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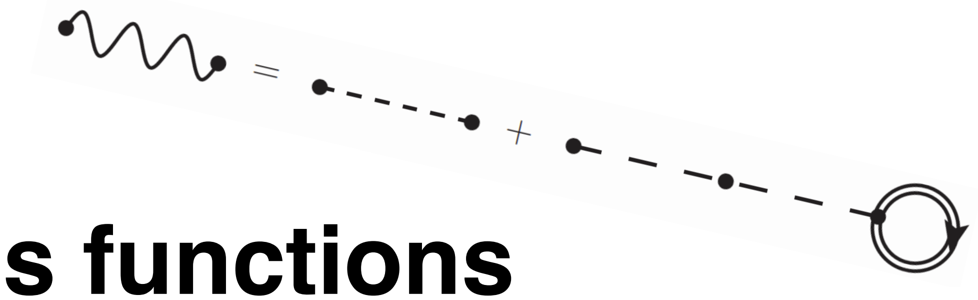
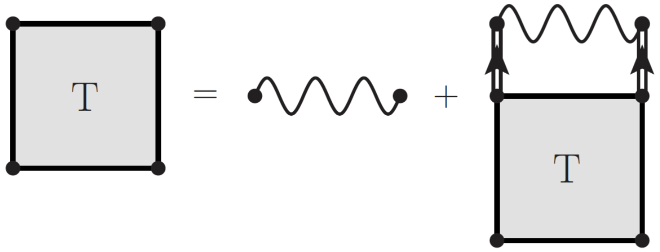
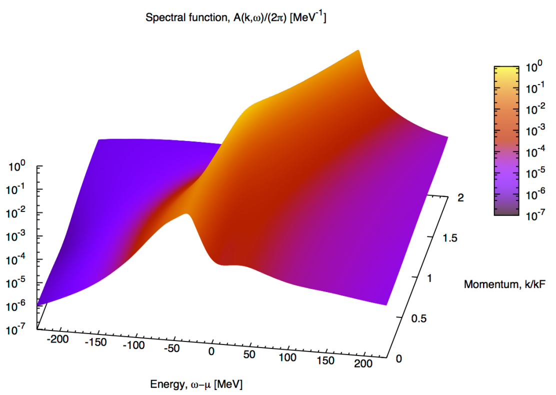


Self-consistent Green's functions with three-nucleon interactions

Arianna Carbone - TU Darmstadt

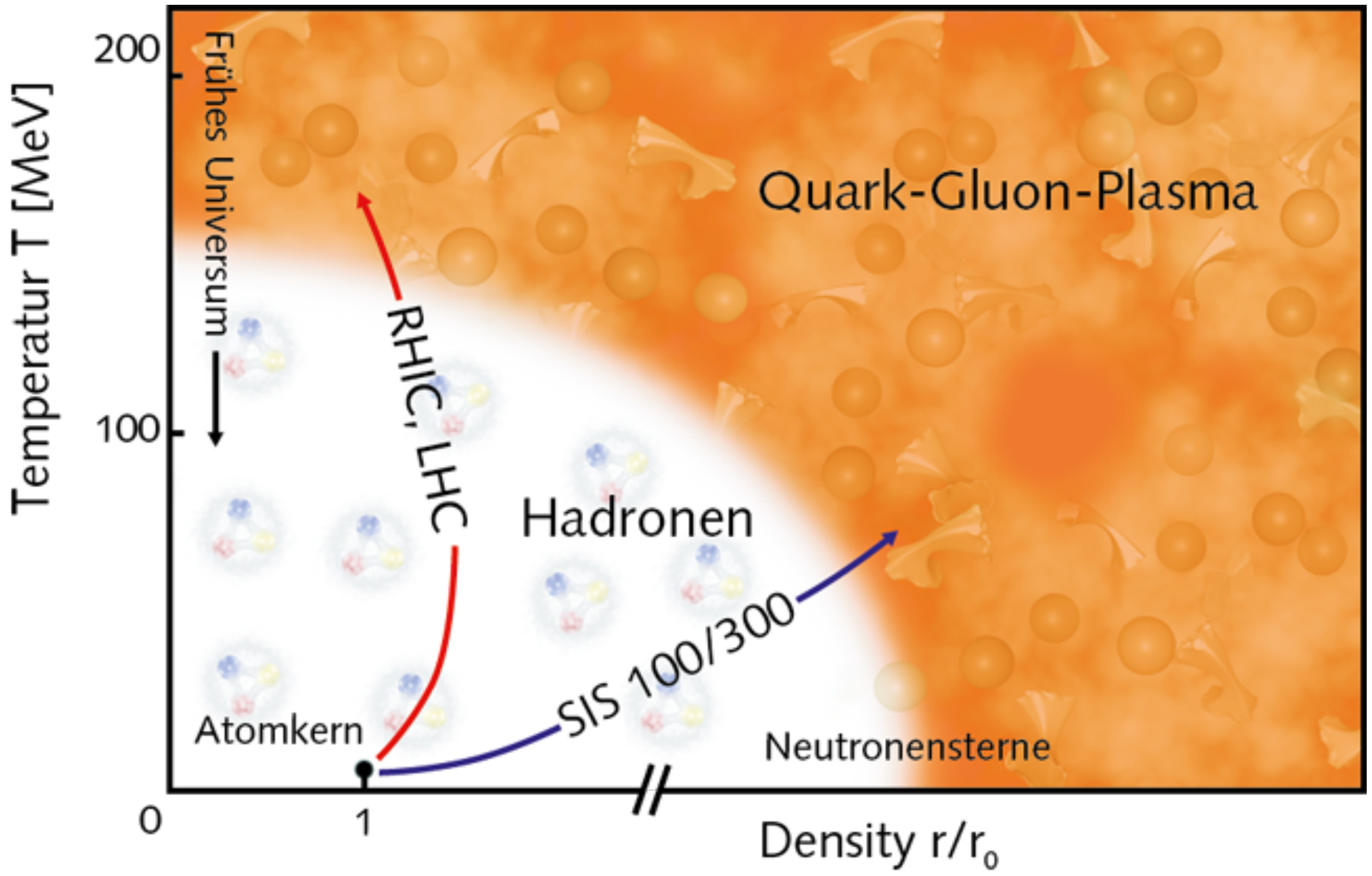
EMMI Workshop 13-16 October 2015
Cold dense nuclear matter: from short-range nuclear correlations to neutron stars





Nuclear matter covers wide ranges of density and temperature

The phase diagram of hadronic matter



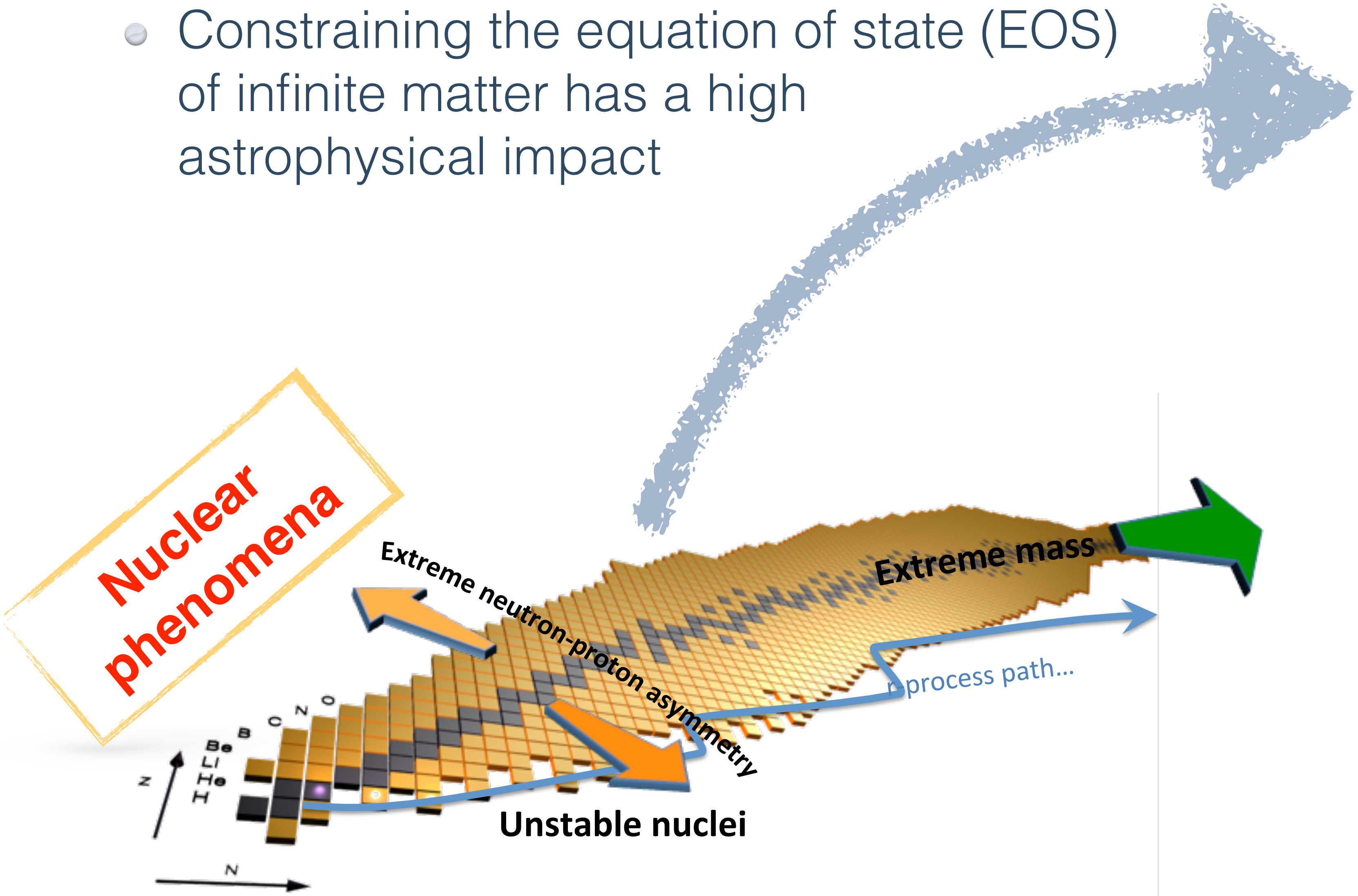
https://www.gsi.de/en/start/fair/forschung_an_fair/kernmateriephysik.htm

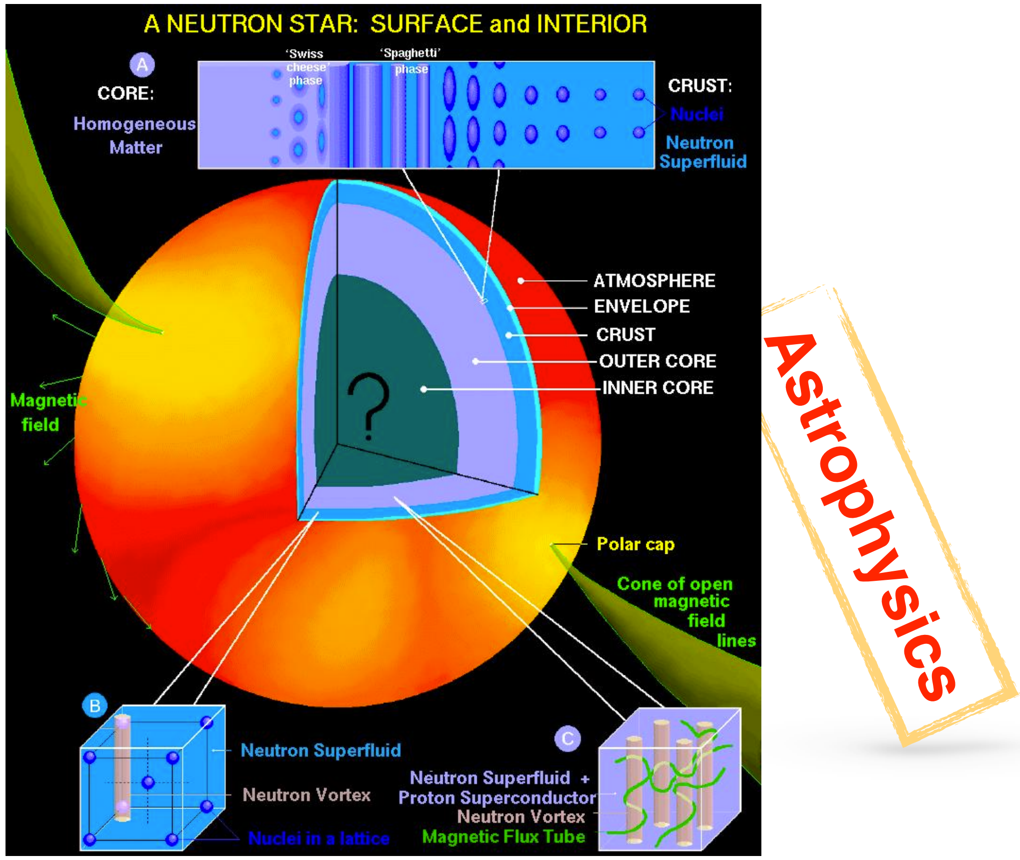
- Matter interacting via the strong force appears in diverse forms
- Experiments try to fill in the pieces of the phase diagram puzzle
- High-energy nucleus-nucleus collisions will probe the neutron-rich region
- What's the contribution from nuclear many-body theory?

