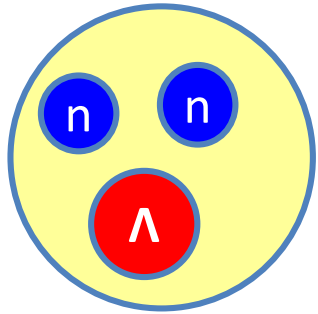


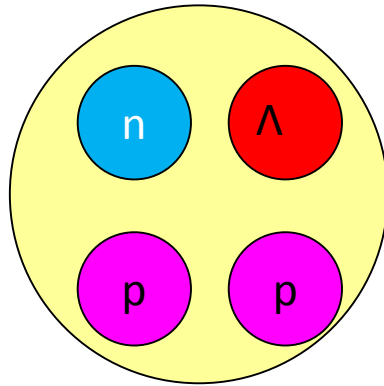
# Few-body aspect of hypernuclei

E. Hiyama (RIKEN)

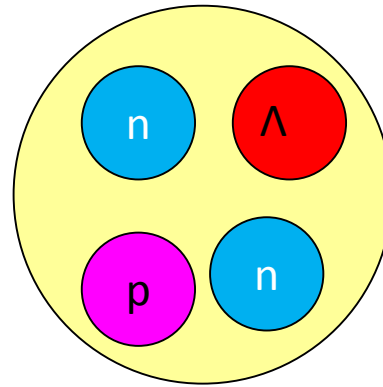
One of the important and interesting subjects :  
to study three- and four-baryon systems



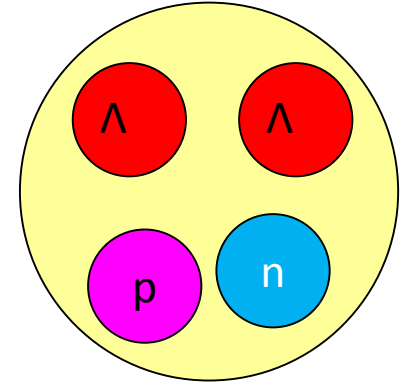
${}^3_{\Lambda}n$



${}^4_{\Lambda}\text{He}$



${}^4_{\Lambda}\text{H}$



${}^4_{\Lambda\Lambda}\text{H}$

C. Rappold et al.,  
HypHI collaboration  
Phys. Rev. C 88,  
041001 (R) (2013)

A. Esser, PRL114,  
232501(2015)

(1) Why is it important to study these three- and four-body systems?

(2) What kind of new understandings do we obtain by solving these systems as three- and four-body problems?

## Our few-body calculational method

Gaussian Expansion Method (GEM) , since 1987 ,

- A variational method using Gaussian basis functions
- Take all the sets of Jacobi coordinates

Developed by Kyushu Univ. Group,  
Kamimura and his collaborators.

Review article :

E. Hiyama, M. Kamimura and Y. Kino,  
Prog. Part. Nucl. Phys. 51 (2003), 223.

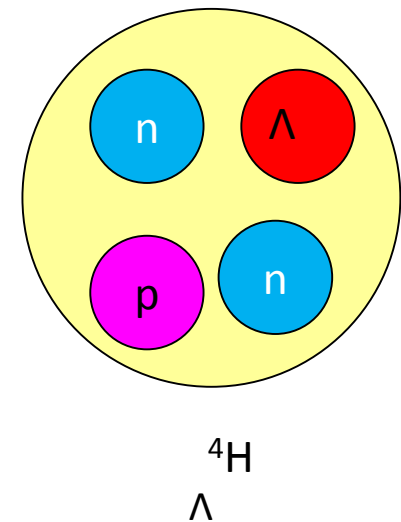
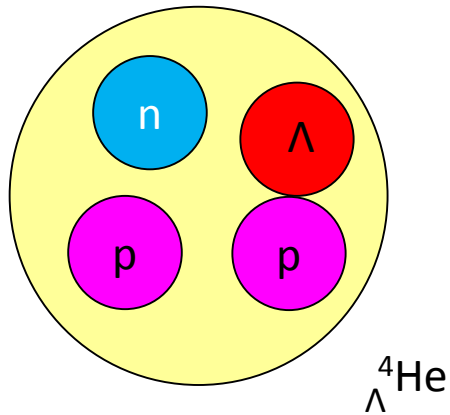
High-precision calculations of various 3- and 4-body systems:

Exsotic atoms / molecules ,	Light hypernuclei,
3- and 4-nucleon systems,	3-quark systems,
multi-cluster structure of light nuclei,	

An example of the accuracy of the method:

# Section 1

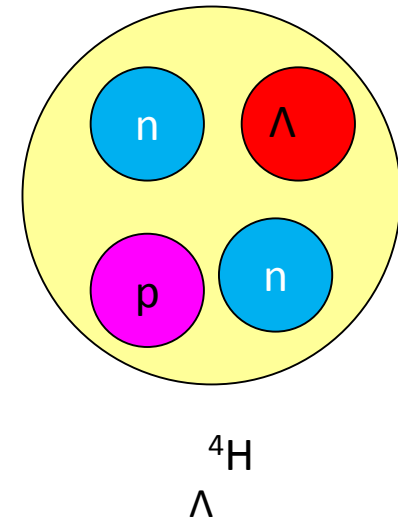
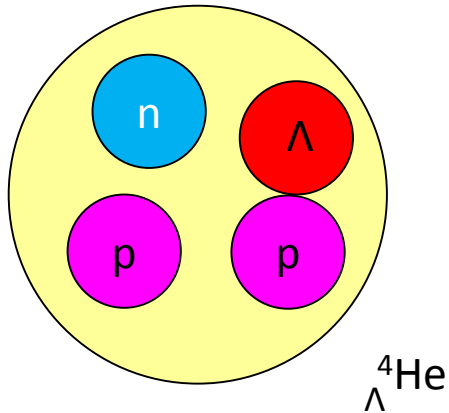
four-body calculation of  ${}^4_{\Lambda}\text{H}$  and  ${}^4_{\Lambda}\text{He}$



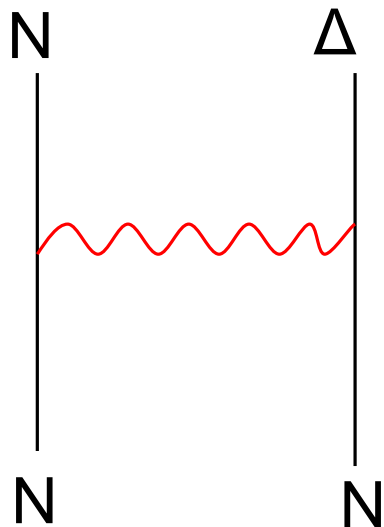
In  $S = -1$  sector,  
important subjects to extract information on YN interaction:

(1) Charge symmetry breaking

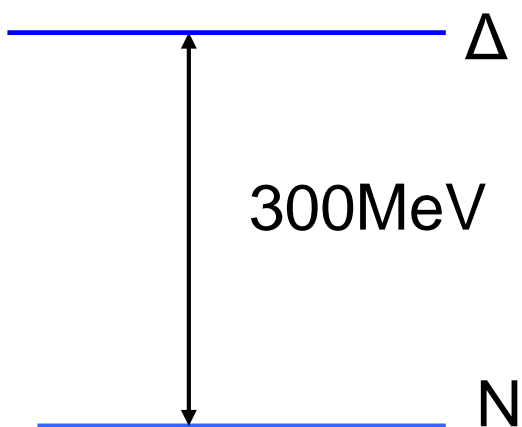
(2)  $\Lambda N - \Sigma N$  coupling



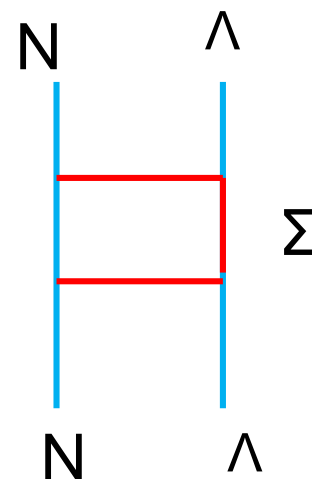
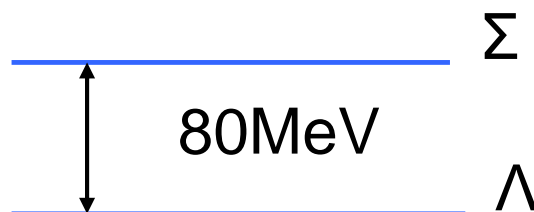
# Non-strangeness nuclei



Nucleon can be converted into  $\Delta$ .  
However, since mass difference between nucleon and  $\Delta$  is large, then probability of  $\Delta$  in nucleus is not large.

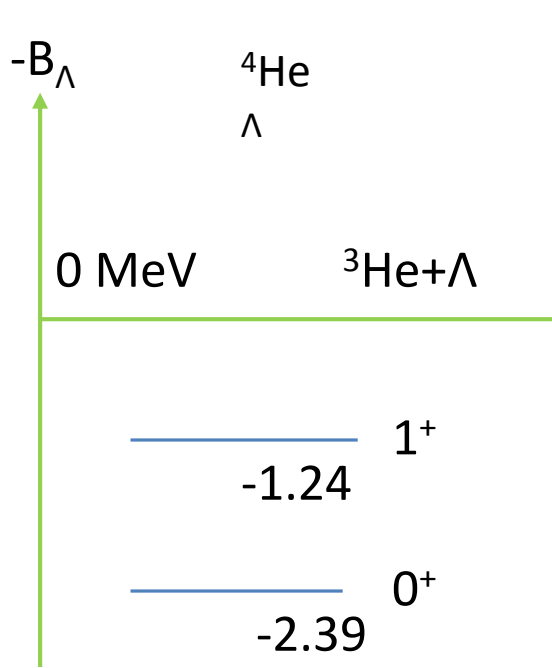


On the other hand, the mass difference between  $\Lambda$  and  $\Sigma$  is much smaller, then  $\Lambda$  can be converted into  $\Sigma$  particle easily.

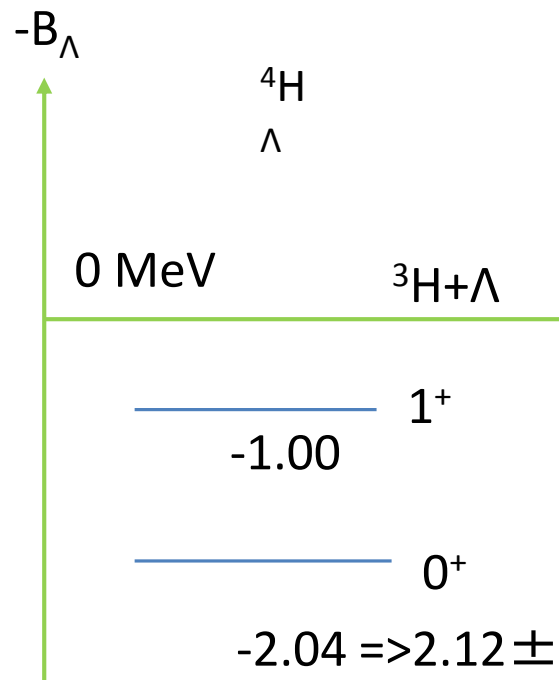


## Interesting Issues for the $\Lambda$ N- $\Sigma$ N particle conversion in hypernuclei

- (1) How large is the mixing probability of the  $\Sigma$  particle in the hypernuclei?
- (2) How important is the  $\Lambda$ N— $\Sigma$ N coupling in the binding energy of the  $\Lambda$  hypernuclei?

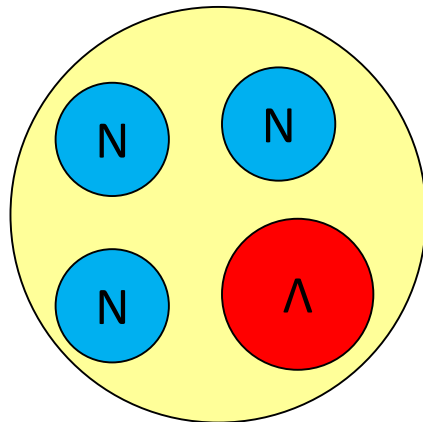


Exp.



Exp.

