Tetraquarks in a Bethe-Salpeter approach

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EMMI Workshop:
Anti-matter, hyper-matter and exotica production at the LHC

CERN, July 21, 2015

thanks to:
Christian Fischer
Walter Heupel

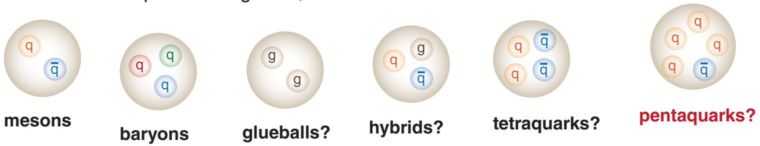
Outline

- **Introduction**
- **Some background:**
  Dyson-Schwinger & Bethe-Salpeter equations,
  applications to mesons and baryons
- **Tetraquarks** as meson-meson / diquark-antidiquark systems
  Heupel, GE, Fischer, PLB 718 (2012)
- **Tetraquarks** as **four-quark** systems
  Heupel, GE, Fischer, in preparation
- **Summary**

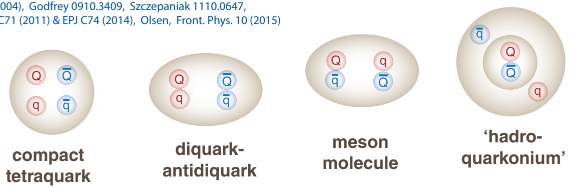
Introduction

QCD Lagrangian: $\mathcal{L} = \bar{\psi}(x)\,(i\partial\!\!\!/ + g A\!\!\!/ - M)\,\psi(x) - \frac{1}{4}\,F^a_{\mu\nu}\,F^{\mu\nu}_a$

- if it were only that simple...
  we don't measure quarks and gluons, but **hadrons**

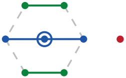


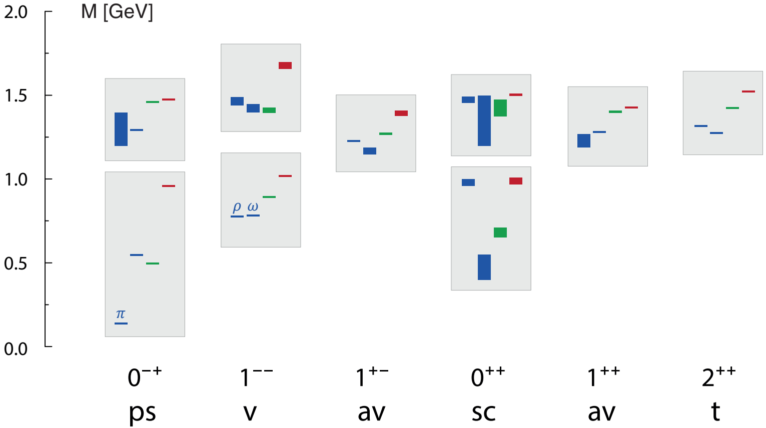
- Growing evidence for four-quark states in **charmonium & bottomonium** spectrum:
  X(3872), Y(4260), charged Z states, ...
  Swanson, PLB 588 (2004), Godfrey 0910.3409, Szczepaniak 1110.0647,
  Brambilla et al., EPJ C71 (2011) & EPJ C74 (2014), Olsen, Front. Phys. 10 (2015)



Introduction

Light meson spectrum (PDG):
grouped with J^{PC} and flavor content





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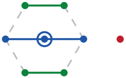

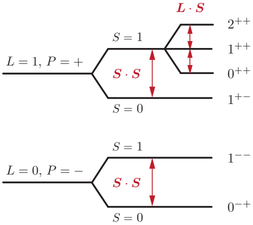

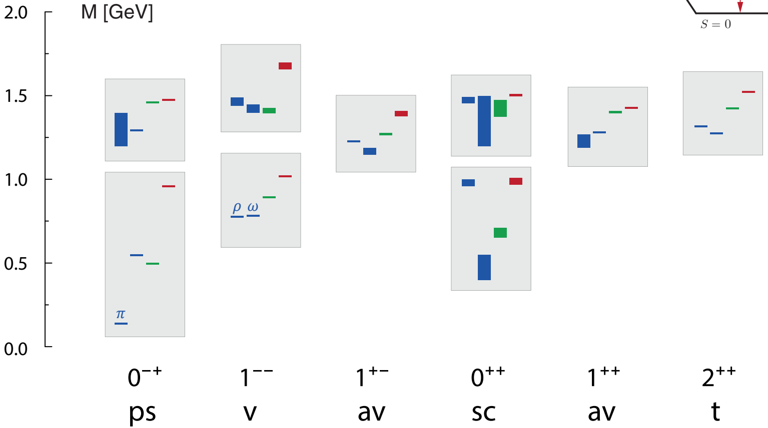

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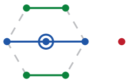

