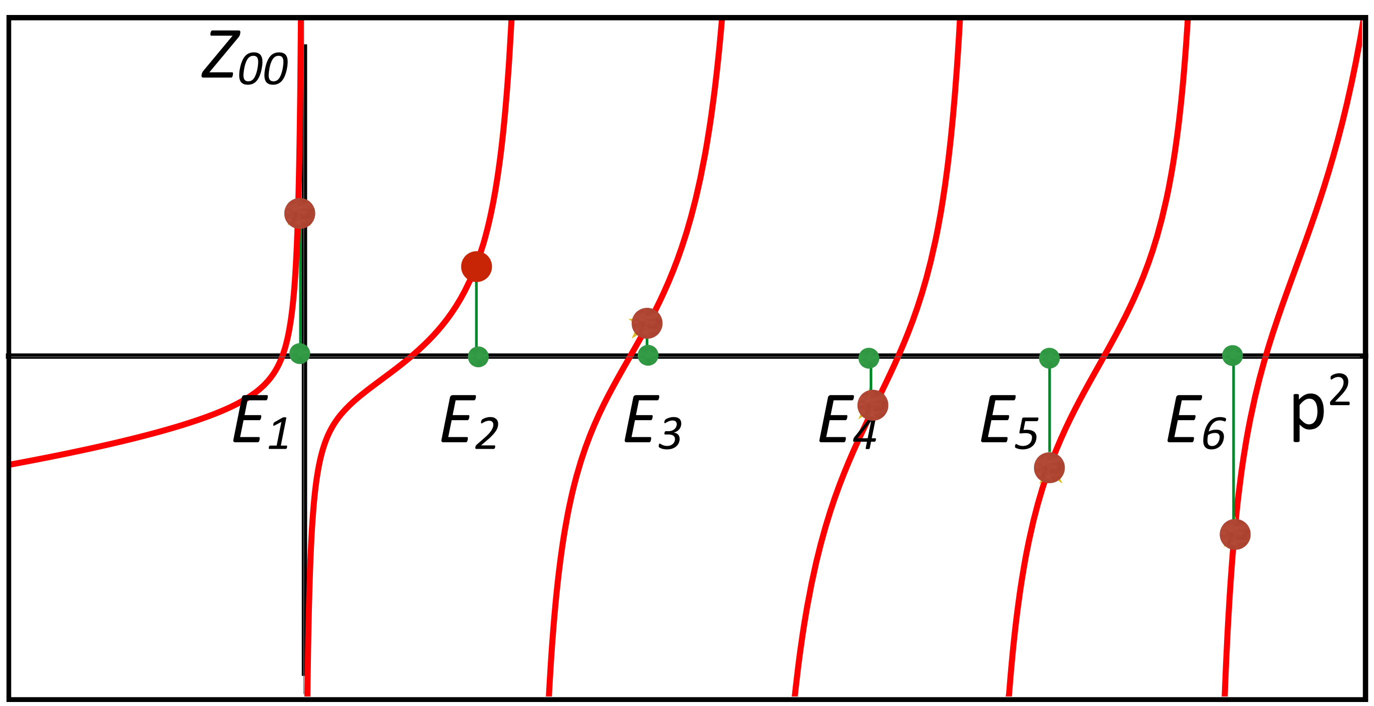
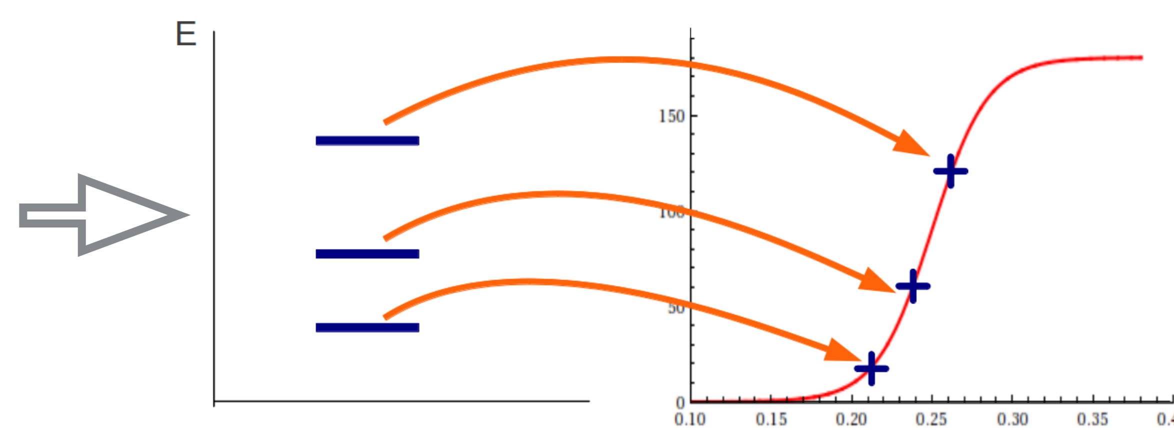
$\rho \cot \delta \rightarrow \delta(E_n)$



“Lüscher curves”



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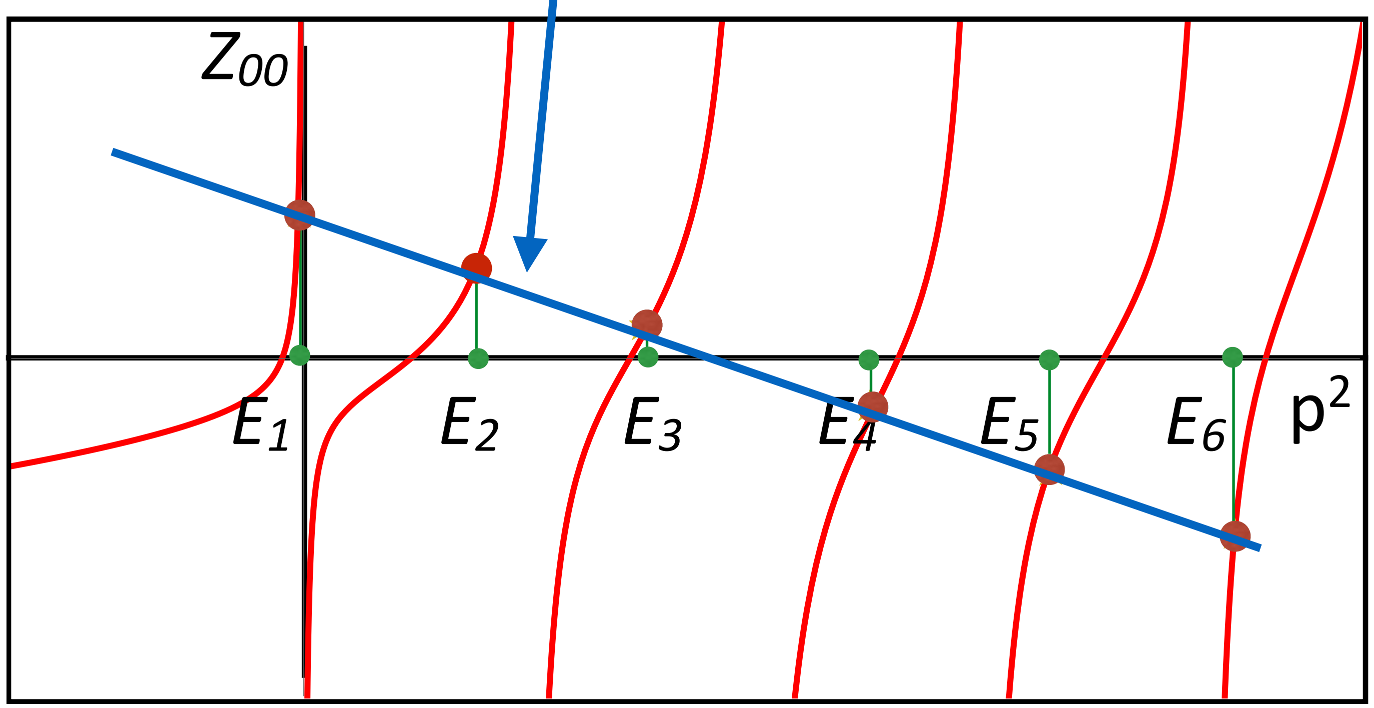
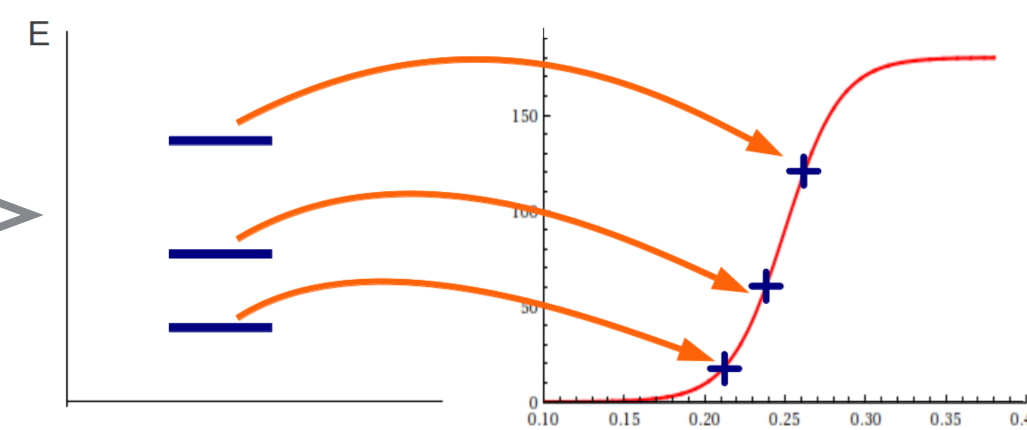


"Lüscher curves"



From energy levels to phase shifts

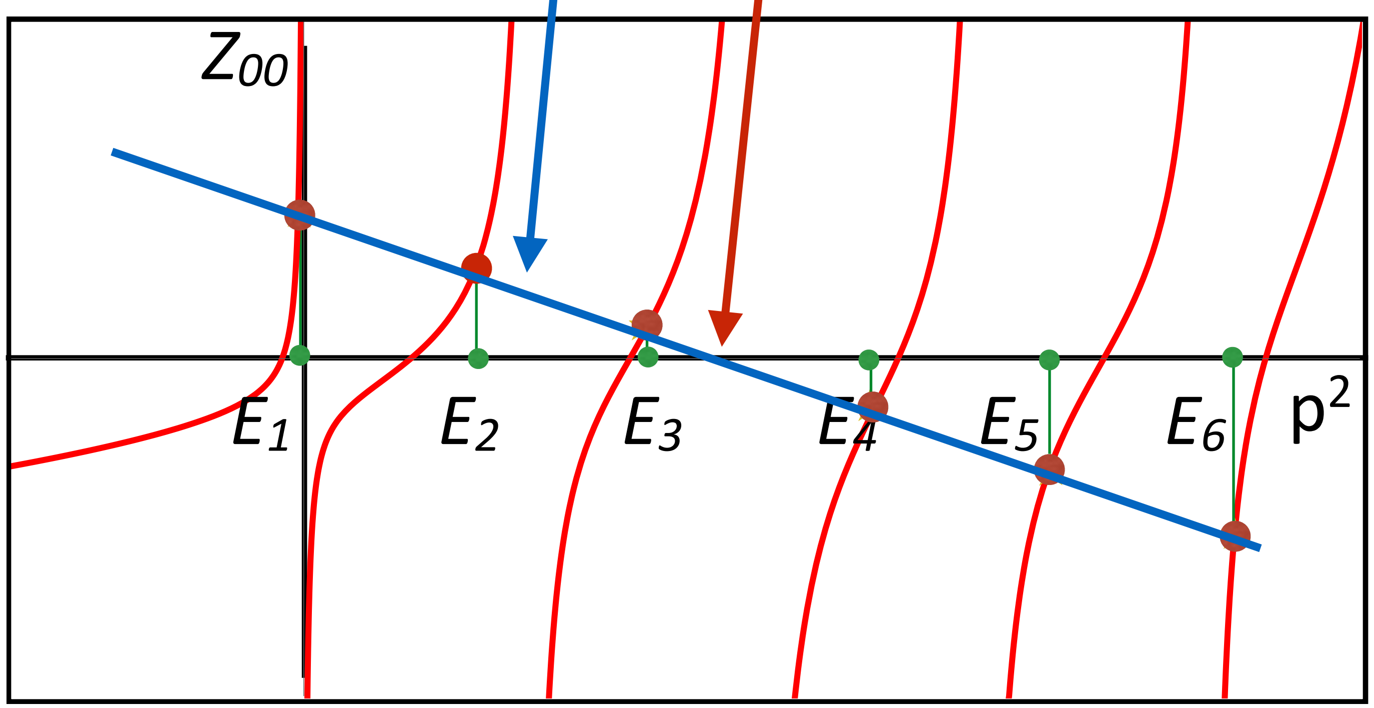
(in the elastic region)

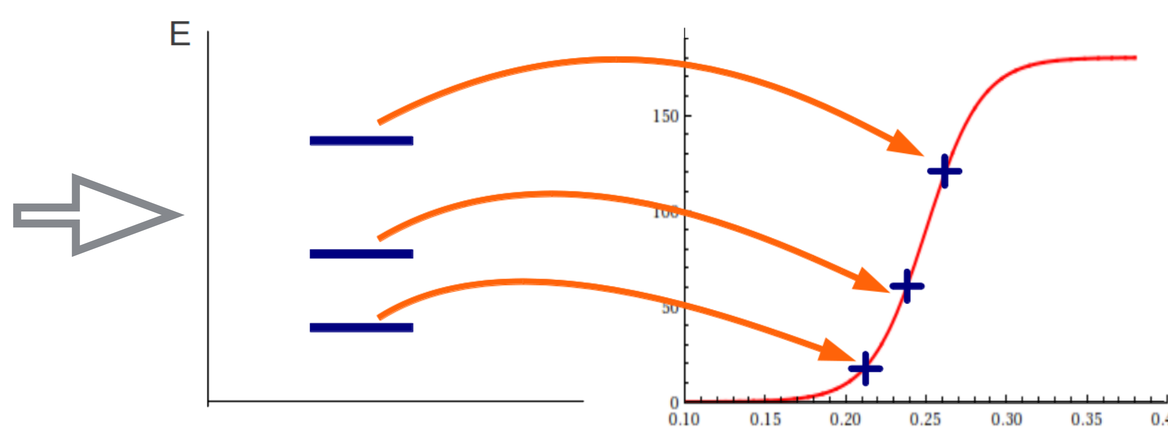
$$\boxed{\mathrm{Re}(f^{-1}) \; - \; c\,\mathcal{Z}_{00}\left(1;\left(\frac{pL}{2\pi}\right)^2\right) = 0}$$