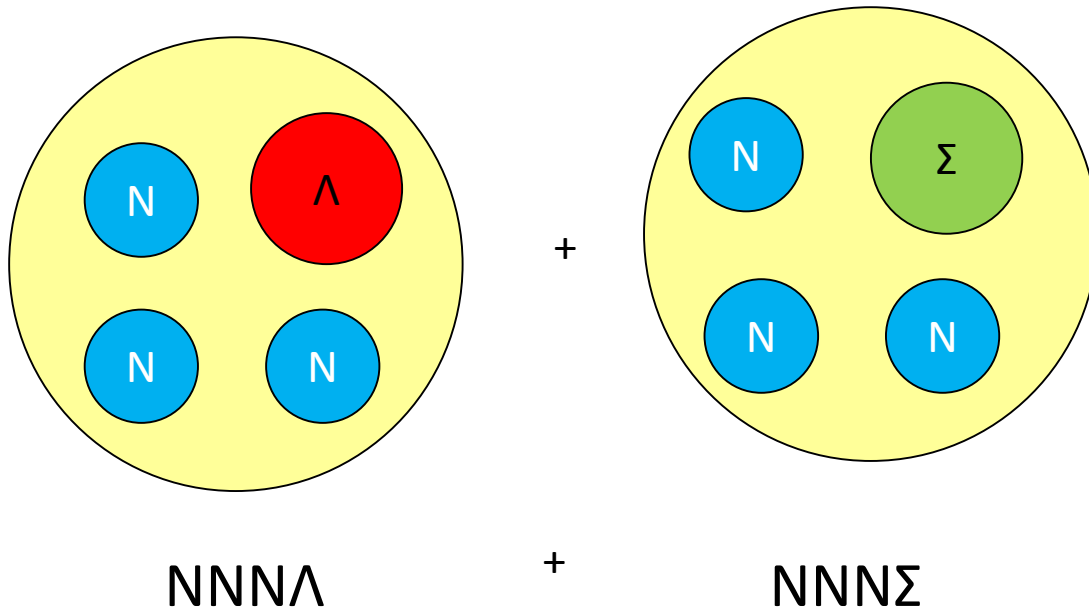
A. Esser, PRL114,  
232501(2015)



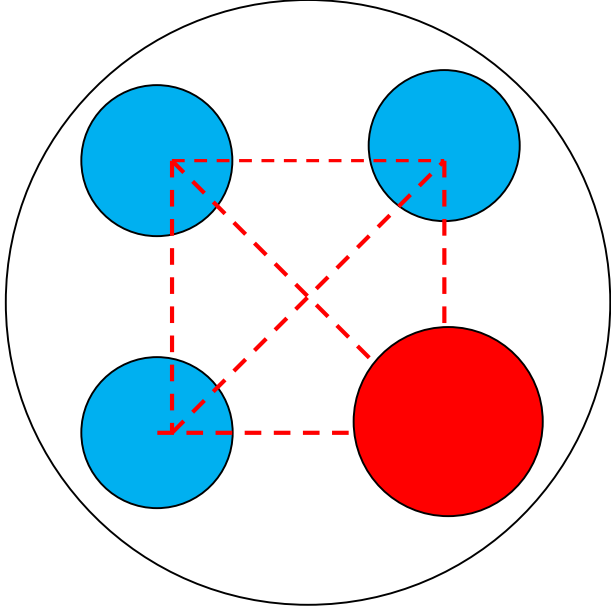
${}^4\text{He}$   
 $\Lambda$

${}^4\text{H}$   
 $\Lambda$





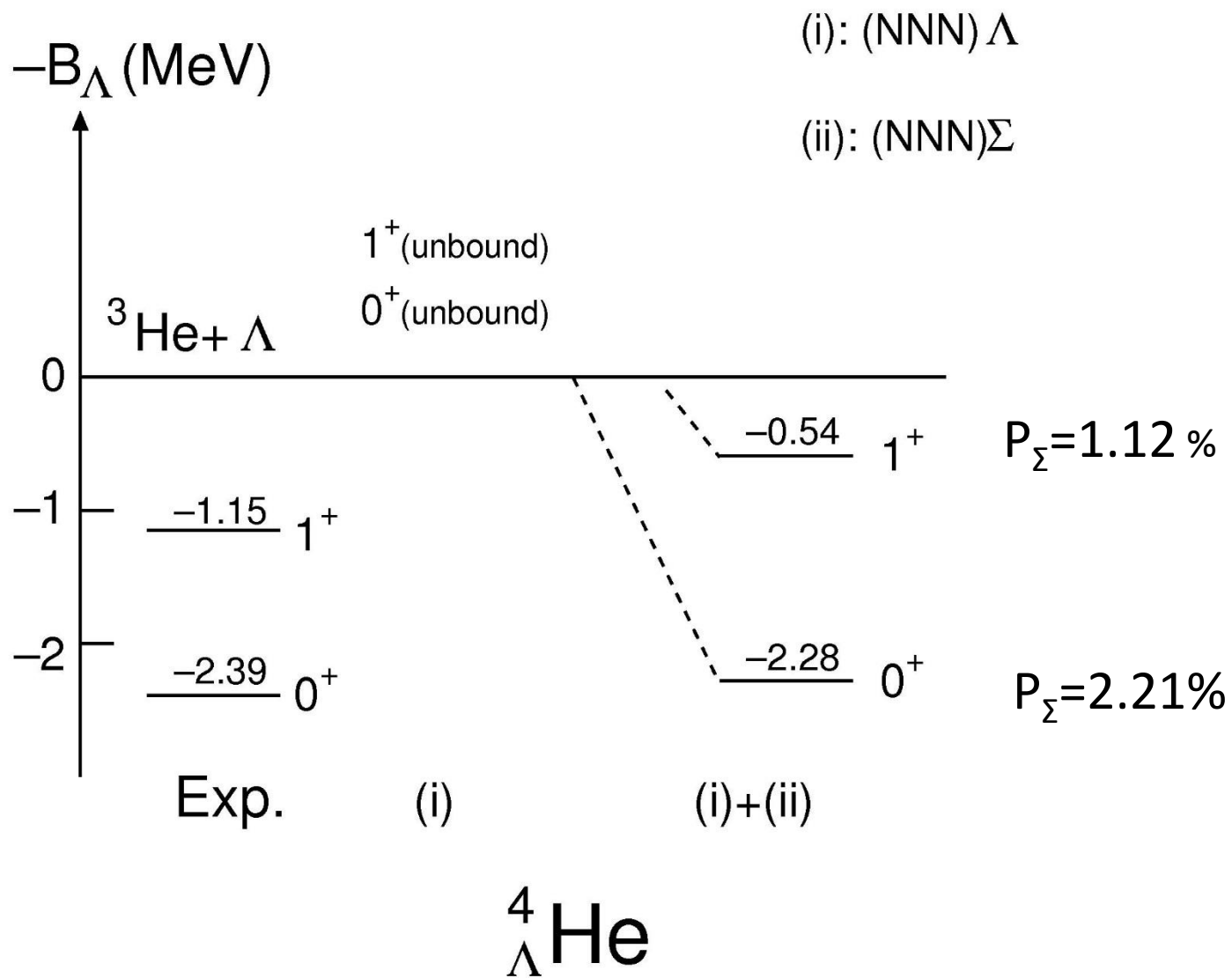
- E. Hiyama et al., Phys. Rev. C65, 011301 (R) (2001).  
 H. Nemura et al., Phys. Rev. Lett. 89, 142502 (2002).  
 A. Nogga et al., Phys. Rev. Lett. 88, 172501 (2002).



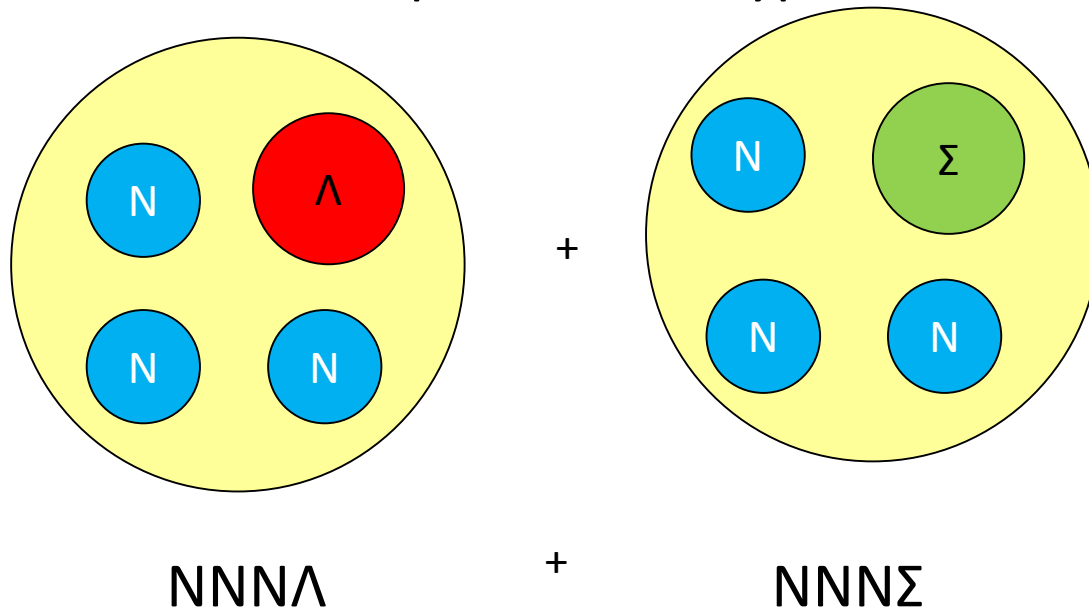
${}^4_{\Lambda}\text{He}, {}^4_{\Lambda}\text{H}$

$V_{NN}$  : AV8 potential

$V_{YN}$  : Nijmegen soft-core '97f potential



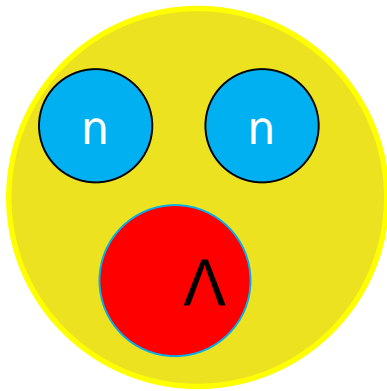
To summarize the part of  $A=4$  hypernuclei



$NNN\Sigma$  channel is essentially important to make  $A=4$  hypernuclei.

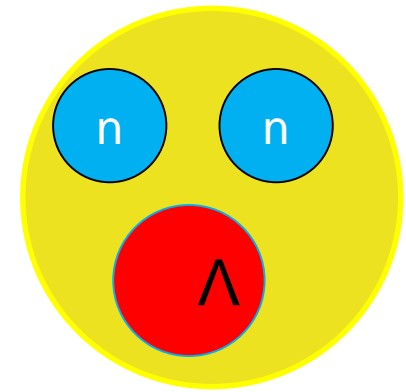
## Section 2

three-body calculation of  ${}^3_{\Lambda}n$



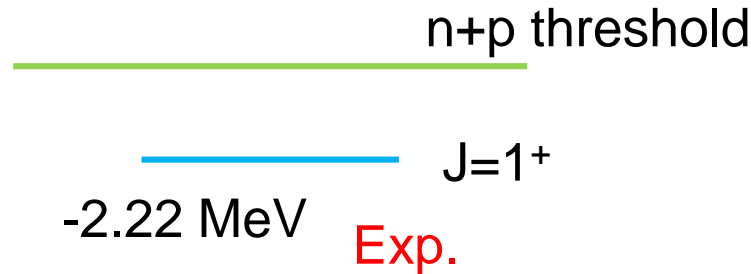
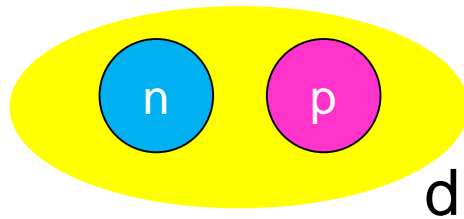
E. Hiyama, S. Ohnishi,  
B.F. Gibson, and T. A. Rijken,  
PRC89, 061302(R) (2014).

What is interesting to study  $nn\Lambda$  system?

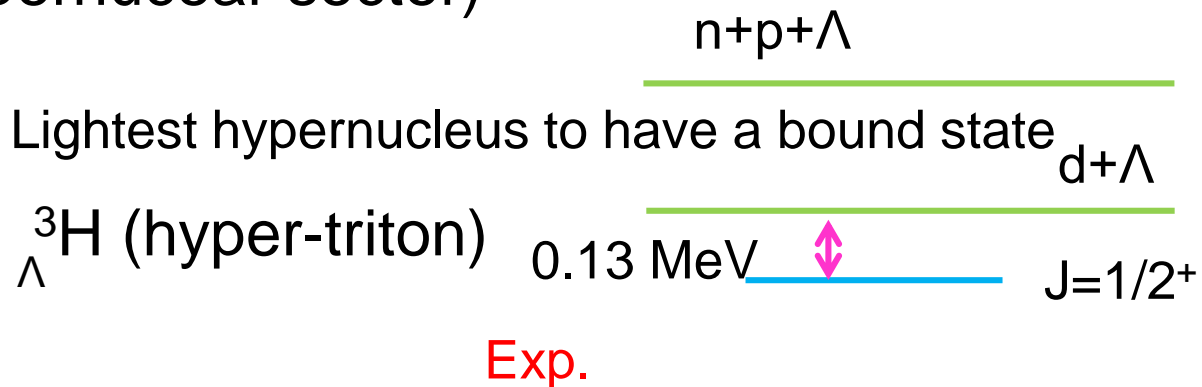
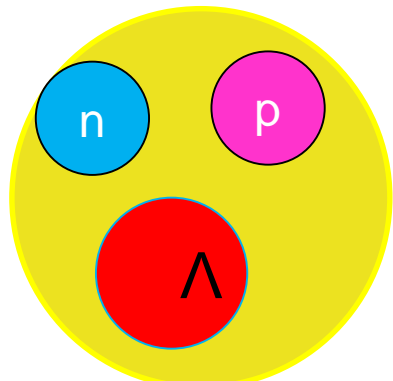


$S=0$

The lightest nucleus to have a bound state is deuteron.

