Results from Nuclear Physics Lattice QCD calculations

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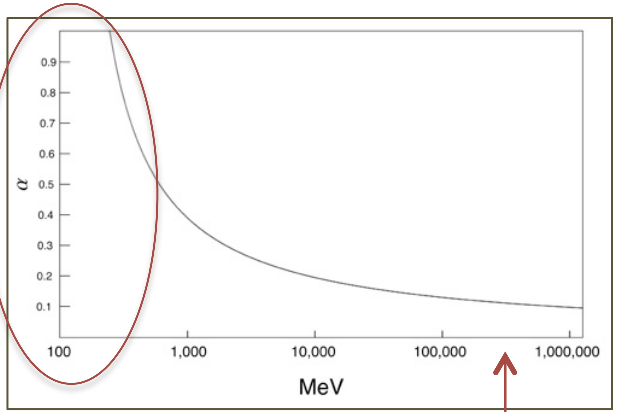
for the NPLQCD Collaboration

EMMI Workshop: Anti-matter, hyper-matter and exotica production at the LHC

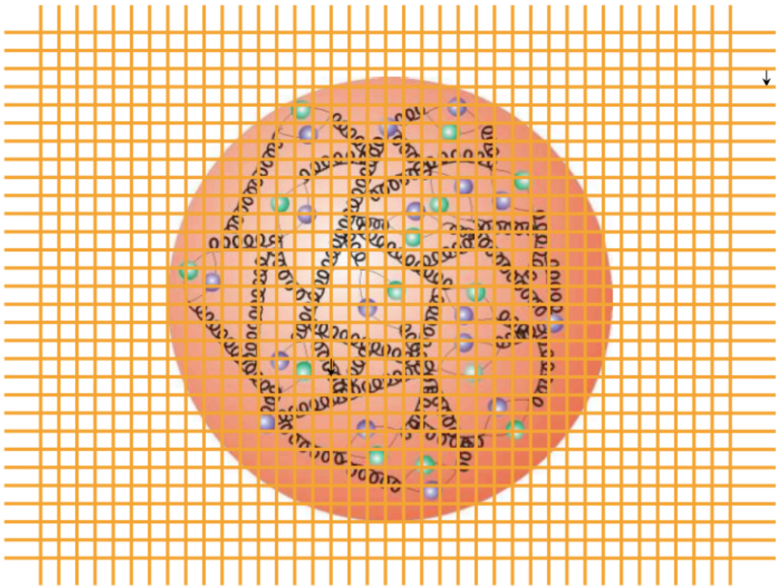
July 20-22, 2015 @ CERN

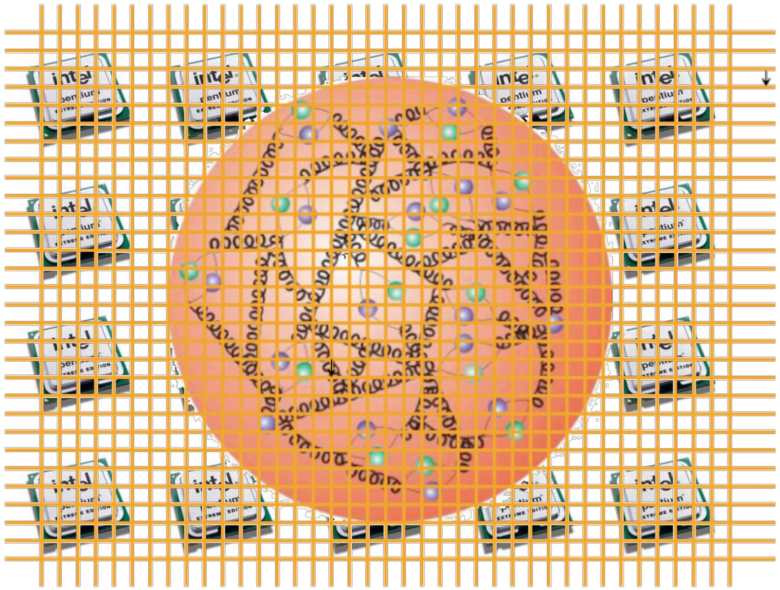
Nuclear physics, the low-energy regime



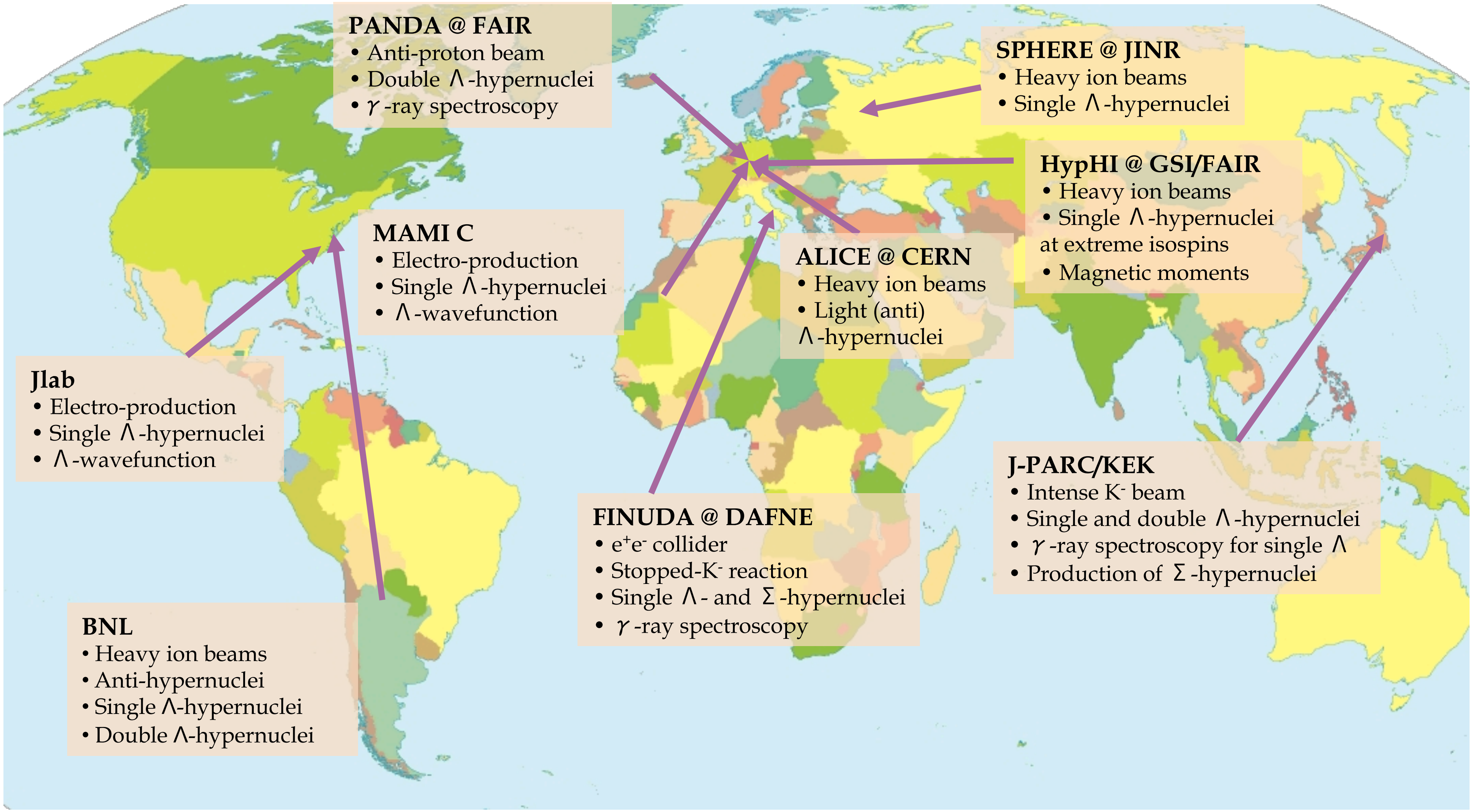
Perturbation theory applicable

- ✓ quarks and gluons confined into coulourless bound states: hadrons (p , n , π…)
- ✓ quarks strongly coupled
- ✓ perturbation theory breaks down: need of other formulations, computational techniques
- ✓ keep the basic degrees of freedom (quarks, gluons) and develop non-perturbative methods (Lattice QCD)

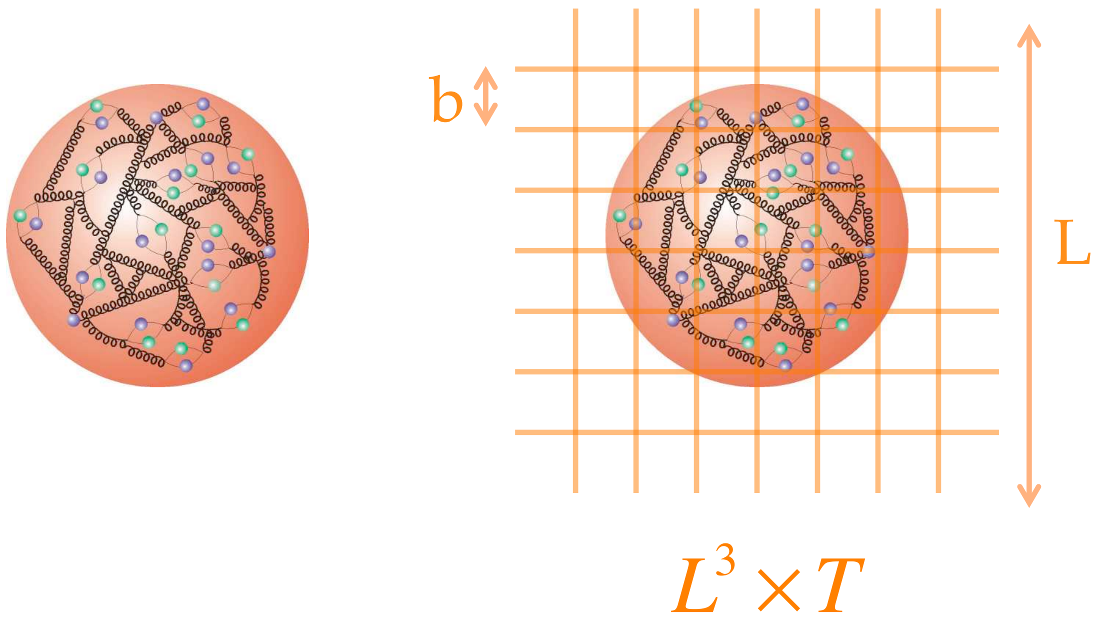




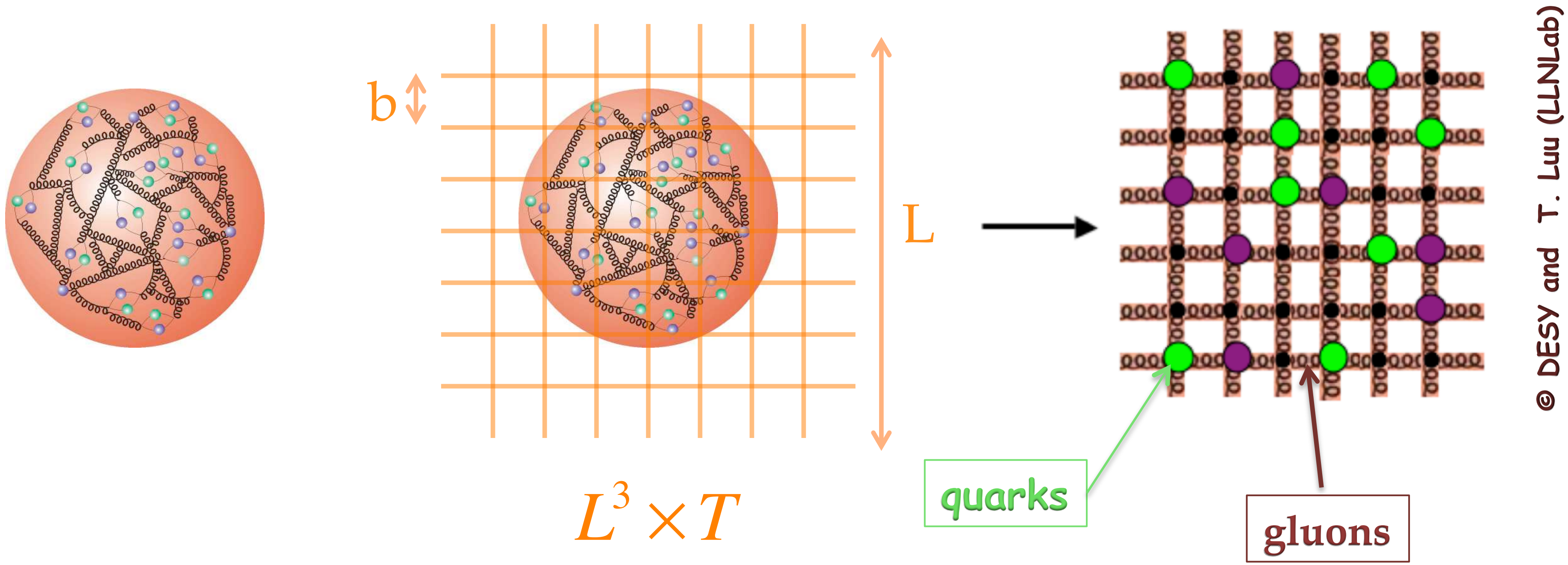
"strange" Experimental program



J. Pochodzalla, Int. Journal Modern Physics E, Vol 16, no. 3 (2007) 925-936

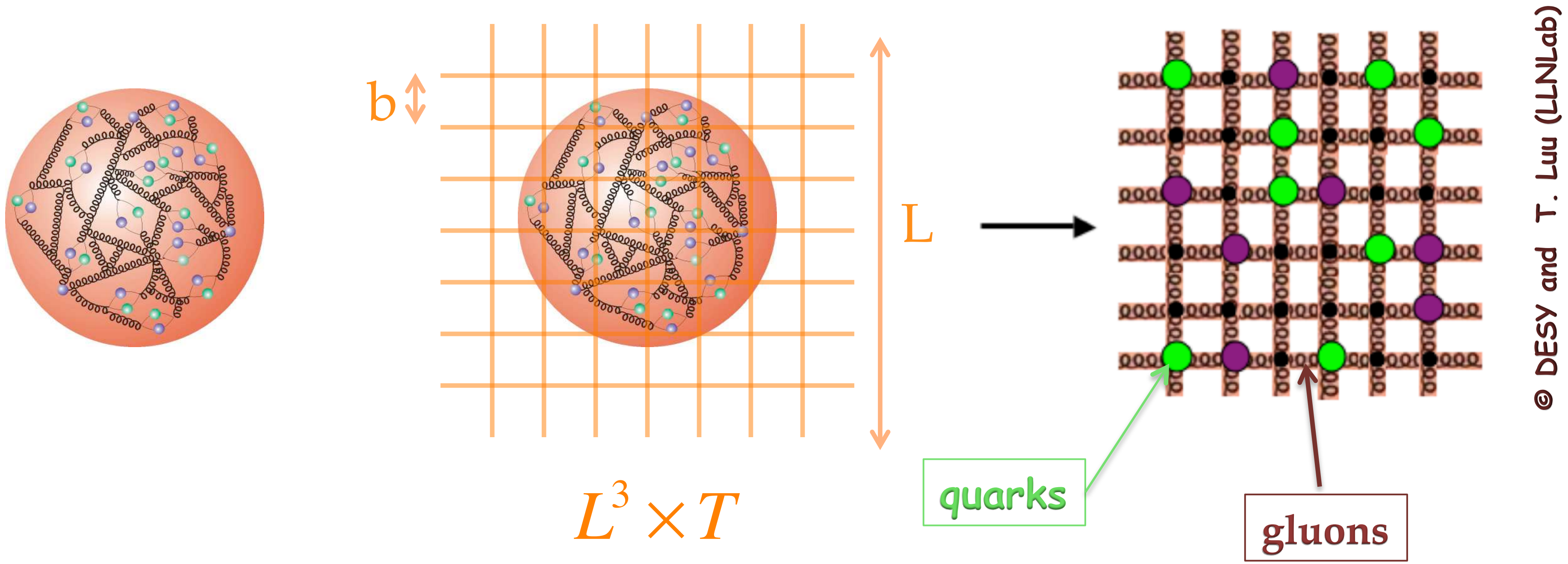


b
L
$L^3 \times T$



For numerical calculations in QCD, the theory is formulated on a (Euclidean) space-time lattice

((anti) periodic (time) spatial boundary conditions) N_s x N_s x N_s x N_t



For numerical calculations in QCD, the theory is formulated on a (Euclidean) space-time lattice

((anti) periodic (time) spatial boundary conditions) $N_s \times N_s \times N_s \times N_t$

