Hypernuclear equation of state for compact stars

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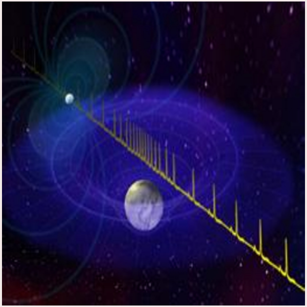
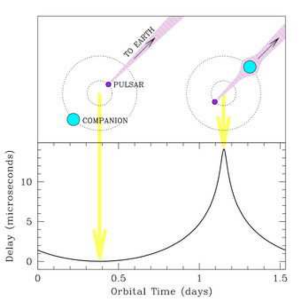
A two-solar-mass neutron star measured

The largest pulsating star yet observed casts doubts on exotic matter theories





The binary millisecond pulsar J1614-223010+11 Shapiro delay signature:

$$\Delta t = -\frac{2GM}{c^3}\log(1 - \vec{R}\cdot\vec{R}'). \quad (1)$$

The pulsars mass 1.97 ± 0.04 solar masses which rules out almost all currently proposed hyperon or boson condensate equations of state. (Demorest et al, 2010, Nature 467, 1081)

Hypernuclear Equation of State for Compact Stars

Approaches to dense hypernuclear matter

- Microscopic non-rel. approaches: NSC potentials + BHF theory
  No massive stars, even with 3B force
- Quantum Monte Carlo + 3-body equations
  Massive stars with 3B force; only Λ so far
- Relativistic density functionals

  Nuclear part is constrained by nuclear phenomenology
  Hyperonic sector constrained by flavor symmetries of strong interactions
  Λ-hyperon interactions can be constrained from Λ-hypernuclei

Relativistic covariant Lagrangians for hypernuclear matter

To generate the DFT start with the Lagrangian :

$$\begin{aligned}\mathcal{L} &= \sum_B \bar{\psi}_B \Big[\gamma^\mu \Big(i\partial_\mu - g_{\omega BB}\omega_\mu - \frac{1}{2} g_{\rho BB} \boldsymbol{\tau}\cdot\boldsymbol{\rho}_\mu\Big) - (m_B - g_{\sigma BB}\sigma)\Big]\psi_B \\ &+ \frac{1}{2}\partial^\mu\sigma\partial_\mu\sigma - \frac{1}{2}m_\sigma^2\sigma^2 - \frac{1}{4}\omega^{\mu\nu}\omega_{\mu\nu} + \frac{1}{2}m_\omega^2\omega^\mu\omega_\mu - \frac{1}{4}\boldsymbol{\rho}^{\mu\nu}\boldsymbol{\rho}_{\mu\nu} + \frac{1}{2}m_\rho^2\boldsymbol{\rho}^\mu\cdot\boldsymbol{\rho}_\mu \\ &+ \sum_\lambda \bar{\psi}_\lambda (i\gamma^\mu\partial_\mu - m_\lambda)\psi_\lambda - \frac{1}{4}F^{\mu\nu}F_{\mu\nu}, \end{aligned} \tag{2}$$

- B-sum is over the baryonic octet $B \equiv p, n, \Lambda, \Sigma^{\pm,0}, \Xi^{-,0}$
- Meson fields include σ meson, $\boldsymbol{\rho}_\mu$-meson and ω_μ-meson
- Leptons include electrons, muons and neutrinos for $T \neq 0$

Fixing the couplings: nucleonic sector

Density dependent parametrization (S. Typel and H. H. Wolter, Nuclear Physics A 656, 331):

$$g_{iN}(\rho_B) = g_{iN}(\rho_0)h_i(x), \qquad i = \sigma, \omega, \qquad h_i(x) = a_i \frac{1 + b_i(x + d_i)^2}{1 + c_i(x + d_i)^2} \tag{3}$$

$$g_{\rho N}(\rho_B) = g_{\rho N}(\rho_0) \exp[-a_\rho(x - 1)]. \tag{4}$$

DD-ME2 parametrization of D. Vretenar, P. Ring et al. Phys. Rev. C 71, 024312 (2005).

