

Light (anti-)nuclei production and flow in relativistic HIC: A microscopic coalescence approach

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- Motivation
- Data on light nuclei production
- Deuteron
 - Elementary deuteron production processes
 - Deuteron production in transport model
 - Deuteron production in coalescence model
- Triton and helium and their hypernuclei
- Summary

Motivation

- Light nuclei production in HIC provides another means for studying the size of emitting system and its space-time evolution as the coalescence model gives

$$E_A \frac{d^3 N_A}{dp_A^3} = B_A \left(E_p \frac{d^3 N_p}{dp_p^3} \right)^A, \quad p_A = A p_p$$

where the coalescence parameter B_A is related to the size and temperature of the emitting system as well as the nuclear radius.

- Testing this idea using a transport model that explicitly takes into account the production and dissociation of deuteron as well as their elastic scattering in hadronic matter.
- Understanding elliptic flow of light nuclei such as if there are nucleon number scaling and negative values at low p_T .

In terms of coalescence radius p_0 , [Gutbrod et al., PRL 37, 667 (1976)]

$$B_A = \left(\frac{4\pi}{3} p_0^3 \right)^{A-1} \frac{M}{m^A}$$

In the density matrix formalism, [Sato & Yazaki, PLB 98, 153 (1981)]

$$\text{Deuteron : } \frac{4\pi}{3} p_0^3 = \frac{3}{4} \cdot 2^{3/2} (4\pi)^{3/2} [\nu_d \nu / (\nu_d + \nu)]^{3/2}$$

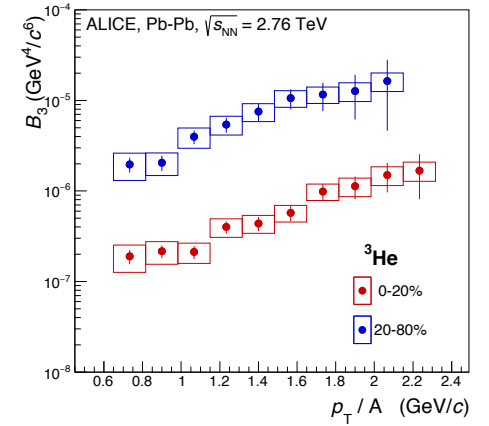
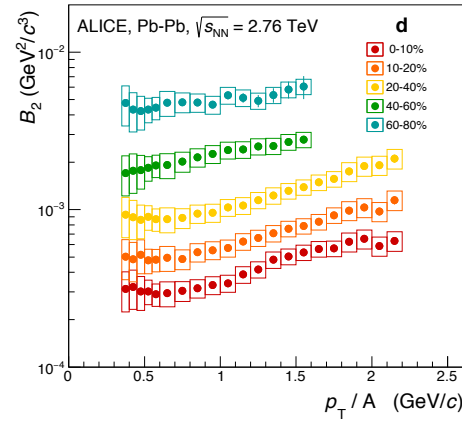
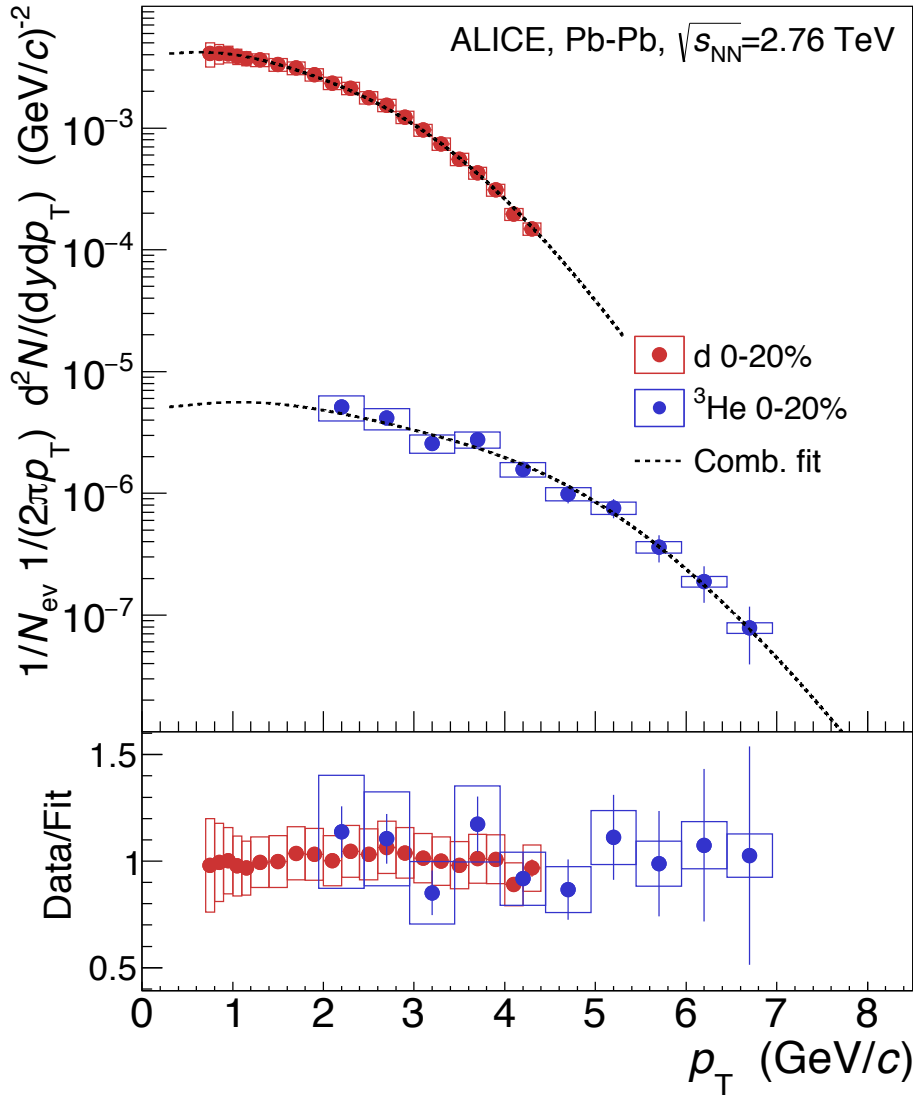
$$\text{Triton : } \frac{1}{2} \left(\frac{4\pi}{3} p_0^3 \right)^2 = \frac{1}{4} \cdot 2^{3/2} (4\pi)^3 [\nu_t \nu / (\nu_t + \nu)]^3$$

$$\text{Alpha : } \frac{1}{4} \left(\frac{4\pi}{3} p_0^3 \right)^3 = \frac{1}{16} \cdot 2^{3/2} (4\pi)^{9/2} [\nu_\alpha \nu / (\nu_\alpha + \nu)]^{9/2}$$

ν and ν_A are size parameters of emitting source and nuclei, respectively

Deuteron and helium spectra and yields from ALICE

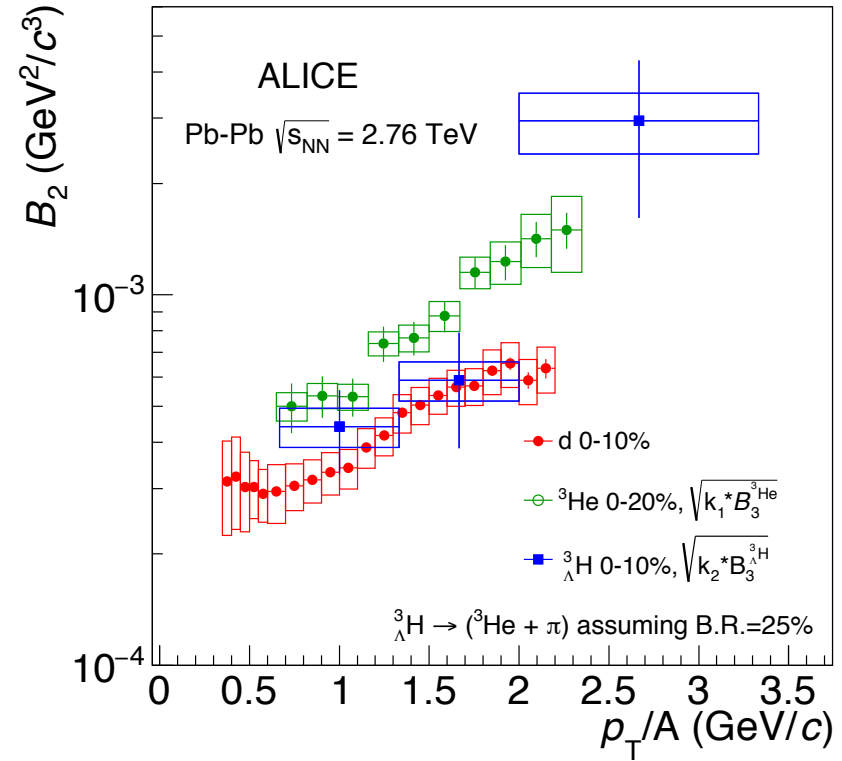
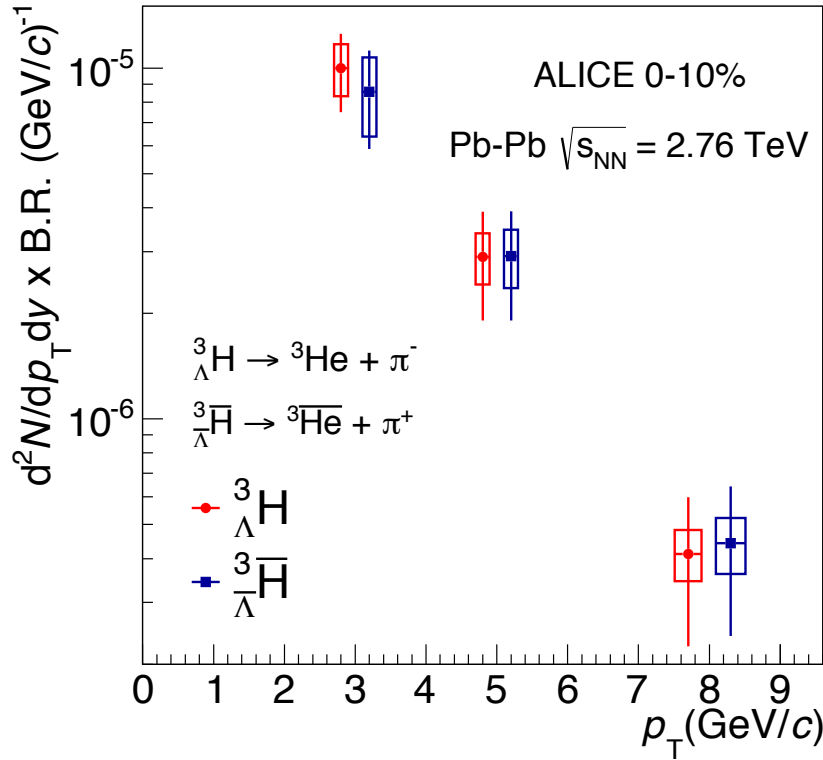
ALICE Collaboration: arXiv:1505.08951 [nucl-ex]



