

# Correlations in Few<sup>\*</sup> Body Systems

EMMI Workshop: Cold dense nuclear matter: from short-range nuclear correlations to neutron stars

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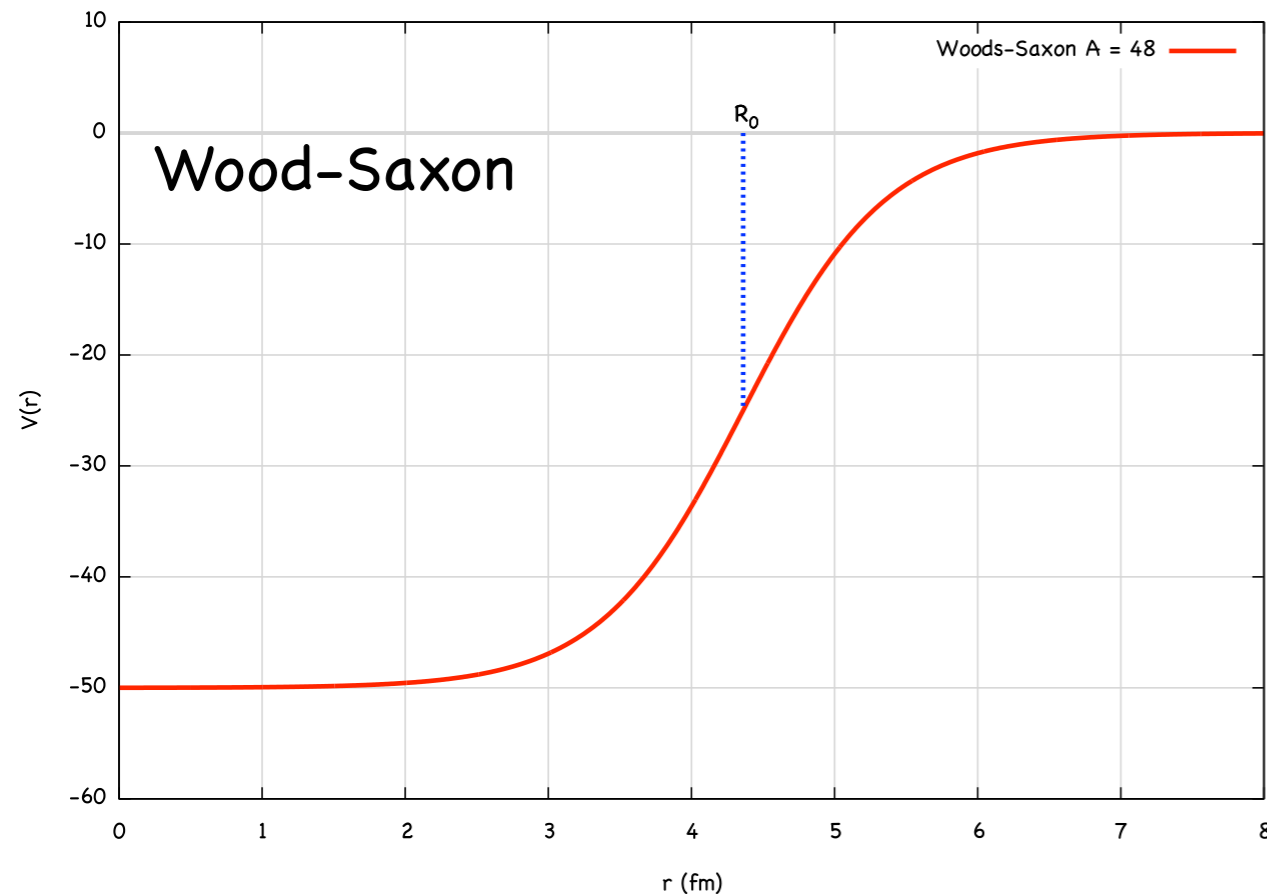
\* and heavier

# Outline

- Introduction
- NN potential responsible
  - Implications
- Correlations exist
  - Some examples
- Two nucleon correlations
  - Inclusive
  - Exclusive – Few body systems
- 3N correlations
  - $4\text{He}/3\text{He}$
  - $^{12}\text{C}/4\text{He}$  etc
- $A_{zz}$ : Scattering from a Tensor Polarized Deuteron
- Finish

# Independent Particle Shell Model

- Independent particle states of a uniform potential – a **mean field**.



- Long mean free paths
- No two-body interactions
- Absence of correlations in<sup>3</sup> ground-state wave function.**

$$V_{WS}(r) = \frac{-V_0}{1 + e^{(r-R)/a}} - V_{ls} \left( \frac{\hbar}{m_{\pi}c} \right)^2 \frac{1}{r} \frac{d}{dr} \left( \frac{1}{1 + e^{(r-R)/a}} \right) \vec{l} \cdot \vec{\sigma} + V_{Coul}$$

- The single-particle energies  $\xi_{\alpha}$  and wave function  $\Phi_{\alpha}$  are the basic quantities – can be accessed **in knockout reactions**
- The spectral function should exhibit a structure at fixed energies with momentum distributions characteristic of the shell (orbit).

$$S(\vec{p}, E) = \sum_a |\Phi_a(p)|^2 \delta(E + \xi_a)$$

- Enormous strong force acting
- So many nucleons to collide with
- How can nucleons possibly complete whole orbits ( $10^{21}/s$ ) without interacting?

**Hard core is small part of the nuclear volume**

$$\frac{V_c}{V_{total}} = \left( \frac{c}{2r_o} \right)^3 \approx 1/100$$

# Case for Correlations

- The nucleon–nucleon (NN) interaction is singularly repulsive at short distances
  - Difficult to find two nucleons close to each other.
  - Loss in configuration space components signals an increase of high-momentum components
- Both the correlation hole and the high- $k$  components are absent in IPMs
- Taken together the loss of configuration space and the strengthening of high of momentum components are “correlations”.
- The NN tensor force also provides high-momentum components; required to obtain the quadrupole moment of the deuteron and predicts a isospin dependence of SRCs.

# Possible Two Nucleon states

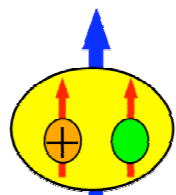
# NN Interaction

L	S	J	$\pi = -1^L$	$T(L+S+T \text{ odd})$	$^{2S+1}L_J$
0	0	0	+	1	$^1S_0$
0	1	1	+	0	$^3S_1$
1	0	1	-	0	$^1P_1$
1	1	0	-	1	$^3P_0$
1	1	1	-	1	$^3P_1$
1	1	2	-	1	$^3P_2$
2	0	2	+	1	$^1D_2$
2	1	1	+	0	$^3D_1$
2	1	2	+	0	$^3D_2$
2	1	3	+	0	$^3D_3$

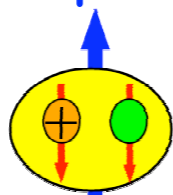
Two-nucleon states

The Pauli principle requires that two-nucleon states be antisymmetric wrt to exchange of the nucleons' space, spin, and isospin coordinates

