Ab-Initio Description of p-Shell Hypernuclei

Anti-matter, hyper-matter & exotica production at the LHC

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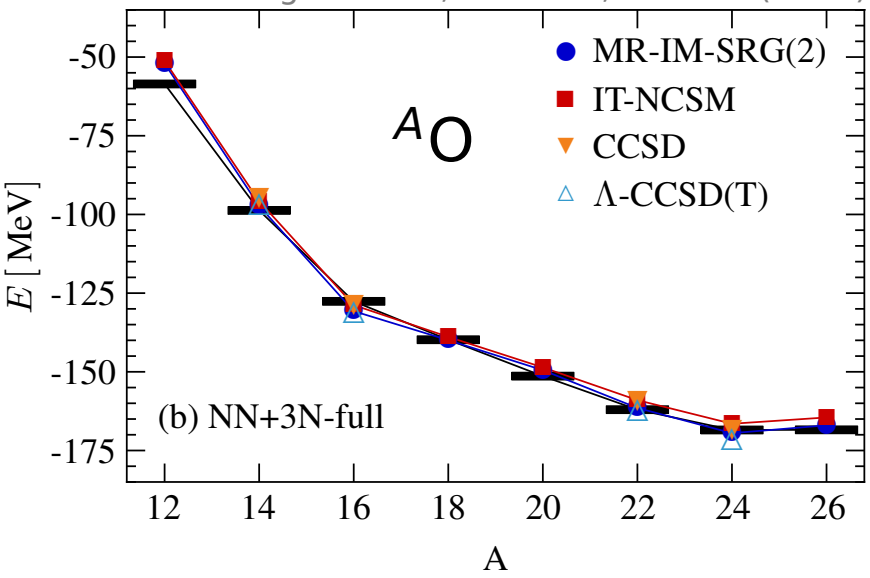
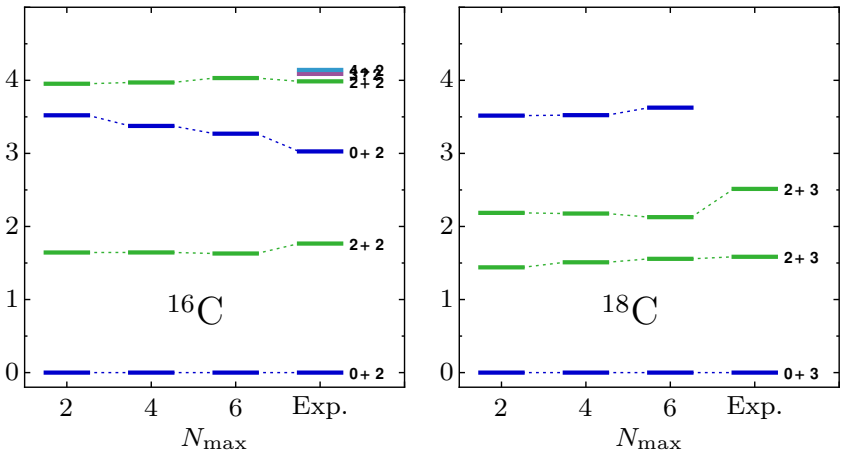
DARMSTADT

Motivation

Hergert *et al.*, PRL **110**, 242501 (2013)





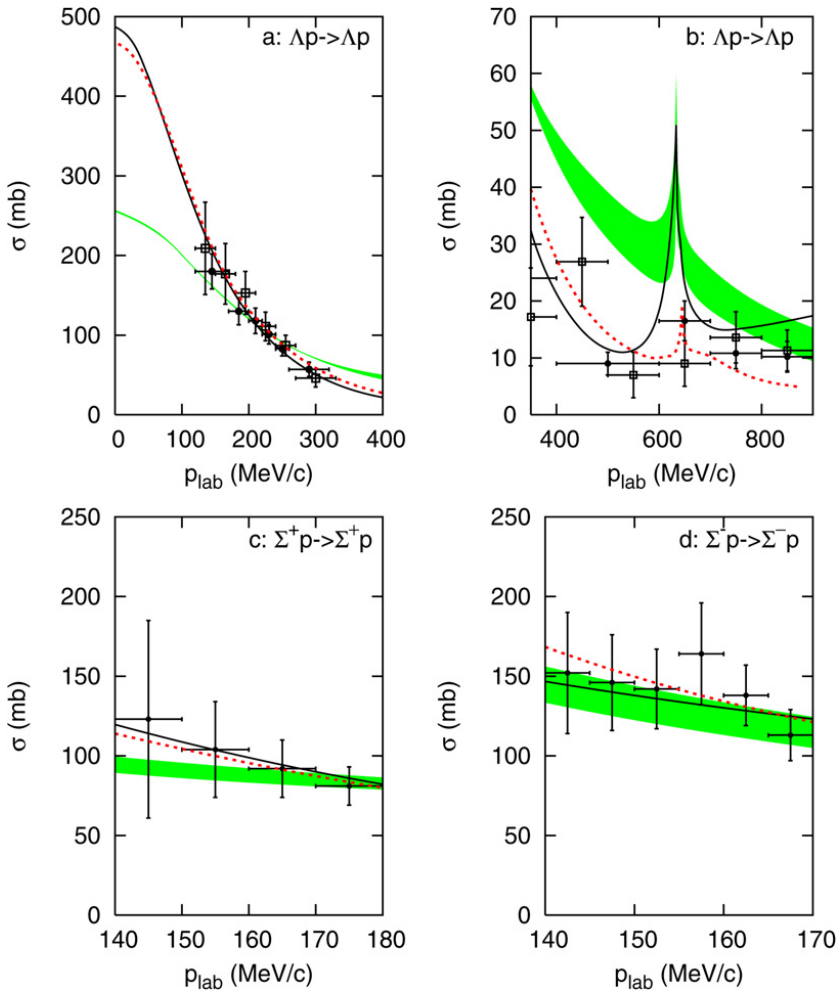
Roth *et al.*, Few-Body Syst. **55**, 659 (2014)

Status of *ab initio* nuclear structure

- Accurate NN+3N Hamiltonians from chiral EFT
- Unitary transformations for converged results
- Many-body methods with controlled uncertainties
- Access to binding energies, spectra, radii, transitions. . .

Extension to hypernuclei?

Hyperon-Nucleon Interaction

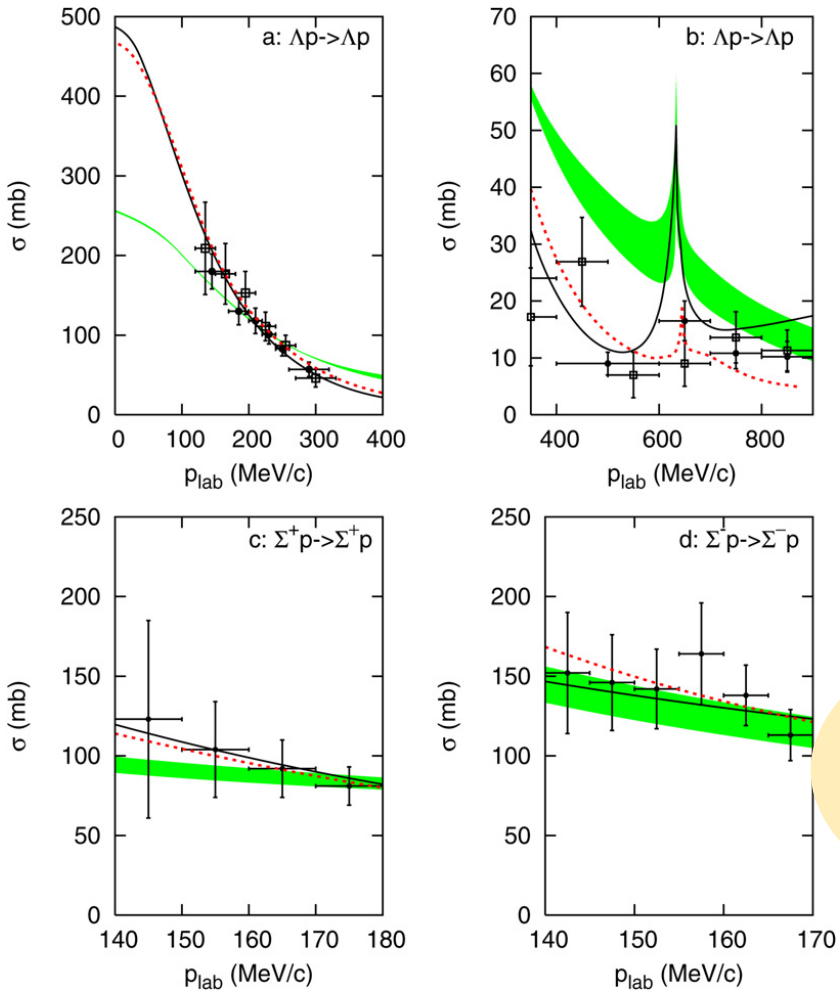


- Weaker than NN ($\sim 1\,\mathrm{MeV}/N$)
- No YN bound state
- $\Lambda N \leftrightarrow \Sigma N$ conversion
- Spin singlet—triplet transition

Meson-exchange models and chiral EFT interactions available

Polinder *et al.*, Nucl. Phys. A **779**, 244 (2006)

Hyperon-Nucleon Interaction



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Meson-exchange models and chiral EFT interactions available

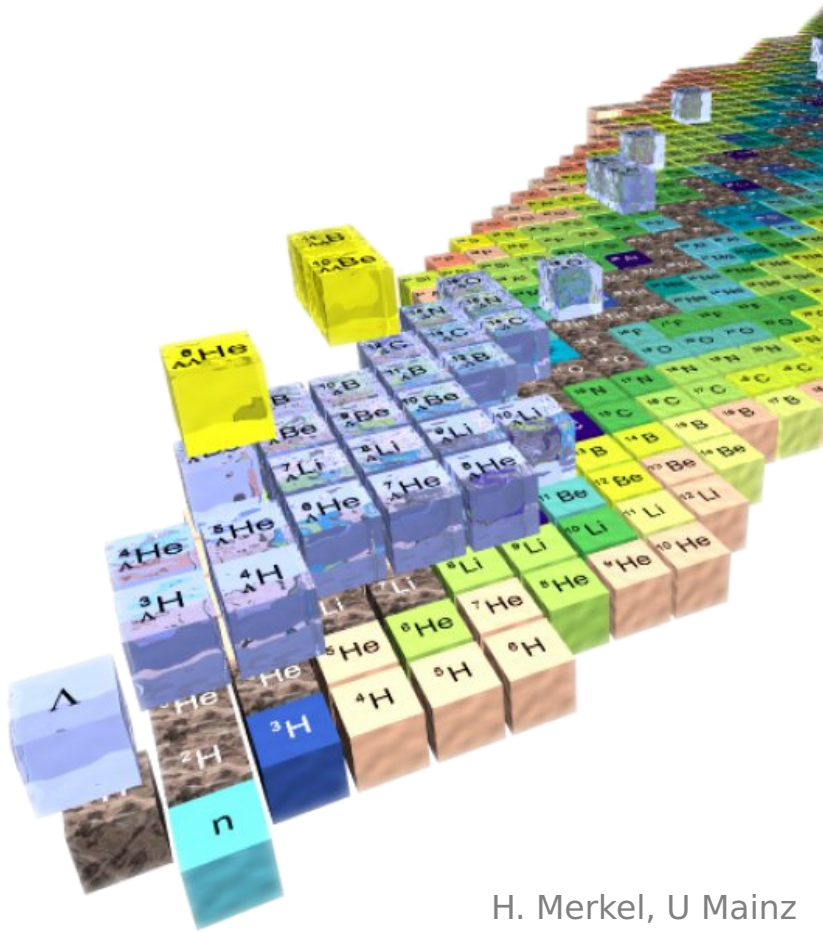
YN scattering experiments are challenging
⇒ Few data points
⇒ Large error bars

Polinder *et al.*, Nucl. Phys. A **779**, 244 (2006)

Necessary Developments

Extend No-Core Shell Model to hypernuclei

- Account for different particle masses
- Include fully active Λ and Σ
  ⇒ Coupled-channel problem
- Adapt unitary transformation framework