- Spectrum can be fitted by coalescence model with coalescence parameters B_2 and B_3 increasing with p_t but decreasing with centrality

Light hypernuclei production from ALICE

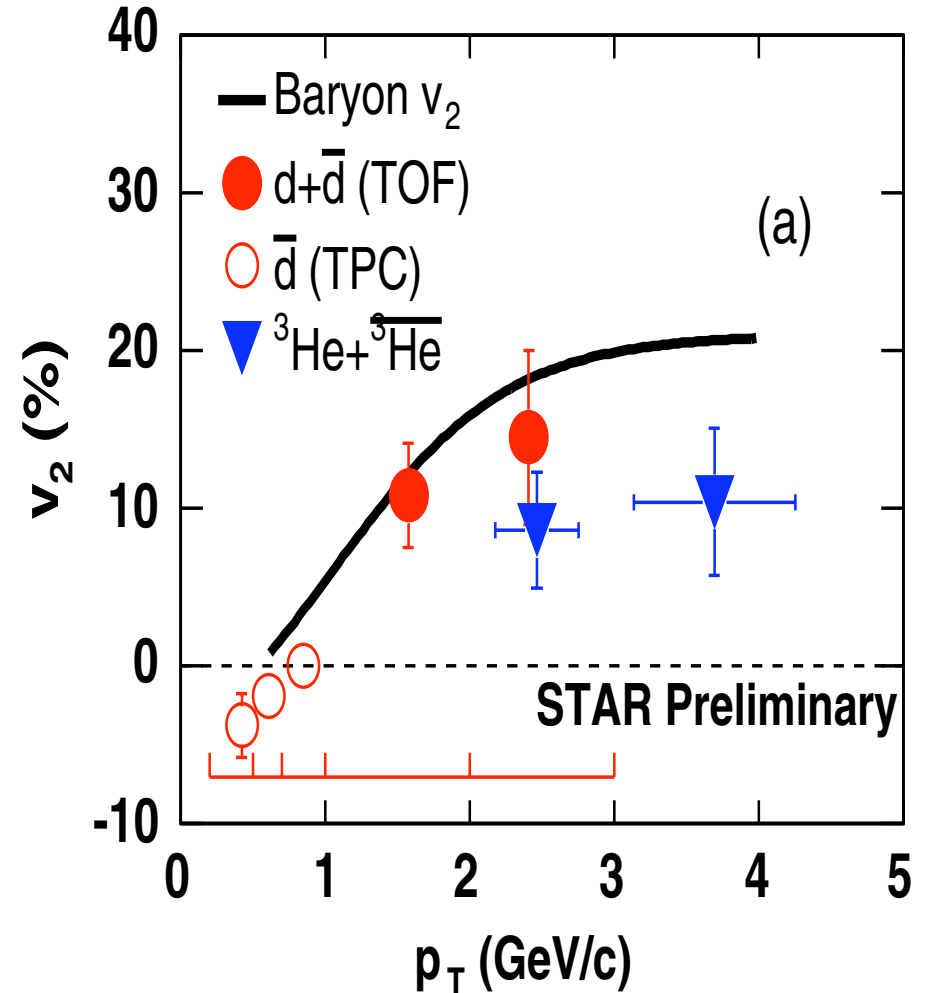
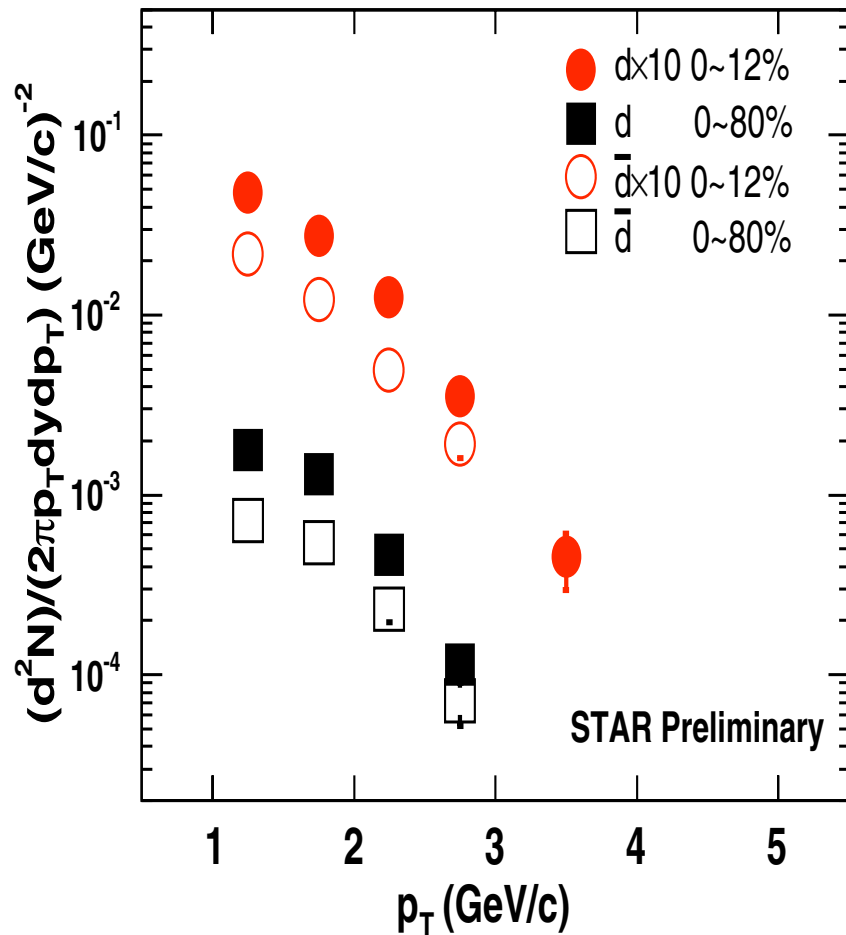
ALICE, arXiv:1506.08453 [nucl-ex]



- Similar yields and spectra for hyperhelium and anti-hyperhelium
- Fitting with coalescence model requires coalescence parameter B_3 also increasing with p_T and decreasing centrality

Deuteron p_T spectrum and v_2 from STAR

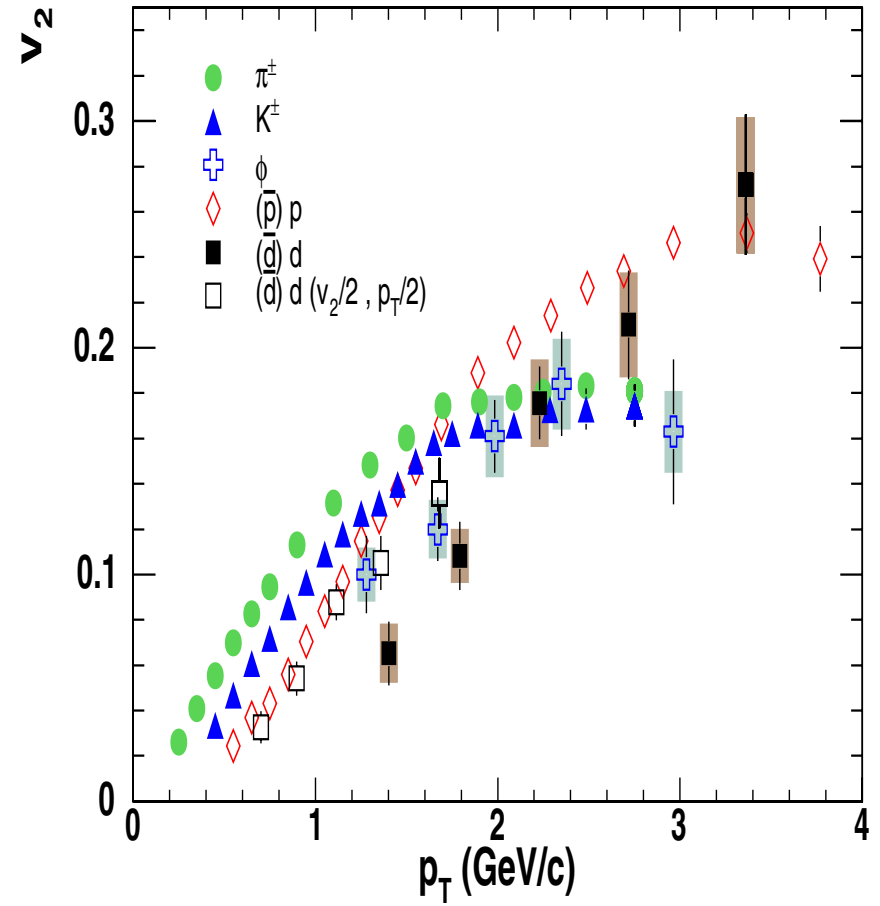
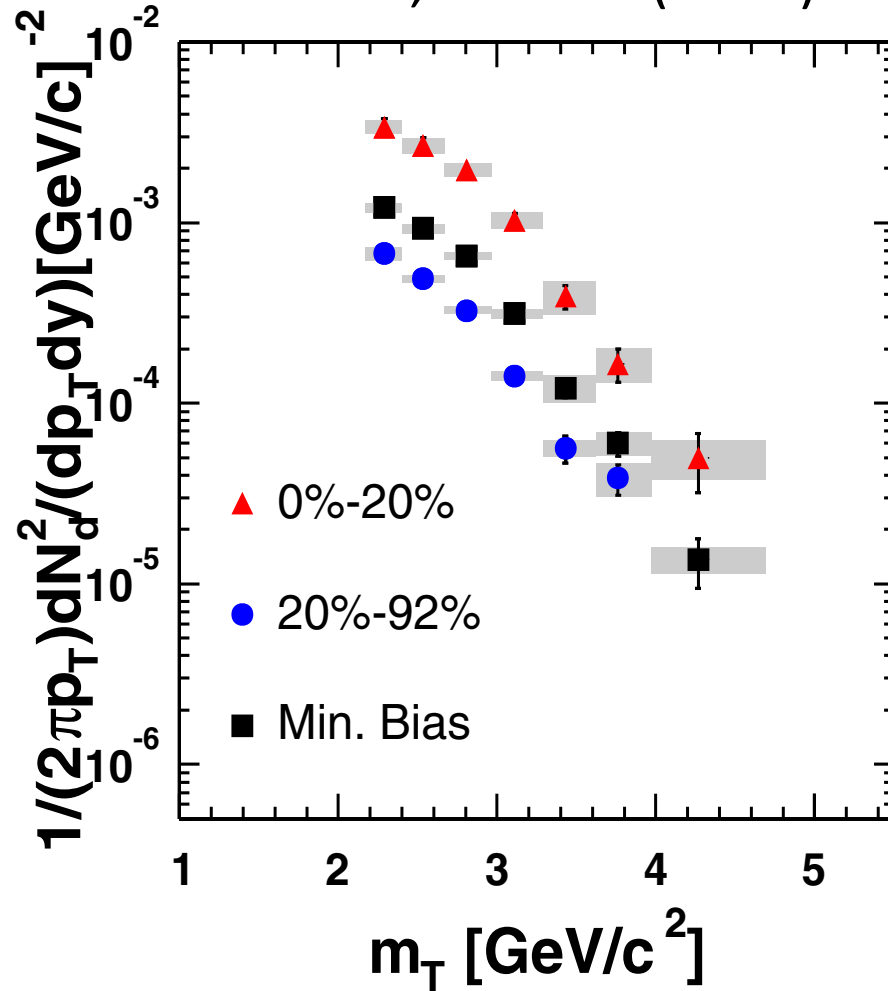
STAR Collaboration, JPG 34, S1087 (2007); arXiv:0909.0566 [nucl-ex]



