Results from HALQCD on light nuclei and exotic states

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for HAL QCD Collaboration

HAL (Hadrons to Atomic nuclei from Lattice) QCD Collaboration

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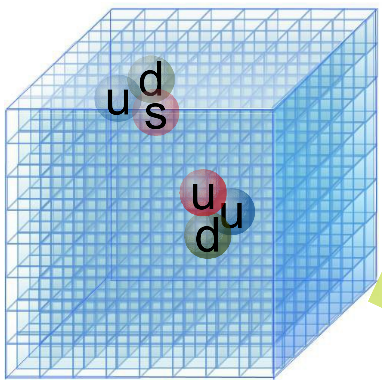
Introduction

Derivation of hadronic interaction from QCD

Start with the fundamental theory,QCD, to obtain a “proper” interaction

Lattice QCD simulation



Lüscher's finite volume method

M. Lüscher, NPB354(1991)531

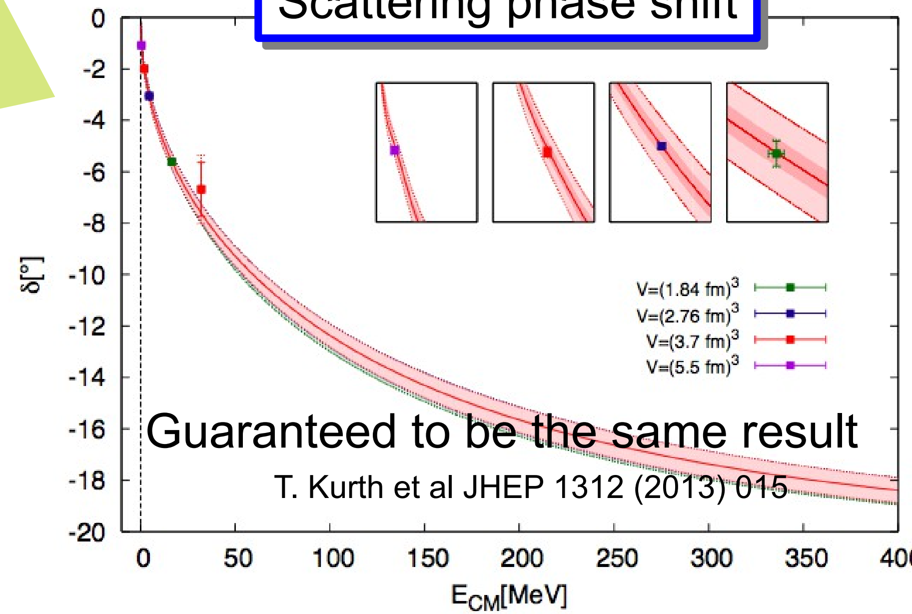
1. Measure the discrete energy spectrum, **E**
2. Put the E into the formula which connects E and **δ**

HAL QCD method

Ishii, Aoki, Hatsuda, PRL**99** (2007) 022001

1. Measure the NBS wave function, **Ψ**
2. Calculate potential, **V**, through Schrödinger eq.
3. Calculate observables by scattering theory

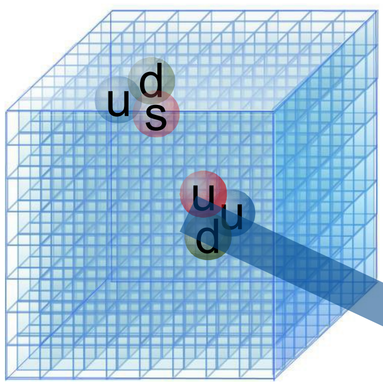
Scattering phase shift



Guaranteed to be the same result

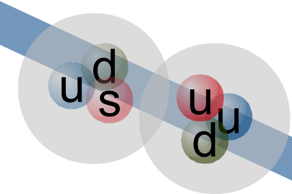
T. Kurth et al JHEP 1312 (2013) 015

Strategy of HAL QCD



Technical improvements

- **Unified Contraction Algorithm,**
- **Time dependent method,**
- **Higher partial waves,**
- **Finite volume energy vs potential**

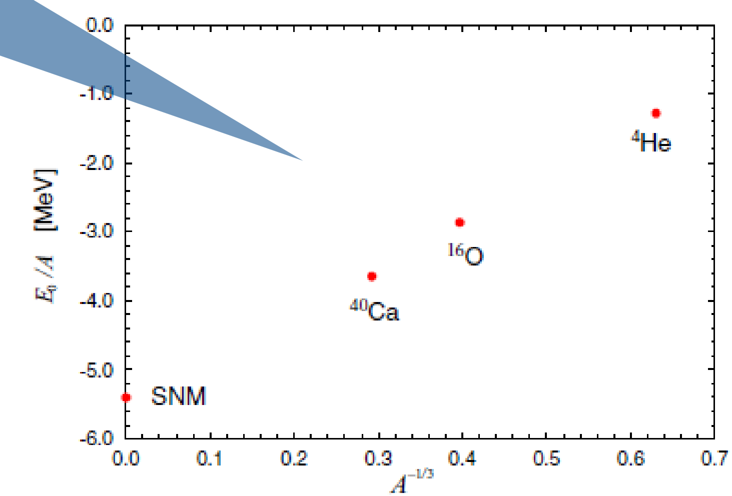


Application

- **Few-body system**
- **Medium heavy system**
- **Neutron star EOS**

Extensions of the method

- **Generalized BB interaction**
- **N–Ω, Ω–Ω interaction**
- **Generalized Dec-Dec interaction**
- **Charmed baryon system**
- **Meson-meson,meson-baryon system**
- **Three-body interaction**



HAL QCD method

HAL QCD method

Definition : equal time NBS w.f.

$$\Psi(E,\vec{r})e^{-E(t-t_0)} = \sum_{\vec{x}} \langle 0|B_i(t,\vec{x}+\vec{r})B_j(t,\vec{x})|E,t_0\rangle$$

E : Total energy of system

- In asymptotic region : $(p^2+\nabla^2)\Psi(E,\vec{r})=0$
- In interaction region : $(p^2+\nabla^2)\Psi(E,\vec{r})=K(E,\vec{r})$

$$\Psi(E,\vec{r}) \simeq A\frac{\sin(pr+\delta(E))}{pr}$$

Aoki, Hatsuda, Ishii, PTP123, 89 (2010).

Modified Schrödinger equation

$$\left(-\frac{\partial}{\partial t}+\frac{\nabla^2}{2\mu}\right)R_I^{B_1B_2}(t,\vec{r}) = \int U(\vec{r},\vec{r}\,')R_I^{B_1B_2}(t,\vec{r})d^3r'$$

$$R_I^{B_1B_2}(t,\vec{r}) = F_{B_1B_2}(\vec{r},t)e^{(m_1+m_2)t}$$

N. Ishii et al Phys. Lett. B**712**(2012)437

Derivative expansion

$$U(\vec{r},\vec{r}\,') = V_C(r)+S_{12}V_T(r)+\vec{L}\cdot\vec{S}_s V_{LS}(r)+\vec{L}\cdot\vec{S}_a V_{ALS}(r)+O(\nabla^2)$$

K. Murano et al Phys.Lett. B735 (2014) 19

Coupled channel Schrödinger equation.

$$\left(\frac{p_\alpha^2}{2\mu_\alpha}+\frac{\nabla^2}{2\mu_\alpha}\right)\underline{\Psi^\alpha}(\vec{r},E) = V^\alpha{}_\alpha(\vec{r})\underline{\Psi^\alpha}(\vec{r},E)+V^\alpha{}_\beta(\vec{r})\underline{\Psi^\beta}(\vec{r},E)$$

S.Aoki [HAL QCD collab.] Proc. Jpn. Acad., Ser. B, **87** 509

Exotica from LQCD

Exotic candidates

Exotic hadrons (multi-hadrons) which we have tackled.

- **H-dibaryon**
  - **R.L.Jaffe PRL38(1977)**
- **N-Ω system**
  - **F.Wang et al. PRC51(1995)**
  - **Q.B.Li, P.N.Shen, EPJA8(2000)**
- **ΔΔ and ΩΩ system**
  - **F.J.Dyson,N-H.Xuong, PRL13(1964)**

- **Tcc**
  - **H.J.Lipkin PLB172(1986)**
  - **S.Zouzou et al. Z.Phys.C30(1986)**
- **Zc(3900)**
  - **M. B. Voloshin, PRD.87.091501**

- **Predicted B.E. and structures are highly depend on the model parameters.**
- **Not yet found in experiments**

Lattice QCD study of hadron interactions is awaited.

H-dibaryon channel

Interests of S=-2 multi-baryon system

H-dibaryon

- The flavor singlet state with J=0 predicted by R.L. Jaffe.
  - Strongly attractive color magnetic interaction.
  - No quark Pauli principle for flavor singlet state.