D-state nucleon flips spin



L=0



L=2

Symmetric triplet  $T = 1$

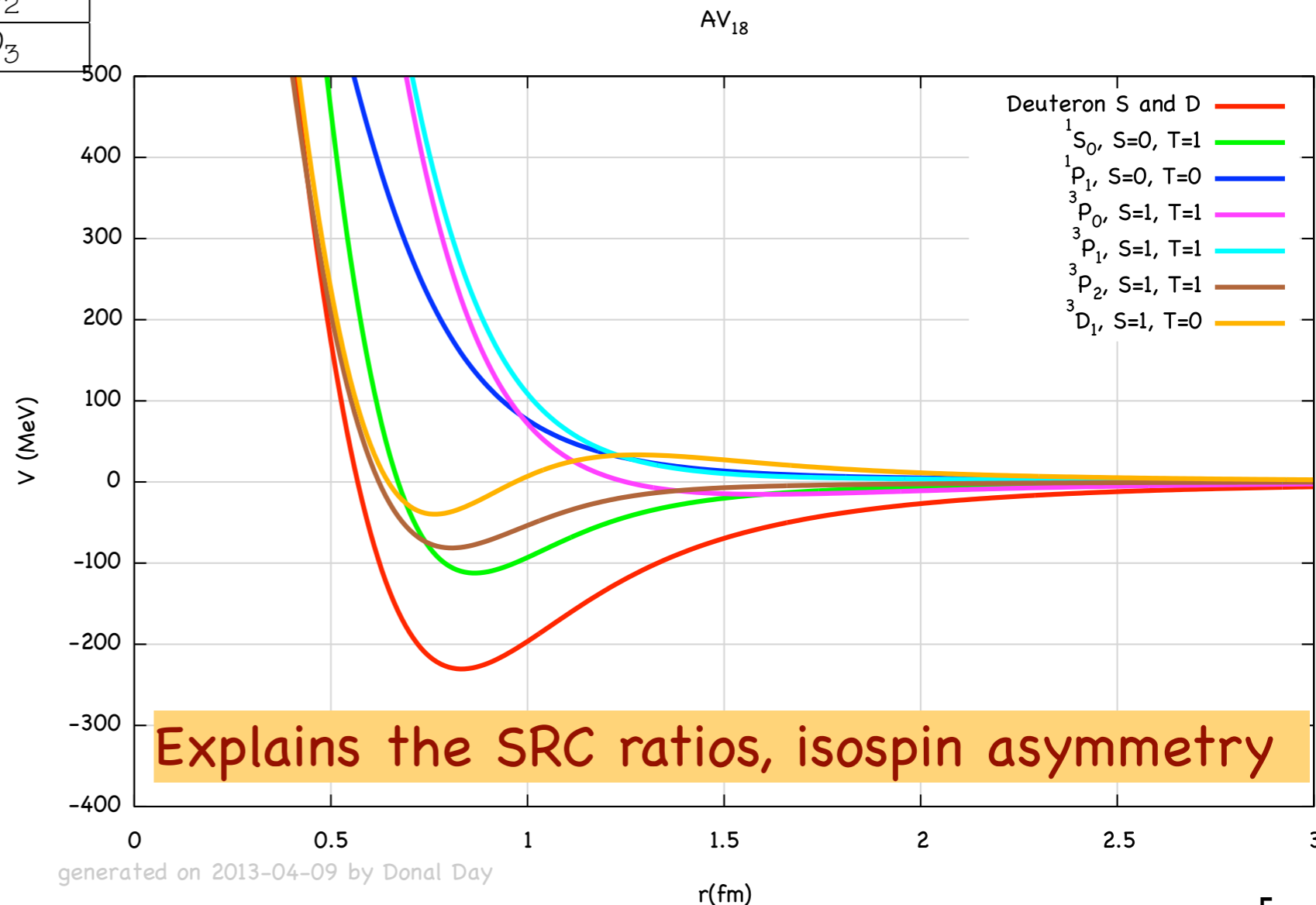
$^3(T)_1 = |p_1\rangle |p_2\rangle$  proton-proton state

$^3(T)_{-1} = |n_1\rangle |n_2\rangle$  neutron-neutron state

$^3(T)_0 = \frac{1}{\sqrt{2}}(|p_1\rangle |n_2\rangle + |p_2\rangle |n_1\rangle)$  neutron-proton state

Antisymmetric singlet  $T = 0$

$^1(T)_0 = \frac{1}{\sqrt{2}}(|p_1\rangle |n_2\rangle - |p_2\rangle |n_1\rangle)$  neutron-proton state