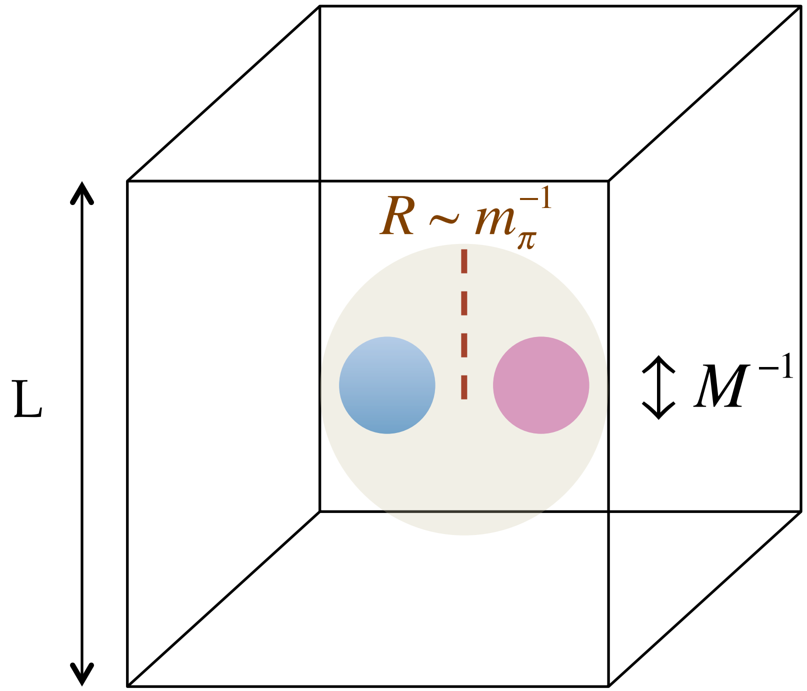
$$L >> \text{ relevant scales } >> b \quad \left(\frac{1}{L} << m_\pi << \Lambda_\chi << \frac{1}{b} \right)$$

→ finite number of d.o.f.
(finite volume)

$$b << \frac{1}{M_N} \text{ (ultraviolet cutoff)}$$

nucleon-nucleon scattering

$$\Rightarrow m_\pi L >> 1 \quad \text{(infrared cutoff)}$$



Lüscher, 1990

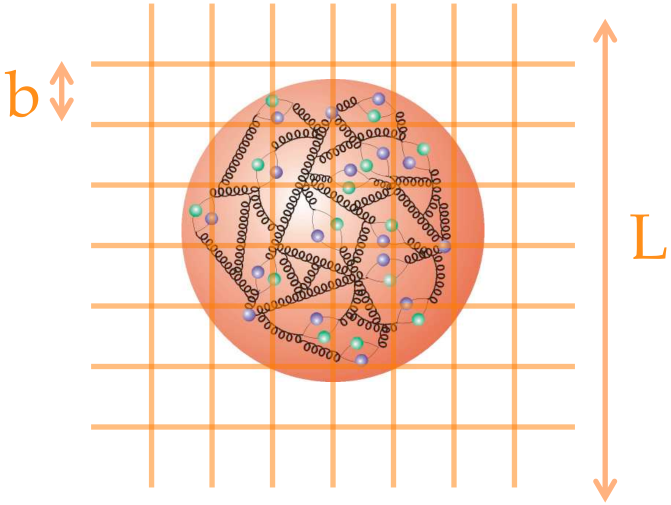
$$R < \frac{L}{2}$$

$$b << \frac{1}{M_N} \text{ (ultraviolet cutoff)}$$

nucleon-nucleon scattering

$$\Rightarrow m_\pi L >> 1 \quad \text{(infrared cutoff)}$$

NPLQCD



$N_s \times N_s \times N_s \times N_t$

$$\text{Cost} \approx \left[\frac{1}{m_q}\right][L]^a\left[\frac{1}{\mathrm{b}}\right]^\gamma$$

Significant computational resources required for calculations @ $\mathrm{m}_{u,d}^{phys}$ (petascale)

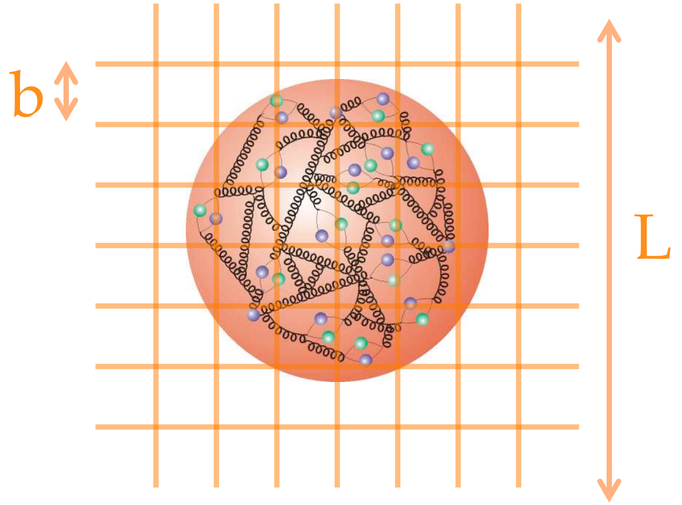
USE UNPHYSICAL VALUES OF THESE PARAMETERS
LATTICE ARTIFACTS

sources of systematic errors in the numerical simulation

finite volume L, discretization (finite spacing) b, value of the light quark masses

EFT Extrapolations

$L \to \infty$ $b \to 0$ $m_q \to m_q^{physical}$



N_s x N_s x N_s x N_t

$$\boxed{\text{Cost} \approx \left[\frac{1}{m_q}\right][L]^a\left[\frac{1}{\mathrm{b}}\right]^\gamma}$$

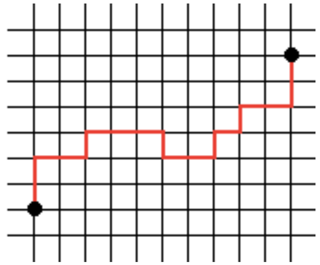
$$\underbrace{64^3 \times 256}_{L^3 \times T} \times 3_{\text{color}} \times 4_{\text{spin}}$$ large number of degrees of freedom

only practical way for this type of computation → **Monte Carlo integration**

Formalism

Direct Lattice QCD extraction $\longleftrightarrow$ Compute correlation functions

LQCD is a non-perturbative implementation of Field Theory, which uses the Feynman path-integral approach to evaluate transition matrix elements



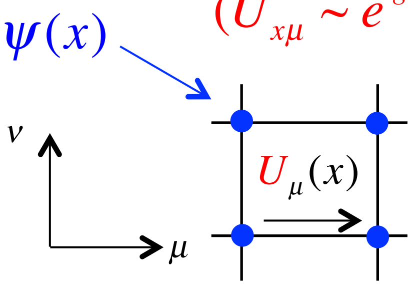
Our continuous Path-Integral (QCD partition function):

$$Z = \int D\varphi(x)\, e^{-iS_{QCD}[\varphi(x)]} = \int DU\, D\bar{\psi}\, D\psi\, e^{-iS_{QCD}[U,\bar{\psi},\psi]} \xrightarrow[t\to -i\tilde{t}]{} \int DU\, D\bar{\psi}\, D\psi\, e^{-\tilde{S}_{QCD}[U,\bar{\psi},\psi]}$$

oscillating phase

decaying exponential

$(U_{x\mu} \sim e^{igbA_{x\mu}})$



The weight of each path is a real positive quantity

$$Z = \int DU\, D\psi\, D\bar{\psi}\, e^{-\bar{\psi}Q(U)\psi - S_g[U]} = \int DU \left\{ \int D\psi\, D\bar{\psi}\, e^{-\bar{\psi}Q(U)\psi} \right\} e^{-S_g[U]}$$

$$= \int DU\, \boxed{\det Q(U)\, e^{-S_g[U]}}$$

~ P(U)
Boltzmann weight

$$\det\left[Q_f(A)\right] \equiv \det\left(\not{D}[A] + m\right)$$

(quark matrix)

BASIS OF NUMERICAL SIMULATIONS

1. Generate an ensemble of N gauge-field configurations $\{U_i\}$ according to the probability distribution P(U)

$$Z = \int DU\ D\psi\ D\bar{\psi}\ e^{-\bar{\psi}Q(U)\psi - S_g[U]} = \int DU \left\{ \int D\psi\ D\bar{\psi}\ e^{-\bar{\psi}Q(U)\psi} \right\} e^{-S_g[U]}$$

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~ P(U)

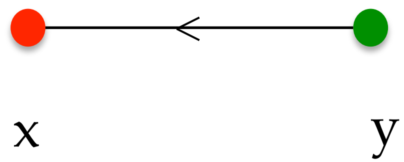
Boltzmann weight

$$\det\left[Q_f(A)\right] \equiv \det\left(\not{D}[A] + m\right)$$

(quark matrix)

2. Use the N gauge-field configurations previously generated to calculate the quark propagators on each configuration $Q^{-1}\,[U_i]$

When computing expectation values of any given operator *O*, the quark fields in *O* are re-expressed in terms of quark propagators using Wick's Theorem: **write all possible contractions for the fields** (removing the dependence of quarks as dynamical fields)



$$Q_u^{-1}(x,y) = u(\overbrace{y)\bar{u}(}x)$$

expectation values

$$\langle \hat{O} \rangle = \frac{1}{Z} \int DU \;\; D\bar{\psi} \;\; D\psi \, \hat{O}\,[\psi, \bar{\psi}, U]\; e^{-\bar{\psi} Q(U) \psi - S_g[U]}$$

$$\langle \hat{O} \rangle = \frac{1}{Z} \int DU \;\; \underbrace{\hat{O}\,[Q(U)^{-1}]}_{\text{propagators}} \underbrace{\det(Q(U))\, e^{-S_g[U]}}_{\text{configurations} \;\; (\sim \text{P (U)})}$$

$$\xleftrightarrow[\lim N \to \infty]{} \langle \hat{O} \rangle = \frac{1}{N} \sum_{\substack{i=1 \\ \text{gluon cfgs}}}^{N} \hat{O}\,[Q(U_i)^{-1}]$$

$$\text{error} \sim \frac{1}{\sqrt{N}}$$

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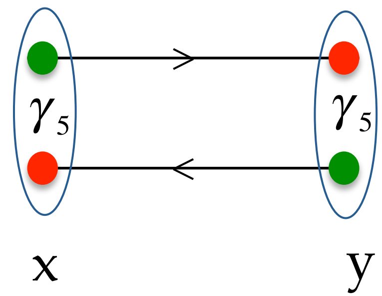
$$\pi^+ = \bar{d}\gamma_5 u$$



$$\langle \pi^\dagger(x)\,\pi(y)\rangle = \langle \bar{u}(x)\gamma_5 d(x)\,\bar{d}(y)\gamma_5 u(y)\rangle$$

expectation values

$$\langle \hat{O} \rangle = \frac{1}{Z}\int DU\;D\bar{\psi}\;D\psi\,\hat{O}[\psi,\bar{\psi},U]\,e^{-\bar{\psi}Q(U)\psi - S_g[U]}$$

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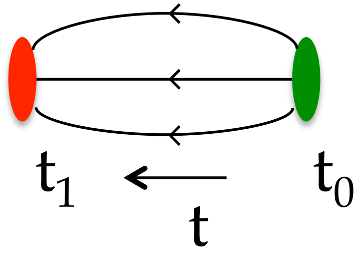
$$\text{error} \sim \frac{1}{\sqrt{N}}$$

3. Contract propagators onto correlation functions C_i (t)



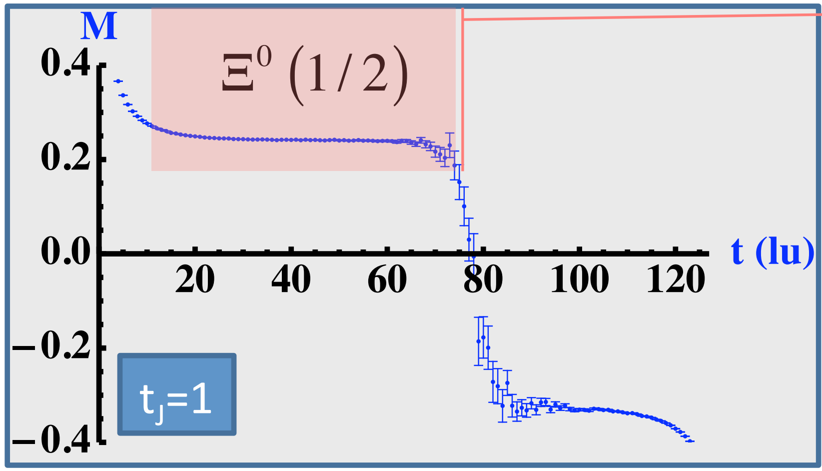
$$C(\Gamma^\nu,\vec{p},t)=\sum_{\vec{x}_1} e^{-i\vec{p}\vec{x}_1}\,\Gamma^\nu \left\langle J(\vec{x}_1,t)\,\bar{J}(\vec{x}_0,0)\right\rangle$$

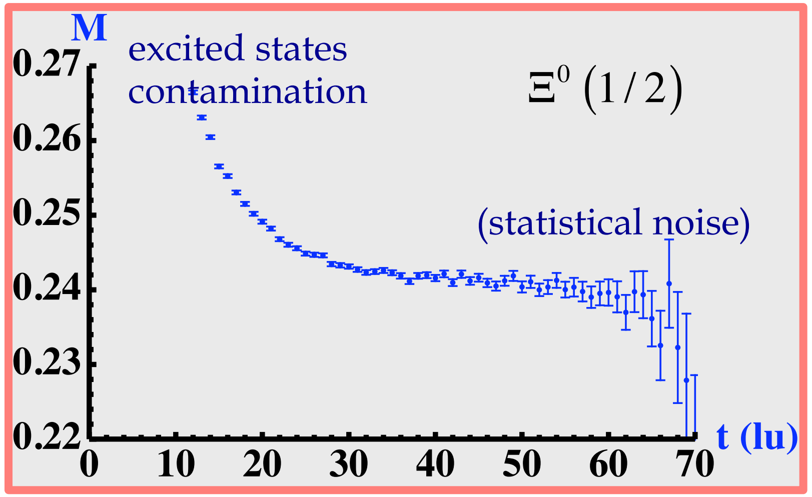
$$C(t)=\langle 0|\phi(t)\phi^\dagger(0)|0\rangle \longrightarrow \langle\phi|e^{-Ht}|\phi\rangle=\sum_n\langle\phi|e^{-Ht}|n\rangle\langle n|\phi\rangle=\sum|\langle\phi|n\rangle|^2 e^{-E_n t}$$

$$\phi(t)=e^{Ht}\phi\, e^{-Ht} \qquad\qquad \xrightarrow{t\to\infty} Z_0\, e^{-E_0 t}$$

$$\frac{1}{t_J}\log\frac{C_A(t)}{C_A(t+t_J)}=E_{0A}\quad (m_A)$$

g.s. energy (mass) from plateau





(anti-periodic bc in time)

$$\Xi^0_\alpha(\vec{x},t)=\varepsilon^{ijk}s^i_\alpha(\vec{x},t)\left(u^{j^T}_\alpha(\vec{x},t)C\gamma_5\, s^k(\vec{x},t)\right)$$

pions:

$$\frac{\sigma}{\langle C \rangle} \to \frac{1}{\sqrt{N}}$$

nucleons:

$$\frac{\sigma}{\langle C \rangle} \sim \frac{1}{\sqrt{N}} \times \exp\left(M_N - \frac{3m_\pi}{2} \right) t$$

Lepage, 1989

for baryons, the noise grows exponentially with time

Expectation is that for A nucleons:

$$\frac{\sigma}{\langle C \rangle} \sim \frac{\exp\left[A \left(M_N - \frac{3m_\pi}{2} \right) t \right]}{\sqrt{N}}$$

$$\frac{1}{t_J} \log \frac{C_A(t)}{C_A(t+t_J)} = E_{0A} \quad (m_A)$$

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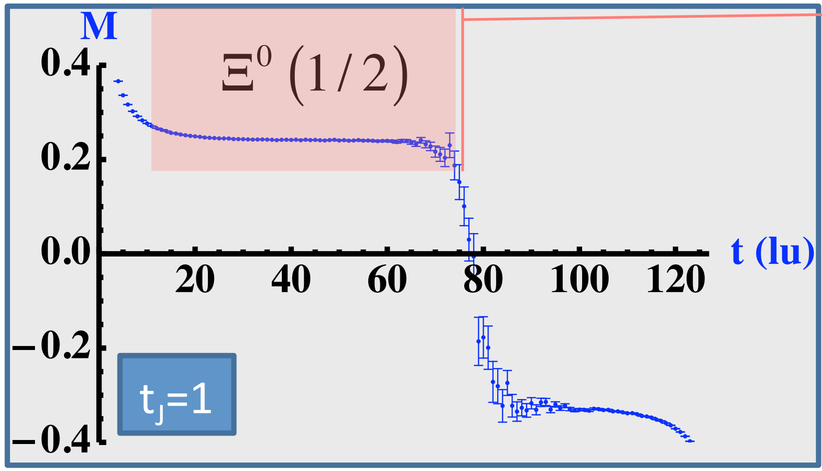