H. Merkel, U Mainz

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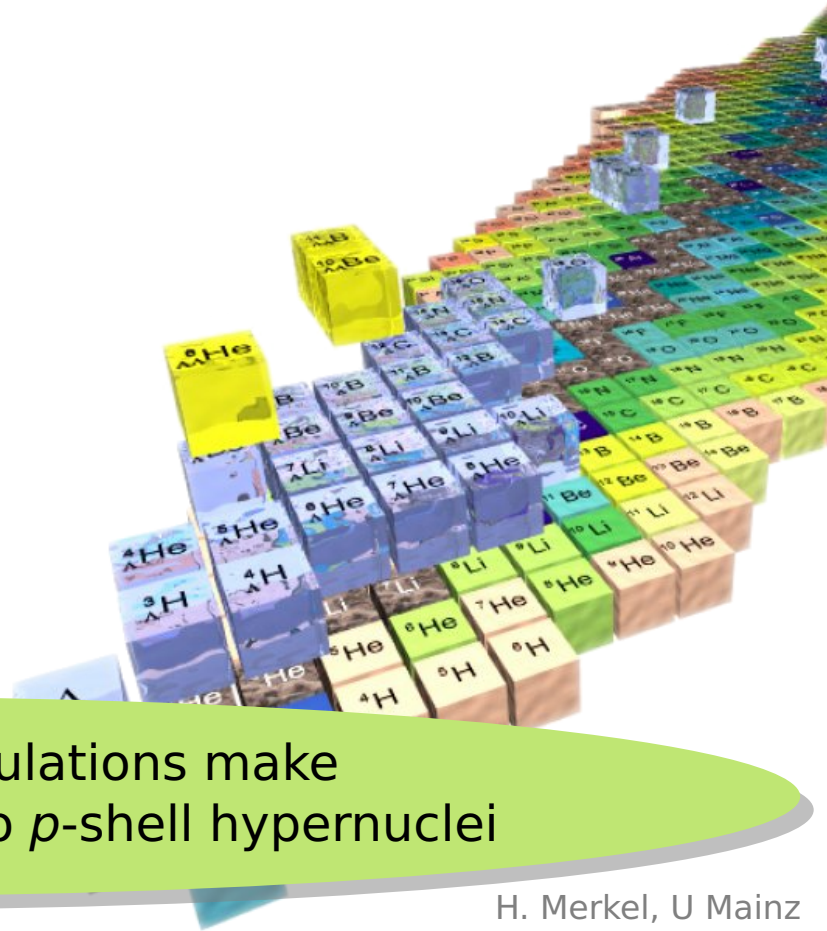

Ab-initio many-body calculations make connection from YN interactions to *p*-shell hypernuclei

H. Merkel, U Mainz

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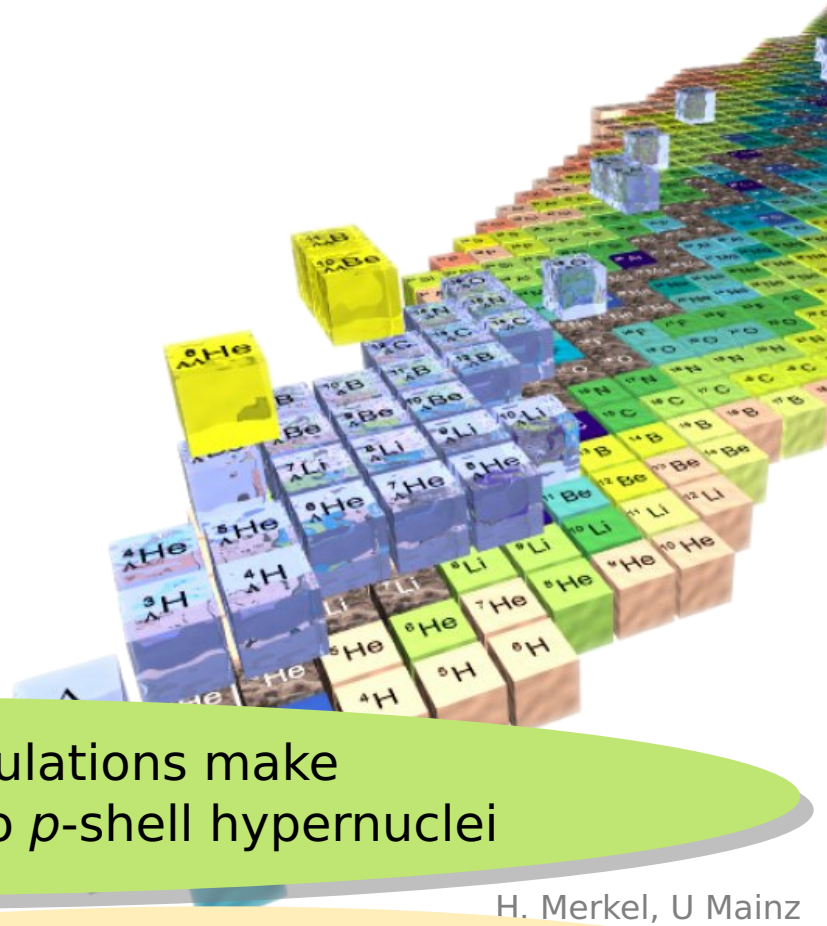


Ab-initio many-body calculations make connection from YN interactions to *p*-shell hypernuclei

H. Merkel, U Mainz

"→" prediction of observables
"←" constraints on YN interactions

Hypernuclear Hamiltonian

$$\boldsymbol{H} = \boldsymbol{\Delta M} + \boldsymbol{T}_{\text{int}} + \boldsymbol{V}_{\text{NN}} + \boldsymbol{V}_{\text{3N}} + \boldsymbol{V}_{\text{YN}}$$

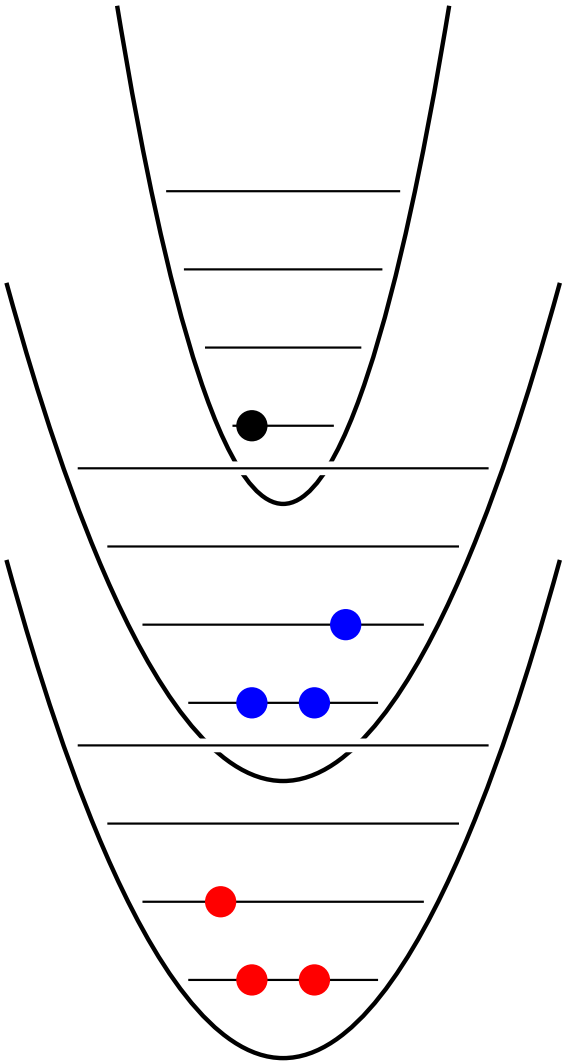
- NN: chiral N^{3}LO
  Entem & Machleidt
  Phys. Rev. C **68**, 041001(R) (2003)
  $\Lambda_{\text{NN}} = 500\,\text{MeV}$
- 3N: chiral N^{2}LO
  Navrátil
  Few-Body Syst. **41**, 117 (2007)
  $\Lambda_{\text{3N}} = 500\,\text{MeV}$

- YN: chiral LO
  Polinder, Haidenbauer & Meißner
  Nucl. Phys. A **779**, 244 (2006)
  $\Lambda_{\text{YN}} = 600\,\text{MeV}, 700\,\text{MeV}$

  OR Jülich'04
  Haidenbauer & Meißner
  Phys. Rev. C **72**, 044005 (2005)
  meson-exchange

NN+3N yields quantitative description of *p*-shell nuclei

Importance-Truncated No-Core Shell Model



- A-body Slater determinants from HO states

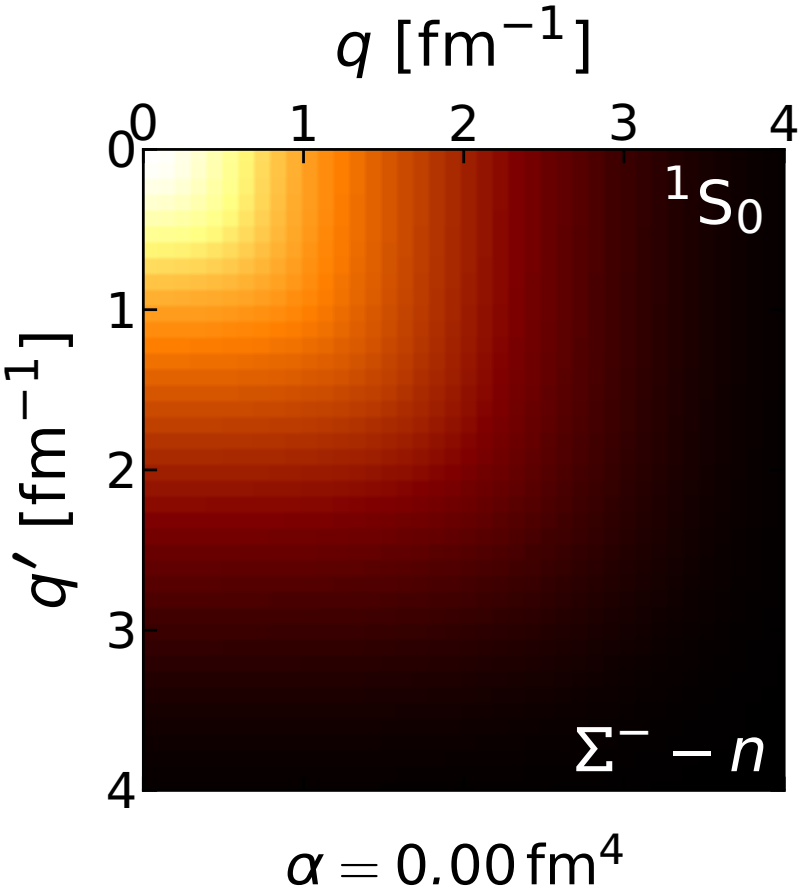
$$|s_1 s_2 \cdots s_A\rangle\,, \quad s_i \equiv |e(l\tfrac{1}{2})j\chi\rangle_i$$

- Λ-Σ conversion, e.g.

$$|pn\Lambda\rangle\,, |pp\Sigma^-\rangle\,, |nn\Sigma^+\rangle \in \mathcal{M}(^{3}_{\Lambda}H)$$

- Impose $N_{\max}$ truncation
- Importance truncation: discard irrelevant states + *a posteriori* extrapolation
- Diagonalize Hamilton matrix ⇒ Energies & wave functions

Similarity Renormalization Group



$$\partial_\alpha \boldsymbol{H}(\alpha) = [\boldsymbol{\eta}(\alpha), \boldsymbol{H}(\alpha)]$$

$$\boldsymbol{\eta}(\alpha) = m_N^2 [\boldsymbol{T}_{\mathrm{int}}, \boldsymbol{H}(\alpha)]$$

- Up to 6 coupled channels
- Decouples high and low momenta
  $\Rightarrow$ Improved $N_{\max}$ convergence
- BUT: Induced many-body terms
  $\Rightarrow$ Assess via α-dependence
- NN+3N: Induced terms negligible up to $A \approx 10$

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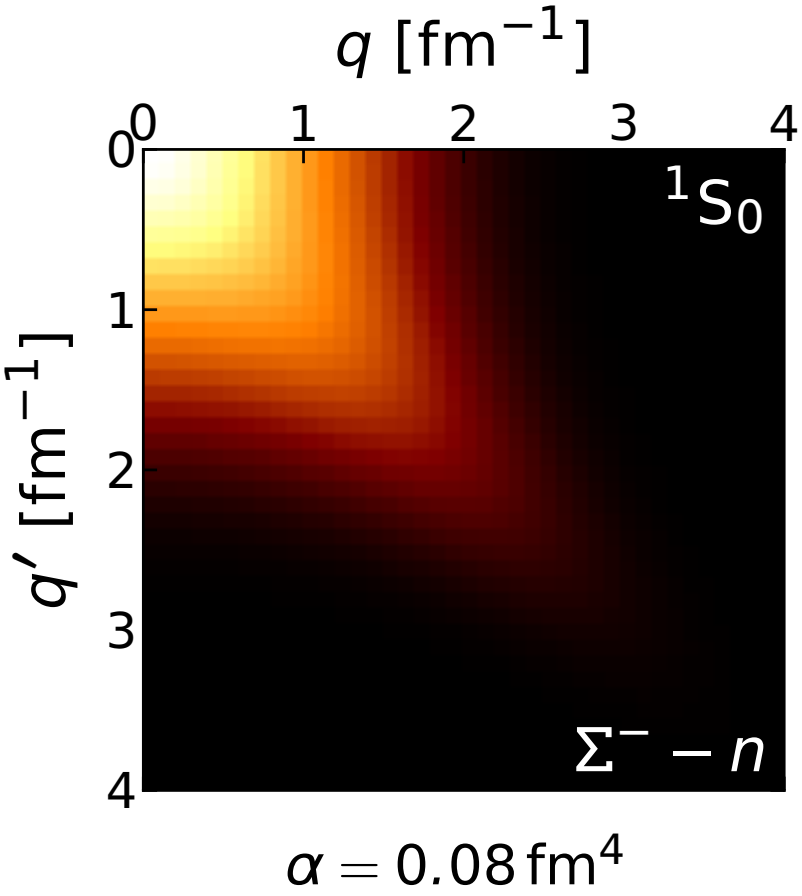