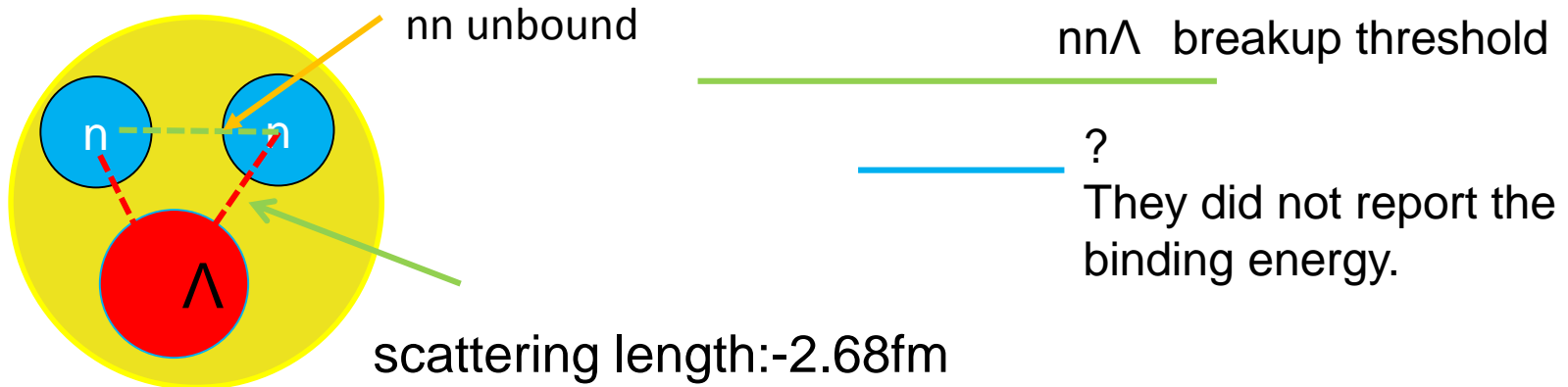


$S=-1$  ( $\Lambda$  hypernuclear sector)



Search for evidence of  ${}^3_{\Lambda}n$  by observing  $d + \pi^-$  and  $t + \pi^-$  final states in the reaction of  ${}^6\text{Li} + {}^{12}\text{C}$  at 2 A GeV

C. Rappold,<sup>1,2,\*</sup> E. Kim,<sup>1,3</sup> T. R. Saito,<sup>1,4,5,†</sup> O. Bertini,<sup>1,4</sup> S. Bianchin,<sup>1</sup> V. Bozkurt,<sup>1,6</sup> M. Kavatsyuk,<sup>7</sup> Y. Ma,<sup>1,4</sup> F. Maas,<sup>1,4,5</sup> S. Minami,<sup>1</sup> D. Nakajima,<sup>1,8</sup> B. Özel-Tashenov,<sup>1</sup> K. Yoshida,<sup>1,5,9</sup> P. Achenbach,<sup>4</sup> S. Ajimura,<sup>10</sup> T. Aumann,<sup>1,11</sup> C. Ayerbe Gayoso,<sup>4</sup> H. C. Bhang,<sup>3</sup> C. Caesar,<sup>1,11</sup> S. Erturk,<sup>6</sup> T. Fukuda,<sup>12</sup> B. Göküzüm,<sup>1,6</sup> E. Guliev,<sup>7</sup> J. Hoffmann,<sup>1</sup> G. Ickert,<sup>1</sup> Z. S. Ketenci,<sup>6</sup> D. Khanef, <sup>1,4</sup> M. Kim,<sup>3</sup> S. Kim,<sup>3</sup> K. Koch,<sup>1</sup> N. Kurz,<sup>1</sup> A. Le Fèvre,<sup>1,13</sup> Y. Mizoi,<sup>12</sup> L. Nungesser,<sup>4</sup> W. Ott,<sup>1</sup> J. Pochodzalla,<sup>4</sup> A. Sakaguchi,<sup>9</sup> C. J. Schmidt,<sup>1</sup> M. Sekimoto,<sup>14</sup> H. Simon,<sup>1</sup> T. Takahashi,<sup>14</sup> G. J. Tambave,<sup>7</sup> H. Tamura,<sup>15</sup> W. Trautmann,<sup>1</sup> S. Voltz,<sup>1</sup> and C. J. Yoon<sup>3</sup>  
(HypHI Collaboration)



Observation of nn $\Lambda$  system (2013)

Lightest hypernucleus to have a bound state

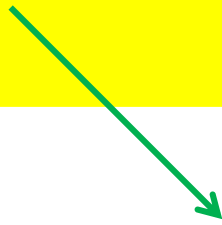
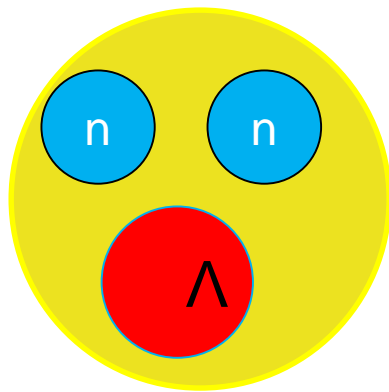
Any two-body systems are unbound. $\Rightarrow$ nn $\Lambda$  system is bound.

Lightest Borromean system.

Theoretical important issue:

Do we have bound state for  $nn\Lambda$  system?

If we have a bound state for this system, how much is binding energy?



$nn\Lambda$  breakup threshold



?

They did not report the binding energy.

NN interaction : to reproduce the observed binding energies of  ${}^3\text{H}$  and  ${}^3\text{He}$

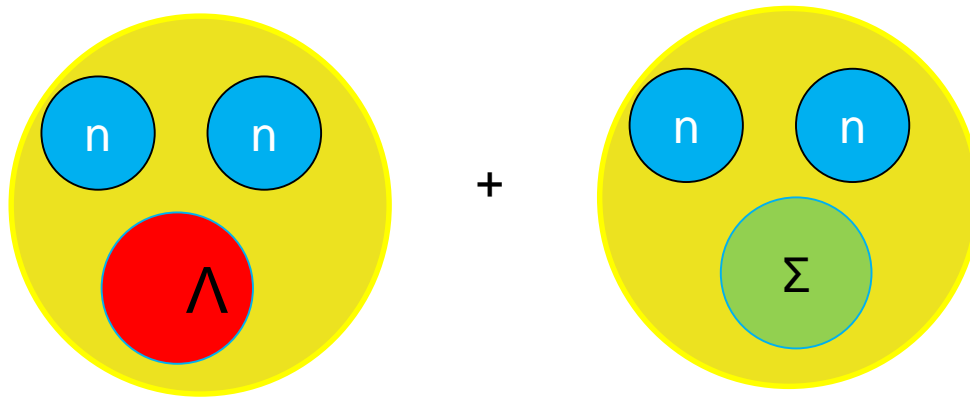
NN: AV8 potential

We do not include 3-body force for nuclear sector.

How about YN interaction?

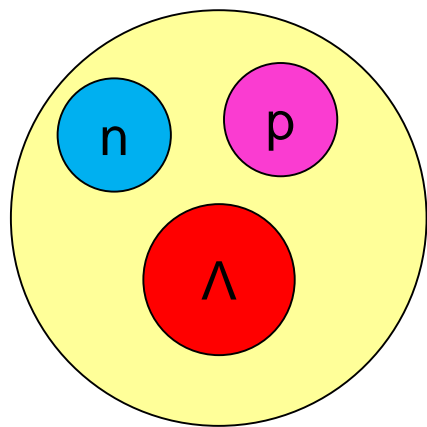


To take into account of  $\Lambda$  particle to be converted into  $\Sigma$  particle, we should perform below calculation using realistic hyperon( $\Lambda$ )-nucleon(N) interaction.



YN interaction: Nijmegen soft core '97f potential (NSC97f)  
proposed by Nijmegen group

reproduce the observed binding energies of  ${}^3_{\Lambda}\text{H}$ ,  ${}^4_{\Lambda}\text{H}$  and  ${}^4_{\Lambda}\text{He}$



${}^3\text{H}_\Lambda$

$-B_\Lambda$

0 MeV

$d+\Lambda$

$1/2^+$

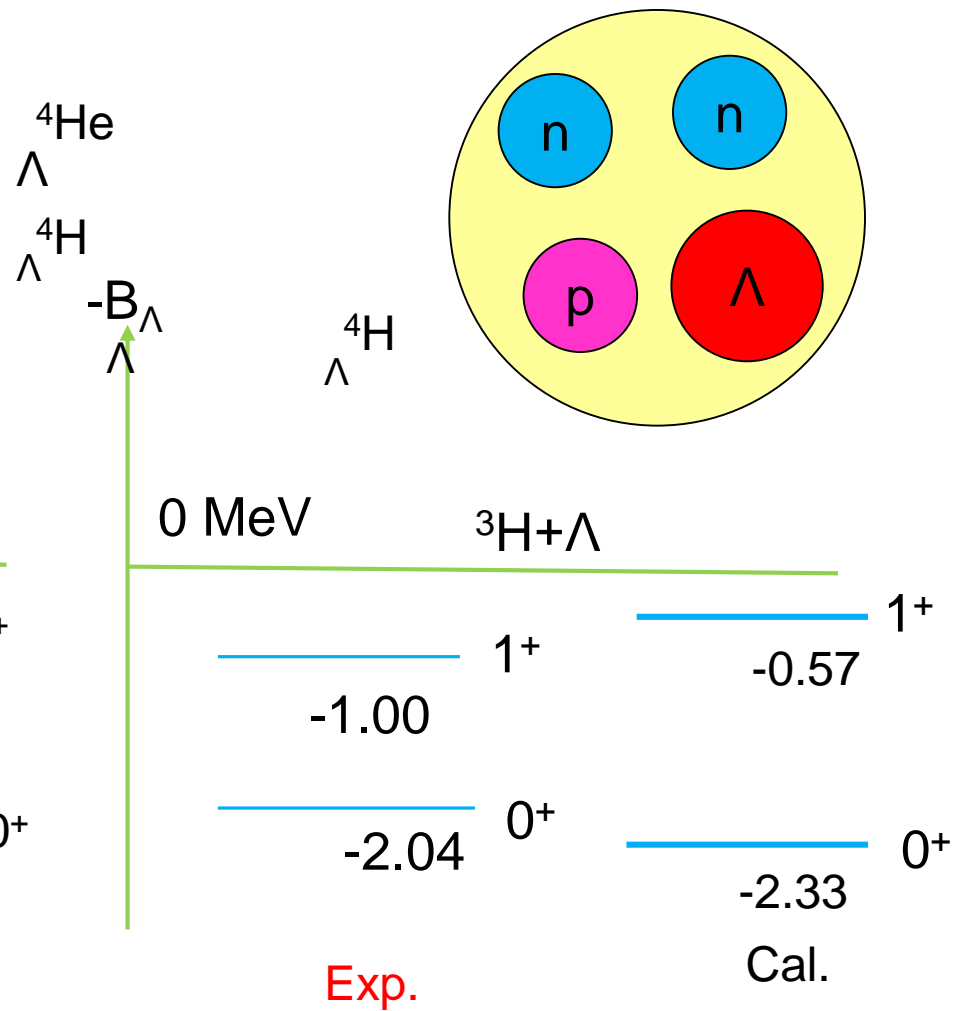
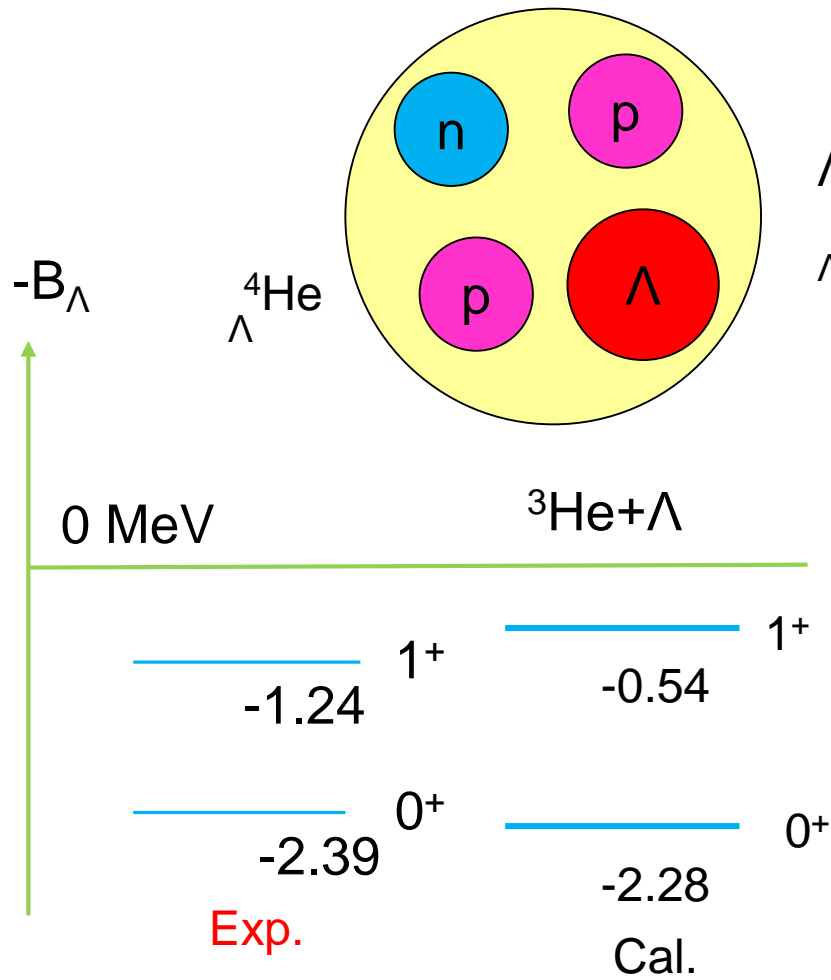
$1/2^+$