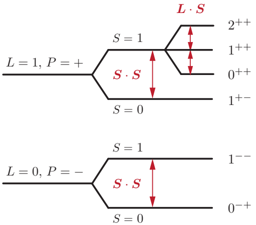

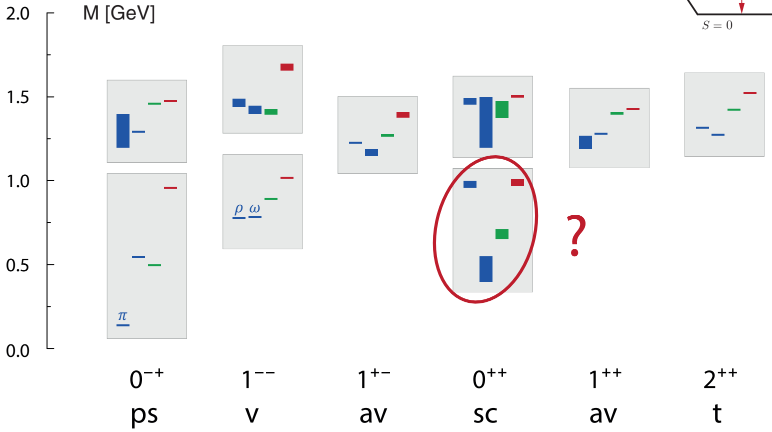

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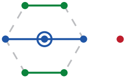
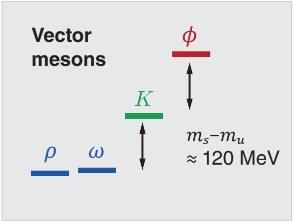
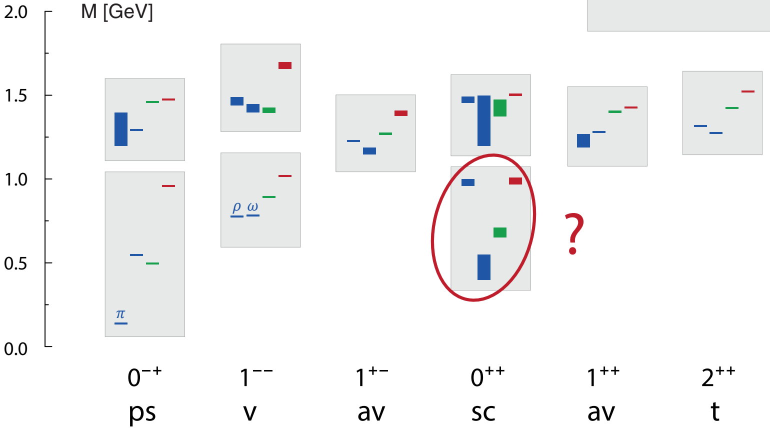
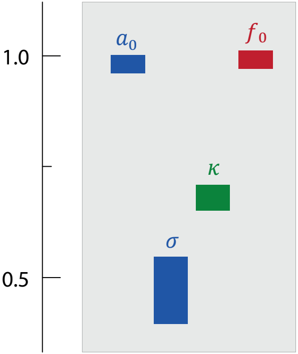


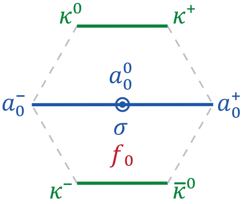




Introduction

But **light scalar** (0^{++}) **mesons** don't fit into the conventional meson spectrum:





f_0 (980 MeV) $s\bar{s}$

κ (680 MeV) $u\bar{s}, d\bar{s}$

a_0 (980 MeV) } $u\bar{u}, d\bar{d}, u\bar{d}$

σ (500 MeV) }

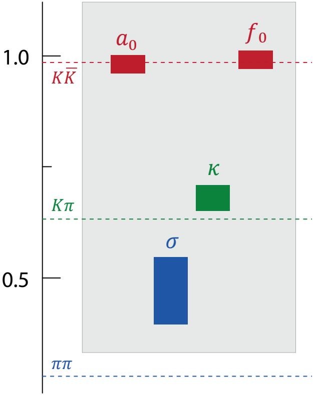
- Why are a_0, f_0 mass-degenerate?
- Why are their **decay widths** so different?

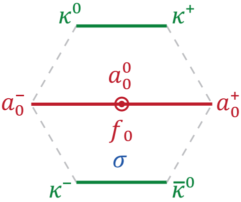
  $\Gamma(\sigma, \kappa) \approx 550$ MeV

  $\Gamma(a_0, f_0) \approx 50$–$100$ MeV
- Why are they so **light**?
  Scalar mesons ~ **p-waves,** should have
  masses similar to axialvector & tensor mesons ~ 1.3 GeV

Introduction

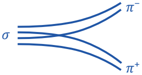
What if they were **tetraquarks** (diquark-antidiquark)? Jaffe 1977, Close, Tornqvist 2002, Maiani, Polosa, Riquer 2004





f_0 (980 MeV) } $us\bar{u}\bar{s}$, ...
a_0 (980 MeV) }
κ (800 MeV) $us\bar{u}\bar{d}$, ...
σ (500 MeV) $ud\bar{u}\bar{d}$

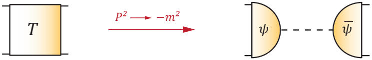
- Explains **mass ordering**: f_0, a_0 have two strange quarks
- Explains **decay widths**:
  f_0 and a_0 couple to $K\bar{K}$,
  large widths for σ, κ



- Alternative: **meson molecules?**
  Weinstein, Isgur 1982, 1990, Close, Isgur, Kumano 1993
- Large Nc, unitarized ChPT, quark models, ELSM, ...
  Pelaez 2004, Weinberg 2013, Cohen, Llanes-Estrada, Pelaez, Ruiz de Elvira 2014,
  Londergan, Nebreda, Pelaez, Szczepaniak 2013, Giacosa 2006, Parganlija, Giacosa, Rischke 2010, . . .

Bethe-Salpeter equations

- Extract hadron properties from **poles** in $q\bar{q}, qqq, qq\overline{qq}$ **scattering matrices:**



- defines onshell **Bethe-Salpeter amplitude.** Simplest example: **pion**

$$\psi(q,P) = \gamma_5 \left(f_1 + f_2 \not{P} + f_3 \not{q} + f_4 [\not{q}, \not{P}] \right) \otimes \text{Color} \otimes \text{Flavor}$$

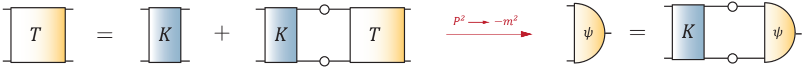
most general Dirac-Lorentz structure,
Lorentz-invariant dressing functions:

$$f_i = f_i(q^2, q \cdot P, P^2 = -m^2)$$

$\Longrightarrow$ pion is made of **s waves** and **p waves!**
(relative momentum ~ orbital angular momentum)

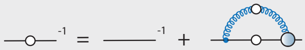
- Use **scattering equation** (inhomogeneous BSE) to obtain T in the first place: $T = K + K\,G_0\,T$

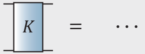
Homogeneous BSE
for **BS amplitude:**



Bethe-Salpeter equations

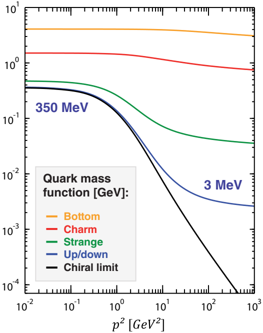
Kernel is closely related to **quark Dyson-Schwinger equation:**





- **Dynamical breaking of chiral symmetry** generates “constituent- quark masses”

$$S_0(p) = \frac{-i\not{p}+m}{p^2+m^2} \longrightarrow S(p) = \frac{1}{A(p^2)}\,\frac{-i\not{p}+M(p^2)}{p^2+M^2(p^2)}$$