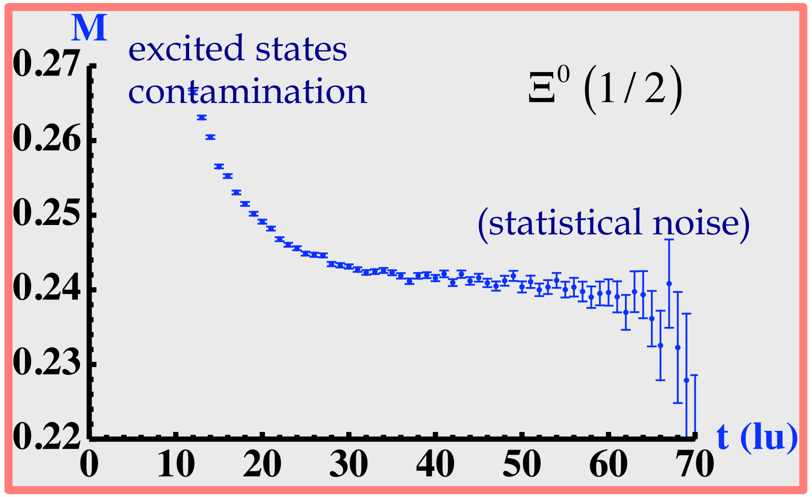

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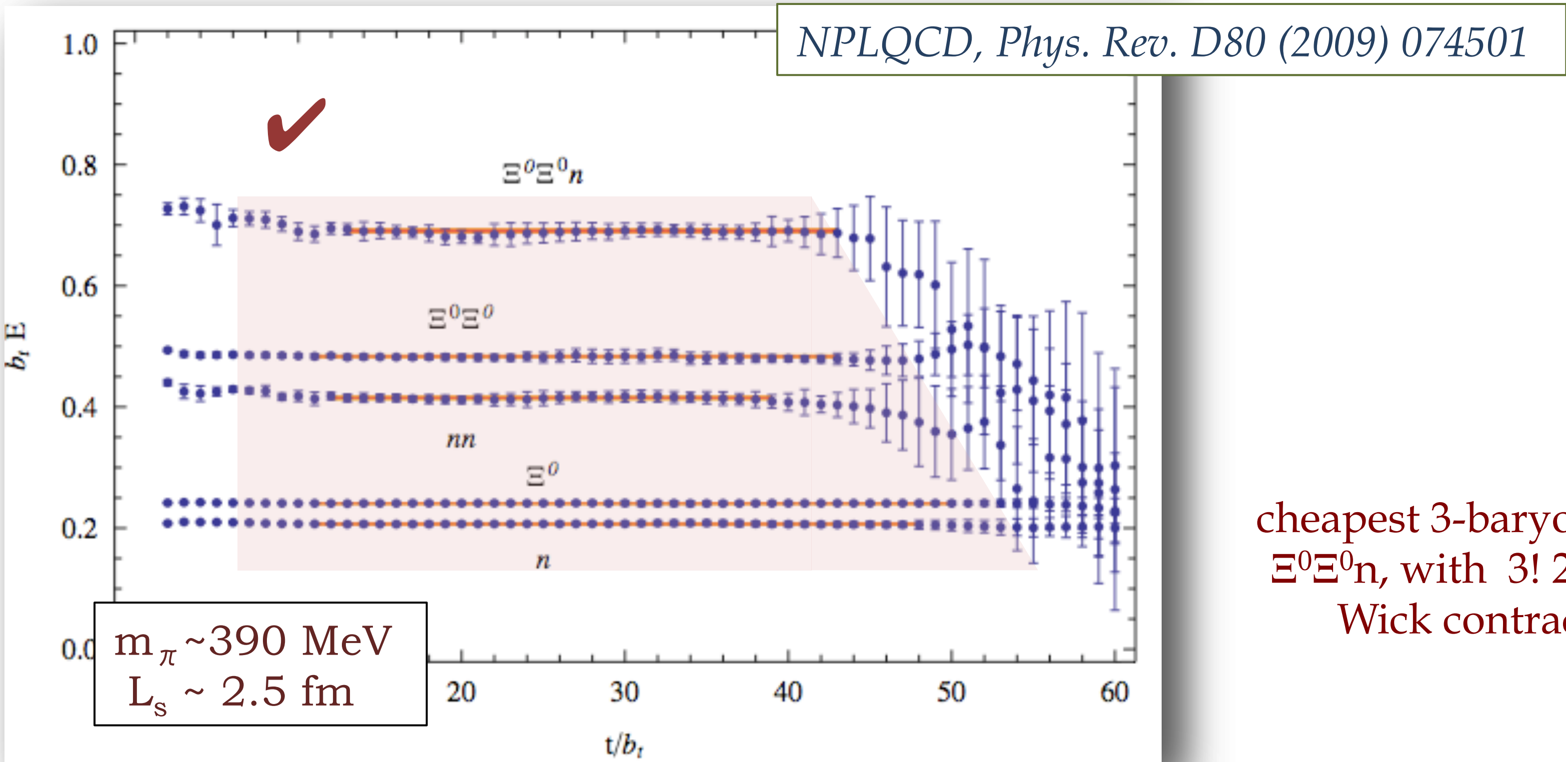
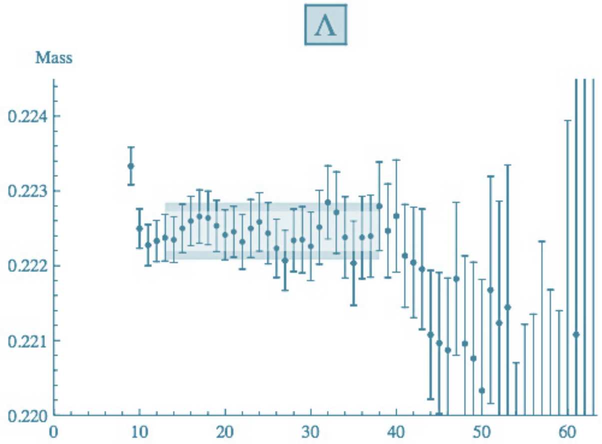
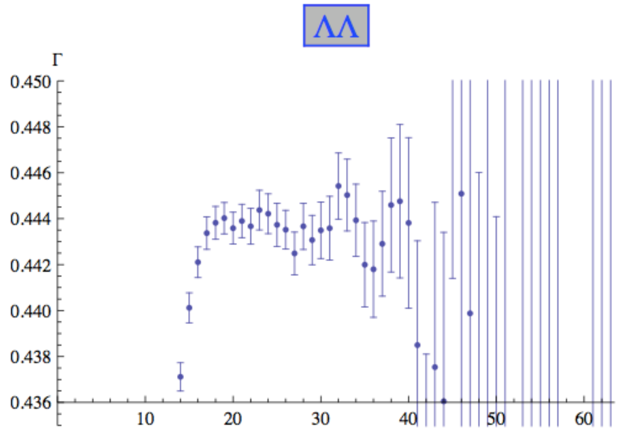


cheapest 3-baryon system:
$\Xi^0\Xi^0$n, with 3! 2! 4! = 288
Wick contractions

We can also extract the energy of the interacting system for a given {m$_\pi$, L, b} set

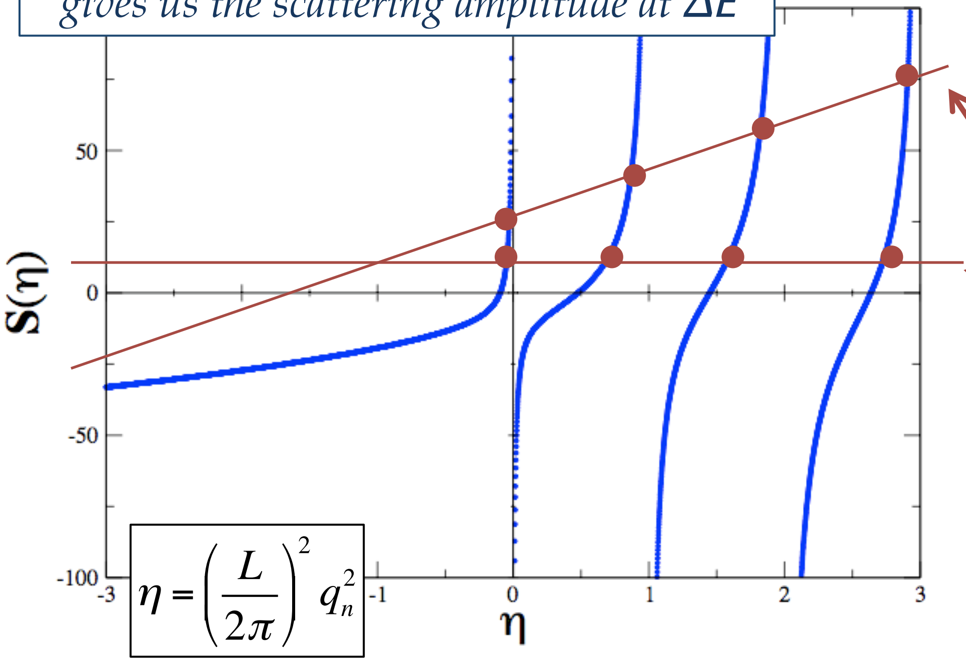




$$G_{\Lambda\Lambda}(t) = \frac{C_{\Lambda\Lambda}(t)}{C_{\Lambda}(t)\, C_{\Lambda}(t)} \rightarrow A_0\, e^{-\Delta E_{\Lambda\Lambda} t} \qquad \rightarrow \qquad \frac{1}{t_J} \log \frac{G(t)}{G(t+t_J)} \rightarrow \text{extract} \quad \Delta E$$

We can also extract the energy of the interacting system for a given {m_π, L, b} set

gives us the scattering amplitude at ΔE



$$\eta = \left(\frac{L}{2\pi}\right)^2 q_n^2$$

$$q_n \cot\delta(q_n) = -\frac{1}{a} + \frac{1}{2} r_0 q_n^{\,2}$$

$$\frac{1}{t_J}\log\frac{G(t)}{G(t+t_J)} \rightarrow \text{extract} \quad \Delta E$$

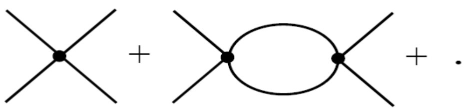
Phys.Lett. B585 (2004) 106-114

$$q_n \cot\delta(q_n) = \frac{1}{\pi L} S\left(q_n^{\,2}\left(\frac{L}{2\pi}\right)^2\right) \equiv \frac{1}{\pi L}\sum_{\vec{j}}^{\Lambda} \frac{1}{|\vec{j}|^2 - \left(\frac{L q_n}{2\pi}\right)^2} - \frac{4\Lambda}{L}$$

● : location of the E eigenstates in the finite volume

(3D Riemann- zeta function)

$$+\,O(e^{-m_\pi L})$$

$\mathcal{A} \sim$  $= \frac{4\pi}{M}\frac{1}{p\cot\delta(p) - ip}$

Scattering states:

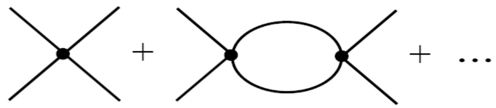
Beane, Bedaque, Parreño, Savage, Phys. Lett. B585 (2004) 106-114

$$\Delta E_0 = \frac{p^2}{M} = \frac{4\pi a}{ML^3}\left[1 - c_1\frac{a}{L} + c_2\left(\frac{a}{L}\right)^2 + \ldots\right]$$ Ground state energy shift

extract the scattering lentgh

Recovering M. Lüscher, Commun. Math. Phys. 105, 153 (1986) (L>>a)

Bound states?

$\mathcal{A} \sim$  $= \dfrac{4\pi}{M}\dfrac{1}{p\cot\delta(p) - ip}$

infinite volume

b.s. $\quad p^2 = -\gamma^2$

$\cot\delta(i\gamma) = i$

finite volume:

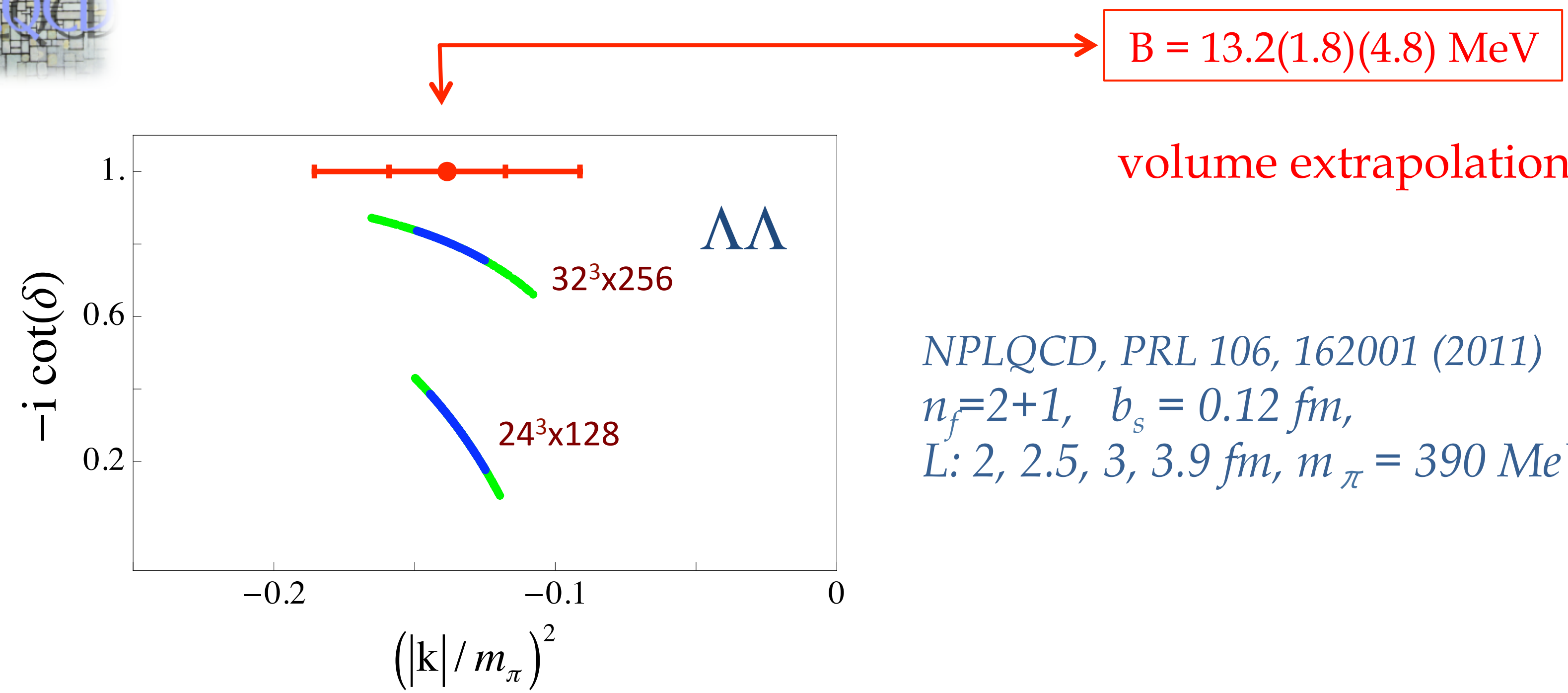
$$\cot\delta(i\gamma)\big|_{k=i\gamma} = i - i\sum_{\vec{m}\neq\vec{0}}\frac{e^{-|\vec{m}|\gamma L}}{|\vec{m}|\gamma L}$$

$k^2 < 0, \quad k = i\kappa$

$$\kappa = \gamma + \frac{g_1}{L}\left(e^{-\gamma L} + \sqrt{2}e^{-\sqrt{2}\gamma L} + \ldots\right)$$

$\kappa \to \gamma$ for large L

$$B_\infty = \frac{\gamma^2}{M}$$



volume extrapolation

NPLQCD, PRL 106, 162001 (2011)
n_f=2+1, b_s = 0.12 fm,
L: 2, 2.5, 3, 3.9 fm, m_π = 390 MeV

infinite volume

b.s. $\quad p^2 = -\gamma^2$

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finite volume:

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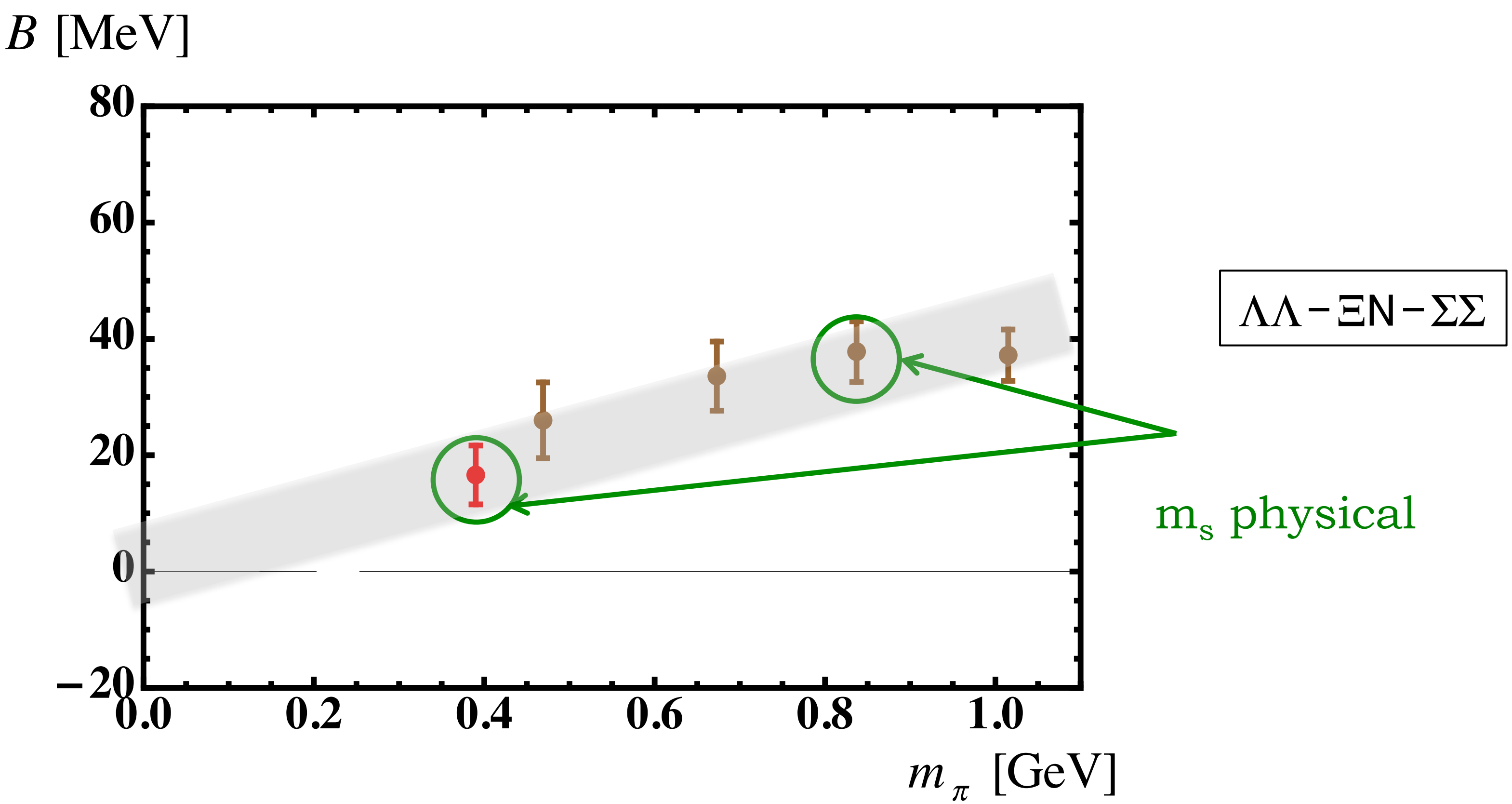
NPLQCD, PRL 106, 162001 (2011) n_f=2+1, b_s = 0.12 fm, L: 2, 2.5, 3, 3.9 fm, m_π = 390 MeV
NPLQCD, Mod.Phys.Lett. A26 (2011) n_f=2+1, b_s = 0.12 fm, L: 4fm, m_π = 230 MeV
NPLQCD, PRD87 (2013) n_f=3, b_s = 0.145 fm, L: 3.4, 4.5, 6.7 fm, m_π = 807 MeV



HALQCD, PRL 106, 162002 (2011), NPA 881 (2012)
n_f=3, b_s = 0.12 fm, L: 4 fm m_π = 469, 670, 830, 1015, 1171 MeV

$A = 2, s = -2, J = 0, I = 0$

NPLQCD, PRL 106, 162001 (2011) n_f=2+1, b_s = 0.12 fm, L: 2, 2.5, 3, 3.9 fm, m_π = 390 MeV
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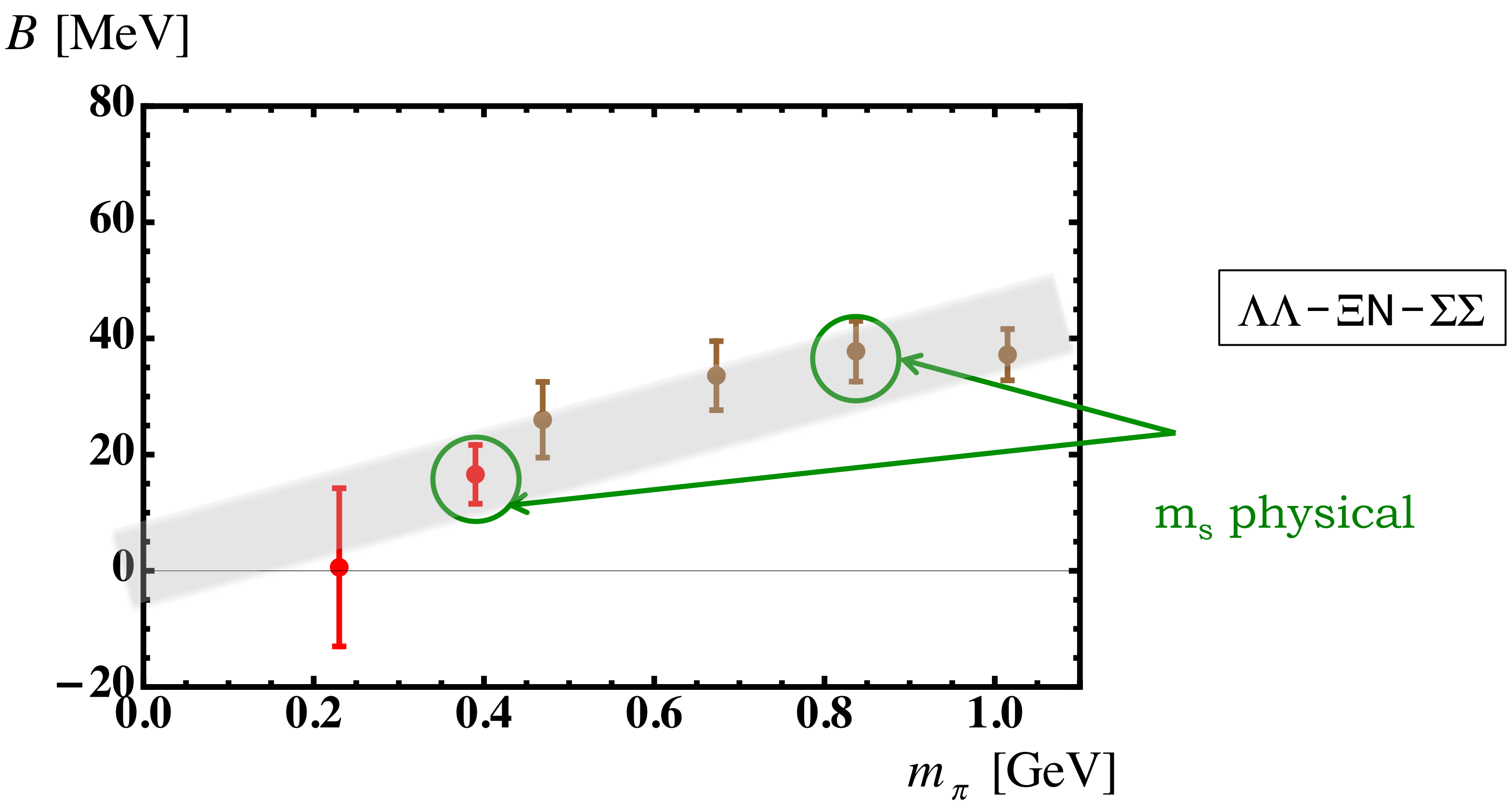

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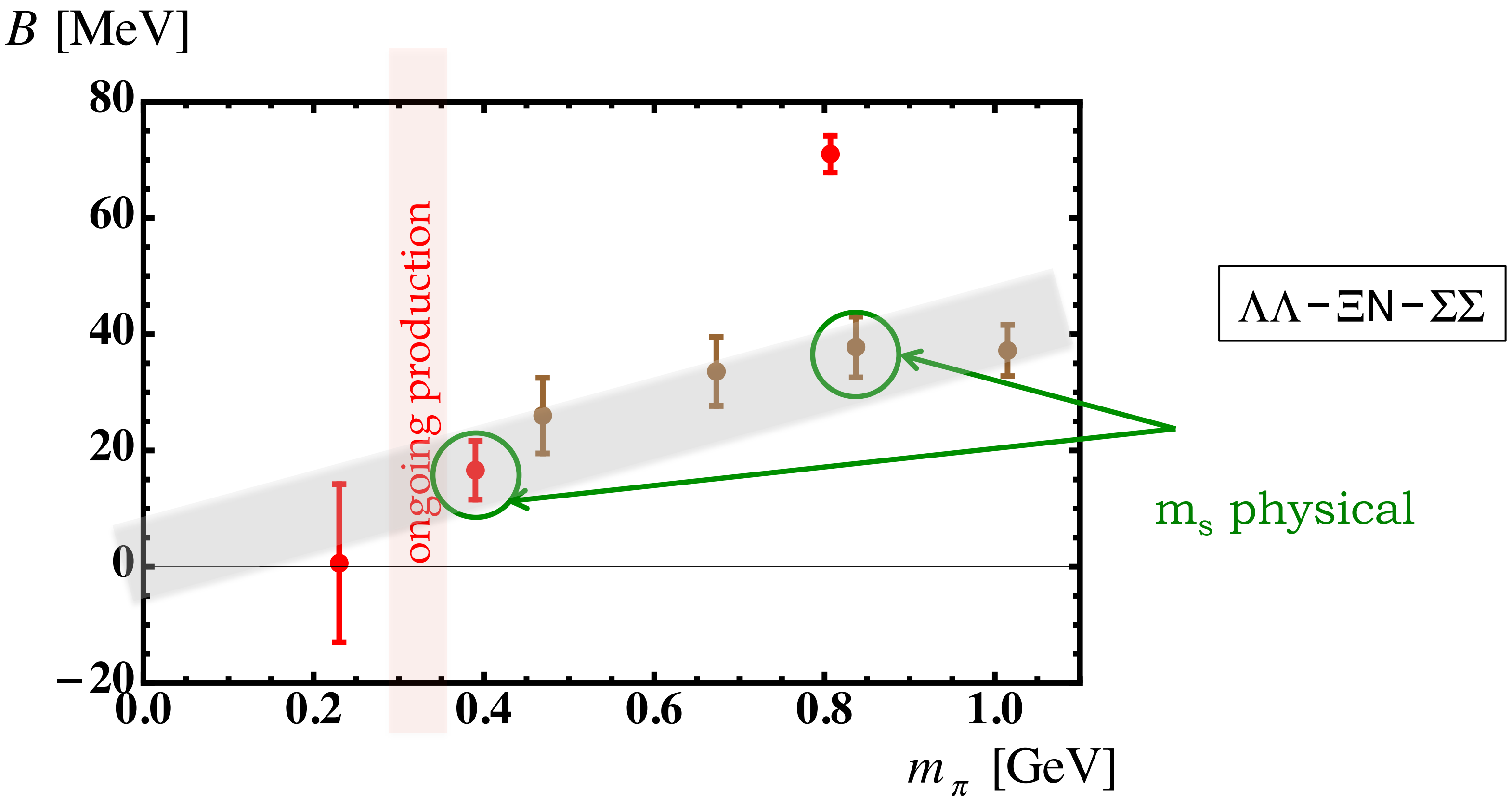

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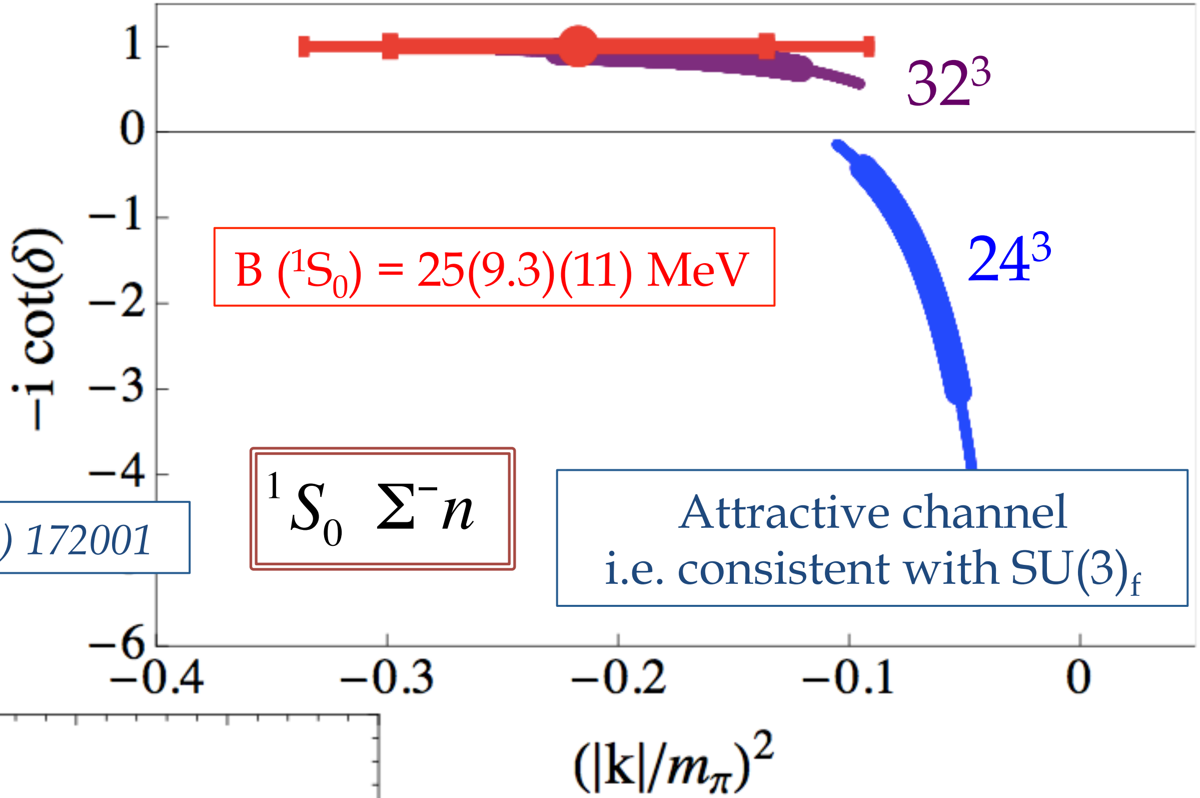
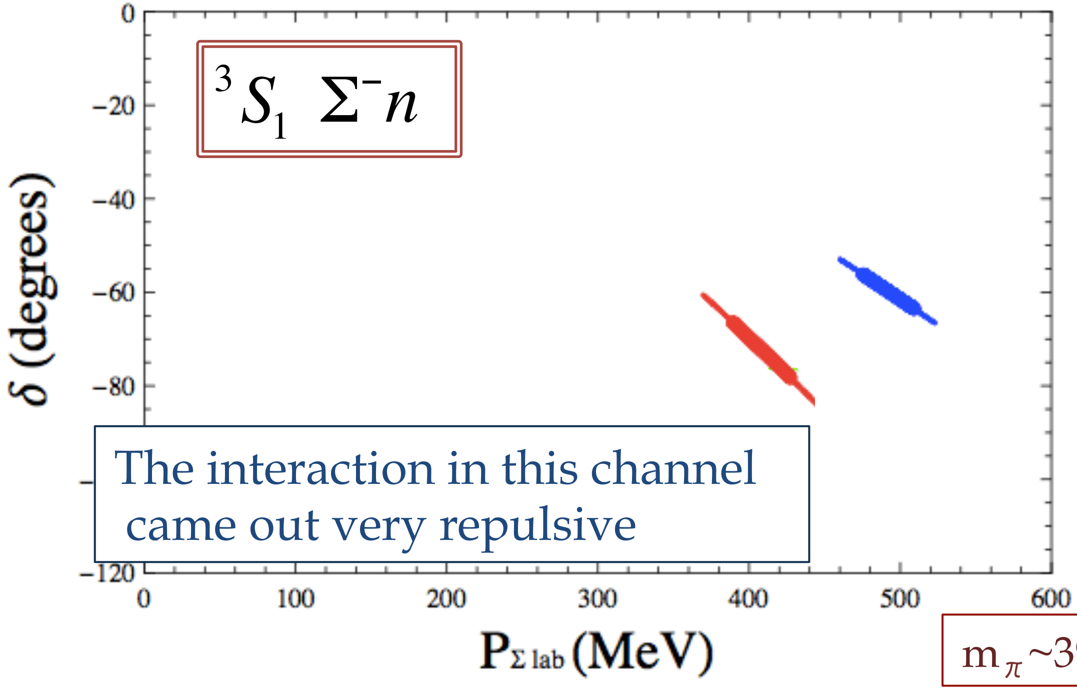
The ΣN channel