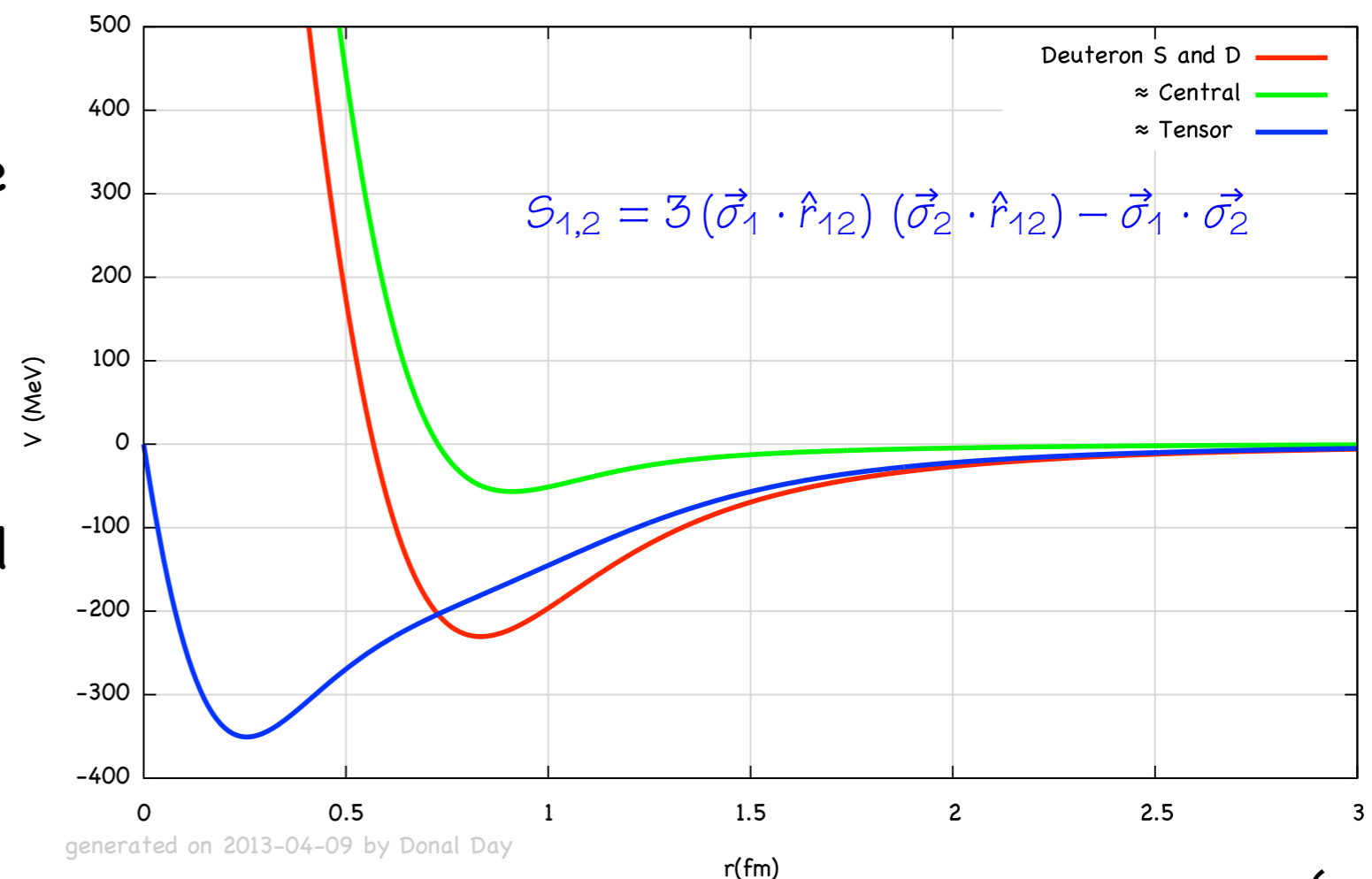


# How can two nucleons combine?

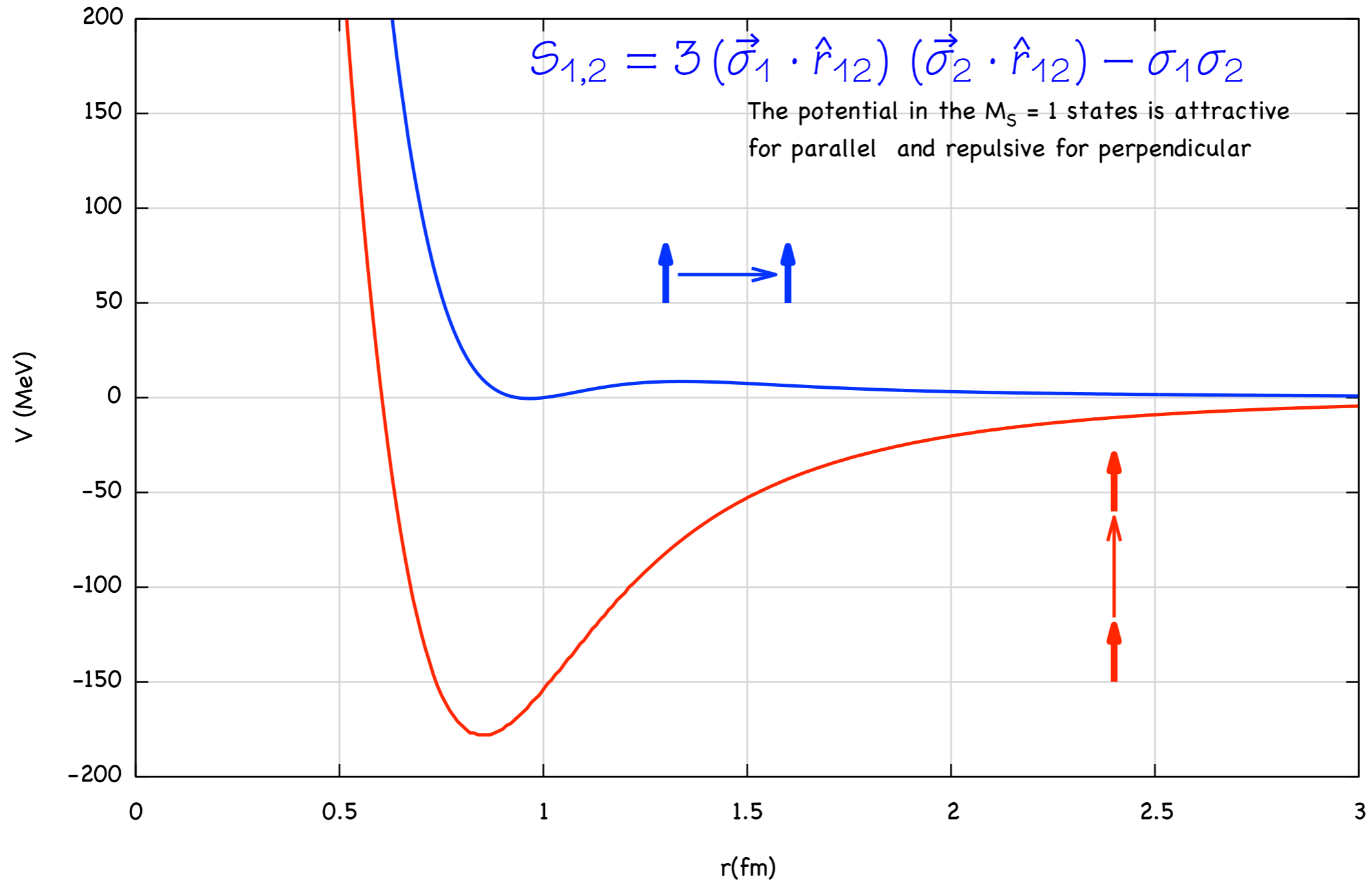
- The SR NN attraction dominated tensor interaction, which yields high momentum iso-singlet (np) pairs.
- Absent in the iso-triplet channel (pp, nn, np).
- The two-body distribution should be identical to the deuteron distribution,  $n_2(k) = n_D(k)$ , and the ratio of scattering cross sections between a heavy nucleus  $A$  and the deuteron to yield  $a_2(A, Z)$
- Without the tensor contribution the deuteron would not be bound

L	S	J	$\pi = -1^L$	$T(L+S+T \text{ odd})$	$^{2S+1}L_J$
0	0	0	+	1	$^1S_0$
0	1	1	+	0	$^3S_1$
1	0	1	-	0	$^1P_1$
1	1	0	-	1	$^3P_0$
1	1	1	-	1	$^3P_1$
1	1	2	-	1	$^3P_2$
2	0	2	+	1	$^1D_2$
2	1	1	+	0	$^3D_1$
2	1	2	+	0	$^3D_2$
2	1	3	+	0	$^3D_3$