- Coalescence parameter $B_2 \sim 10^{-3} \text{ GeV}^2/\text{c}^3$, comparable to ALICE
- Appreciable elliptic flow at high p_T but negative at low p_T

Deuteron p_T spectrum and v_2 from PHENIX

PRL 94, 122302 (2004)



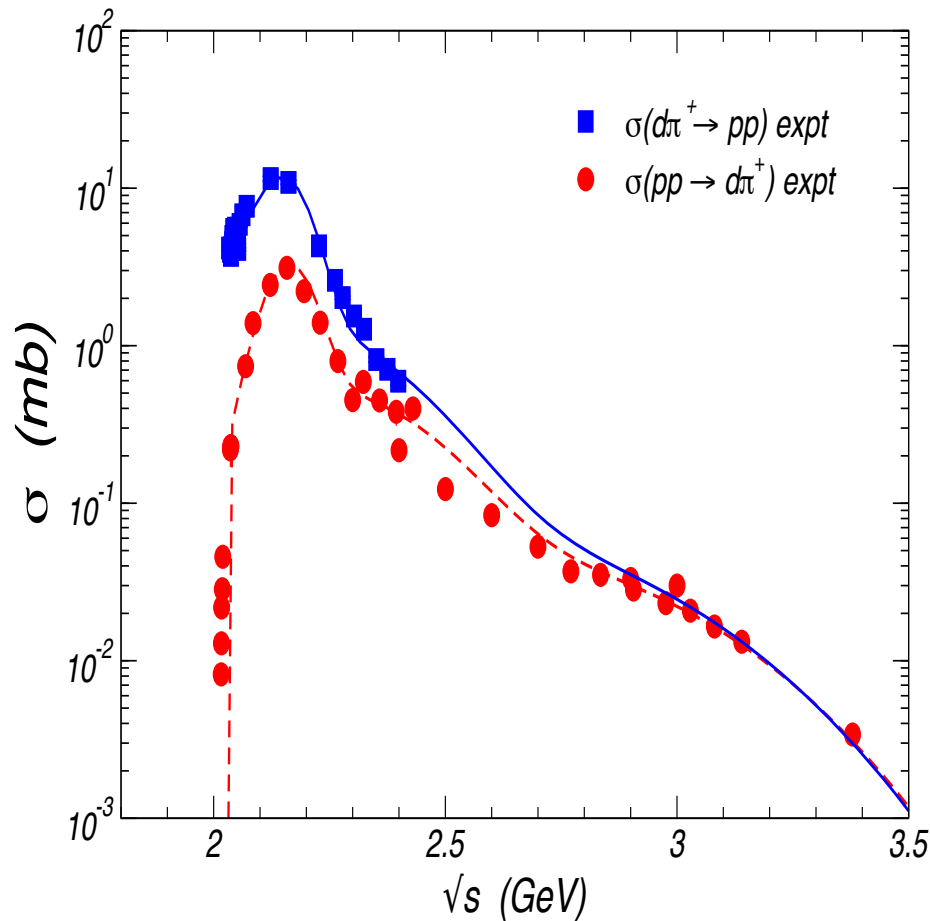
- $B_2 \sim 10^{-3} \text{ GeV}^2/c^3$, $T_{\text{eff}} \sim 520 \text{ MeV}$, $dN_d/dy \sim 0.025$
- Approximate nucleon number scaling of v_2 at low p_T

A relativistic transport (ART) model for HIC

Li & Ko, PRC 52, 2037 (1995)

- Based on BUU model with explicit isospin dependence
- Including baryons N , $\Delta(1232)$, $N^*(1440)$, $N^*(1535)$, Λ , Σ and mesons π , ρ , ω , η , K
- Including baryon-baryon, meson-baryon and meson-meson elastic and inelastic scattering with empirical cross sections if available, otherwise from theoretical models
- Effects of higher nucleon and delta resonances up to 2 GeV are included as intermediate states in meson-baryon scattering
- Very successful in describing many experimental results at AGS
- Used as a hadronic afterburner in the AMPT model
- Extended to include deuteron production and annihilation as well as elastic scattering

Deuteron production and annihilation processes



■ $pp \rightarrow d\pi^+$
Shimizu et al., NPA 386, 571 (1982)

■ $\pi^+d \rightarrow pp$
Pasyuk et al., PRC 55, 1026 (1997)

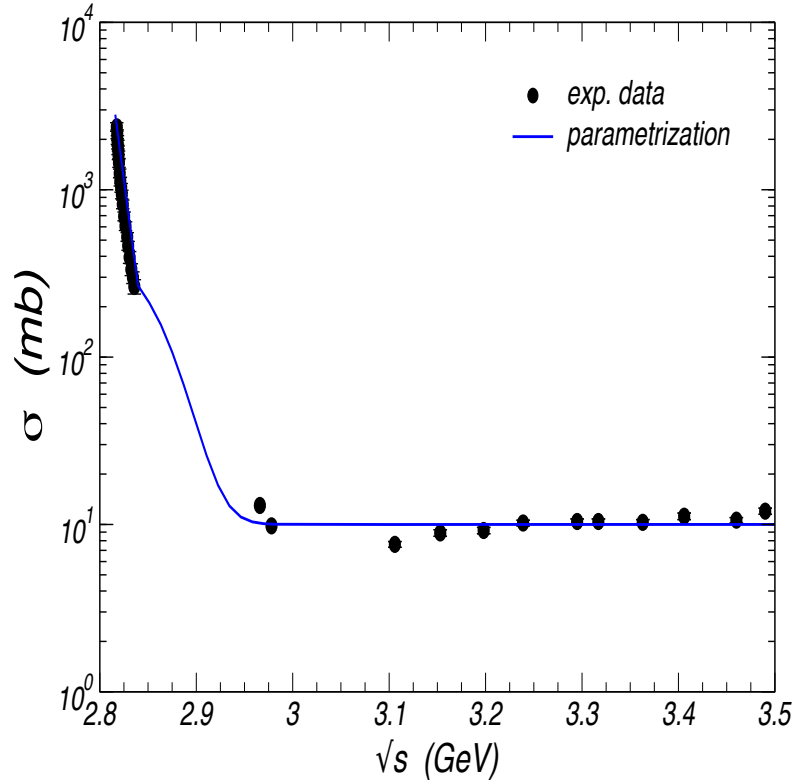
$$\sigma(pp \rightarrow d\pi^+) = \frac{1}{4} \frac{p_\pi}{p_p} f(s)$$

$$f(s) = 26 \exp\left[-(s - 4.65)^2 / 0.1\right] \\ + 4 \exp\left[-(s - 4.65)^2 / 2\right] \\ + 0.28 \exp\left[-(s - 6)^2 / 10\right]$$

- Cross sections for $BB \rightarrow d\pi$ and $Md \rightarrow BB$ are assumed to be same as those for $pp \rightarrow d\pi$ and $\pi d \rightarrow pp$ at same center of mass energy, respectively.

■ $Nd \rightarrow Nd$

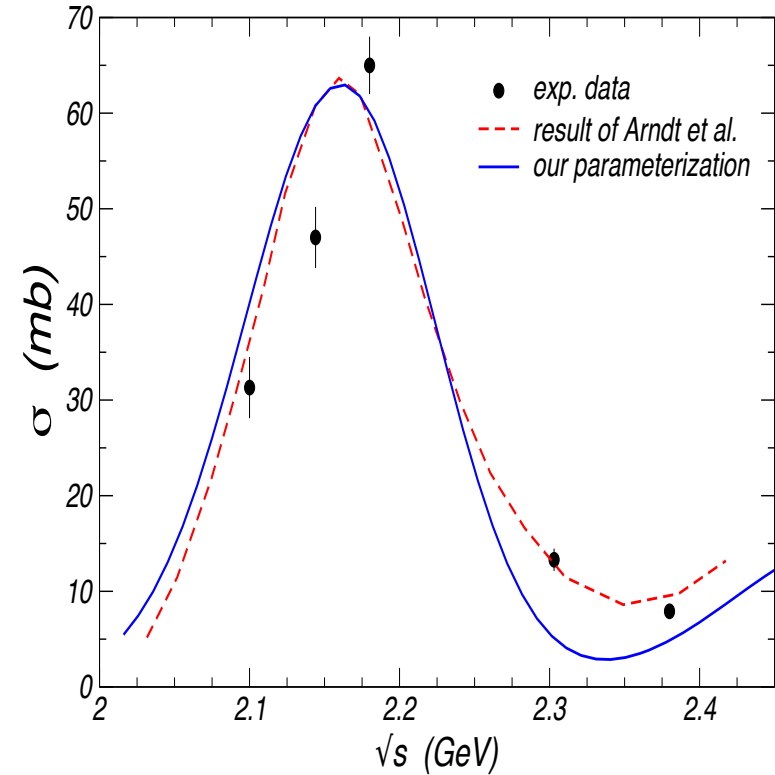
Hatanaka et al., PRC 66,044002 (2002)



$$\sigma(Nd \rightarrow Nd) = 2500 \exp\left[-(s - 7.93)^2 / 0.003\right] \\ + 300 \exp\left[-(s - 7.93)^2 / 0.1\right] + 10$$

■ $\pi d \rightarrow \pi d$

Khandaker et al., PRC 44, 21 (1991)



$$\sigma(\pi d \rightarrow \pi d) = 63 \exp\left[-(s - 4.67)^2 / 0.15\right] \\ + 15 \exp\left[-(s - 6.25)^2 / 0.3\right]$$

- Cross sections for $Bd \rightarrow Bd$ and $Md \rightarrow Md$ are assumed to be same as those for $Nd \rightarrow Nd$ and $\pi d \rightarrow \pi d$ at same c.m. energy, respectively.