$-0.13 \pm 0.05$  MeV

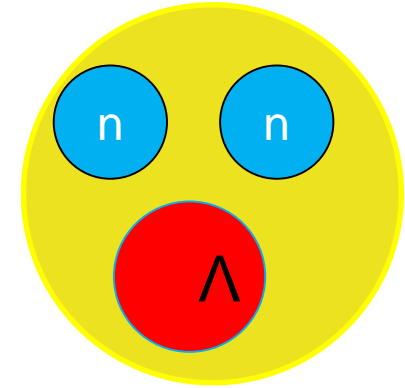
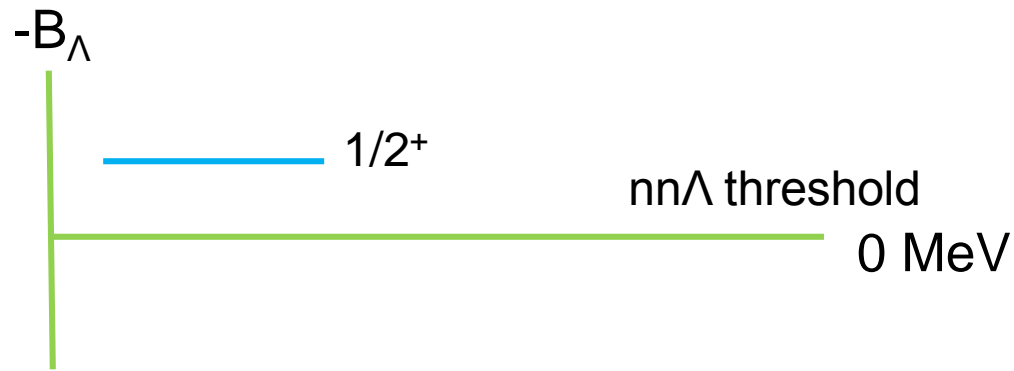
-0.19 MeV

Exp.

Cal.



What is binding energy of  $nn\Lambda$ ?



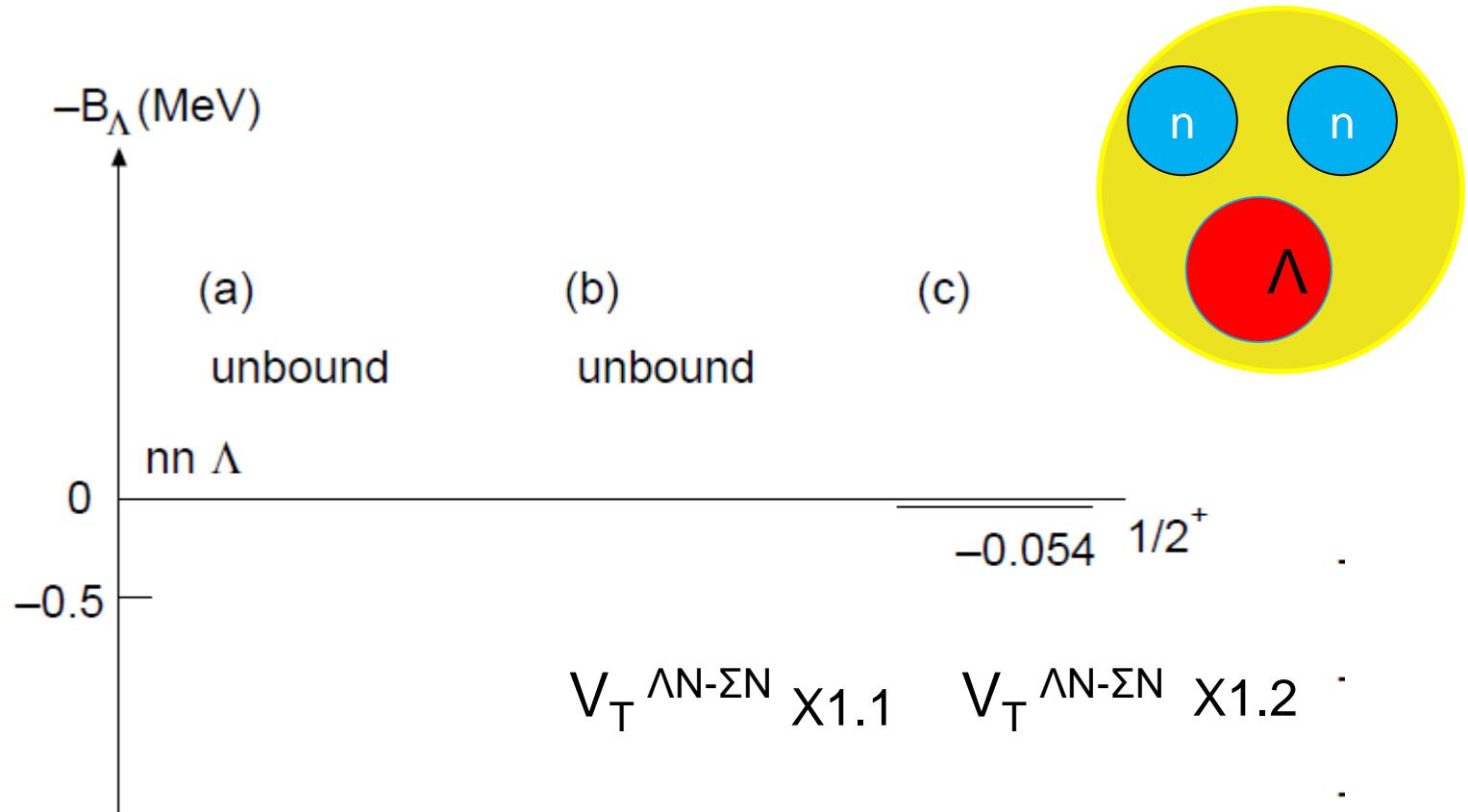
We have no bound state in nn $\Lambda$  system.  
This is inconsistent with the data.

Now, we have a question.

Do we have a possibility to have a bound state in nn $\Lambda$  system tuning strength of YN potential ?

It should be noted to maintain consistency with the binding energies of  ${}^3_\Lambda\text{H}$  and  ${}^4_\Lambda\text{H}$  and  ${}^4_\Lambda\text{He}$ .

$$V_T^{\Lambda N-\Sigma N} \quad \times 1.1, 1.2$$



When we have a bound state in  $nn\Lambda$  system, what are binding energies of  ${}^3_{\Lambda}\text{H}$  and  $A=4$  hypernuclei?

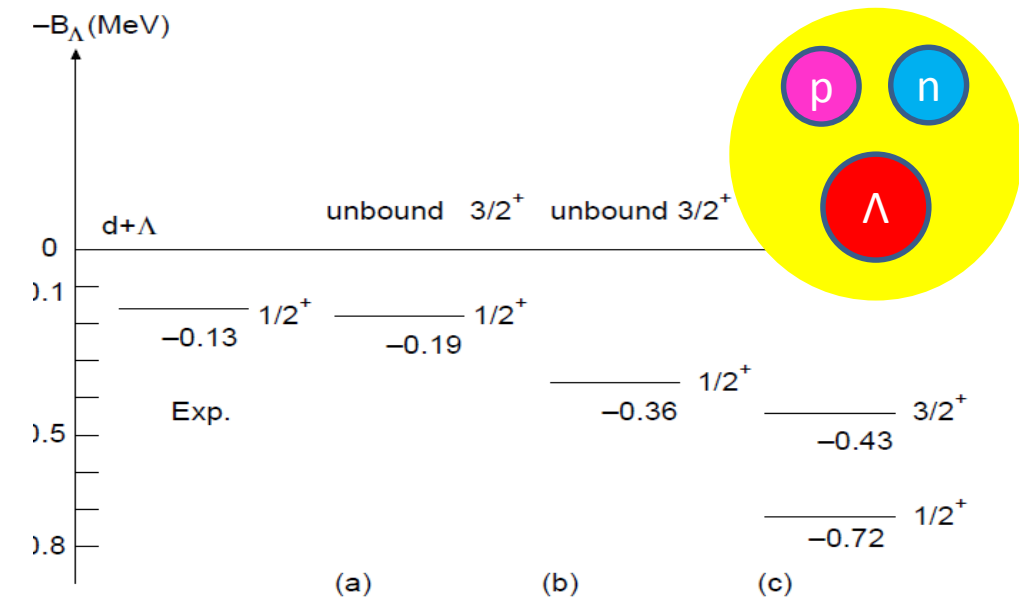
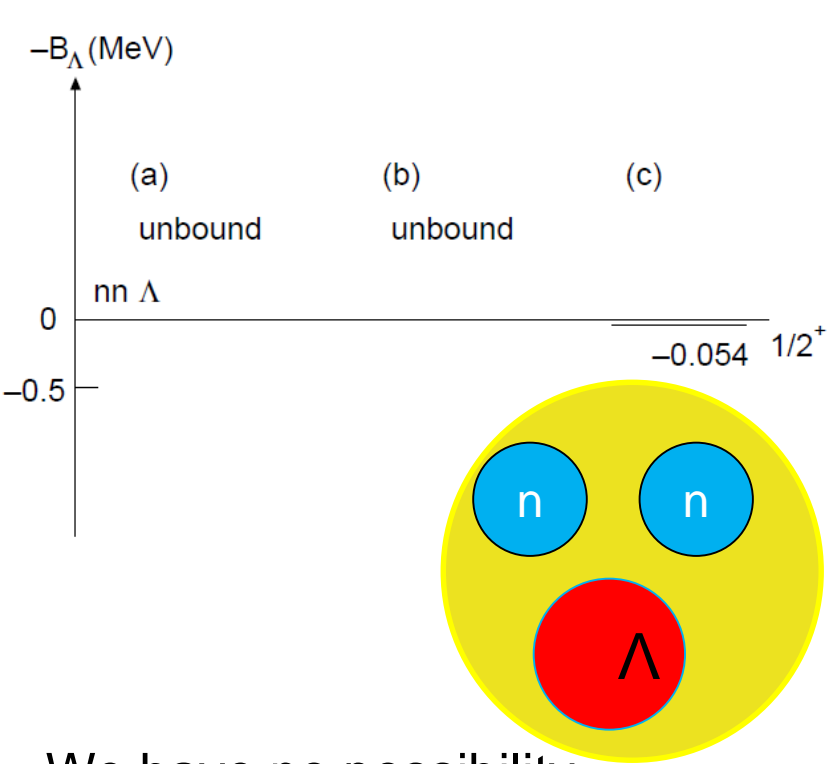
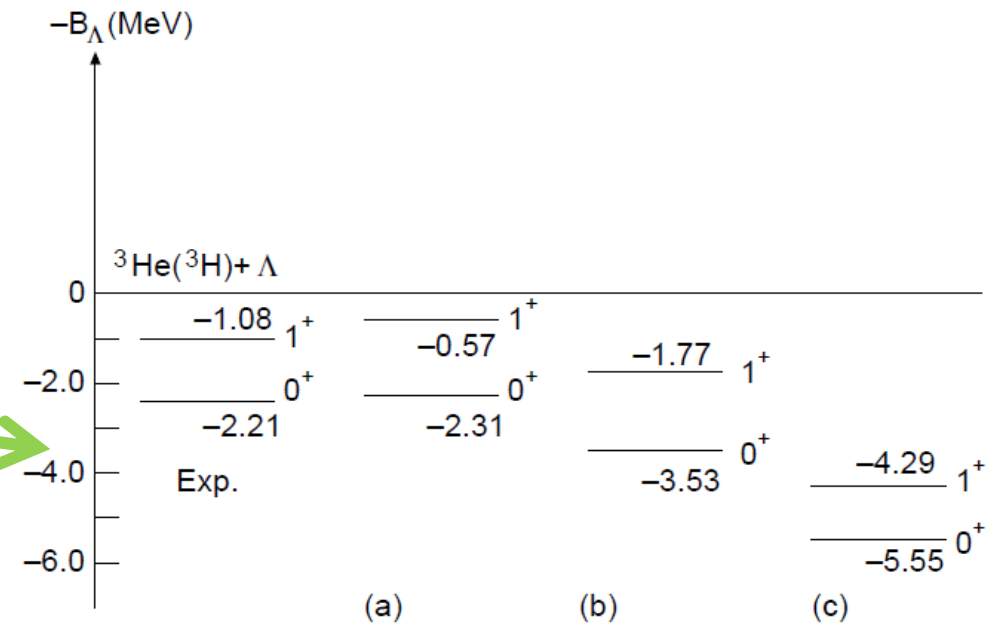
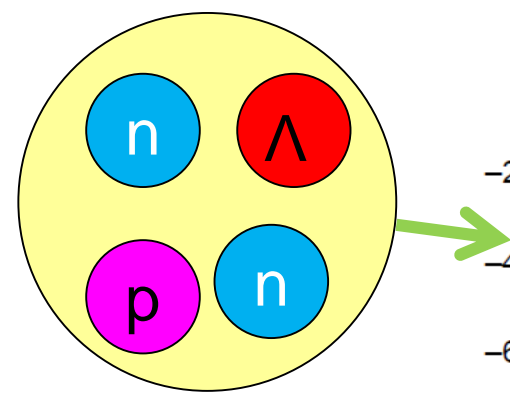


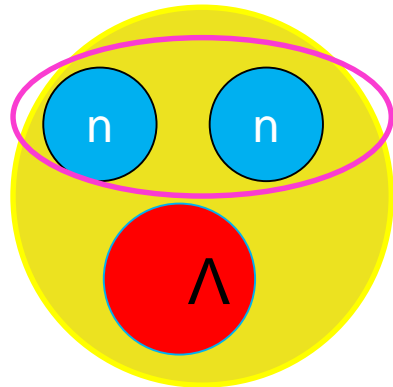
FIG. 3: Calculated  $\Lambda$ -separation energy for  ${}^3_{\Lambda}\text{H}$  with (a)  ${}^3V_{N\Lambda-N\Sigma}^T \times 1.00$ , (b)  ${}^3V_{N\Lambda-N\Sigma}^T \times 1.10$ , and (c)  ${}^3V_{N\Lambda-N\Sigma}^T \times$

We have no possibility to have a bound state in nn $\Lambda$  system.