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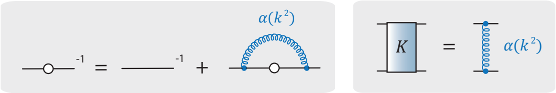

Rainbow-ladder:
tree-level vertex +
effective coupling

$$\alpha(k^2) = \alpha_{\mathrm{IR}}\left(\tfrac{k^2}{\Lambda^2}, \eta\right) + \alpha_{\mathrm{UV}}(k^2)$$

Maris, Roberts, Tandy,
PRC 56 (1997), PRC 60 (1999)

- **Dynamical breaking of chiral symmetry** generates "constituent- quark masses"

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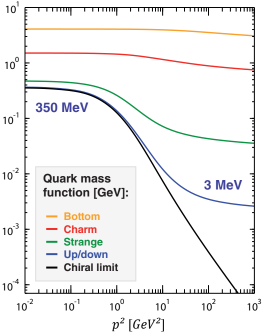
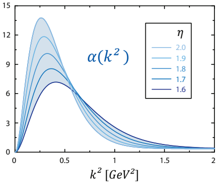
- **Vector & axial symmetries** automatically preserved:

  ⇒ Goldstone theorem, massless pion in χL

  ⇒ em. current conservation

  ⇒ Goldberger-Treiman

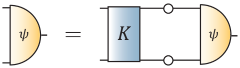




Adjust scale Λ to observable, keep width η as parameter

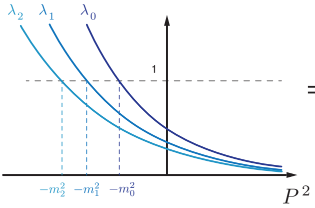
Bethe-Salpeter equations

- BS amplitude makes only sense **onshell**, but homogeneous BSE = **eigenvalue equation**, can be solved for offshell momenta:

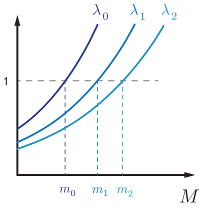


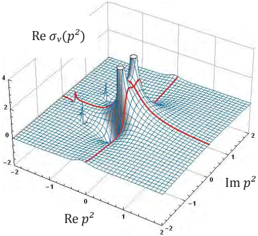
$$K\,\psi_i = \lambda_i(P^2)\,\psi_i\,, \qquad \lambda_i \xrightarrow{P^2 \,\to\, -m_i^2} 1$$

- Largest eigenvalue ⇔ ground state, smaller ones ⇔ excitations



⇒





- Restricted by singularity structure in **quark propagator** (but no **physical threshold!**): mesons: $M < 2m_p$, baryons: $M < 3m_p\,,\;\; m_p \sim 500MeV$

⇒ include residues (numerically difficult) or **extrapolate eigenvalue**

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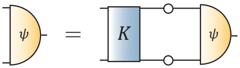

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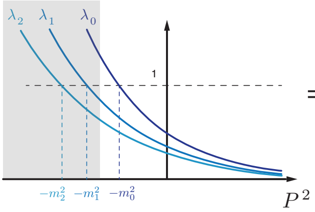
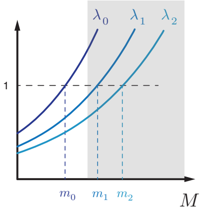
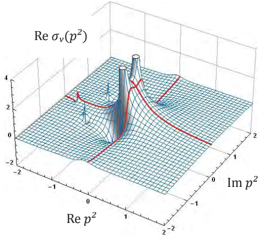


⇒





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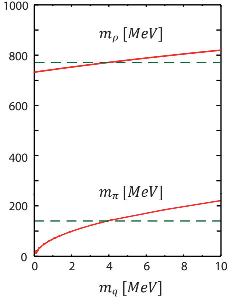
⇒ include residues (numerically difficult) or **extrapolate eigenvalue**

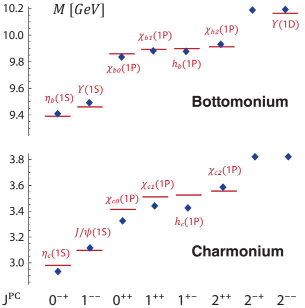
Mesons

- Rainbow-ladder works well for **pseudoscalar & vector mesons:** masses, form factors, decays, ...

  Maris, Roberts, Tandy, PRC 56 (1997), PRC 60 (1999);
  Bashir et al., Commun. Theor. Phys. 58 (2012)

Pion is Goldstone boson, satisfies GMOR: $m_\pi{}^2 \sim m_q$





- **Heavy mesons**

  Blank, Krassnigg, PRD 84 (2011),
  Hilger et al., PRD 91 (2015),
  Fischer, Kubrak, Williams, EPJ A 51 (2015)

- Rainbow-ladder good for **'s-wave'** dominated states

- Need to go **beyond rainbow-ladder** for scalar & axialvector mesons, excited states, η-η', ...

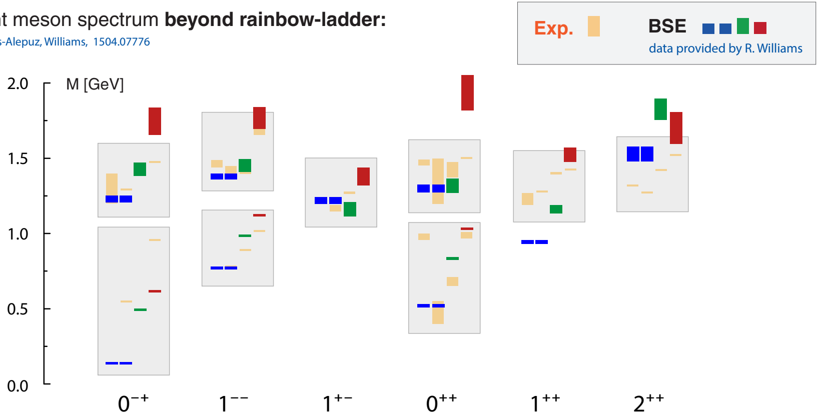
  Fischer, Williams & Chang, Roberts, PRL 103 (2009)
  Alkofer et al., EPJ A38 (2008), ...

e.g. σ meson: 600-700 MeV in RL ⟶ ?