Double-Λ hypernucleus

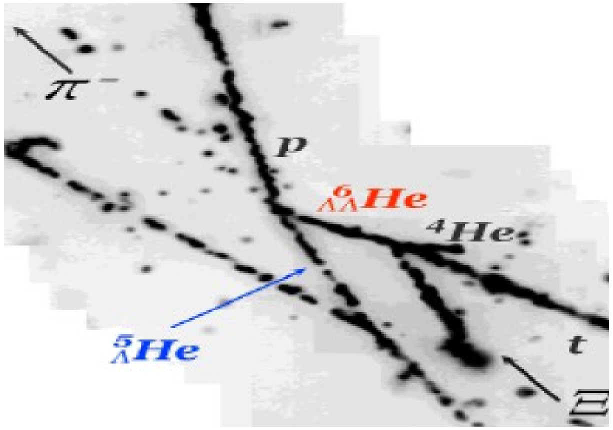
- Conclusions of the "NAGARA Event"

K.Nakazawa and KEK-E176 & E373 Collaborators

Λ–N attraction

Λ–Λ weak attraction

$m_H \geq 2m_\Lambda$ - 6.9MeV

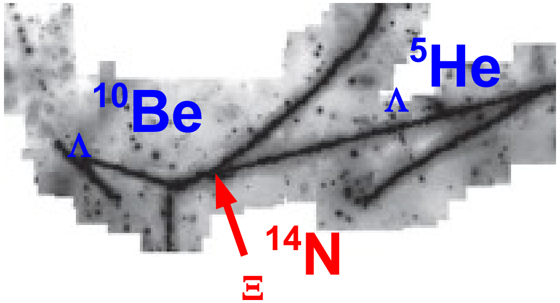


Ξ hypernucleus

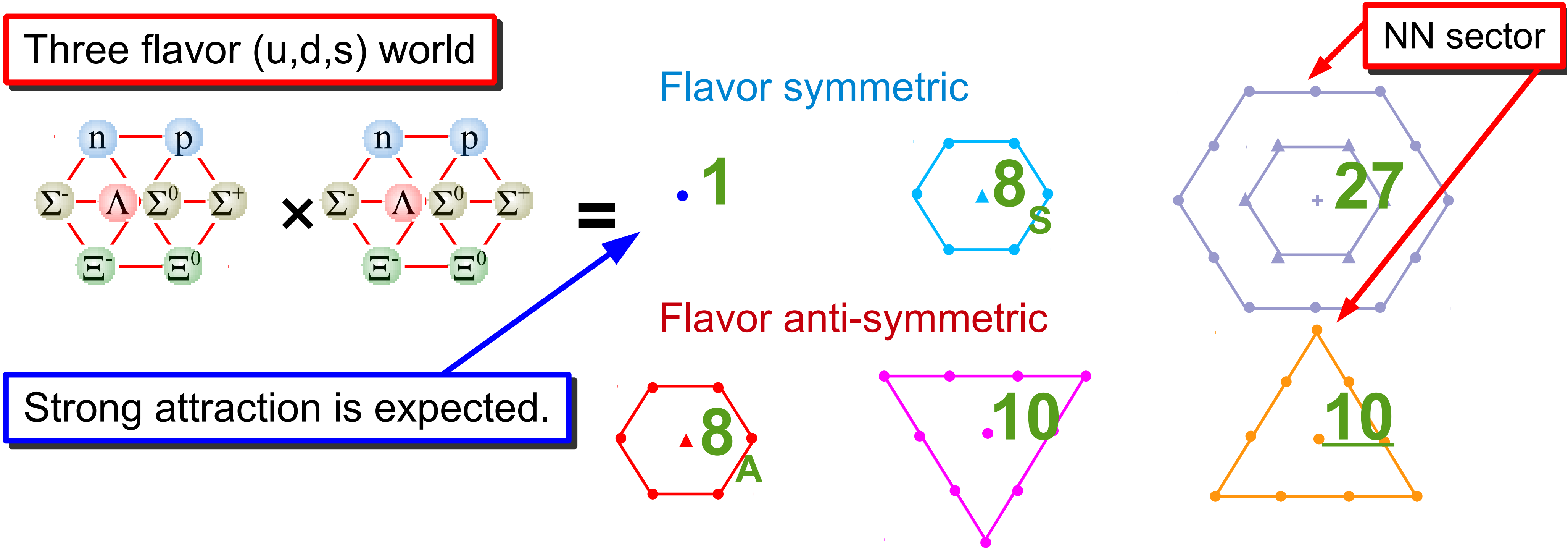
- Conclusions of the "KISO Event"

K.Nakazawa and KEK-E373 Collaborators

Ξ–N attraction



SU(3) feature of BB interaction



In view of quark degrees of freedom

Oka, Shimizu and Yazaki NPA464 (1987)

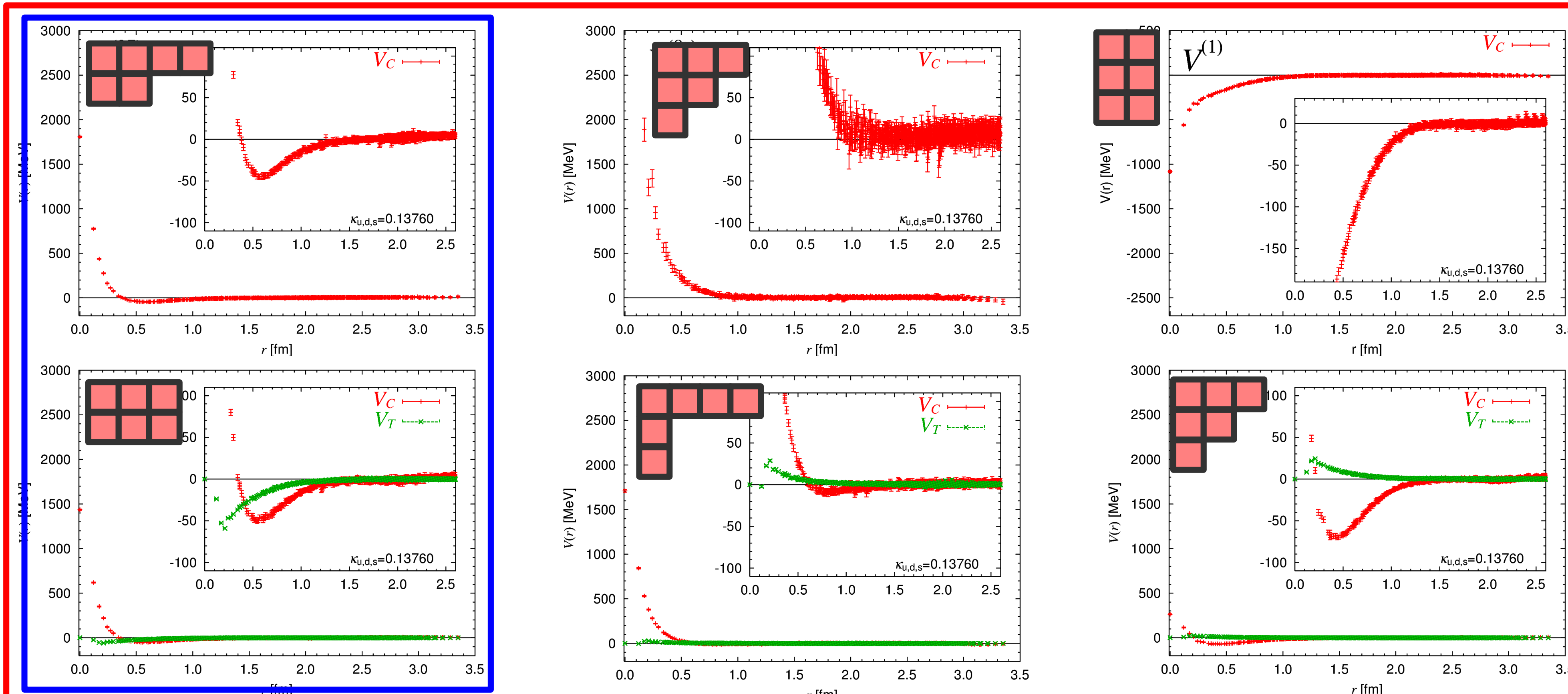
- Short range repulsion in BB interaction could be a result of Pauli principle and color-magnetic interaction for the quarks.
- Strengths of repulsive core in YN and YY interaction are largely depend on their flavor structures.
- For the s-wave BB system, no repulsive core is predicted in flavor singlet state which is known as H-dibaryon channel.