Question: If we tune  $^1S_0$  state of nn interaction, Do we have a possibility to have a bound state in nn $\Lambda$ ? In this case, the binding energies of  $^3\text{H}$  and  $^3\text{He}$  reproduce the observed data?

Some authors pointed out to have dineutron bound state in nn system. Ex. H. Witala and W. Gloeckle, Phys. Rev. C85, 064003 (2012).



$T=1, ^1S_0$  state

I multiply component of  $^1S_0$  state by 1.13 and 1.35. What is the binding energies of nn $\Lambda$  ?

PHYSICAL REVIEW C 85, 064003 (2012)

#### Di-neutron and the three-nucleon continuum observables

H. Witala

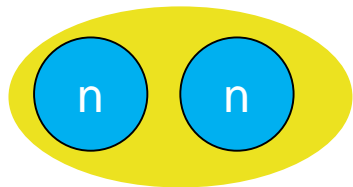
*M. Smoluchowski Institute of Physics, Jagiellonian University, PL-30059 Kraków, Poland*

W. Glöckle

*Institut für Theoretische Physik II, Ruhr-Universität Bochum, D-44780 Bochum, Germany*

(Received 24 April 2012; published 25 June 2012)

We investigate how strongly a hypothetical  $^1S_0$  bound state of two neutrons would affect observables in neutron-deuteron reactions. To that aim we extend our momentum-space scheme of solving the three-nucleon Faddeev equations and incorporate in addition to the deuteron also a  $^1S_0$  di-neutron bound state. We discuss effects induced by a di-neutron on the angular distributions of the neutron-deuteron elastic scattering and deuteron breakup cross sections. A comparison to the available data for the neutron-deuteron total cross section and elastic scattering angular distributions cannot decisively exclude the possibility that two neutrons can form a  $^1S_0$  bound state. However, strong modifications of the final-state-interaction peaks in the neutron-deuteron breakup reaction seem to disallow the existence of a di-neutron.



nn unbound

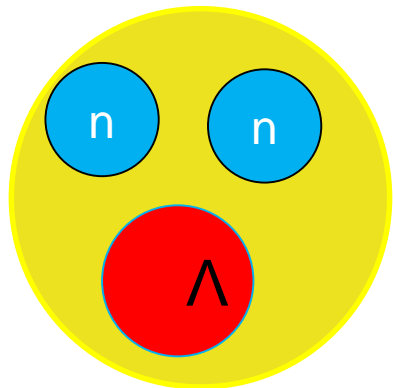
0 MeV

-0.066 MeV

$^1S_0 \times 1.13$

-1.269 MeV

$^1S_0 \times 1.35$



nnΛ unbound

unbound

0 MeV

$1/2^+$

-1.272 MeV

We do not find any possibility to have a bound state in nnΛ.

N+N+N

$^3\text{H}$  ( $^3\text{He}$ )  
-8.48 (-7.72)

-7.77 (-7.12)

-9.75 (-9.05)

-13.93 (-13.23) MeV

Exp.

Cal.

Cal.

Cal.

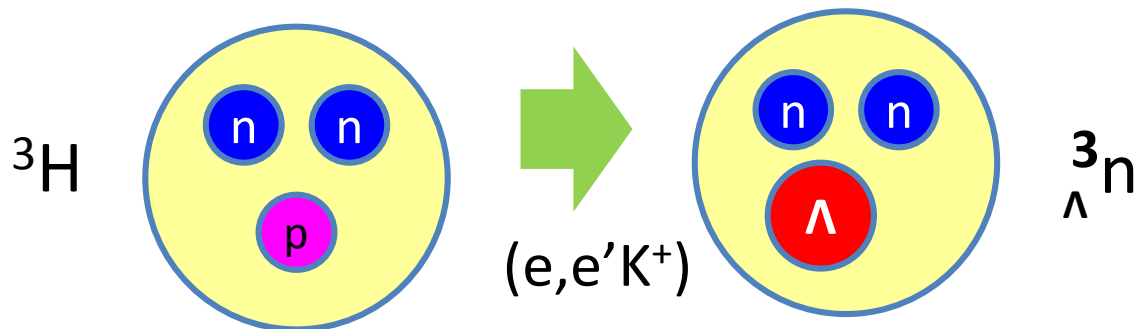
$1/2^+$

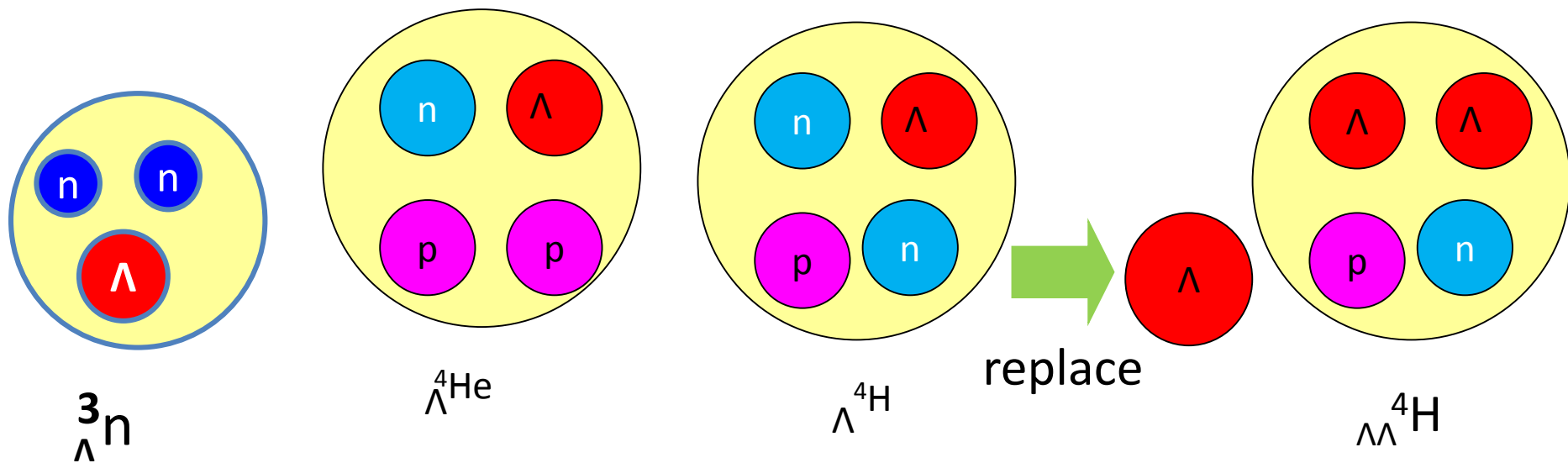


## Summary of $nn\Lambda$ system:

Motivated by the reported observation of data suggesting a bound state  $nn\Lambda$ , we have calculated the binding energy of this hypernucleus taking into account  $\Lambda N$ - $\Sigma N$  explicitly. We did not find any possibility to have a bound state in this system. However, the experimentally they reported evidence for a bound state. As long as we believe the data, we should consider additional missing elements in the present calculation. I need more data for  $nn\Lambda$  system. For this purpose, I am waiting for ALICE data.

It is planned to perform search experiment of  $nn\Lambda$  system at JLab to conclude whether or not the system exists as bound state experimentally. Also, it is planned to perform search experiment of  $nn\Lambda$  system at HypHI collaboration+Super FRS in 2018.

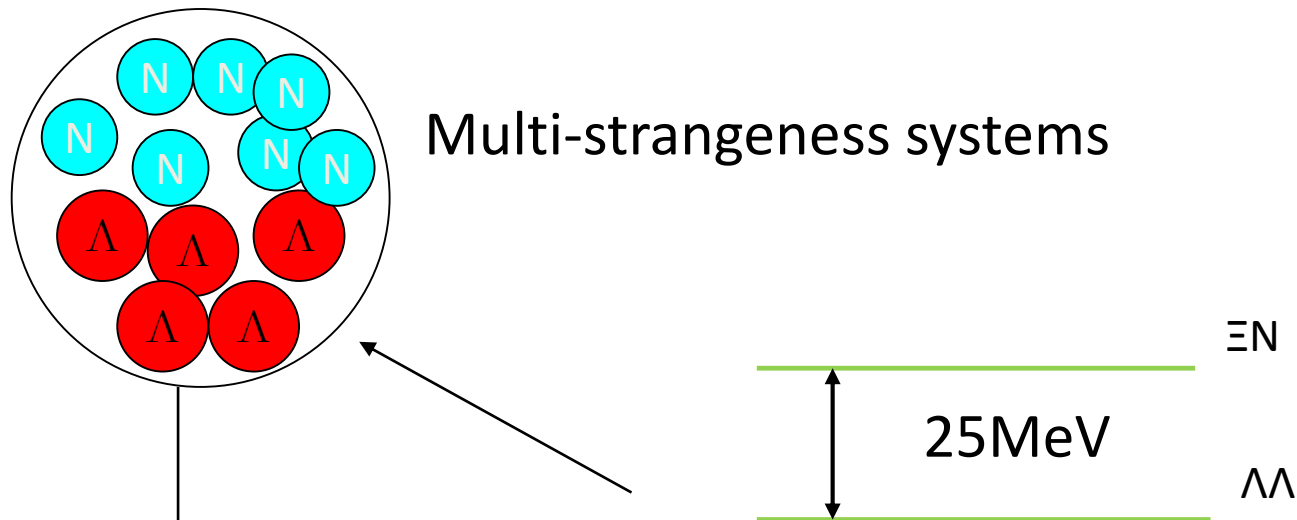




Interesting issue:  
 $\Lambda\Lambda - \Xi N$  coupling

# $\Lambda\Lambda$ - $\Xi$ N coupling

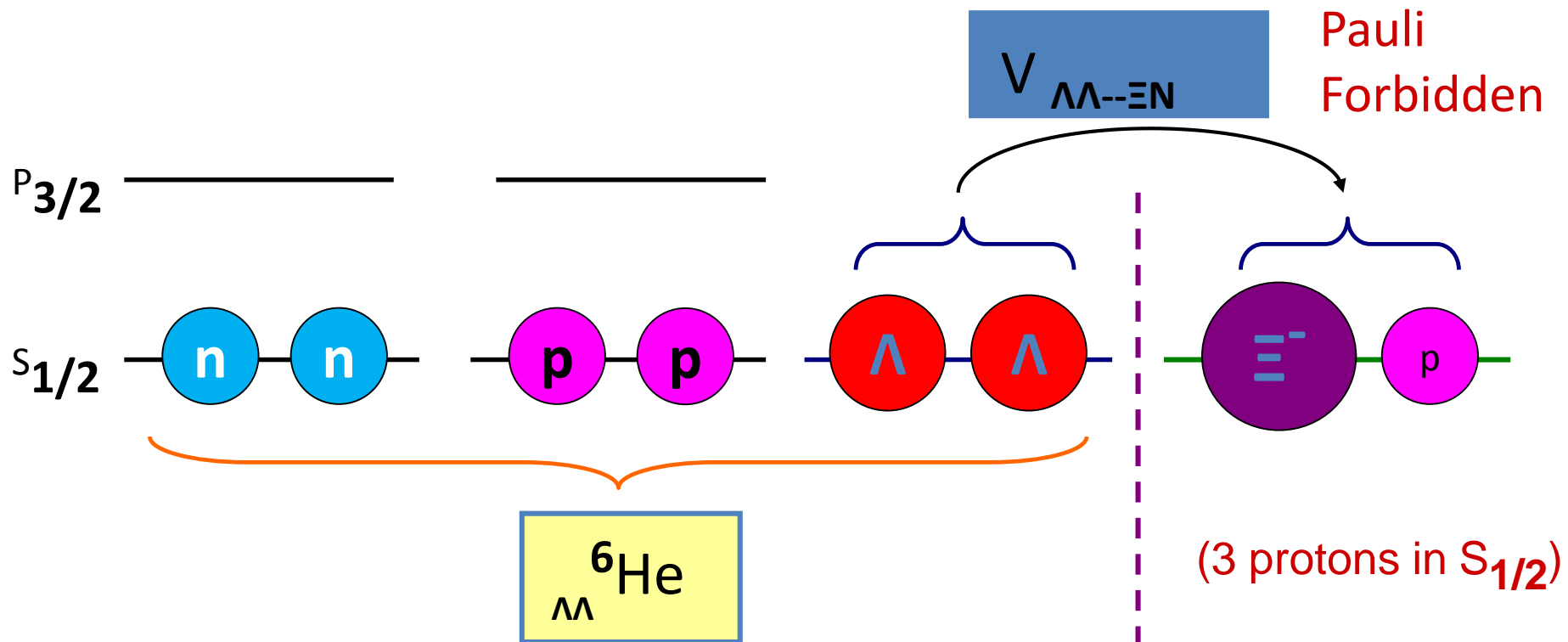
One of the major goals in hypernuclear physics  
To study structure of multi-strangeness systems  
(extreme limit : neutron star)



threshold energy difference is very small!