The nuclear many-body problem

- Build reliable methods with predictive power
- The study of exotic nuclei is probing the limits of the nuclear landscape
- Constraining the equation of state (EOS) of infinite matter has a high astrophysical impact

From nuclei to nuclear matter

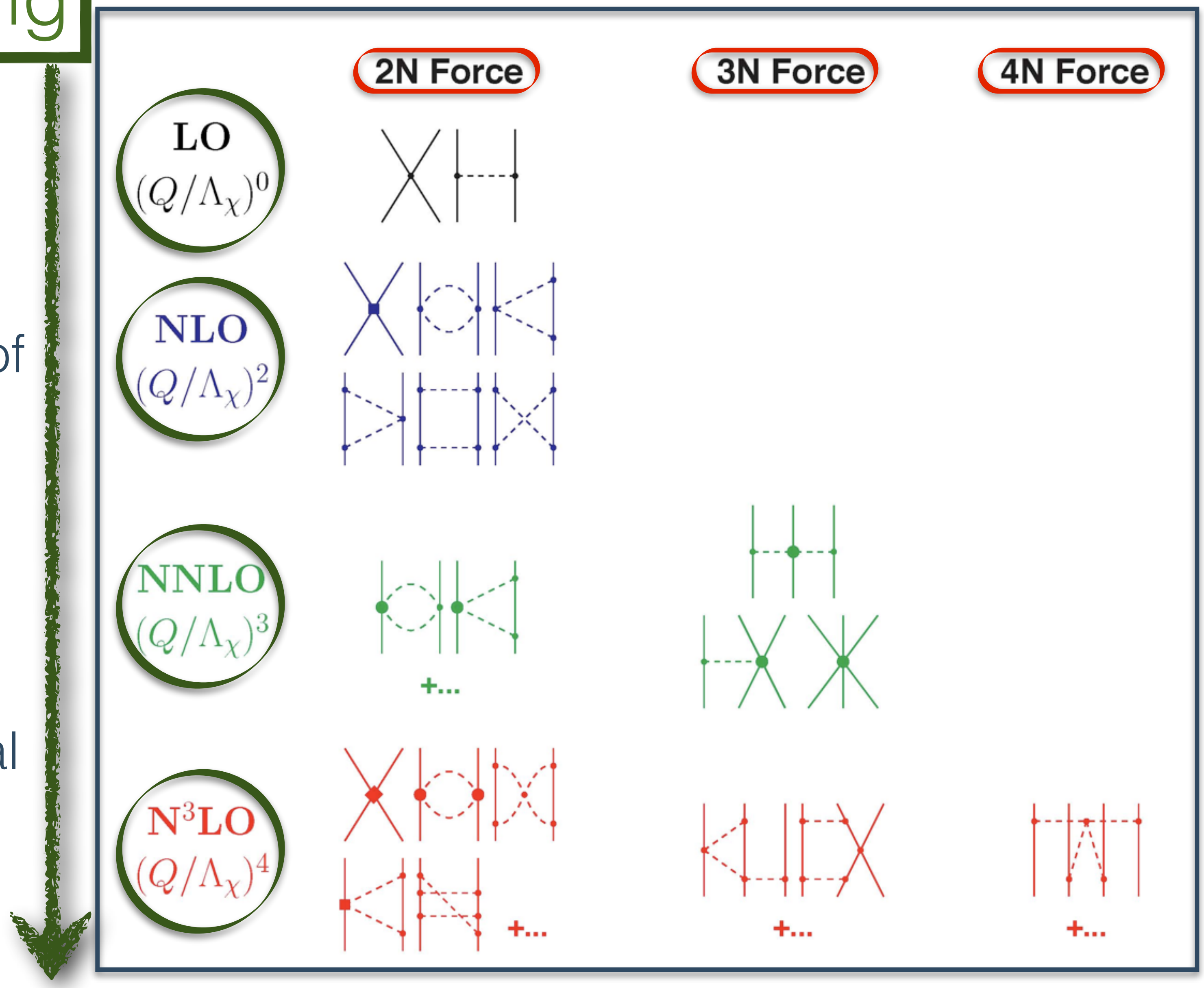




Why nuclear matter from chiral EFT?

Power counting

- Effective theory of QCD
- Nucleons & pions as degrees of freedom
- Power counting expansion
- Hierarchy of many-body forces
- Enables estimates of theoretical uncertainties



Epelbaum *et al.*, Rev. Mod. Phys. 81, 1773(2009)
Machleidt *et al.*, Phys. Rep. 503, 1 (2011)

Chiral nuclear forces

Power counting

Approach used:

2NF: N2LO or N3LO

Entem & Machleidt, PRC 68, 041001 (2003)

Epelbaum *et al.*, NPA **747**, 362 (2005)

Ekström *et al.*, PRL **110**, 192502 (2013)

Recent improvements:

Epelbaum *et al.*, EPJA **51**, 53 (2015)

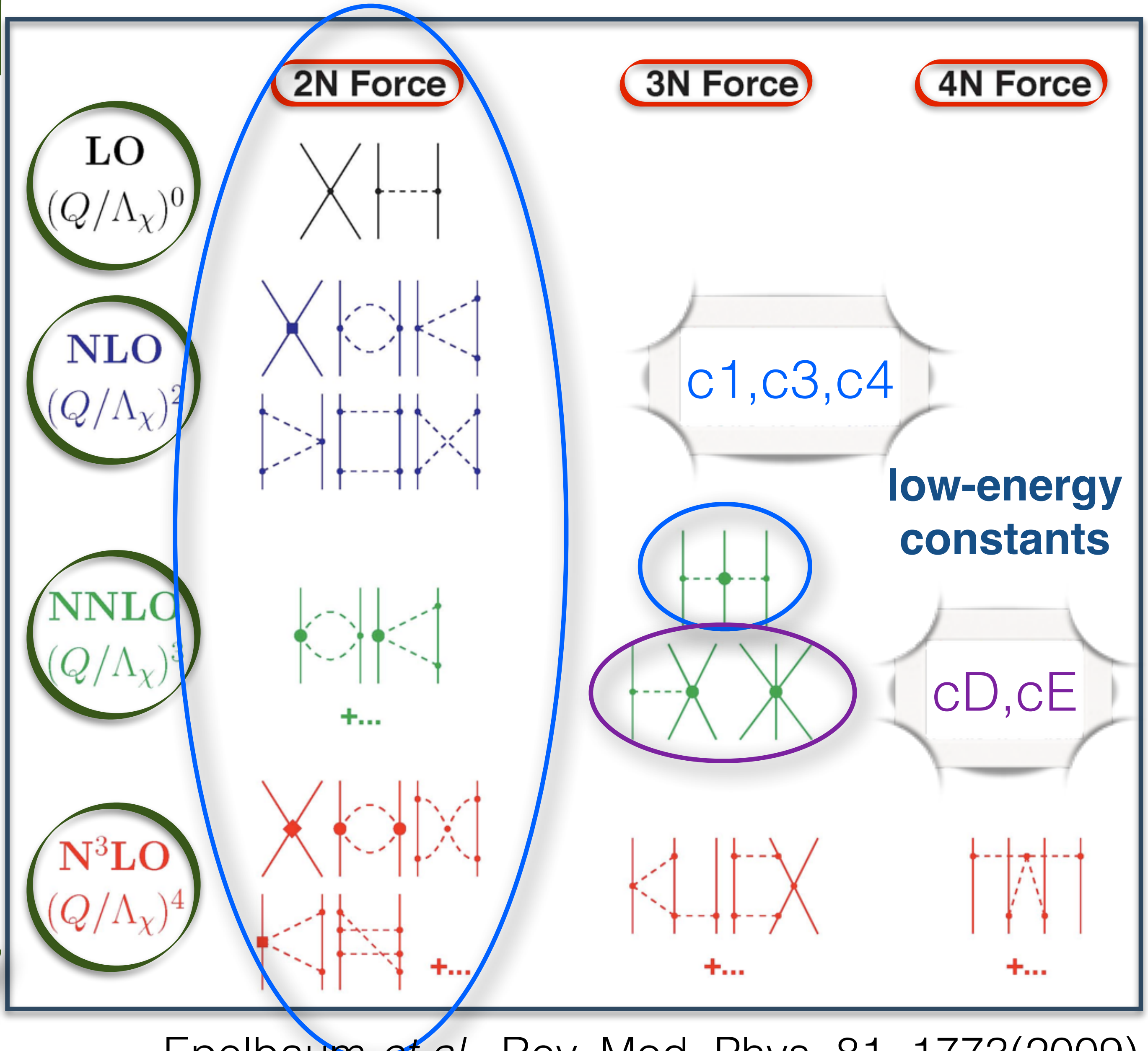
Ekström *et al.*, PRC **91**, 051301(R) (2015)

3NF: N2LO

Van Kolck, PRC **49,** 2932 (1994)

Epelbaum et al., PRC 66, 064001 (2002)

At N2LO two new couplings need to be fit to few-body data



Epelbaum *et al.*, Rev. Mod. Phys. 81, 1773(2009)

Machleidt *et al.*, Phys. Rep. 503, 1 (2011)

Chiral nuclear forces

Power counting

First calculations with full N3LO:

3NF: N3LO

Ishikawa *et al.*, PRC **76**, 014006 (2007)
Bernard *et al.*, PRC **77**, 064004 (2008)
Bernard *et al.*, PRC **84**, 054001 (2011)

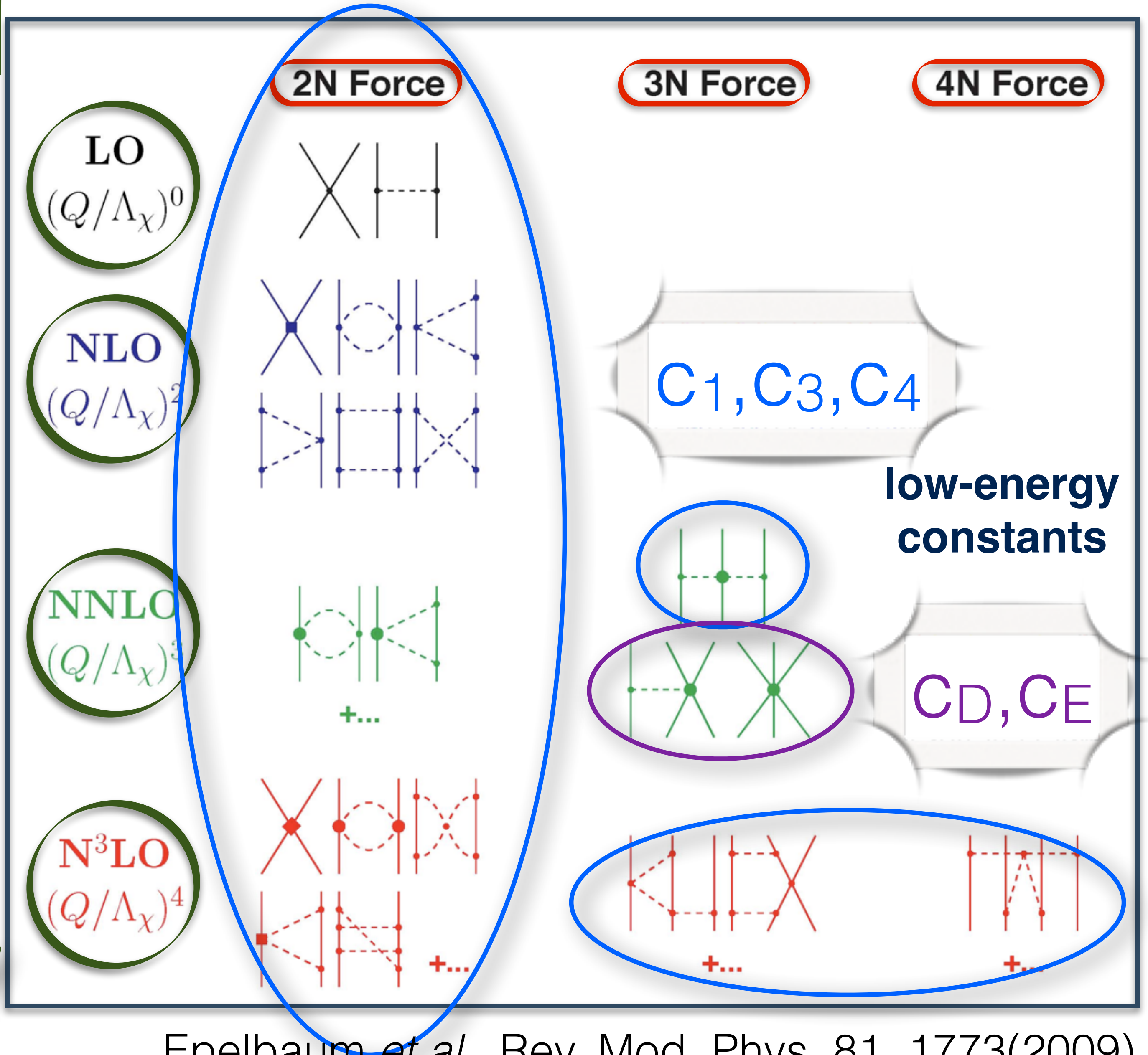
Important recent improvements

Hebeler *et al.*, PRC **91**, 044001 (2015)

4NF: N3LO

Epelbaum, PLB **639**, 456 (2006)
Epelbaum, EPJA **34**, 197 (2007)

Many-body forces have to be included because they are part of the theory!