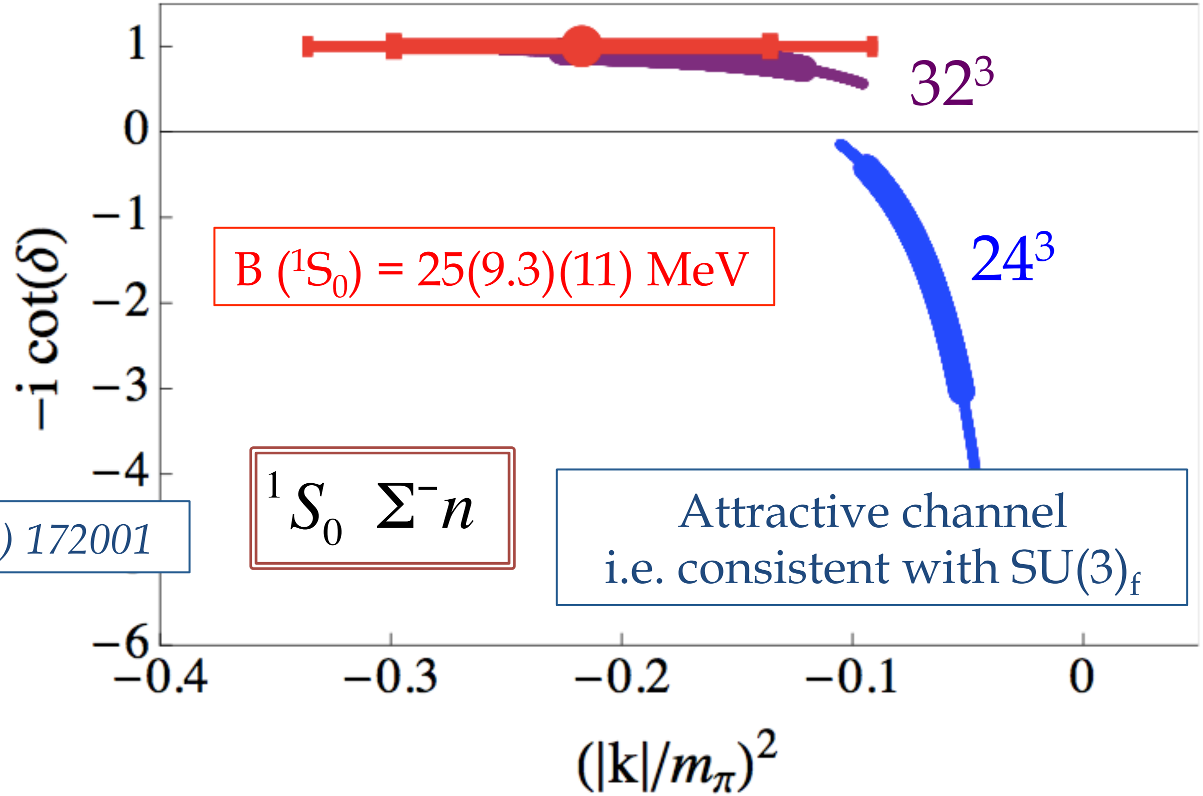
NPLQCD, Phys.Rev.Lett. 109 (2012) 172001

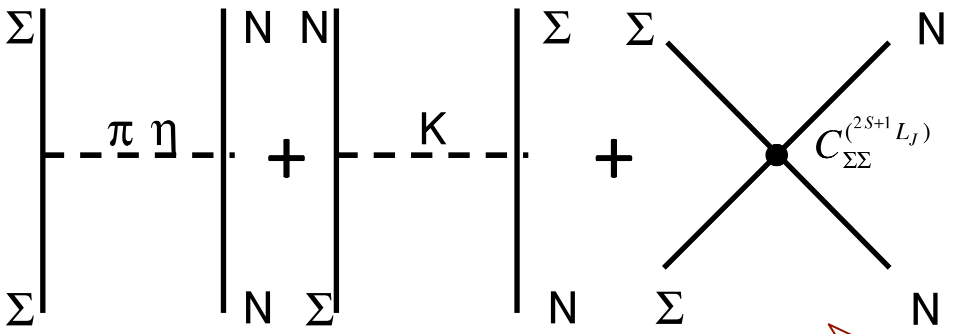


m_π ~390 MeV, L_s ~ 2, 2.5, 3, 4 fm, b=0.123 fm

The ΣN channel

NPLQCD, Phys.Rev.Lett. 109 (2012) 172001





LO ΣN interactions
(only S-wave)

quark-mass dependence

$$V_{REG}(r) \sim g e^{-\Lambda^2 r^2}, \quad g e^{-\Lambda^4 r^4}$$

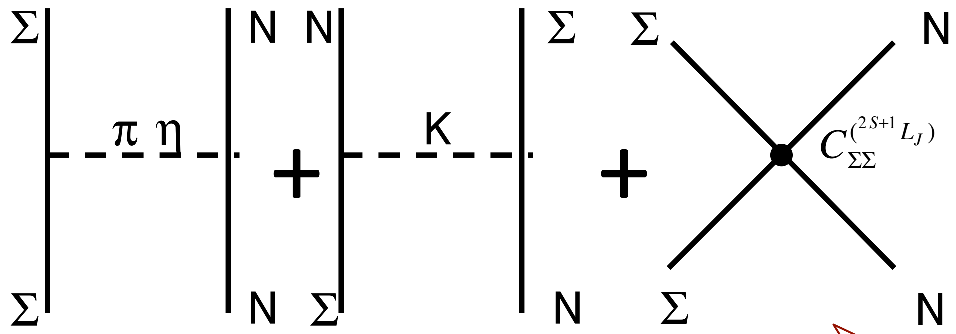
NPLQCD

The $C_{\Sigma\Sigma}^{({}^1S_0)}$ coefficient is fit to reproduce the B.E calculated in the lattice

B $({}^1S_0)$ = 25(9.3)(11) MeV

LO EFT potential (for any m_π)

We find that the channel becomes unbound for $m_\pi \sim 300$ MeV, consistent with EFT constrained by experimental data *(Haidenbauer & Meissner, NPA881 (2012) 44-61)*



LO ΣN interactions

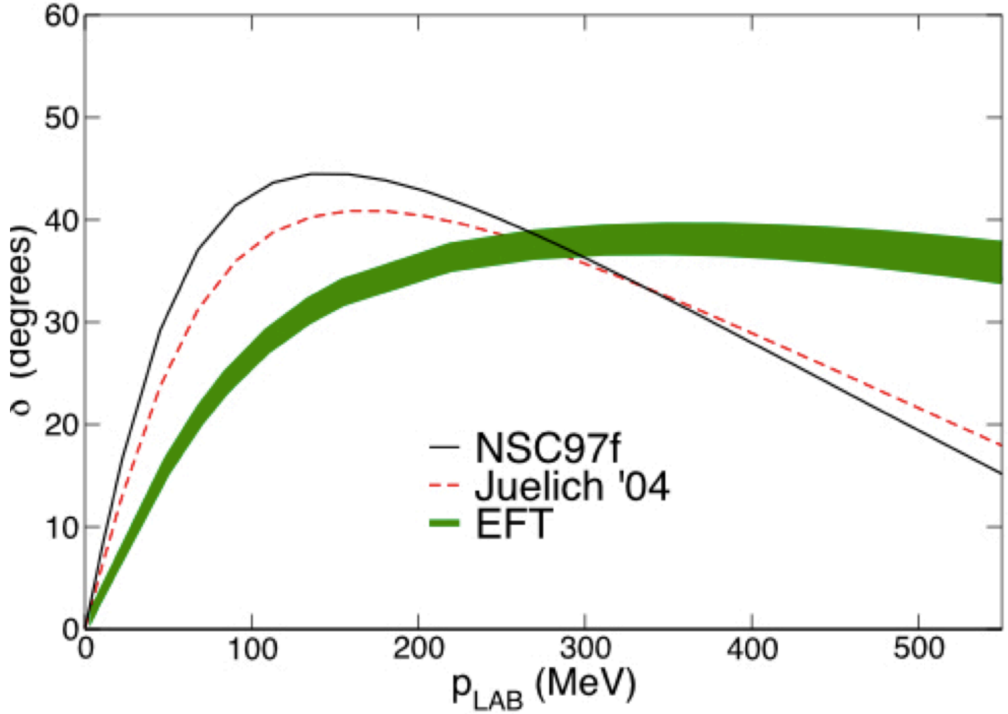
(only S-wave)

quark-mass dependence

$$V_{REG}(r) \sim g e^{-\Lambda^2 r^2}, \quad g e^{-\Lambda^4 r^4}$$

The ΣN channel

$^1S_0 \ \Sigma^- n$



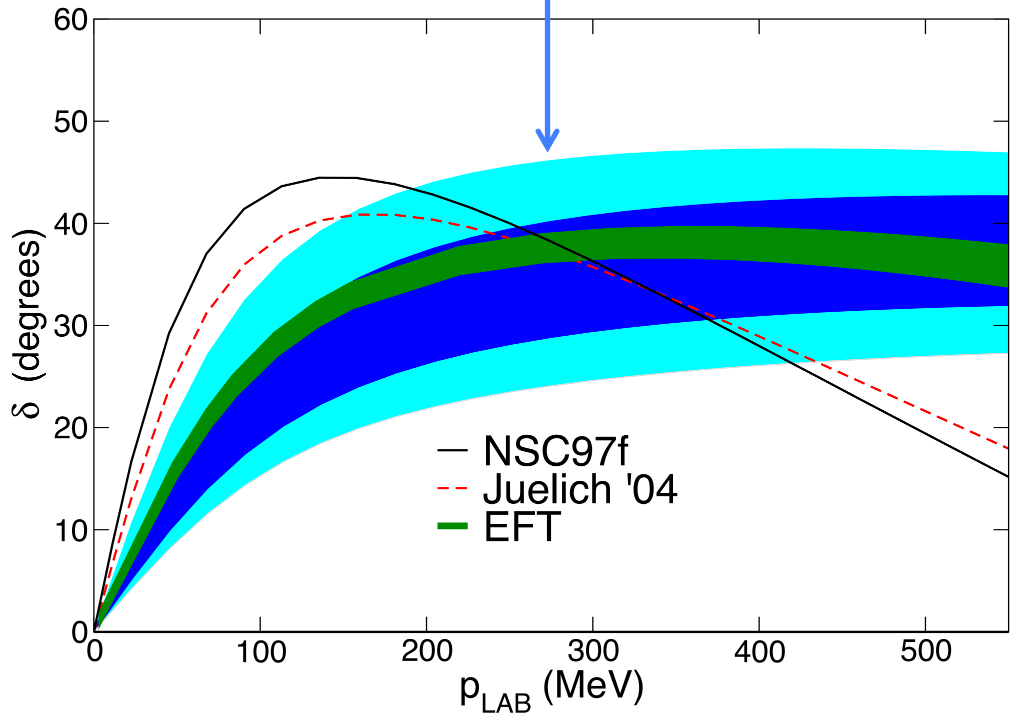
J. Haidenbauer, U.-G. Meissner, A. Nogga and H. Polinder, Lect. Notes Phys. **724** (2007) 113

The ΣN channel

$^1S_0 \ \Sigma^- n$

LQCD predicition @ m_π^{phys}

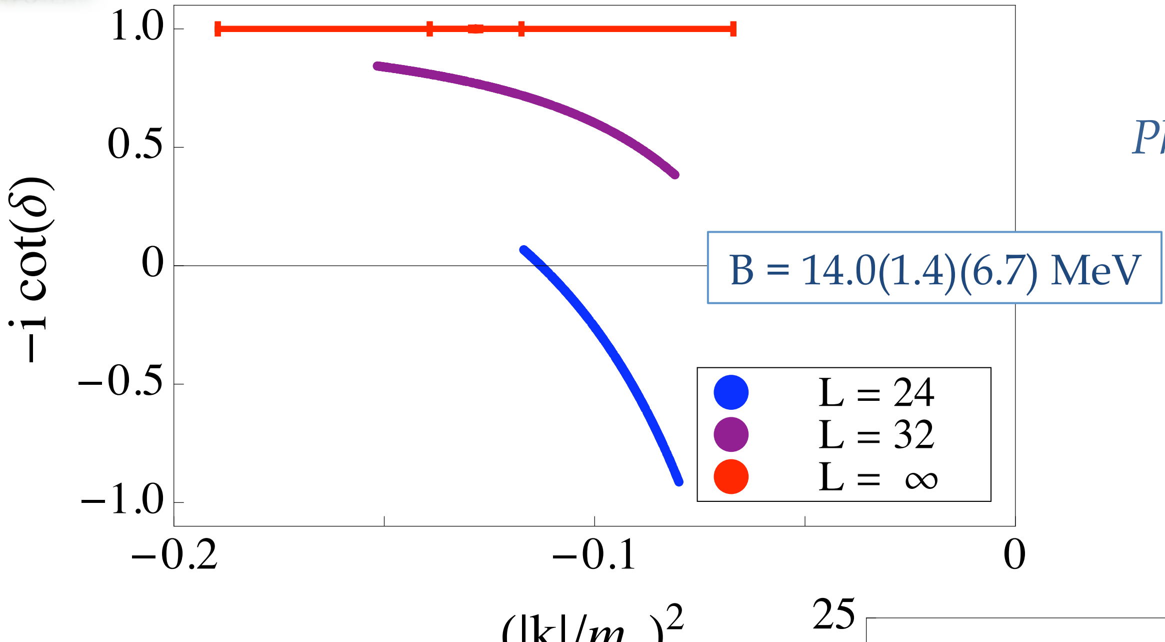
NPLQCD, Phys.Rev.Lett. 109 (2012) 172001

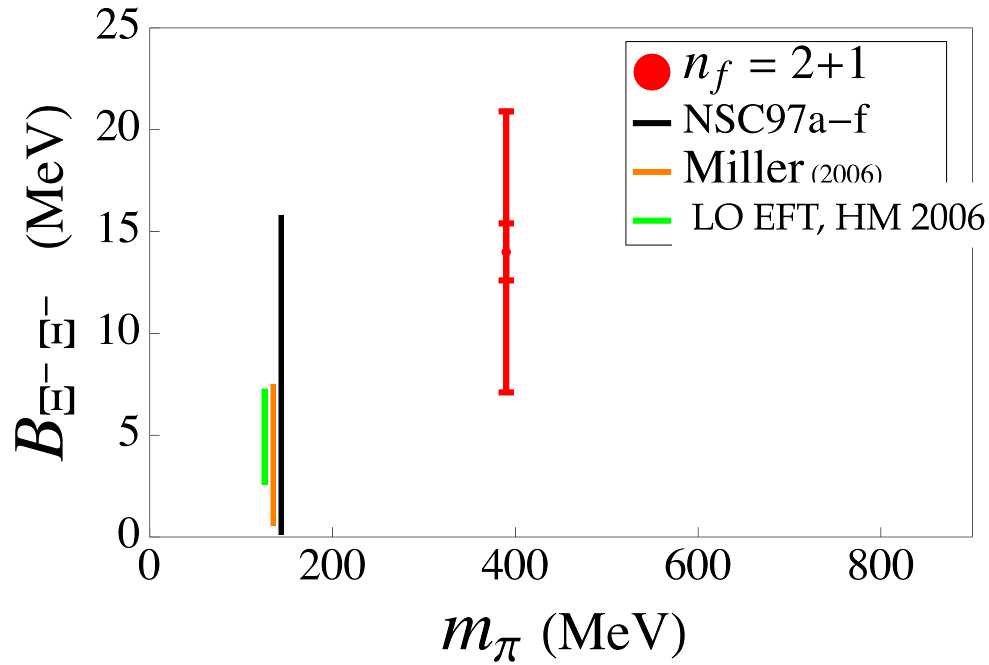


J. Haidenbauer, U.-G. Meissner, A. Nogga and H. Polinder, Lect. Notes Phys. **724** (2007) 113

The $\Xi^- \Xi^-$ channel

Phys.Rev. D85 (2012) 054511



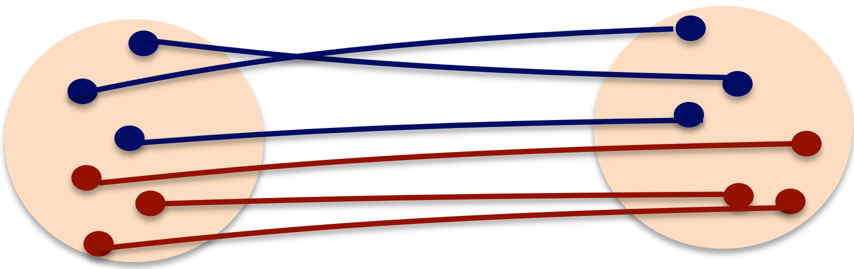


Going beyond A=2

✓ Larger complexity as compared to calculations for single hadrons

$$\left((A+Z)!\,(2A-Z)!\right)$$

$^{3}\mathrm{H} \rightarrow 2880 \qquad {}^{4}\mathrm{He} \rightarrow 518\,400$



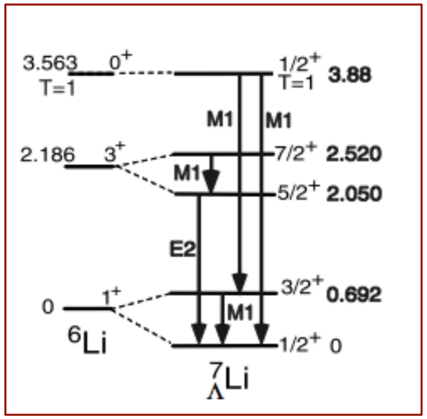
\# Wick contractions ~ $N_u!\, N_d!\, N_s!$

Detmold & Savage, PRD82 (2010) 014511
Detmold & Orginos, PRD87 (2013) 11, 114512
Doi & Endres, Comput.Phys.Commun. 184 (2013) 117

✓ Demand larger lattice volumes

✓ Demand better accuracy

$$\boxed{\frac{\sigma}{\langle C\rangle} \sim \frac{1}{\sqrt{N}} \exp\left[A\left(M_N - \frac{3m_\pi}{2}\right)t\right]}$$



✓ Small energy splittings in nuclear physics

↓

Need of high statistics calculations

Perform calculations at heavier light-quark masses:

✓ signal-to-noise ratio improves

✓ reduced computational resources to generate LQCD configurations, *i.e.*, larger statistics

$\boxed{SU(3)_f}$

no continuum extrapolation

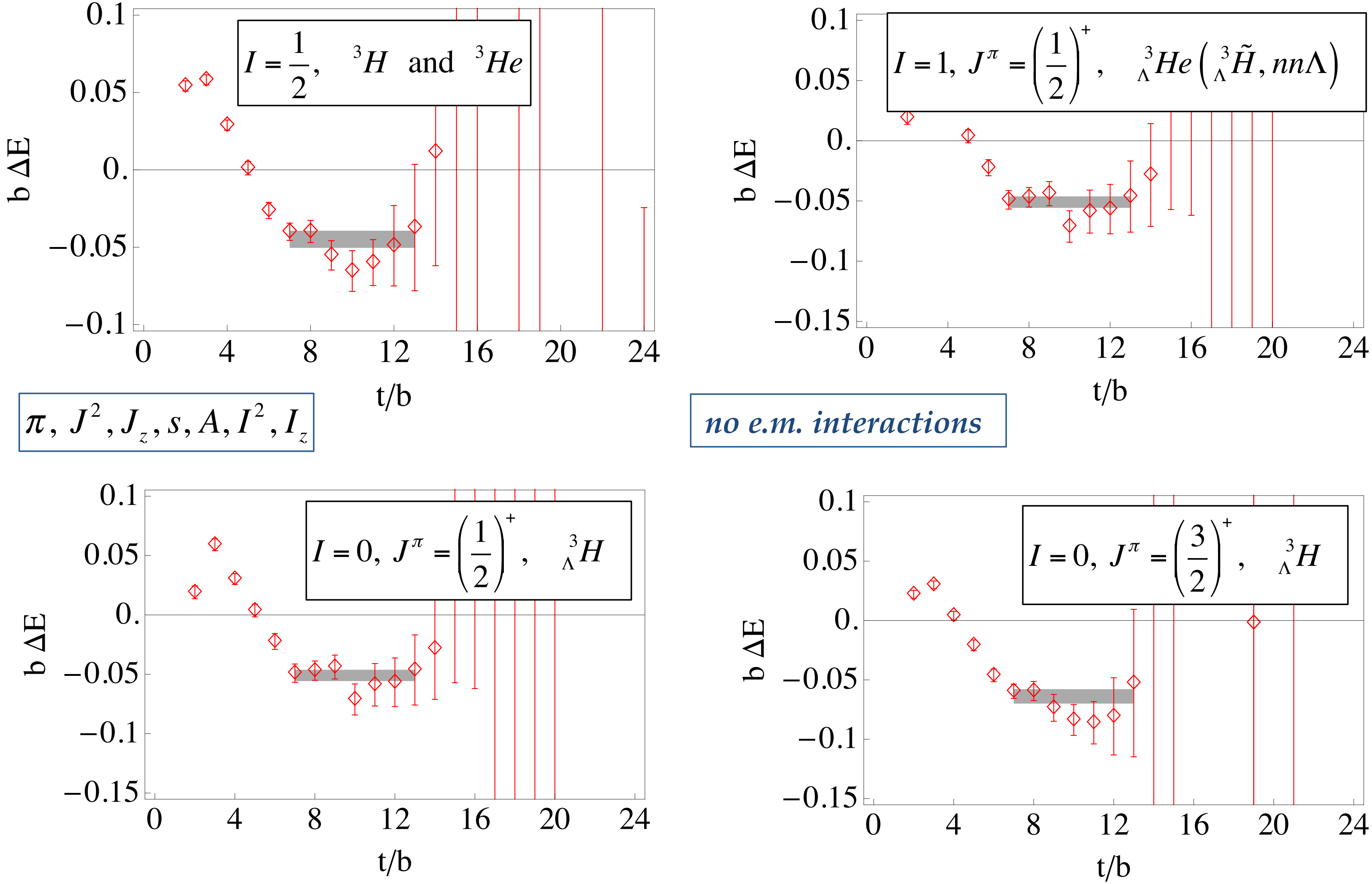
L/b	T/b	β	$b\,m_q$	b (fm)	L (fm)	T (fm)	m_π (MeV)	$m_\pi L$	$m_\pi T$	$N_{\rm cfg}$	$N_{\rm src}$
24	48	6.1	−0.2450	0.145	3.4	6.7	806.5(0.3)(0)(8.9)	14.3	28.5	3822	96
32	48	6.1	−0.2450	0.145	4.5	6.7	806.9(0.3)(0.5)(8.9)	19.0	28.5	3050	72
48	64	6.1	−0.2450	0.145	6.7	9.0	806.7(0.3)(0)(8.9)	28.5	38.0	1905	54

infinite volume extrapolation

Example, A=3 system:

NPLQCD Phys.Rev. D87 (2013), 034506

$(48^3 \times 64)$



$\pi, J^2, J_z, s, A, I^2, I_z$

no e.m. interactions

The nucleon-nucleon system

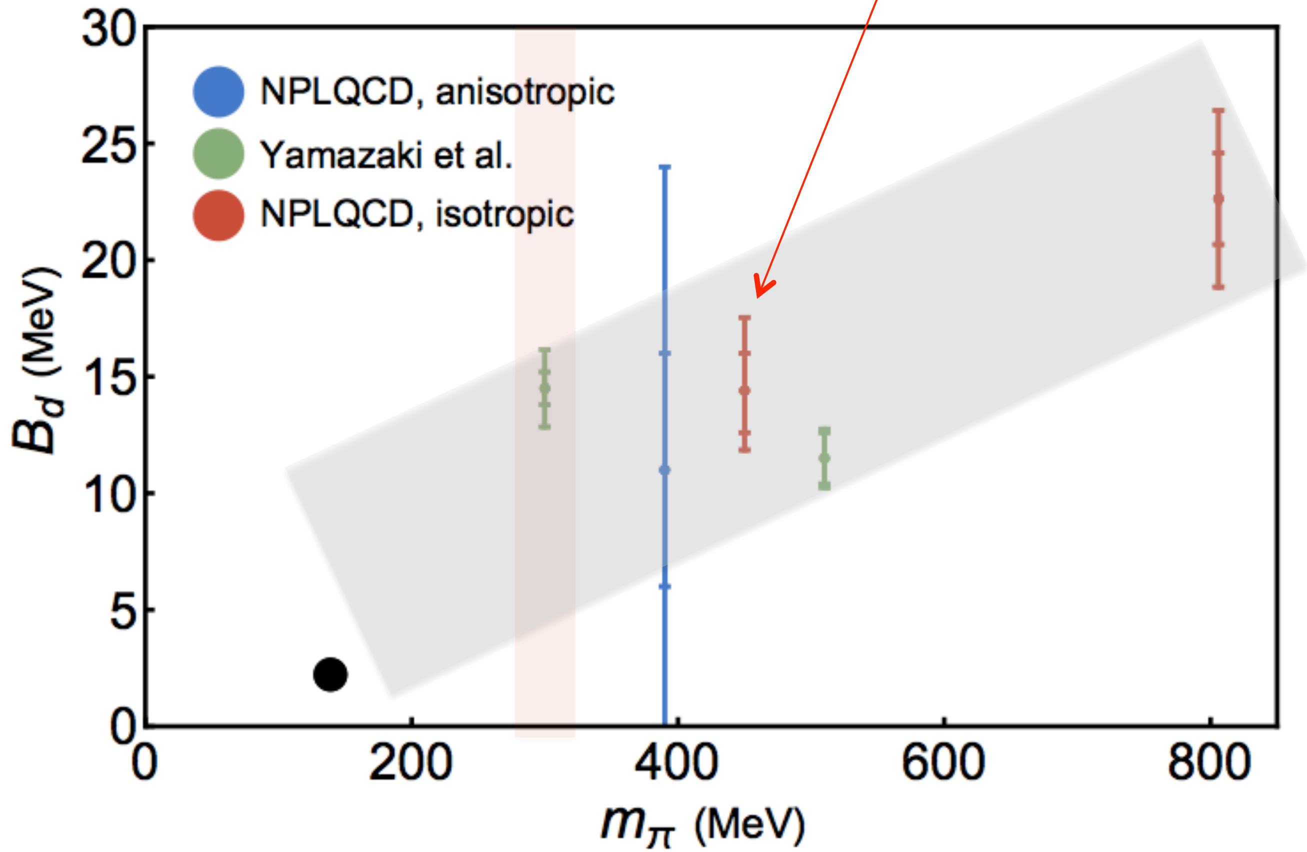
Major challenge in LQCD calculations for Nuclear Physics:

recover the experimentally known properties of the two-nucleon systems

The deuteron

ongoing production @ this pion mass

(preliminary)

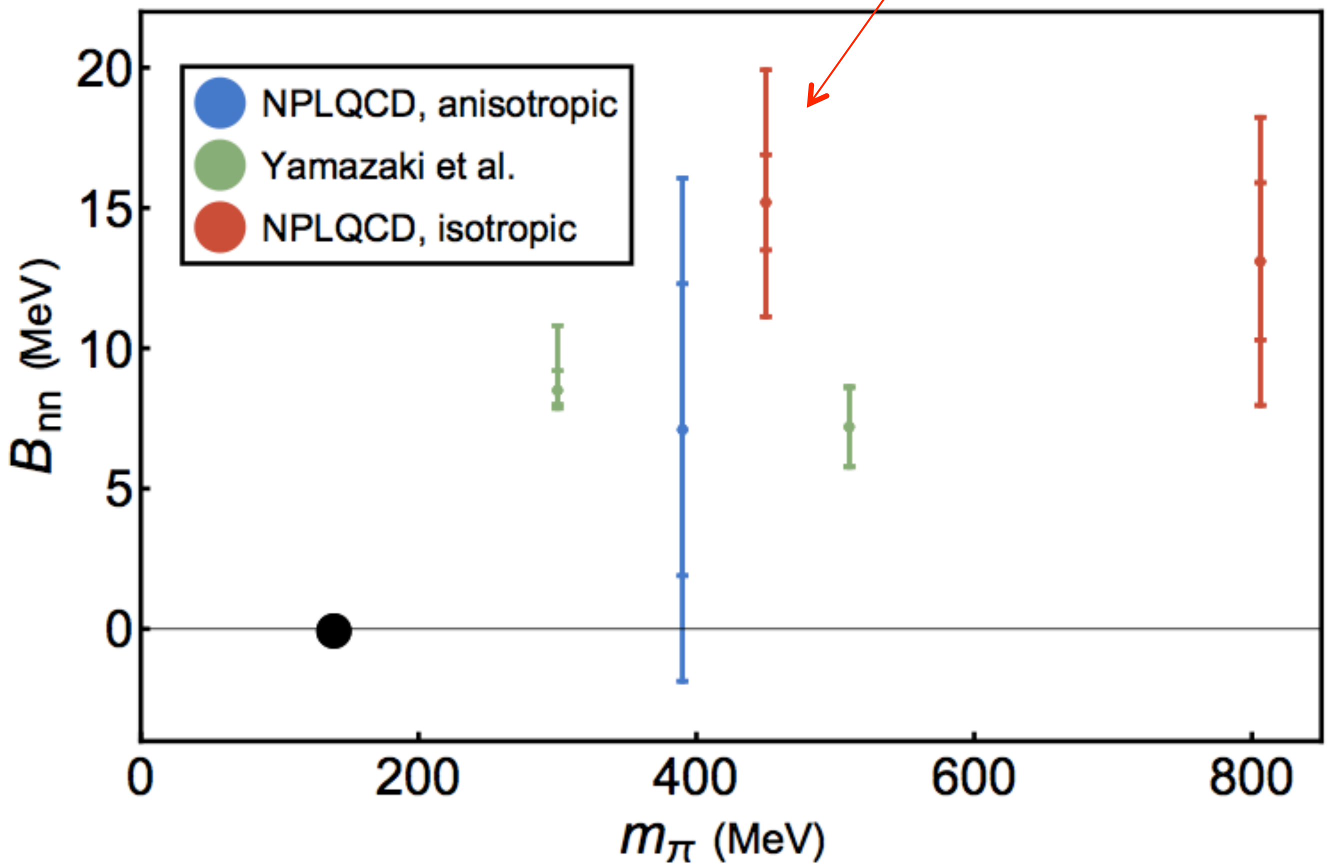


.

The nn system

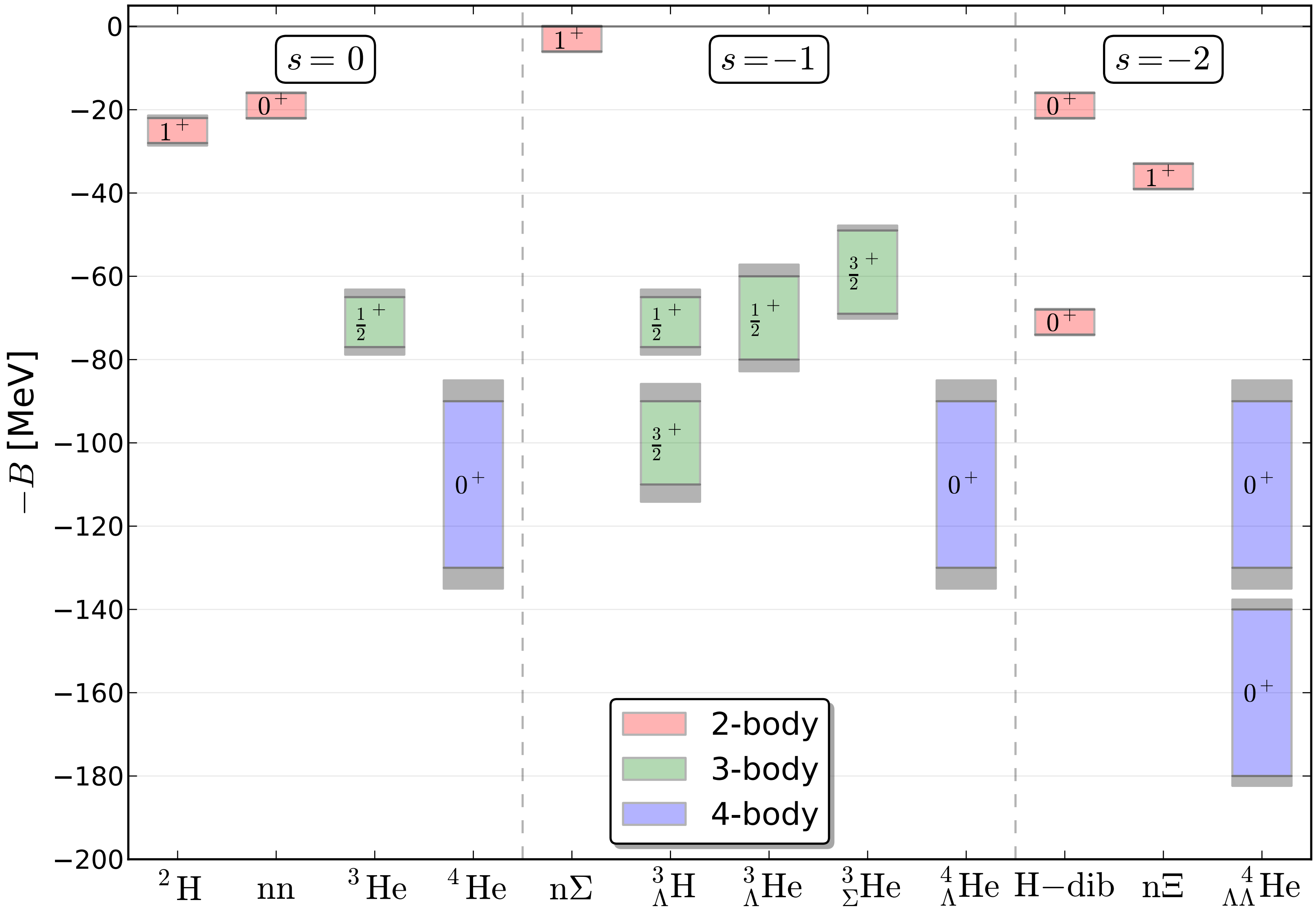
no clear pattern for the nn binding energy

(preliminary)