	σ	ω	ρ
m_i [MeV]	550.1238	783.0000	763.0000
$g_{Ni}(\rho_0)$	10.5396	13.0189	3.6836
a_i	1.3881	1.3892	0.5647
b_i	1.0943	0.9240	—
c_i	1.7057	1.4620	—
d_i	0.4421	0.4775	—

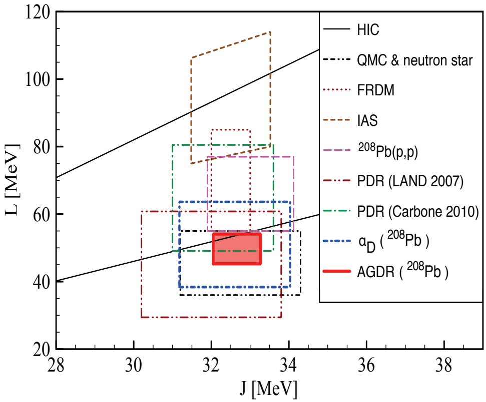
Total number of parameters 8: boundary conditions on $h(x)$ at $x = 1$.

Nuclear phenomenology

Our density functional is consistent with the following parameters of nuclear systems

- saturation density $\rho_0 = 0.152 \text{ fm}^{-3}$
- binding energy per nucleon $E/A = -16.14$ MeV,
- incompressibility $K_0 = 250.90$ MeV,
- symmetry energy $J = 32.30$ MeV,
- symmetry energy slope $L = 51.24$ MeV,
- symmetry incompressibility $K_{sym} = -87.19$ MeV
- of the energy and symmetry energy with respect to density taken at saturation have the following values $Q_0 = 478.30$ and $Q_{sym} = 777.10$ MeV.

The allowed parameter space



Experiments predict symmetry energy at saturation $J = 32.7 \pm 0.6$ MeV and the slope of the symmetry energy $L = 49.7 \pm 4.4$ MeV), see arXiv:1311.1456 [nucl-ex]. Our DFT values are: symmetry energy $J = 32.30$ MeV, symmetry energy slope $L = 51.24$ MeV.

Fixing the couplings: hyperonic sector

Hyperon-vector mesons couplings: $SU(3)$-flavor symmetry and vector dominance model

$$g_{\Xi\omega} = \frac{1}{3} g_{N\omega}, \quad g_{\Sigma\omega} = g_{\Lambda\omega} = \frac{2}{3} g_{N\omega}. \tag{5}$$
$$g_{\Xi\rho} = g_{\rho N}, \quad g_{\Sigma\rho} = 2g_{\rho N}, \quad g_{\Lambda\rho} = 0. \tag{6}$$

And no strange mesons (for example ϕ) in the theory; see Schaffner et al, Bednarek et al

Scalar σ-meson – hyperon couplings

$$g_{N\sigma} = \cos\theta_S\, g_1 + \sin\theta_S\, (4\alpha_S - 1) g_S/\sqrt{3}, \tag{7}$$
$$g_{\Lambda\sigma} = \cos\theta_S\, g_1 - 2\sin\theta_S\, (1 - \alpha_S) g_S/\sqrt{3}, \tag{8}$$
$$g_{\Sigma\sigma} = \cos\theta_S\, g_1 + 2\sin\theta_S\, (1 - \alpha_S) g_S/\sqrt{3}, \tag{9}$$
$$g_{\Xi\sigma} = \cos\theta_S\, g_1 - \sin\theta_S\, (1 + 2\alpha_S) g_S/\sqrt{3}. \tag{10}$$

Inequalities for σ-meson couplings

For arbitrary underlying quark model we have

$$\boxed{2(g_{N\sigma} + g_{\Xi\sigma}) = 3g_{\Lambda\sigma} + g_{\Sigma\sigma}\,.} \tag{11}$$

e.g., for $SU(6)$ symmetric quark model $g_{\Xi\sigma} = (1/3)g_{N\sigma}$ and $g_{\Lambda\sigma} = g_{\Sigma\sigma} = (2/3)g_{N\sigma}$.