Initial hadron distributions after hadronization

- Approximated by a blast wave model with temperature T

$$f(x, p) \propto \exp(-p^\mu u_\mu / T)$$

and transverse flow $\beta = \beta(r)[1 + \varepsilon(p_T)\cos(2\phi)]\mathbf{n}$

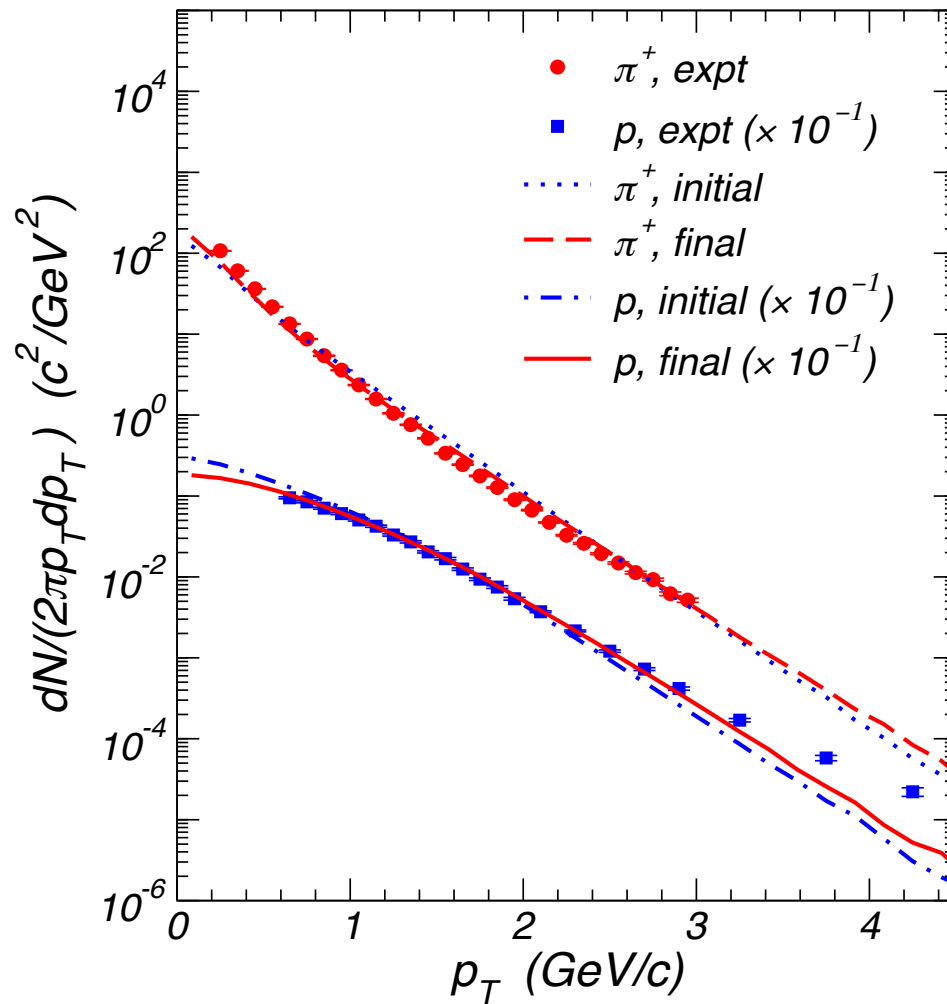
where $\varepsilon(p_T) = c_1 \exp(-p_T/c_2)$, $\beta(r) = \beta_0(r/R_0)$

and spatial eccentricity $s_2 = \left\langle \frac{y^2 - x^2}{y^2 + x^2} \right\rangle$

- For minimum biased Au+Au @ $S_{NN}^{1/2} = 200$ GeV

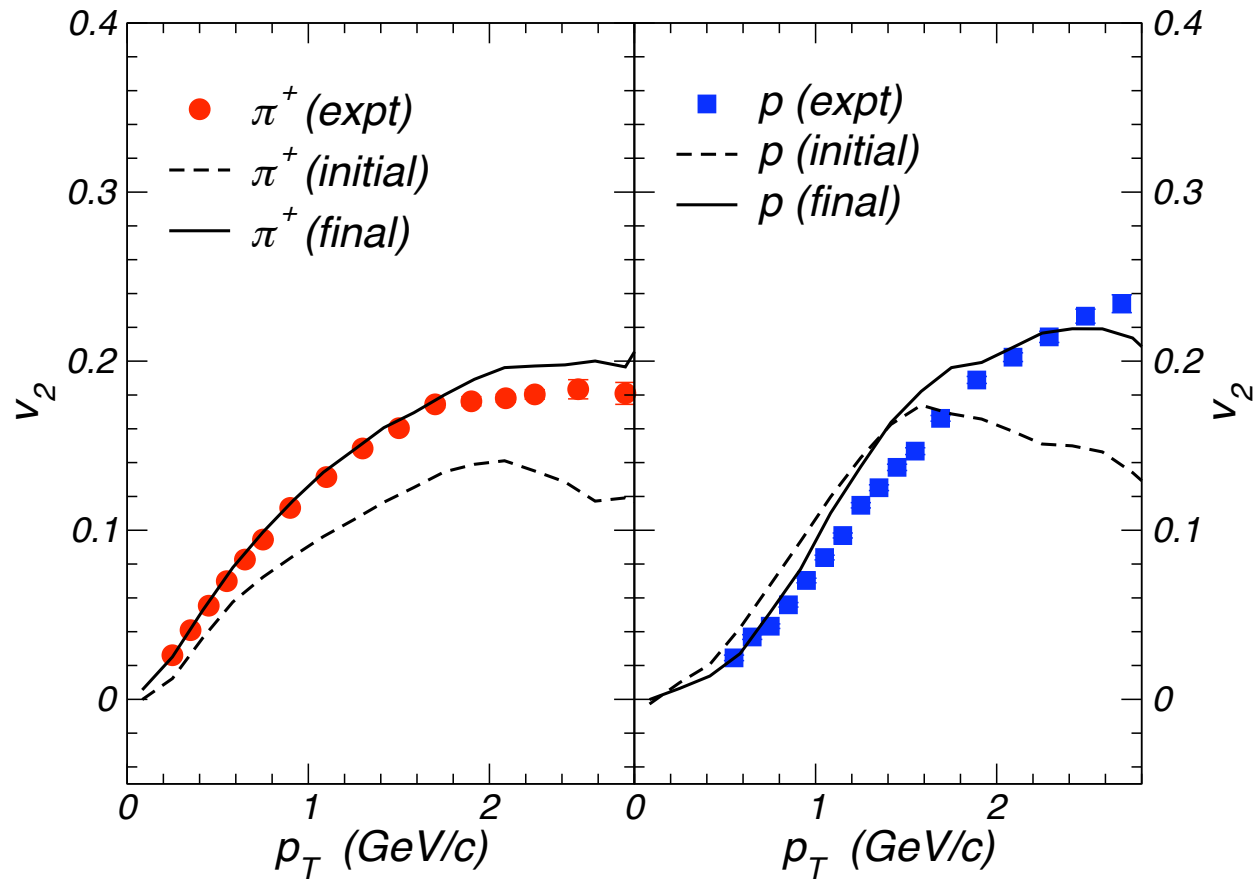
$T = 175$ MeV, $dN_{\pi^+}/dy = 25$, $dN_p/dy = 0.6$, $\beta_0 = 0.55$, $c_1 = 0.7$,
 $c_2 = 0.55$ GeV, $R_0 = 5$ fm, $s_2 = 0.05$

Initial and final pion and proton p_T spectra



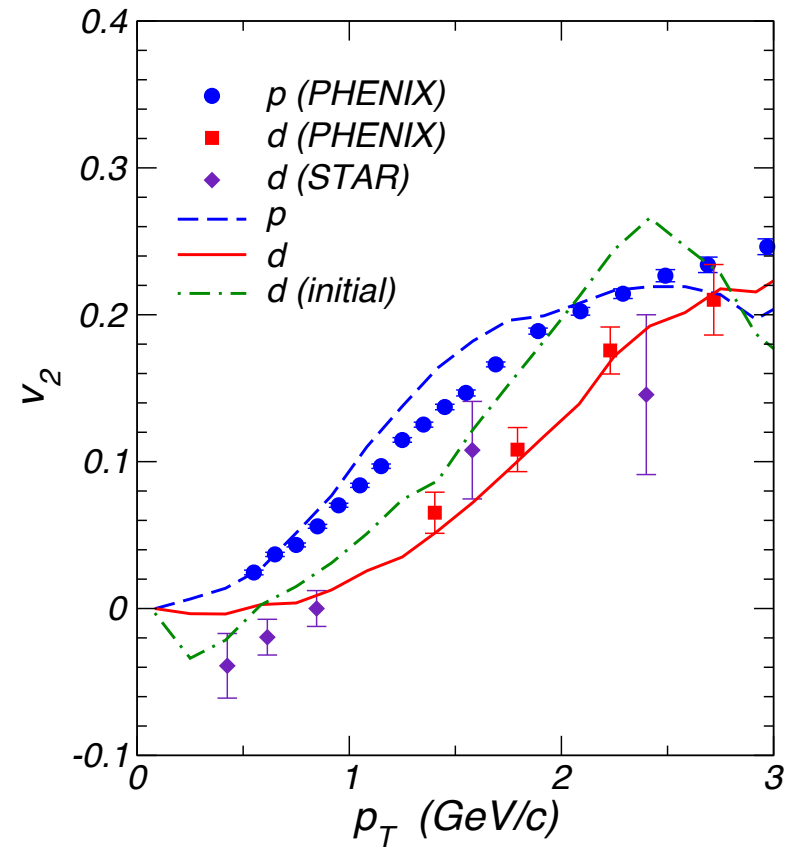
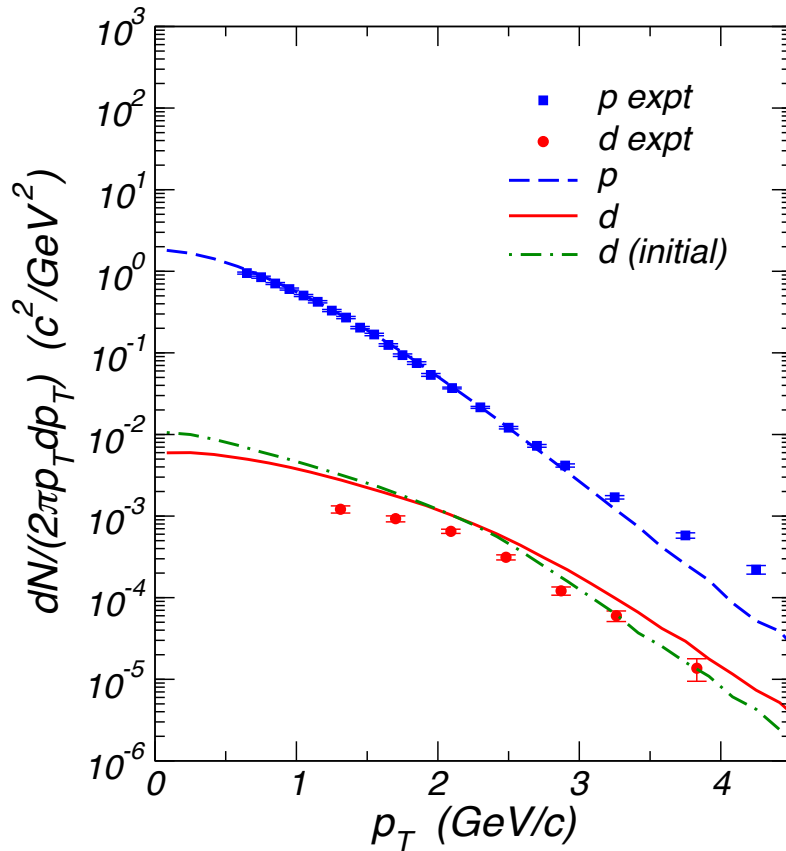
- Very similar initial and final p_T spectra due to canceling effects of decreasing temperature as a result of expansion and increasing collective flow velocity.