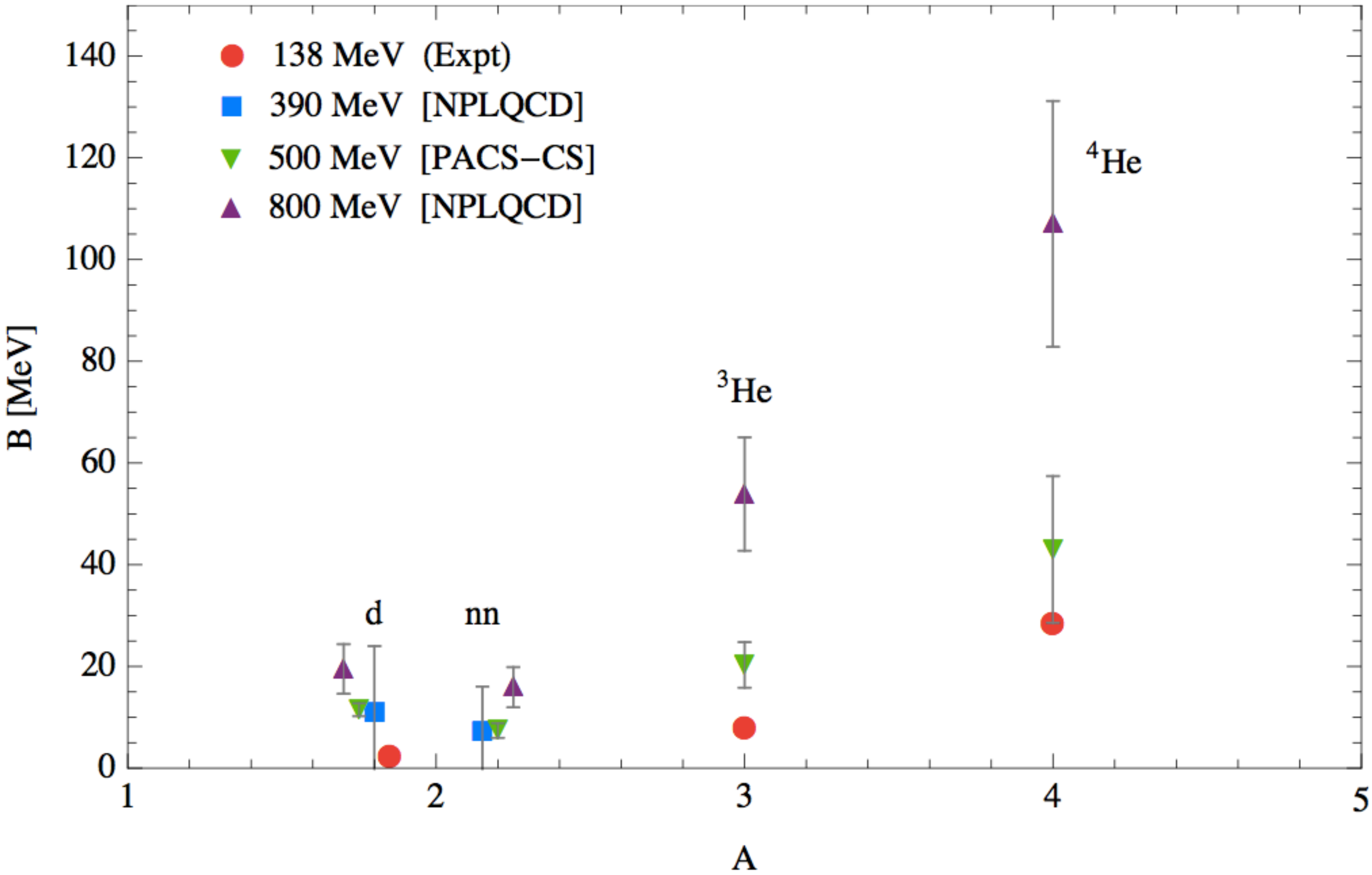
NPLQCD Phys. Rev. D87 (2013) 3, 034506

$SU(3)_f$

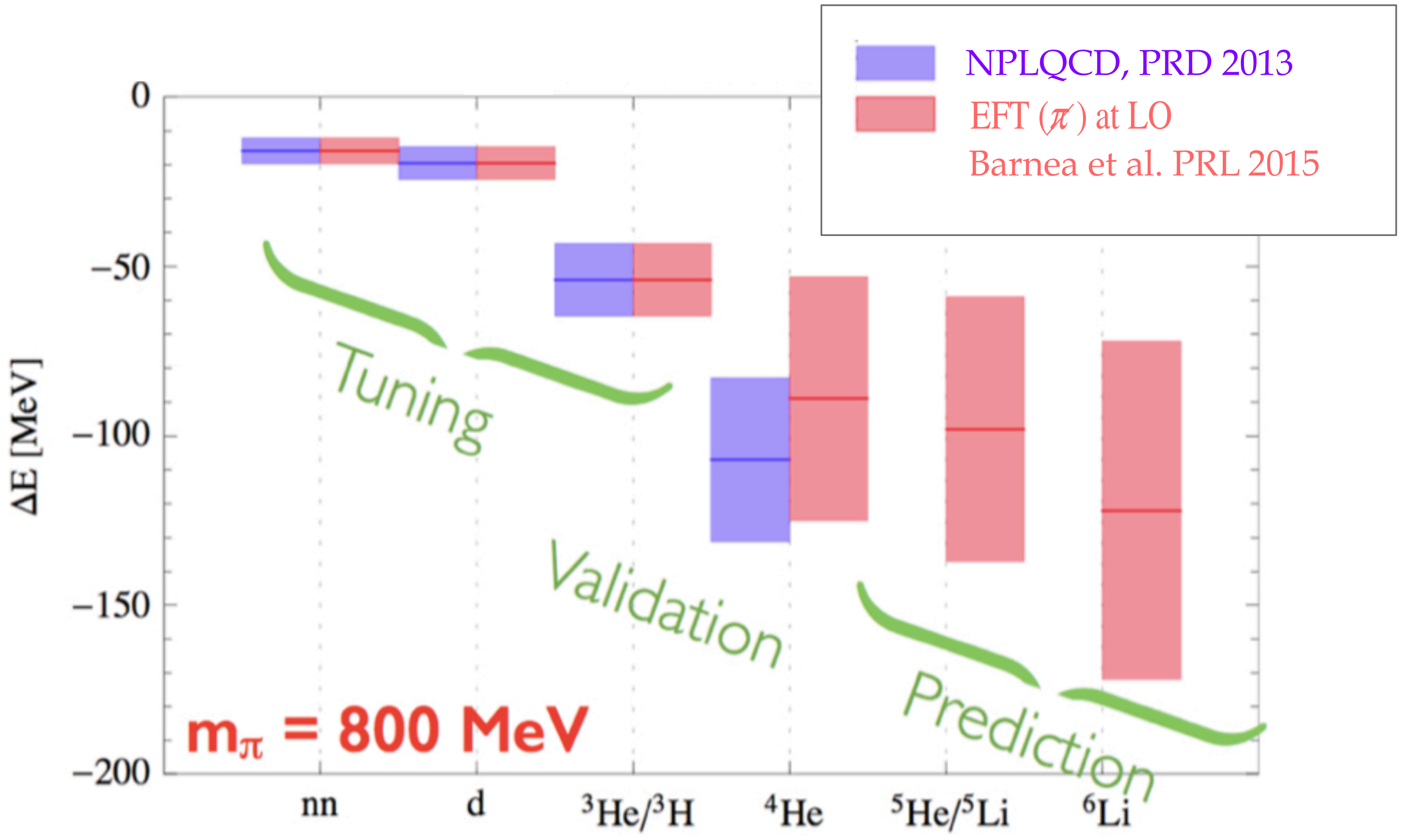


$(\pi,\ J^2, J_z, s, A, I^2, I_z)$

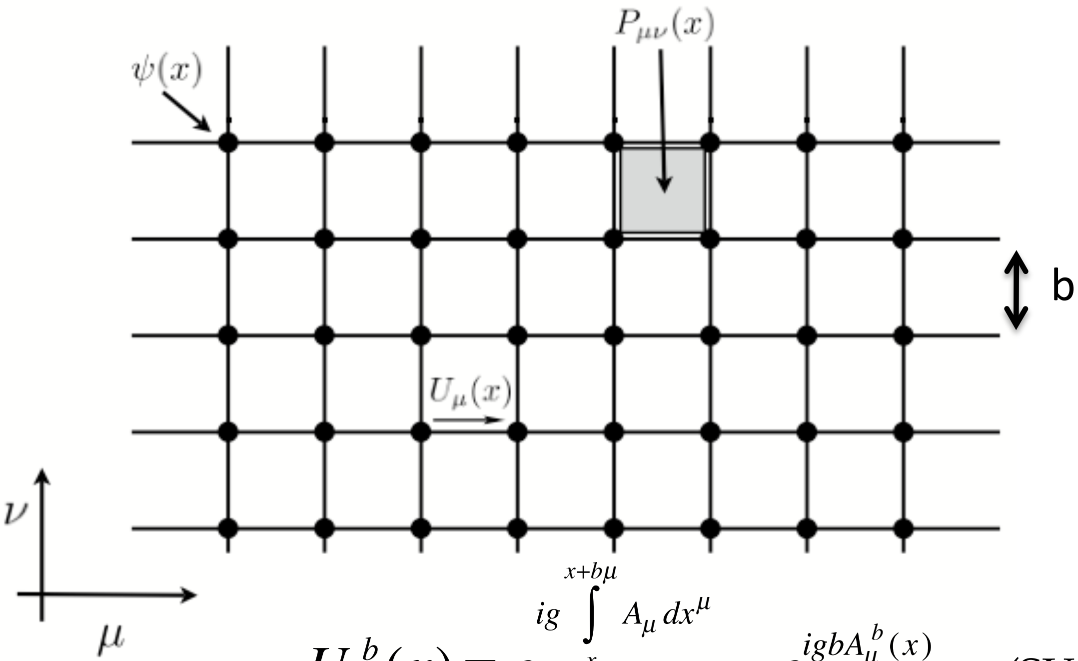
no e.m. interactions


138 MeV (Expt)
390 MeV [NPLQCD]
500 MeV [PACS–CS]
800 MeV [NPLQCD]
B [MeV]
A
d
nn
³He
⁴He
0
20
40
60
80
100
120
140
1
2
3
4
5

many body: EFT + EIHH/AFDMC



LQCD calculations of magnetic moments of light nuclei



Uniform, time-independent background magnetic field

$$e|B| = \frac{6\pi}{L^2}\tilde{n}, \quad \vec{B} = \hat{z}\cdot B$$

G. t'Hooft, 1979

$$U_\mu(x + L\hat{\nu}) = U_\mu(x)$$

$$b^2|eB| = \frac{2\pi n}{L^2}, \quad n \in Z$$

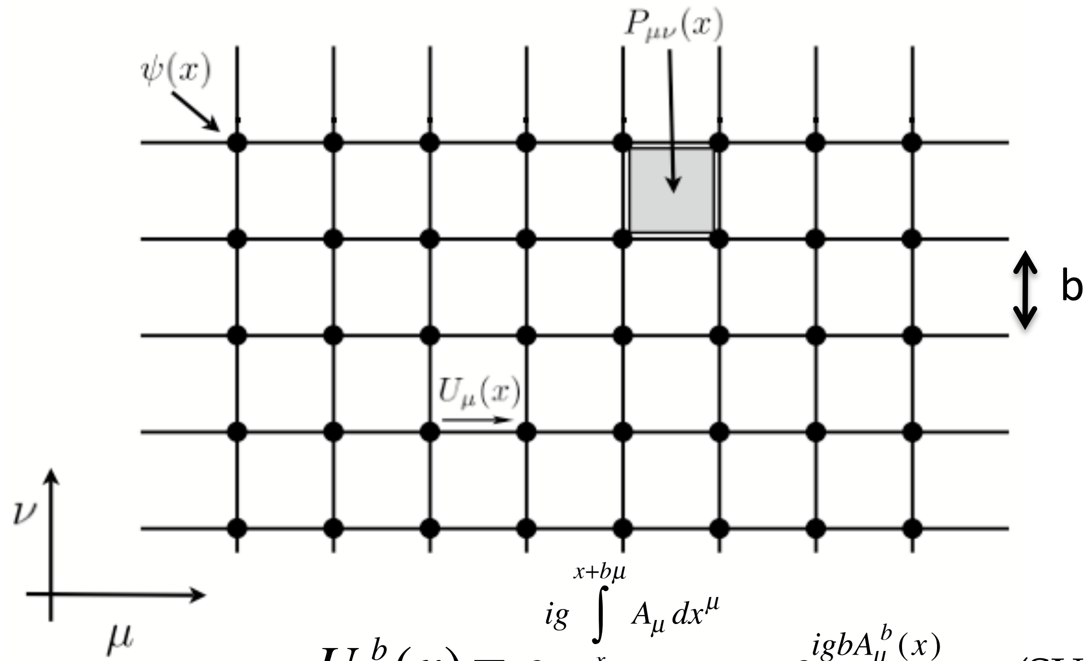
$$U_\mu^b(x) \equiv e^{ig\int_x^{x+b\mu} A_\mu dx^\mu} = e^{igbA_\mu^b(x)}$$ (SU(3)$_c$ matrices, "links")

$$U_\mu^b(x) \to U_\mu^b(x) U_\mu^{\text{ext}}(x)$$

$$U_0^{\text{ext}}(x) = U_3^{\text{ext}}(x) = 1 \qquad U_1^{\text{ext}}(x) = \begin{cases} 1 & x_1 \neq L - b \\ e^{-i\beta x_2} & x_1 = L - b \end{cases} \qquad U_2^{\text{ext}}(x) = e^{i\beta x_1}$$

$$\beta \sim q_q B b^2$$

LQCD calculations of magnetic moments of light nuclei



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$$U_\mu^b(x) \equiv e^{ig\int_x^{x+b\mu} A_\mu dx^\mu} = e^{igbA_\mu^b(x)}$$ (SU(3)$_c$ matrices, "links")

$$\boxed{E_{h;j_z}(B) = \underbrace{\sqrt{M_h^2 + P_{\|}^2 + (2n_L+1)|Q_h eB|}}_{\substack{\text{Landau levels} \\ \textit{(charged particles)}}} - {\color{red}\mu_h}\cdot B - 2\pi\, {\color{red}\beta_h^{(M0)}}\,|B|^2 - 2\pi\, {\color{red}\beta_h^{(M2)}}\left\langle \hat{T}_{ij} B_i B_j \right\rangle + \ldots}$$

LQCD calculations of magnetic moments of light nuclei $SU(3)_f$

Use background magnetic fields

$m_\pi \sim 800$ MeV

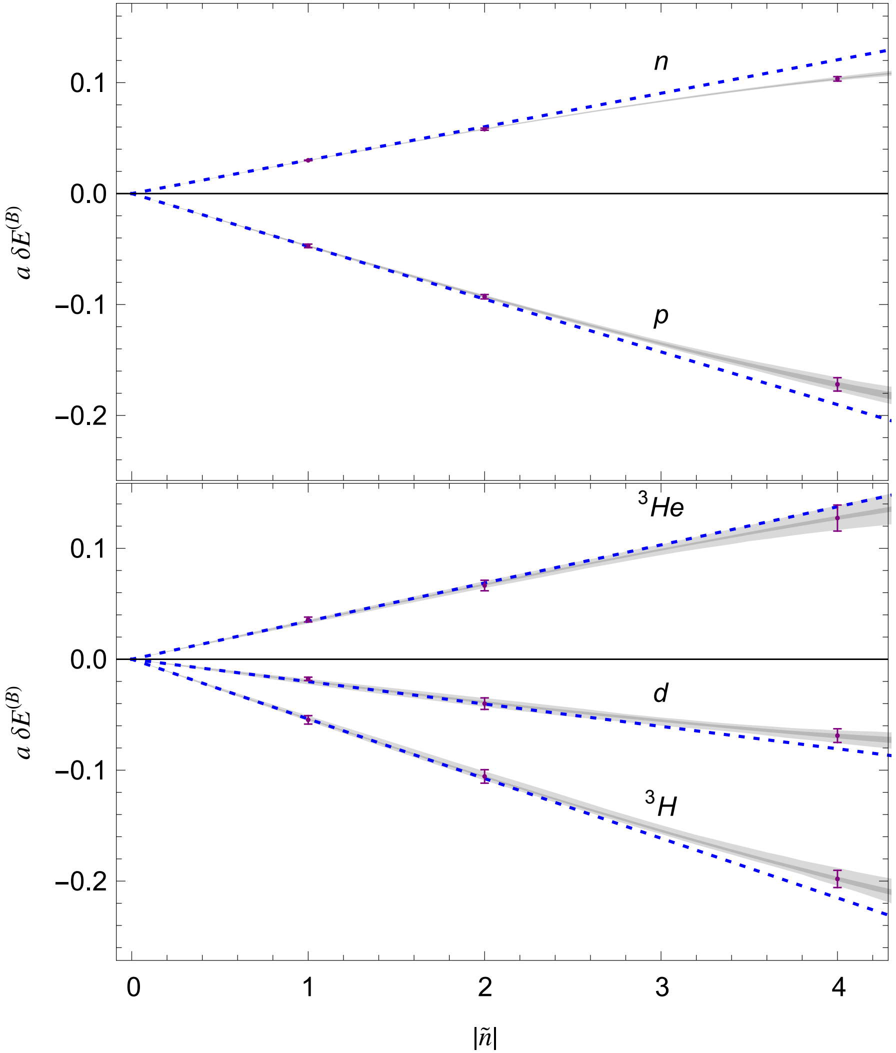
$$e|B| = \frac{6\pi}{L^2}\tilde{n}, \quad \vec{B} = \hat{z}\cdot B \quad (e|B| \sim 0.046\,\tilde{n}\ \mathrm{GeV}^2)$$

$$E(B) = M + \frac{|QeB|}{2M} - \mu\cdot B - 2\pi\beta|B|^2 + \ldots$$

$$\delta E^B = E^B_{+j} - E^B_{-j}$$

$$R(B) = \frac{C^B_{j_z}(t)\,C^0_{-j_z}(t)}{C^B_{-j_z}(t)\,C^0_{j_z}(t)} \xrightarrow{t\to\infty} Ze^{-\delta E^B t}$$