Mesons

Light meson spectrum **beyond rainbow-ladder:**

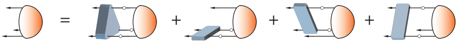
Sanchis-Alepuz, Williams, 1504.07776



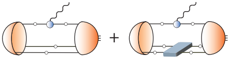
- Gluon propagator & three-gluon vertex **consistent with QCD**, quark-gluon vertex solved in the process. No need for model interaction!
- **Radial excitations** and **exotics** now in the right ballpark. Scalars?

Baryons

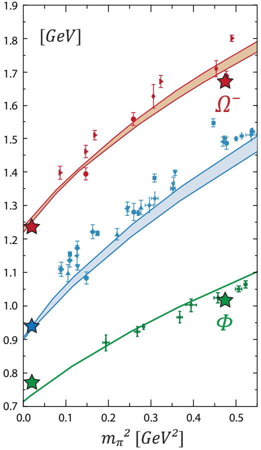
- Covariant Faddeev equation for **baryons:**
  keep 2-body interactions & rainbow-ladder,
  but no further approximations: $M_N = 0.94\,\text{GeV}$
  GE, Alkofer, Krassnigg, Nicmorus, PRL 104 (2010), GE, PRD 84 (2011),
  Sanchis-Alepuz, Fischer, PRD 90 (2014), Sanchis-Alepuz, Fischer, Kubrak, PLB 733 (2014)



- **Baryon form factors:**
  nucleon and Δ FFs, $N\to\Delta\gamma$ transition
  GE, PRD 84 (2011), Sanchis-Alepuz, Williams, Alkofer, PRD 87 (2013),
  Alkofer, GE, Sanchis-Alepuz, Williams, 1412.8413



- **Scattering amplitudes:**
  Compton scattering
  GE & Fischer, PRD 85 (2012) & PRD 87 (2013)
  hadronic light-by-light for muon g-2
  GE, Fischer, Heupel 1505.06336



Delta:
Sanchis-Alepuz et al., PRD 84 (2011)

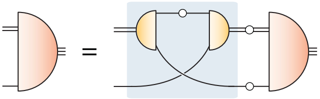
Nucleon:
GE, Alkofer, Krassnigg, Nicmorus, PRL 104 (2010);
GE, PRD 84 (2011)

ρ–meson:
Maris & Tandy, PRC 60 (1999)

Tetraquarks: two-body equation

Use **quark-diquark model** as template:

- Assumption: separable qq scattering matrix ⇒
  Faddeev equation simplifies to **quark-diquark BSE**



Oettel, Hellstern, Alkofer, Reinhardt, PRC 58 (1998),
Cloet, GE, El-Bennich, Klahn, Roberts, Few Body Syst. 46 (2009)
. . .

- **Quark exchange** between quark & diquark binds nucleon

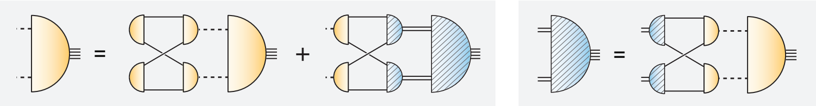
- All quark and diquark properties calculated from quark level,
  same rainbow-ladder interaction:
  scalar diquark $\sim$ **800 MeV, axialvector diquark** $\sim$ **1 GeV**

- N and Δ masses & form factors very similar:
  quark-diquark model is good approximation for three-body equation
  Nucleon and Δ electromagnetic FFs, $N \to \Delta\pi$ decay, $N \to \Delta\gamma$ transition
  GE, Cloet, Alkofer, Krassnigg, Roberts, PRC 79 (2009), Nicmorus, GE, Alkofer, PRD 82 (2010),
  Mader, GE, Blank, Krassnigg, PRD 84 (2011), GE, Nicmorus, PRD 85 (2012)

Tetraquarks: two-body equation

Use **quark-diquark model** as template:

- Assumption: separable qq, $q\bar{q}$ scattering matrices ⇒ coupled **diquark-antidiquark / meson-meson** equations: Heupel, GE, Fischer, PLB 718 (2012)



- **Quark exchange** between mesons and diquarks binds tetraquark
- Coupled equations can be contracted into single **meson-meson equation**, where diquarks appear only internally (not vice versa!)

  ⇒ **meson molecule** with **diquark-antidiquark admixture!**

So far:

- 0^{++}, isoscalar, 4 identical quarks: $nn\overline{nn}$, $ss\overline{ss}$, $cc\overline{cc}$, ...
- keep only **pseudoscalar meson** and **scalar diquark**, calculated in rainbow-ladder

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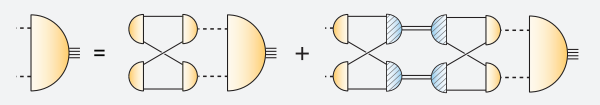

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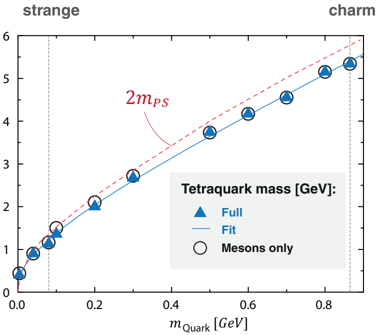
Tetraquarks: two-body equation

Tetraquark masses:
Heupel, GE, Fischer, PLB 718 (2012)

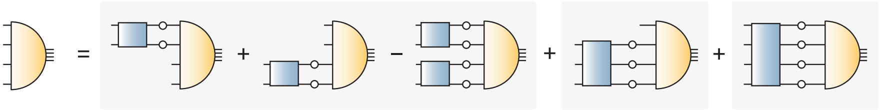
- up/down: $m \sim 400$ MeV $\Leftrightarrow$ σ / f_0 (500)?
- The σ is so light because it 'feels' **Goldstone nature of the pion** - diquarks completely irrelevant!
- Resolves problem with diquark-antidiquark interpretation: ' 2 x 800 MeV - binding energy ' ~ 500 MeV?!
- **All-strange** tetraquark: $m \sim 1.2$ GeV
  all-charm tetraquark: $m \sim 5.3$ GeV
  (below $2\eta_c$ threshold)



$\Rightarrow$ Artifact of 2-body approximation or genuine result?
What about κ, a_0 / f_0 ?