Initial and final pion and proton elliptic flows



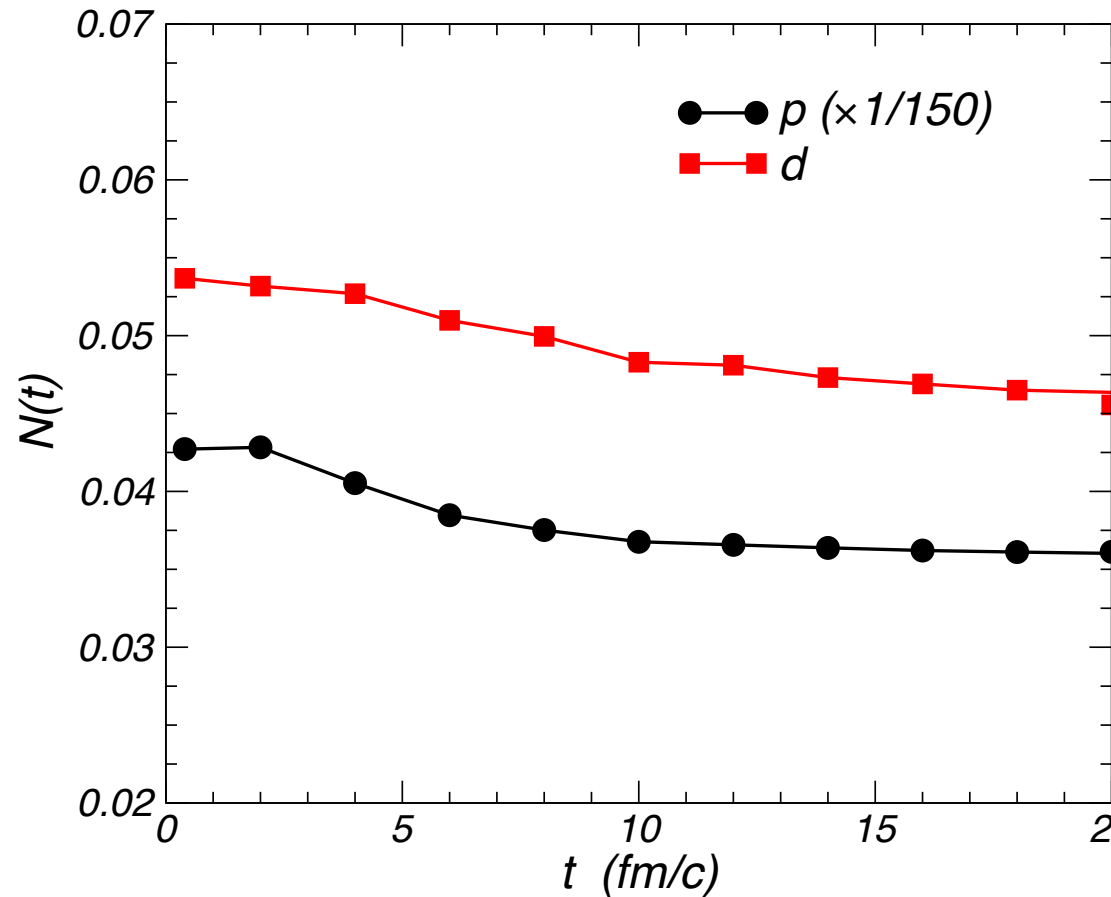
- Effect of hadronic scattering on elliptic flow is appreciable

Initial and final deuteron p_T spectrum and elliptic flow



- Hadronic scattering has some effect on deuteron p_T spectrum
- Large hadronic scattering effect on deuteron elliptic flow

Time evolution of proton and deuteron numbers



- Both proton and deuteron numbers decrease only slightly with time \rightarrow early chemical equilibration

Coalescence model for deuteron production

$$\frac{d^3 N_d}{dp_d^3} = \frac{3}{4} \int d^3 x_1 d^3 p_1 d^3 x_2 d^3 p_2 f_p(x_1, p_1) f_n(x_2, p_2) \\ \times f_d^W(x'_1, x'_2; p'_1, p'_2) \delta^{(3)}(p_d - p_1 - p_2)$$

$f_{p/n}(x, p)$: proton/neutron phase-space distribution function
at freeze out

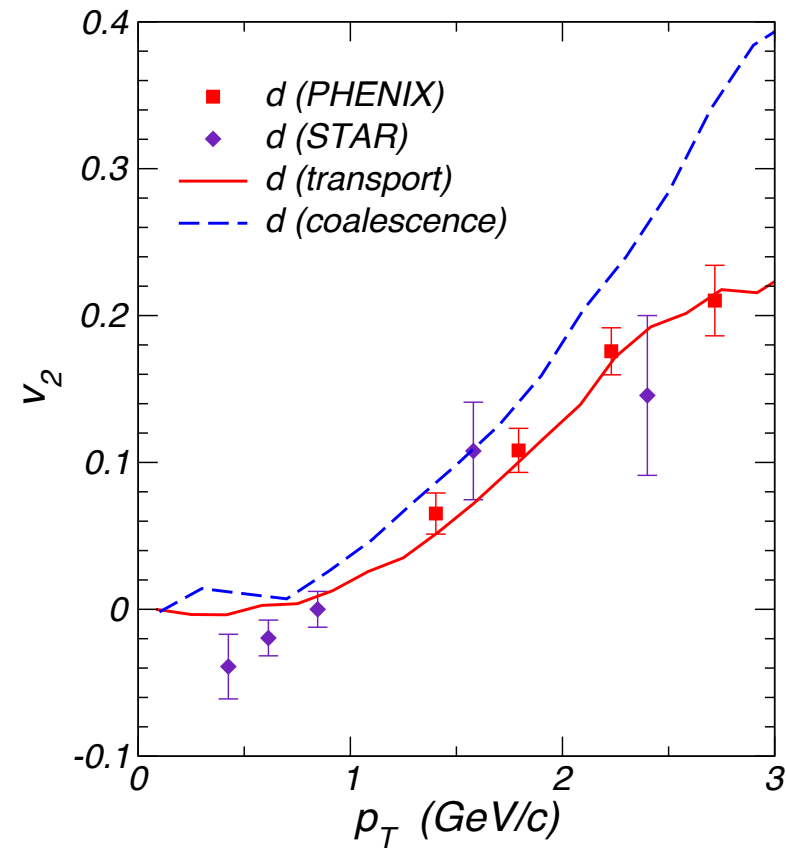
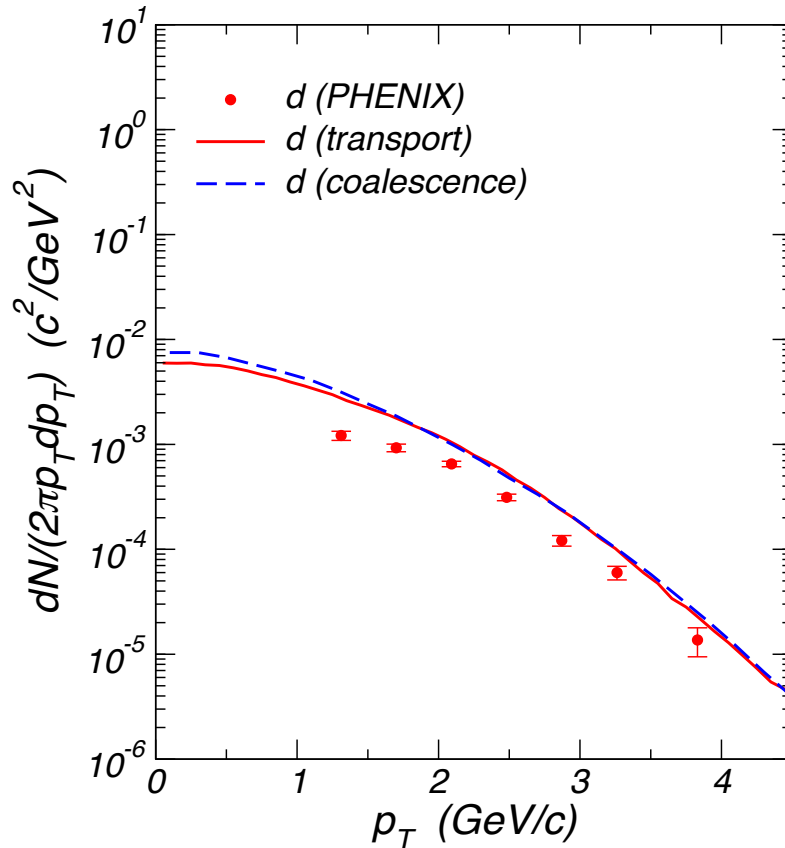
f_d^W : deuteron Wigner distribution based on harmonic oscillator
wave function

$$f_d^W(x'_1, x'_2; p'_1, p'_2) = 8 \exp \left[- (p'_1 - p'_2)^2 \sigma^2 / 4 - (x'_1 - x'_2)^2 / \sigma^2 \right]$$

with $\sigma = 1 / \sqrt{\mu \omega}$, $\mu = m_N / 2$

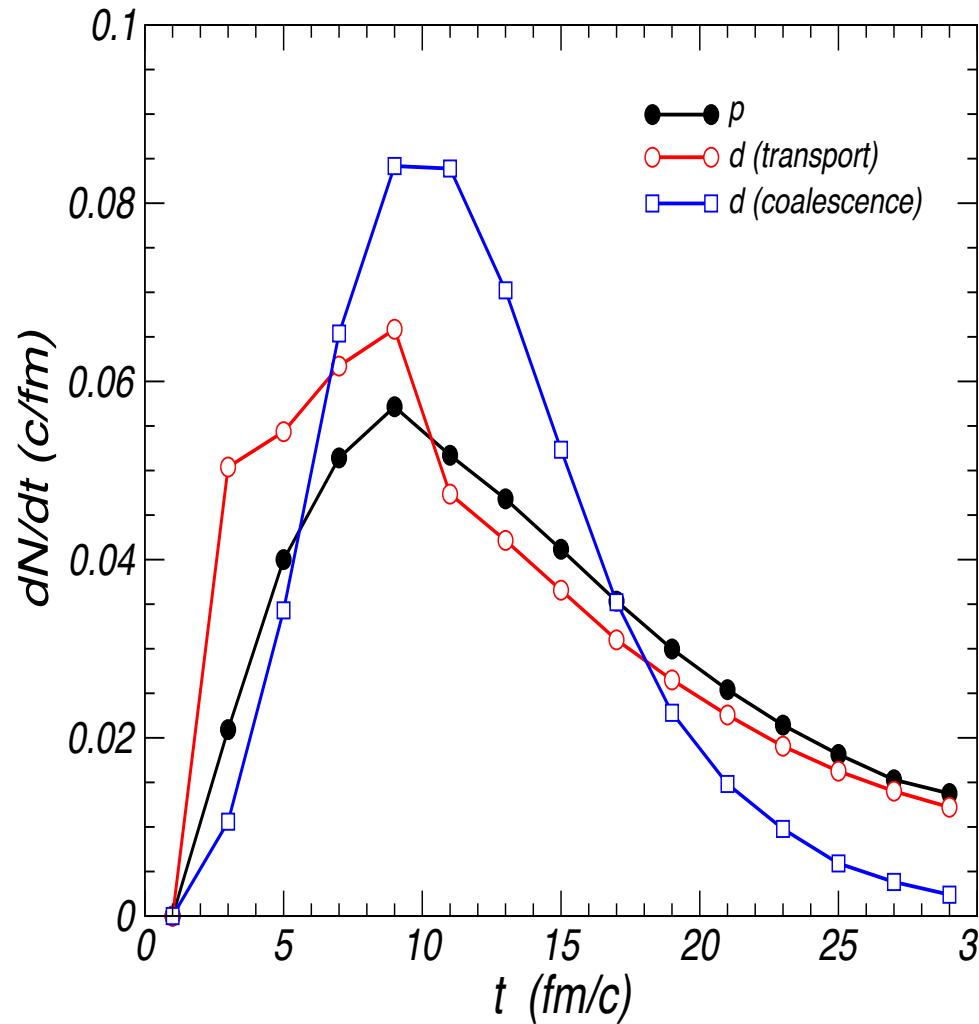
$$\sqrt{\langle r_d^2 \rangle} = 1.96 \text{ fm} \rightarrow \omega = 8.06 \times 10^{-3} \text{ GeV}$$