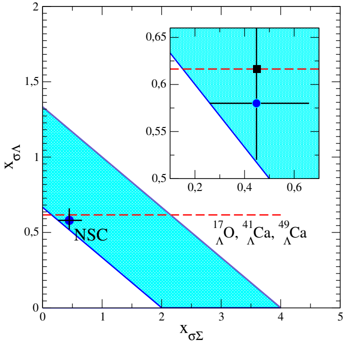
$$g_{\Xi\sigma} = \frac{1}{2}(3g_{\Lambda\sigma} + g_{\Sigma\sigma}) - g_{N\sigma}\,, \qquad 0 \le g_{\Xi\sigma} \le g_{N\sigma}\,, \tag{12}$$

$$\boxed{1 \le \frac{1}{2}(3x_{\Lambda\sigma} + x_{\Sigma\sigma}) \le 2x_{N\sigma}\,, \quad x_{i,\sigma} = \frac{g_{i\Sigma}}{g_{N\sigma}}} \tag{13}$$

Nijmegen soft-core potential implies that

$$x_{\Lambda\sigma} = 0.58, \qquad x_{\Sigma\sigma} = 0.448. \tag{14}$$

The allowed parameter space



- Blue region - values allowed by the inequality
- NSC values are shown by dot, red line - Λ-hypernuclei, cross - our parameter space hypernuclear couplings

Hypernuclear calculatoins

	Λ 1s$_{1/2}$ state [MeV]	E/A [MeV]	r_p [fm]	r_n [fm]	r_Λ [fm]
$^{17}_{\Lambda}$O					
a	0.846	−7.443	2.609	2.579	8.313
b	−4.564	−7.760	2.606	2.576	3.203
c	−27.279	−9.035	2.563	2.534	1.977
$^{41}_{\Lambda}$C					
a	0.934	−8.336	3.372	3.319	8.710
b	−8.519	−8.565	3.370	3.317	3.168
c	−35.224	−9.199	3.347	3.294	2.298
$^{49}_{\Lambda}$C					
a	0.973	−8.442	3.389	3.576	8.825
b	−9.882	−8.662	3.387	3.571	3.140
c	−37.257	−9.207	3.365	3.548	2.419

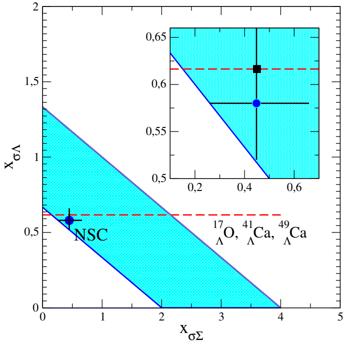
Table : Properties of Λ-hypernuclei $^{17}_{\Lambda}$O, $^{41}_{\Lambda}$C, and $^{49}_{\Lambda}$C for the models *a*, *b*, and *c*. The columns list the single-particle energy of the Λ 1s$_{1/2}$ state, the binding energy and the rms radii for neutrons, protons and Λ-hyperon.
model *a* with $x_{\sigma\Lambda} = 0.52$, model *b* with $x_{\sigma\Lambda} = 0.59$, and model *c* with $x_{\sigma\Lambda} = 0.66$; all three models have $x_{\omega\Lambda} = 2/3$ and $x_{\rho\Lambda} = 0$.

Hypernuclear calculatoins

	$E_{Mass}[\Lambda\ 1s_{1/2}]$ [MeV]	$E[\Lambda\ 1s_{1/2}]$ [MeV]	E/A [MeV]	r_p [fm]	r_n [fm]	r_Λ [fm]
$^{17}_{\Lambda}$O	−12.109	−11.716	−8.168	2.592	2.562	2.458
^{16}O	–	–	−8.001	2.609	2.579	–
$^{41}_{\Lambda}$C	−17.930	−17.821	−8.788	3.362	3.309	2.652
^{40}Ca	–	–	−8.573	3.372	3.320	–
$^{49}_{\Lambda}$Ca	−19.215	−19.618	−8.858	3.379	3.562	2.715
^{48}Ca	–	–	−8.641	3.389	3.576	–