B-B potentials in SU(3) limit

$m_\pi = 469 MeV$

$^1S_0$

$^3S_1$--$^3D_1$

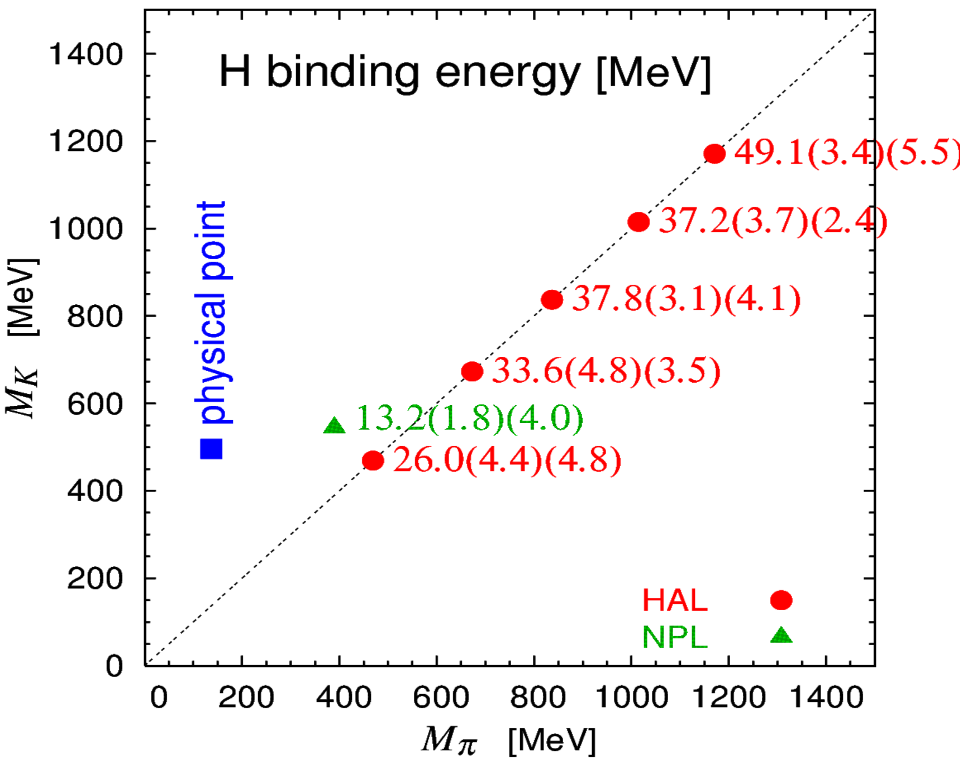


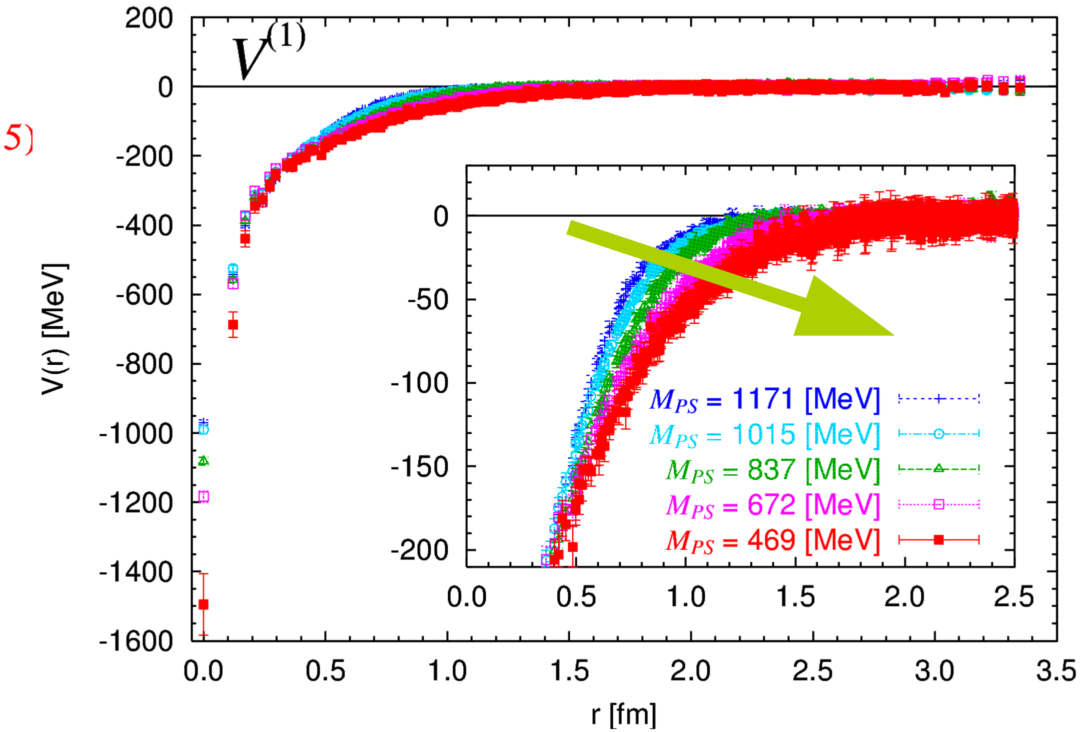
Two-flavors

Three-flavors

- Quark Pauli principle can be seen at around short distances
  - No repulsive core in flavor singlet state
  - Strongest repulsion in flavor 8s state
- Possibility of bound H-dibaryon.

H-dibaryon (unphysical situation)





HAL : PRL106(2011)162002
NPL : PRL106(2011)162001

- Both results shows the bound H-dibaryon state in heavy pion region.
- Potential in flavor singlet channel is getting more attractive as decreasing quark masses

Does the H-dibaryon state survive on the physical point?

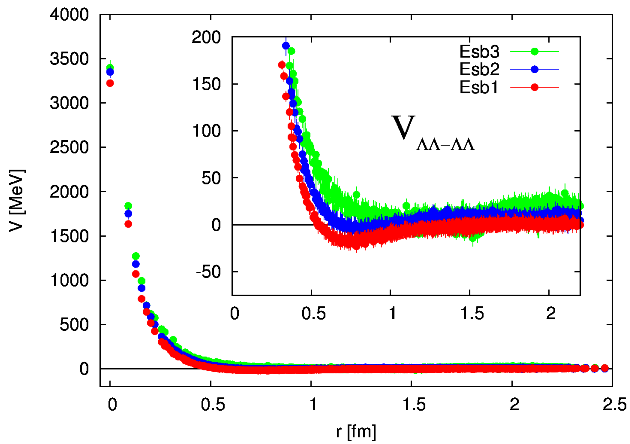
→ **Go to the SU(3) broken situation.**

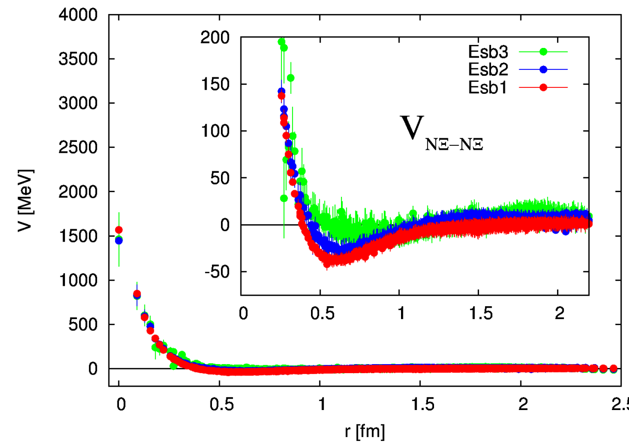
$\Lambda\Lambda$, $N\Xi$, $\Sigma\Sigma$ (I=0) $^1S_0$ channel

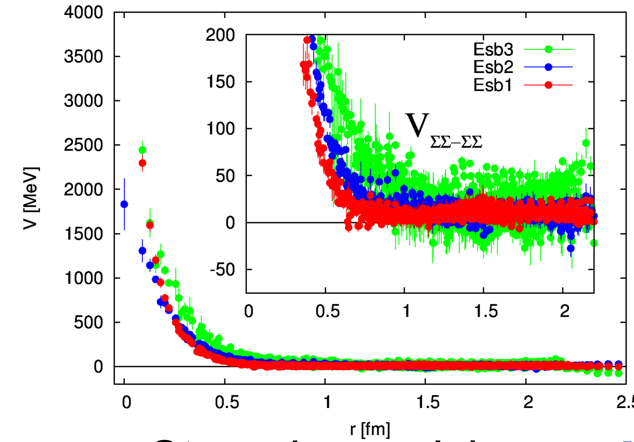
Esb1 : m_π= 701 MeV
Esb2 : m_π= 570 MeV
Esb3 : m_π= 411 MeV