Two-nucleon states

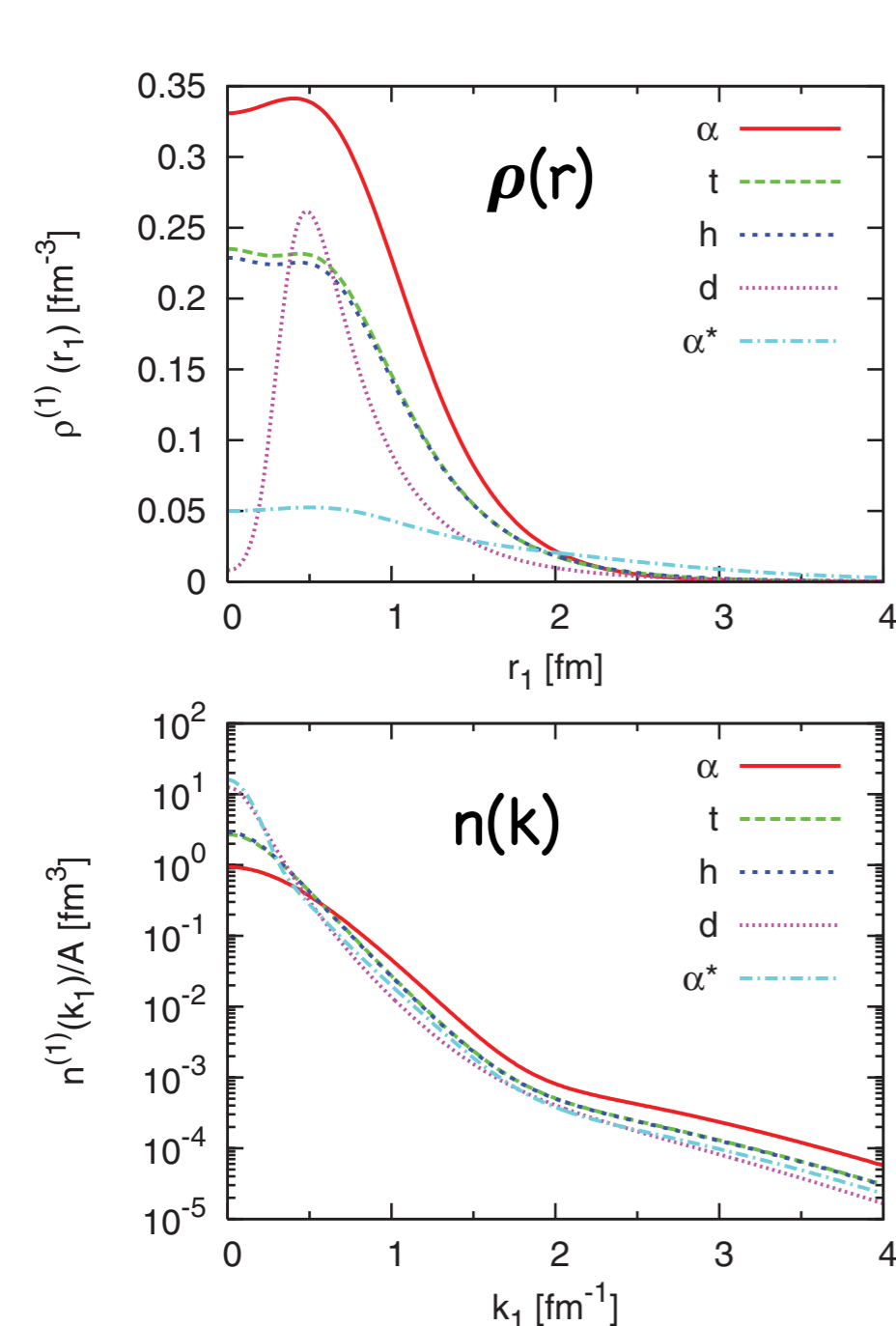


# $AV_{18}$ - static part



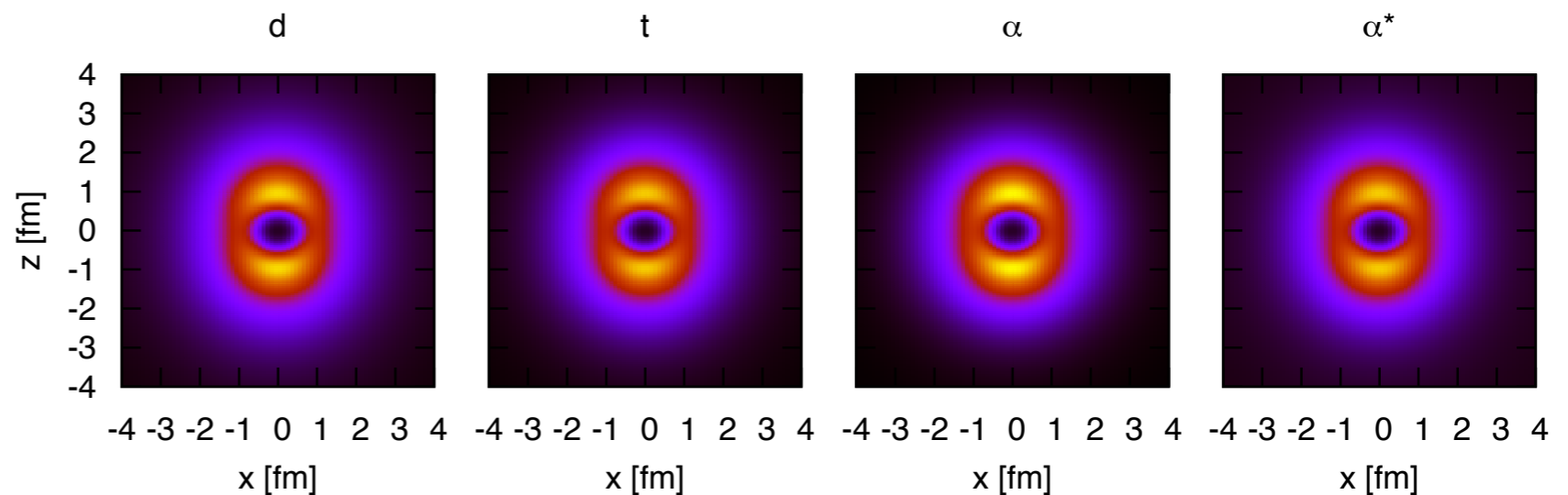
The tensor interaction causes a quadrupole type dependence as a function of the angle between the total spin direction (which we aligned along the  $z$  axis) and the direction of the distance vector  $\mathbf{r}$ . The main attraction is obtained when the spins of the nucleons are aligned with the distance vector  $\mathbf{r}$  while almost no attraction exists in the  $x$  direction where the spins are orthogonal to  $\mathbf{r}$ .

# Configuration and Momentum space



$$\rho^{(1)}(\mathbf{r}_1) = \frac{1}{2J+1} \sum_{\mathbf{M}} \langle \Psi; JM | \sum_{i=1}^A \delta^3(\hat{\mathbf{r}}_i - \mathbf{r}_1) | \Psi; JM \rangle$$

$$n^{(1)}(\mathbf{k}_1) = \frac{1}{2J+1} \sum_{\mathbf{M}} \langle \Psi; JM | \sum_{i=1}^A \delta^3(\hat{\mathbf{k}}_i - \mathbf{k}_1) | \Psi; JM \rangle$$