Deuteron p_T spectrum and elliptic flow from coalescence model



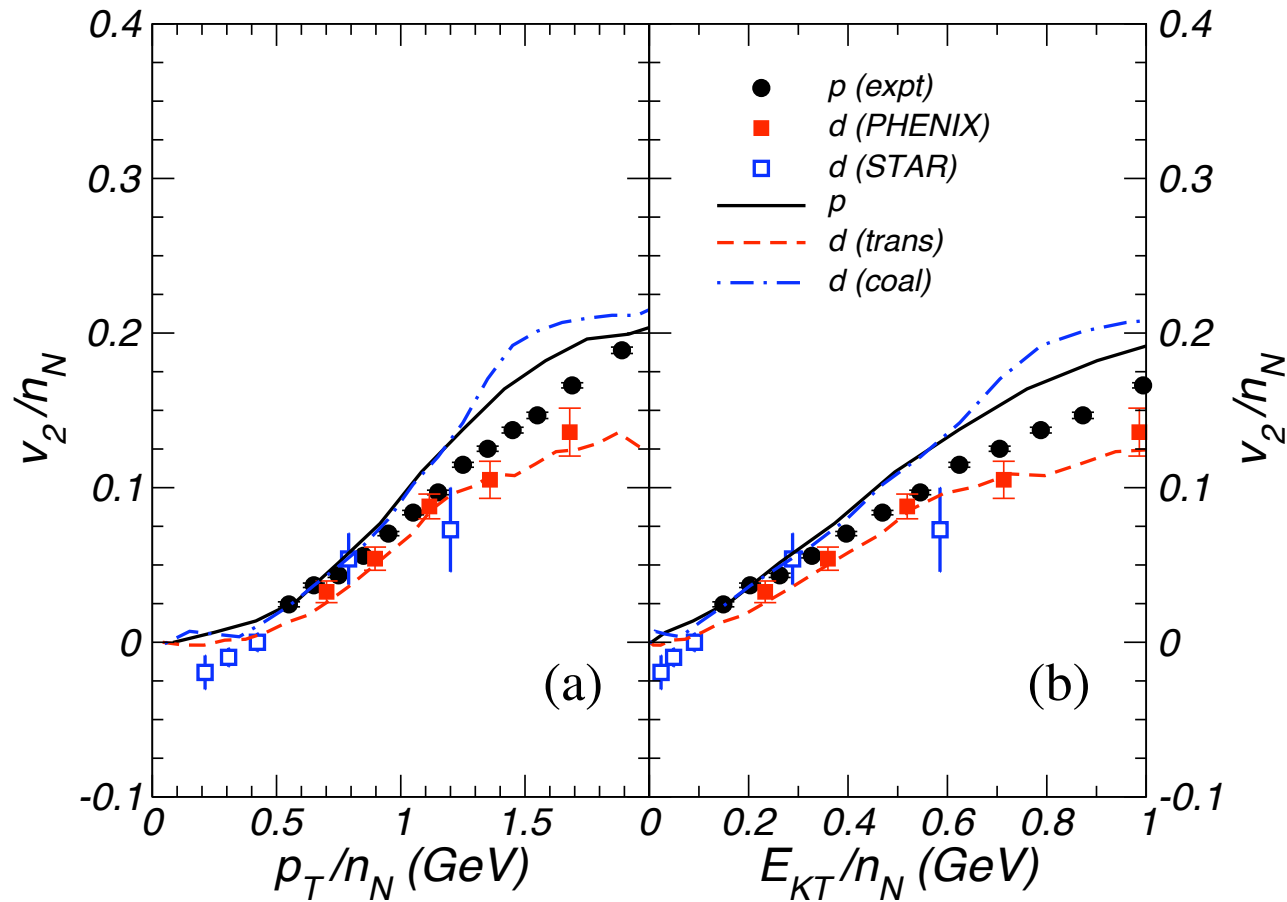
- Similar p_T spectrum from transport and coalescence models
- Smaller elliptic flow at large p_T from transport model than from coalescence model

Deuteron emission time distributions



- Similar emission time distributions for protons and deuterons in coalescence model
- Slight different deuteron early emission time distribution in transport and coalescence models

Nucleon number scaling of deuteron elliptic flow



- Approximate nucleon number scaling of proton and deuteron elliptic flows at low p_T

Light clusters production

Chen, Ko & Li, PRC 68, 017601 (03);
NPA 729, 809 (03)

Coalescence model

$$\begin{aligned} \frac{dN_M}{d^3K} = & G \binom{A}{M} \binom{M}{Z} \frac{1}{A^M} \int \left[\prod_{i=1}^Z f_p(\mathbf{r}_i, \mathbf{k}_i) \right] \\ & \times \left[\prod_{i=Z+1}^M f_n(\mathbf{r}_i, \mathbf{k}_i) \right] \rho^W(\mathbf{r}_{i_1}, \mathbf{k}_{i_1} \cdots \mathbf{r}_{i_{M-1}}, \mathbf{k}_{i_{M-1}}) \\ & \times \delta(\mathbf{K} - (\mathbf{k}_1 + \cdots + \mathbf{k}_M)) d\mathbf{r}_1 d\mathbf{k}_1 \cdots d\mathbf{r}_M d\mathbf{k}_M \end{aligned}$$

$f_p(\vec{r}, \vec{k})$: nucleon phase-space distribution function

$\rho^W(\vec{r}_i, \vec{k}_i)$: Wigner phase-space distribution function of clusters

G: statistical factor; 3/8 for deuteron, 1/3 for triton and helium3

Wigner phase-space distribution function for triton and helium3

Gaussian wave function $\psi(\vec{r}_1, \vec{r}_2, \vec{r}_3) = (3\pi^2 b^4)^{-3/4} e^{-\frac{\rho^2 + \lambda^2}{2b^2}}$

Jacobi coordinates

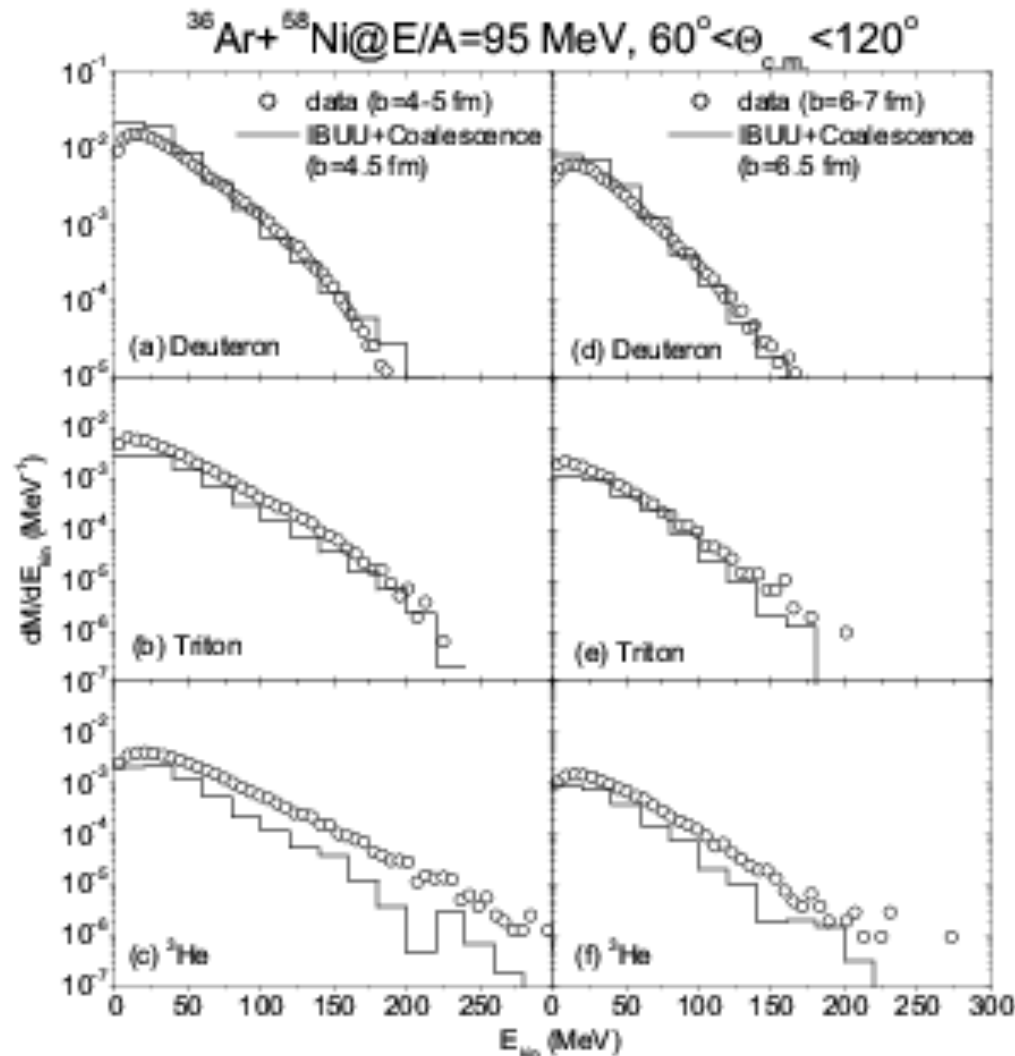
$$\vec{R} = \frac{1}{3}(\vec{r}_1 + \vec{r}_2 + \vec{r}_3), \quad \vec{\rho} = \frac{1}{\sqrt{2}}(\vec{r}_1 - \vec{r}_2), \quad \vec{\lambda} = \frac{1}{\sqrt{6}}(\vec{r}_1 + \vec{r}_2 - 2\vec{r}_3)$$

$$\vec{K} = \vec{k}_1 + \vec{k}_2 + \vec{k}_3, \quad \vec{k}_\rho = \frac{1}{\sqrt{2}}(\vec{k}_1 - \vec{k}_2), \quad \vec{k}_\lambda = \frac{1}{\sqrt{6}}(\vec{k}_1 + \vec{k}_2 - 2\vec{k}_3)$$

$$\Rightarrow \rho_{t(\text{He})}^W(\rho, k_\rho; \lambda, k_\lambda) = 8^2 e^{-\frac{\rho^2 + \lambda^2}{b^2}} e^{-(k_\rho^2 + k_\lambda^2)b^2}$$