- **N_f = 2+1 full QCD with L = 2.9fm**

Diagonal elements



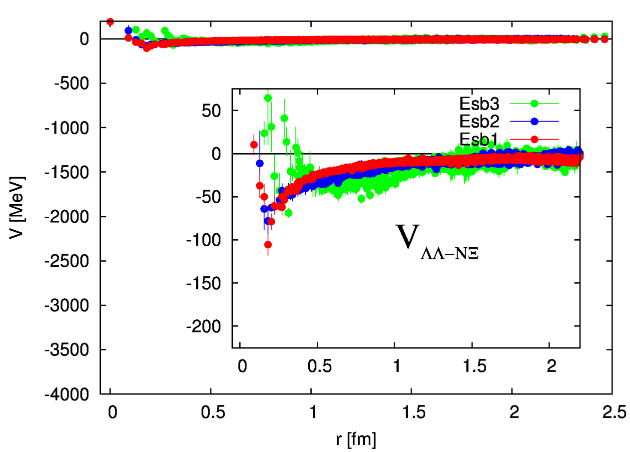


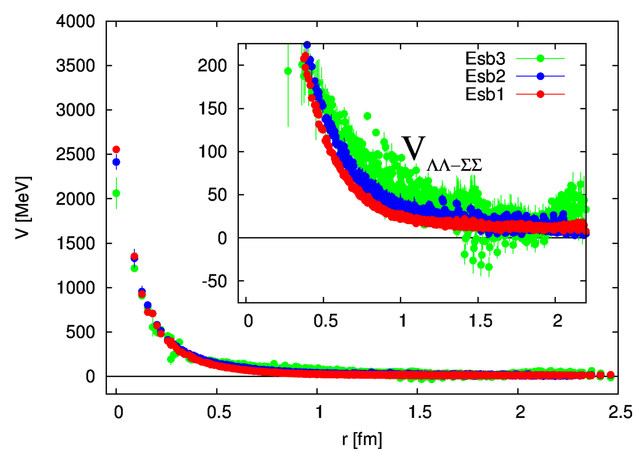


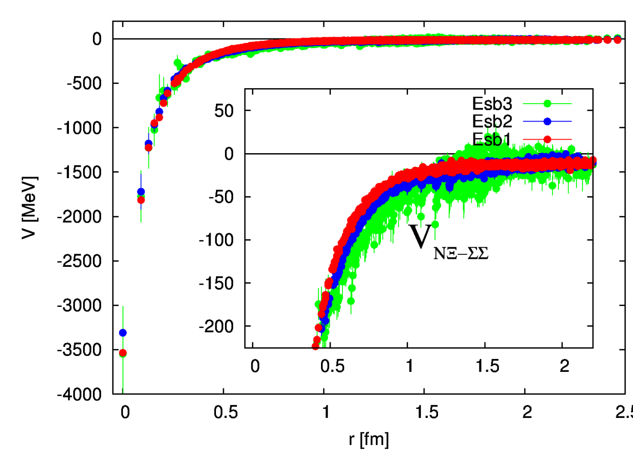
shallow attractive pocket — Deeper attractive pocket — Strongly repulsive

All channels have repulsive core

Off-diagonal elements





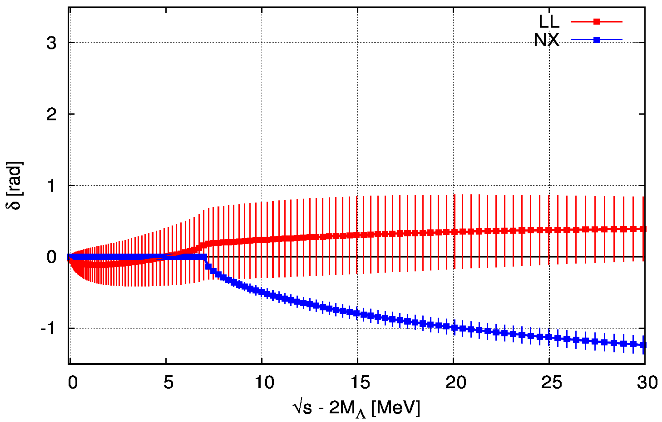


ΛΛ and NΞ phase shifts

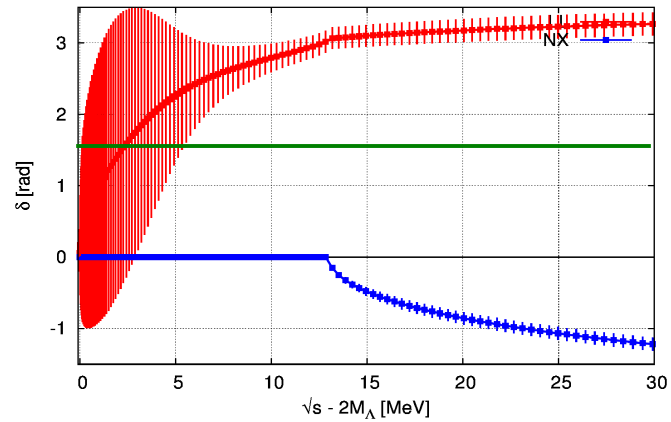
▶ N_f = 2+1 full QCD with L = 2.9fm

Preliminary!

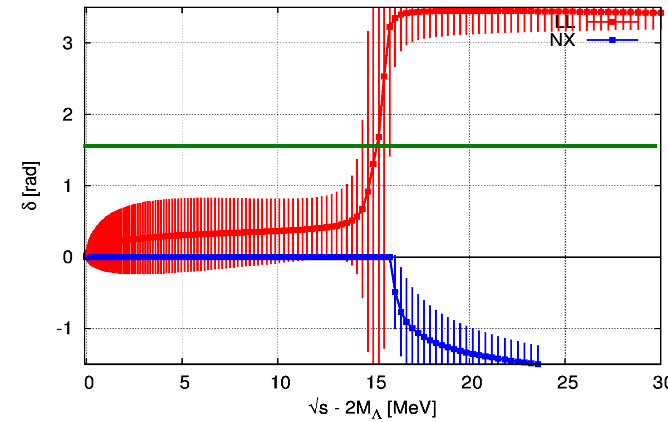
mπ = 700 MeV



mπ = 570 MeV

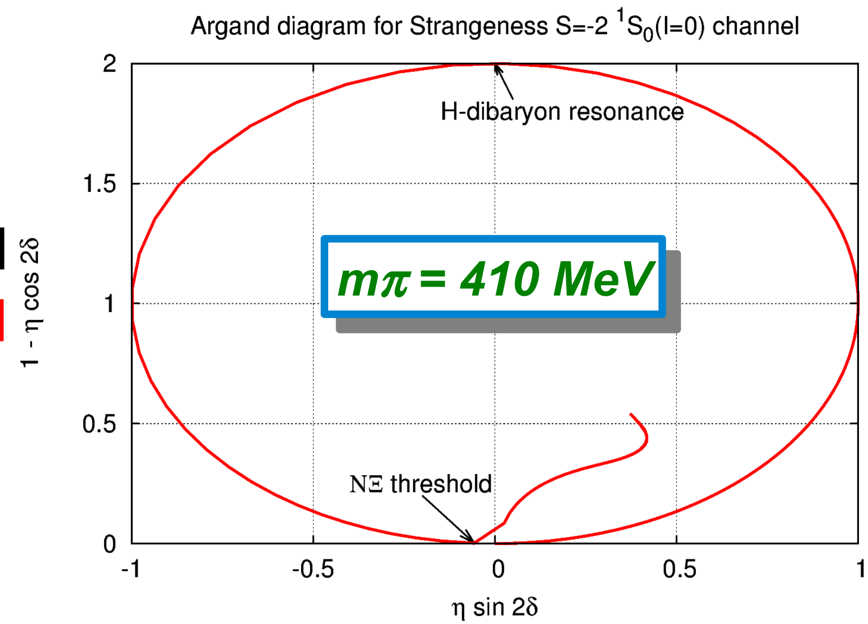


mπ = 410 MeV



- ***mπ = 700 MeV :*** bound state
- ***mπ = 570 MeV*** : resonance near ΛΛ threshold
- ***mπ = 410 MeV*** : resonance near NΞ threshold

H-dibaryon is unlikely bound state

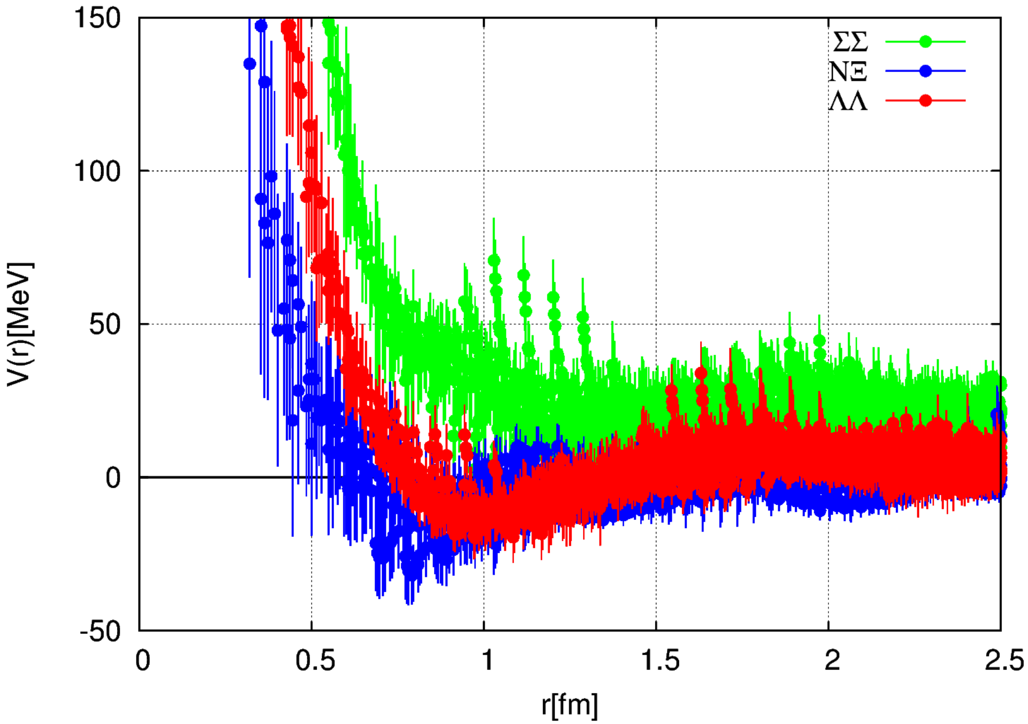


$\Lambda\Lambda$, $N\Xi$, $\Sigma\Sigma$ (I=0) $^1S_0$ channel near the physical point