Epelbaum *et al.*, Rev. Mod. Phys. 81, 1773(2009)
Machleidt *et al.*, Phys. Rep. 503, 1 (2011)

Self-consistent Green's functions

Dickhoff & Barbieri, PPNP **52**, 377 (2004)

- The Green's function as a tool to solve the nuclear many-body problem:

$$G_{\alpha\beta}(\omega) = \sum_m \frac{\langle \Psi_0^N | a_\alpha | \Psi_m^{N+1} \rangle \langle \Psi_m^{N+1} | a_\beta^\dagger | \Psi_0^N \rangle}{\omega - (E_m^{N+1} - E_0^N) + i\eta} + \sum_n \frac{\langle \Psi_0^N | a_\beta^\dagger | \Psi_n^{N-1} \rangle \langle \Psi_n^{N-1} | a_\alpha | \Psi_0^N \rangle}{\omega - (E_0 - E_n^{N-1}) + i\eta}$$

- Self-consistent nonperturbative method:

- Nonrelativistic Hamiltonian:

$$\hat{H} = \sum_\alpha \varepsilon_\alpha^0 \, a_\alpha^\dagger a_\alpha - \sum_{\alpha\beta} U_{\alpha\beta} \, a_\alpha^\dagger a_\beta + \frac{1}{4} \sum_{\substack{\alpha\gamma \\ \beta\delta}} V_{\alpha\gamma,\beta\delta} \, a_\alpha^\dagger a_\gamma^\dagger a_\delta a_\beta + \frac{1}{36} \sum_{\substack{\alpha\gamma\epsilon \\ \beta\delta\eta}} W_{\alpha\gamma\epsilon,\beta\delta\eta} \, a_\alpha^\dagger a_\gamma^\dagger a_\epsilon^\dagger a_\eta a_\delta a_\beta$$

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- The Green's function as a tool to solve the nuclear many-body problem:

particle exits · particle enters · hole exits · hole enters

$$G_{\alpha\beta}(\omega) = \sum_m \frac{\langle\Psi_0^N|a_\alpha|\Psi_m^{N+1}\rangle\langle\Psi_m^{N+1}|a_\beta^\dagger|\Psi_0^N\rangle}{\omega-(E_m^{N+1}-E_0^N)+i\eta} + \sum_n \frac{\langle\Psi_0^N|a_\beta^\dagger|\Psi_n^{N-1}\rangle\langle\Psi_n^{N-1}|a_\alpha|\Psi_0^N\rangle}{\omega-(E_0-E_n^{N-1})+i\eta}$$

energy with an added particle · energy with a removed particle

- Self-consistent nonperturbative method:

Green's function → T-matrix → Self-energy

- Nonrelativistic Hamiltonian:

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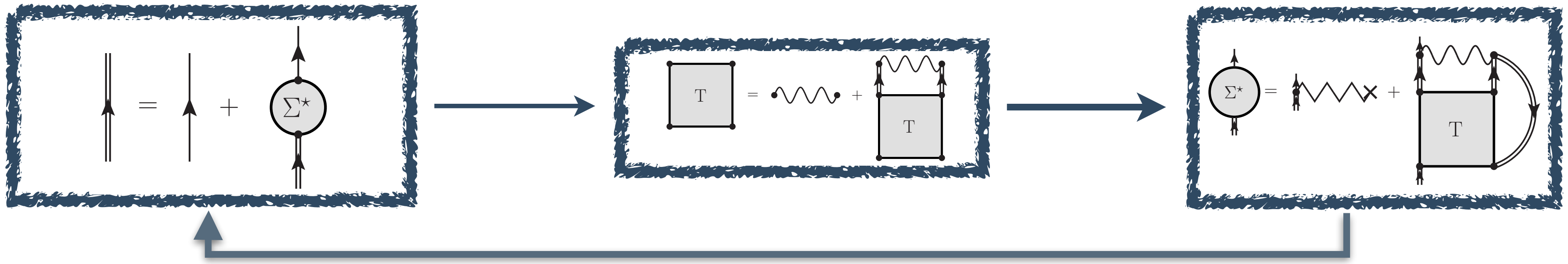
energy with an added particle

energy with a removed particle

- Self-consistent nonperturbative method:



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particle exits *particle enters* *hole exits* *hole enters*

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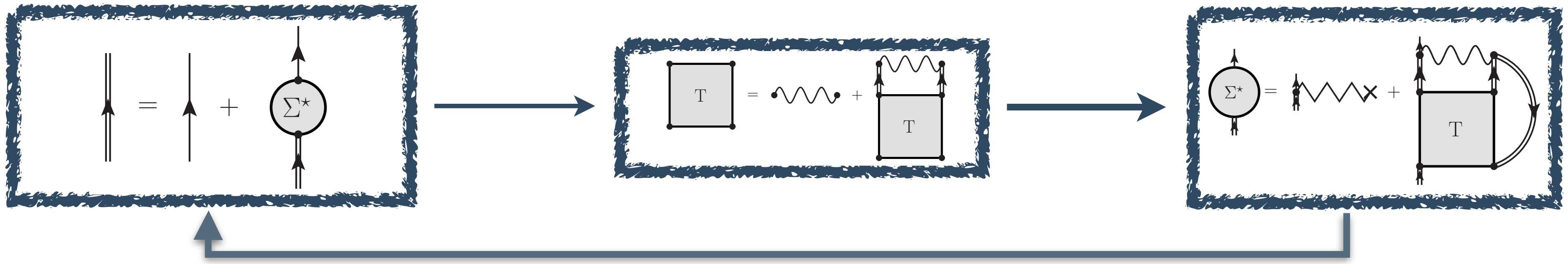
energy with an added particle *energy with a removed particle*

- Self-consistent nonperturbative method:



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$\hat{H}_0$

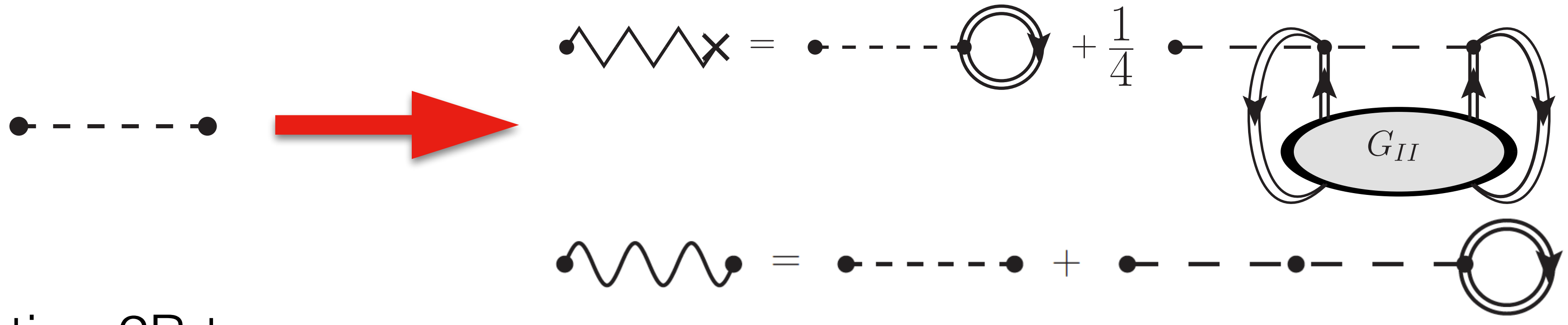
Extend the SCGF approach

Carbone, Cipollone, Barbieri, Rios, Polls, PRC **88**, 054326 (2013)

2B → 2B + 3B

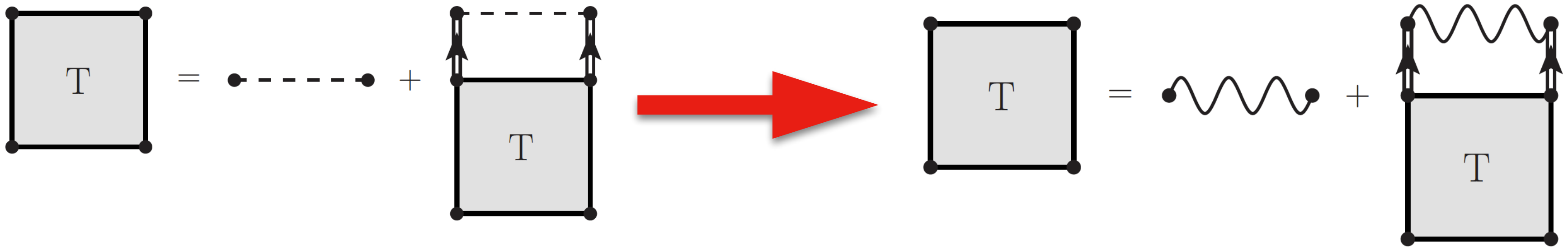
1. define **effective interactions** to include correctly 3B terms:

Interaction



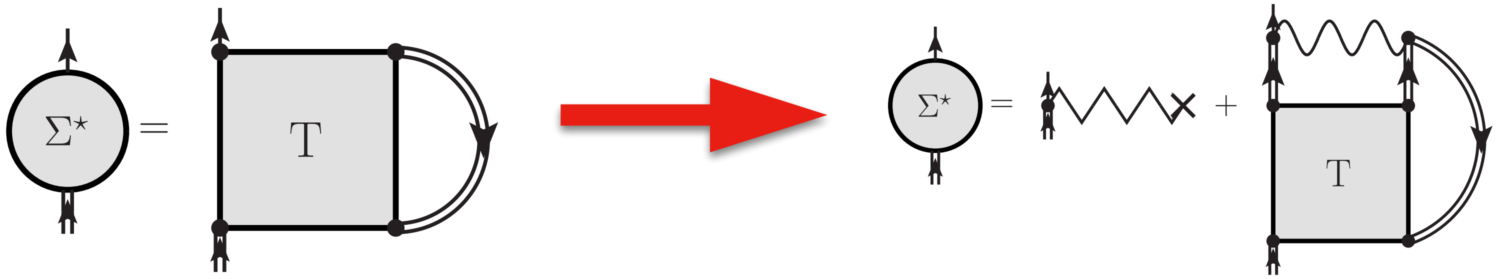
2. calculate T-matrix with effective 2B term:

T-matrix



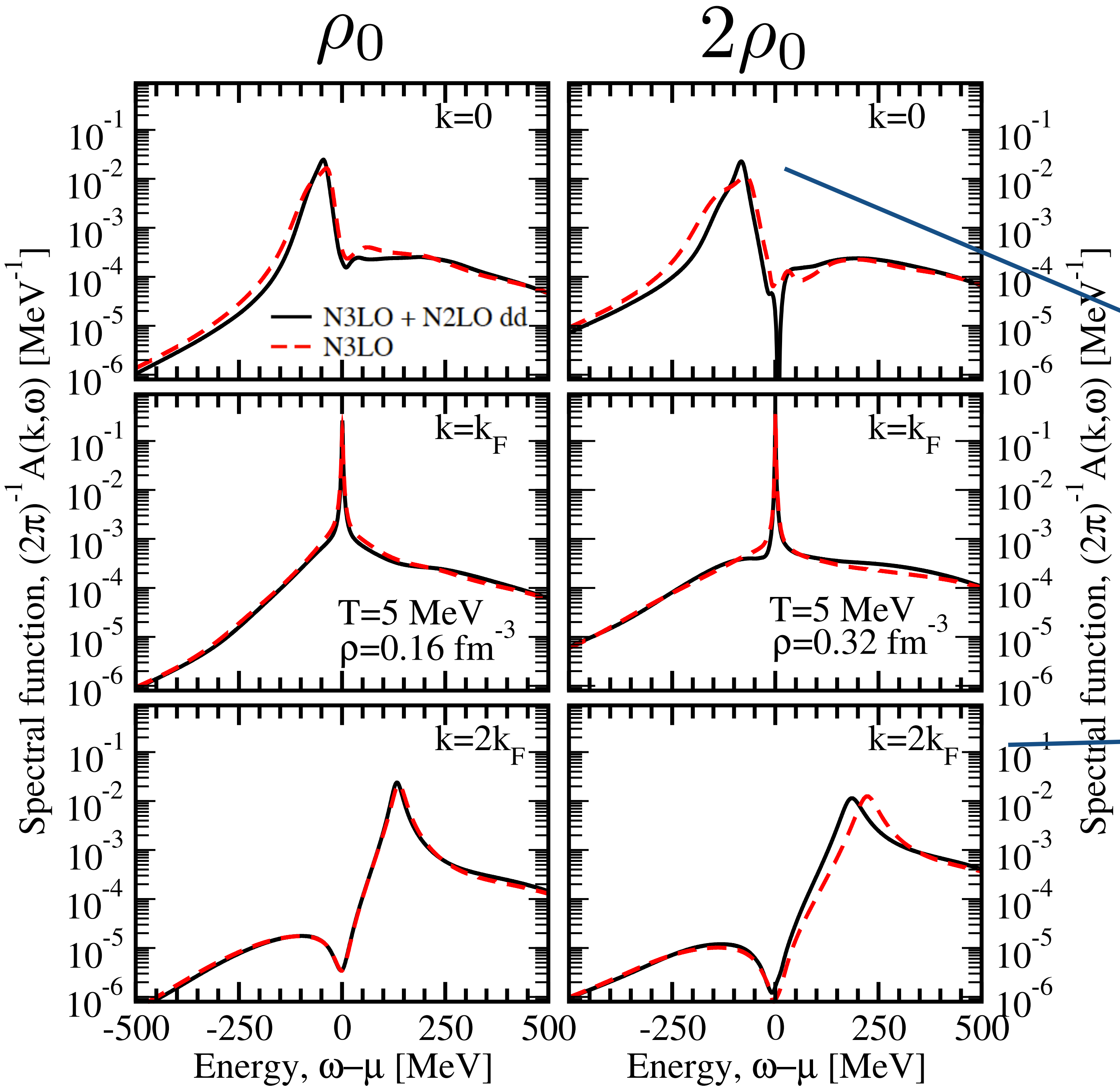
3. calculate self-energy distinguishing the effective terms:

Self-energy



Symmetric nuclear matter

The spectral function



Carbone, Rios, Polls, PRC 88, 044302 (2013)

Slight 3BF effect in general....