$$f_\ell^{-1}(s) = \boxed{\rho(s)\;\cot\delta_\ell(s)} - \mathrm{i}\,\rho(s)$$

resonance

Energy levels $E_n \rightarrow$
$\rho \cot \delta \rightarrow \delta(E_n)$

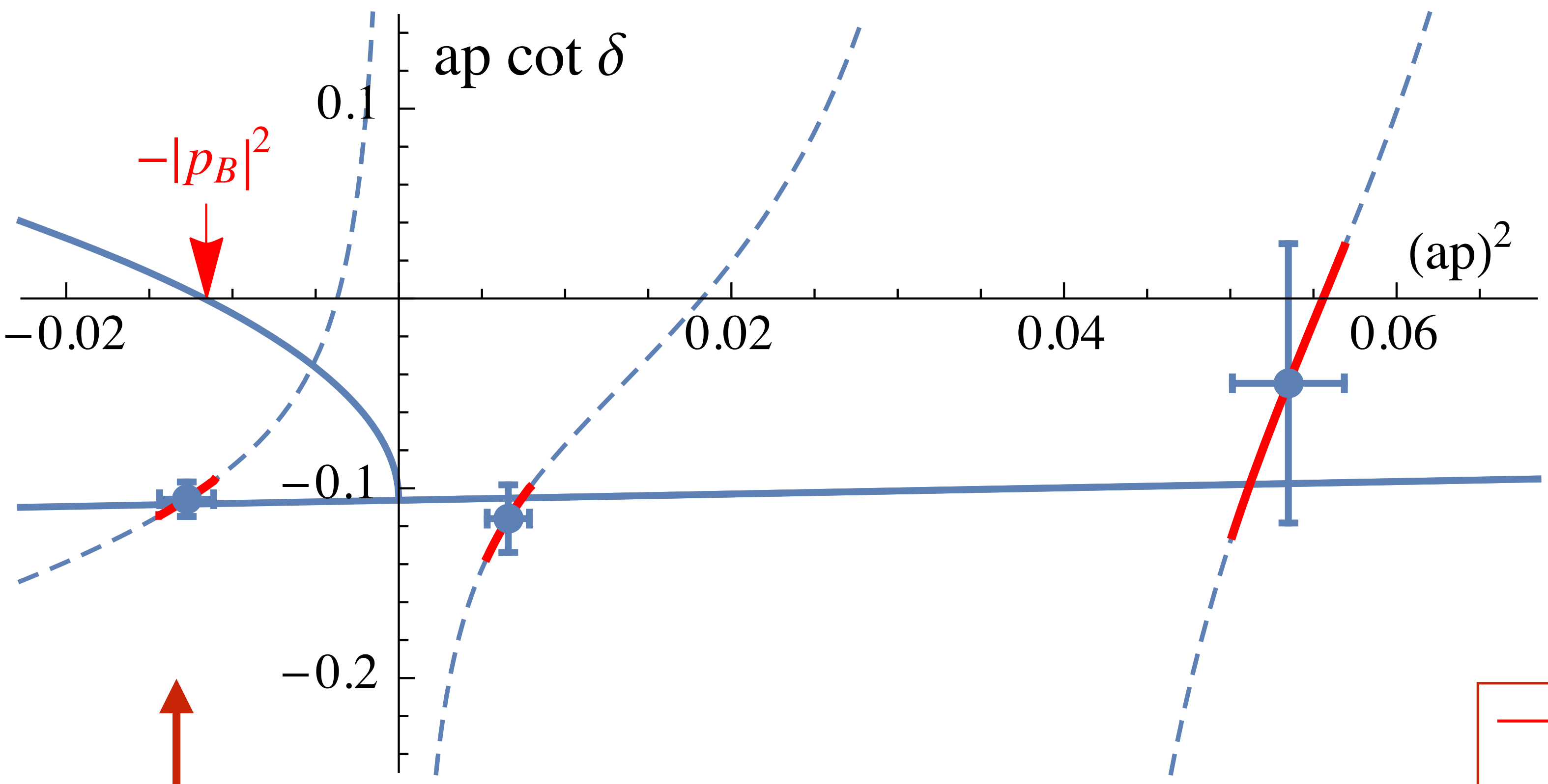




"Lüscher curves"

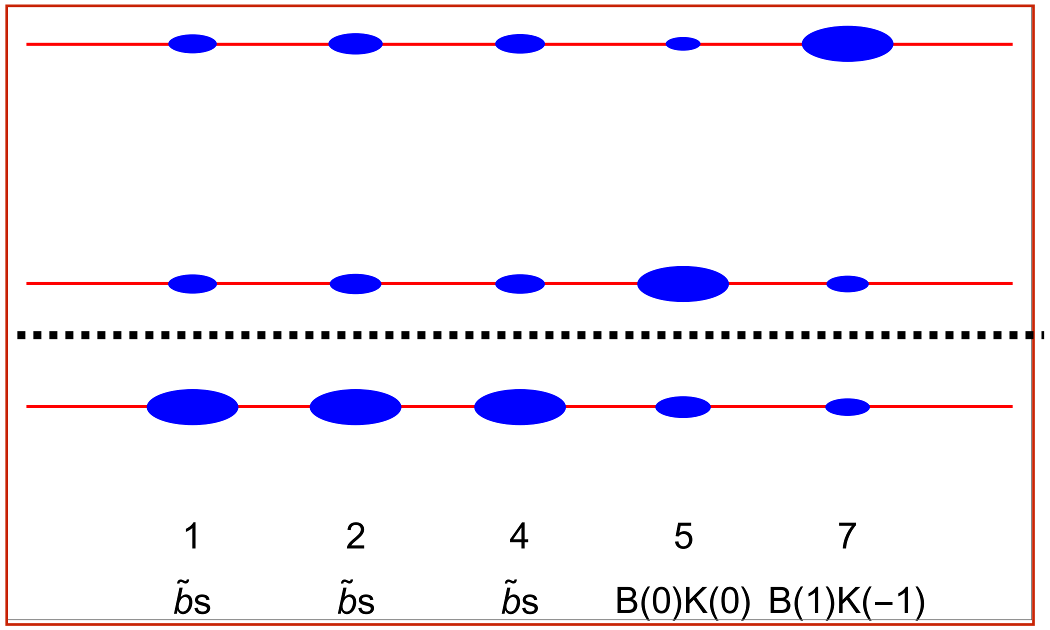
Continuation below threshold: Bound states?

Example: DK scattering in $J^{PC}=0^{++}$ near threshold



Predicting positive parity B_S mesons from lattice QCD
CBL, Mohler et al., Physics Letters B 750 (2015) 17

state composition



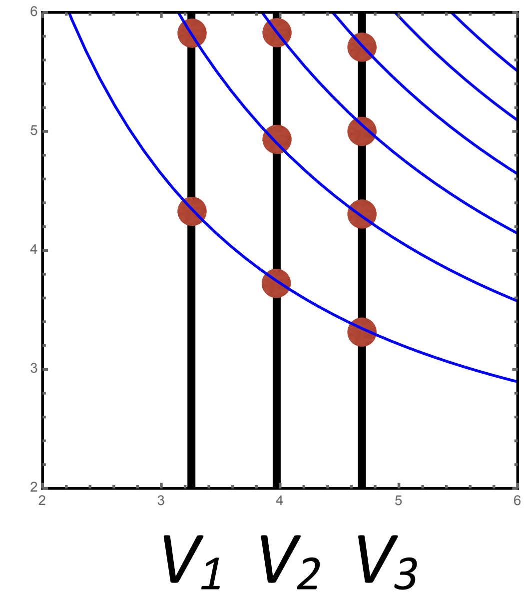
Bound state B_{s0} with
m(B_{s0}) = 5.711(13)(19) GeV

(E_B=64(13)(19) MeV)

How to get more points?

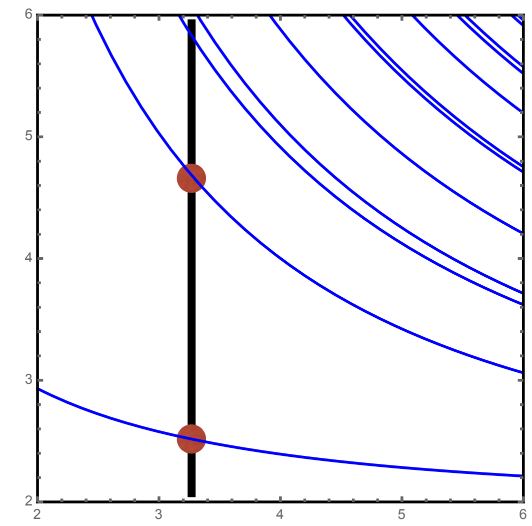
Low lying levels have smaller statistical errors

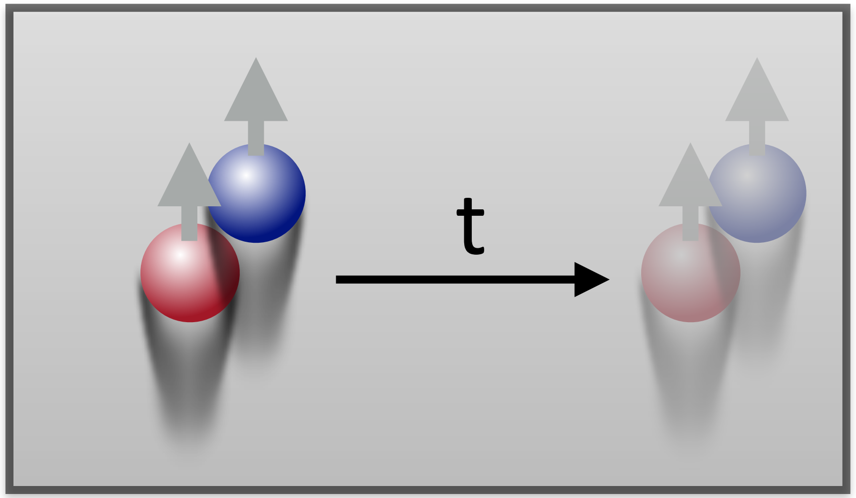
Several volumes (expensive)



Modified boundary conditions

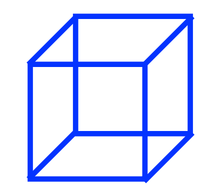
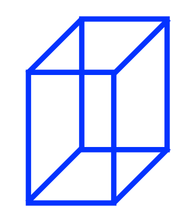
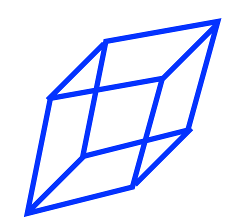
Moving frames (operators with momentum)





Moving frames

Rummukainen, Gottlieb: NP B 450(1995) 397
Kim, Sachrajda,Sharpe: NP B 727 (2005) 218
Feng, Jansen, Renner: PoS LAT10 (2010) 104
Fu, PR D85 (2012) 014506
Leskovec, Prelovsek, PR D85 (2012) 114507
Göckeler et al., PR D 86, 094513 (2012)
Döring et al., Eur.Phys.J.A48 (2012)114

	Relativistic distortion	Symmetry group
$\vec{p} = (0,0,0)$		O_h
$\vec{p} = (0,0,1)$		D_{4d}
$\vec{p} = (1,1,0)$		D_{2d}

Results: light quarks

Example: ππ ➛ ρ ➛ ππ in the elastic region

Xu Feng et al. (ETMC), PR D83, 094505 (2011), arXiv:1011.5288 [hep-lat].
J. Frison et al. (BMW-c), PoS LATTICE2010, 139 (2010), arXiv:1011.3413 [hep-lat].
CBL, D. Mohler et al., PR D84, 054503 (2011),
[Err.: PR D 89 (2014) 059903(E)], arXiv:1105.5636 [hep-lat].
S. Aoki et al. (PACS-CS), PR D84, 094505 (2011), arXiv:1106.5365 [hep-lat].
C. Pelissier et al., PR D87, 014503 (2013), arXiv:1211.0092 [hep-lat].
J. J. Dudek et al. (HSC), PR D87, 034505 (2013),
[Err.: PRD90 (2014) 099902(E)], arXiv:1212.0830 [hep- ph].
B. Fahy et al., PoS LATTICE2014, 077 (2015), arXiv:1410.8843 [hep-lat].
D. J. Wilson et al., (HSC), (2015), arXiv:1507.02599 [hep-ph].

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[Err.: PRD90 (2014) 099902(E)], arXiv:1212.0830 [hep- ph].
B. Fahy et al., PoS LATTICE2014, 077 (2015), arXiv:1410.8843 [hep-lat].
D. J. Wilson et al., (HSC), (2015), arXiv:1507.02599 [hep-ph].

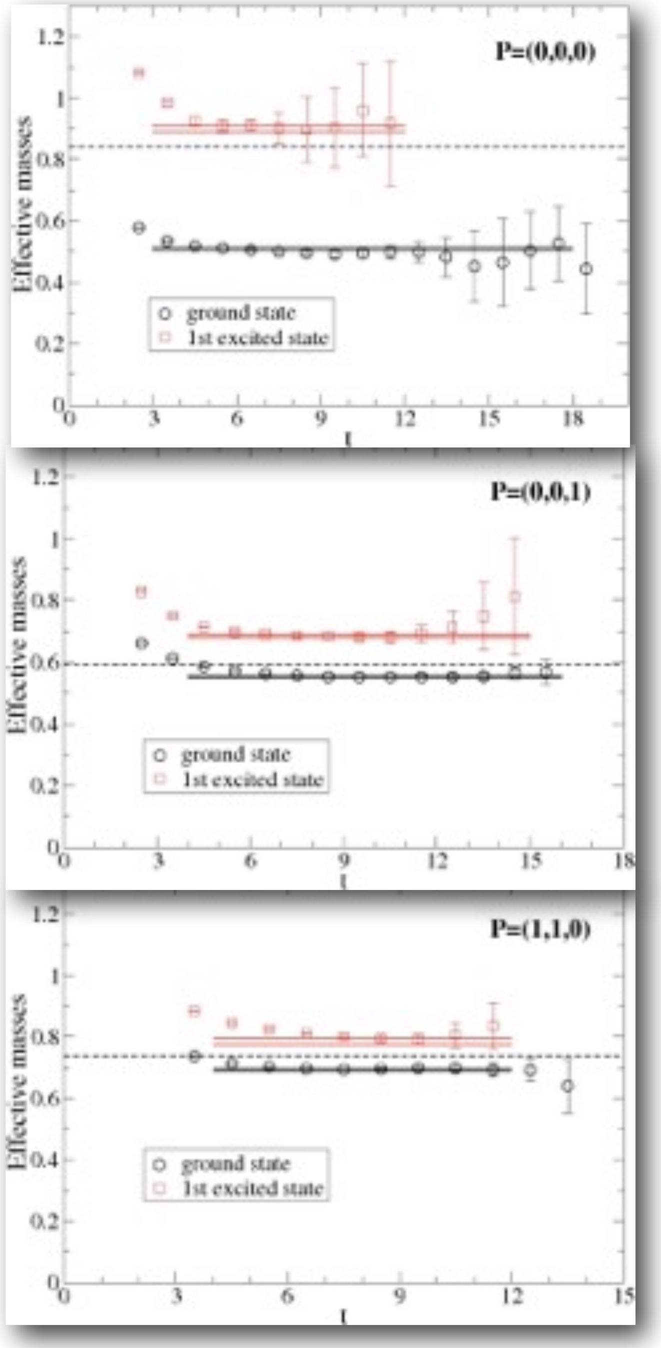
Moving frames, 18 lattice operators

Coupled ππ, K$\underline{\text{K}}$ system

Example: ππ ➛ ρ ➛ ππ

2011

Up to 18 ρ and ππ operators
P=(000), (001), (011)