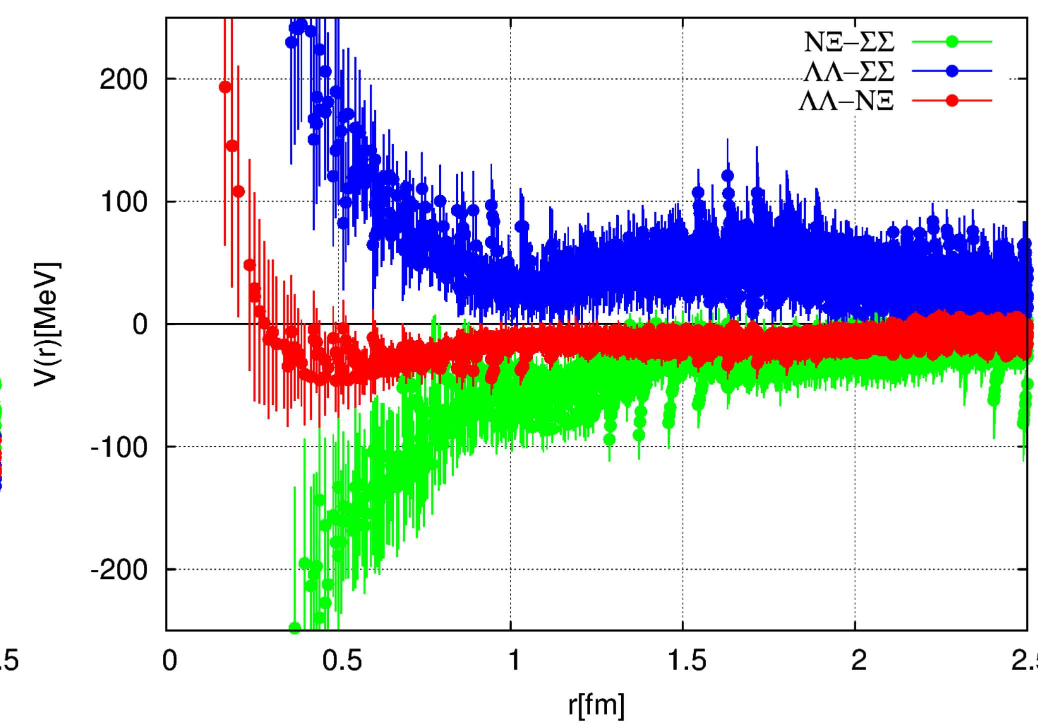
- N_f = 2+1 full QCD with L = 8fm, ***m_π = 145 MeV***

Preliminary!

Diagonal elements



Off-diagonal elements



- All diagonal element have a repulsive core ΣΣ–ΣΣ potential is strongly repulsive.
- Off-diagonal potentials are relatively strong except for ΛΛ–NΞ transition
- We need more statistics to discuss physical observables through this potential.

Comparison of potential matrices

Transformation of potentials
from the particle basis to the SU(3) irreducible representation (irrep) basis.

SU(3) Clebsh-Gordan coefficients

$$\begin{pmatrix} |1\rangle \\ |8\rangle \\ |27\rangle \end{pmatrix} = U \begin{pmatrix} |\Lambda\Lambda\rangle \\ |N\Xi\rangle \\ |\Sigma\Sigma\rangle \end{pmatrix}, \quad U \begin{pmatrix} V^{\Lambda\Lambda} & V^{\Lambda\Lambda}_{N\Xi} & V^{\Lambda\Lambda}_{\Sigma\Sigma} \\ V^{N\Xi}_{\Lambda\Lambda} & V^{N\Xi} & V^{N\Xi}_{\Sigma\Sigma} \\ V^{\Sigma\Sigma}_{\Lambda\Lambda} & V^{\Sigma\Sigma}_{N\Xi} & V^{\Sigma\Sigma} \end{pmatrix} U^t \rightarrow \begin{pmatrix} V_1 & & \\ & V_8 & \\ & & V_{27} \end{pmatrix}$$

In the SU(3) irreducible representation basis,
the potential matrix should be diagonal in the SU(3) symmetric configuration.

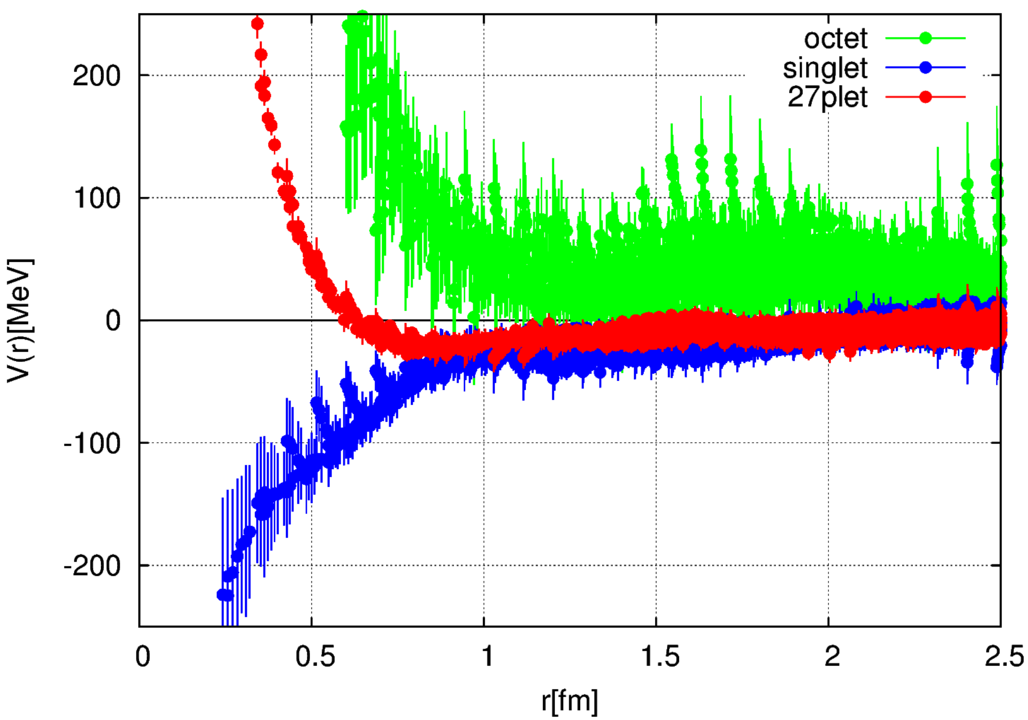
Off-diagonal part of the potential matrix in the SU(3) irrep basis
would be an effectual measure of the SU(3) breaking effect.

Potentials in $^1S_0$ channel with SU(3) basis

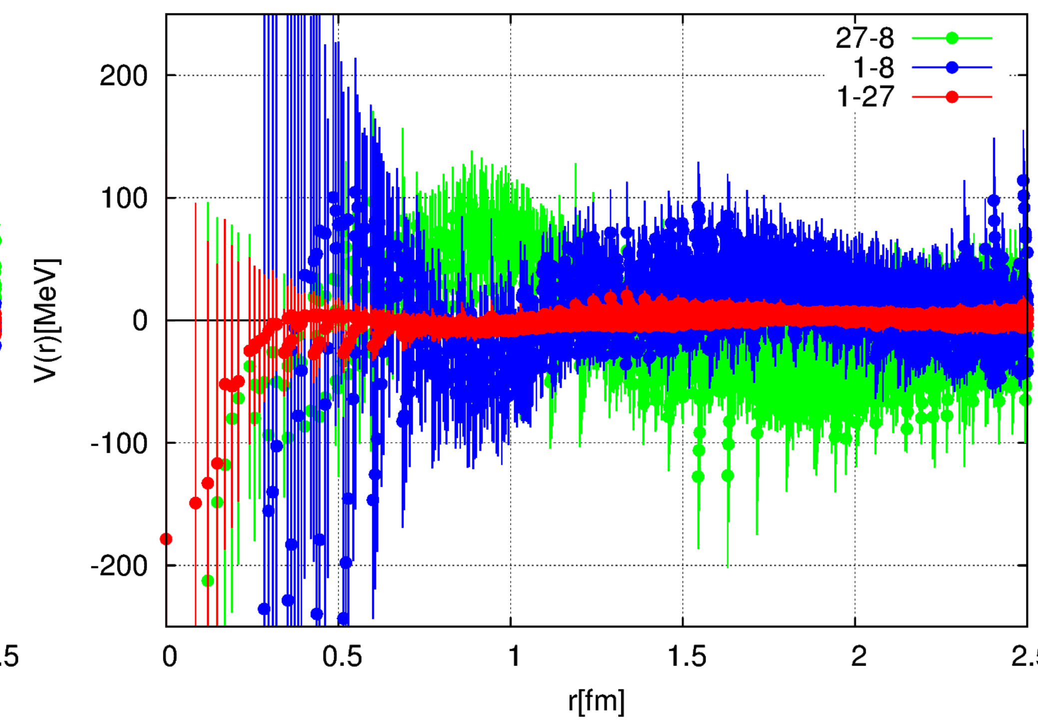
- N_f = 2+1 full QCD with L = 8fm, **m_π = 145 MeV**

Preliminary!