It is considered that  
 $\Lambda\Lambda \rightarrow \Xi$ N particle conversion  
is strong in multi-strangeness system.

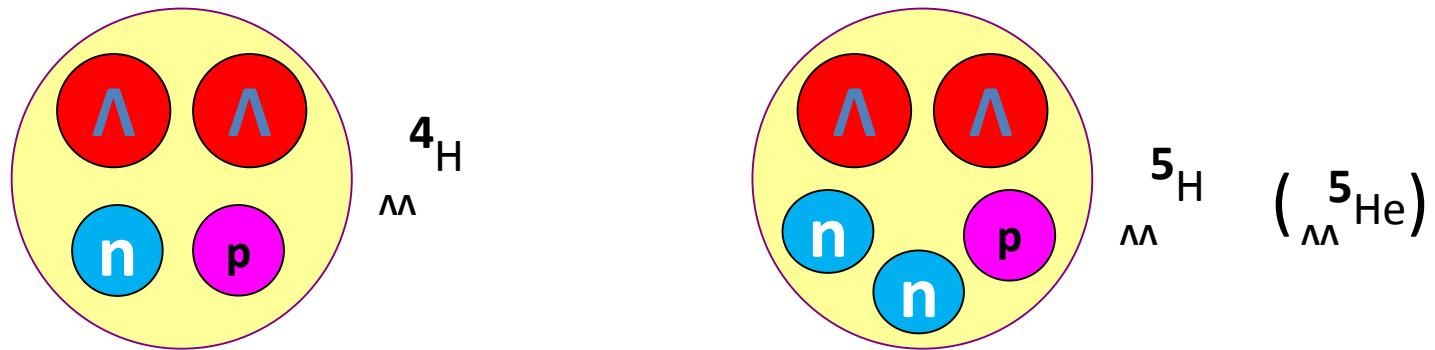
Effect of  $\Lambda\Lambda-\Xi N$  coupling is small in  ${}_{\Lambda\Lambda}{}^6\text{He}$  which was observed as NAGARA event.



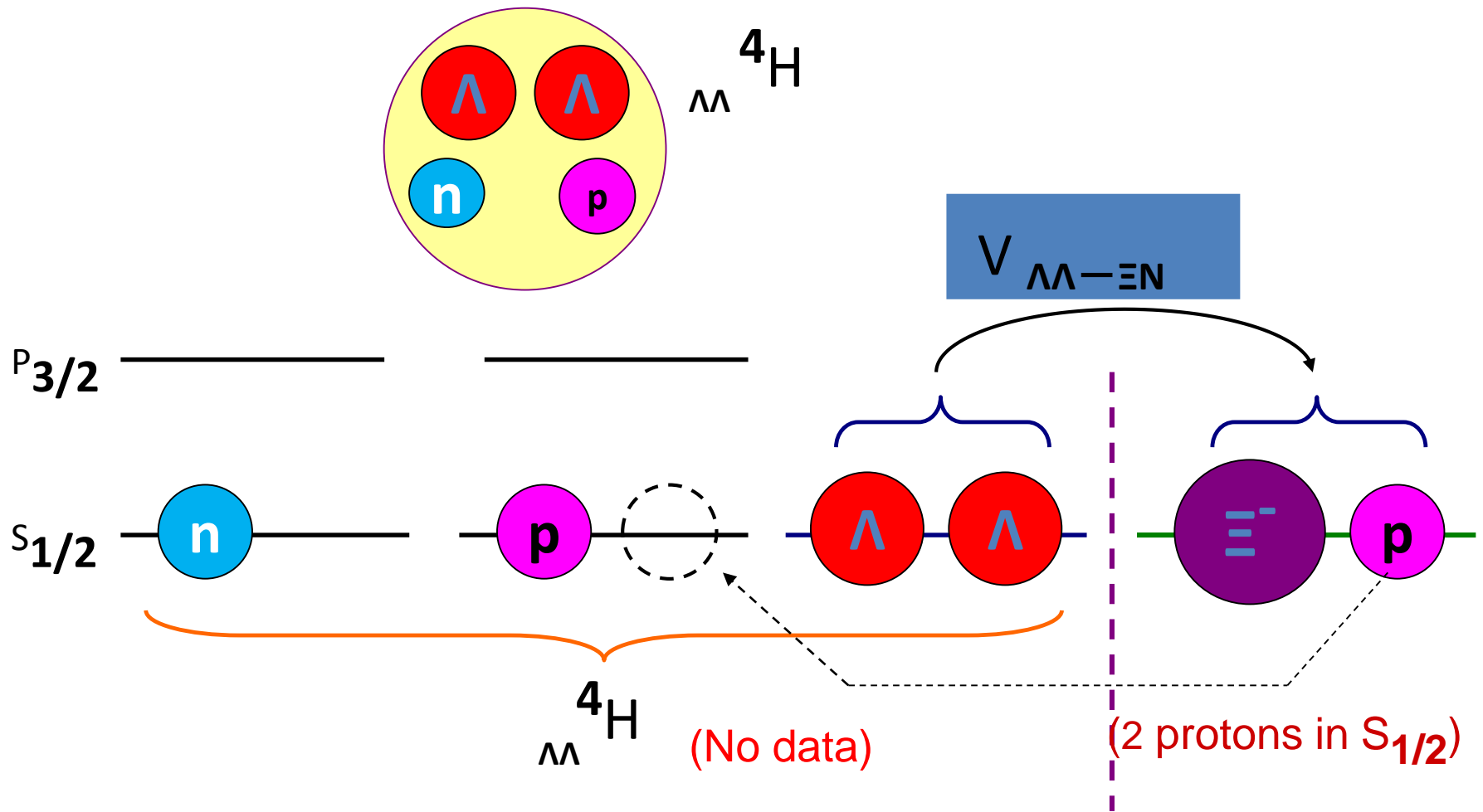
- I.R. Afnan and B.F. Gibson, Phys. Rev. C67, 017001 (2003).
- Khin Swe Myint, S. Shinmura and Y. Akaishi, nucl-th/029090.
- T. Yamada and C. Nakamoto, Phys. Rev.C62, 034319 (2000).

For the study of  $\Lambda\Lambda-\Xi N$  coupling interaction,  
 s-shell double  $\Lambda$  hypernuclei such as

${}_{\Lambda\Lambda}^4\text{H}$  and  ${}_{\Lambda\Lambda}^5\text{H}$  ( ${}_{\Lambda\Lambda}^5\text{He}$ ) are very suitable.

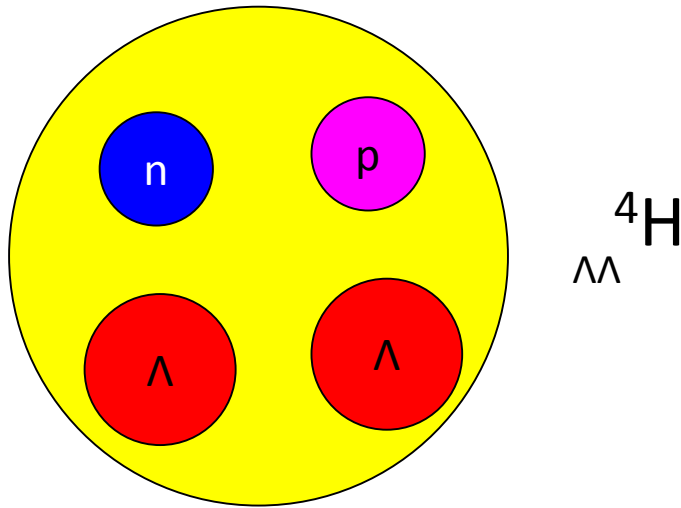


- I.N. Filikhin and A. Gal, Phys. Rev. Lett. 89, 172502 (2002)
- Khin Swe Myint, S. Shinmura and Y. Akaishi, Eur. Phys. J. A16, 21 (2003).
- D. E. Lanscoy and Y. Yamamoto, Phys. Rev. C69, 014303 (2004).
- H. Nemura, S. Shinmura, Y. Akaishi and Khin Swe Myint, Phys. Rev. Lett. 94, 202502 (2005).



Due to NO Pauli plocking, the  $\Lambda\Lambda-\Xi N$  coupling can be large in  ${}_{\Lambda\Lambda}^4\text{H}$

B.F. Gibson, I.R. Afnan, J.A. Carlson and D.R. Lehman,  
 Prog. Theor. Phys. Suppl. 117, 339 (1994).



The important issue:

Does the  $YY$  interaction which designed to reproduce the binding energy of  ${}_{\Lambda\Lambda}^6\text{He}$  make  ${}_{\Lambda\Lambda}^4\text{H}$  bound?

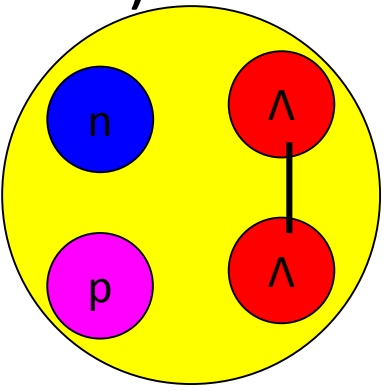
And how does the effect of  $\Lambda\Lambda-\Xi\text{N}$  coupling play important role in the binding energy of  ${}_{\Lambda\Lambda}^6\text{He}$  and  ${}_{\Lambda\Lambda}^4\text{H}$ ?

1) I.N. Filikhin and A. Gal, Phys. Rev. Lett. 89, 172502

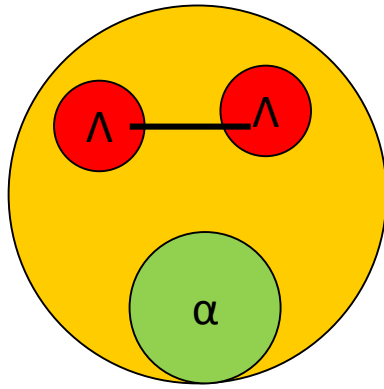
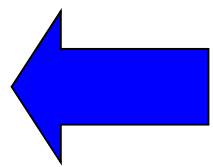
(2002)

2) H. Nemura, Y. Akaishi et al., Phys. Rev. C67, 051001

(2002)



$V_{\Lambda\Lambda}$

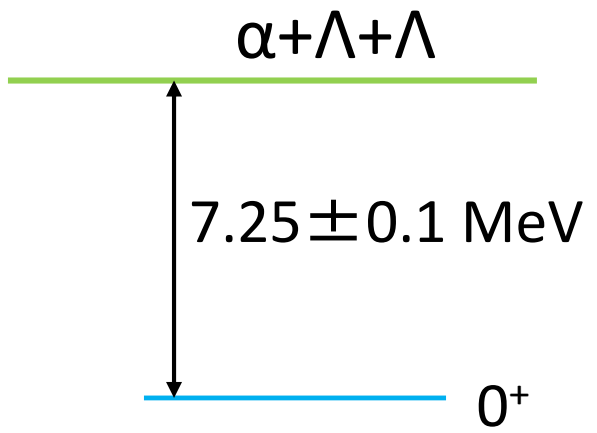


${}_{\Lambda\Lambda}^6\text{He}$

NAGARA event

NOT BOUND !