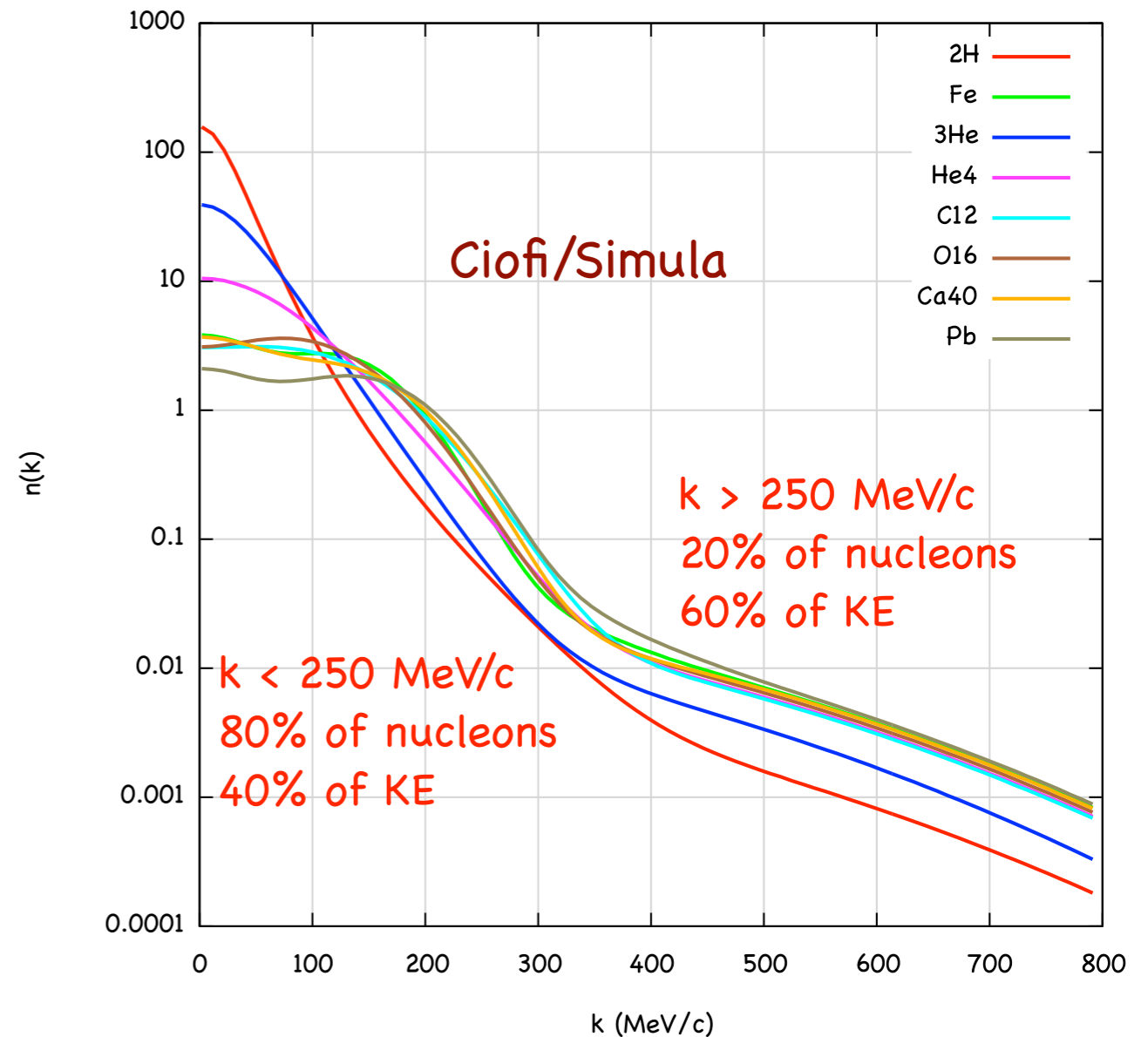
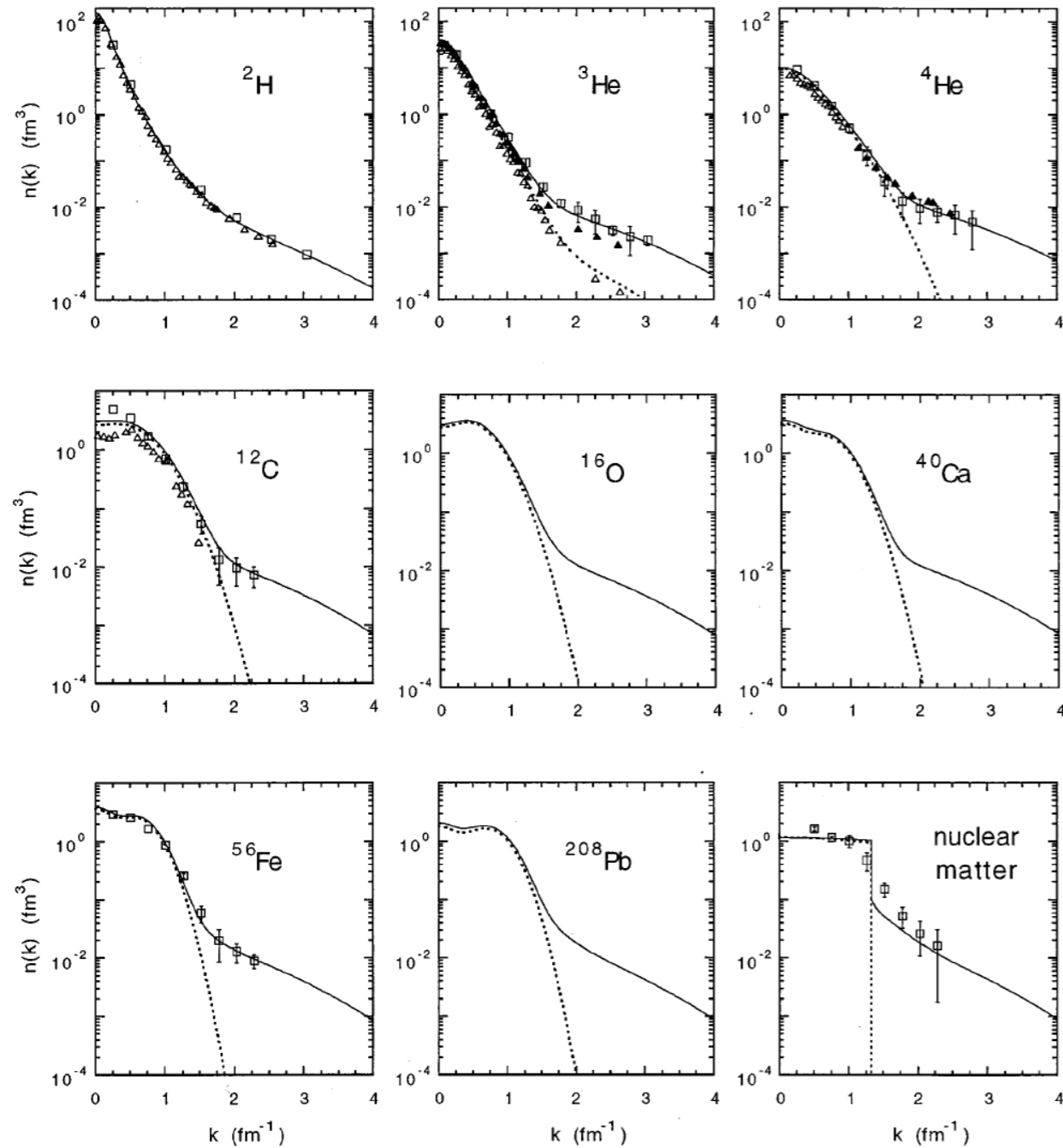


**Two-body densities** in coordinate space for a pair of nucleons with  $S = 1$ ,  $M_S = 1$ , and  $T = 0$  in the ground states of  $^2\text{H}$ ,  $^3\text{H}$ , and  $^4\text{He}$  and the 20.21 MeV excited state of  $^4\text{He}$

Where the potential is attractive,  $r \approx (0,0,\pm 1 \text{ fm})$ , the densities are  $\gg$  and in regions where the interaction is repulsive or close to zero the probability of finding the particle pair is small.