Diagonal elements



Off-diagonal elements

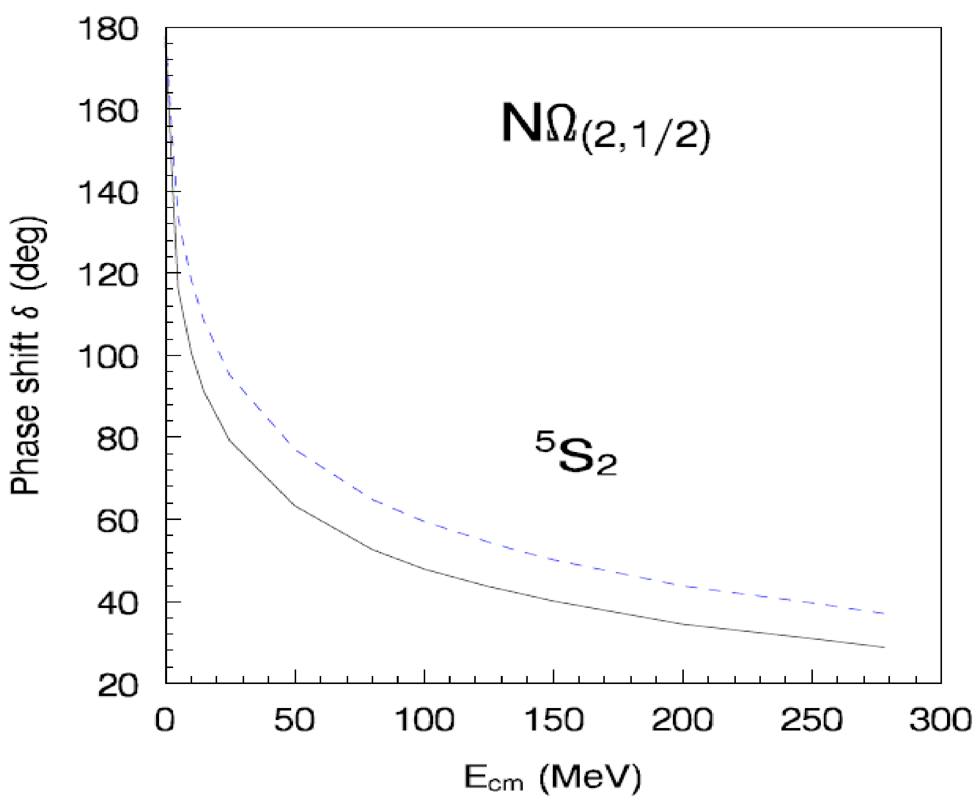


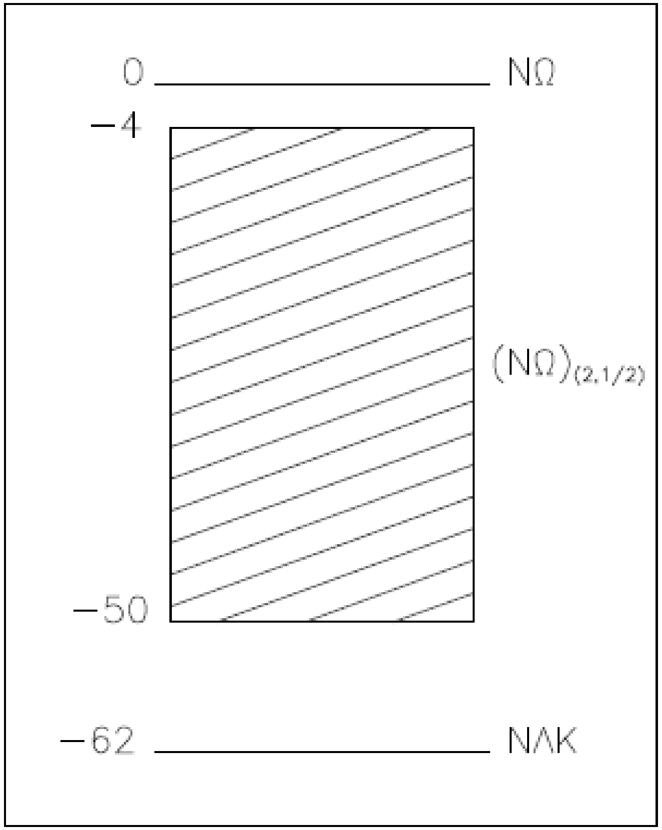
- Potential of flavor singlet channel does not have a repulsive core
- Potential of flavor octet channel is strongly repulsive which reflect a Pauli effect.
- Off-diagonal potentials are visible only in r<1fm region.

$N\Omega$ state

NΩ system from chiral quark model

- **One of di-baryon candidate**
- **Bound state is reported with S=2, I=1/2**
  - **Binding energy is highly depend on model parameter**



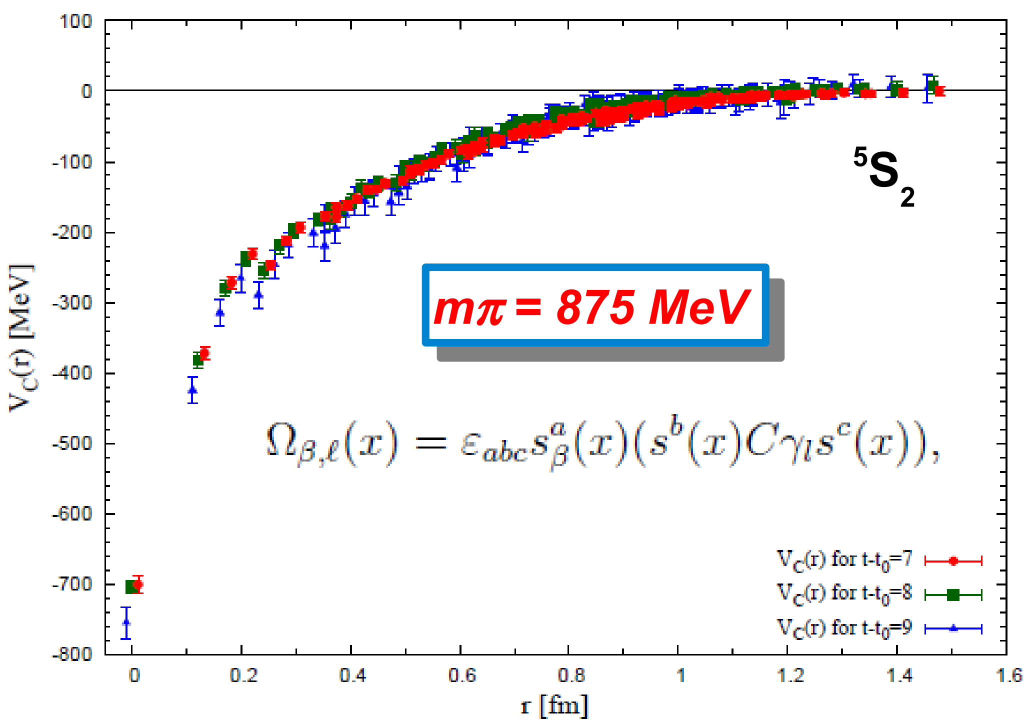


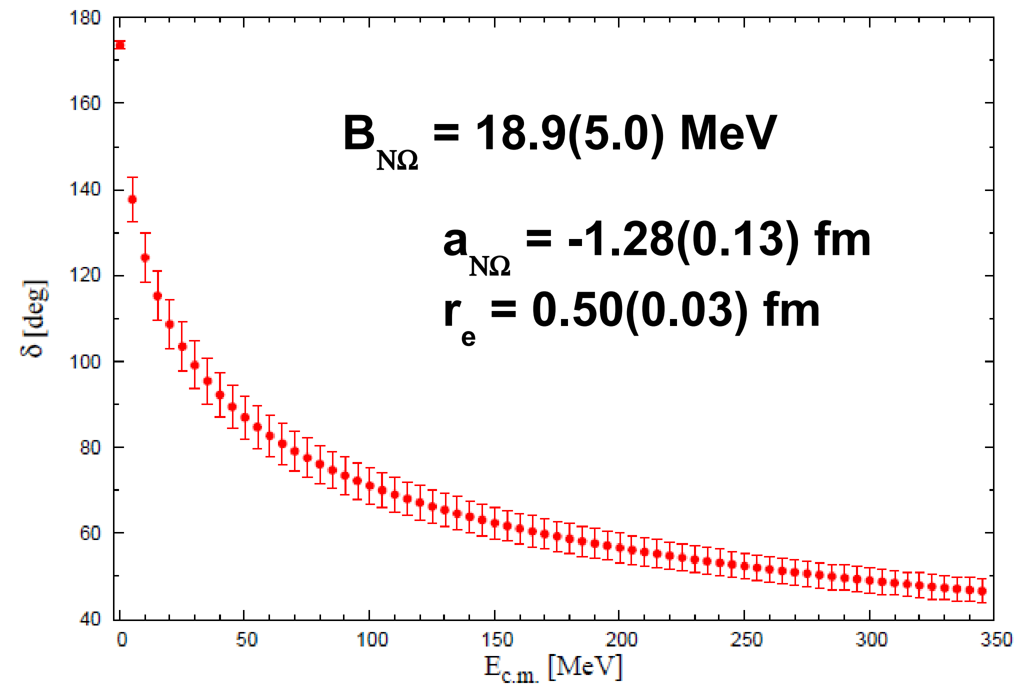
Q.B.Li, P.N.Shen, EPJA8(2000)