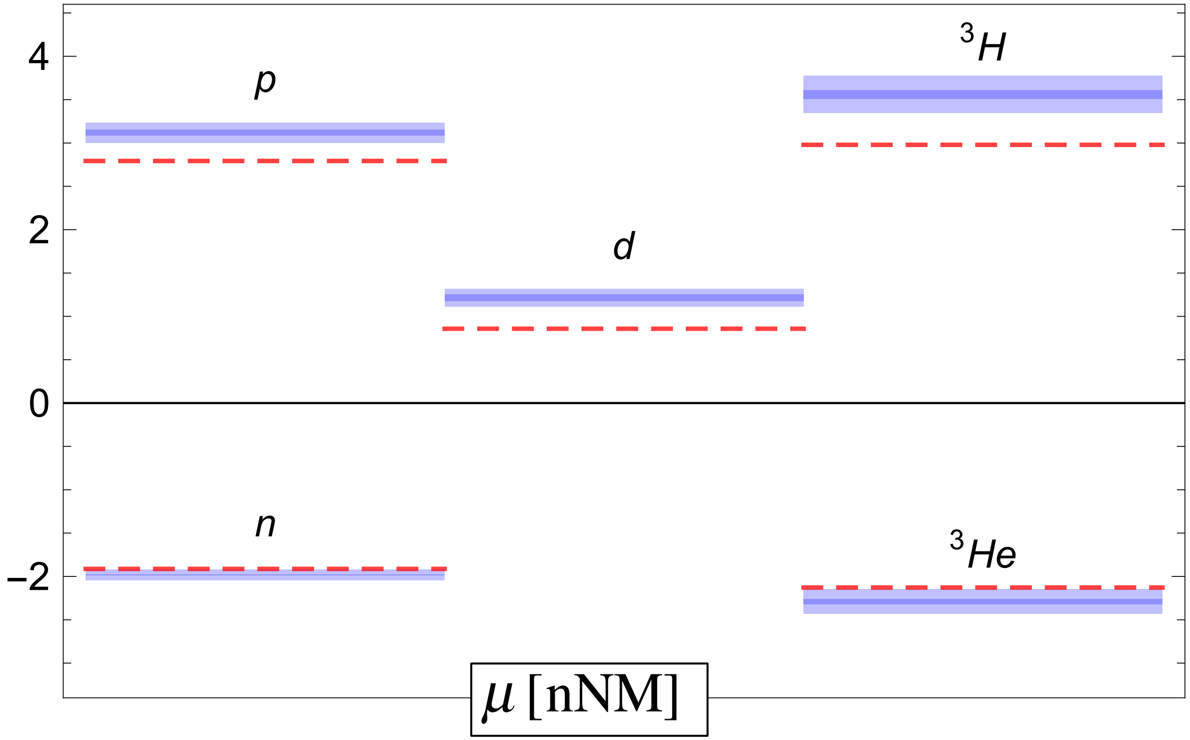
$$\delta E^B = -2\mu|B| + \gamma_3|B|^3$$



NPLQCD, Phys. Rev .Lett. 113 (2014) 25, 252001

LQCD calculations of magnetic moments of light nuclei $SU(3)_f$

NPLQCD, Phys. Rev .Lett. 113 (2014) 25, 252001



$$\mathrm{nNM} = \frac{e}{2M_N^{\mathrm{latt}}} = \frac{e}{2M_N(m_\pi^{\mathrm{latt}})}$$

LQCD @ $m_\pi \sim 800$ MeV

experiment

Shell-model predictions

$$\mu(^3H) = \mu_p$$

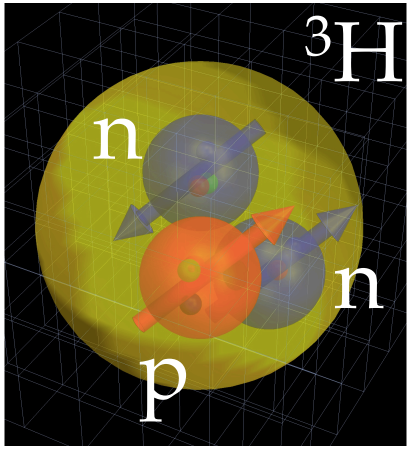
$$\mu(^3He) = \mu_n$$

$$\mu_d = \mu_n + \mu_p$$

LQCD @ $m_\pi \sim 800$ MeV

experiment

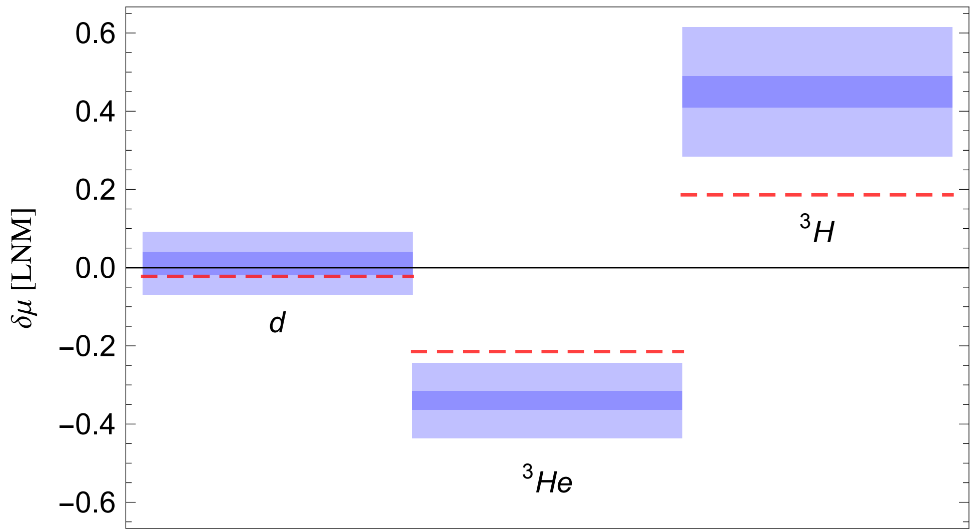
NPLQCD, Phys. Rev .Lett. 113 (2014) 25, 252001



$j_{nn} = 0$

© www.nersc.gov
Artist's impression of a triton.
Neutrons in blue and protons in red, with quarks inside; the arrows indicate the alignments of the spins.
Image: William Detmold, MIT

Differences between the shell-model predictions and the LQCD calculations for the magnetic moments



$np \to d\gamma$ cross section

e-Print: arXiv:1505.02422 [hep-lat]

Illustration of how the long-distance and short-distance contributions can be isolated with lattice QCD.

$$\sigma(np \to d\gamma) = \frac{e^2\left(\gamma_0^2 + \left|\vec{p}^2\right|\right)^3}{M^4 \gamma_0^3 \left|\vec{p}\right|} \left|\tilde{X}_{M1}\right|^2 + \ldots$$

E1, M2, …

Bethe, Longmire (1950)
Noyes (1965)

$$\tilde{X}_{M1} = \frac{Z_d}{-\frac{1}{a_1} + \frac{1}{2} r_1 \left|\vec{p}\right|^2 - \mathrm{i}\left|\vec{p}\right|} \times \left[\frac{\kappa_1 \gamma_0^2}{\gamma_0^2 + \left|\vec{p}\right|^2}\left(\gamma_0 - \frac{1}{a_1} + \frac{1}{2} r_1 \left|\vec{p}\right|^2\right) + \frac{\gamma_0^2}{2} l_1\right]$$

$$l_1 = \tilde{l}_1 - \sqrt{r_1 r_3}\,\kappa_1$$

with $\kappa_1 = \frac{\kappa_p - \kappa_n}{2}$ isovector nucleon magnetic moment

$$Z_d = \frac{1}{\sqrt{1 - \gamma_0 r_3}}$$

EFT ($\pi\!\!\!/$)
Kaplan (1997)
Kaplan, Savage, Wise (1998, 1999)
van Kolck (1999)
Beane, Savage (2001)

$np \rightarrow d\gamma$ cross section

e-Print: arXiv:1505.02422 [hep-lat]

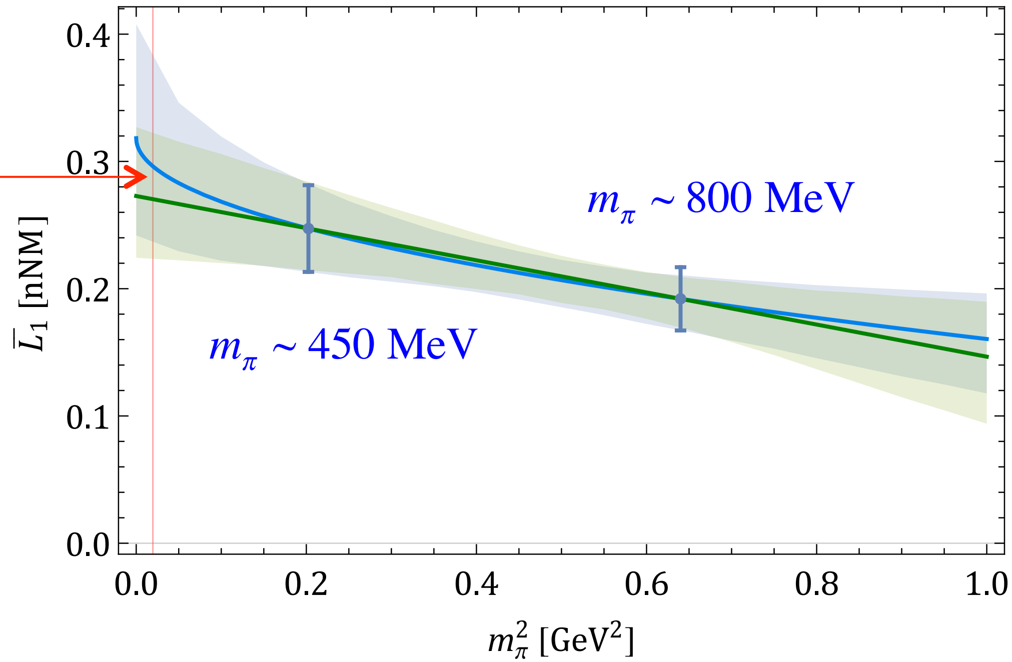
$\bar{L}_1^{lqcd} = 0.285^{+63}_{-60}$ nNM

$l_1^{lqcd} = -4.48^{+16}_{-15}$ fm

$\sigma^{lqcd} = 332.4^{+5.4}_{-4.7}$ mb

$(\sigma^{\exp} = 334.2 \pm 0.5$ mb)

($v_{\text{neutron}} = 2200$ m/s)



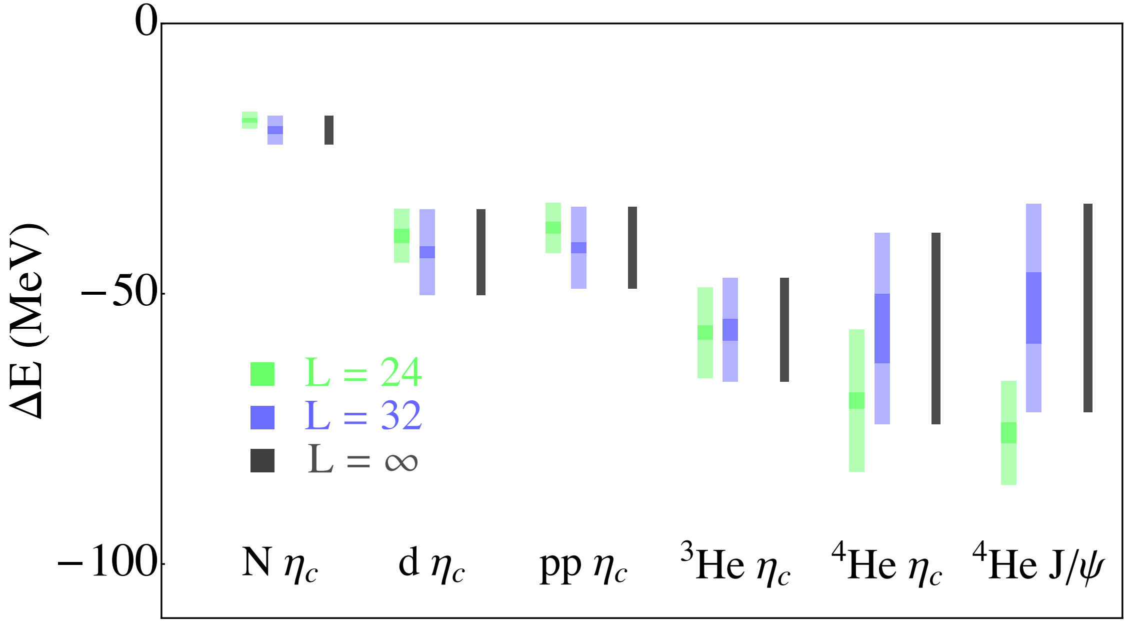
$V = 32^3 \times 96$, b ~ 0.12 fm

$$\Delta E_{^3S_1, ^1S_0}(\vec{B}) = \mp Z_d^2 (\kappa_1 + \gamma_0\, l_1) \frac{|e\vec{B}|}{M} + O\left(|\vec{B}|^2\right) = \mp (\kappa_1 + \bar{L}_1) \frac{|e\vec{B}|}{M}$$

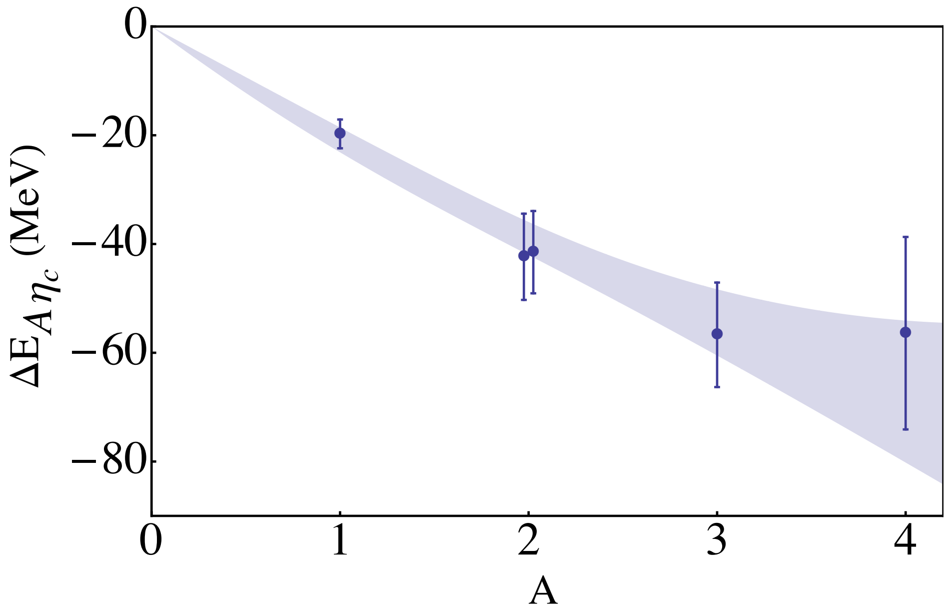
with $\quad \bar{L}_1 = \gamma_0 Z_d^2 (l_1 + r\kappa_1)$

$$\delta E_{^3S_1, ^1S_0}(\vec{B}) \equiv \Delta E_{^3S_1, ^1S_0}(\vec{B}) - \left[E_{p,\uparrow} - E_{p,\downarrow}\right] + \left[E_{n,\uparrow} - E_{n,\downarrow}\right] \rightarrow 2\bar{L}_1 \frac{|e\vec{B}|}{M} + O\left(|\vec{B}|^2\right)$$

A magnetic field mixes the $I_z = j_z = 0$ np states in the $^1S_0$ and $^3S_1 - {}^3D_1$


ΔE (MeV)
0
−50
−100
L = 24
L = 32
L = ∞
N η_c
d η_c
pp η_c
^{3}He η_c
^{4}He η_c
^{4}He J/ψ

$SU(3)_f$


$\Delta E_{A\eta_c}$ (MeV)
0
−20
−40
−60
−80
0
1
2
3
4
A

NPLQCD, summary

Calculation of important quantities in nuclear physics with LQCD is only now becoming practical, with first calculations of simple multibaryon interactions being performed, although at unphysical quark masses.

Present day computing power begins to be sufficient to calculate nuclear properties in lattice QCD with near physical quark masses (completed calculations @ 430 MeV & on-going calcs. @ 300 MeV)

A chiral extrapolation to the physical pion mass may be possible if sufficiently many results at various pion masses are available.

Also, calculations at different lattice spacings are now at reach (b ~ 0.1, 0.08 fm)

LQCD calculations would be specially of interest for systems that are not accessible experimentally → Complementary information to experimental programs not too far in the future.

"This is a long term project that can be done given sufficient resources. Once completed will definitely be transformative although each individual step may not be."
(encouraging referee's comment)

Thanks to the NPLQCD Collaboration

Silas R. Beane & Emmanuel Chang (U of Washington, Seattle)
William Detmold (MIT), Kostas Orginos (William and Mary & JLab),
Assumpta Parreño (Barcelona), Martin J. Savage (INT, Seattle)

and

Saul Cohen, Huey-Wen Lin, Thomas C. Luu, Parikshit Junnarkar,
Brian Tiburzi, André Walker-Loud