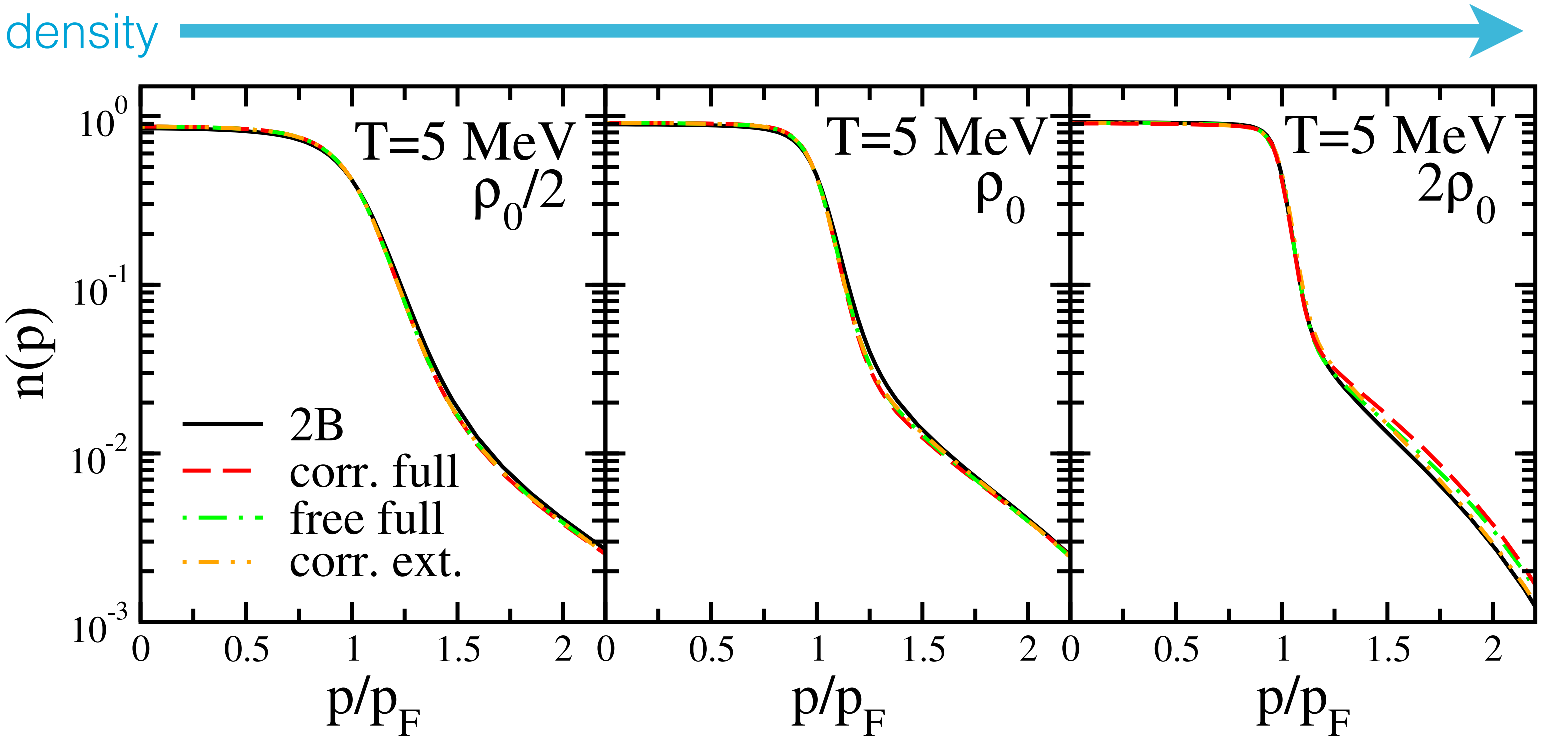
Narrower quasi-parcicle peak at low momenta

Lower energy for quasi-particle with 3NF because of the rescaling

Symmetric nuclear matter

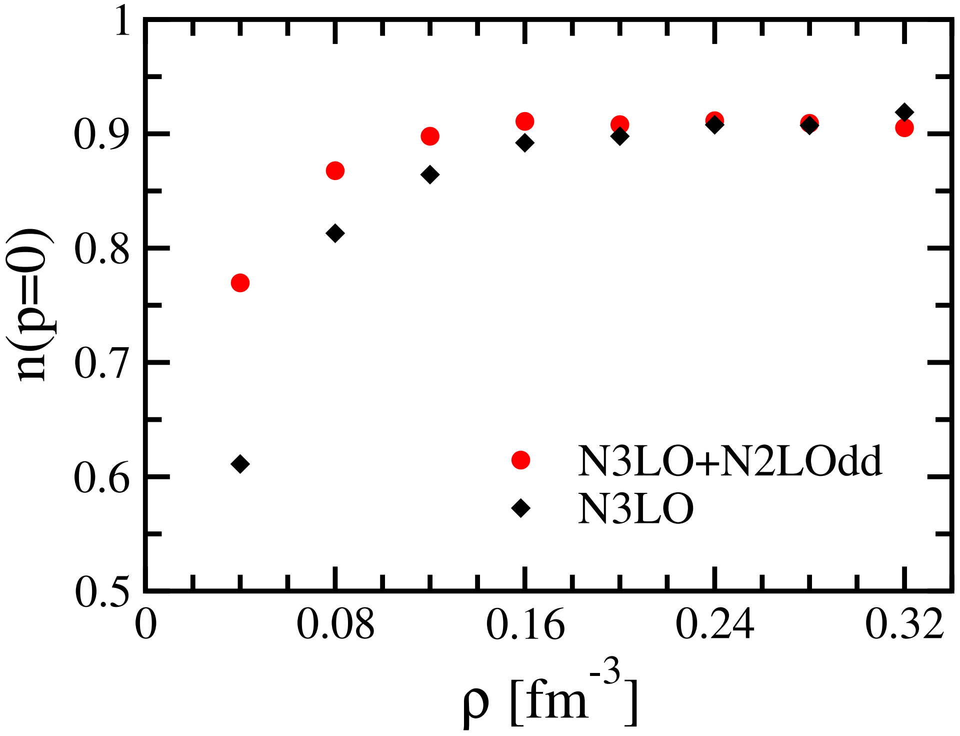
Momentum distribution

Carbone, Rios, Polls, PRC 90, 054322 (2014)



$$n(p) = \int \frac{d\omega}{2\pi} \mathcal{A}(p,\omega) f(\omega)$$

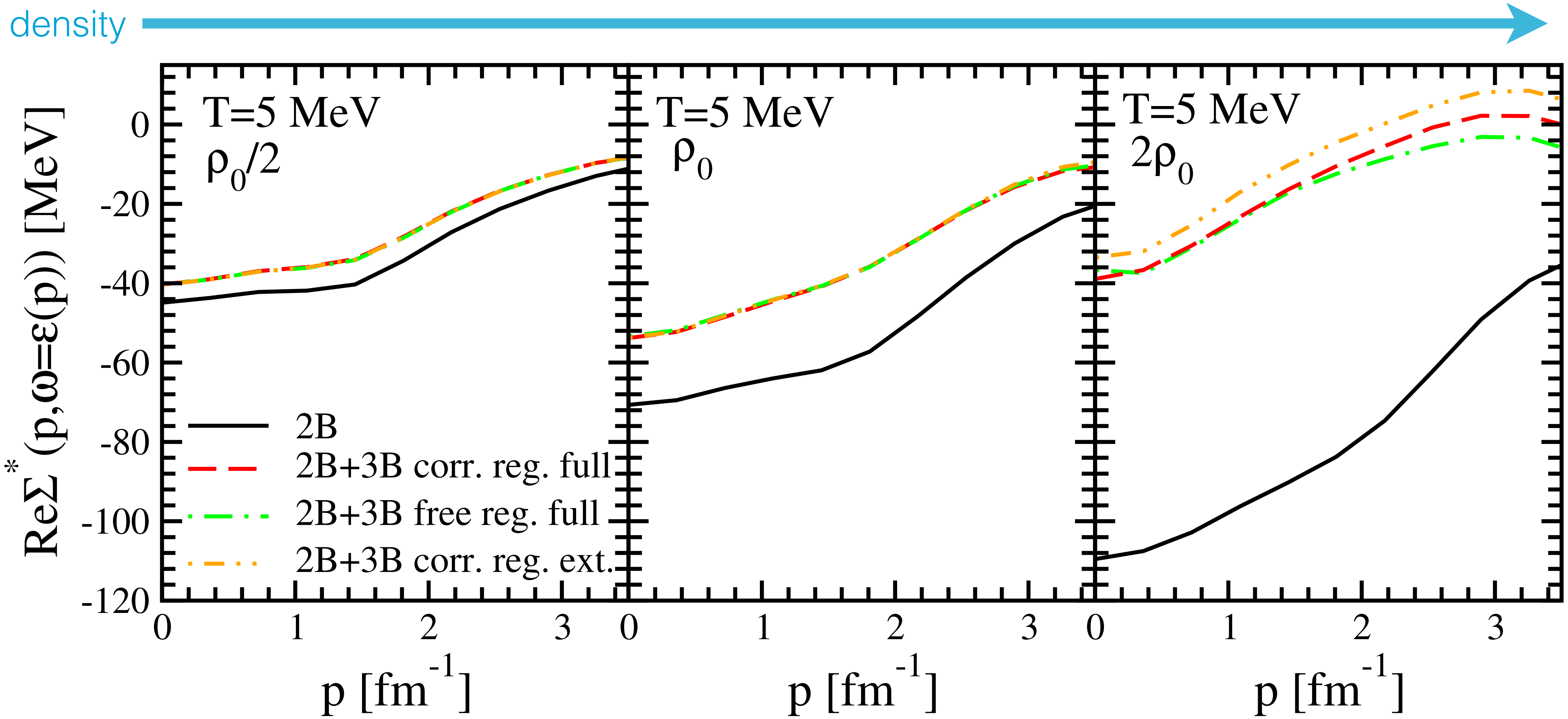
- small changes due to 3BF
- high-momentum components
- depletion density-dependent
- small averaging differences



Symmetric nuclear matter

Single-particle potential

density



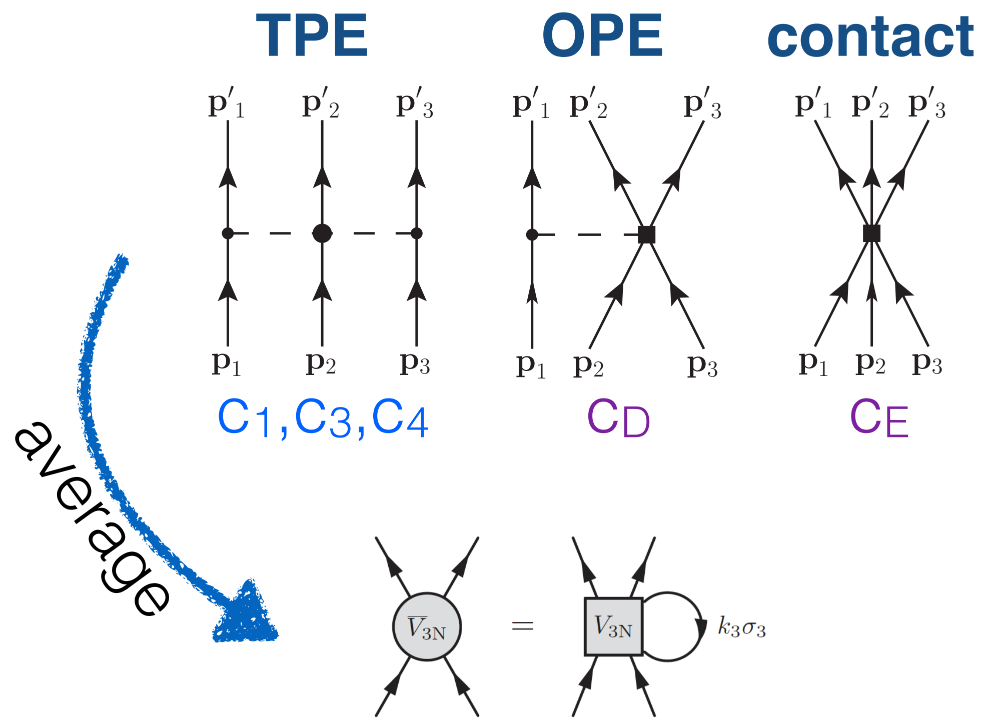
$$\varepsilon_{qp}(p) = \frac{p^2}{2m} + \mathrm{Re}\Sigma^\star(p, \varepsilon_{qp}(p))$$

Single-particle spectra

- strong effect of 3-body forces
- repulsion rises with density
- modifications due to averaging procedure visible at high density

Symmetric nuclear matter

The need for 3-body nuclear forces

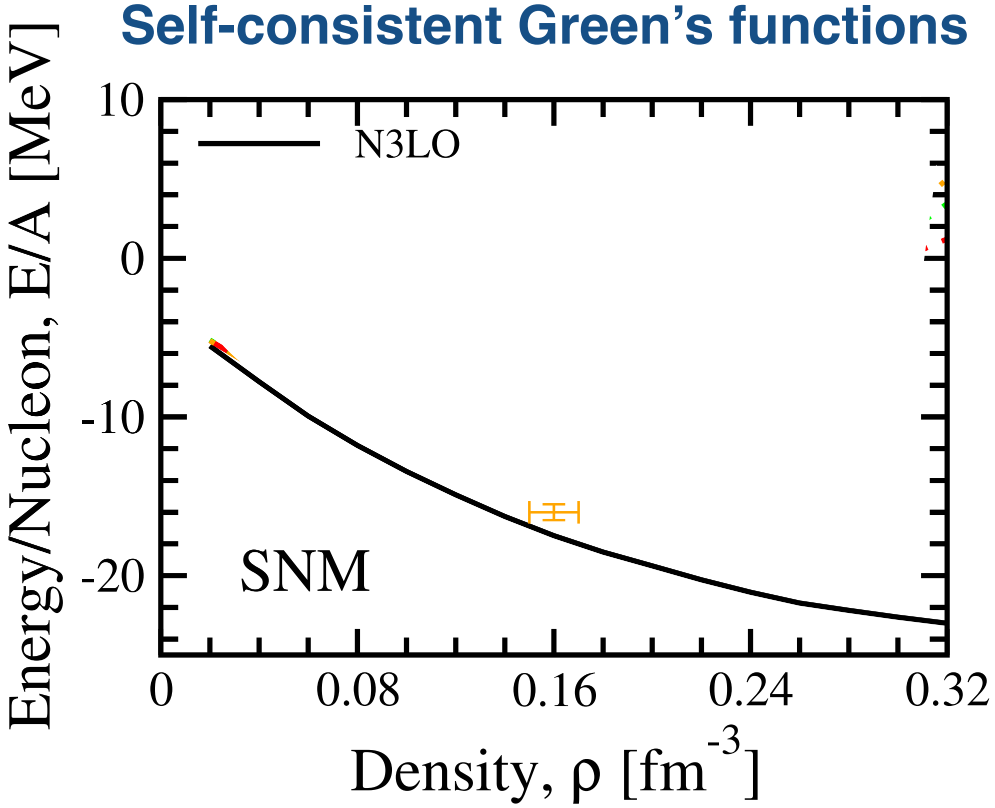


J.W. Holt *et al.*, PRC 81, 024002 (2010)
Hebeler *et al.*, PRC 82, 014314 (2010)
Carbone *et al.*, PRC 90, 054322 (2014)

The Koltun sumrule

$$\frac{E}{A} = \frac{\nu}{\rho}\int \frac{\mathrm{d}^3p}{(2\pi)^3}\int \frac{\mathrm{d}\omega}{2\pi}\frac{1}{2}\left\{\frac{p^2}{2m}+\omega\right\}\mathcal{A}(p,\omega)f(\omega) - \frac{1}{2}\langle\Psi^N|\hat{W}|\Psi^N\rangle$$