Tetraquarks: four-body equation

Start from **four-quark bound-state equation:**



Two-body interactions:

- $K \otimes I + I \otimes K - K \otimes K$ structure necessary to prevent overcounting in T-matrix $T = K + K\,G_0\,T$

  Kvinikhidze & Khvedelidze, Theor. Math. Phys. 90 (1992)

- plus permutations:

  $(qq)(\bar{q}\bar{q}),\ (q\bar{q})(q\bar{q}),\ (q\bar{q})(q\bar{q})$

  (12) (34)   (23) (14)   (13) (24)

Three-body interactions (+ permutations)

Four-body interactions

Keep **two-body interactions** with **rainbow-ladder kernel:** well motivated by many other studies, tetraquark is **s-wave**

Structure of the amplitude

General structure of **Bethe-Salpeter amplitude** $\Gamma(p,q,k,P)$ complicated:

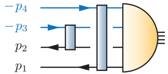
$$\Gamma(p,q,k,P) = \sum_i f_i\left(p^2, q^2, k^2, \quad \ldots \quad\right) \quad \tau_i(p,q,k,P) \quad \otimes \quad \text{2 Color tensors} \quad \otimes \quad \text{Flavor}$$

9 Lorentz invariants

256 Dirac-Lorentz tensors

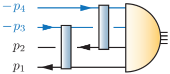
2 Color tensors: $3 \otimes \bar{3}, \quad 6 \otimes \bar{6}$ or $1 \otimes 1, \quad 8 \otimes 8$ (Fierz-equivalent)

$$P = p_1 + p_2 + p_3 + p_4$$



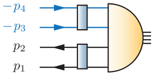
$$p = \tfrac{1}{2}\left(p_2 + p_3 - p_1 - p_4\right)$$

's channel'



$$q = \tfrac{1}{2}\left(p_3 + p_1 - p_2 - p_4\right)$$

'u channel'



$$k = \tfrac{1}{2}\left(p_1 + p_2 - p_3 - p_4\right)$$

't channel'

Structure of the amplitude

General structure of **Bethe-Salpeter amplitude** $\Gamma(p,q,k,P)$ complicated:

$$\Gamma(p,q,k,P) = \sum_i f_i\left(p^2, q^2, k^2, \quad \dots \quad\right) \quad \tau_i(p,q,k,P) \quad \otimes \quad \text{2 Color tensors} \quad \otimes \quad \text{Flavor}$$

9 Lorentz invariants

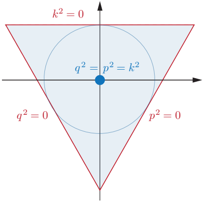
256 Dirac-Lorentz tensors

2 Color tensors: $3 \otimes \bar{3}$, $6 \otimes \bar{6}$ or $1 \otimes 1$, $8 \otimes 8$ (Fierz-equivalent)

Arrange Lorentz invariants into
multiplets of permutation group S4:
GE, Fischer, Heupel, 1505.06336

$\Rightarrow f_i\left(\mathcal{S}_0, \bigtriangledown,$  $,$  $\right)$

- **Singlet:** $\mathcal{S}_0 = \frac{1}{4}(p^2+q^2+k^2)$
- **Doublet:** $\mathcal{D}_0 = \frac{1}{4\mathcal{S}_0}\begin{bmatrix}\sqrt{3}\,(q^2-p^2) \\ p^2+q^2-2k^2\end{bmatrix}$
- **2 Triplets:**  , 



Keep **s waves** only:
Fierz-complete, **16** tensors:

e.g. $\left\{\begin{matrix} C^T\gamma_5 \otimes \gamma_5 C \\ C^T\gamma^\mu \otimes \gamma^\mu C \\ \dots \end{matrix}\right\}$ in (12)(34)

automatically includes also
$\gamma_5 \otimes \gamma_5$ in (23)(14), (31)(24)

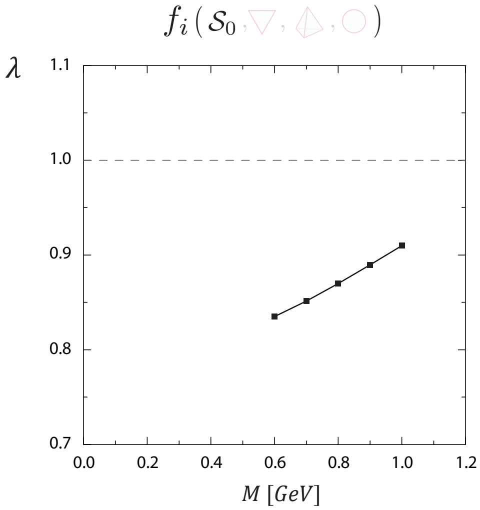
BSE eigenvalue

Homogeneous BSE = **eigenvalue equation**, solve for offshell momenta:

$$K\,\psi_i = \lambda_i(P^2)\,\psi_i\,,$$

Largest eigenvalue ⇔ ground state

$$\lambda_i \xrightarrow{P^2 \,\to\, -M_i^2} 1$$



BSE eigenvalue

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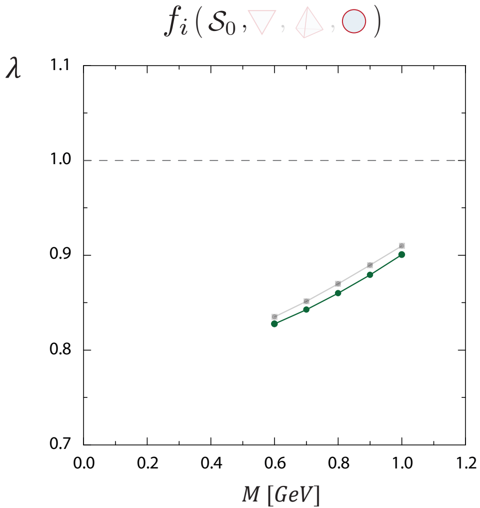

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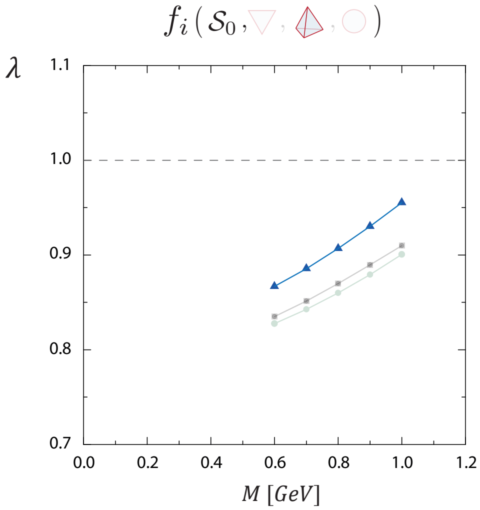