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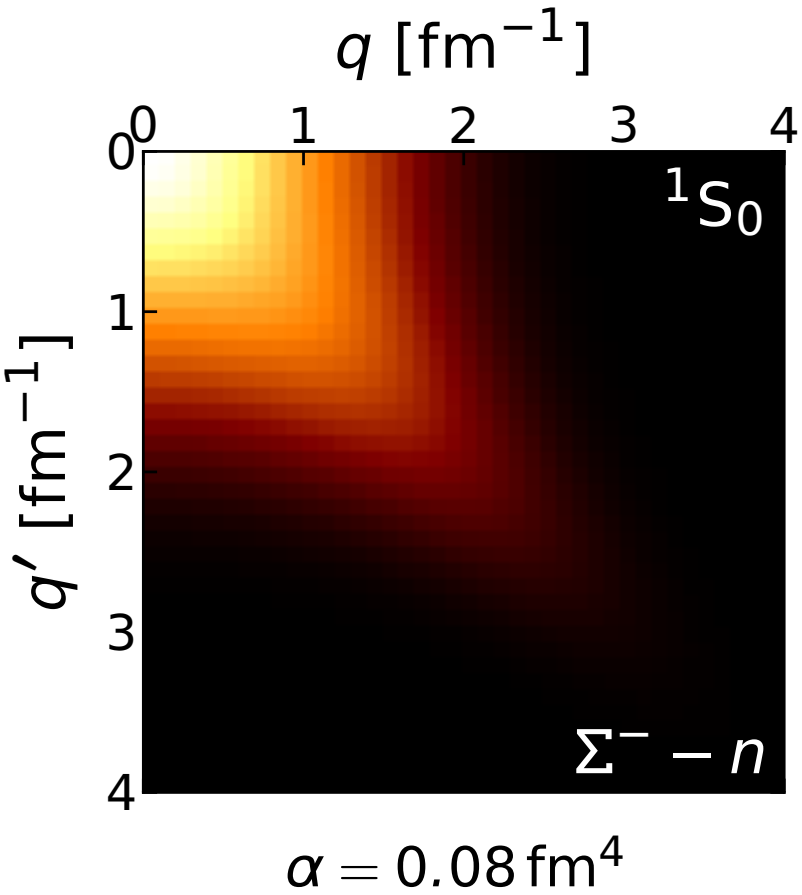

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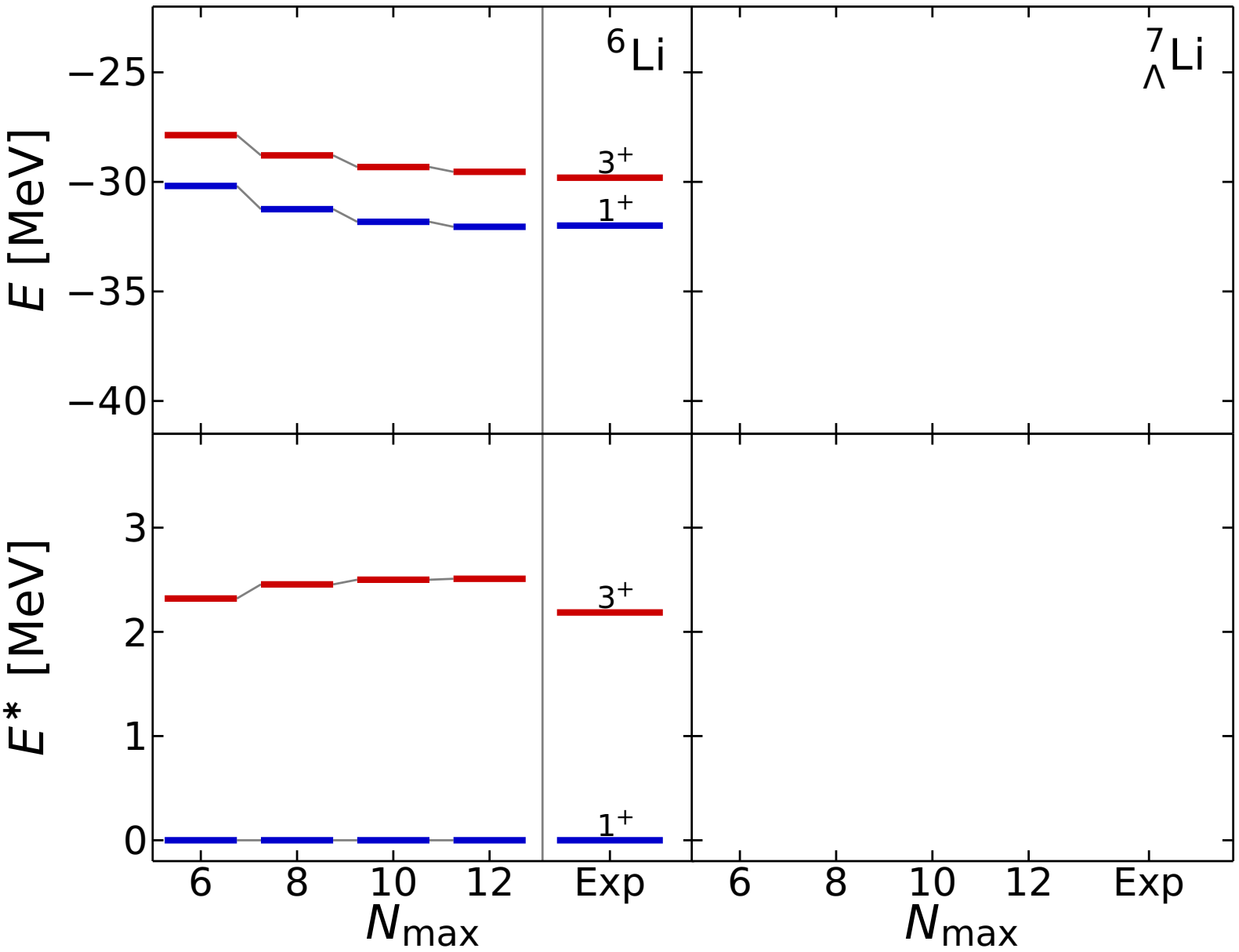
Use $\alpha_N = 0.08\,\mathrm{fm}^4$ for NN+3N
and $\alpha_Y = 0\,\mathrm{fm}^4$ for YN

Results for Selected Hypernuclei

RW, D. Gazda, P. Navrátil, A. Calci, J. Langhammer, R. Roth, PRL 113, 192502 (2014)

${}^{7}_{\Lambda}$Li — Energies and Spectra



$\alpha_N = 0.08\,\mathrm{fm}^4$
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$\hbar\Omega = 20\,\mathrm{MeV}$

NN+3N
reproduces ^{6}Li

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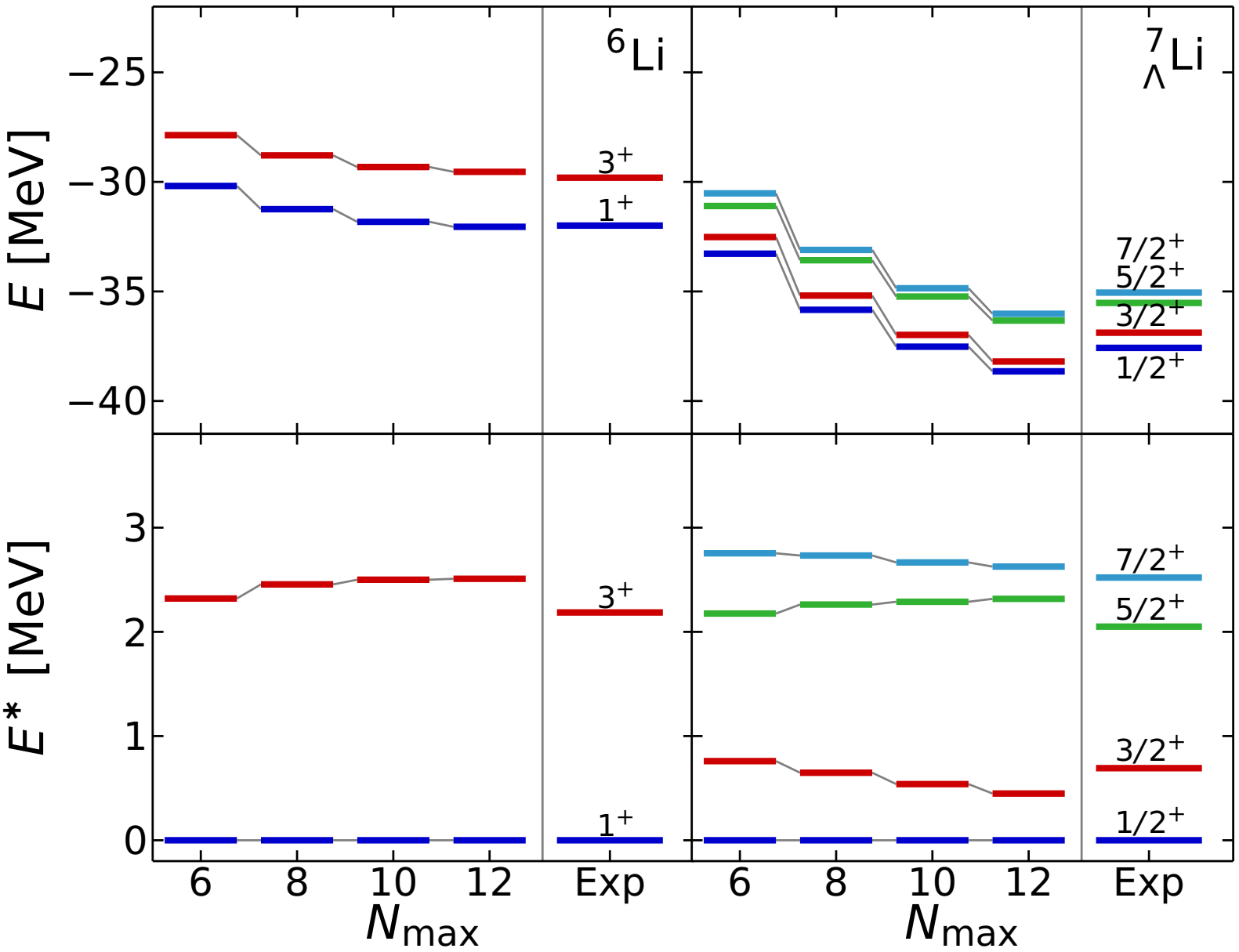