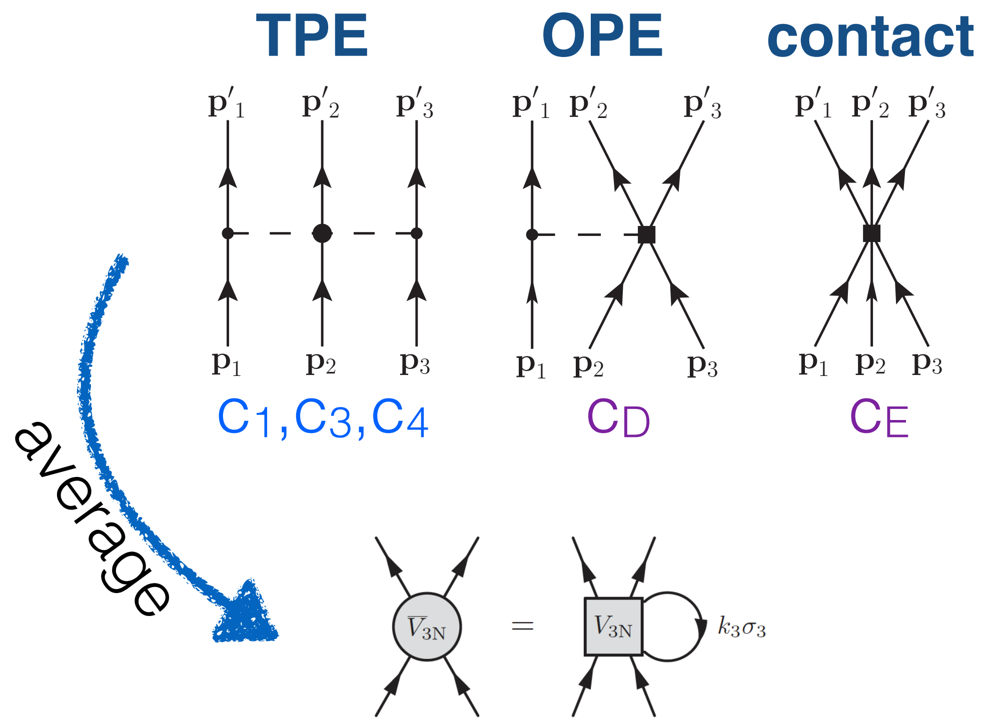
Self-consistent Green's functions



Carbone *et al.*, PRC 90, 054322 (2014)

- Repulsion due to 3BF
- Improved prediction of saturation density
- Small averaging dependence
- However saturation energy underbound

The need for 3-body nuclear forces



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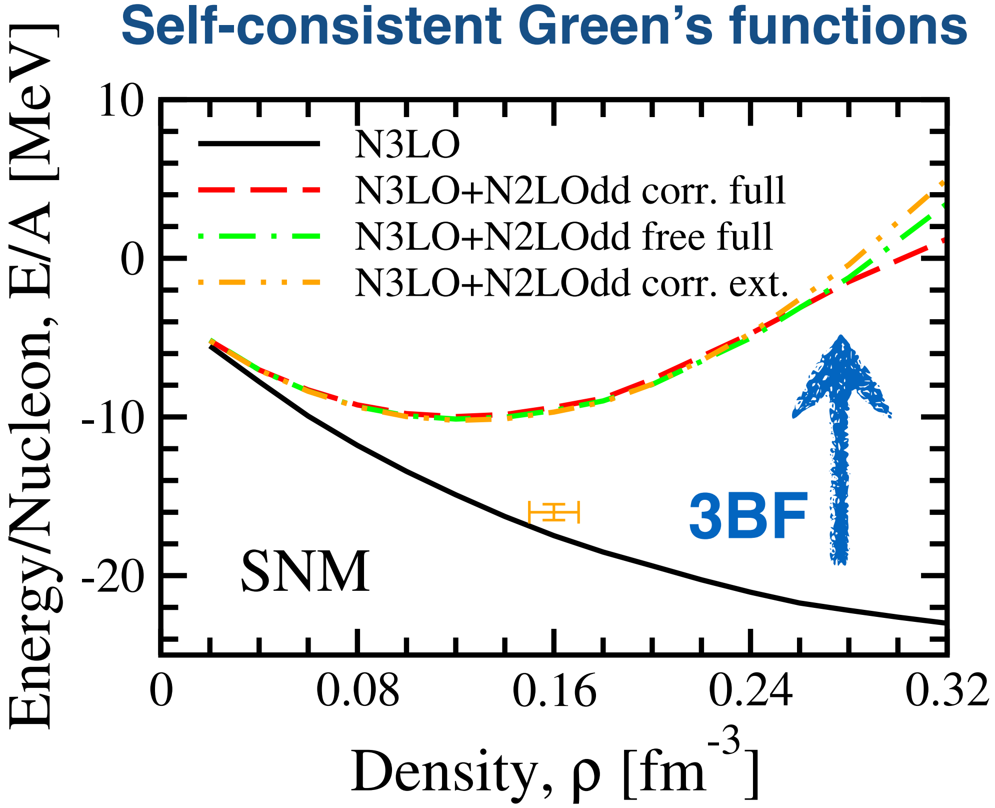

Carbone *et al.*, PRC 90, 054322 (2014)

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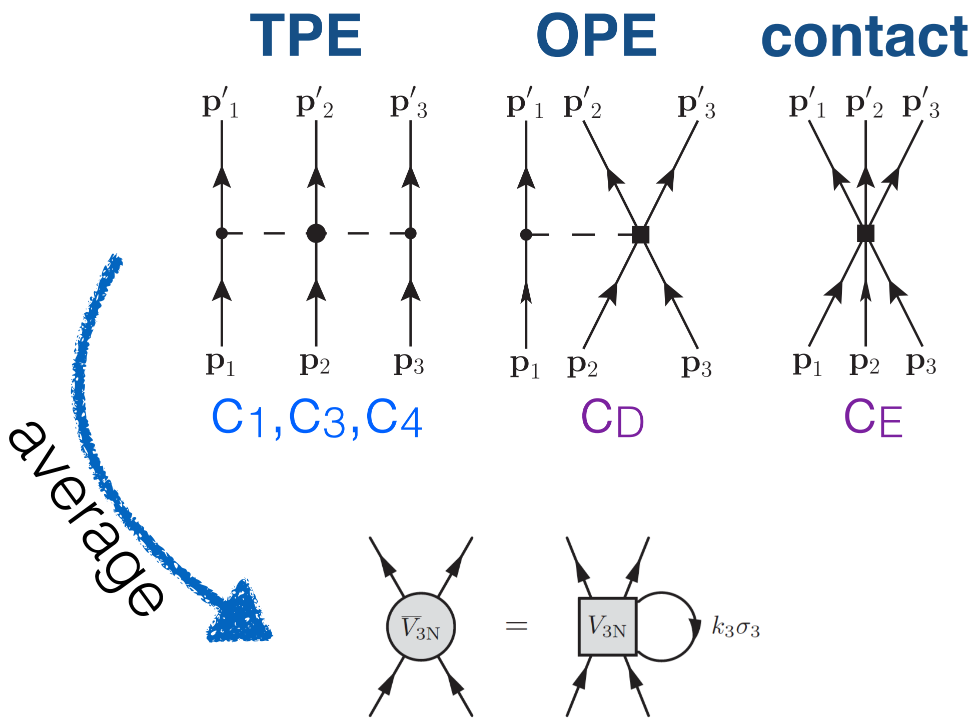
Symmetric nuclear matter

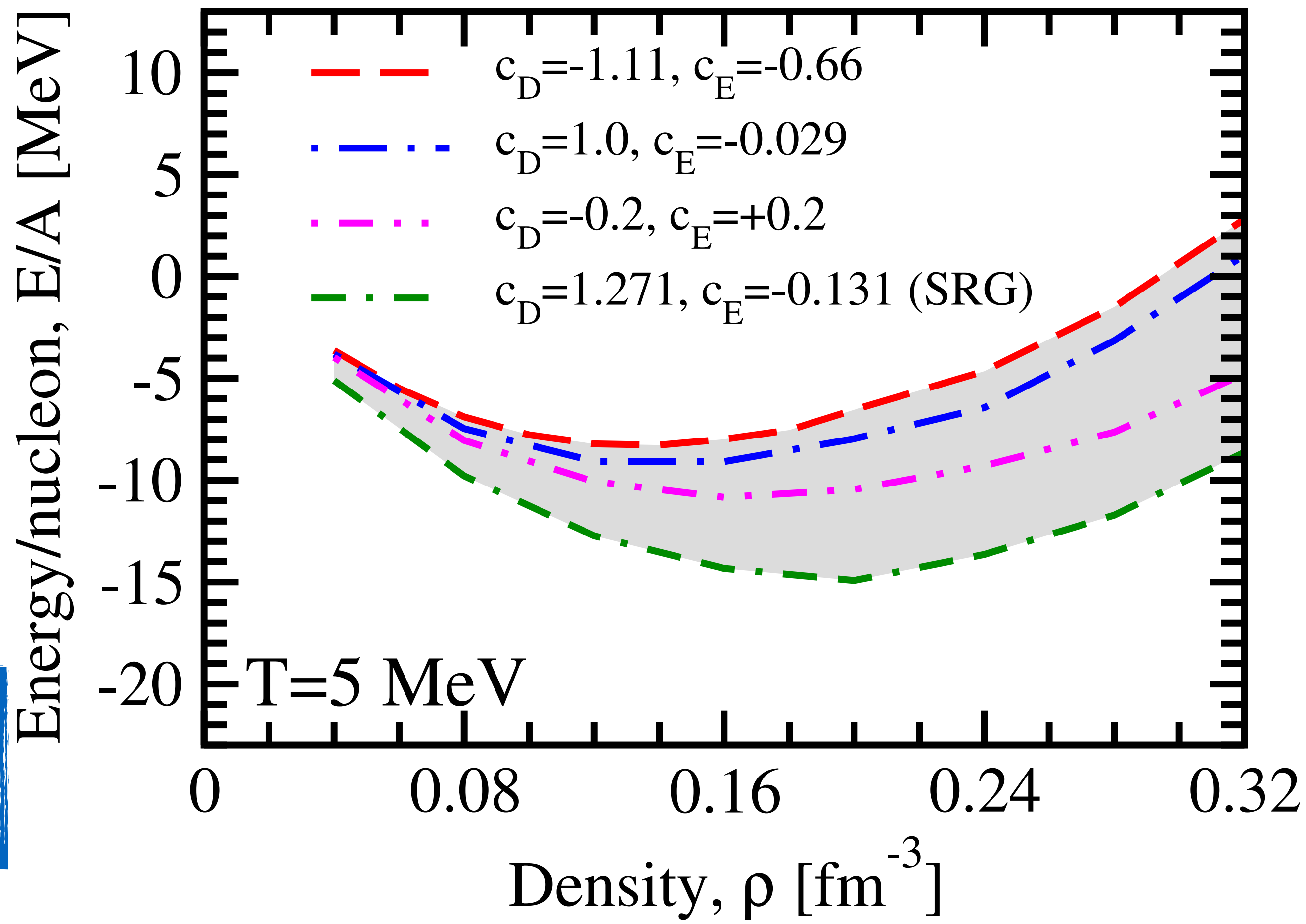
Theoretical uncertainties



- Band represents the theoretical uncertainties
- The uncertainty dependence increases with density as the effect of 3NF

J.W. Holt *et al.*, PRC 81, 024002 (2010)
Hebeler *et al.*, PRC 82, 014314 (2010)
Carbone *et al.*, PRC 90, 054322 (2014)