$N\Omega$ system $J^p(I) = 2^+(1/2)$

▶ **N_f = 2+1 full QCD with L = 1.9fm**

F.Etminan et al(HAL QCD), NPA928(2014)89





$N\Omega$ state cannot decay into $\Lambda\Xi$ D-wave state in this setup

- **Strongly attractive S-wave effective potential in $J^p(I) = 2^+(1/2)$**
- **Good baseline to explore S=-3 baryonic system**

$\Omega\Omega$ and $\Delta\Delta$ state

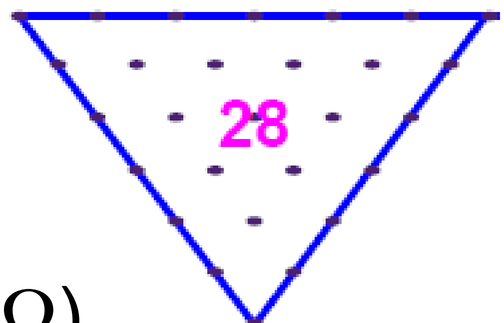
Decuplet-Decuplet interaction

- **Flavor symmetry aspect**

Decuplet-Decuplet interaction can be classified as

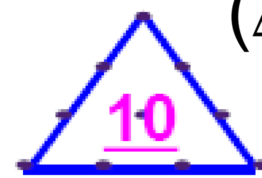
$$10\otimes 10 = 28 \oplus \cancel{27} \oplus \cancel{35} \oplus \overline{10}$$

$(\Delta\Delta)_{ST=03}$



$(\Omega\Omega)_{ST=00}$

$(\Delta\Delta)_{ST=30}$



	28plet (0^+)	28plet (2^+)	10*plet (1^+)	10*plet (3^+)
Pauli	allowed	forbidden	---	allowed
CMI	repulsive	---	---	Not attractive

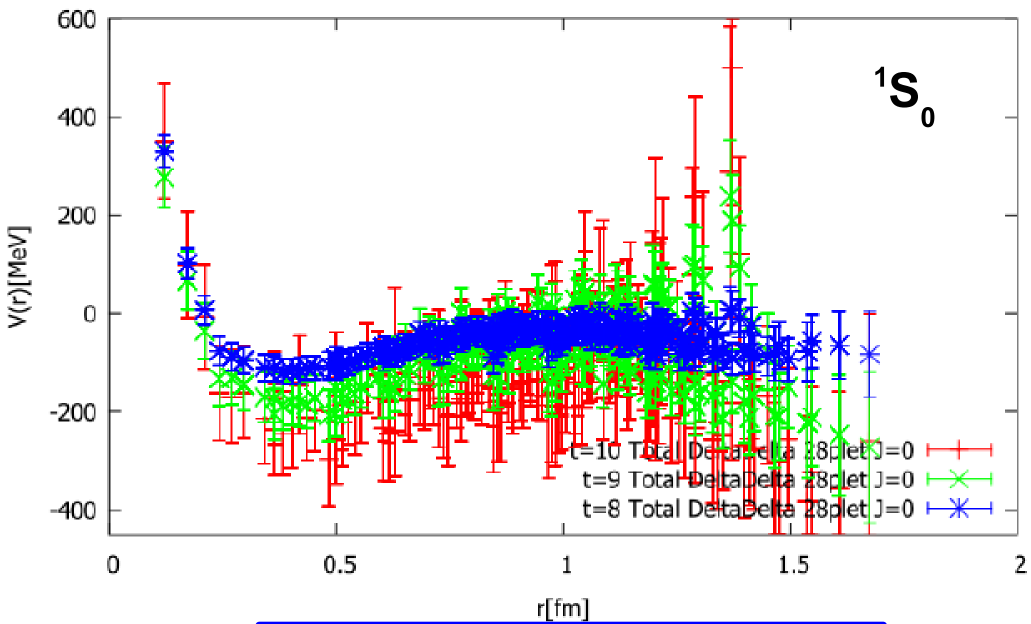
- **Δ–Δ(J=3) : Bound (resonance) state was found in experiment.**
- **Δ–Δ(J=0) [and Ω–Ω(J=0)] : Mirror of Δ–Δ(J=3) state**

Decuplet-Decuplet interaction in SU(3) limit

- **N_f = 2+1 full QCD with L = 1.93fm, *$m\pi$ = 1015 MeV***

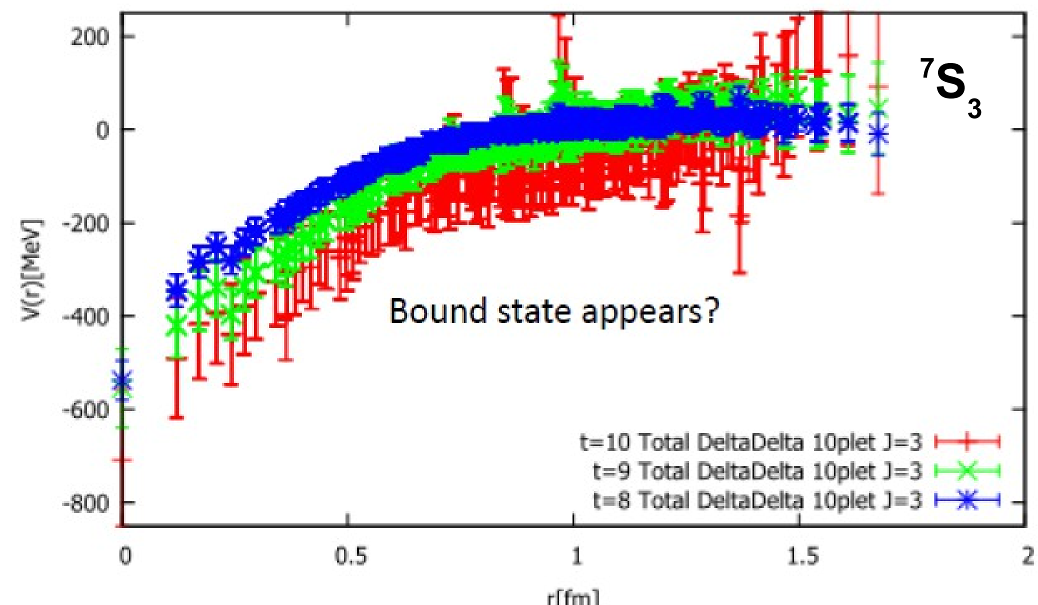
Preliminary!

28plet



Δ–Δ(J=0) and Ω–Ω(J=0)

10*plet



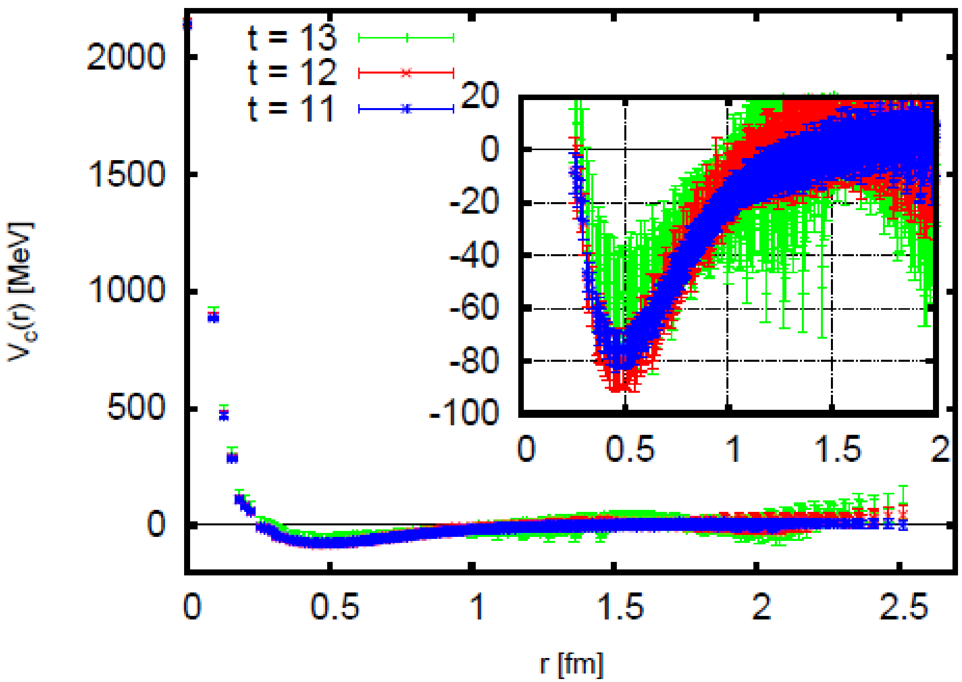
Δ–Δ(J=3)