Chiral LO

— $\Lambda = 600\,\mathrm{MeV}$

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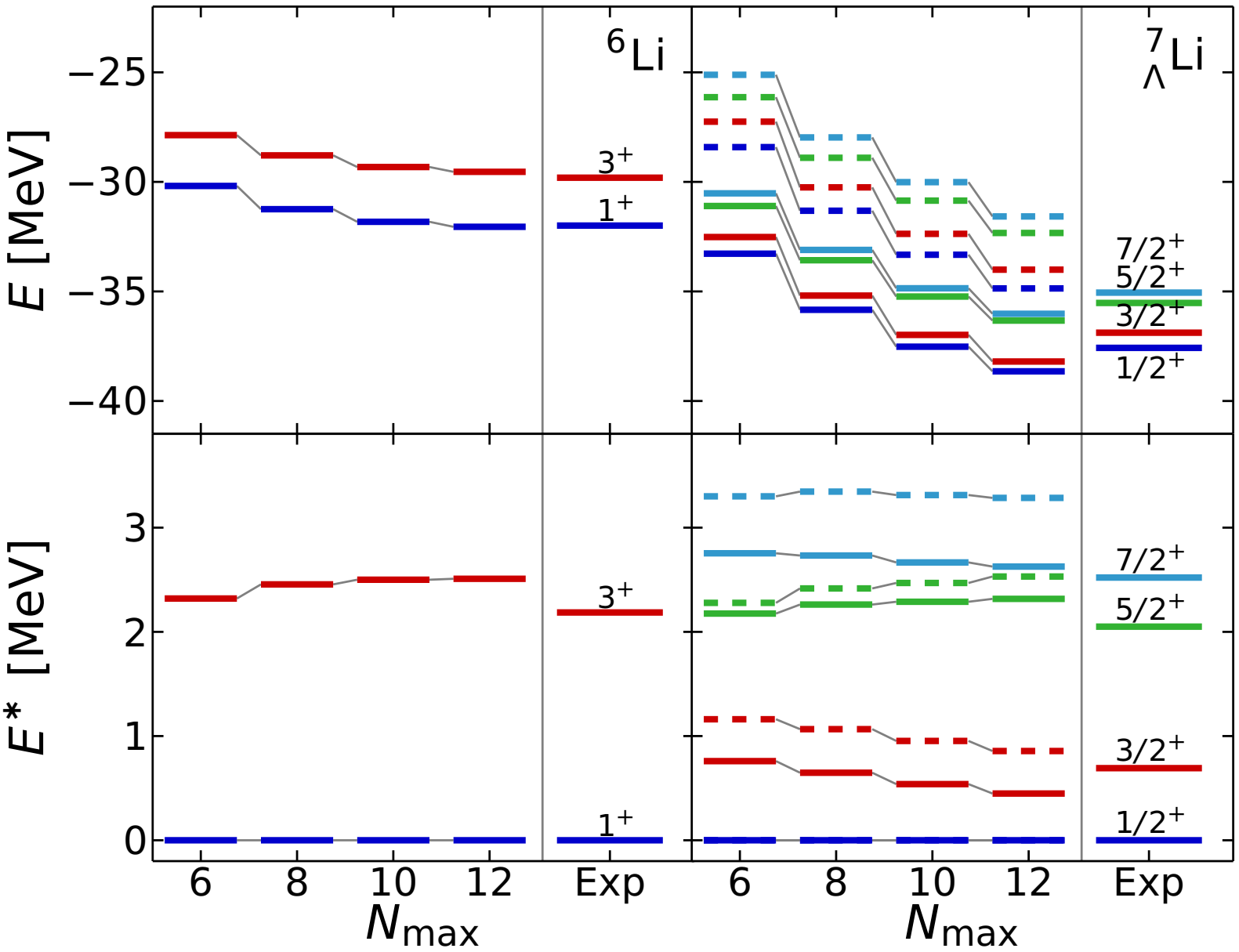

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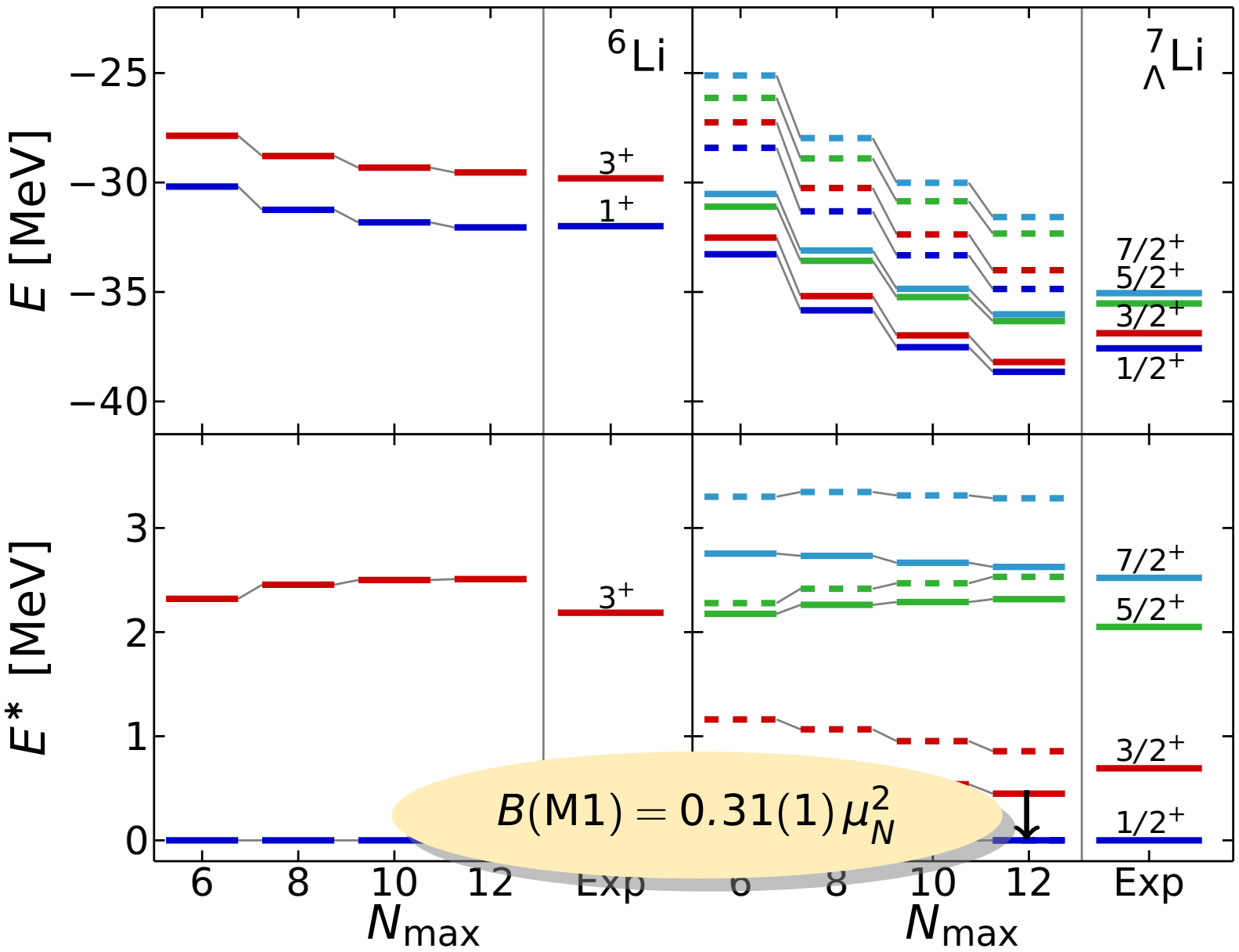
$^{7}_{\Lambda}$Li: Sizable cutoff dependence

${}^{7}_{\Lambda}$Li — Energies and Spectra



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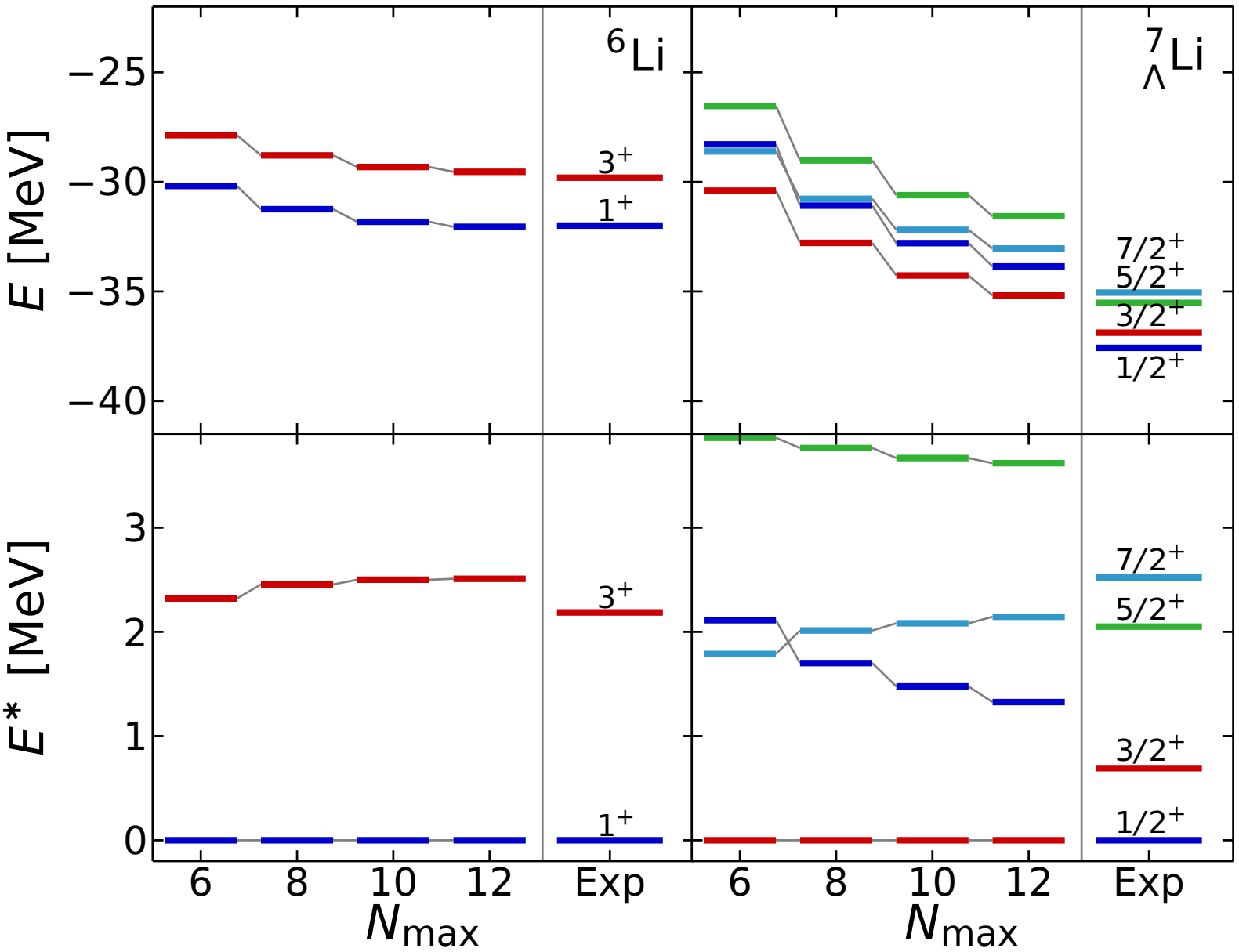


Jülich'04

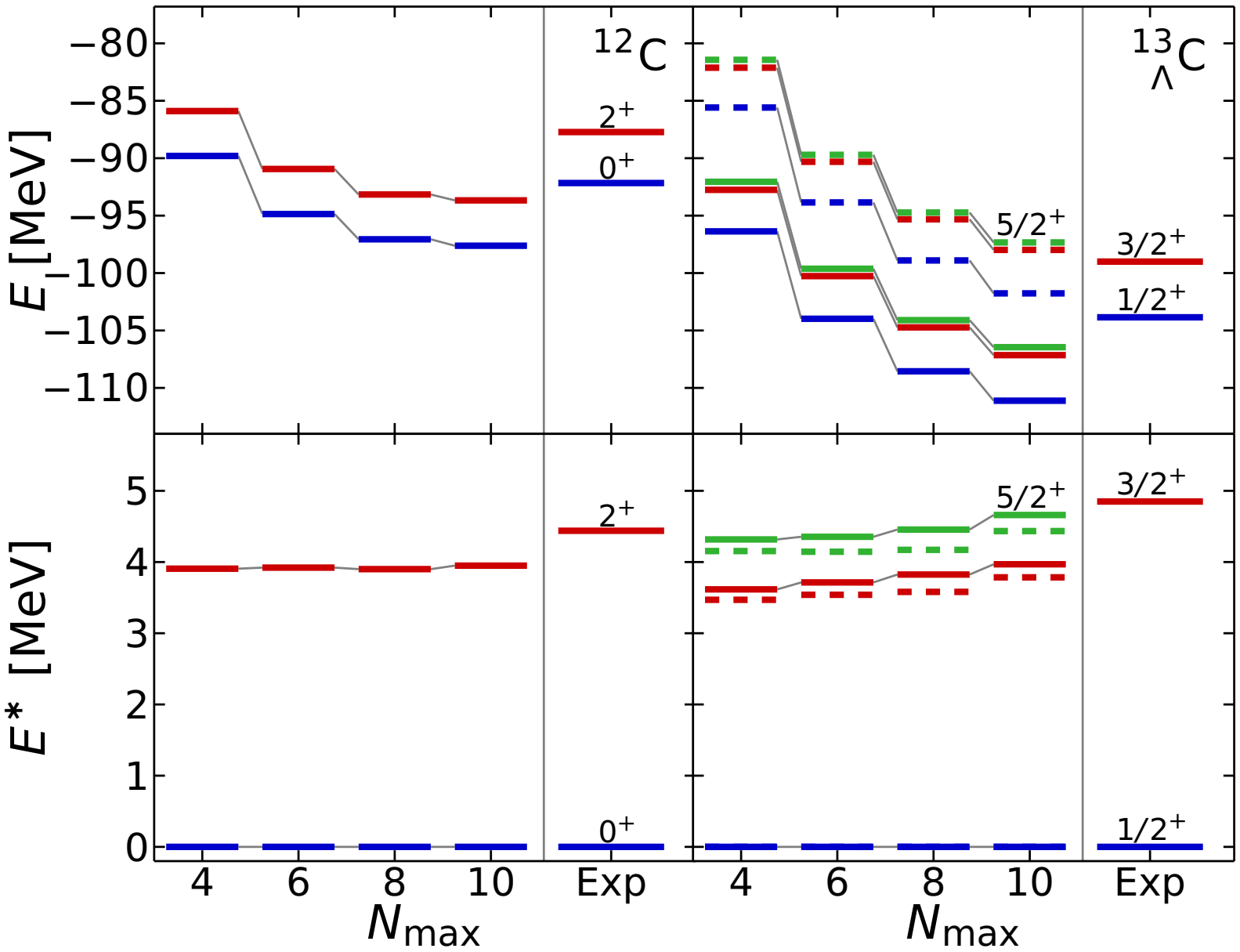
$\alpha_N = 0.08\,\mathrm{fm}^4$
$\alpha_Y = 0\,\mathrm{fm}^4$
$\hbar\Omega = 20\,\mathrm{MeV}$