Self-consistent Green's functions

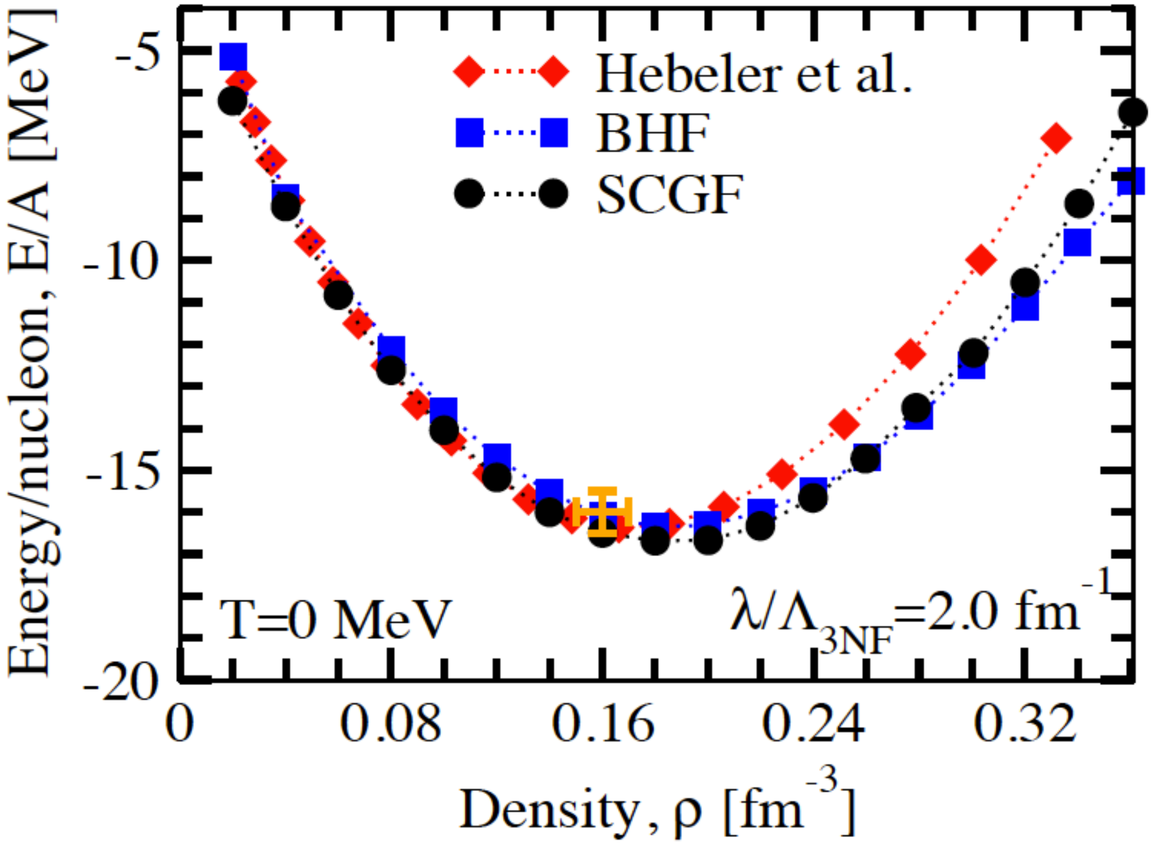


c_D and c_E fit to few-body properties

Symmetric nuclear matter

Many-body methods comparison

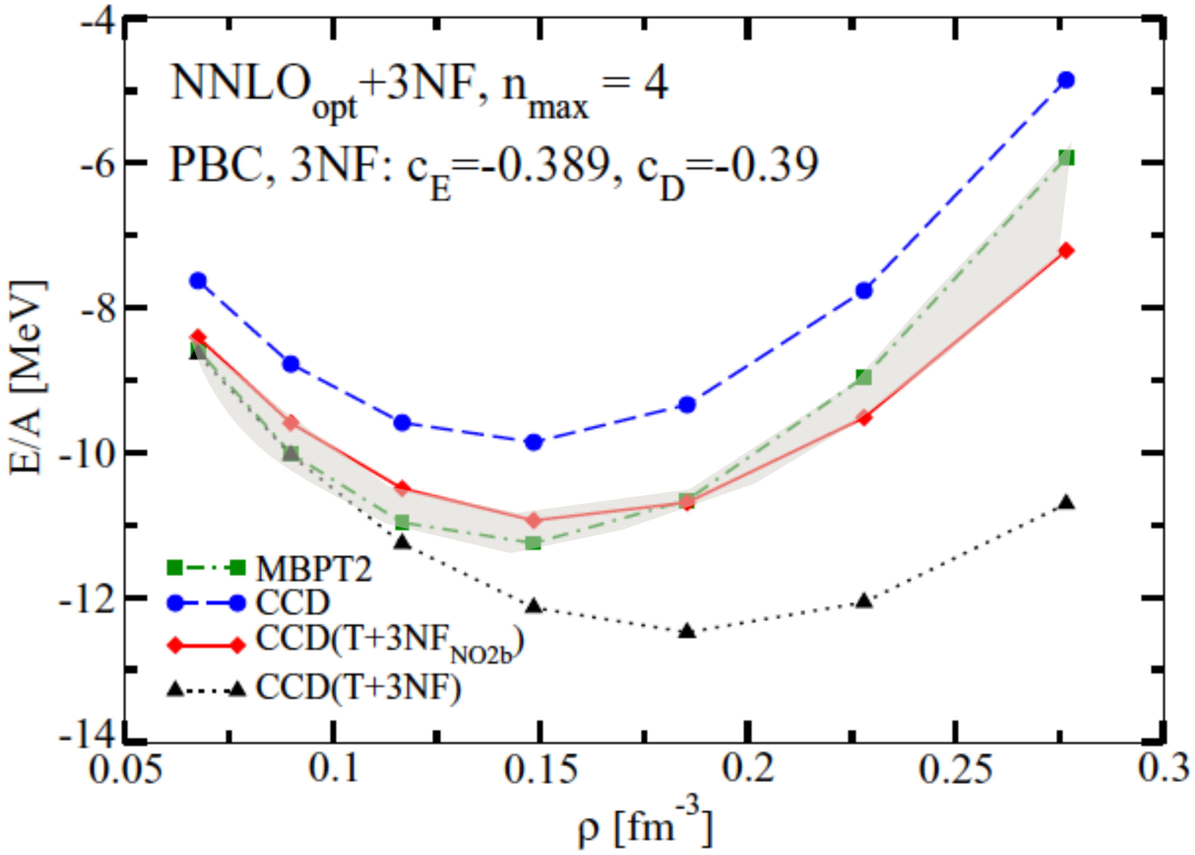
Many-body perturbation theory
Brueckner-Hartree-Fock
Self-consistent Green's functions



Carbone *et al.*, PRC 88, 044302 (2013)

Hebeler *et al.*, PRC 83, 031301(R) (2011)

Coupled Cluster
Many-body perturbation theory



Hagen *et al.*, PRC 90, 054322 (2014)

Further results:

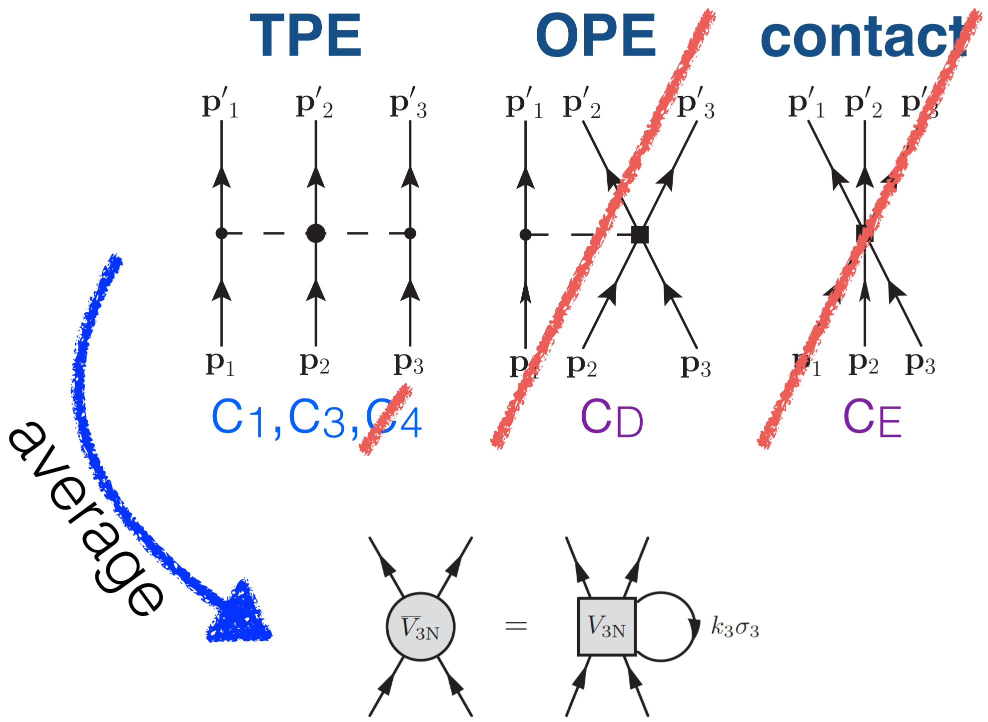
In-medium Chiral PT J.W. Holt et al., PPNP 73, 35 (2013),
Lacour et al., Ann. Phys. 326, 241 (2011)

MBPT Coraggio et al., PRC 89, 044321 (2014)

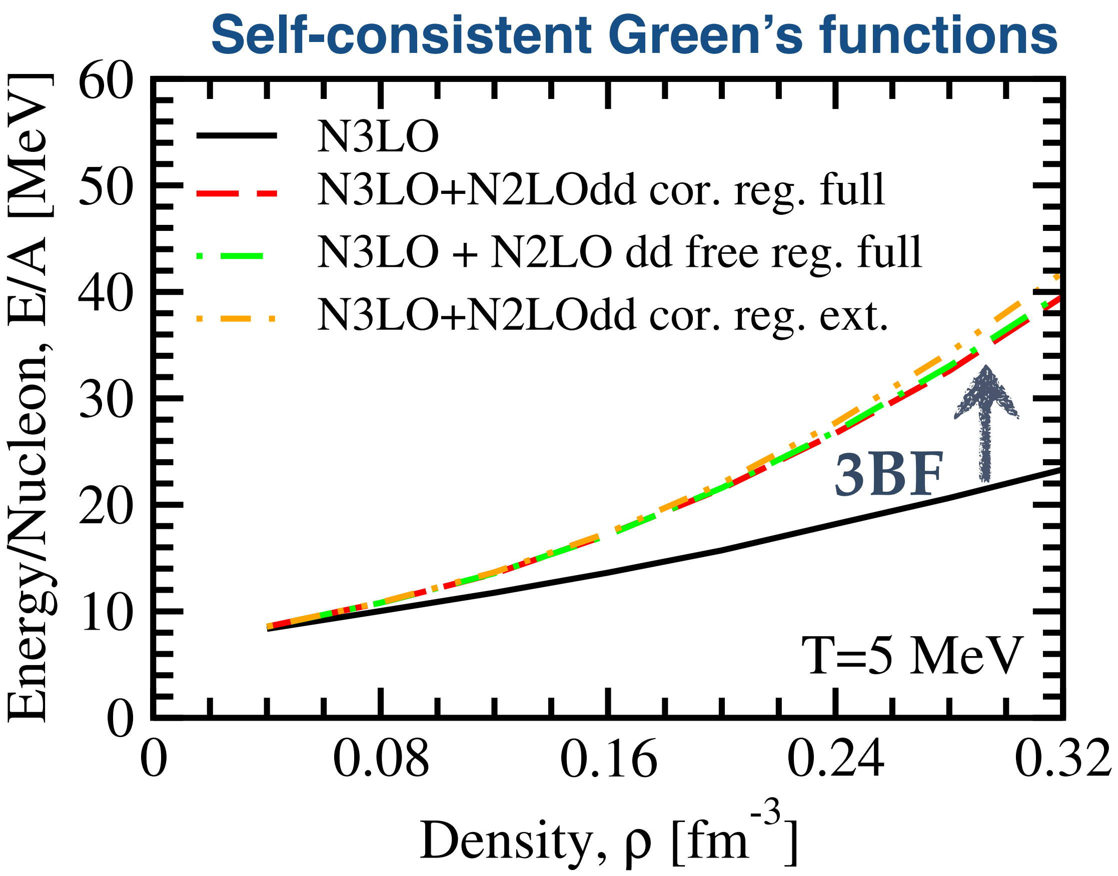
Uncertainty from the many-body calculations is smaller than the one estimated from chiral EFT

Pure neutron matter

How neutron matter energy stiffens



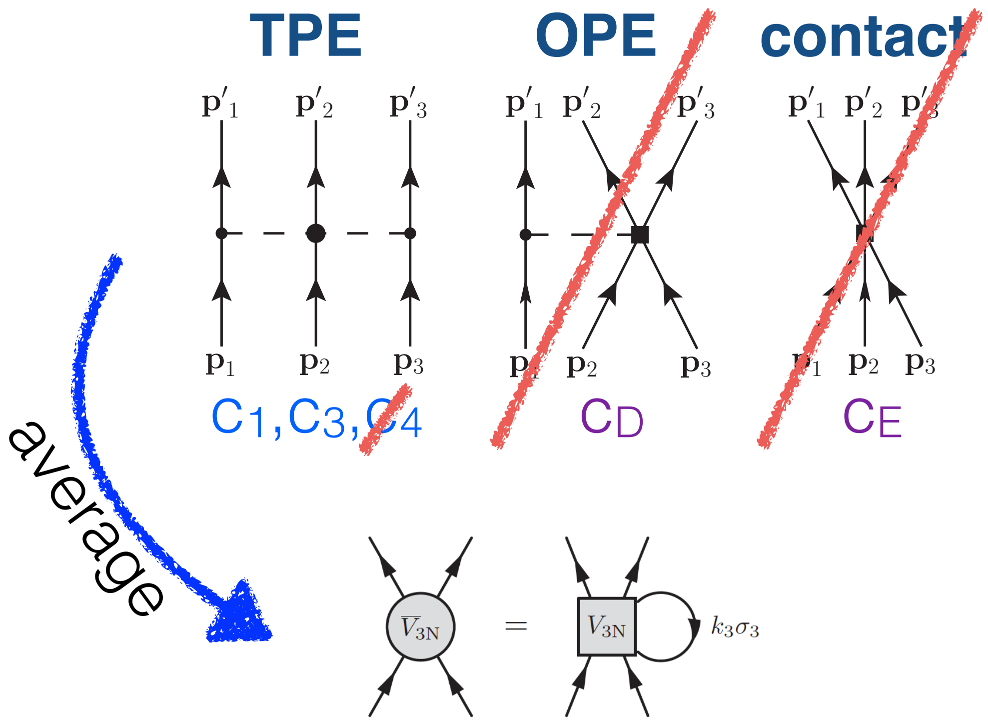
- 3NFs fully predicted
- no need to fit to few-body properties



Carbone *et al.*, PRC 90, 054322 (2014)

- Global repulsive effect due to 3bf
- Repulsion of 4 MeV at 0.16 fm^{-3} to 15 MeV at 0.32 fm^{-3}
- Small dependence on averaging procedures