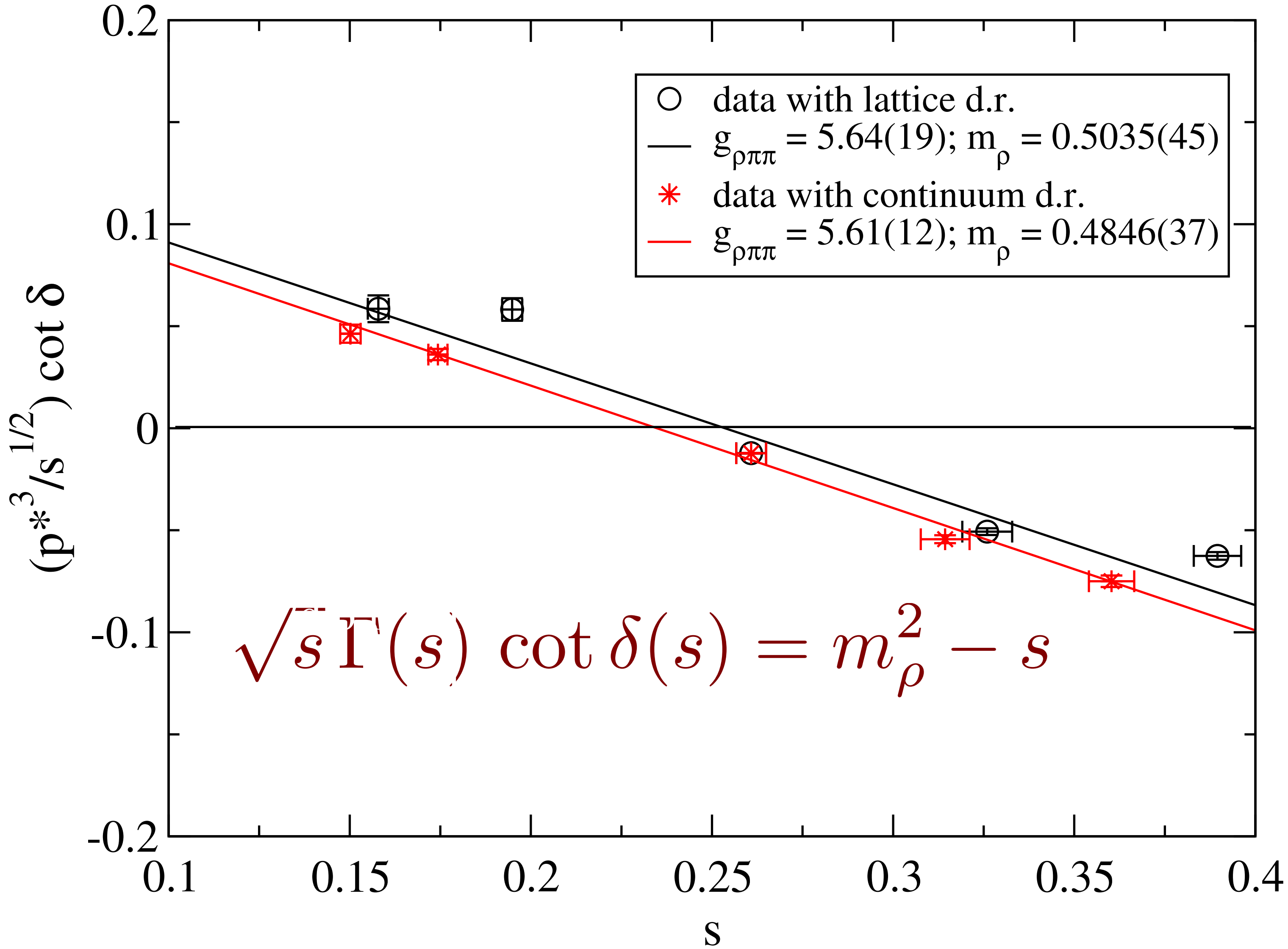


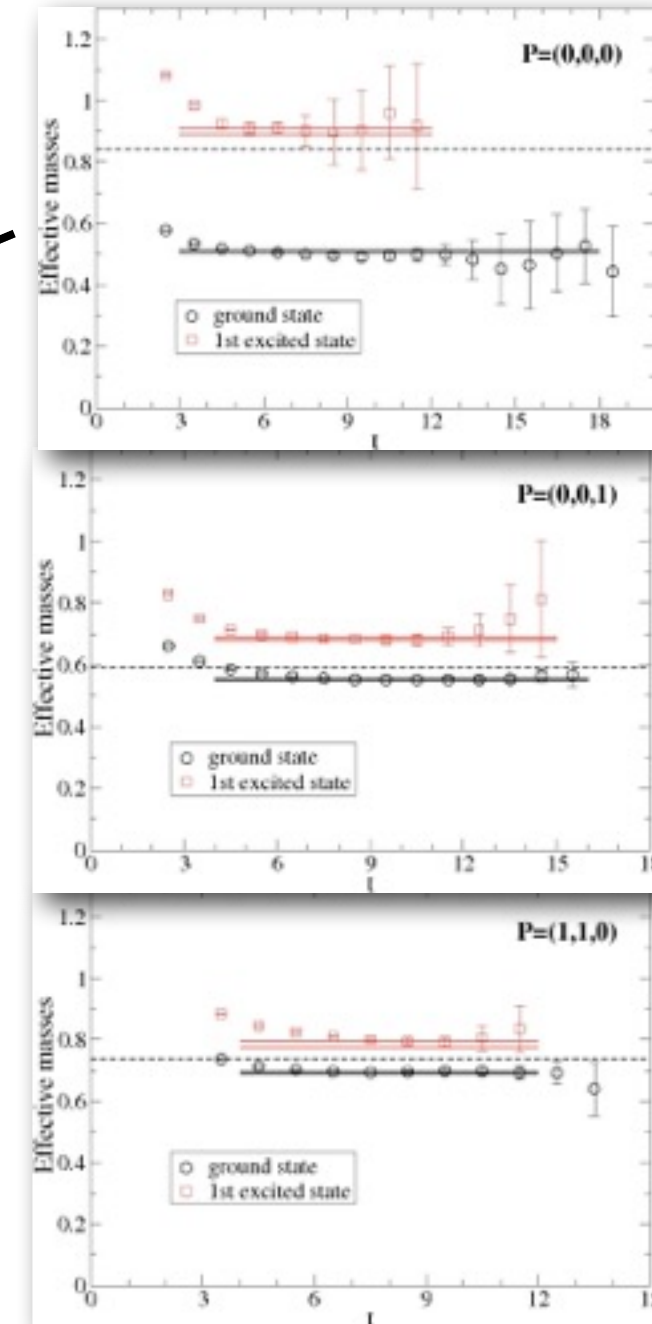
CBL, Mohler, Prelovsek, Vidmar;
PR D **84**, 054503 (2011)
Erratum PR D 89 (2014) 059903(E)

Example: ππ ➛ ρ ➛ ππ

2011

Up to 18 ρ and ππ operators
P=(000), (001), (011)

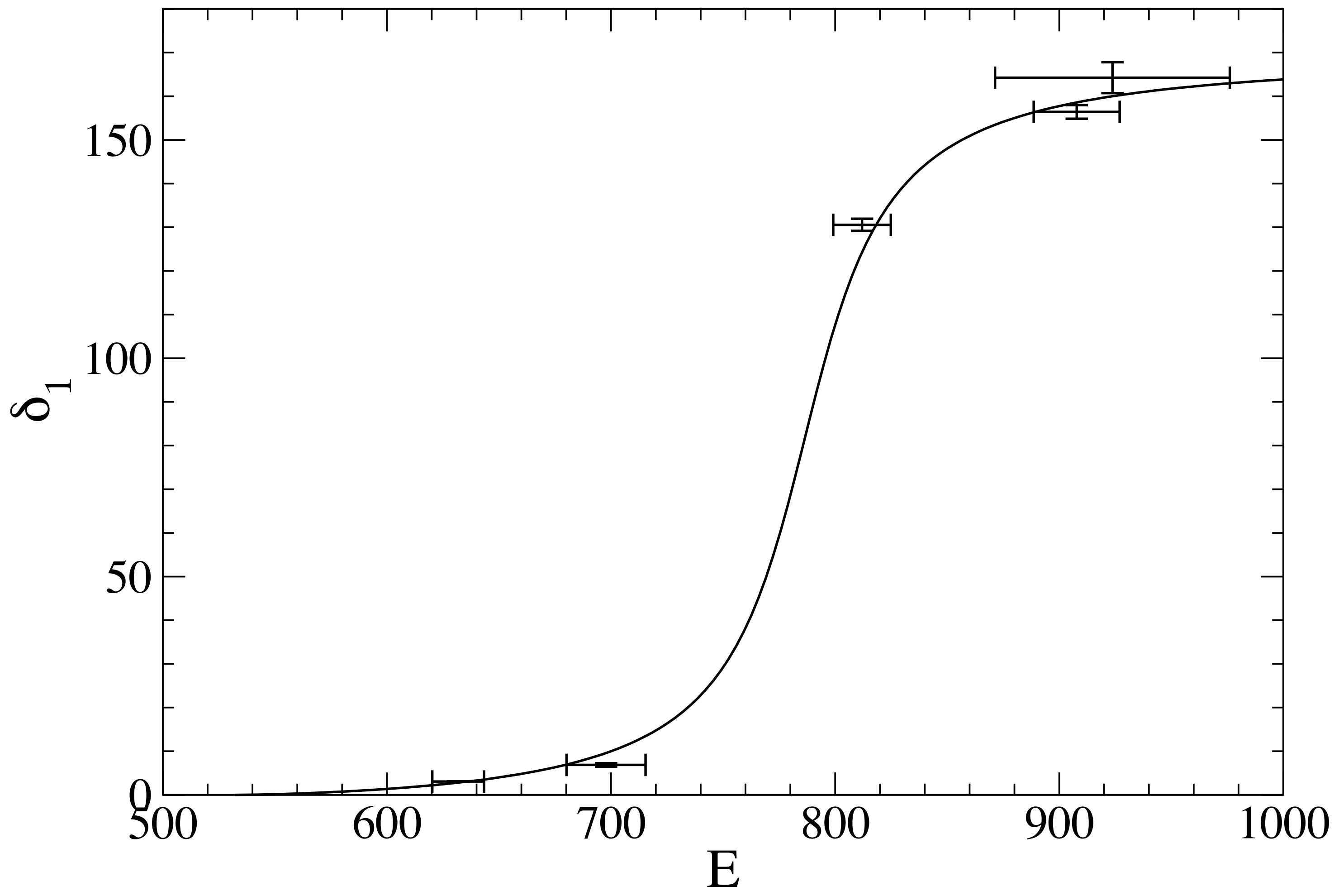




CBL, Mohler, Prelovsek, Vidmar;
PR D **84**, 054503 (2011)
Erratum PR D 89 (2014) 059903(E)

Example: ππ ➛ ρ ➛ ππ

2011



$$m_\pi = 266(3)(3)\ \mathrm{MeV}$$
$$m_\rho = 772(6)(8)\ \mathrm{MeV}$$
$$g_{\rho\pi\pi} = 5.61(12)$$
$$g_{\rho\pi\pi,exp} = 5.96$$

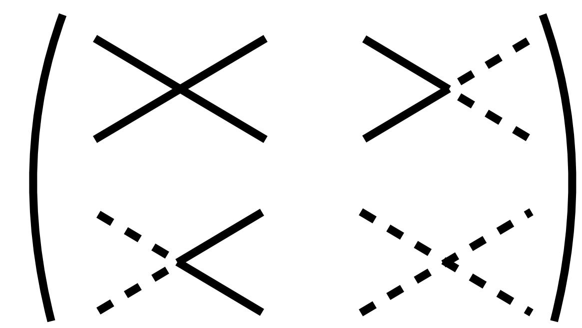
CBL, Mohler, Prelovsek, Vidmar;
PR D **84**, 054503 (2011)
Erratum PR D 89 (2014) 059903(E)

Beyond the elastic region: coupled channels

"..to boldly go, where.."

Extension to several coupled channels

Matrices T, Z: $$\boxed{\det\left[T^{-1} - Z\right] = 0}$$



Bernard et al ., JHEP 1101 (2011) 019 [arXiv:1010.6018]

Briceno et al ., PR D 88, 034502 (2013) — two nucleons

Briceno et al , PR D 88, 094507 (2013) — moving multichannels

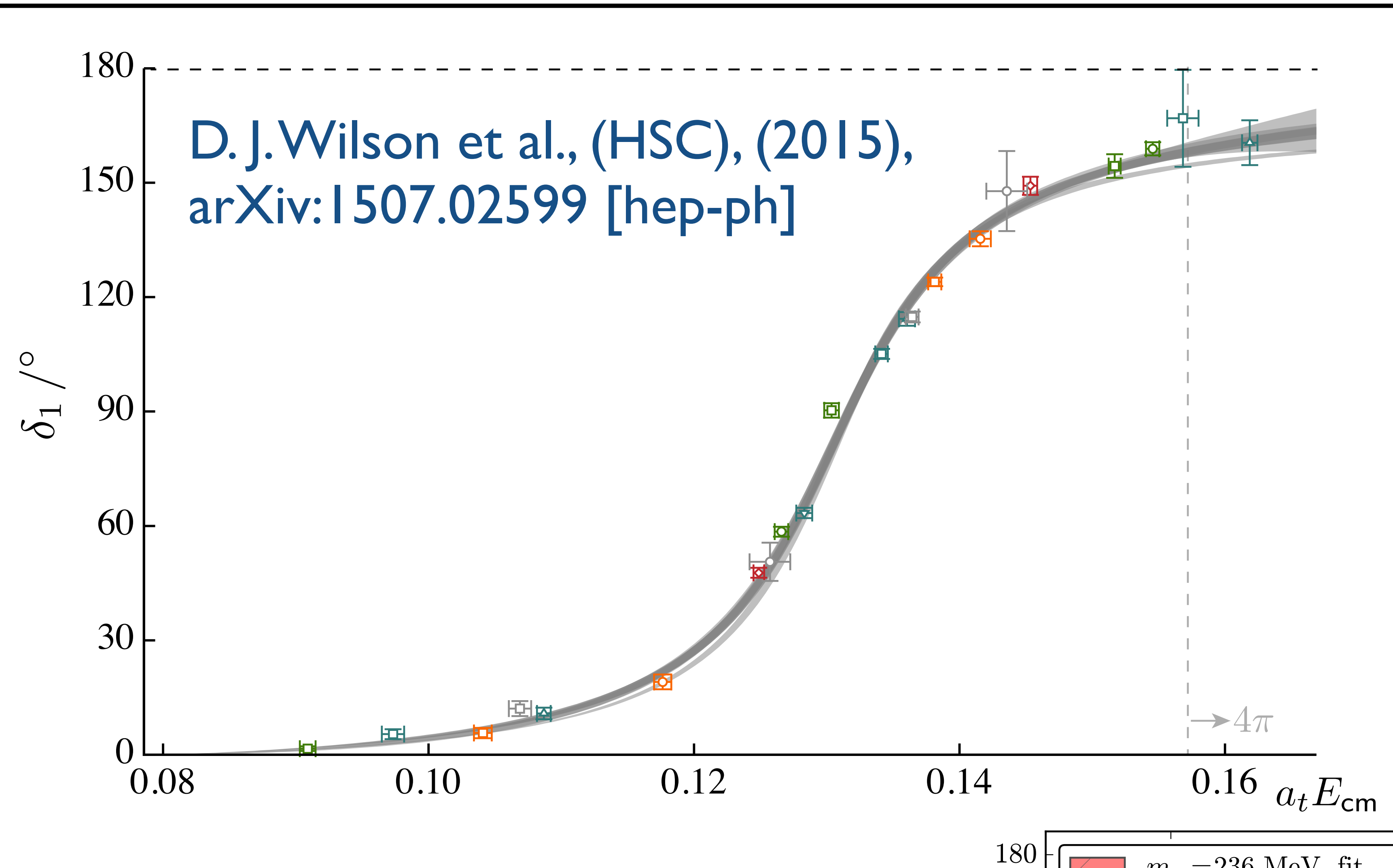
Briceno et al ., PR D 89, 074507 (2014) — arbitrary spin

Hansen & Sharpe, PR D86 (2012) 016007[arXiv:1204.0826]

Briceno et al., PR D 91, 034501 (2015) — $1 \rightarrow 2$ transitions

Example: (ππ,KK) ➤ ρ ➤ (ππ,KK)