Table : Single-particle energies of the Λ $1s_{1/2}$ states, binding energies, and rms radii of the Λ-hyperon, neutron, and proton of $^{17}_{\Lambda}$O, $^{41}_{\Lambda}$C, and $^{49}_{\Lambda}$Ca are presented for optimal model. In addition, single-particle energies of the Λ $1s_{1/2}$ states, i.e. separation energies of the Λ-particle, obtained from the mass formula of Levai et al (1998) are given for these Λ-hypernuclei. Furthermore, the properties of ^{16}O, ^{40}Ca, and ^{48}Ca are given for the optimal model. The optimal model obtained in this way has $x_{\sigma\Lambda} = 0.6164$.

The allowed parameter space



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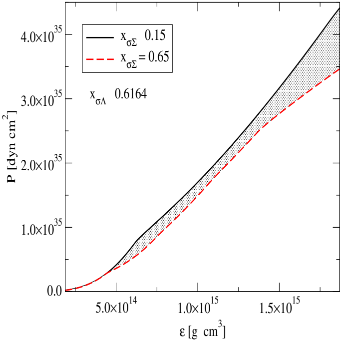
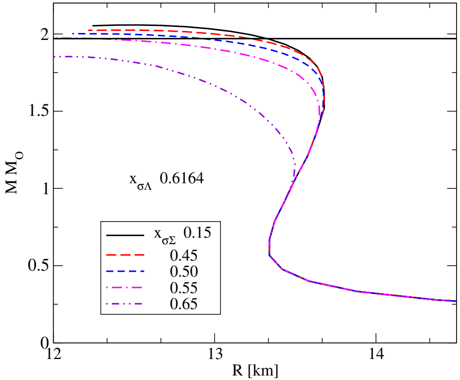
EOS of hypernuclear matter



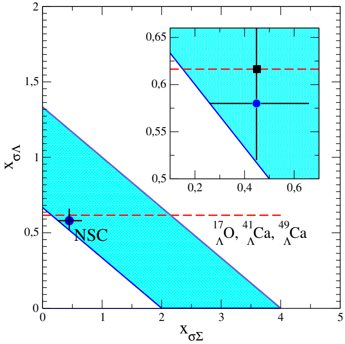
Zero temperature equations of state of hypernuclear matter for fixed $x_{\sigma\Lambda} = 0.6164$ and a range of values $0.15 \leq x_{\sigma\Sigma} \leq 0.65$. These values generate the shaded area, which is bound from below by the softest EoS (dashed red line) corresponding to $x_{\sigma\Sigma} = 0.65$ and from above by the hardest EoS (solid line) corresponding to $x_{\sigma\Sigma} = 0.15$.

Mass vs Radius relationship



The solid (blue) lines show the cases $x_{\Sigma\sigma} = 0.26$ and $x_{\Lambda\sigma} = 0.58$ (upper panel) and $x_{\Sigma\sigma} = 0.52$ and $x_{\Lambda\sigma} = 0.448$ (lower panel). The (red) dots show the cases $x_{\Sigma\sigma} = 0.66$ and $x_{\Lambda\sigma} = 0.58$ (upper panel) and $x_{\Sigma\sigma} = 0.66$ and $x_{\Lambda\sigma} = 0.448$ (lower panel).

The allowed parameter space



- Blue region - values allowed by the inequality
- NSC values are shown by dot, red line - Λ-hypernuclei, cross - our parameter space hypernuclear couplings, square - limit set by $2M_{\odot}$.

Conclusions

- We used a relativistic density functional theory of hypernuclear matter to extract bounds on the density-dependent couplings of a hypernuclear DFT
- Did simultaneous fits to the medium-heavy Λ-hypernuclei and used the requirement that the maximum mass of a hyperonic compact star is at least two-solar masses.
- Narrowed down significantly the parameter space of couplings of DFT - the range of optimal values of parameters is given by

$$x_{\sigma\Lambda} = 0.6164, \qquad 0.15 \leq x_{\sigma\Sigma} \leq 0.5. \tag{15}$$

- Our work provides a proof-of-principle of the method for constraining any theoretical framework that describes hypernuclear systems using current laboratory and astrophysical data.