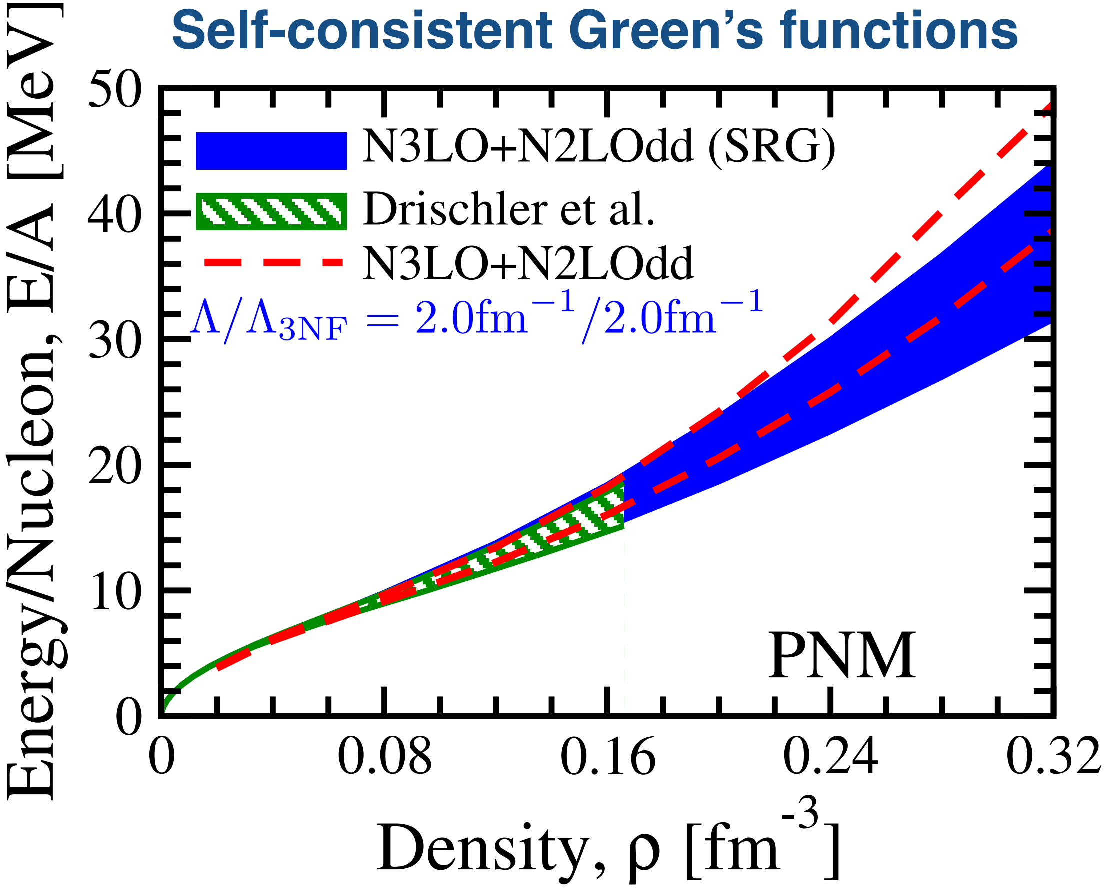
Many-body methods comparison



- SCGF: agreement up to 0.20 fm^{-3} with the use of different Hamiltonians
- MBPT vs SCGF: agreement up to saturation density
- Low-density neutron matter perturbative

- Bands from c1 and c3 uncertainties

Many-body perturbation theory
Self-consistent Green's functions

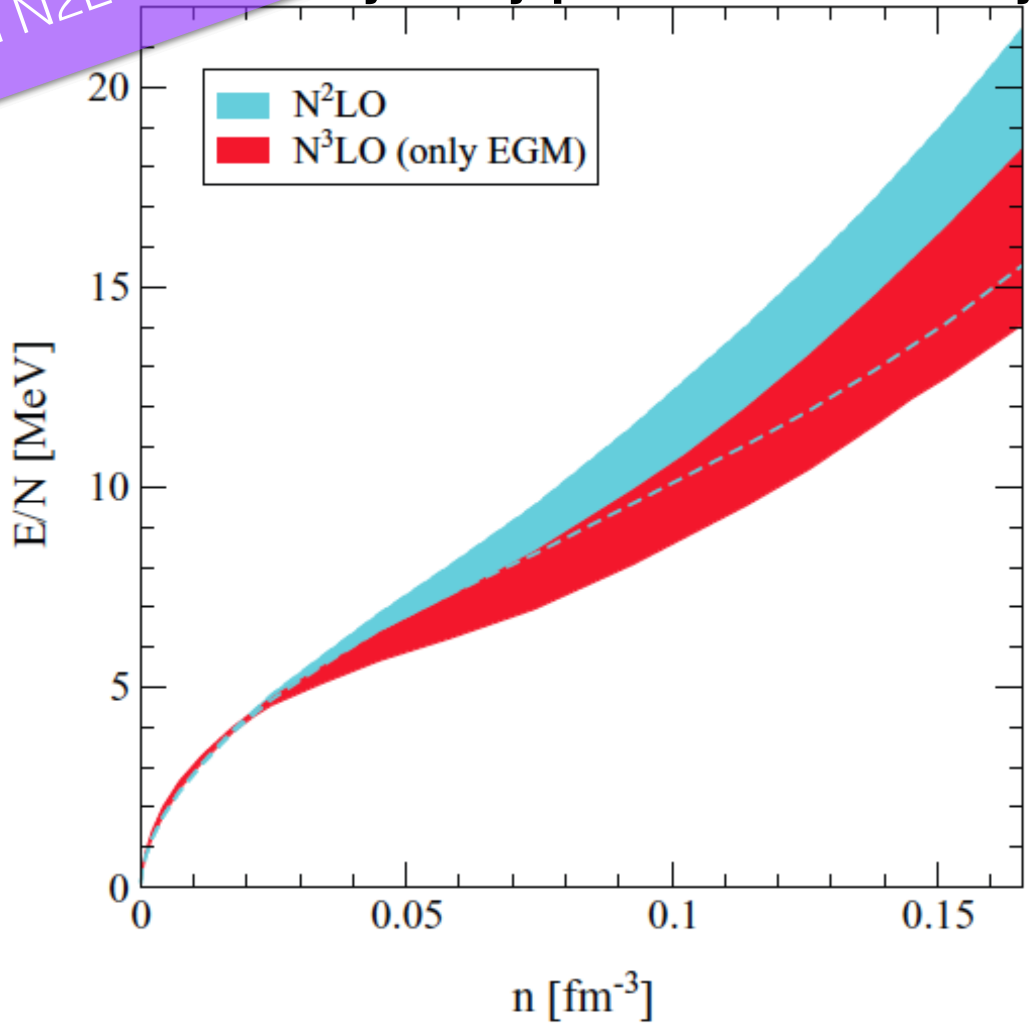


Carbone *et al.*, PRC 90, 054322 (2014)

Low-density neutron matter

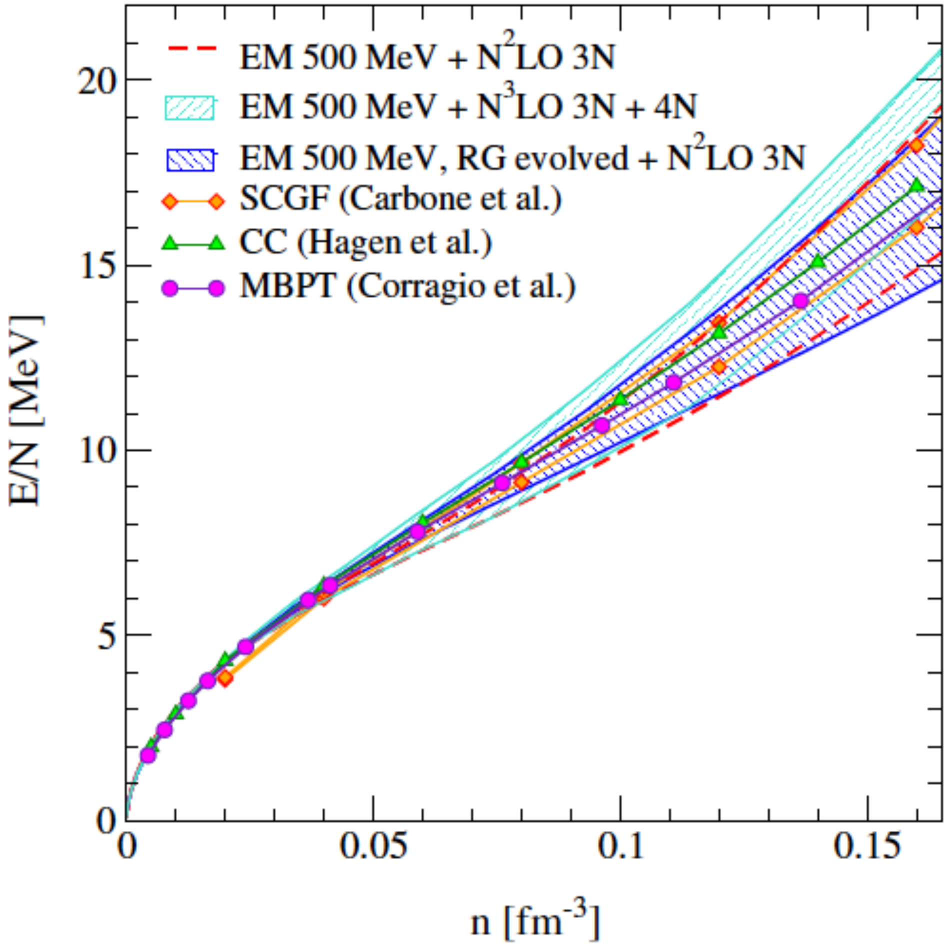
Full N2LO vs N3LO

Many-body perturbation theory



Krüger *et al.*, PRC 88, 025802 (2013)

Remarkable agreement between many-body methods and different Hamiltonians

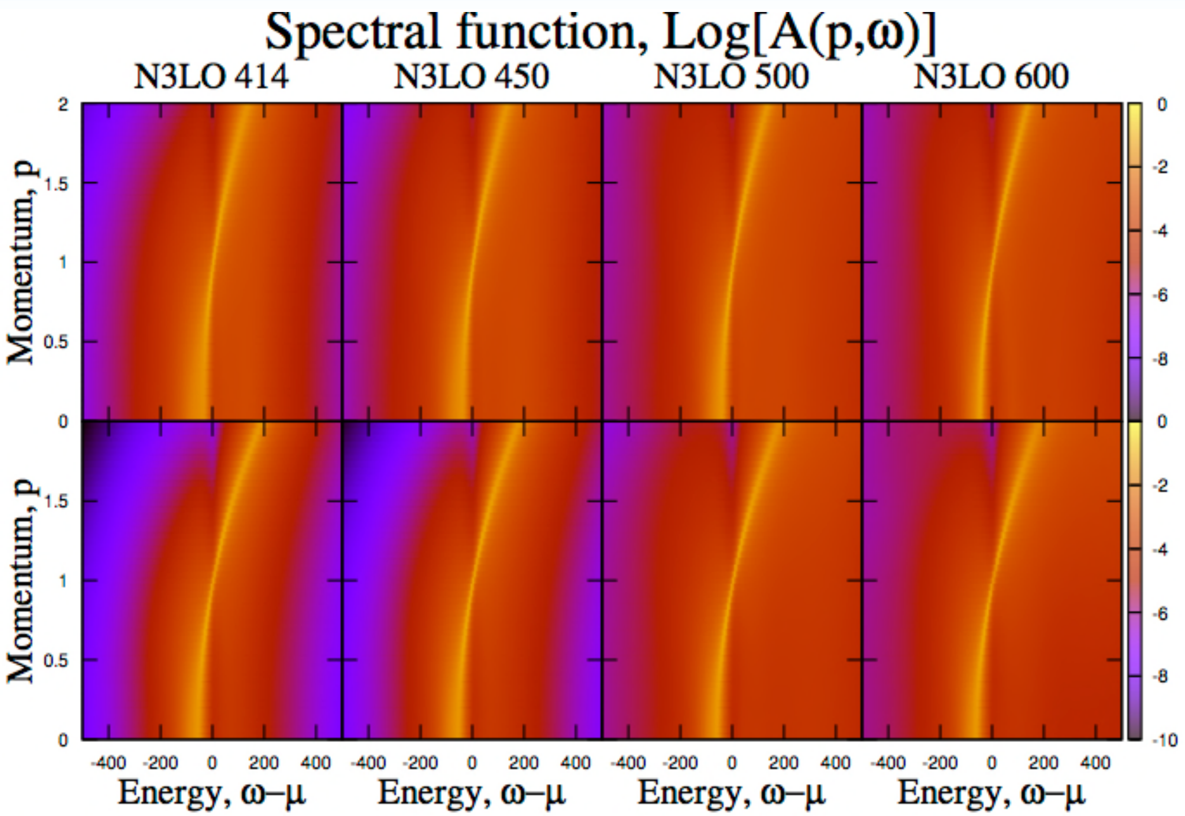


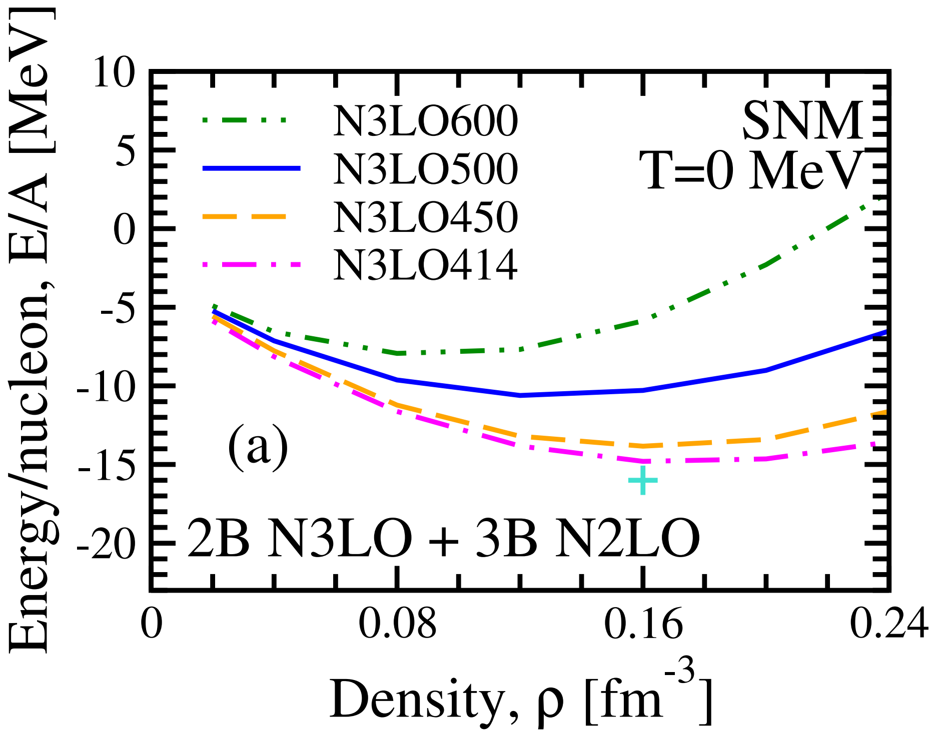
Hebeler *et al.*, Ann. Rev. Nucl. Part. Sci. 65, 457 (2015)