2015



$$m_\pi = 236(2)\ \mathrm{MeV}$$

$$m_\rho = 790(2)\ \mathrm{MeV}$$

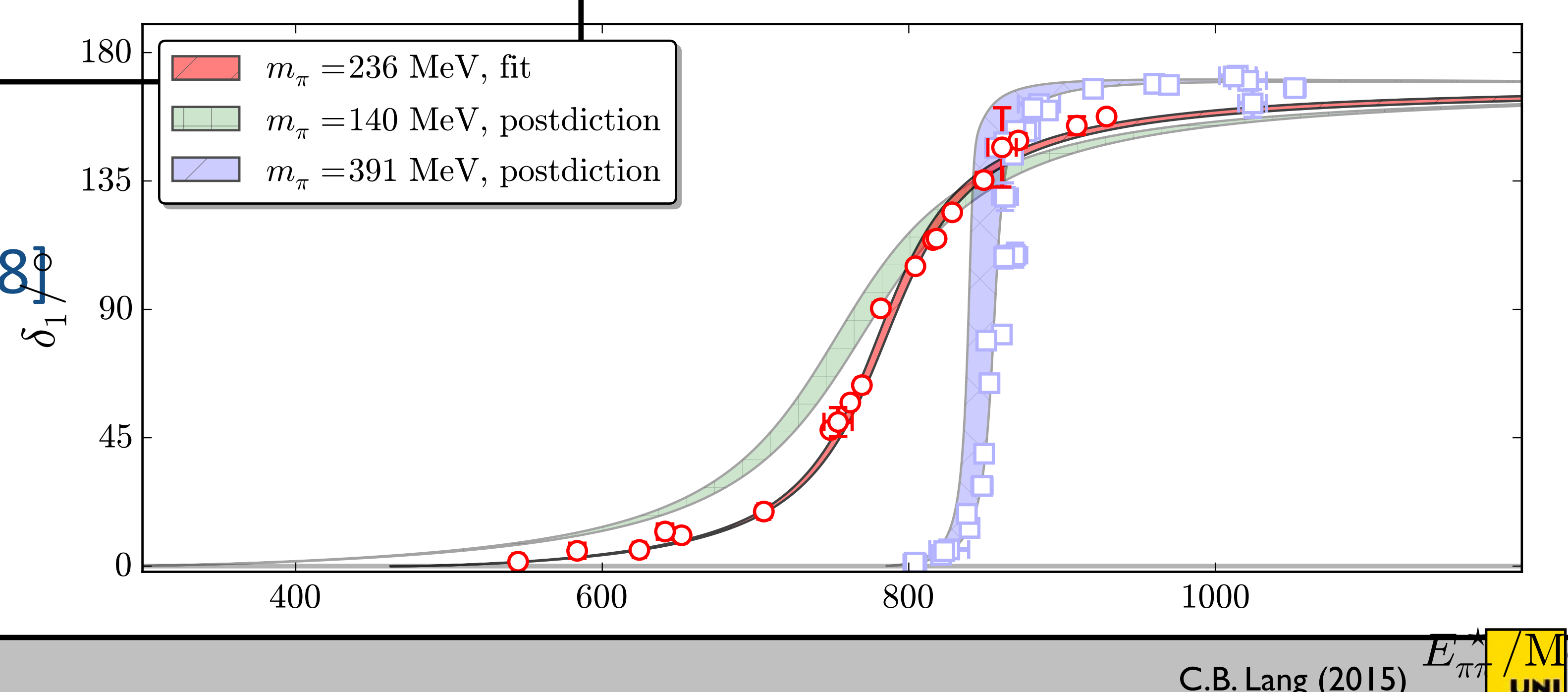
$$\Gamma_\rho = 87(2)\ \mathrm{MeV}$$

$$g_{\rho\pi\pi} = 5.69(7)(3)$$

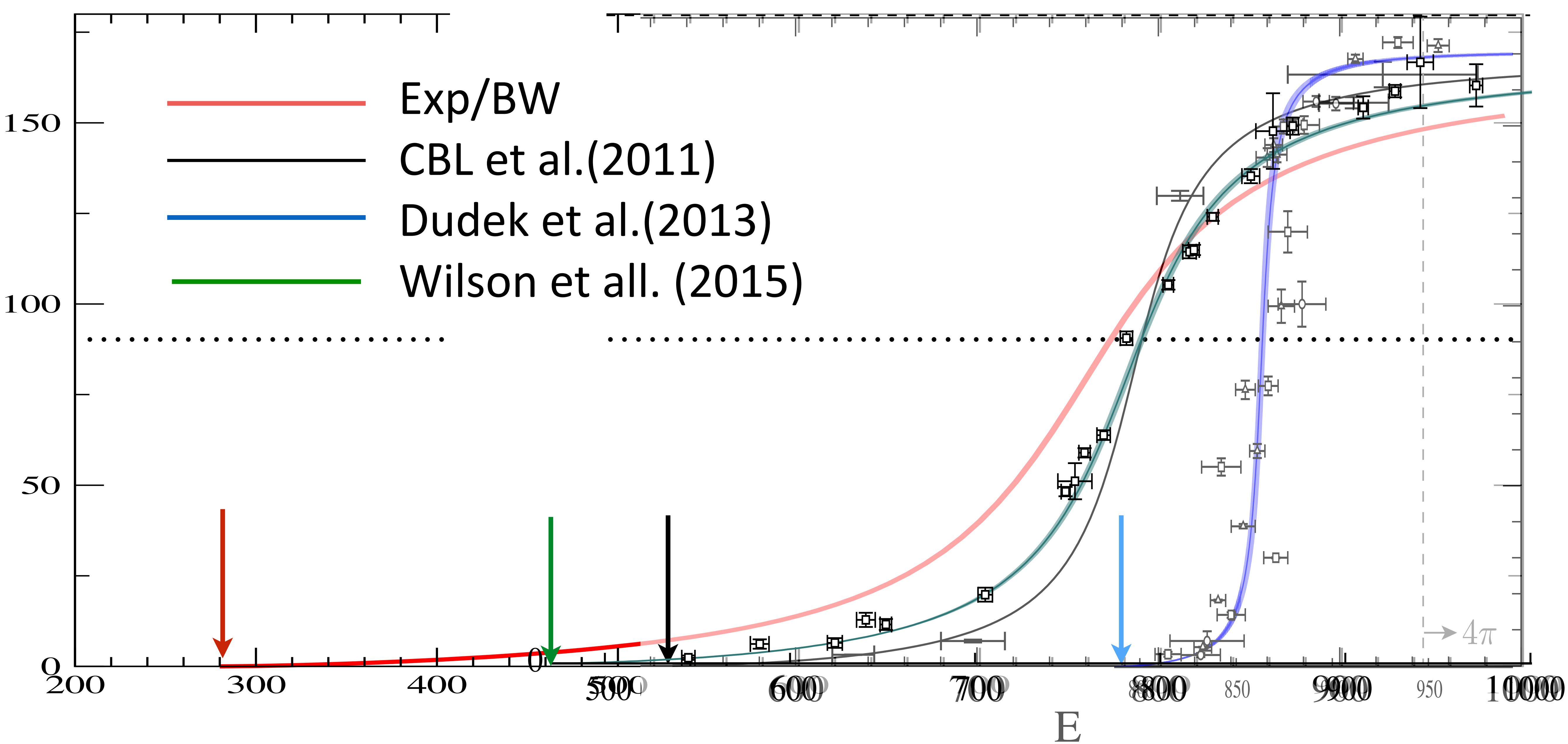
$$g_{\rho\pi\pi,exp} = 5.96$$

Bolton et al. [arXiv:1507.07928]

extrapolation to the physical point

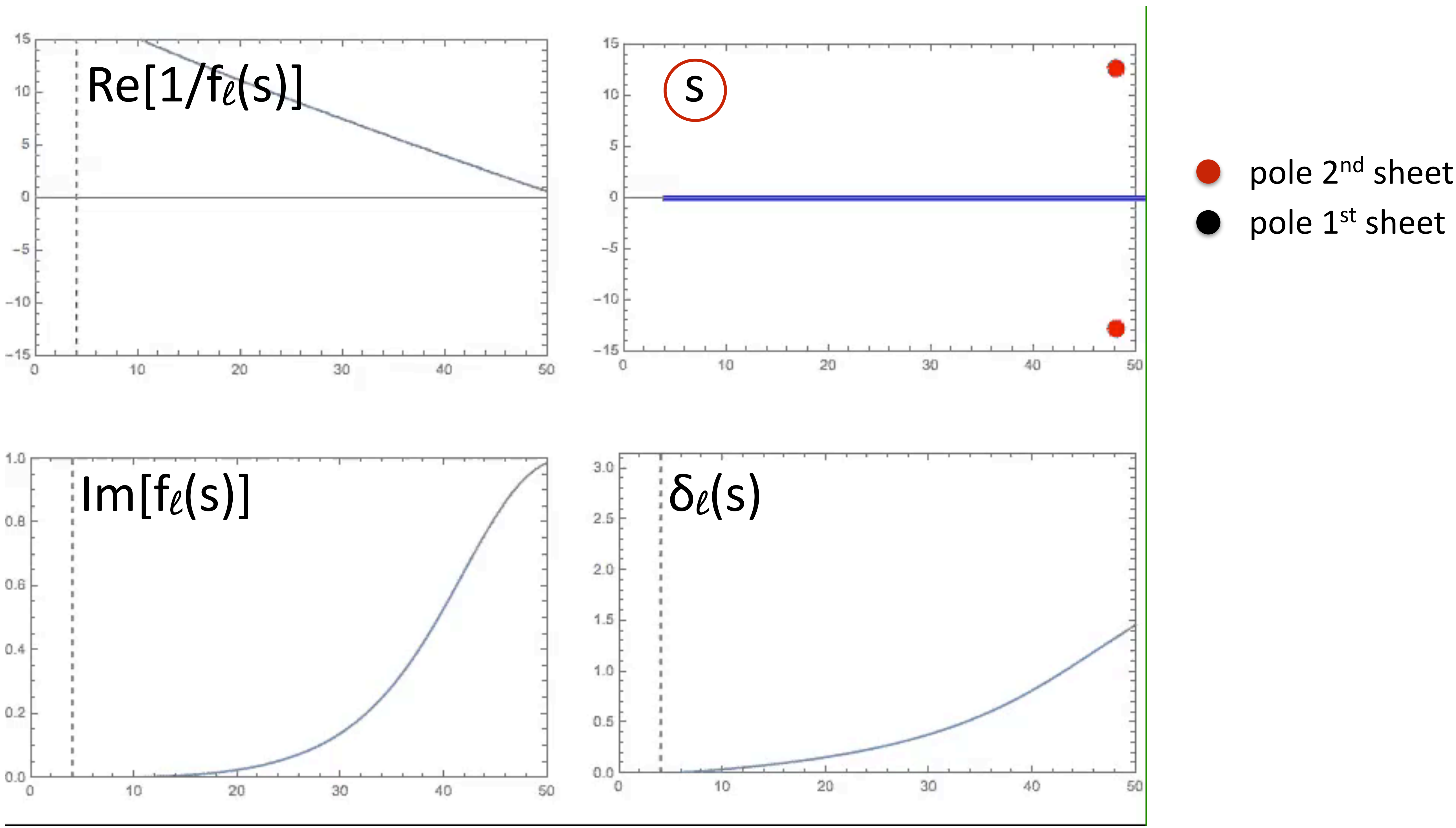


Comparison with BW-fit to experiment



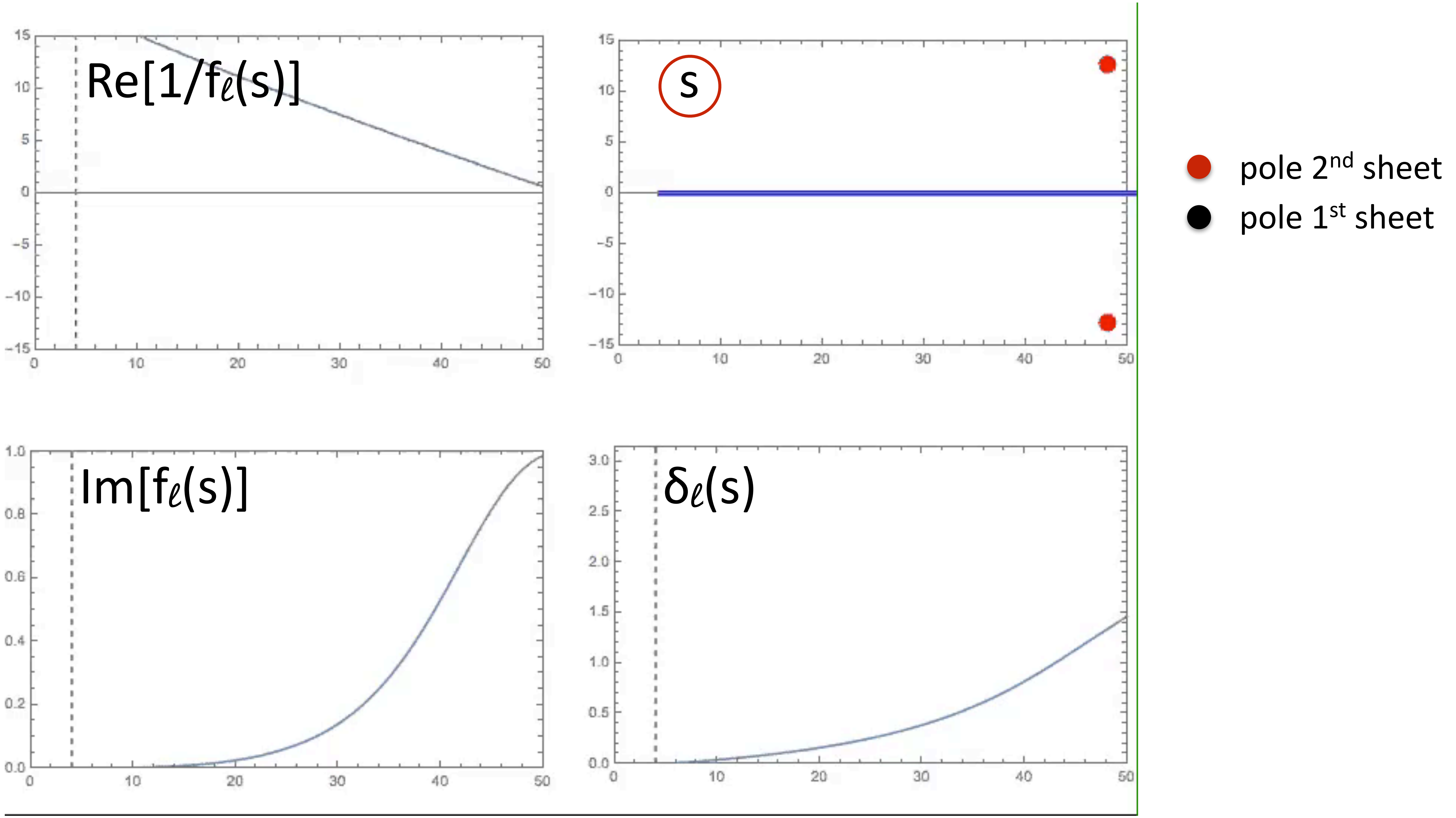
Resonance or bound state

$$p\cot\delta \sim \frac{1}{g^2}\left(m_R^2 - s\right)$$



Resonance or bound state

$$p \cot \delta \sim \frac{1}{g^2}\,(m_R^2 - s)$$



Light quark sector (with meson-meson operators)

($\rho\pi$,a_1), ($\omega\pi$,b_1)

Scattering lengths and resonance parameters:

CBL, Leskovec et al., JHEP 04 (2014) 162; arXiv:1401.2088 [hep-lat]

(πK, κ, K^*)

Scattering lengths:

S.R.Beane et al.,PR D 74, 114503 (2006).