**These correlations can not be represented in a shell model and the two-body densities have their maximum at relative distance  $r = 0$ .**

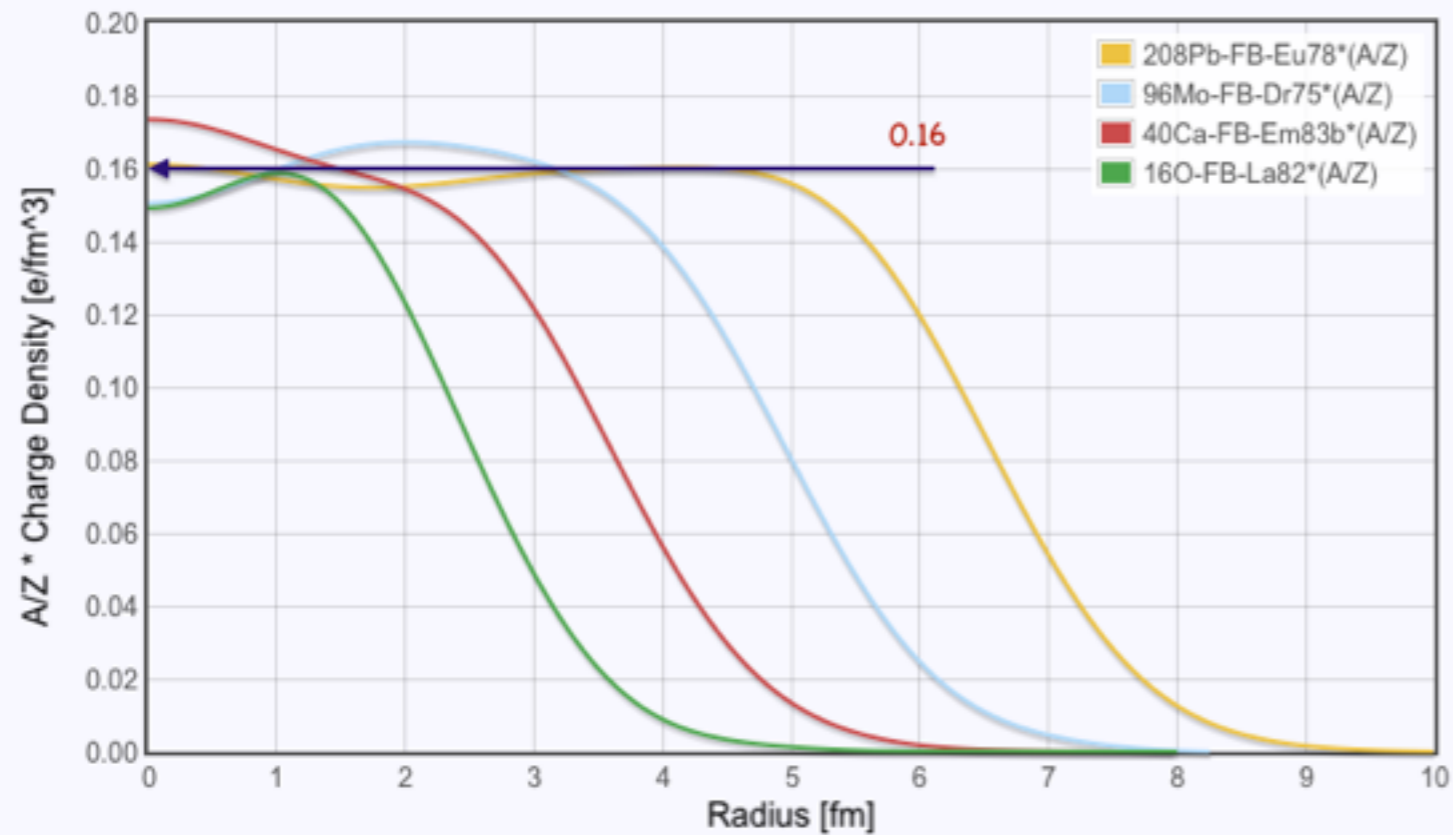
# Momentum Distributions



$n(k)$  is dominated by SRCs at large  $k$  and  $n(k)$  exhibits the same shape for all nuclei for  $k > k_{\text{fermi}}$

# Evidence of SRC

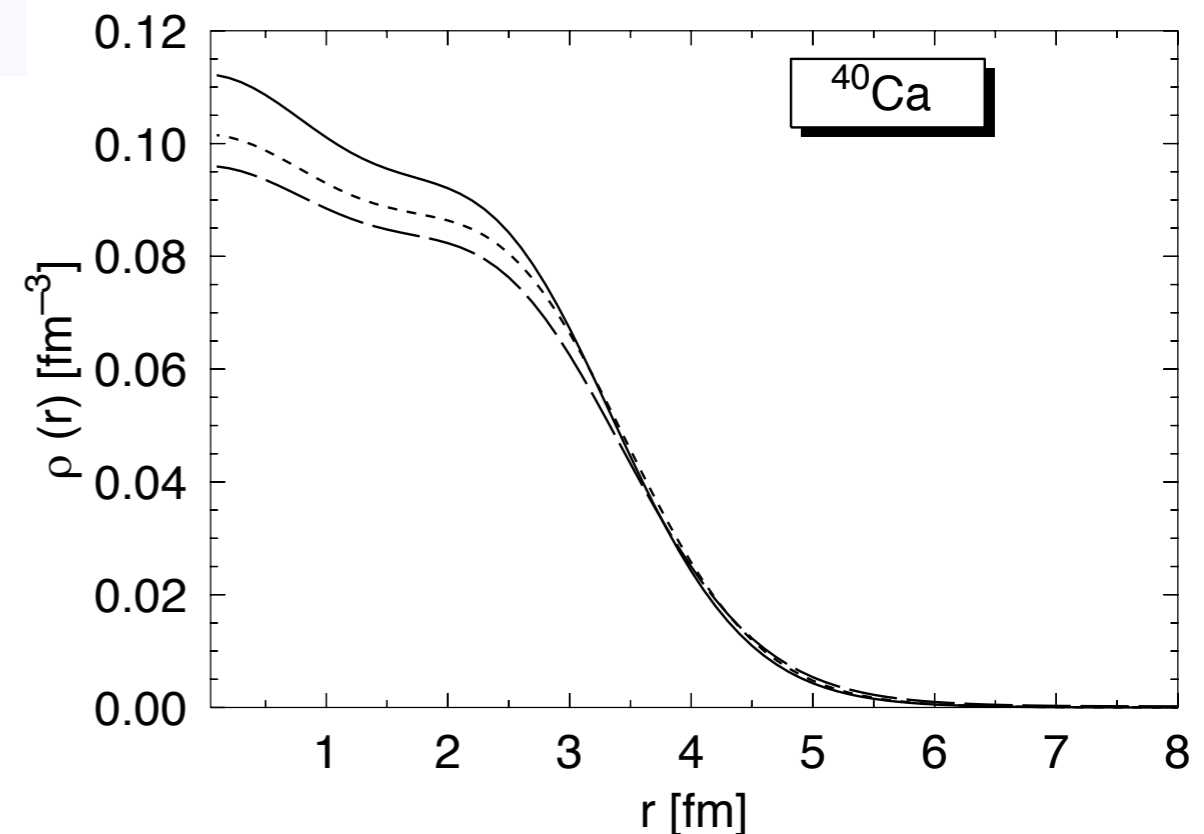
## Correlations and charge distributions