Slight underbinding

Reversed level ordering

Large doublet splittings

$^{13}_{\Lambda}$C — Energies and Spectra



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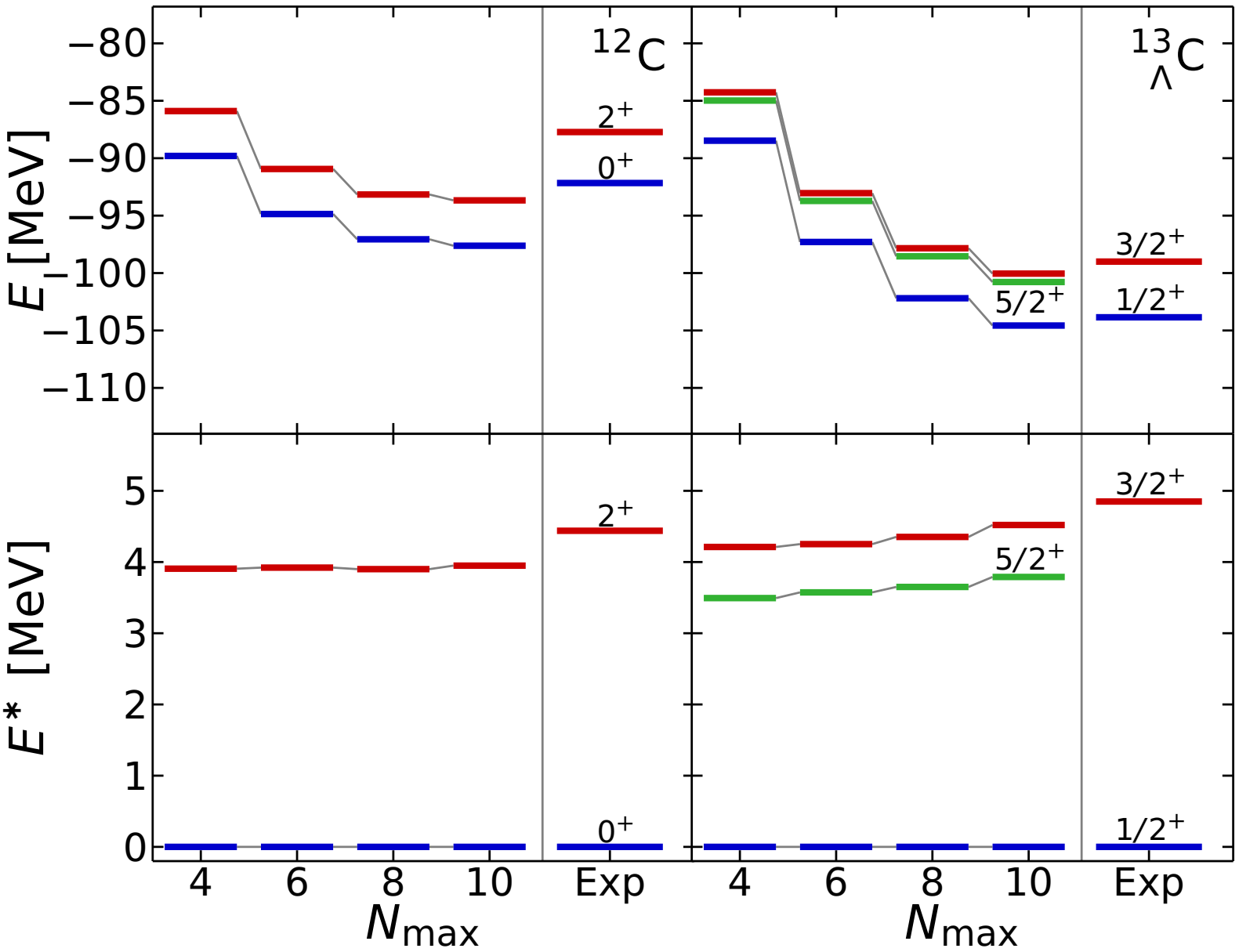
Overbinding due to induced 4N

E^* lower for $\Lambda = 700\,\mathrm{MeV}$

0.7 MeV splitting

$^{13}_{\Lambda}$C — Energies and Spectra



Jülich'04

$\alpha_N = 0.08\,\mathrm{fm}^4$
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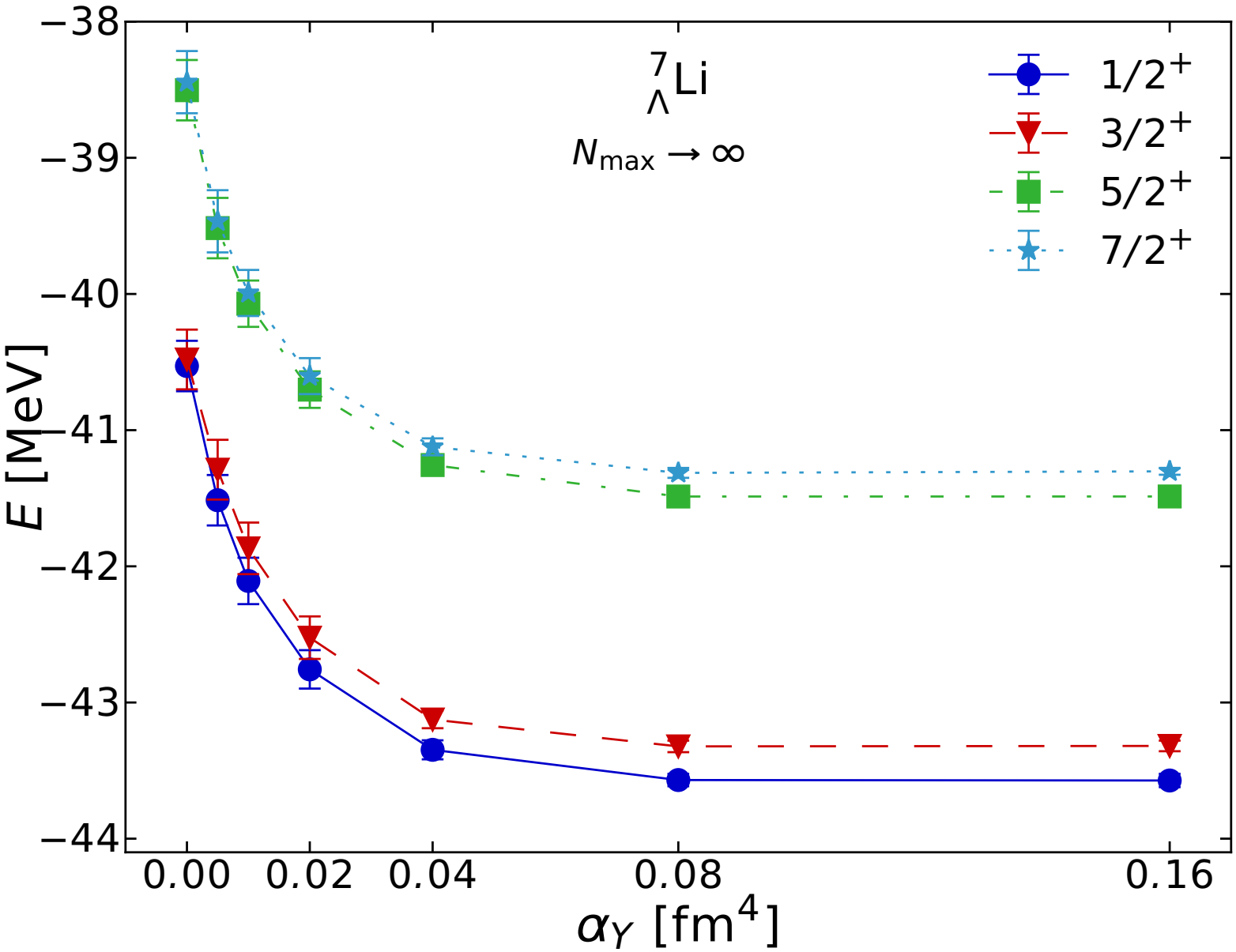
Very similar to chiral LO