Z. Fu and K. Fu, PR D86, 094507 (2012), arXiv:1209.0350 [hep-lat].

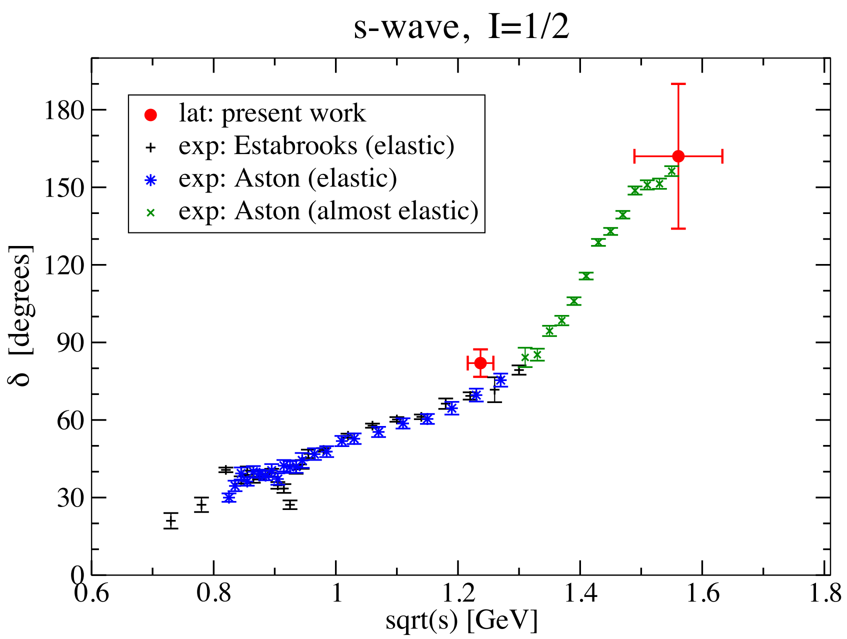
Phase shifts:

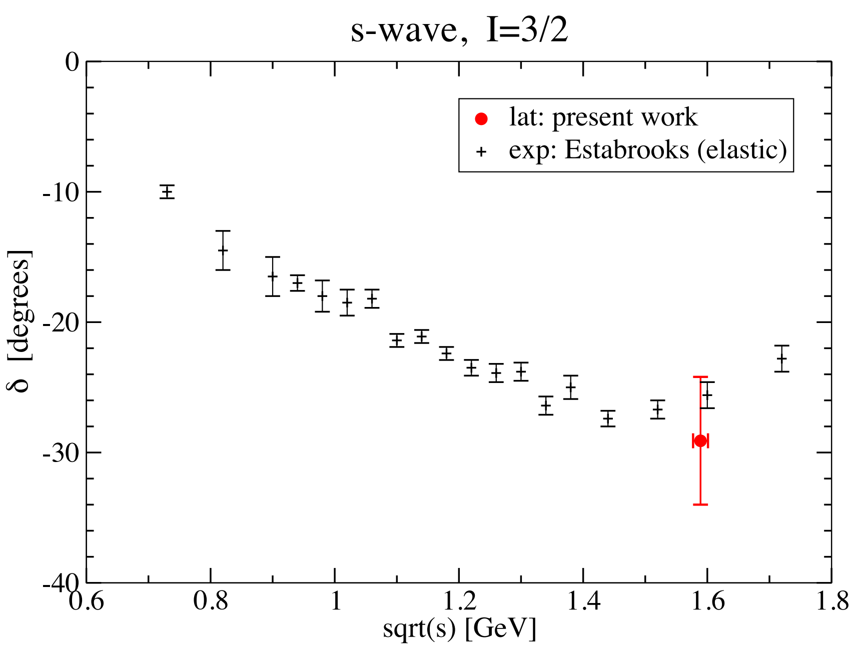
S. Prelovsek et al., PR D88, 054508 (2013), arXiv:1307.0736 [hep-lat].

J. J. Dudek et al. (HSC), PRL 113, 182001 (2014), arXiv:1406.4158 [hep-ph].

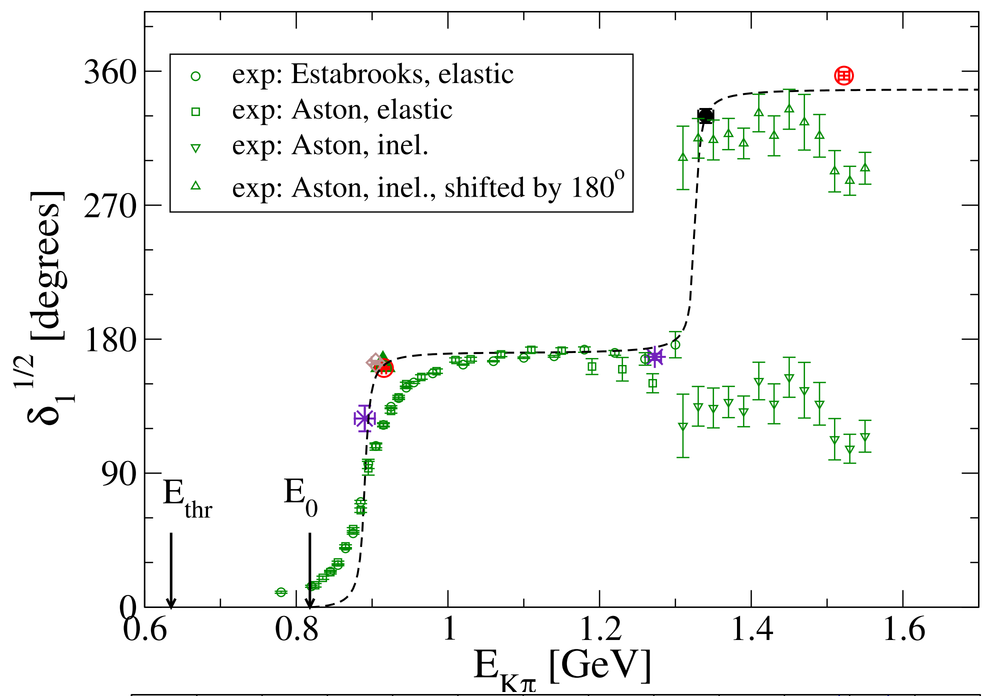
D. J. Wilson et al. (HSC), PR D91, 054008 (2015), arXiv:1411.2004 [hep-ph].

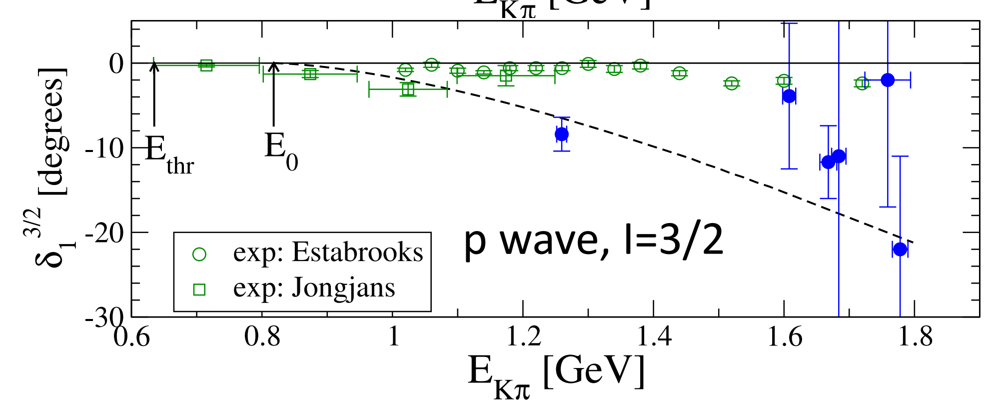
Kπ scattering and the K* width





p wave, I=1/2

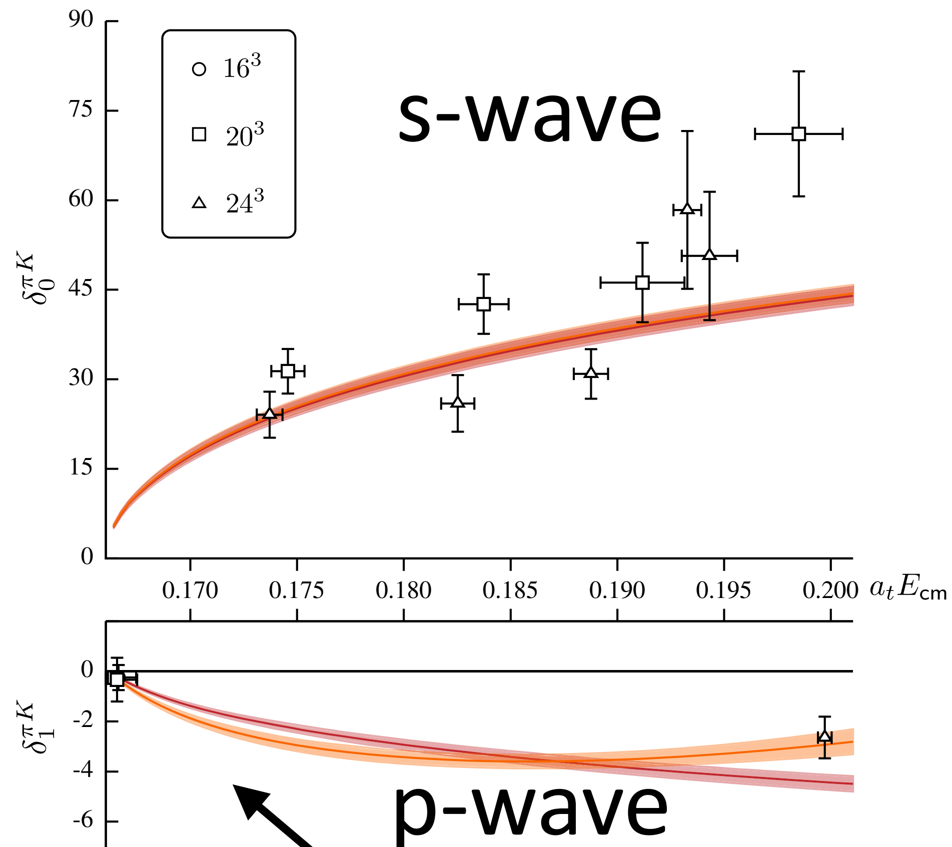




	$m_{K^*(892)}$ [MeV]	$g_{K^*(892)}$ [no unit]	$m_{K^*(1410)}$ [GeV]	$g_{K^*(1410)}$ [no unit]
lat	891 ± 14	5.7 ± 1.6	1.33 ± 0.02	input
exp	891.66 ± 0.26	5.72 ± 0.06	1.414 ± 0.0015	1.59 ± 0.03

CBL, Leskovec, Prelovsek, Mohler,
PR D86 (2012) 054508; arXiv:1207.3204
PR D88 (2013) 054508, arXiv: 1307.0736
and PoS Lattice2013 (2013) 260; arXiv:1310.4958

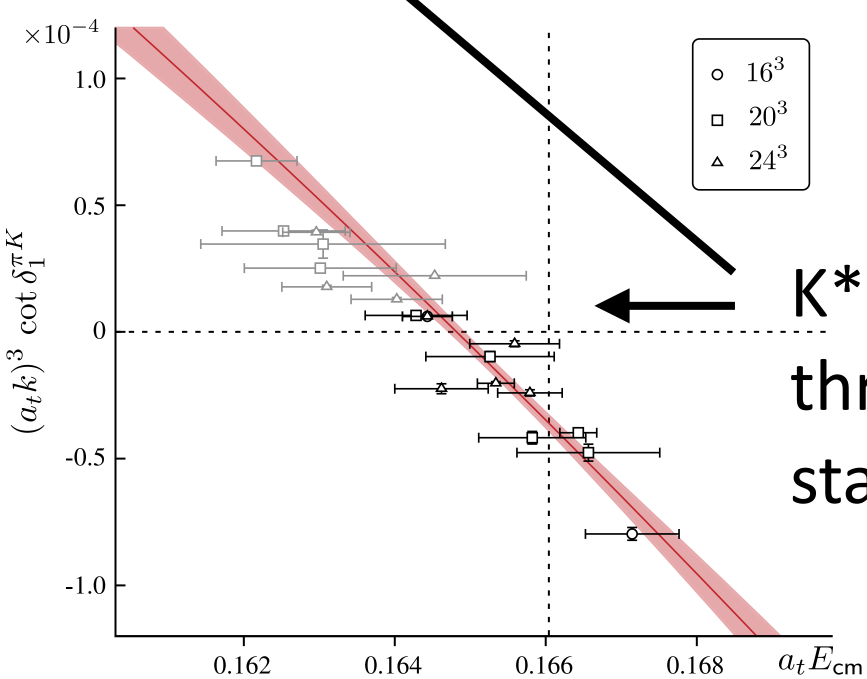
Resonances in coupled πK,ηK scattering



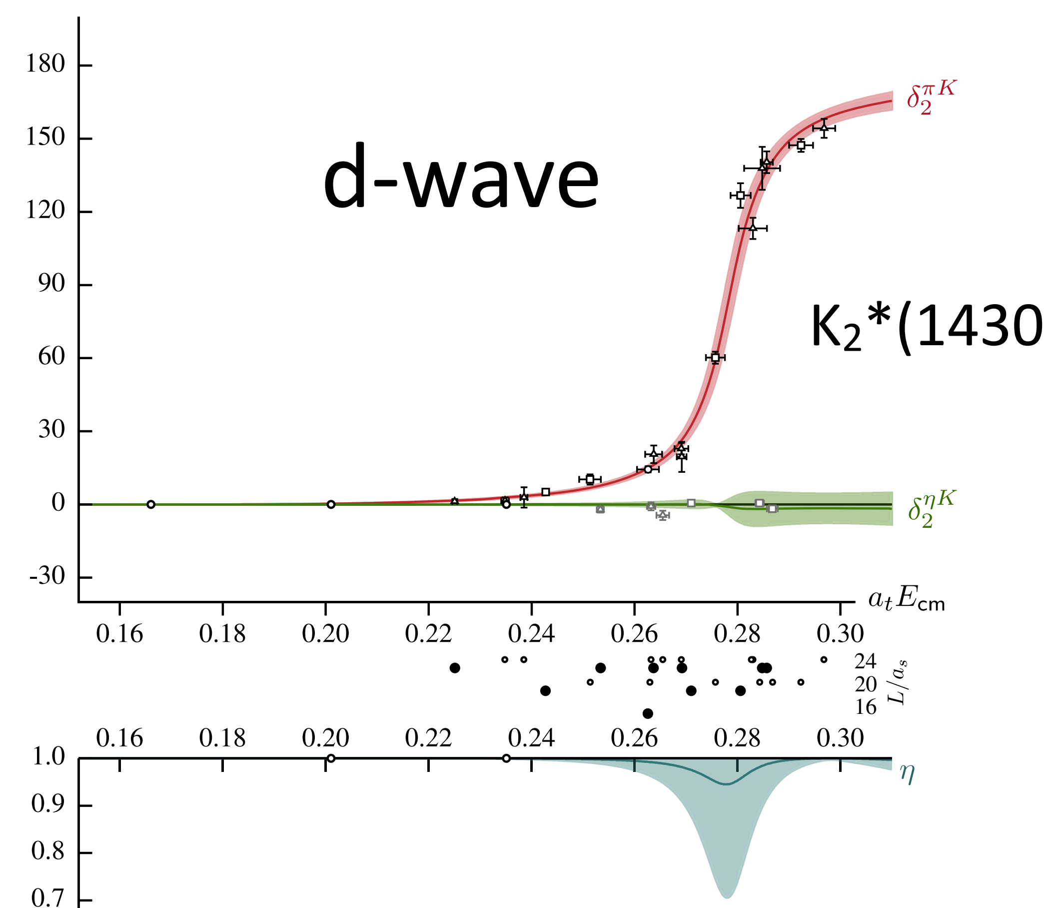
J. J. Dudek et al. (HSC), PRL 113, 182001 (2014), arXiv:1406.4158 [hep-ph].