BSE eigenvalue

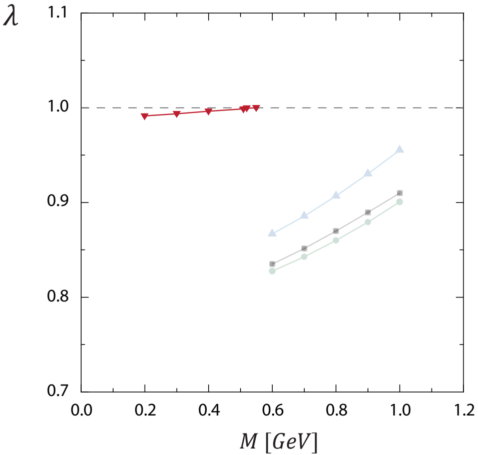
Homogeneous BSE = **eigenvalue equation**, solve for offshell momenta:

$$K\,\psi_i = \lambda_i(P^2)\,\psi_i\,,$$

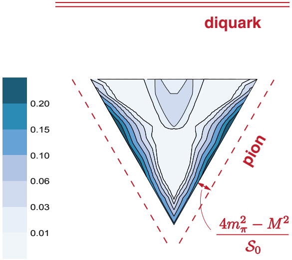
Largest eigenvalue ⇔ ground state

$$\lambda_i \xrightarrow{P^2 \,\longrightarrow\, -M_i^2} 1$$

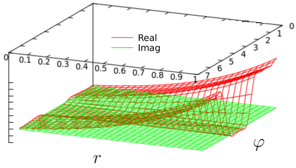
$f_i\,(\,\mathcal{S}_0\,,\nabla\,,\ \ ,\ \ )$



Pion poles in $f_i\,(\mathcal{S}_0, \triangledown, \triangle, \bigcirc)$



Gap in Mandelstam triangle due to **pion poles,** diquarks far away ⇒ irrelevant

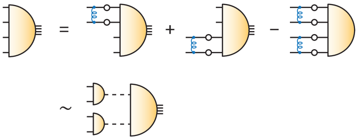


- Four-body equation dynamically generates **pion poles outside the integration domain,** although equation knows nothing about pions

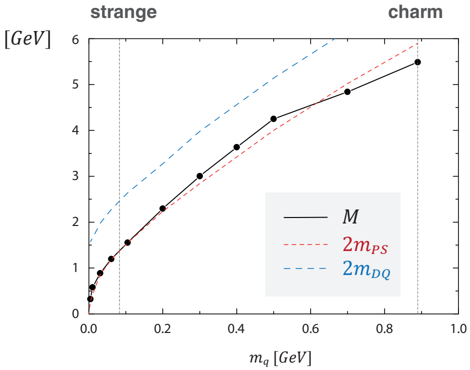
- drive tetraquark mass from 1.4 GeV to ~500 MeV

- **Poles enter integration domain** above threshold $M > 2m_\pi$: the tetraquark becomes a **resonance**

- Four-quark equation produces **bound state** together with its **decay channels!**



Tetraquark mass



Evolution with
current-quark mass:

Resonance close to
$\pi\pi$ threshold, becomes
bound state in charm region

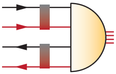
Same pattern for multiplet partners:

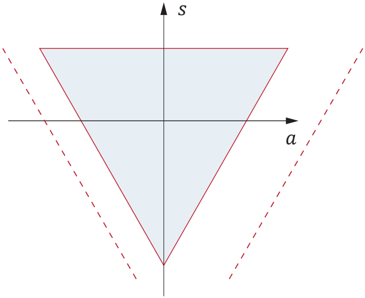
$\sigma \sim$ **380 MeV**, $\kappa \sim$ **700 MeV**, $a_0 / f_0 \sim$ **920 MeV**

Outlook?

Diquark-antidiquark:

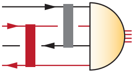
$(n\,c)\,(\bar{n}\,\bar{c})$





Hadro-charmonium:

$(n\,\bar{n})\,(c\,\bar{c})$



Molecule:

$(\bar{n}\,c)\,(n\,\bar{c})$

Summary

- Two-body and four-body equations give consistent results, suggest **light scalar mesons are tetraquarks**

- $\sigma \sim$ **380 MeV**, $\kappa \sim$ **700 MeV**, $a_0 / f_0 \sim$ **920 MeV**

- Dominated by pseudoscalar Goldstone bosons, diquarks irrelevant: **‘meson molecule’** (but resonance)

- Extract **widths?**
  Maybe, not sure yet (look for poles in complex plane)

- Tetraquarks in **heavy-quark regime?**
  Maybe, but rainbow-ladder problematic for heavy-light systems

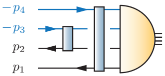
- First solution of **genuine four-quark BSE** (which is also a **resonance!**)

Backup slides

Structure of the amplitude

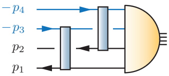
Bethe-Salpeter amplitude $\Gamma(p,q,k,P)$
depends on **four independent momenta:**

$$P = p_1 + p_2 + p_3 + p_4$$



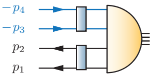
$p = \frac{1}{2}(p_2 + p_3 - p_1 - p_4)$

's channel'



$q = \frac{1}{2}(p_3 + p_1 - p_2 - p_4)$

'u channel'



$k = \frac{1}{2}(p_1 + p_2 - p_3 - p_4)$

't channel'

General structure quite complicated:

$$\Gamma(p,q,k,P) = \sum_i f_i\left(p^2, q^2, k^2, \{\omega_j\}, \{\eta_j\}\right) \tau_i(p,q,k,P) \quad \otimes \quad \text{Color} \quad \otimes \quad \text{Flavor}$$