Opposite level ordering

SRG-Induced YNN Interactions

Variation of α_Y

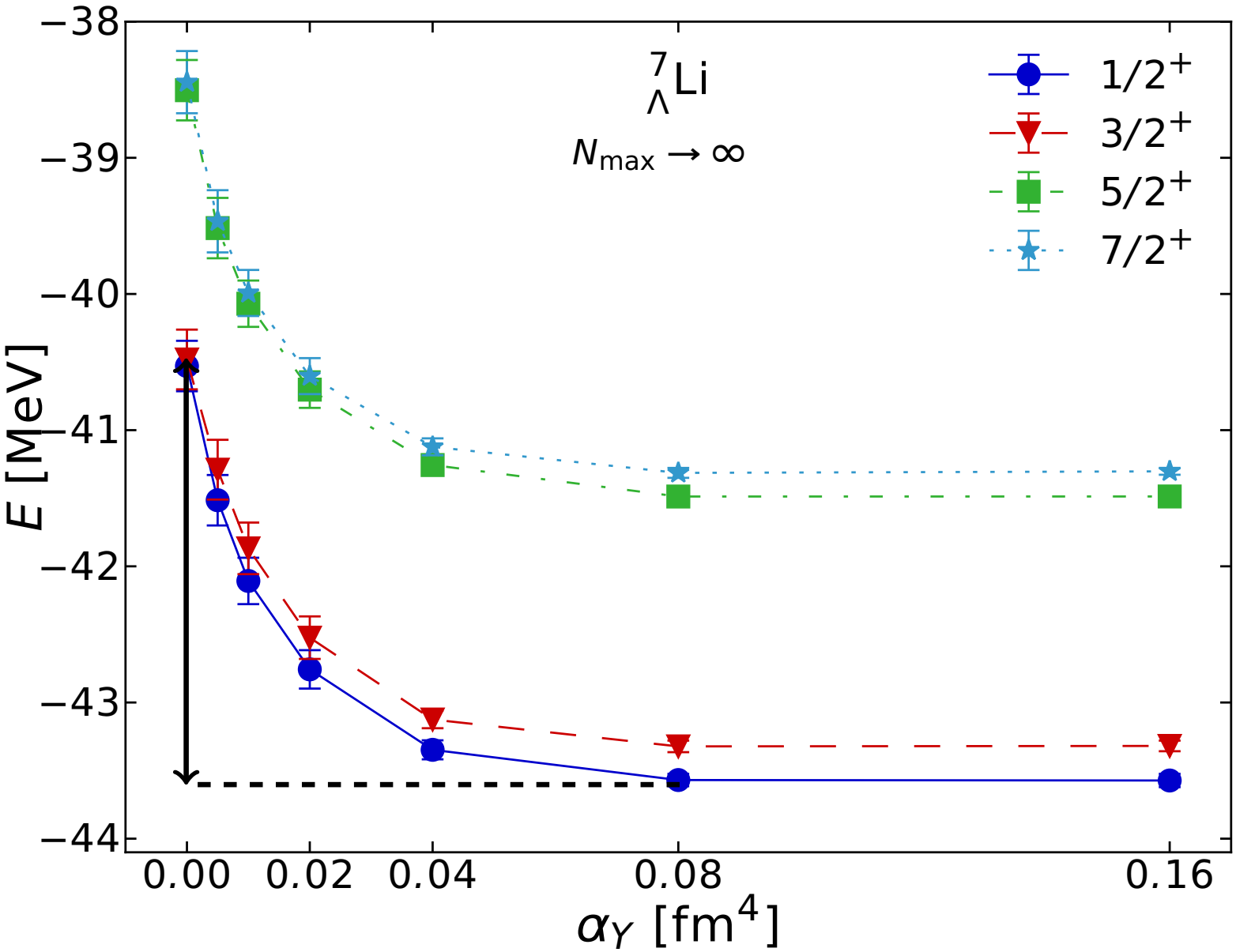


Strong α_Y-dependence of extrapolated energies

Induced YNN strongly repulsive

Spectra stable

Variation of α_Y



Strong α_Y-dependence of extrapolated energies

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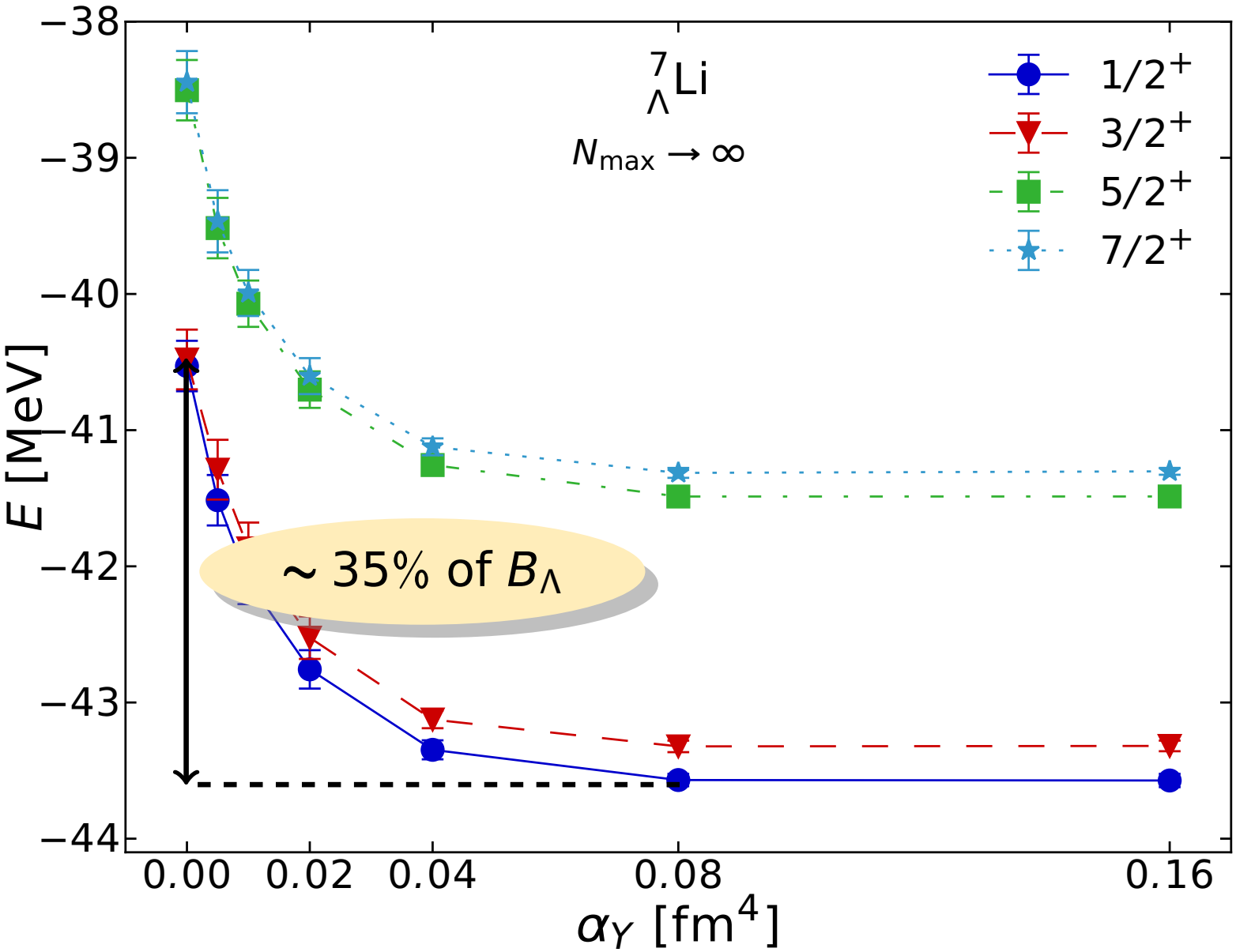
Variation of α_γ



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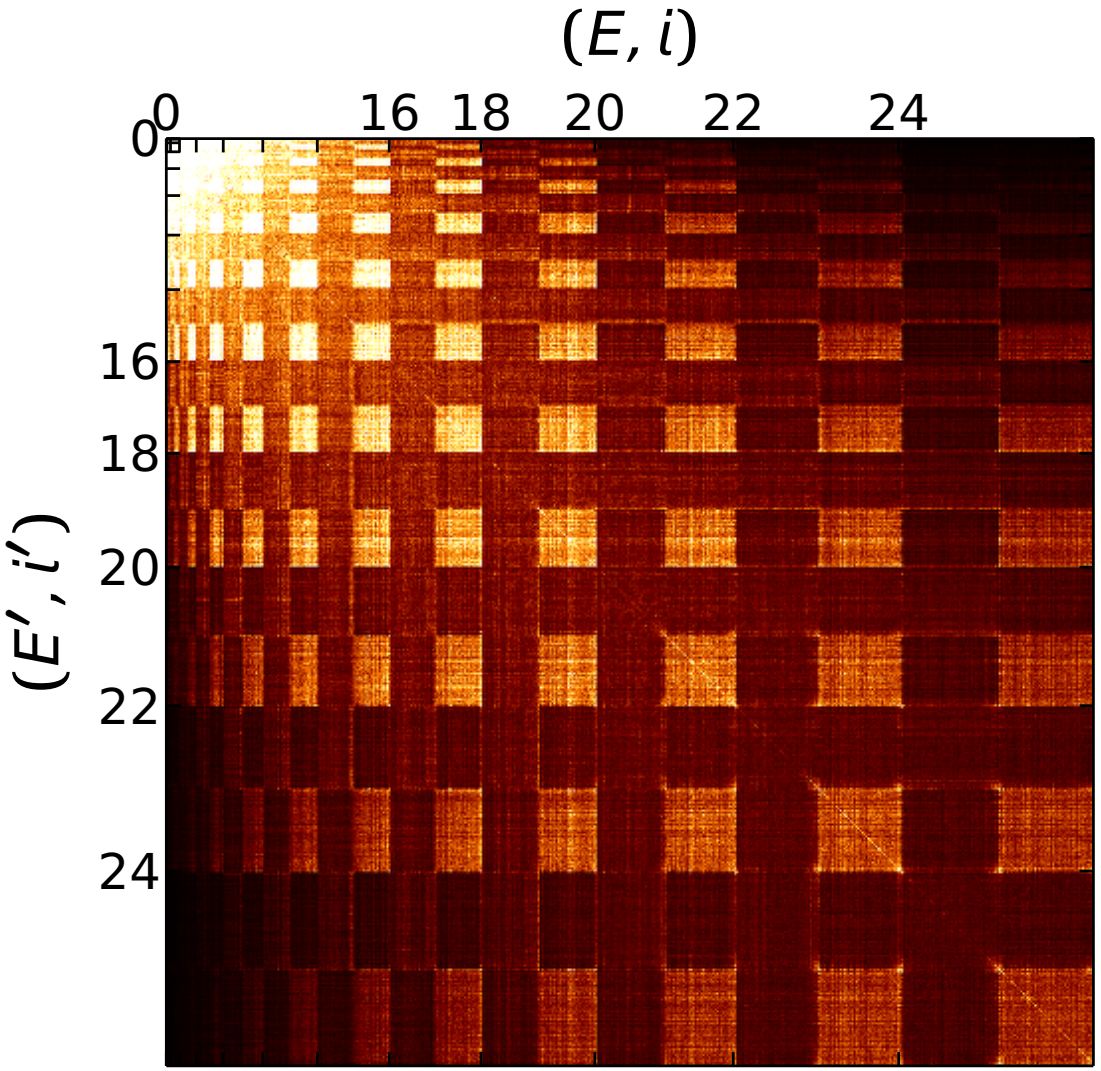
Induced YNN strongly repulsive

Spectra stable

Solution: SRG at the Three-Body Level

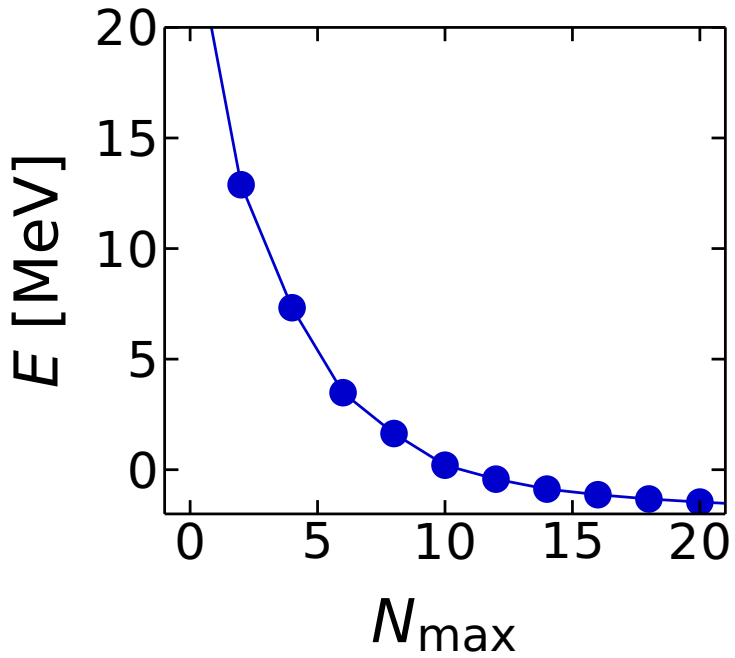


$$ {}_a\langle Ei|\boldsymbol{H} - \boldsymbol{T}_{\text{int}} - \boldsymbol{M}|E'i'\rangle_a $$

$$ \hbar\Omega = 32\,\text{MeV} $$

$$ J^\Pi T = \tfrac{1}{2}^+ 0 \quad ({}^3_\Lambda H) $$

$$ \alpha = 0.00\,\text{fm}^4 $$



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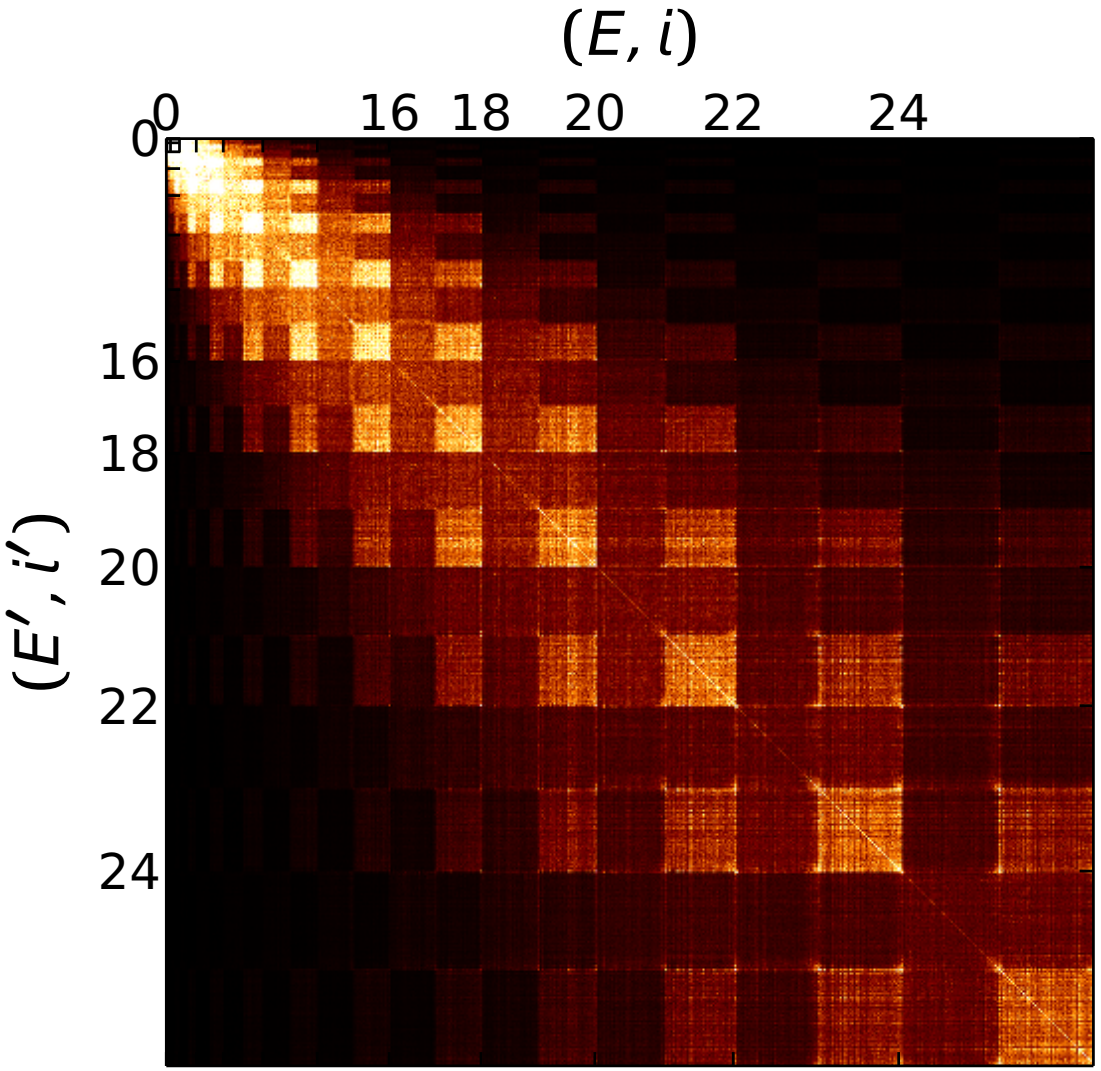