D. J. Wilson et al. (HSC), PR D91, 054008 (2015), arXiv:1411.2004 [hep-ph].

K-matrix parametrisation to lattice spectrum (m_π=391 MeV, m_K=549 MeV)



K* lies below threshold (bound state)



Baryons: Nπ
(negative parity)

CBL&Verduci, PRD87 (2013) 054502
[arXiv:1212.5055]

5-quark operators
and 29 contraction terms

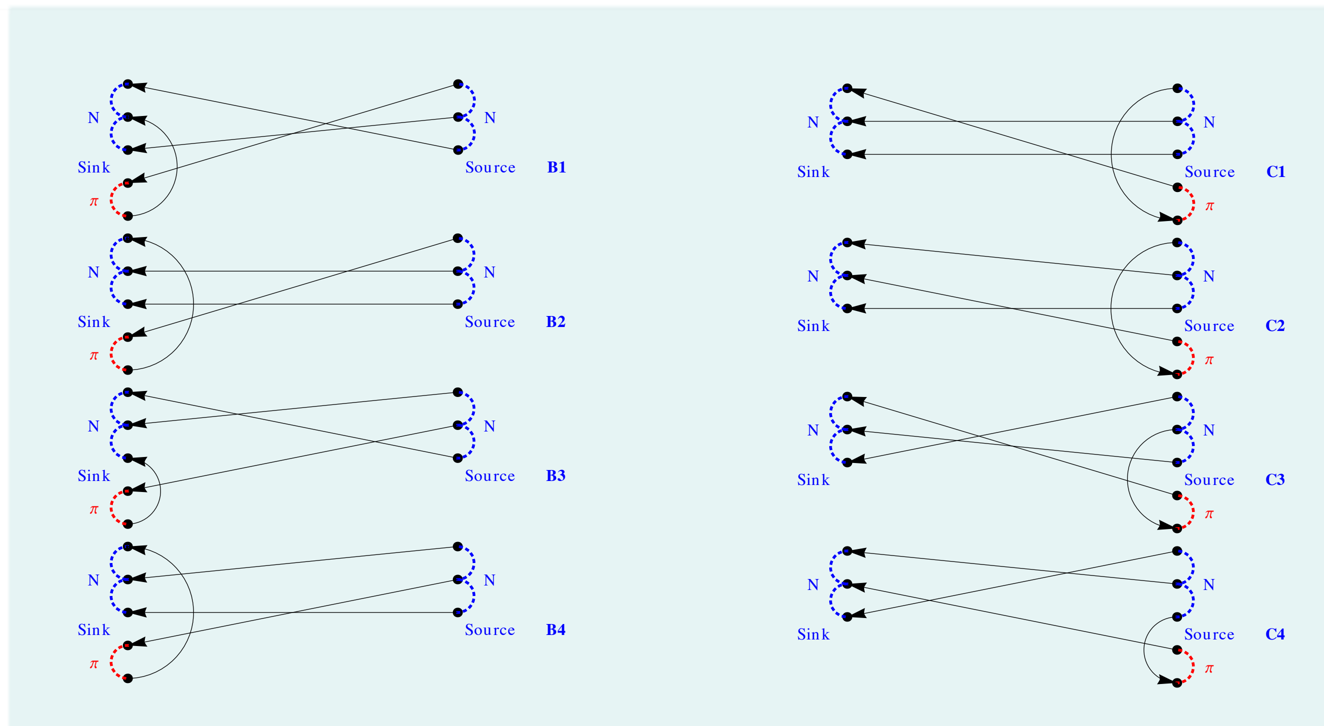
Baryons: Nπ

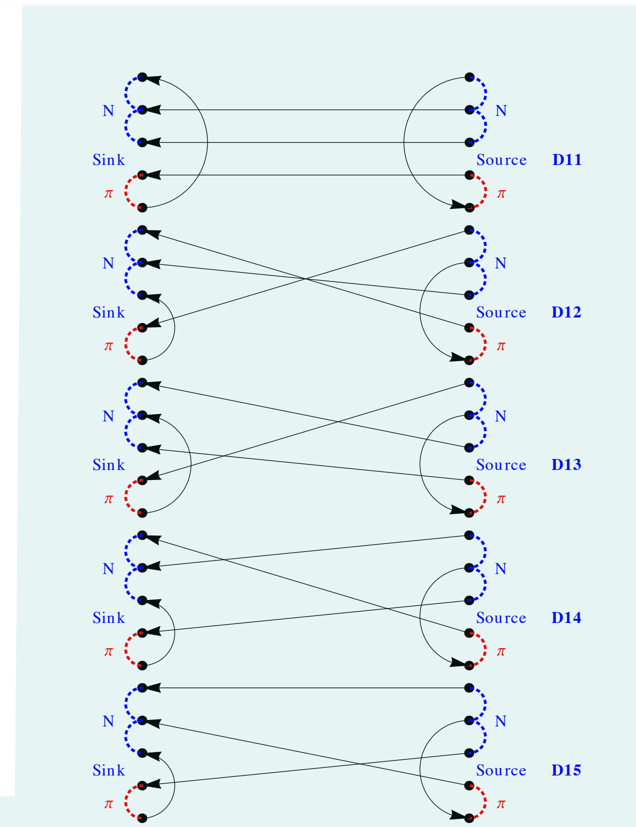
(negative parity)

CBL&Verduci, PRD87 (2013) 054502 [arXiv:1212.5055]

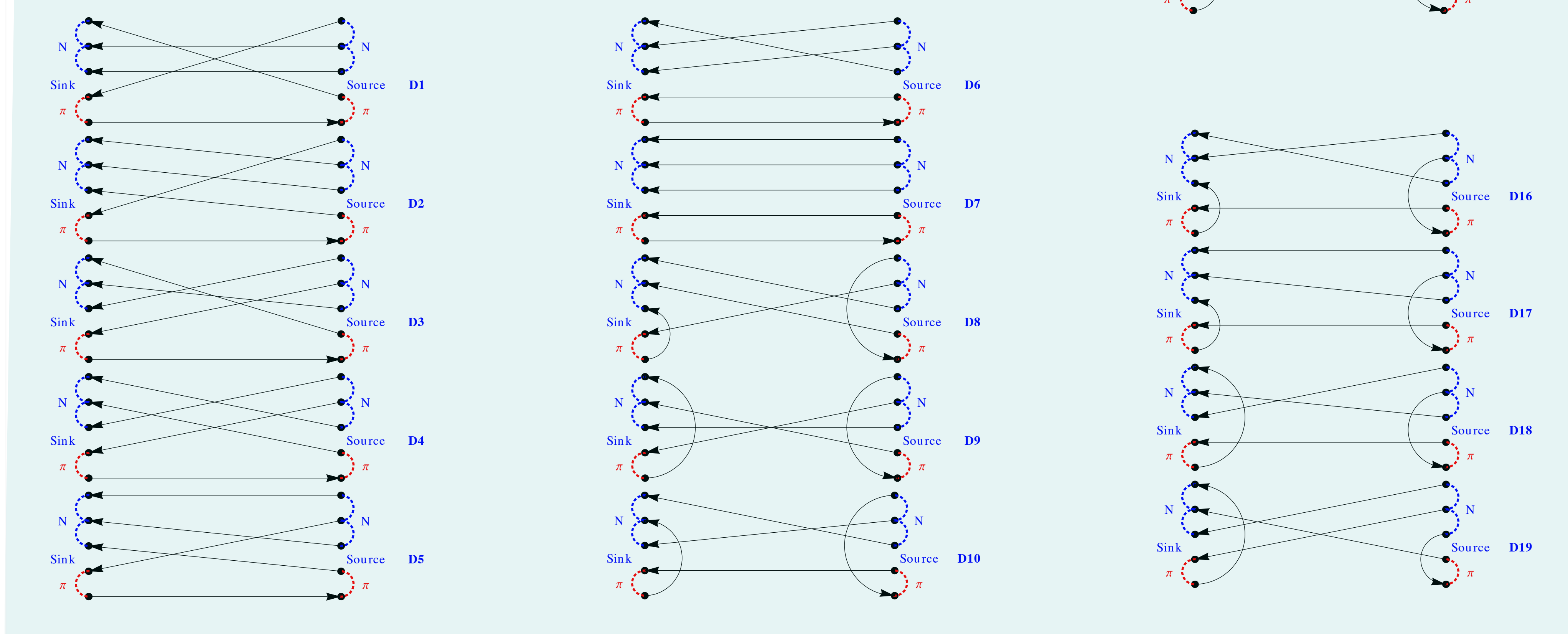
5-quark operators and 29 contraction terms

$N \to N\pi, N\pi \to N$ $\qquad$ $N\pi \to N\pi$





The background is fully dynamical quarks and gluons

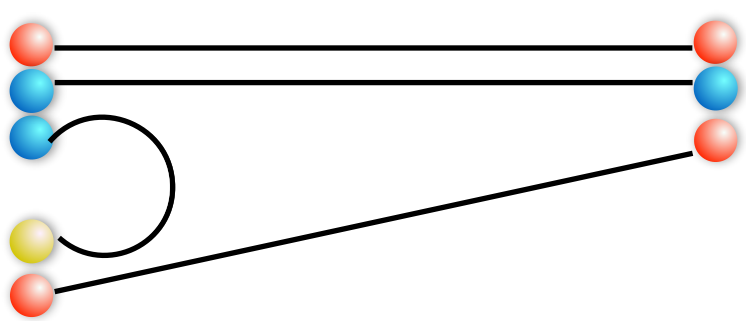


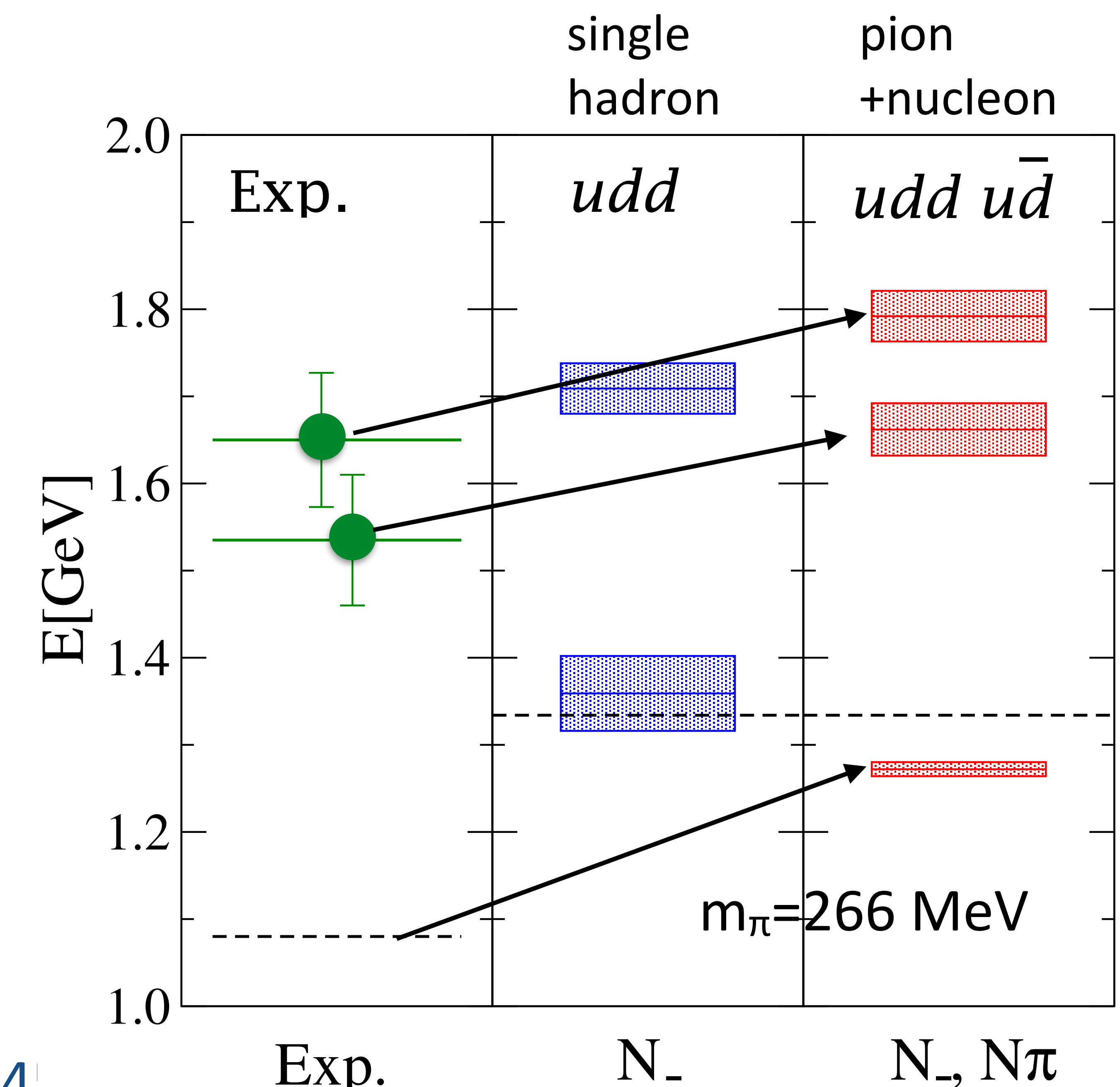
πN: Effect of open 2-hadron channel?

N*(1535),N*(1650)
Nπ negative parity

CBL&Verduci, PRD87 (2013) 054502 [arXiv:1212.5055]

needs annihilation terms





See also Kiratidis et al., PR D 91, 094 (2015) [arXiv: 1501.07667]

Wanted: Coupled channel analysis (Nη, ΛK, Δπ)...

Results: heavy quarks

Heavy quark results

see also Sinead Ryan's talk

(with meson-meson operators)

- What is the effect of nearby thresholds?

  Example: DD threshold and ψ(3770)
  Example:DK and D*K in $D_{sn}^{(*)}$

- Are there new states?

  Example: "level hunting": X(3872) and Z(3900)

Charmonium

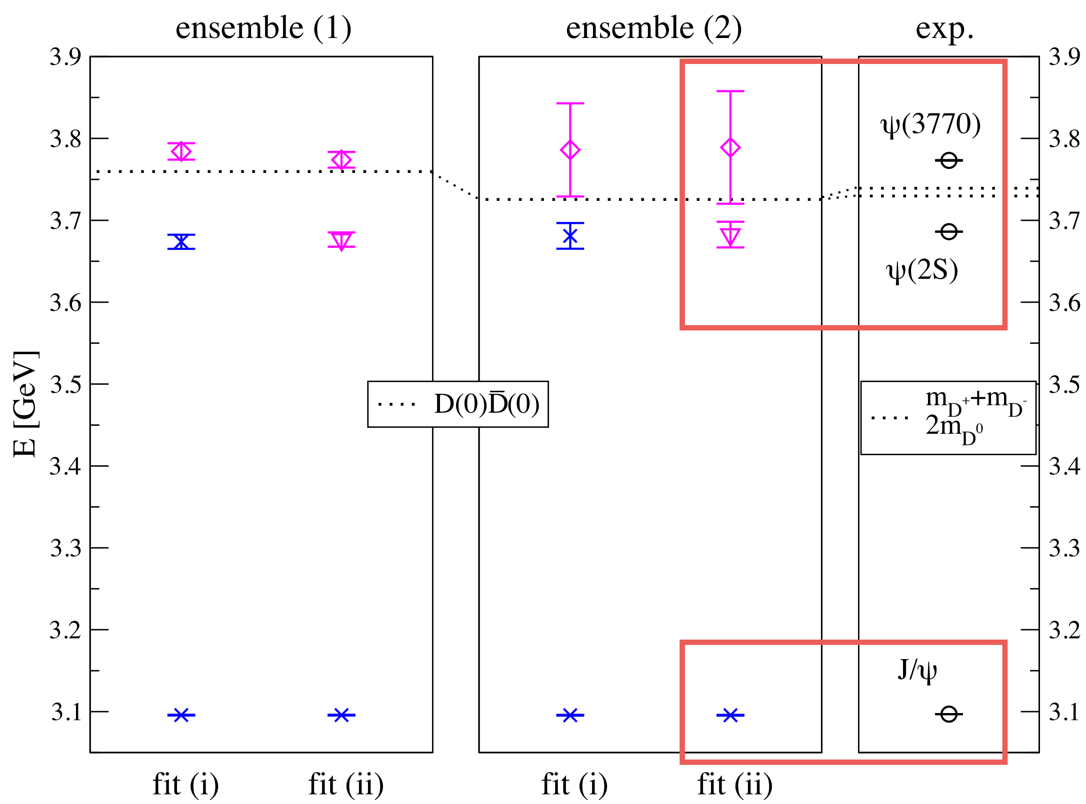
ψ(3770): resonance close to D$\underline{\text{D}}$ threshold

Lattice study: D$\underline{\text{D}}$ scattering on two volumes and m_π=266 and 157 MeV

15 interpolators of c$\underline{\text{c}}$ type
2 operators of type D$\underline{\text{D}}$

CBL et al., JHEP (2015) [arXiv:1503.05363]