9 Lorentz invariants:

$$p^2, \quad q^2, \quad k^2$$

$$\omega_1 = q \cdot k \qquad \eta_1 = p \cdot P$$
$$\omega_2 = p \cdot k \qquad \eta_2 = q \cdot P$$
$$\omega_3 = p \cdot q \qquad \eta_3 = k \cdot P$$

$$P^2 = -M^2$$

256 Dirac-Lorentz tensors

2 Color tensors:

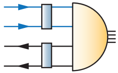
$3 \otimes \bar{3}, \quad 6 \otimes \bar{6}$ or
$1 \otimes 1, \quad 8 \otimes 8$
(Fierz-equivalent)

Structure of the amplitude

Keep **s waves** only:
Fierz-complete, **16** Dirac-Lorentz tensors

#	Structure
1	$(\mathcal{C}^T\gamma_5)_{2,1}\otimes(\gamma_5\mathcal{C})_{3,4}$
2	$\mathcal{C}^T\gamma_5\slashed{P}\otimes\gamma_5\mathcal{C}+\mathcal{C}^T\gamma_5\otimes\gamma_5\slashed{P}\mathcal{C}$
3	$\mathcal{C}^T\gamma_5\slashed{P}\otimes\gamma_5\mathcal{C}$ - $\mathcal{C}^T\gamma_5\otimes\gamma_5\slashed{P}\mathcal{C}$
4	$\mathcal{C}^T\gamma_5\slashed{P}\otimes\gamma_5\slashed{P}\mathcal{C}$
5	$\mathcal{C}^T\gamma^\mu_T\otimes\gamma^\mu_T\mathcal{C}$
6	$\mathcal{C}^T\gamma^\mu_T\slashed{P}\otimes\gamma^\mu_T\mathcal{C}+\mathcal{C}^T\gamma^\mu_T\otimes\gamma^\mu_T\slashed{P}\mathcal{C}$
7	$\mathcal{C}^T\gamma^\mu_T\slashed{P}\otimes\gamma^\mu_T\mathcal{C}$ - $\mathcal{C}^T\gamma^\mu_T\otimes\gamma^\mu_T\slashed{P}\mathcal{C}$
8	$\mathcal{C}^T\gamma^\mu_T\slashed{P}\otimes\gamma^\mu_T\slashed{P}\mathcal{C}$
9	$\mathcal{C}^T\mathbb{1}\otimes\mathbb{1}\mathcal{C}$
10	$\mathcal{C}^T\gamma^\mu_T\slashed{P}\otimes\gamma^\mu_T\mathcal{C}+\mathcal{C}^T\gamma^\mu_T\otimes\gamma^\mu_T\slashed{P}\mathcal{C}$
11	$\mathcal{C}^T\gamma^\mu_T\slashed{P}\otimes\gamma^\mu_T\mathcal{C}$ - $\mathcal{C}^T\gamma^\mu_T\otimes\gamma^\mu_T\slashed{P}\mathcal{C}$
12	$\mathcal{C}^T\gamma^\mu_T\slashed{P}\otimes\gamma^\mu_T\slashed{P}\mathcal{C}$
13	$\mathcal{C}^T\gamma^\mu_T\gamma_5\otimes\gamma^\mu_T\gamma_5\mathcal{C}$
14	$\mathcal{C}^T\gamma^\mu_T\gamma_5\slashed{P}\otimes\gamma^\mu_T\gamma_5\mathcal{C}+\mathcal{C}^T\gamma^\mu_T\gamma_5\otimes\gamma^\mu_T\gamma_5\slashed{P}\mathcal{C}$
15	$\mathcal{C}^T\gamma^\mu_T\gamma_5\slashed{P}\otimes\gamma^\mu_T\gamma_5\mathcal{C}$ - $\mathcal{C}^T\gamma^\mu_T\gamma_5\otimes\gamma^\mu_T\gamma_5\slashed{P}\mathcal{C}$
16	$\mathcal{C}^T\gamma^\mu_T\gamma_5\slashed{P}\otimes\gamma^\mu_T\gamma_5\slashed{P}\mathcal{C}$

Table 2.8: Symmetrized Momentum independent s-wave tensor structures.



$$k=\tfrac{1}{2}\,(p_1+p_2-p_3-p_4)$$

't channel'

$\tau_i(p,q,k,P)$ $\otimes$ Color $\otimes$ Flavor

256 Dirac-Lorentz tensors

2 Color tensors:

$3\otimes\bar{3},\quad 6\otimes\bar{6}$ or
$1\otimes 1,\quad 8\otimes 8$
(Fierz-equivalent)

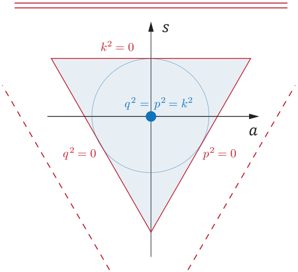
Structure of the amplitude

- **Singlet:** symmetric variable, carries overall scale:

$$\mathcal{S}_0 = \frac{1}{4}\left(p^2 + q^2 + k^2\right)$$

- **Doublet:** $\mathcal{D}_0 = \frac{1}{4\mathcal{S}_0}\begin{bmatrix} \sqrt{3}\,(q^2 - p^2) \\ p^2 + q^2 - 2k^2 \end{bmatrix}$

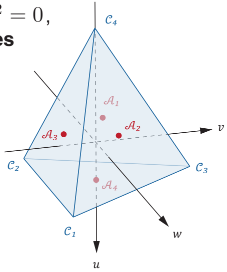
  Mandelstam triangle,
  outside: **meson and diquark poles!**



Lorentz invariants can be grouped into
multiplets of the permutation group S4:
GE, Fischer, Heupel, Williams, 1411.7876

- **Triplet:** $\mathcal{T}_0 = \frac{1}{4\mathcal{S}_0}\begin{bmatrix} 2\,(\omega_1 + \omega_2 + \omega_3) \\ \sqrt{2}\,(\omega_1 + \omega_2 - 2\omega_3) \\ \sqrt{6}\,(\omega_2 - \omega_1) \end{bmatrix}$

  tetrahedron bounded by $p_i^2 = 0$,
  outside: **quark singularities**



- Second **triplet:**
  3dim. sphere

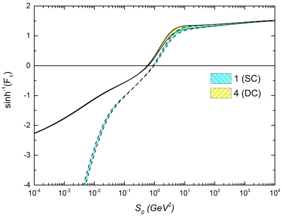
$$\mathcal{T}_1 = \frac{1}{4\mathcal{S}_0}\begin{bmatrix} 2\,(\eta_1 + \eta_2 + \eta_3) \\ \sqrt{2}\,(\eta_1 + \eta_2 - 2\eta_3) \\ \sqrt{6}\,(\eta_2 - \eta_1) \end{bmatrix}$$