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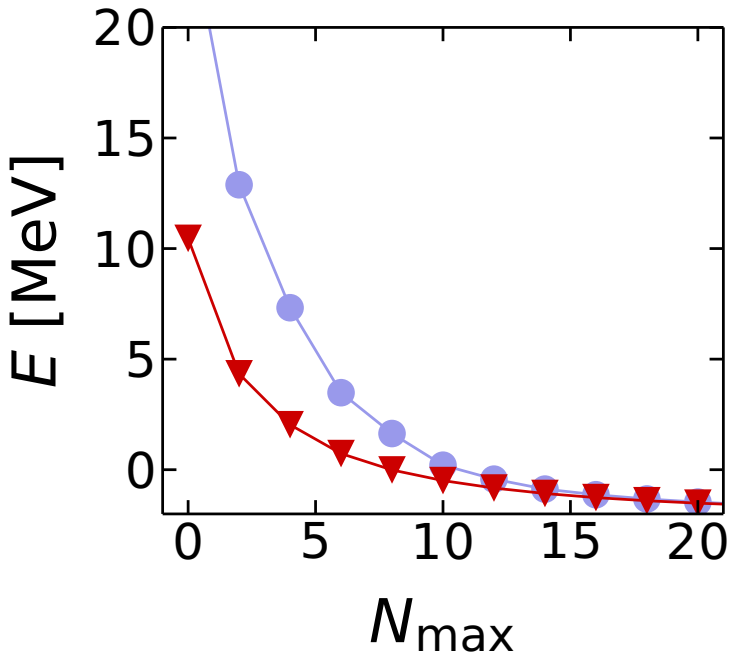

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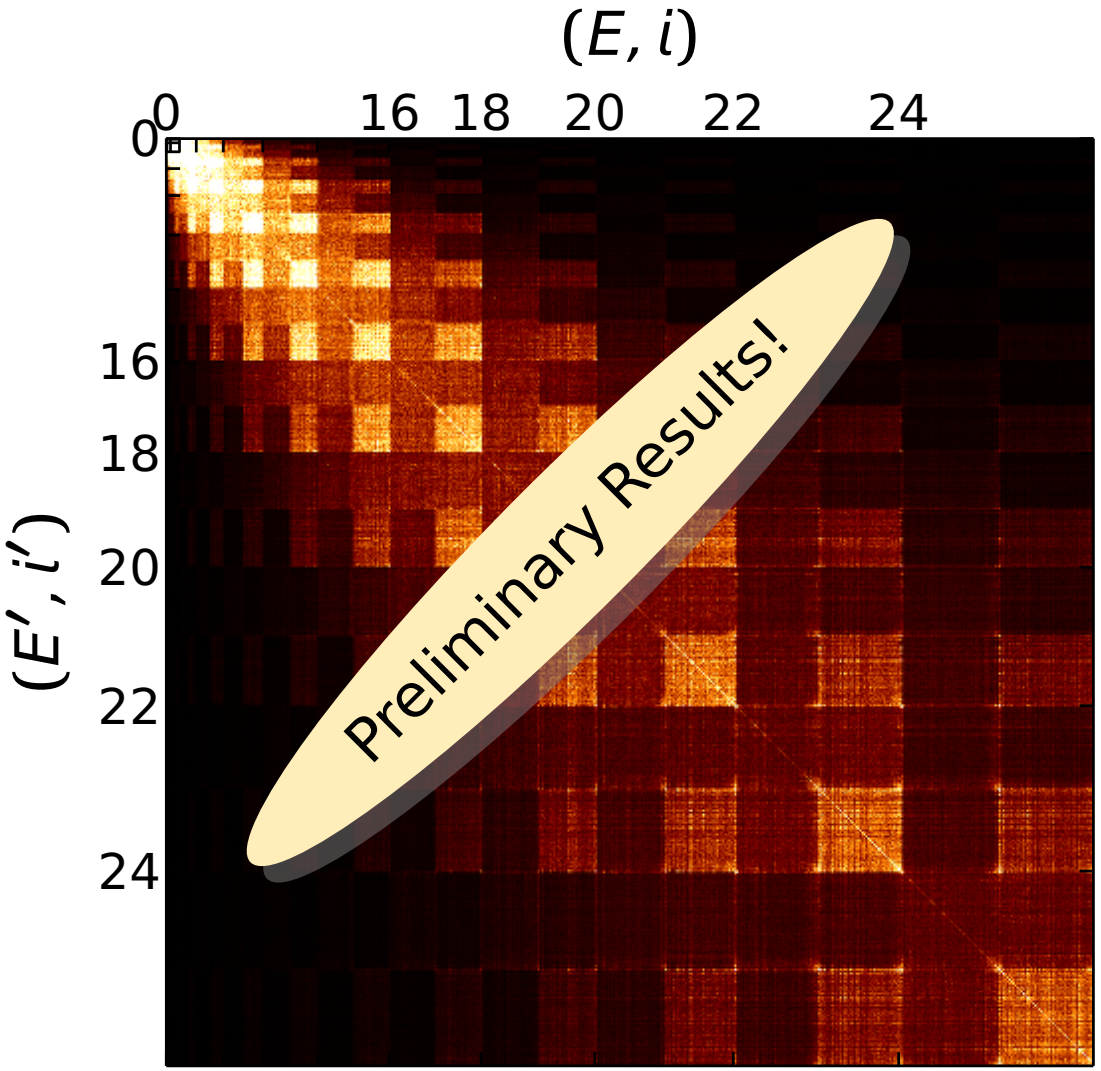

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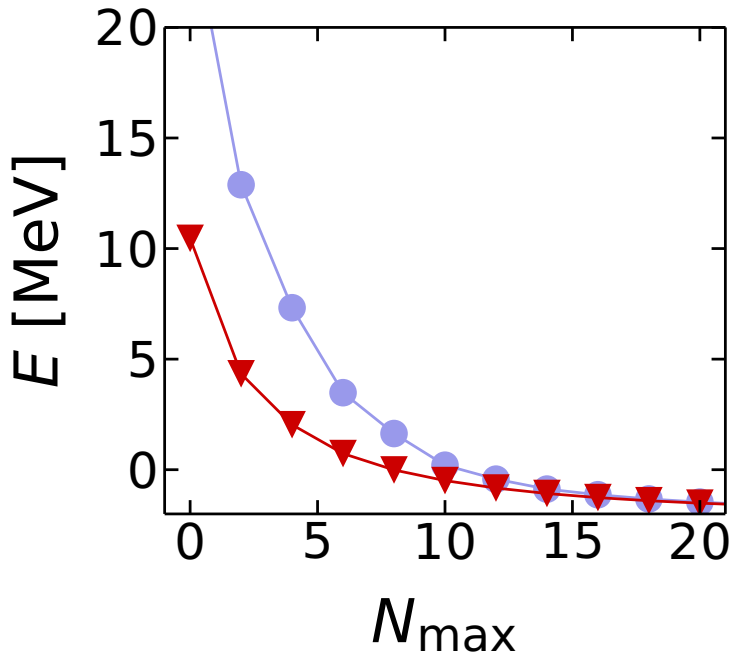
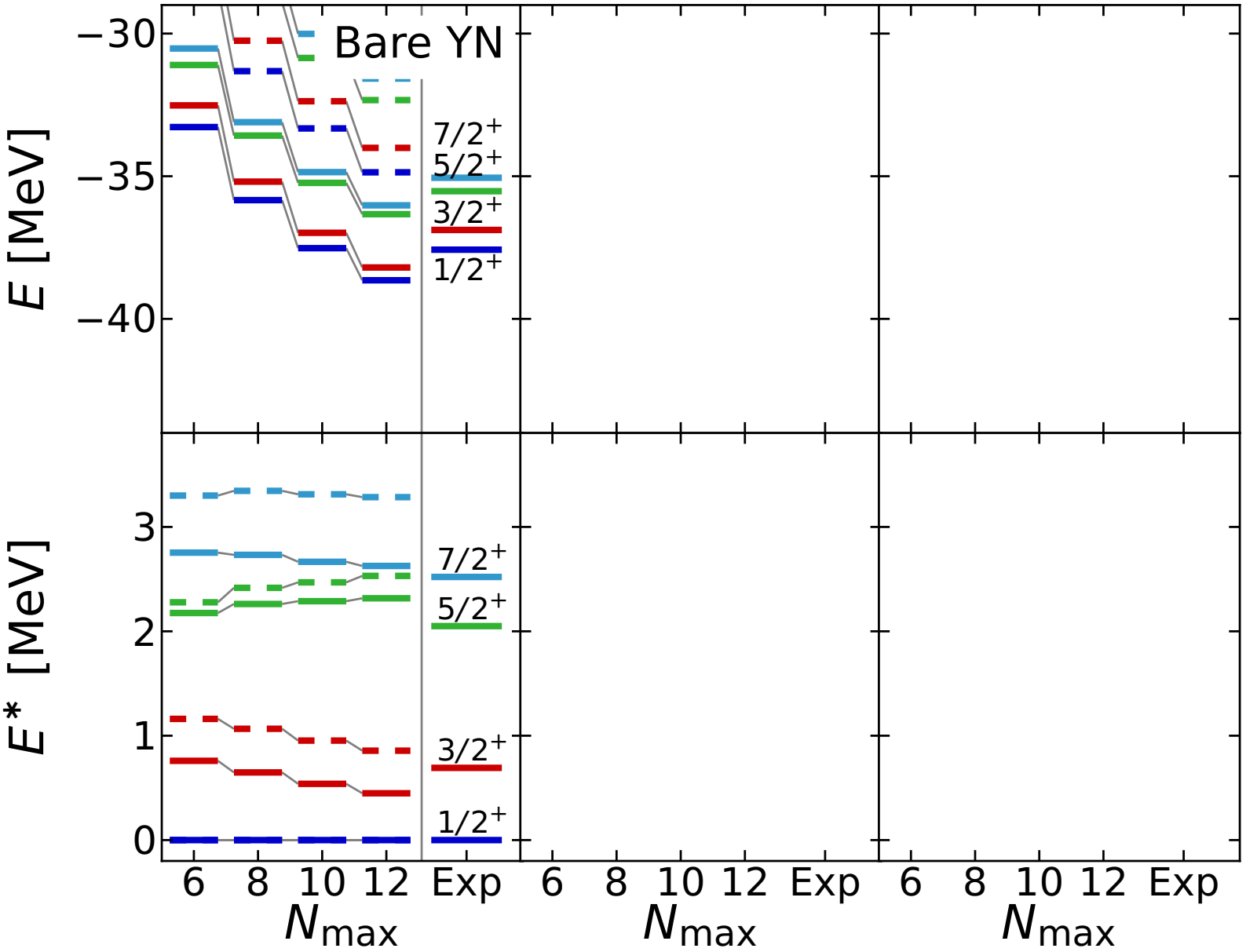