[same paper:
η_{c0}(2P) or X(3915): 0^{++}
controversial signal]



	ψ(3770), m_R	g(no unit)	ψ(2S), m_R
m_π=266 MeV	3774(6)(10)	9.7(1.4)	3676(6)(9)
m_π=157 MeV	3789(68)(10)	28(21)	3682(13)(9)
Exp.	3773.15(33)	18.7(1.4)	3686.11(1)

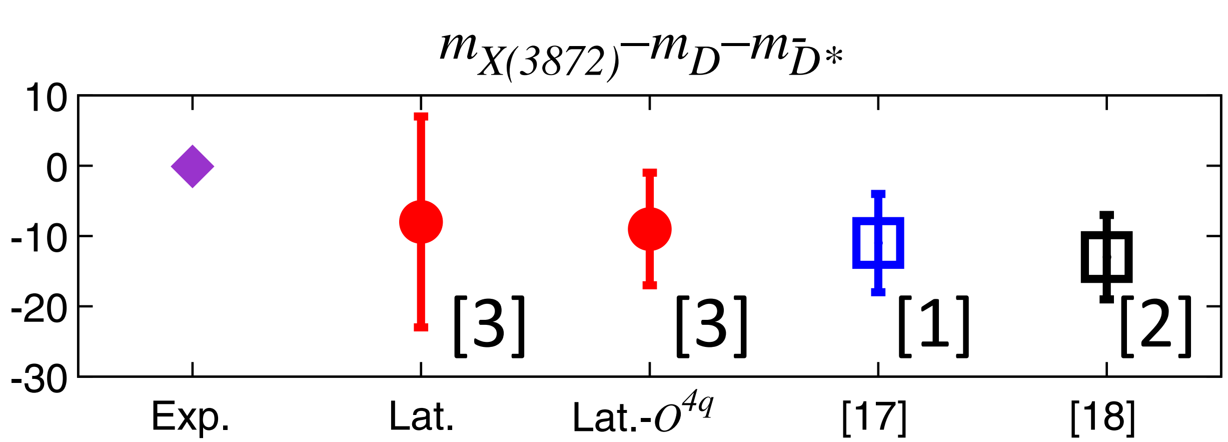
X(3872)

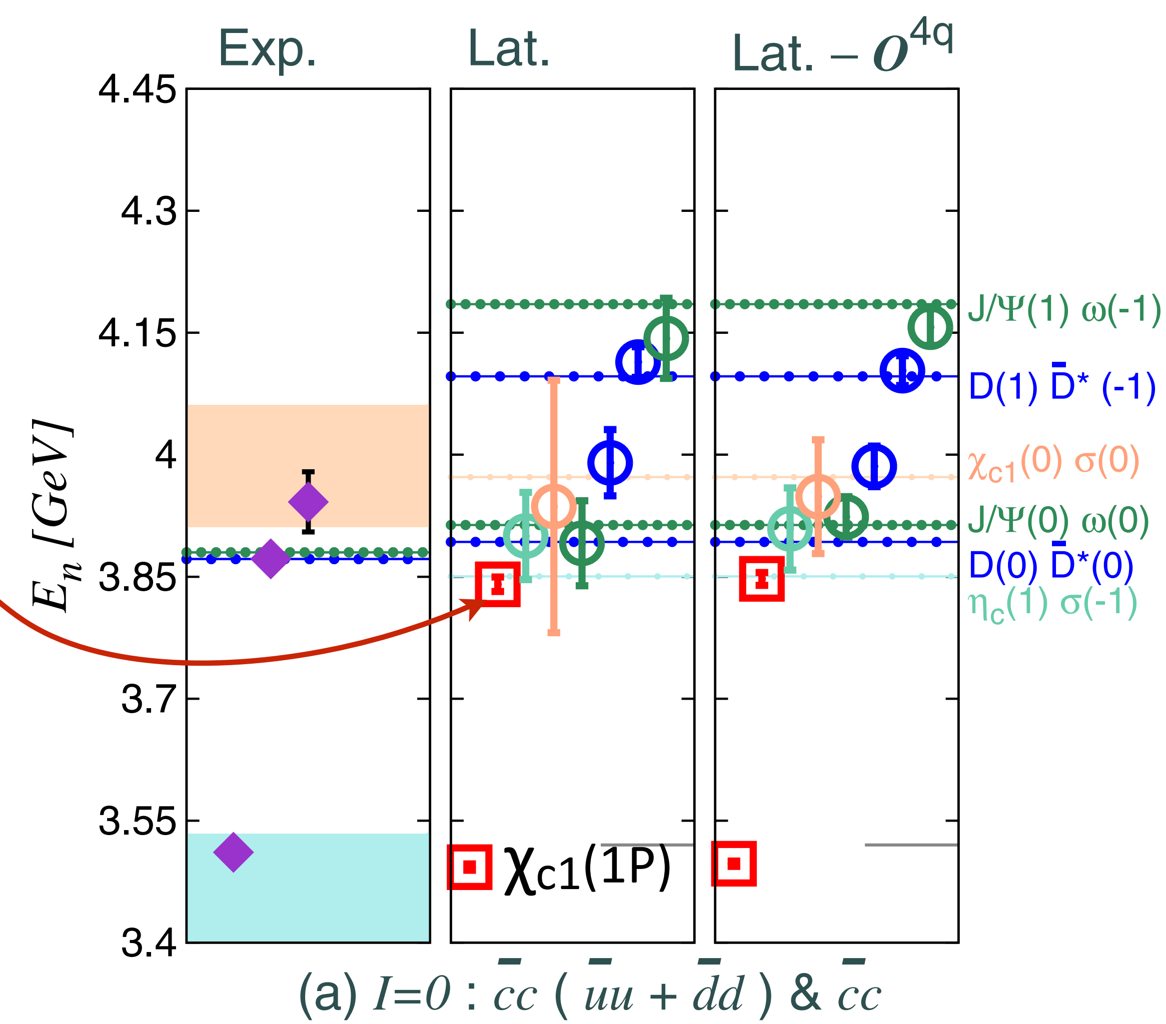
X(3872) $0^+(1^{++})$

[1] Prelovsek/Leskovec, Phys. Rev. Lett. 111, 192001 (2013)
.   [2] Lee et al. (FNAL/MILC), [arXiv:1411.1389]
[3] Padmanath et al,Phys. Rev. D 92 (2015) 034501
[arXiv:1503.03257]

at m_π=266 MeV
22 $c\underline{c}$ and $c\underline{c}u\underline{u}$,$c\underline{c}u\underline{d}$,..interpolators
for I=0 and 1
($D\underline{D}^*$, $J/\psi\ \rho$, $J/\psi\ \omega$, $\eta_c\ \sigma$, $\chi_{c0}\ \pi$, $\chi_{c2}\ \pi$, 4q)

all observe X(3872) closely below $D\underline{D}^*$
(with strong $c\underline{c}$ component)





(large scatt.length 1.1 fm)

Heavy-light sector: D_s (0^+, 1^+, 2^+)

CHARMED, STRANGE MESONS

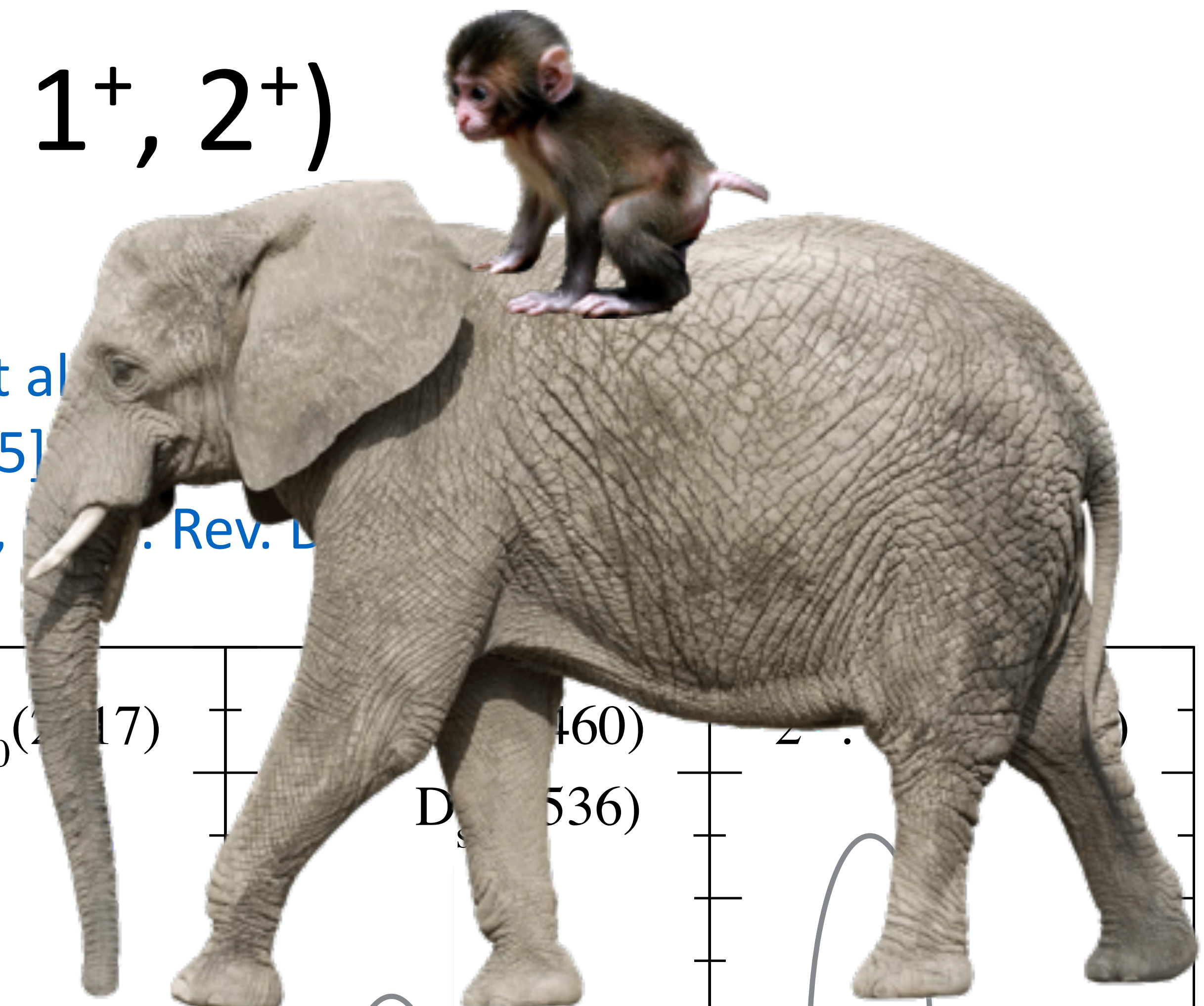
Particles

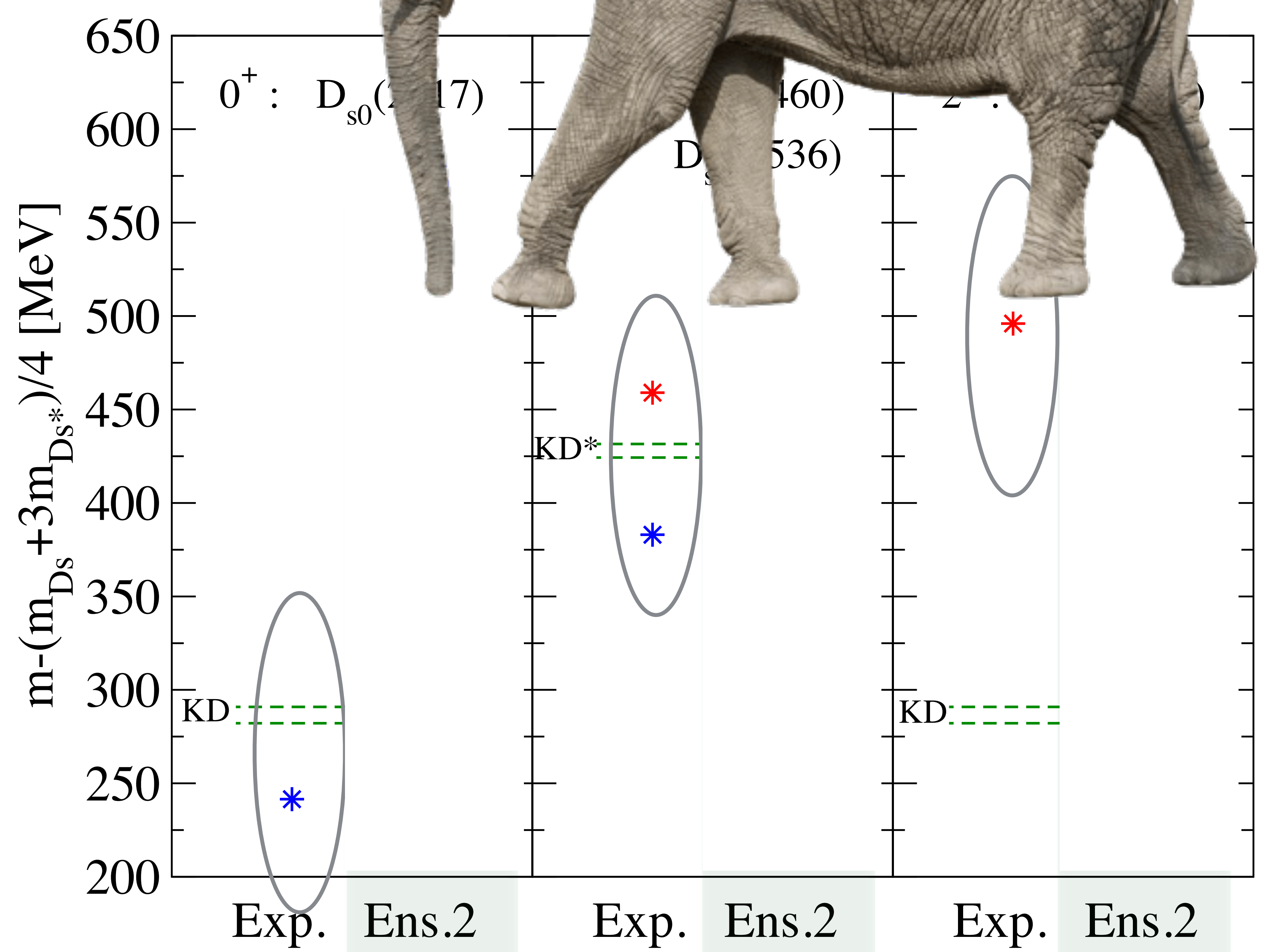
Particles	HQL:
$D_s^{\pm}$	
$D_s^{*\pm}$	
$D_{s0}^*(2317)^{\pm}$	s-wave
$D_{s1}(2460)^{\pm}$	s-wave
$D_{s1}(2536)^{\pm}$	d-wave
$D_{s2}^*(2573)$	d-wave
$D_{s1}^*(2700)^{\pm}$	
$D_{sJ}^*(2860)^{\pm}$	
$D_{sJ}(3040)^{\pm}$	

(PACS-CS lattices.
m_π=157 MeV
Lattice operators-
$c\underline{s}$,DK,D*K)

See also Martinez Torres et al.,
JHEP 1505 (2015) 153

Mohler et al
1308.3175]
CBL et al., . Rev. L





Heavy-light sector: D_s (0^+, 1^+, 2^+)