Central density is saturated – nucleons can be packed only so close together:

$$\rho_{\text{ch}} * (A/Z) = \text{constant}$$

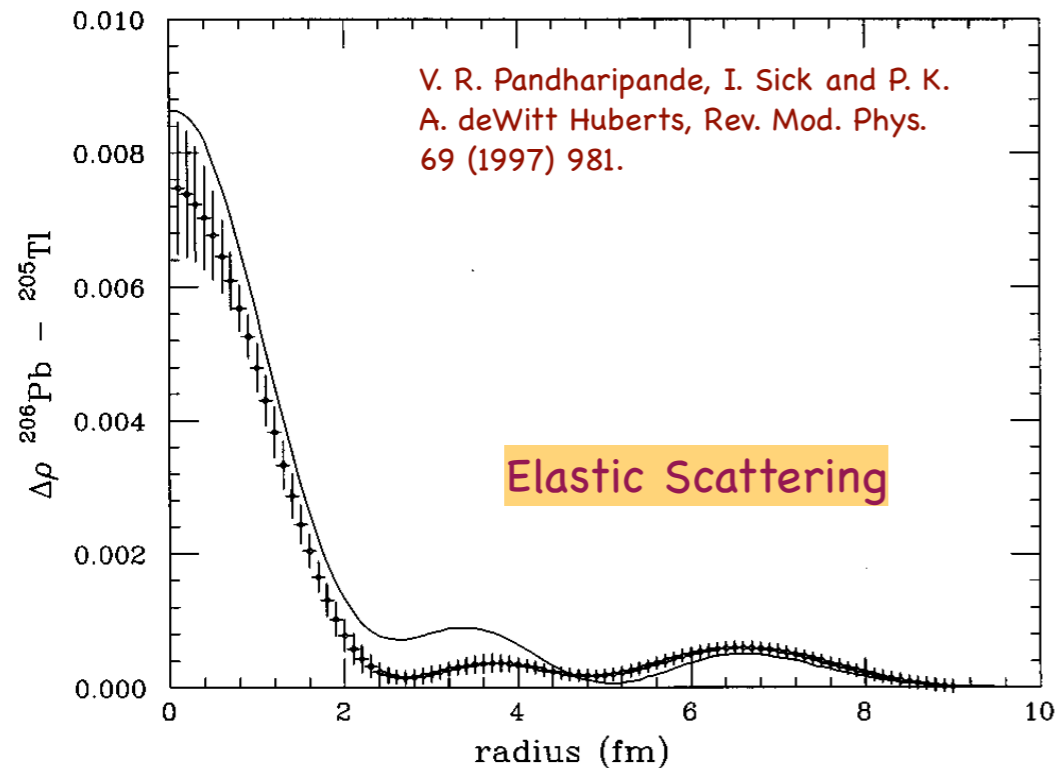
Charge density archive



IPM (full lines), LRC (long dashed lines), SRC (short dashed lines).

Correlations and charge distributions of medium heavy nuclei, Marta Anguiano and Giampaolo Co'