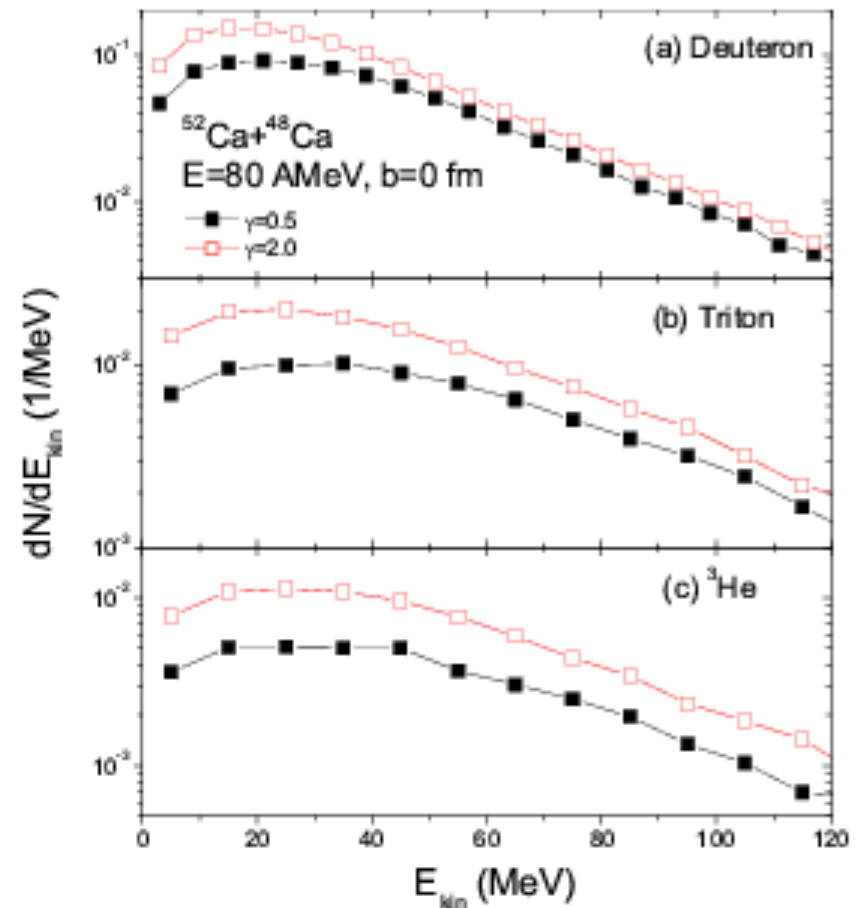
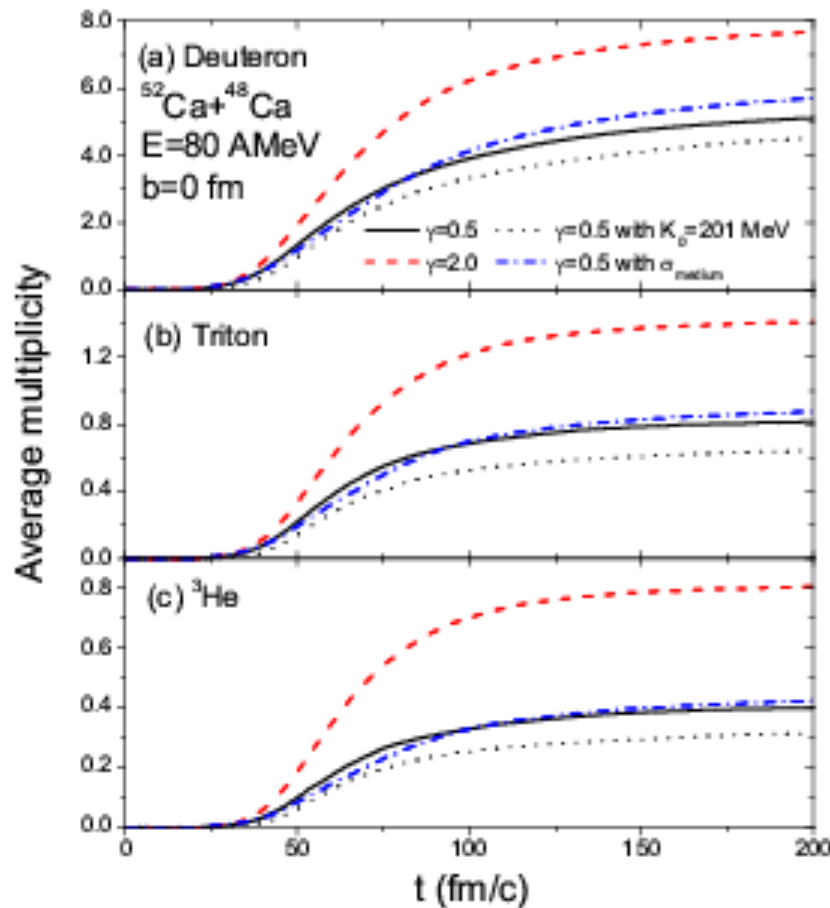
$b=1.61$ fm for triton and 1.74 for He \rightarrow correct radii

Light clusters production from collisions of symmetric nuclei



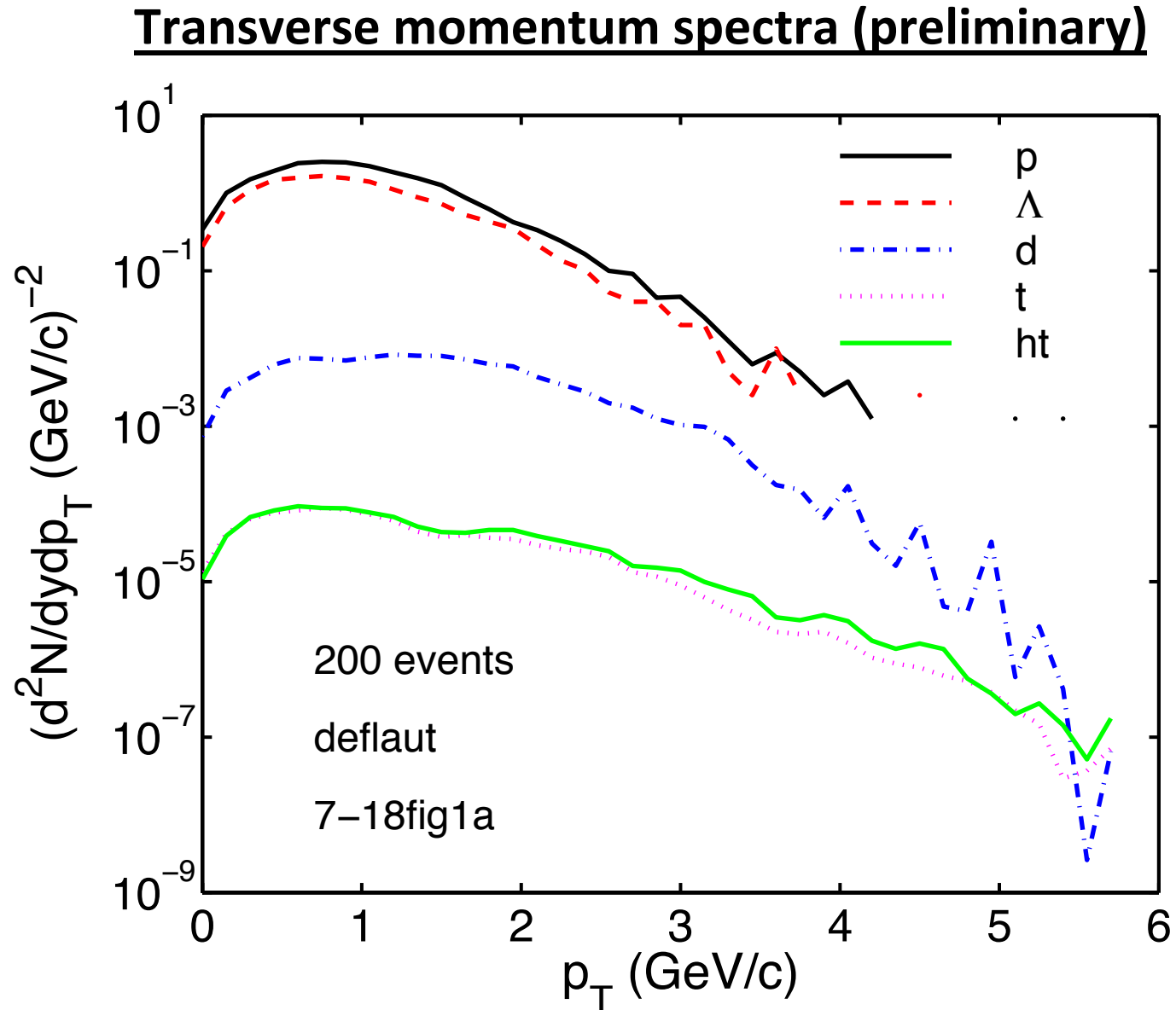
- Deuteron energy spectra reproduced
- Low energy tritons slightly underestimated
- Inverse slope parameter of He underestimated; probably due to neglect of
 - Larger binding effect
 - Stronger Coulomb effect