${}^{7}_{\Lambda}$Li — Effect of Induced YNN



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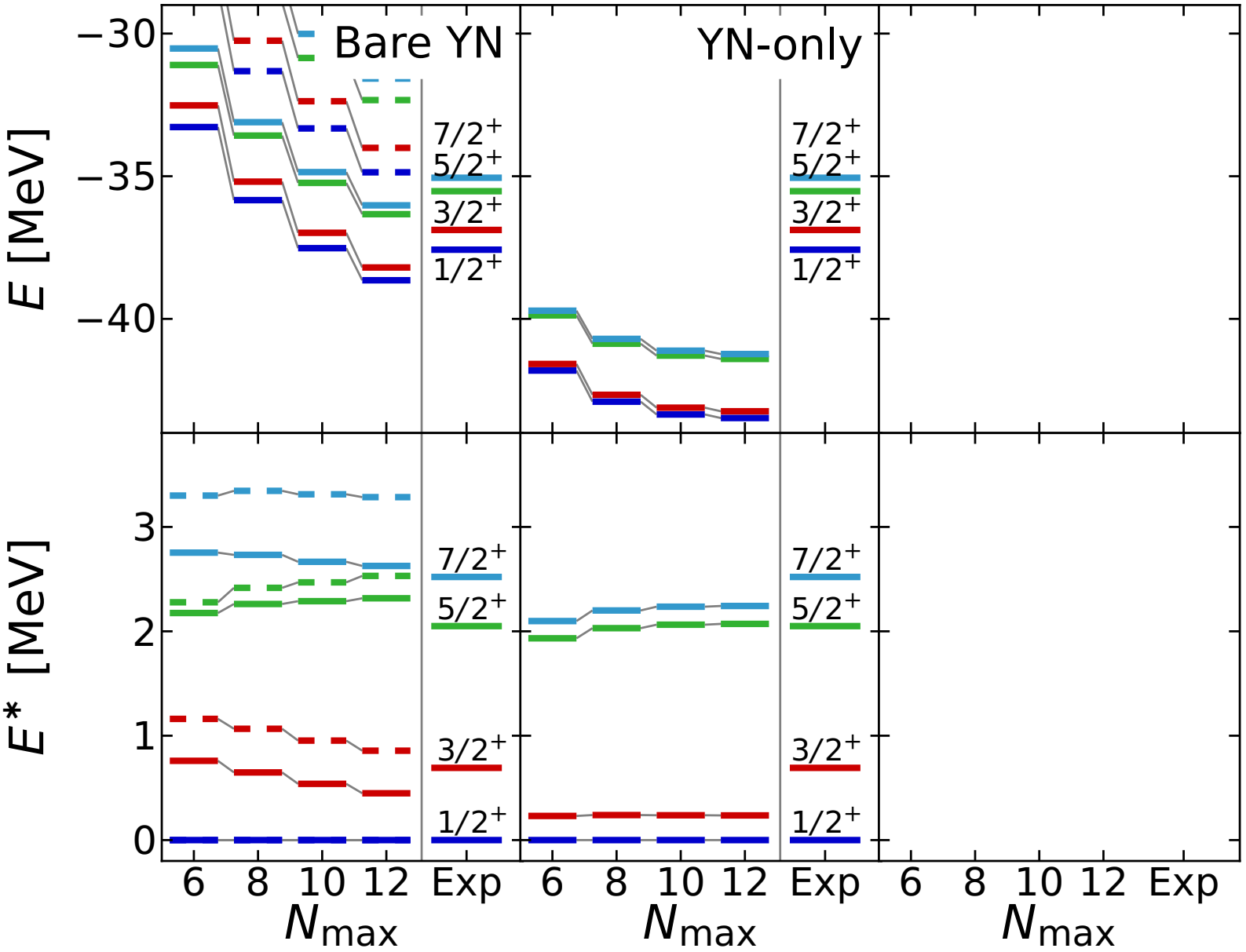

Chiral LO

— $\Lambda = 600\,\mathrm{MeV}$

- - $\Lambda = 700\,\mathrm{MeV}$

$\alpha_N = 0.08\,\mathrm{fm}^4$

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YN-only overbinds

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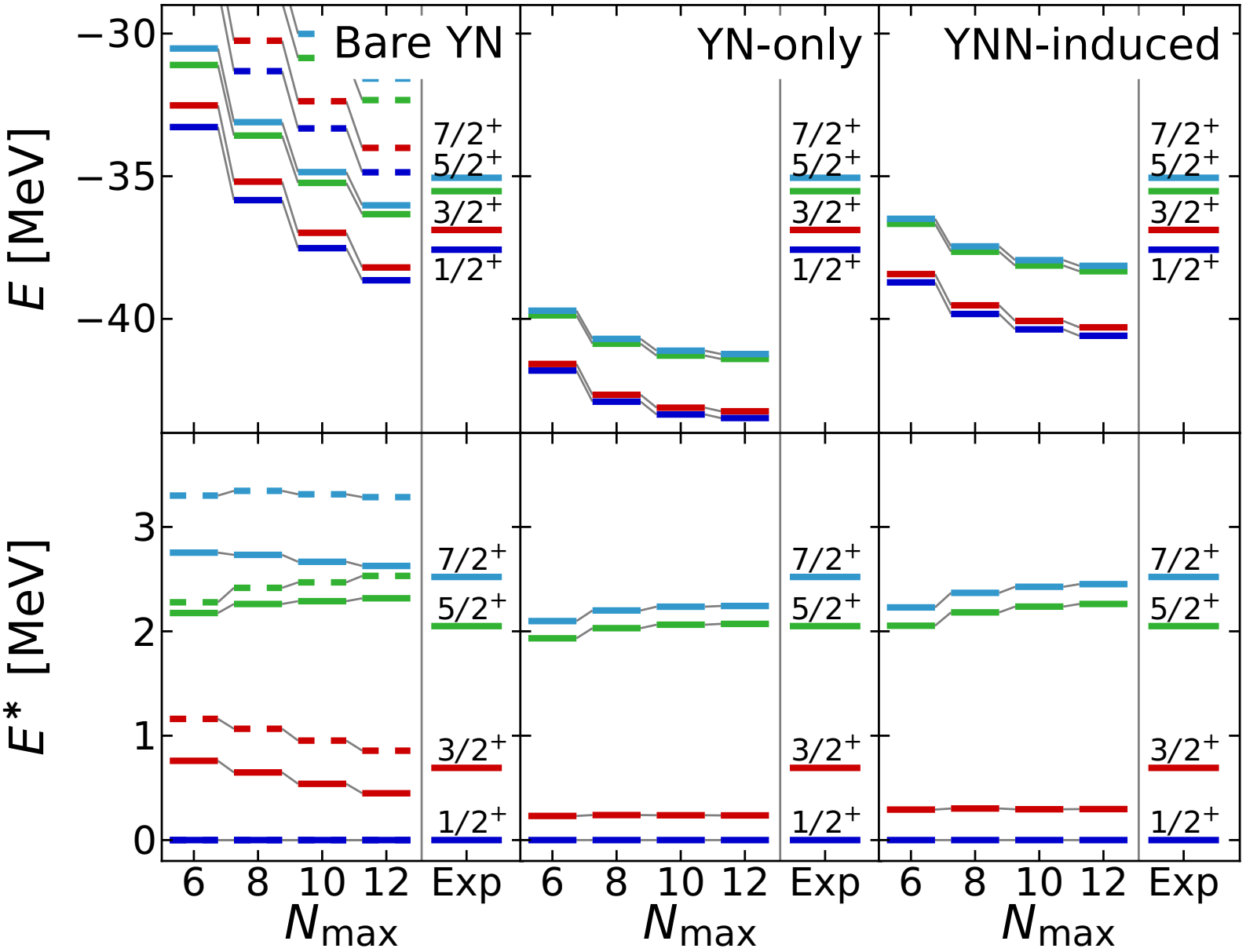

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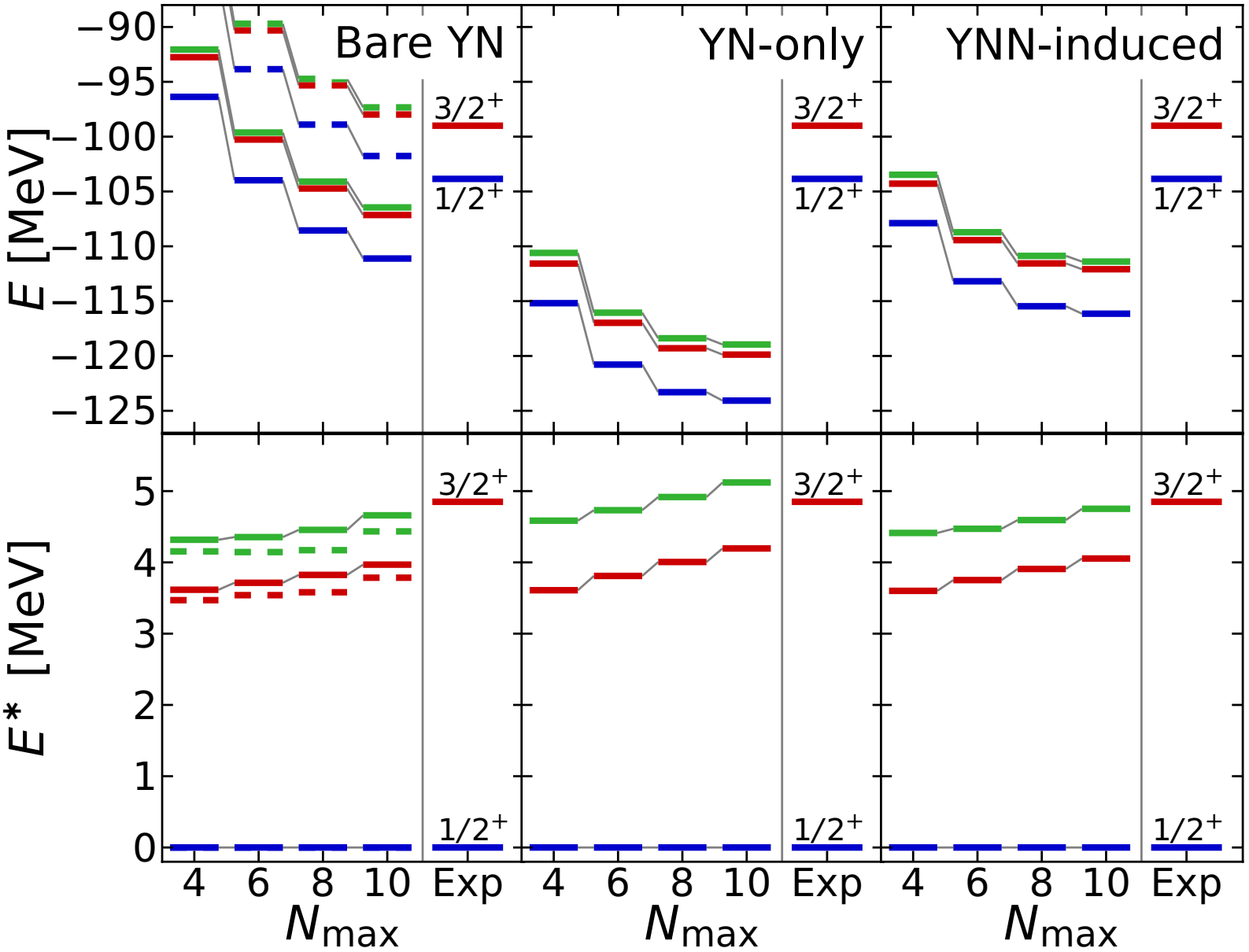
YN-only overbinds

Induced YNN reduces overbinding

Splittings too small, within cutoff uncertainties

$^{13}_{\Lambda}$C — Effect of Induced YNN



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Overbinding partially due to neglected 4N

Induced YNN have slight effect on spectra

Summary & Outlook

- First ab-initio calculations in the *p* shell
- Prediction of observables
- Dependence on YN interaction ⇒ constraints
- SRG of YN induces strong YNN terms
  Possible implications for soft ΛN interactions and the hyperon puzzle
- YNN handled via explicit inclusion
  Preliminary results: reduced overbinding, interaction largest source of uncertainty
- Next: NLO chiral YN interaction & check YNN calculations: convergence of SRG model space, other truncations, other flow parameters

Epilog

Thanks to my group & collaborators

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- **D. Gazda**
  ECT* Trento, Italy
- H. Hergert
  Michigan State University, USA
- H. Feldmeier, T. Neff
  GSI Helmholtzzentrum

JUR{OP,EC}A JÜLICH FORSCHUNGSZENTRUM

LOEWE-CSC CSC Center for Scientific Computing Frankfurt

LICHTENBERG HRZ Hochschulrechenzentrum

COMPUTING TIME

SFB 634

Deutsche Forschungsgemeinschaft DFG

HIC for FAIR Helmholtz International Center

LOEWE Exzellente Forschung für Hessens Zukunft

HELMHOLTZ GEMEINSCHAFT

Bundesministerium für Bildung und Forschung