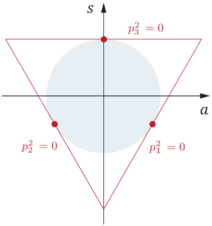
Structure of the amplitude

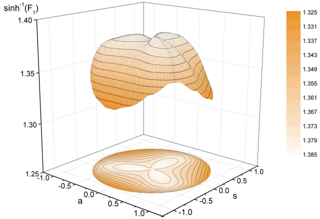
Idea: use symmetries to figure out **relevant** momentum dependence:

- cf. **photon four-point function** ⇔ hadronic LbL scattering contribution to **muon g-2**
  GE, Fischer, Heupel, Williams, 1411.7876

- cf. **three-gluon vertex:** angular variation in Mandelstam plane is negligible, only $\mathcal{S}_0$ relevant GE, Williams, Alkofer, Vujinovic, PRD 89 (2014)

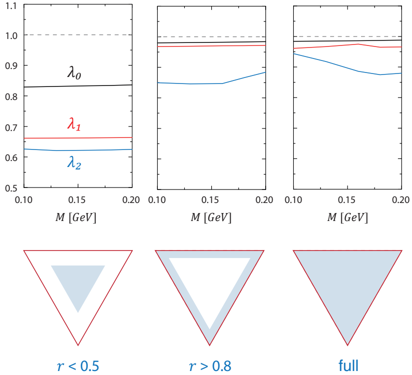






Tetraquark mass

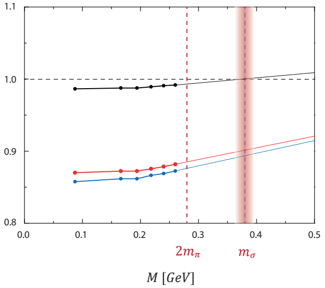
Tetraquark mass driven by momentum dependence close to **r = 1:** visible from phase space cuts (larger eigenvalue ⇔ smaller mass)



But dense eigenvalue spectrum: **spurious states?**

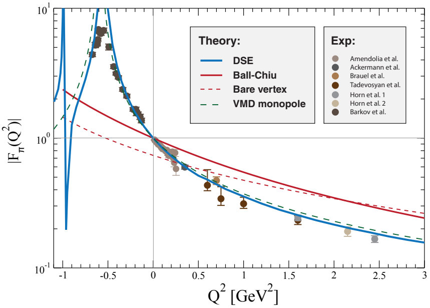
No, just numerical artifact: pion poles at large $\mathcal{S}_0$ (UV!) not properly resolved

⇒ Implement pion (and diquark) poles analytically: ground state unchanged, but low-lying excitations disappear

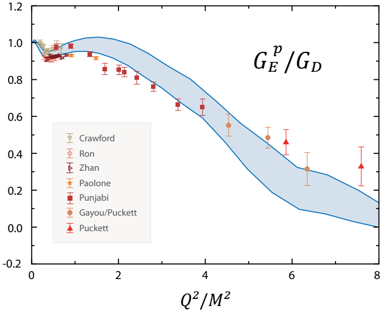


Electromagnetic form factors

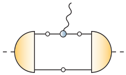
Pion: Maris & Tandy 2006, A. Krassnigg (Schladming 2010)



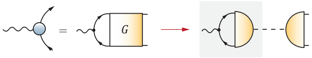
Proton: GE, PRD 84 (2011)



- Form factor from



- **Timelike vector meson poles** automatically generated by quark-photon vertex BSE!

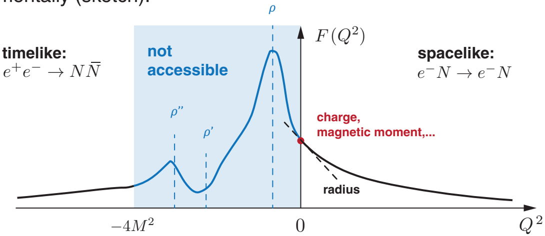


$\Rightarrow \Gamma^\mu =$ **Ball-Chiu** (em. gauge invariance)

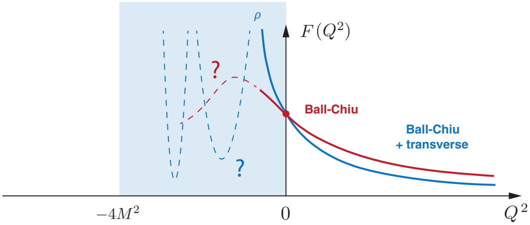
+ **Transverse part** (vm. poles & dominance)

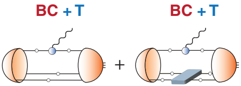
Quark-photon vertex

Structure of quark-photon vertex is reflected in form factors.
Experimentally (sketch):



Calculated:
(Sketch)

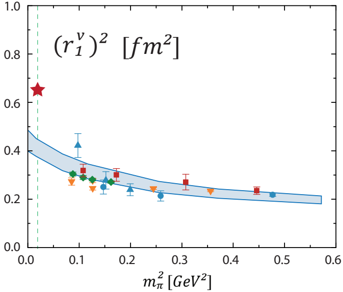




- Ball-Chiu part is dominant **(em. gauge invariance):** charge, magnetic moments
- Transverse part changes slope and charge radii. No pion cloud in RL ⇒ timelike ρ-**meson poles**

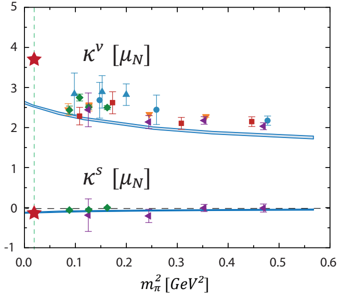
Electromagnetic form factors

Nucleon charge radii:
isovector (p-n) Dirac (F1) radius



- **Pion-cloud effects** missing in chiral region (⇒ divergence!), agreement with lattice at larger quark masses.

Nucleon magnetic moments:
isovector (p-n), isoscalar (p+n)



- **But:** pion-cloud **cancels** in κ^s ⇔ **quark core**

Exp: $\kappa^s = -0.12$
Calc: $\kappa^s = -0.12(1)$ **!!**

GE, PRD 84 (2011)