CHARMED, STRANGE MESONS

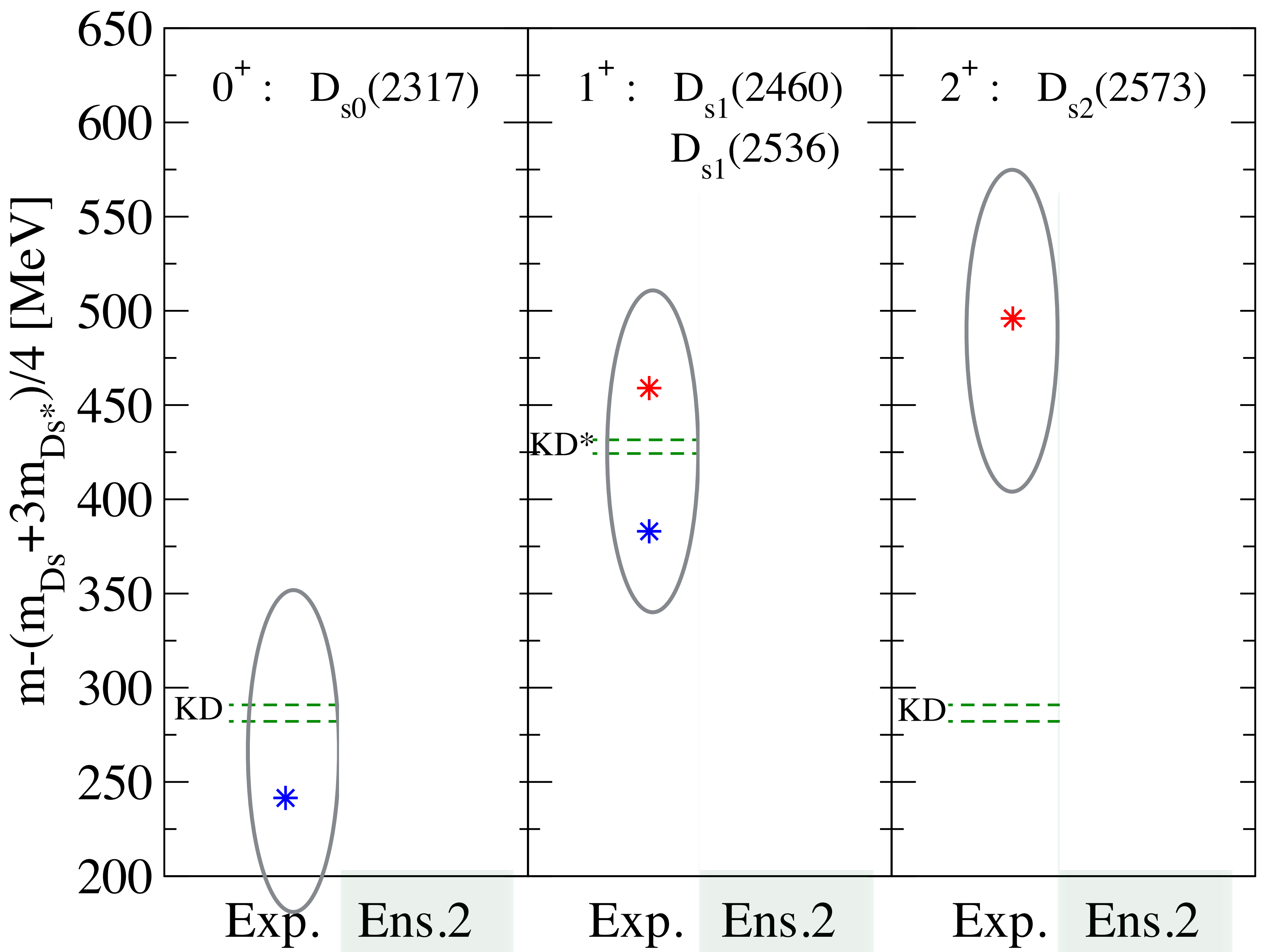
Particles

Particles	HQL:
$D_s^\pm$	
$D_s^{*\pm}$	
$D_{s0}^*(2317)^\pm$	s-wave
$D_{s1}(2460)^\pm$	s-wave
$D_{s1}(2536)^\pm$	d-wave
$D_{s2}^*(2573)$	d-wave
$D_{s1}^*(2700)^\pm$	
$D_{sJ}^*(2860)^\pm$	
$D_{sJ}(3040)^\pm$	

(PACS-CS lattices.
m_π=157 MeV
Lattice operators-
c$\underline{s}$,DK,D*K)

See also Martinez Torres et al., JHEP 1505 (2015) 153

Mohler et al.,PRL. 111, 222001; (2013)[arXiv 1308.3175];
CBL et al., Phys. Rev. D 90, 034510 (2014)



Heavy-light sector: D_s (0^+, 1^+, 2^+)

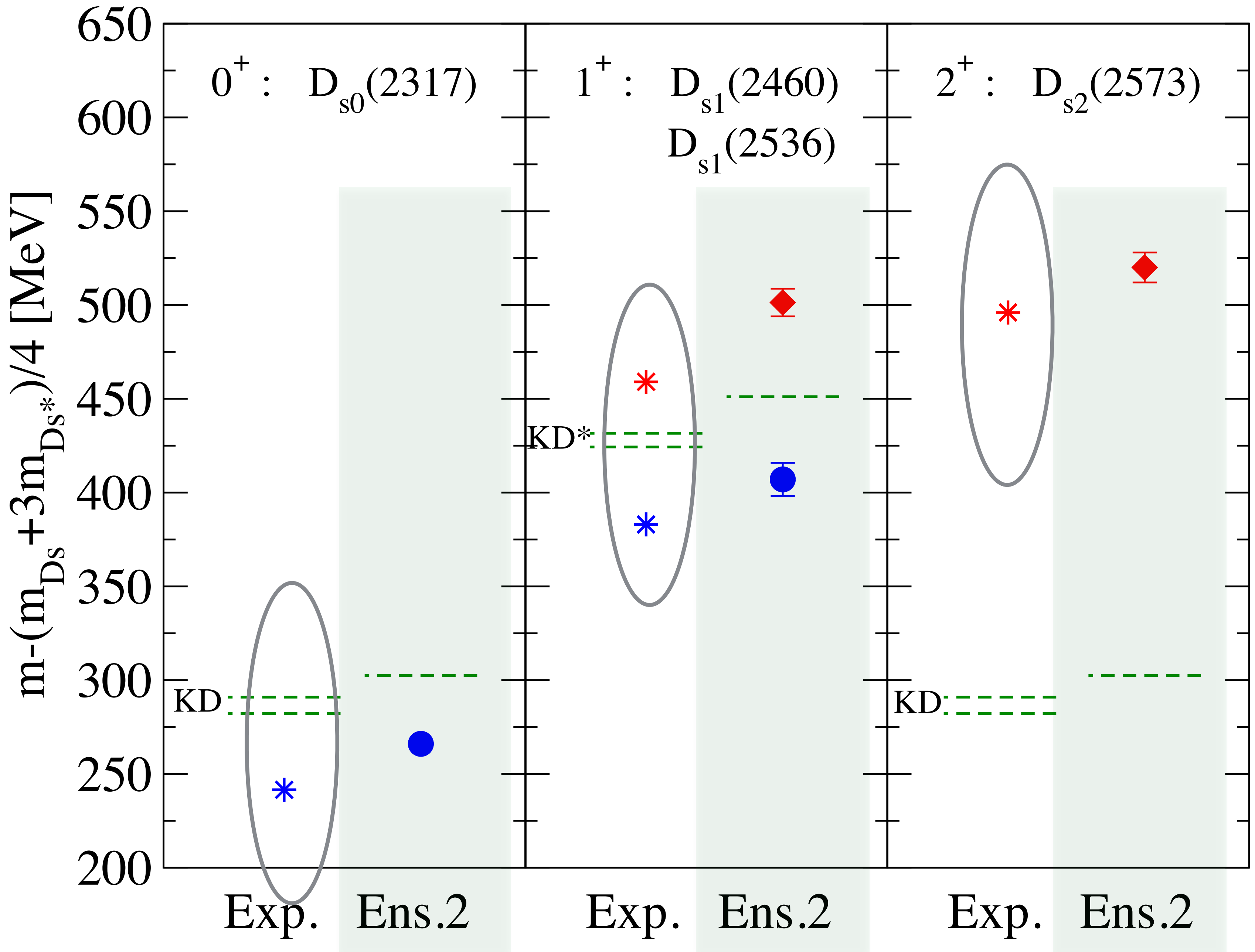
CHARMED, STRANGE MESONS

Particles

Particles	HQL:
$D_s^{\pm}$	
$D_s^{*\pm}$	
$D_{s0}^*(2317)^{\pm}$	s-wave
$D_{s1}(2460)^{\pm}$	s-wave
$D_{s1}(2536)^{\pm}$	d-wave
$D_{s2}^*(2573)$	d-wave
$D_{s1}^*(2700)^{\pm}$	
$D_{sJ}^*(2860)^{\pm}$	
$D_{sJ}(3040)^{\pm}$	

Mohler et al.,PRL. 111, 222001; (2013)[arXiv 1308.3175];
CBL et al., Phys. Rev. D 90, 034510 (2014)

(PACS-CS lattices.
m_π=157 MeV
Lattice operators- $c\underline{s}$,DK,D*K)



See also Martinez Torres et al., JHEP 1505 (2015) 153

Heavy quark sector: B_s (0^+, 1^+, 2^+)

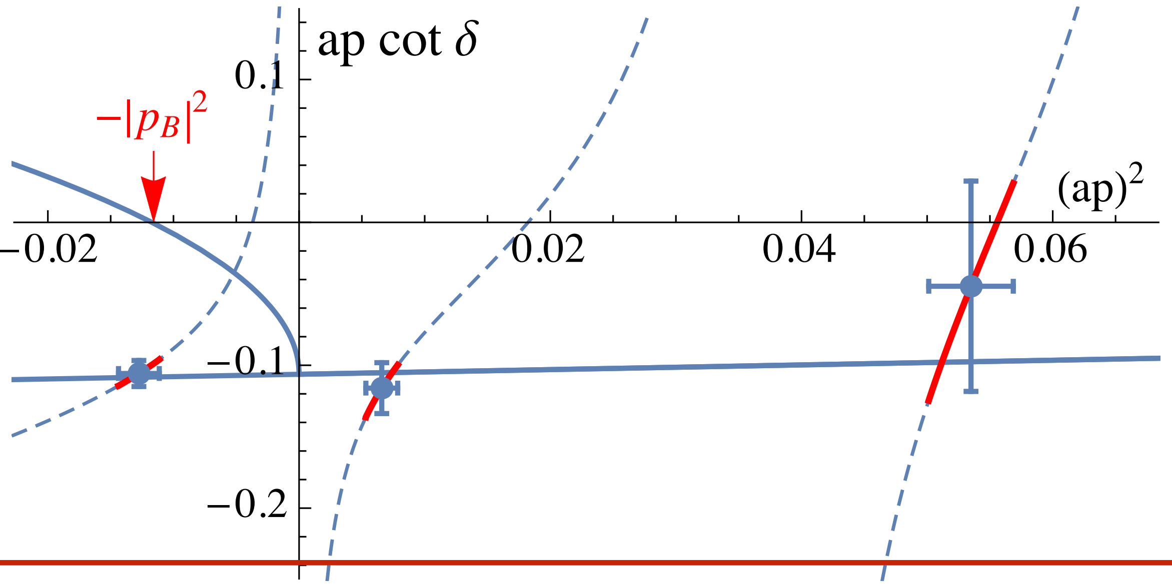
CBL et al., Phys. Lett. B 750 (2015) 17 [arXiv:1501.01646]

BK, B*K scattering (PACS-CS lattices, m_π=157 MeV)

0^+: Bound state B_{s0} with

$m(B_{s0})$ = 5.711(13)(19) GeV

(prediction)



1^+: Bound state B_{s1} with $m(B_{s1})$= 5.750(17)(19) GeV (prediction)

Close to threshold weakly coupled state B_{s1}' at m=5.831(9)(6) GeV
(Exp: B_{s1}(5830) at 5.8287(4) GeV)

The future

…has started already

Two nucleon scattering

Berkowitz et al. (CalLat),[arXiv:1508.00886]

m_π=800 MeV (u,d,s flavor symmetric limit)
spatial extent up to 4.6 fm
partial-waves: S, P, D, F

no backtracking quarks!

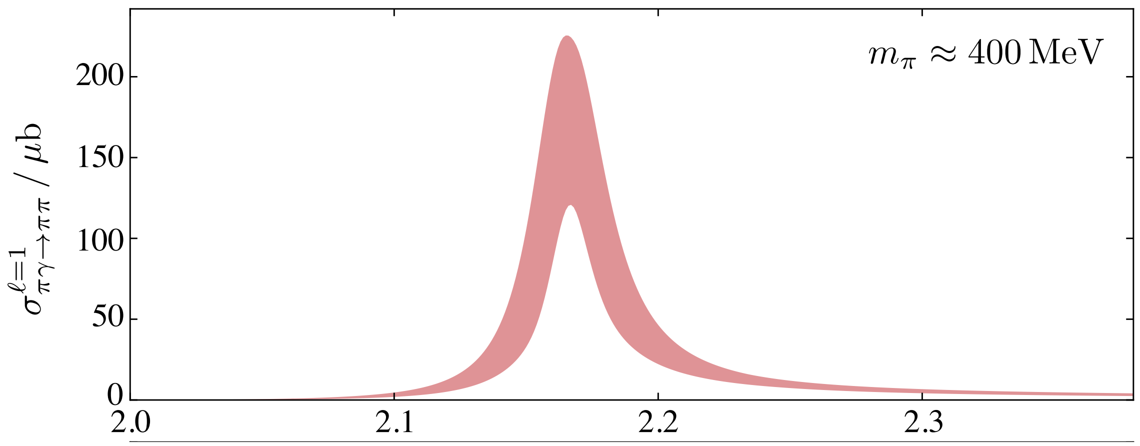
Radiative decays

Briceño et al. [arXiv:1507.06622]

$$\pi\,\gamma^* \to \rho \to \pi\pi$$

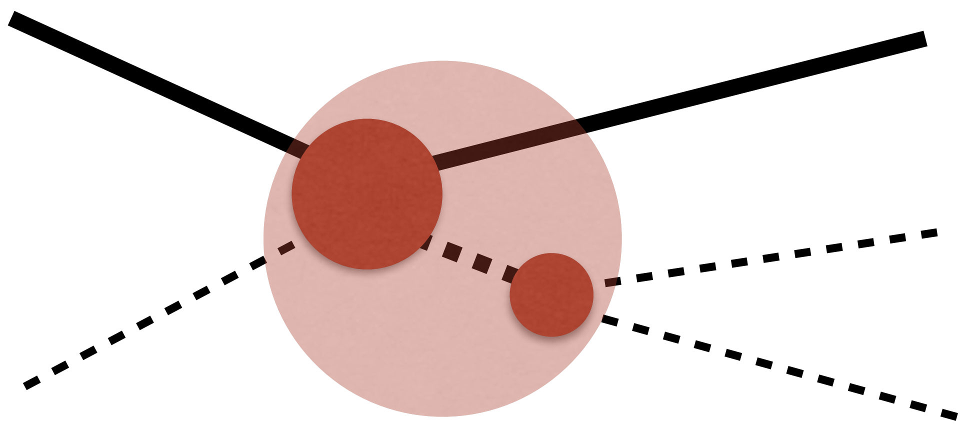
Here ρ is a resonance

m_π=400 MeV



...has started already

More than two particle states



Extension to 3-particle channels

Hansen & Sharpe,PR D 90, 116003 (2014) [arXiv:1408.5933] quantization condition

Meißner et al.,PRL 114, 091602 (2015) [arXiv:1412.4969] shallow bound states

Hansen & Sharpe, [arXiv:1504.04248]

But: No numerical results yet!

The Future?

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Prediction is very difficult, especially about the future (Niels Bohr)

(according to https://en.wikiquote.org/wiki/Niels_Bohr)

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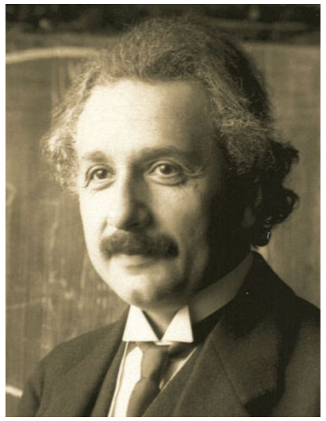
You shouldn't believe all, that you find in the internet (Albert Einstein)



Thank you!