Light clusters from collisions of asymmetric nuclei

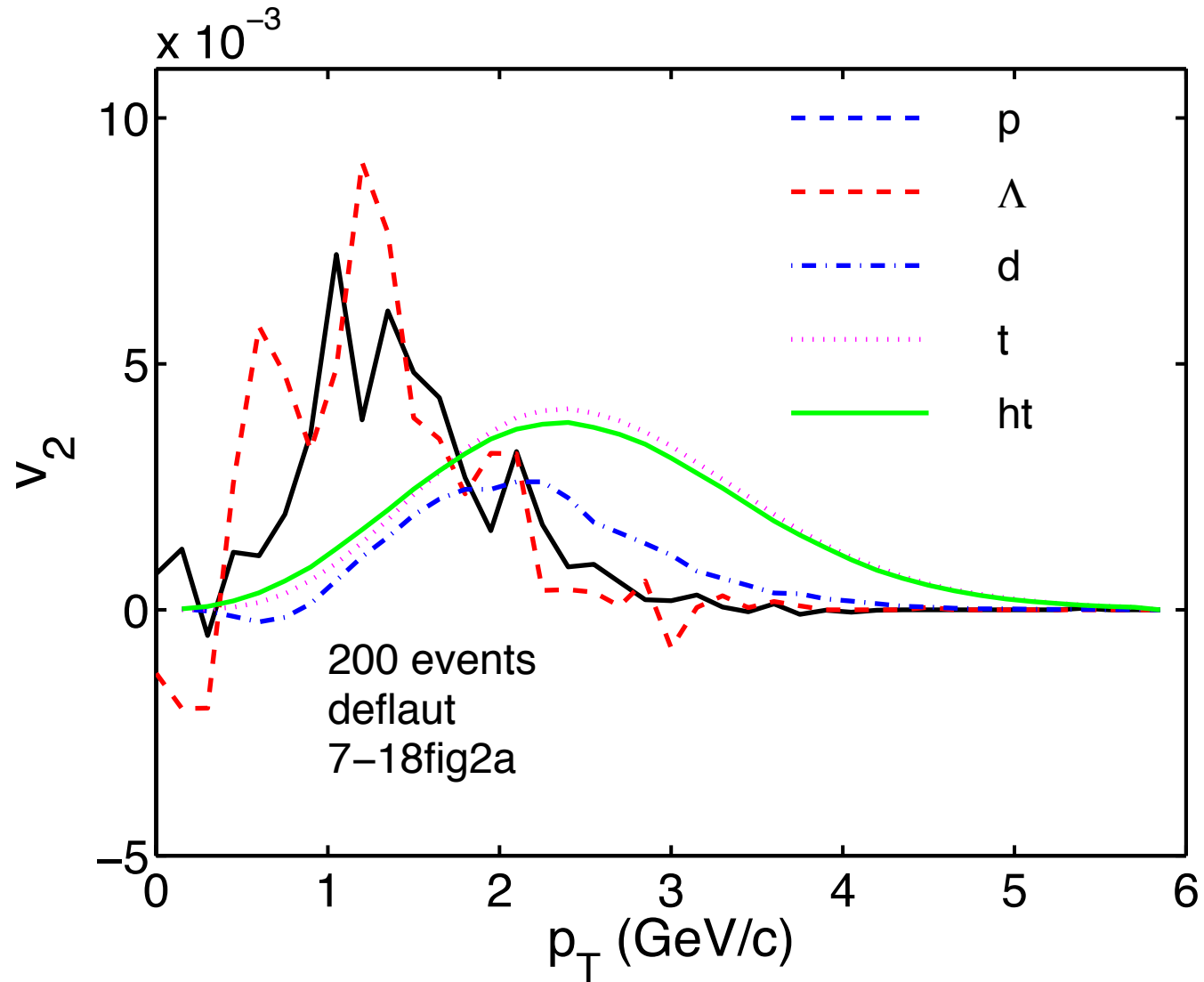


- Symmetry energy effects are about 51%, 73%, and 100% on deuteron, triton and He yields with stiffer one producing more
- Symmetry energy effects stronger on lower energy light clusters
- Effects of isoscalar energy and NN cross sections small

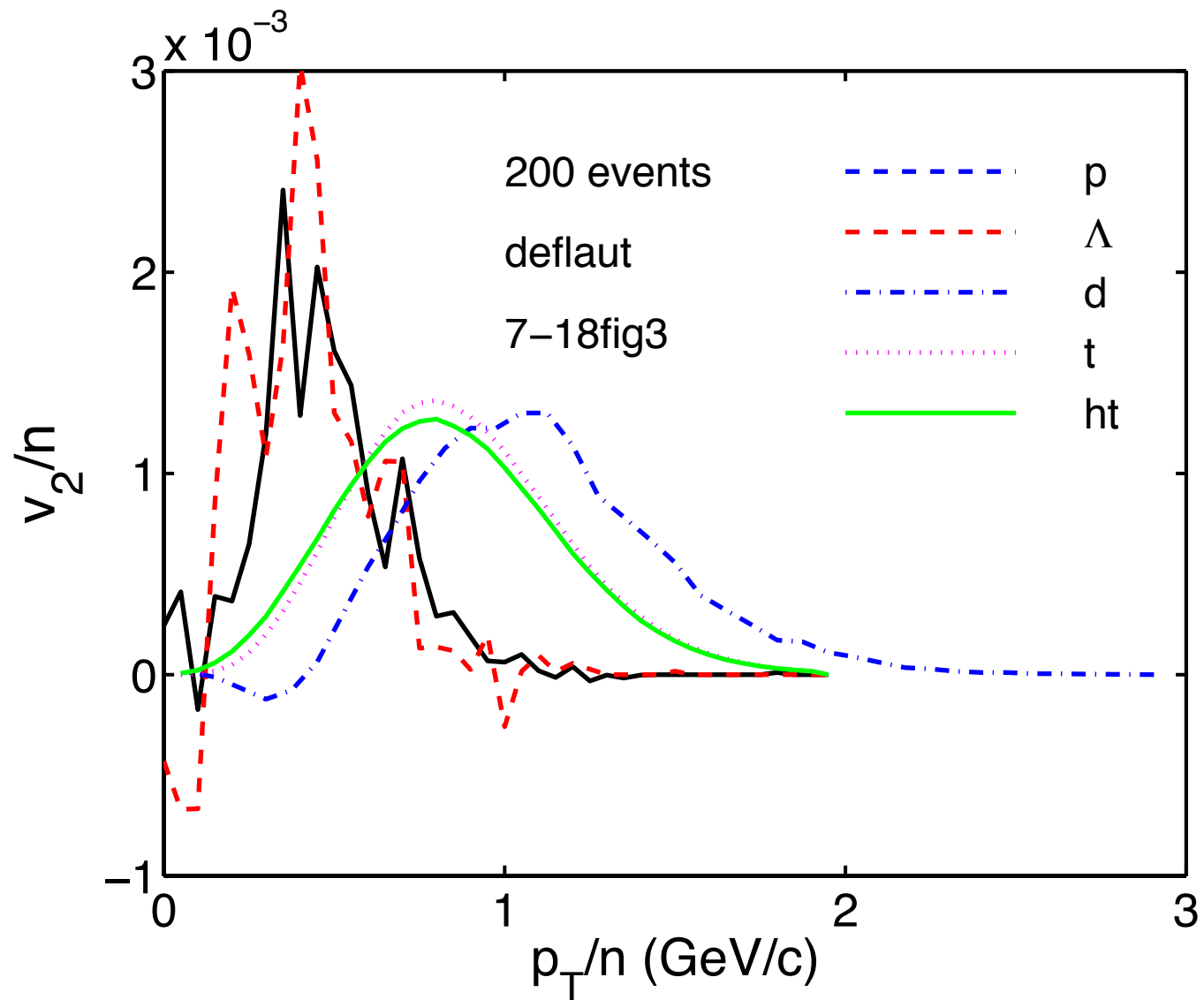
Preliminary results for Pb+Pb @ 2.76 GeV from AMPT



Elliptic flows (preliminary)



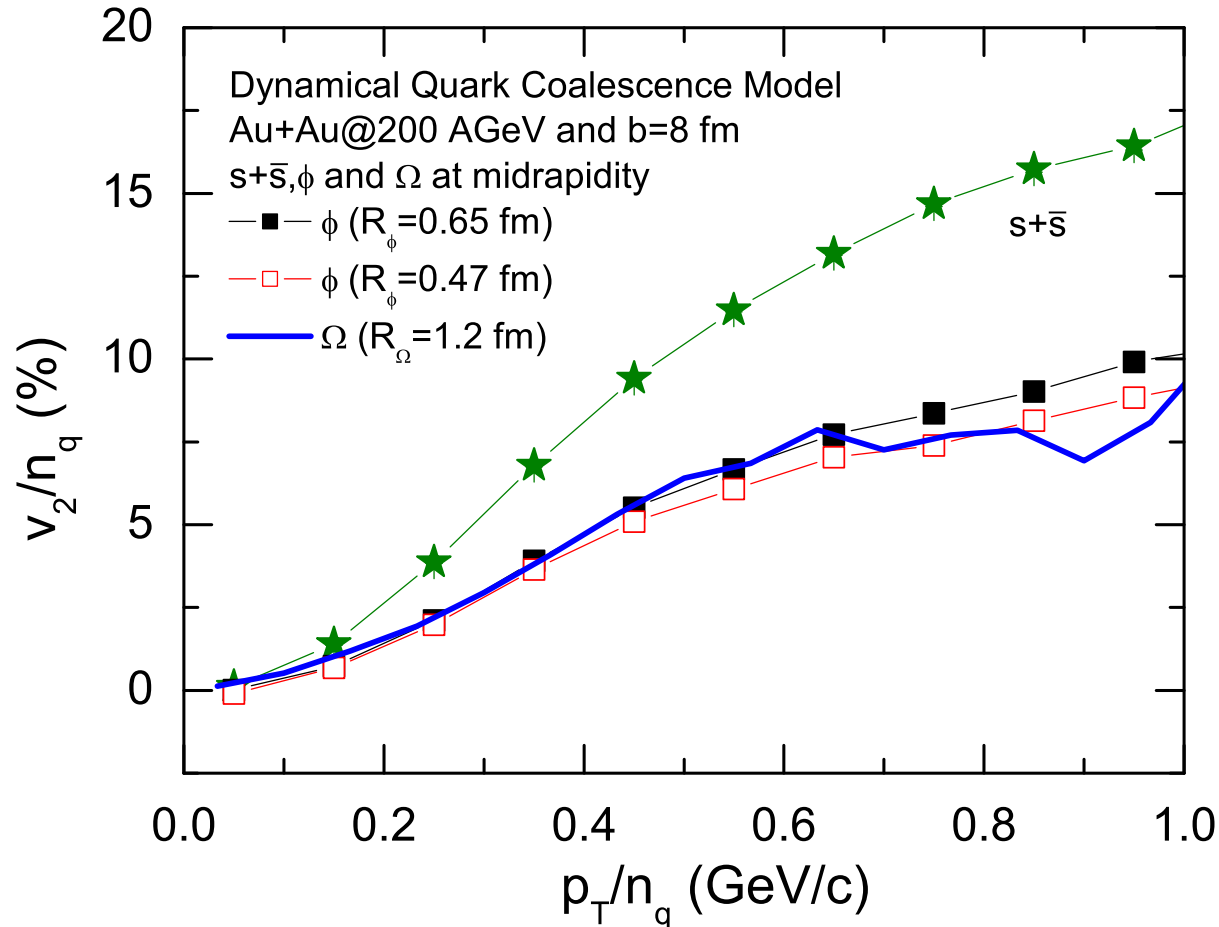
Nucleon number scaled elliptic flows (preliminary)



Dynamical quark coalescence model

Chen & Ko, PRC 73,
044903 (06)

Based on the phase-space distribution of strange quarks from AMPT and including quark spatial and momentum distributions in hadrons



Although scaled phi and Omega satisfy constituent quark number scaling, they are smaller than the strange quark elliptic flow.

Summary and conclusions

- Deuteron
 - Both transport and coalescence models are used in studying deuteron transverse momentum spectrum and elliptic flow in relativistic HIC
 - Final-state hadronic scattering has small effect on deuteron transverse momentum spectrum but appreciable effect on elliptic flow
 - Very similar p_T spectrum from transport and coalescence models
 - Somewhat different elliptic flows from transport and coalescence models → only approximate nucleon number scaling
 - Negative elliptic flow at low p_T is not obtained in either transport nor coalescence model
- Triton and helium and their hypernuclei
 - Coalescence model describes qualitatively their p_T spectra at LHC
 - Nucleon number scaling of v_2 is likely