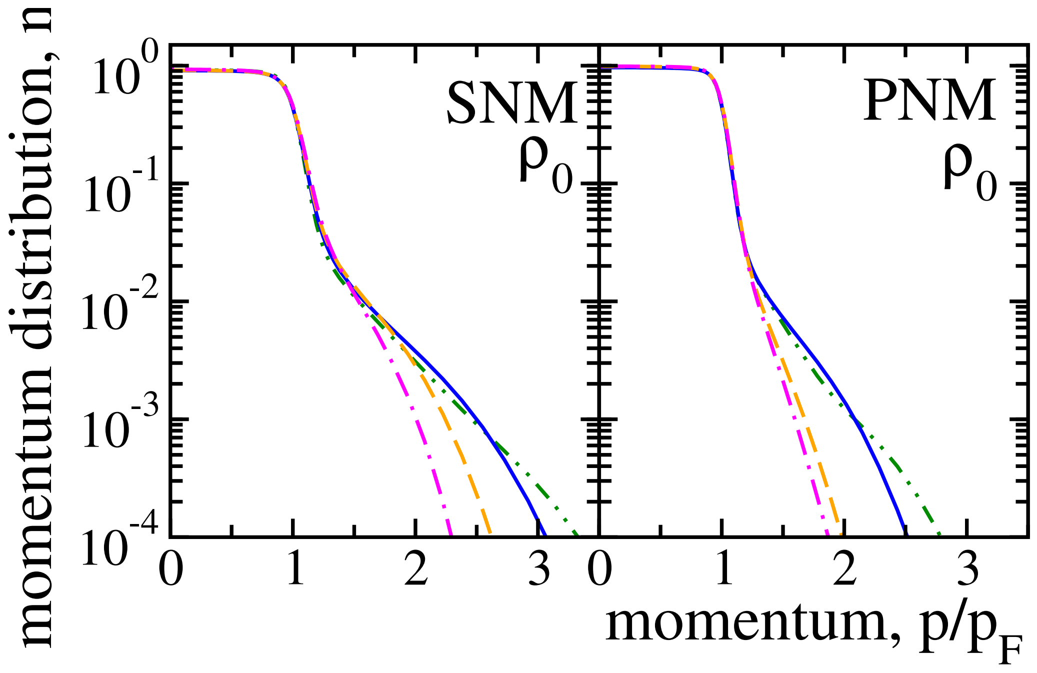
- 3NF N3LO subleading terms important
- Uncertainty band narrows
- Inclusion of explicit Δ will improve band

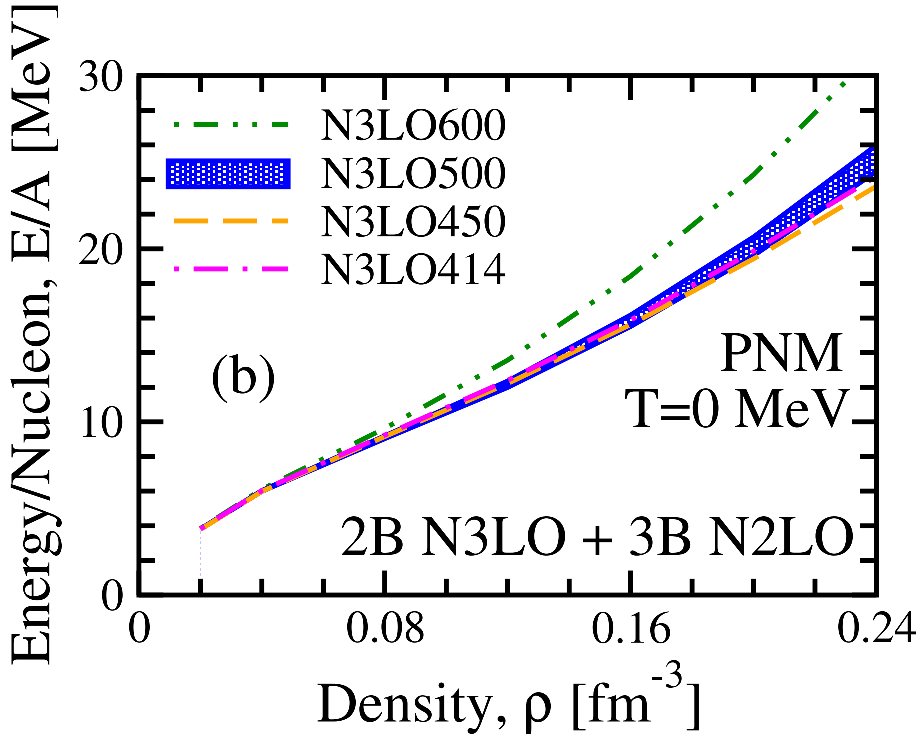
Further results:
AFDMC - Gezerlis et al., PRC 90, 054323 (2014)
Lattice EFT - Epelbaum et al., EPJA 40, 199 (2009)
In-medium Chiral PT - J.W. Holt et al., PPNP 73, 35 (2013),
Lacour et al., Ann. Phys. 326, 241 (2011)

Cutoff dependence

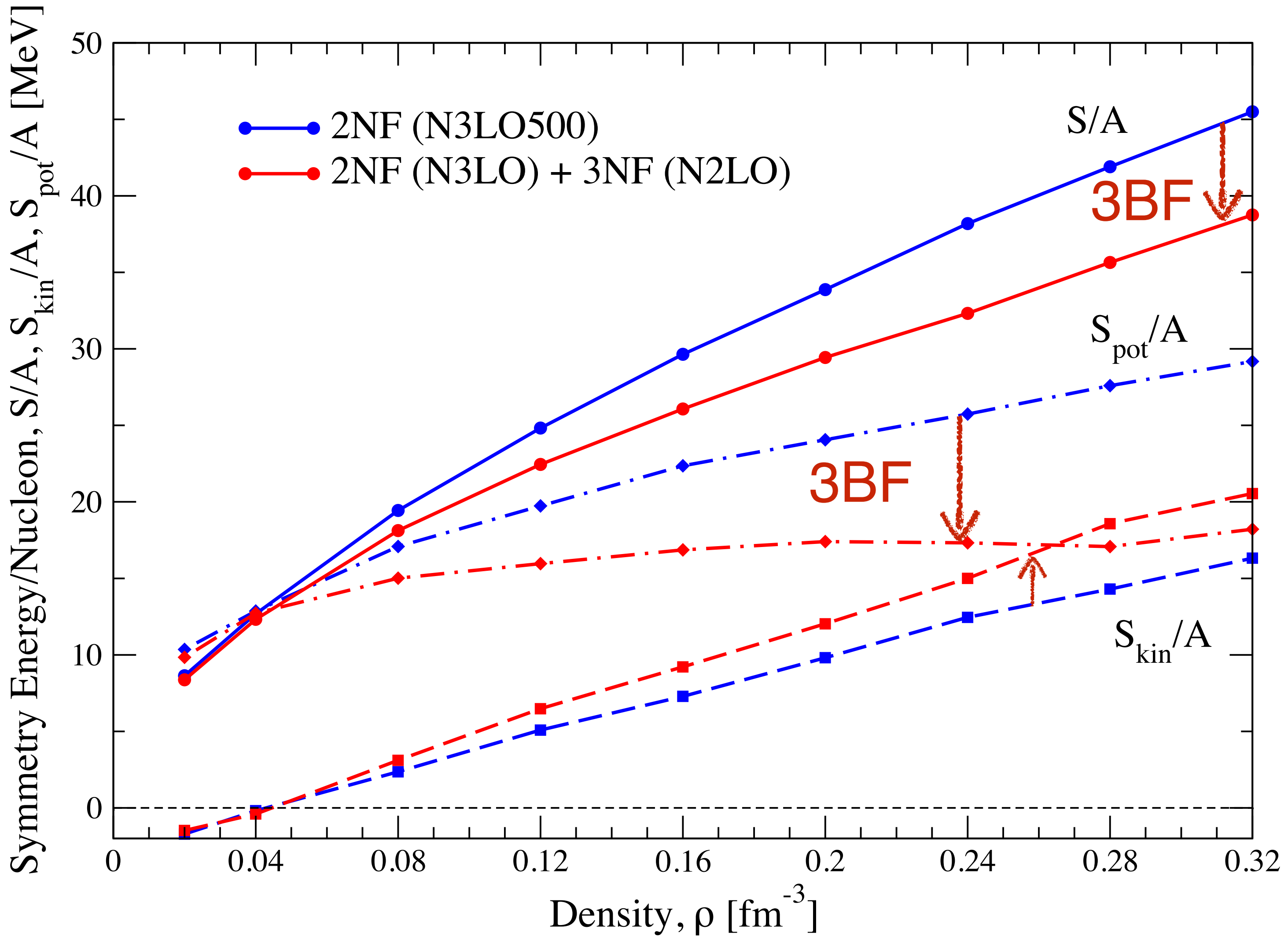








The symmetry energy

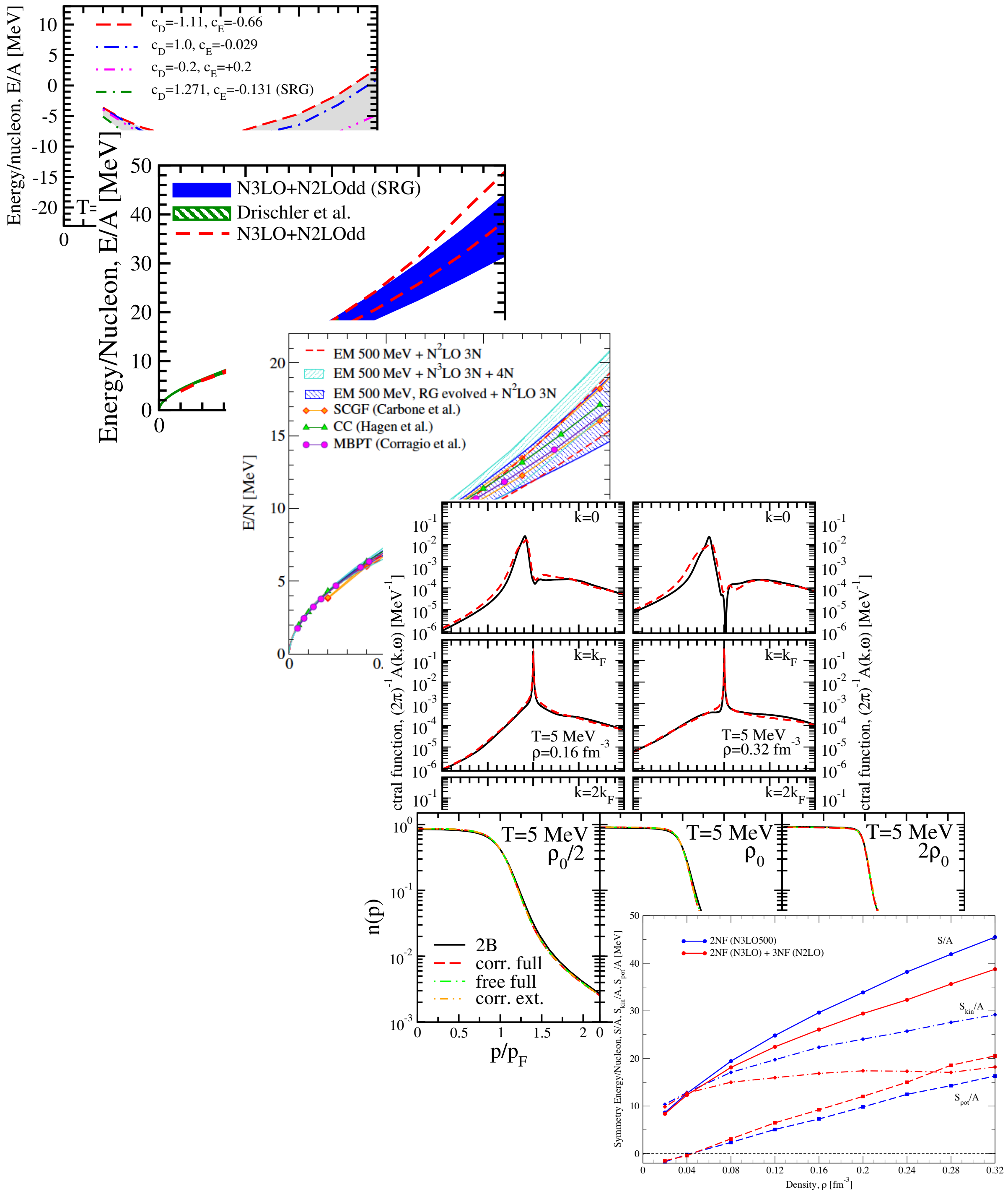


$$\frac{S}{A}(\rho) \simeq \frac{E_{pnm}}{A}(\rho) - \frac{E_{snm}}{A}(\rho)$$

- 3NFs decrease symmetry energy
- Main effect from potential term
- The kinetic term stiffens
- 3NFs lowers high momentum components in SNM

$\rho_0 = 0.16\text{fm}^{-3}$	S_{tot}	E_{pnm}	E_{snm}	S_{kin}	K_{pnm}	K_{snm}	S_{pot}	P_{pnm}	P_{pnm}
2NF [MeV]	29.64	13.64	-16.00	7.29	40.17	32.88	22.36	-26.53	-48.89
2NF+3NF [MeV]	26.07	17.20	-8.87	9.21	41.21	32.00	16.86	-24.00	-40.87

Summary



- A consistent extension of the SCGF method to include 3NFs is accomplished
- Nuclear and neutron matter with theoretical uncertainties can be calculated reliably using *ab initio* methods based on chiral Hamiltonians
- Uncertainties in the many-body methods are under control
- Small overall effect of 3NFs on the momentum distribution
- 3NFs lowers the correlations in SNM

Outlook

- Chiral EFT Hamiltonian: power counting, theoretical uncertainties, limits of chiral EFT, etc.
- Many-body approximation methods: include irreducible 3B terms, improve the effective interactions, include particle-hole diagrams, asymmetric matter, etc.
- Reliable finite temperature results from *ab initio* theory: high astrophysical impact (EOSs at finite T, dynamics of neutron star merger simulations, core-collapse supernovae, etc.)

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Thank you for your attention!

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