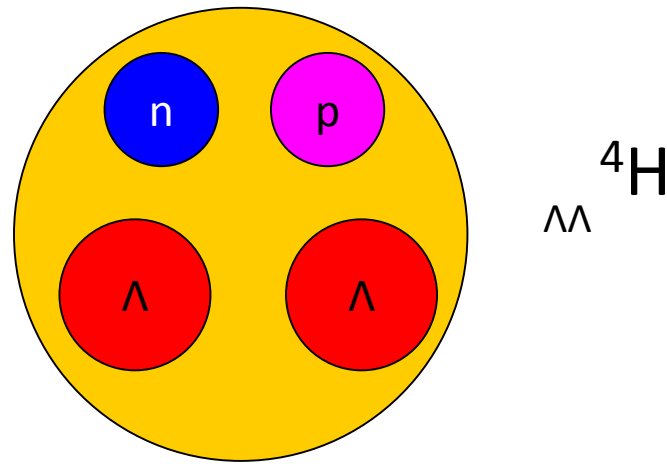
${}_{\Lambda\Lambda}^4\text{H}$



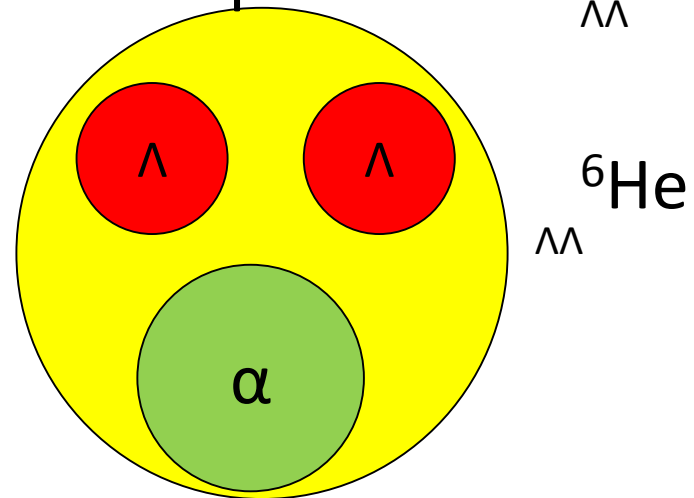


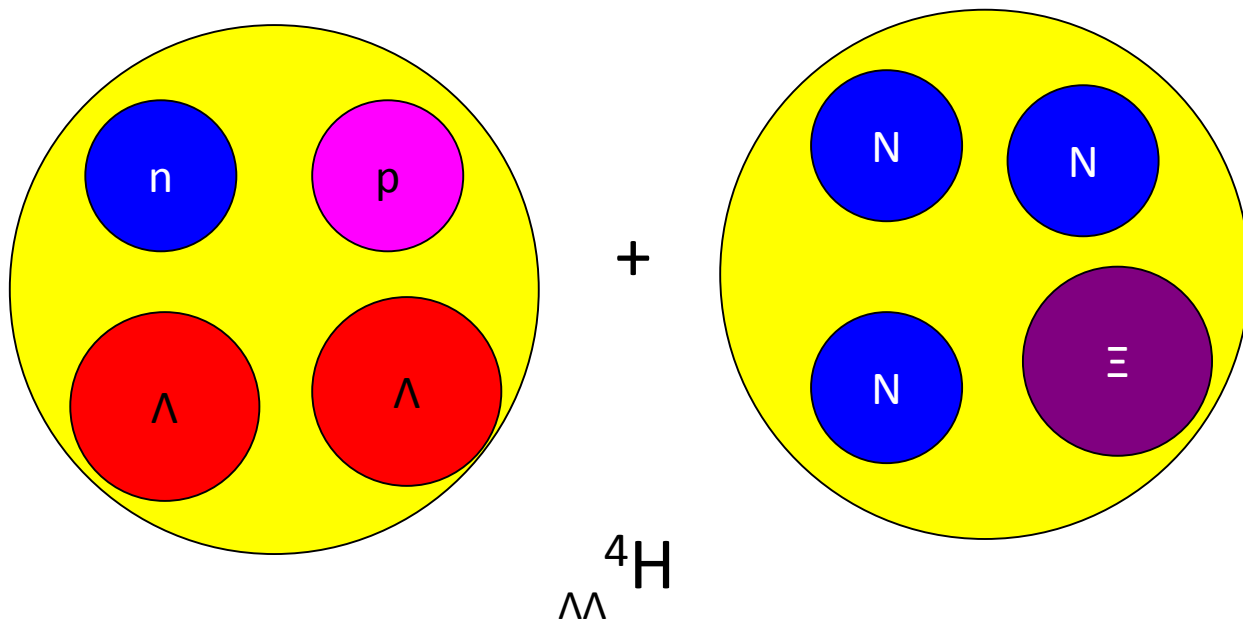
Did not include  $\Lambda\Lambda$ - $\Xi$ N coupling

$\Lambda\Lambda$ - $\Xi$ N coupling  $\Rightarrow$  ▪ significant in  ${}_{\Lambda\Lambda}^4\text{H}$

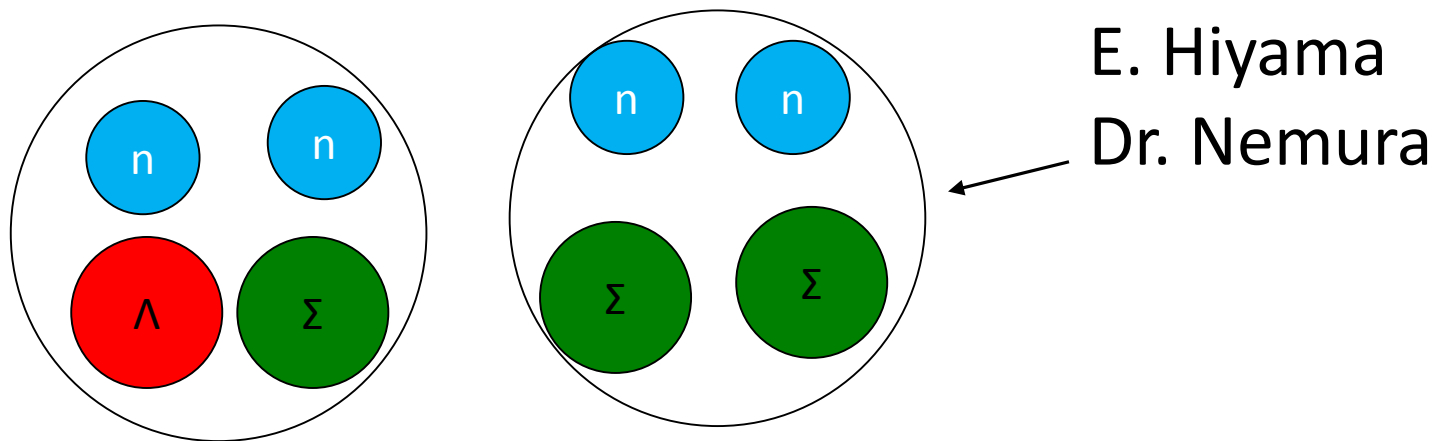


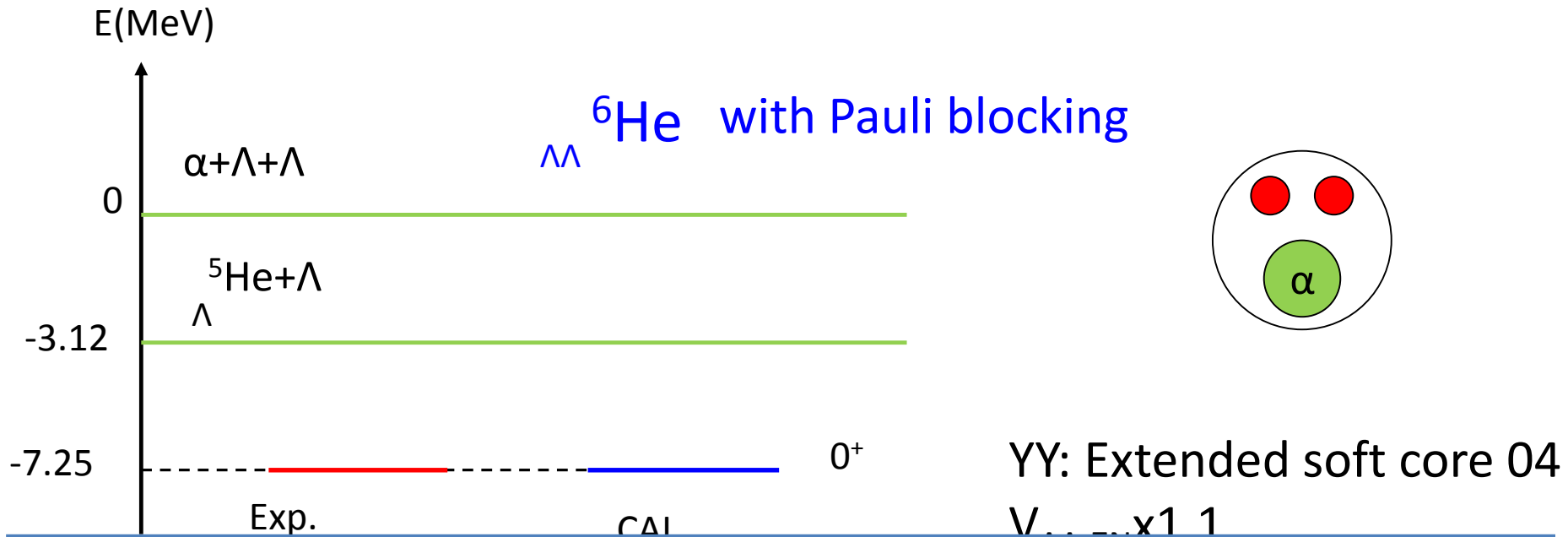
▪ Not so important in  ${}_{\Lambda\Lambda}^6\text{He}$



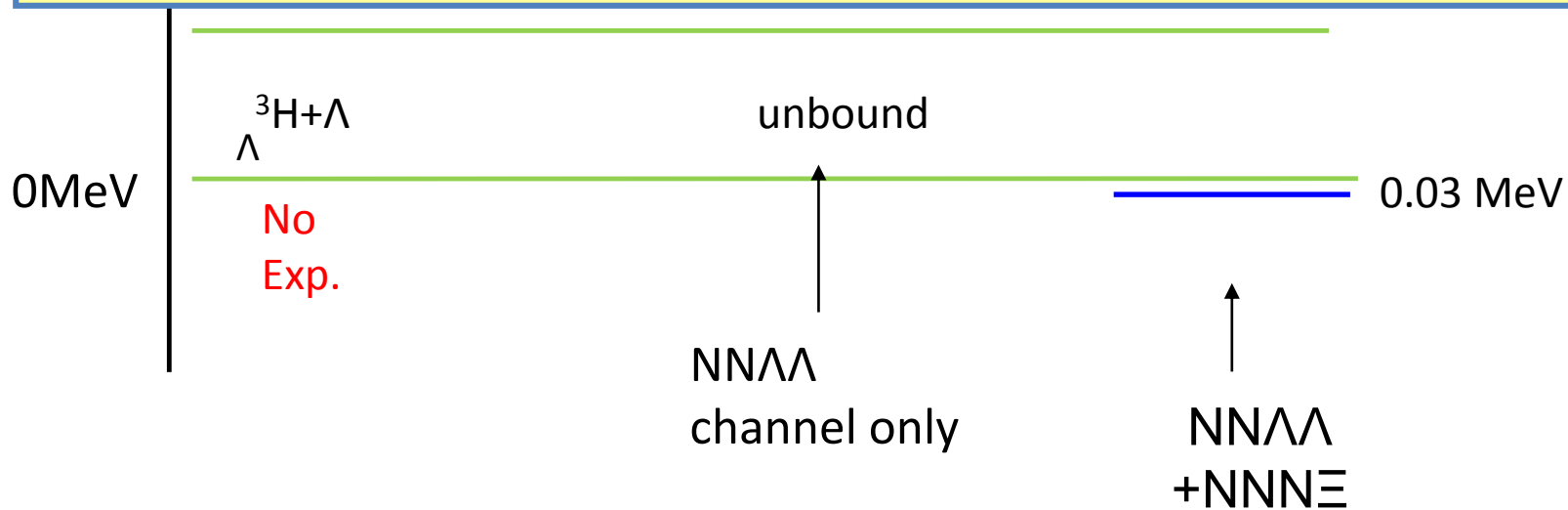


One of the most numerically difficult 4-body problem





If the bound state of  ${}^4_{\Lambda\Lambda}\text{H}$  is observed in the future, we can obtain useful information about  $\Lambda\Lambda$ - $\Xi\text{N}$  coupling mechanism.



E(MeV)

28MeV

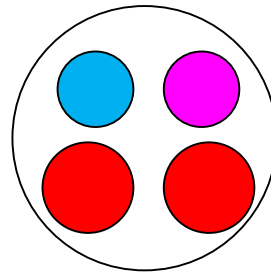
NN $\Xi$ ? ?

n+p+ $\Lambda$ + $\Lambda$

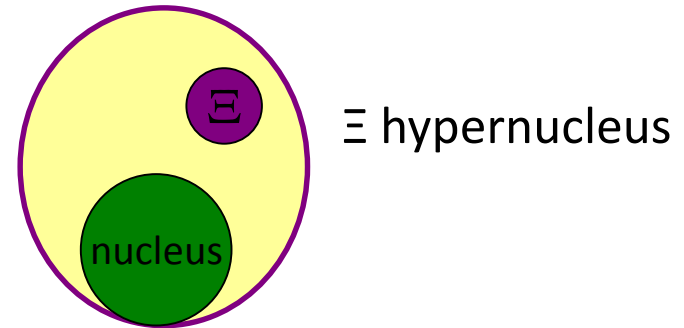
$\Lambda\Lambda$   ${}^4\text{H}$

0  
 ${}^3\text{H}+\Lambda$   
 $\Lambda$

0.03MeV



$\Xi N - \Xi N$  interaction



For the study of  $\Xi N$  interaction, it is important to study the structure of  $\Xi$  hypernuclei.

Then, it is important to predict theoretically what kinds of  $\Xi$  hypernuclei will exist as bound states.

Important issue:

What kind of  $\Xi N$  interaction should we employ?

Since there is no information about  $\Xi N$  interaction, we cannot use phenomenological  $\Xi N$  interaction.

We have realistic interactions although with large ambiguity.