- **Short range repulsion and attractive pocket are found in 28plet.**
- **10*plet [$J^p(I)=3^+(0)$] is strongly attractive in whole region.**
- **Existence of bound state??**

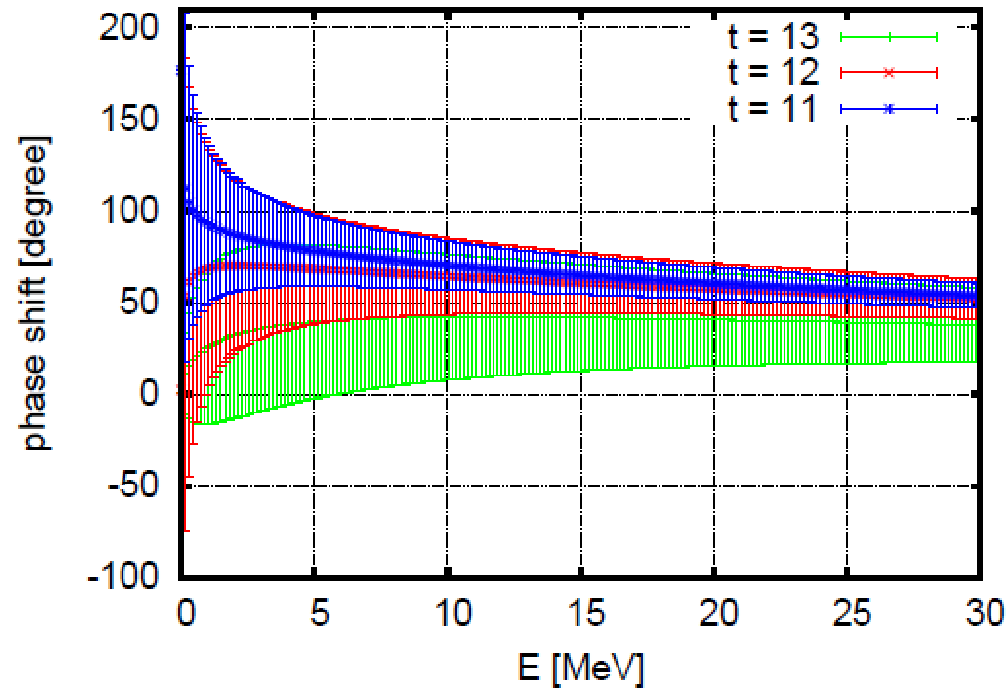
$\Omega\Omega$ $J^p(I) = 0^+(0)$ state in unphysical region

- **N_f = 2+1 full QCD with L = 3fm, *m_π = 700 MeV***

Potential



Phase shift

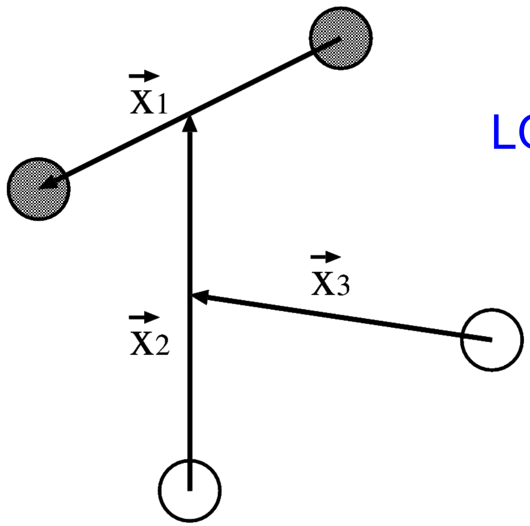


- **Short range repulsion and attractive pocket are found.**
- **Potential is nearly independent on "t" within statistical error.**
- **The system may appear close to the unitary limit.**

The $\Omega\Omega$ state is stable against the strong interaction.

Helium nucleus from QCD

^{4}He nucleus



Schrodinger equation

$$\left[K + V\right]\Psi\left(\vec{x}_1, \vec{x}_2, \vec{x}_3\right) = E\,\Psi\left(\vec{x}_1, \vec{x}_2, \vec{x}_3\right)$$

LQCD M_N (→ K) HALQCD (→ V)

Correlated Gaussian basis ($L=0$)

$$f_A\left(\vec{x}_1, \vec{x}_2, \vec{x}_3\right) = \exp\left[-\frac{1}{2} X \cdot A\, X^t\right]$$

$$w/\ \ X = \left(\vec{x}_1, \vec{x}_2, \vec{x}_3\right),\ \ A = 3\times 3 \text{ matrix}$$

$$\Psi\left(\vec{x}_1, \vec{x}_2, \vec{x}_3\right) = \sum_{i=1}^{N} C_i\, f_{A_i}\left(\vec{x}_1, \vec{x}_2, \vec{x}_3\right)$$

- One can solve the eq. of $4N$ system exactly w/ some method.
- Here, we employ the Stochastic Variational Method.

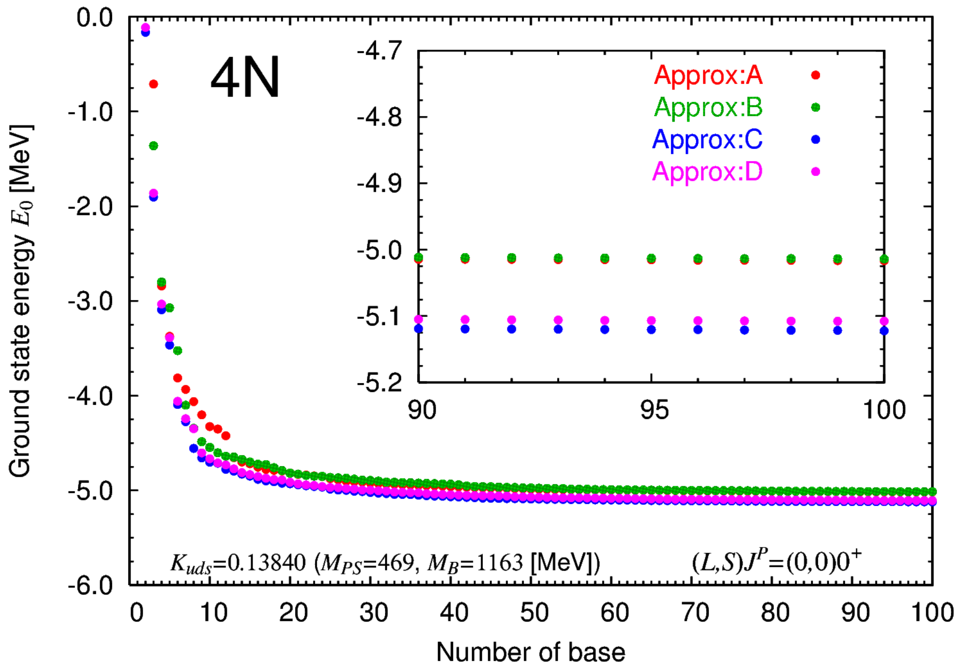
K. Varga and Y. Suzuki, Comp. Phys. Comm. 106 (1997) 157-168

- By generating matrix A randomly, many function f_A are examined.
- Most efficient f_A is add to the basis set. = Competing selection.
- Number of basis gradually increases but remains small. No need to prepare a huge basis set. It is easy to solve the eq.

E converges rapidly.

Ground state of ^{4}He

@SU(3)$_F$ limit
Mps = 469 MeV



	E_0 [MeV]	
Approx A	− 5.0167	Wigner wo/ spin-dep. Serber
Approx B	− 5.0140	
Approx C	− 5.1222	Wigner w/ spin-dep. Serber
Approx D	− 5.1078	

- Number of basis v.s. energy of ^{4}He g.s. at the lightest quark mass.
- We tested 4 approx for unknown odd parity force. But, no diff.

^{4}He = S-shell

- There definitely exists ^{4}He nucleus at MPS = 469 MeV!!

- No ^{4}He nucleus at quark mass of MPS = 632, 837 MeV.
- No 2N, 3N nuclei at all our five values of quark mass.

in contrast to other groups

T. Inoue etal [HAL QCD Colla.] Nucl. Phys. A881 (2012) 28

Summary and outlook

- We have investigated hadronic interactions from lattice QCD.

- We have studied exotic candidate states
  - H-dibaryon channel
    - There is strongly attractive potential in flavor singlet state.
    - It is not enough statistics to calculate several observables and to discuss the fate of H-dibayon.

  - NΩ state with $J^p=2^+$
    - Bound in heavy quark mass point

  - ΔΔ and ΩΩ states
    - ΔΔ(I=0) have strongly attractive potential
    - ΔΔ(I=3) and ΩΩ potential hae repulsive core and attractive pocket

- We have applied a nuclear potential to many body calculation.
  - We find that 4He can be bound even in mπ=470Mev situation
  - No 2N and 3N nuclei in the same situation.