- Nijmegen group
- Ehime group
- Kyoto-Niigata group

# BNL-E885

PHYSICAL REVIEW C 61 054603

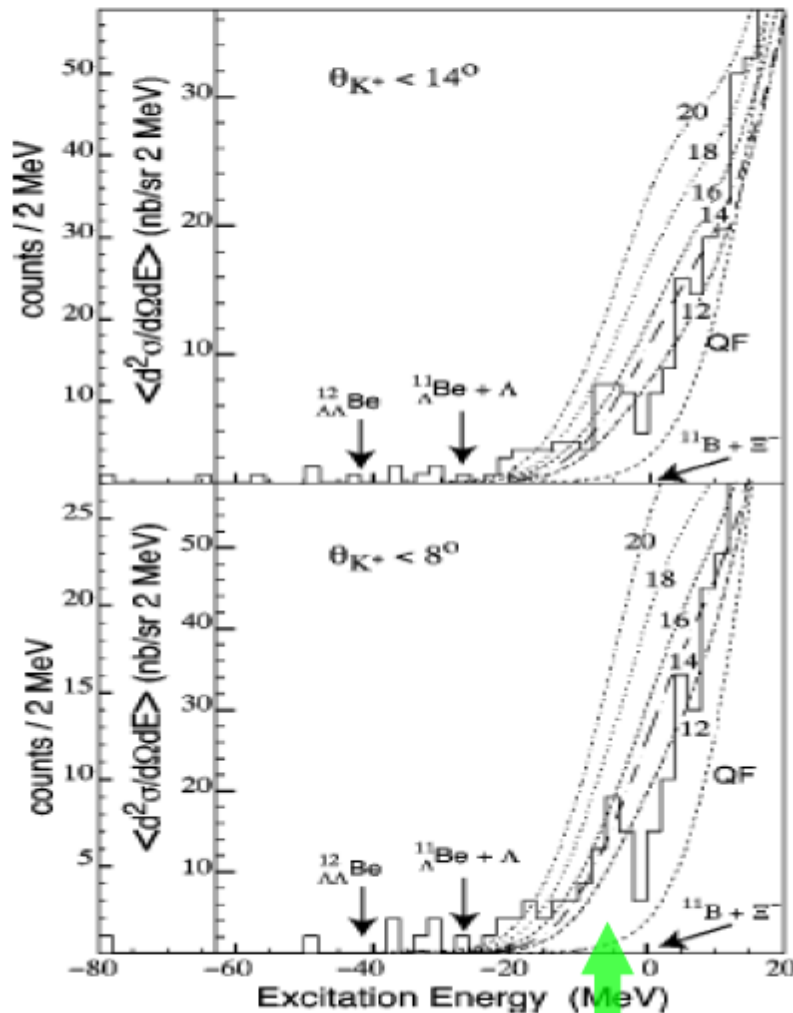
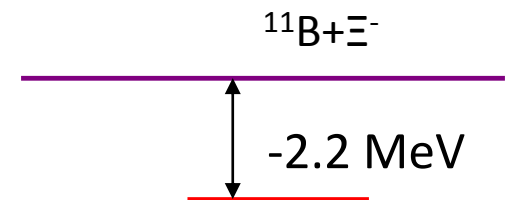


FIG. 6. Excitation-energy spectra from E885 for  $^{12}\text{C}(K^-, K^+)X$

Only experimental data

By assuming a  $\Xi$ -nucleus Woods-Saxon potential with a depth of  $\sim -14$  MeV, we reproduce the experimental data.

This WS potential leads to be bound by  $-2.2$  MeV in  $^{12}\text{Be}$  when the Coulomb interaction is switched off.



We use this information.

## The $\Xi N$ interaction to reproduce the data

- Extended soft core 04d (ESC04d)

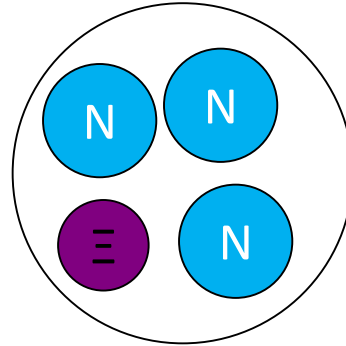
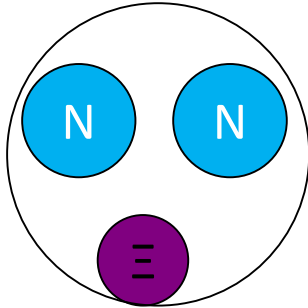
Th. A. Rijken, and Y. Yamamoto, *Phys. Rev. C* **73**, 044008(2006).

Extended soft core '08(ESC08)

Three- and four-body calculation  
using this potential is in progress.



ESC potential leads to give bound states in s-shell  $\Xi$  hypernuclei such as  $NN\Xi$  and  $NNN\Xi$ .



# Results

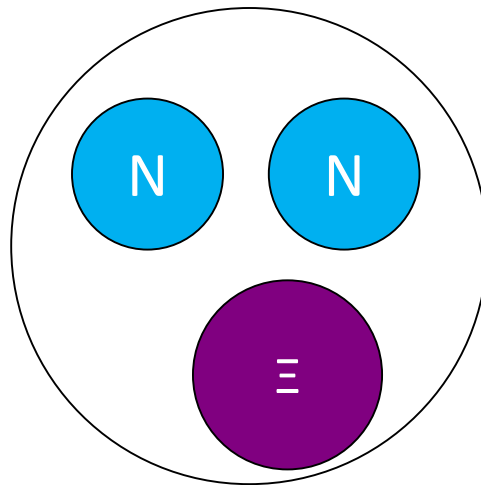
## ESC04

0 MeV

(np)- $\Xi$

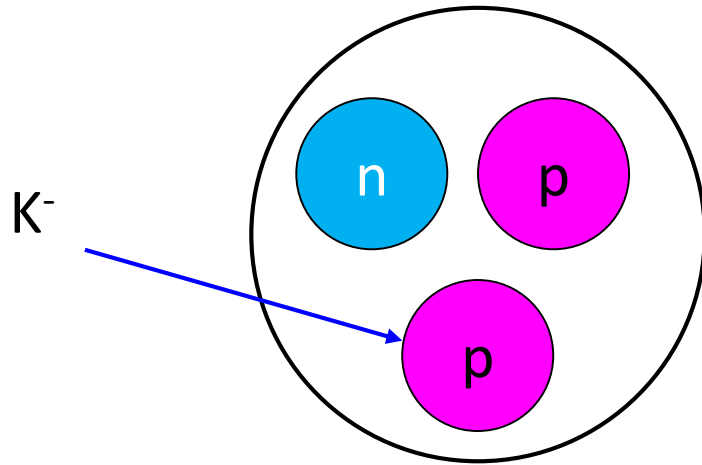
$1/2^+$

-0.15 MeV



NNE

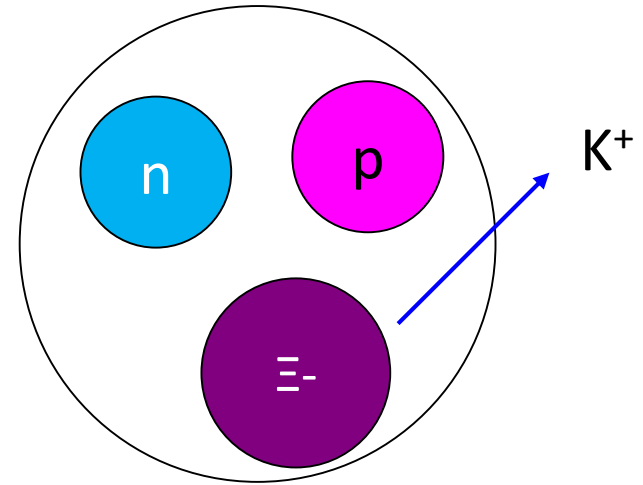
$$T=1/2, T_z=1/2$$



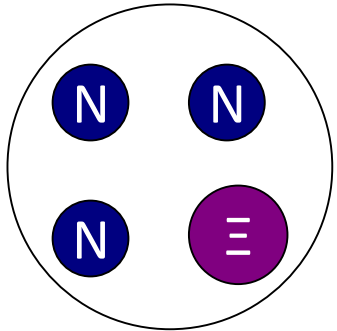
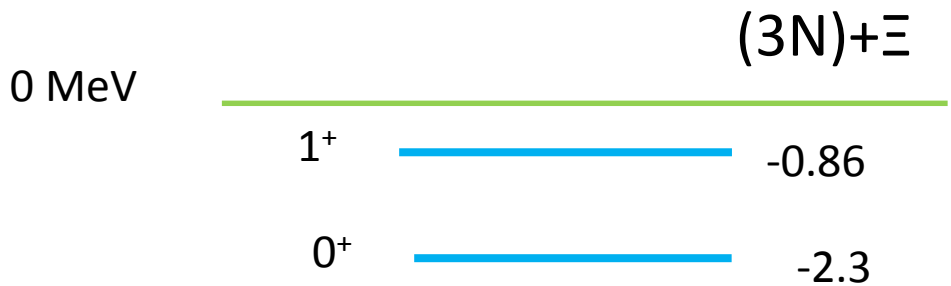
${}^3\text{He}$



$$T=1/2, T_z=-1/2$$



Using  ${}^3\text{He}$  target, it might be produced this  $\Xi$  hypernucleus.  
If this  $\Xi$  hypernucleus exist as bound state, what is isotope of this  $\Xi$  hypernucleus?



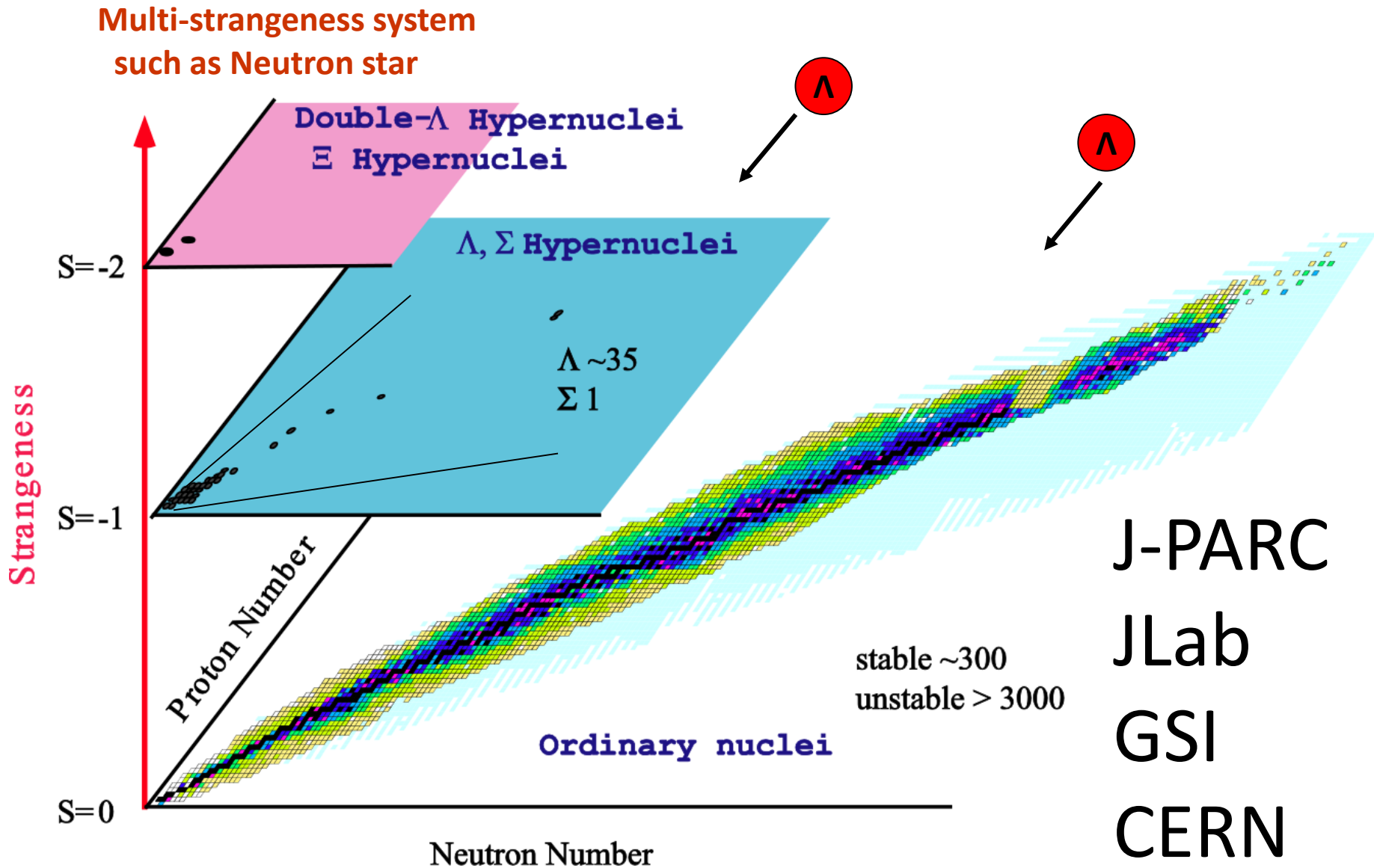
T,S repulsive **strongly attractive**

$$1^+ : [12V(1,1) + \overset{\text{repulsive}}{\underbrace{V(1,0)}} + \overset{\text{strongly attractive}}{\underbrace{10V(0,1)}} + \overset{\text{repulsive}}{\underbrace{3V(0,0)}}] / 26$$

$$0^+ : [\underbrace{V(1,0)}_{\text{weakly repulsive}} + \underbrace{V(0,1)}_{\text{strongly attractive}}] / 2$$

repulsive

# Nuclear chart with strangeness



Thank you!