# Evidence of SRC

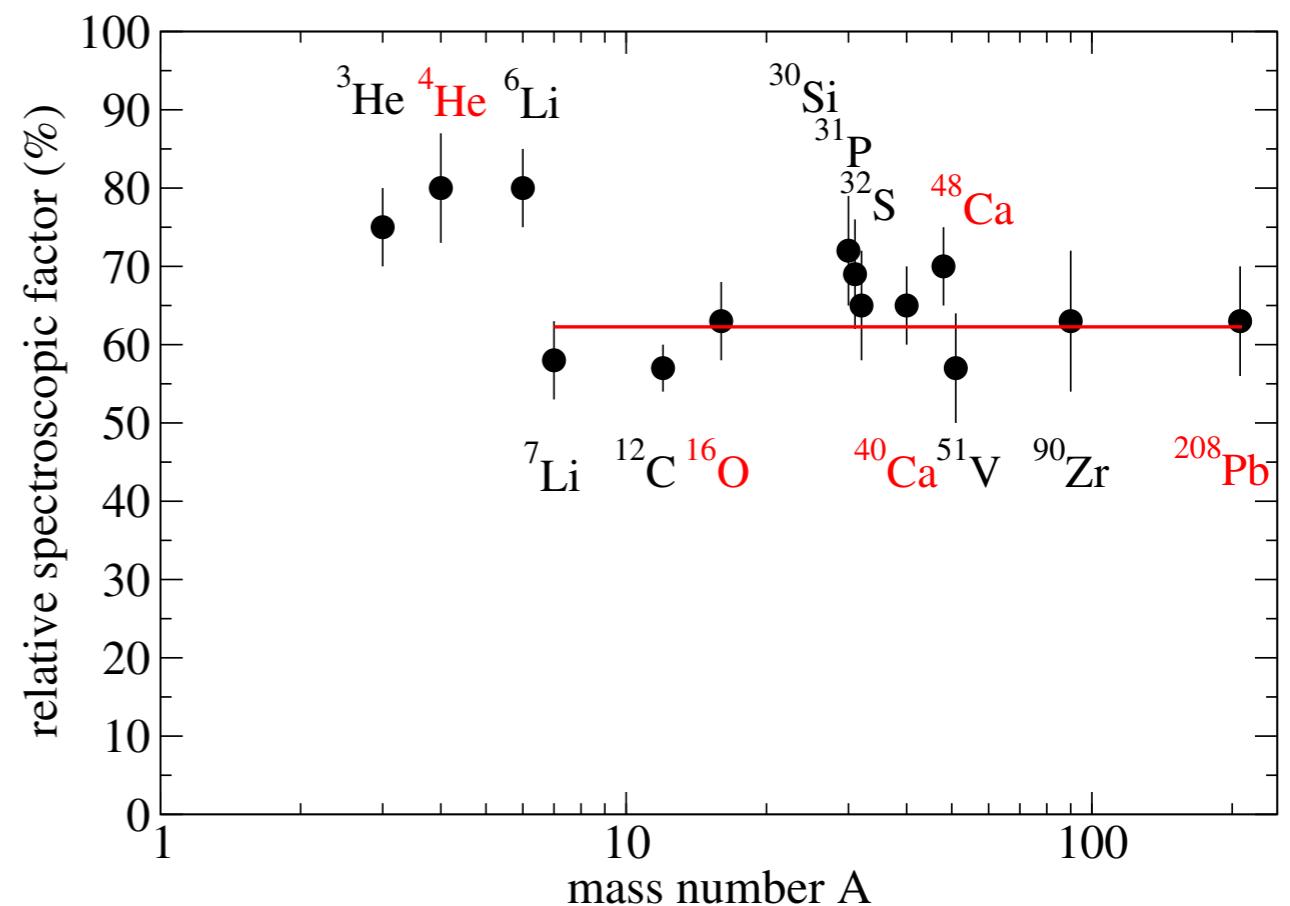
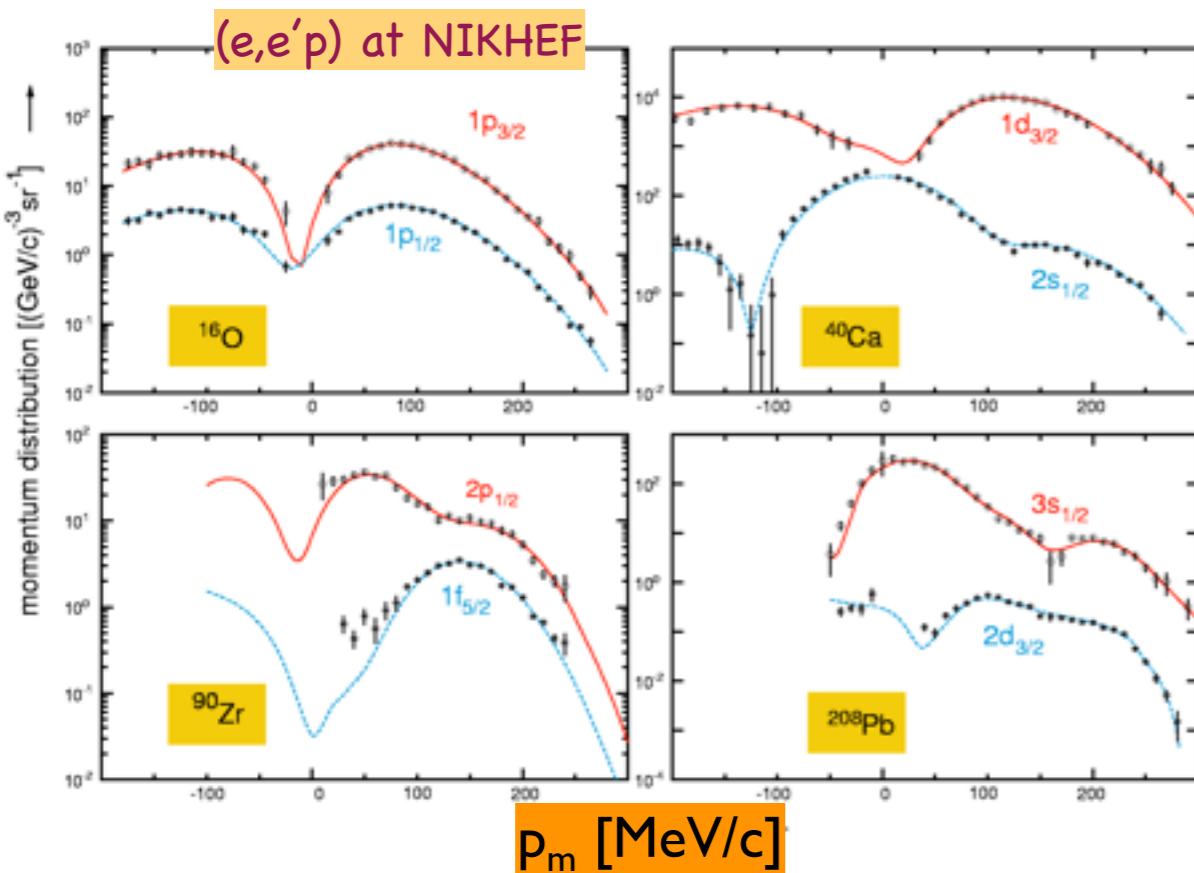


Density difference between  $^{206}\text{Pb}$  and  $^{205}\text{Tl}$ :  
differ by a single  $3s^{1/2}$  proton

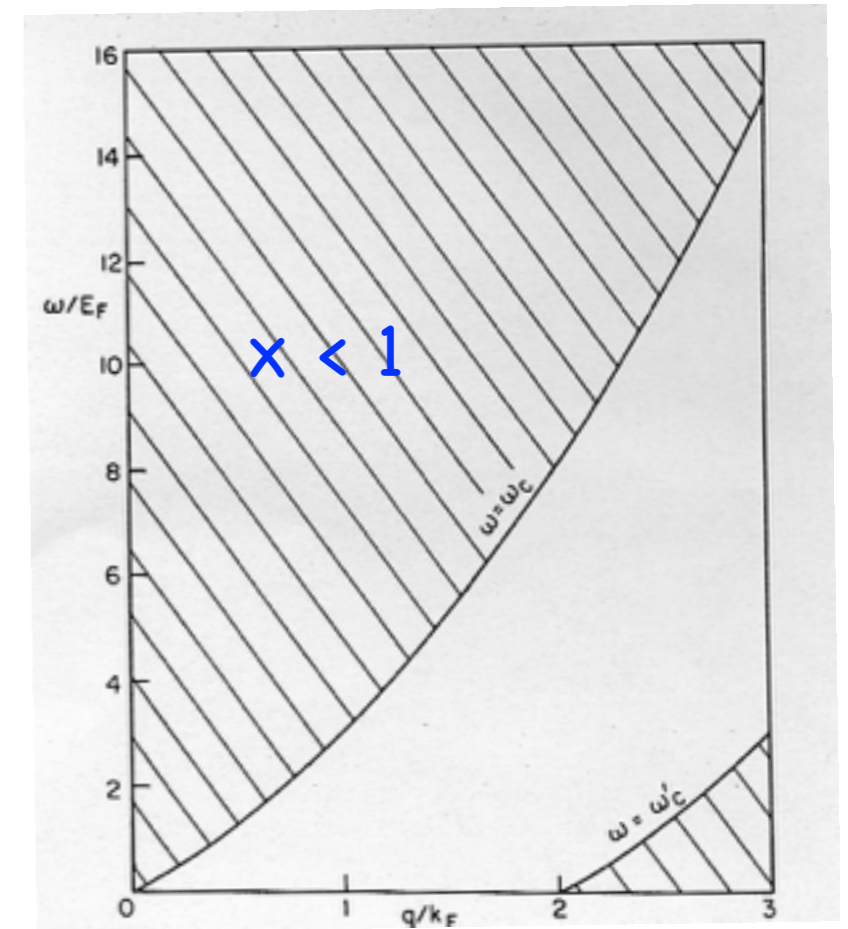
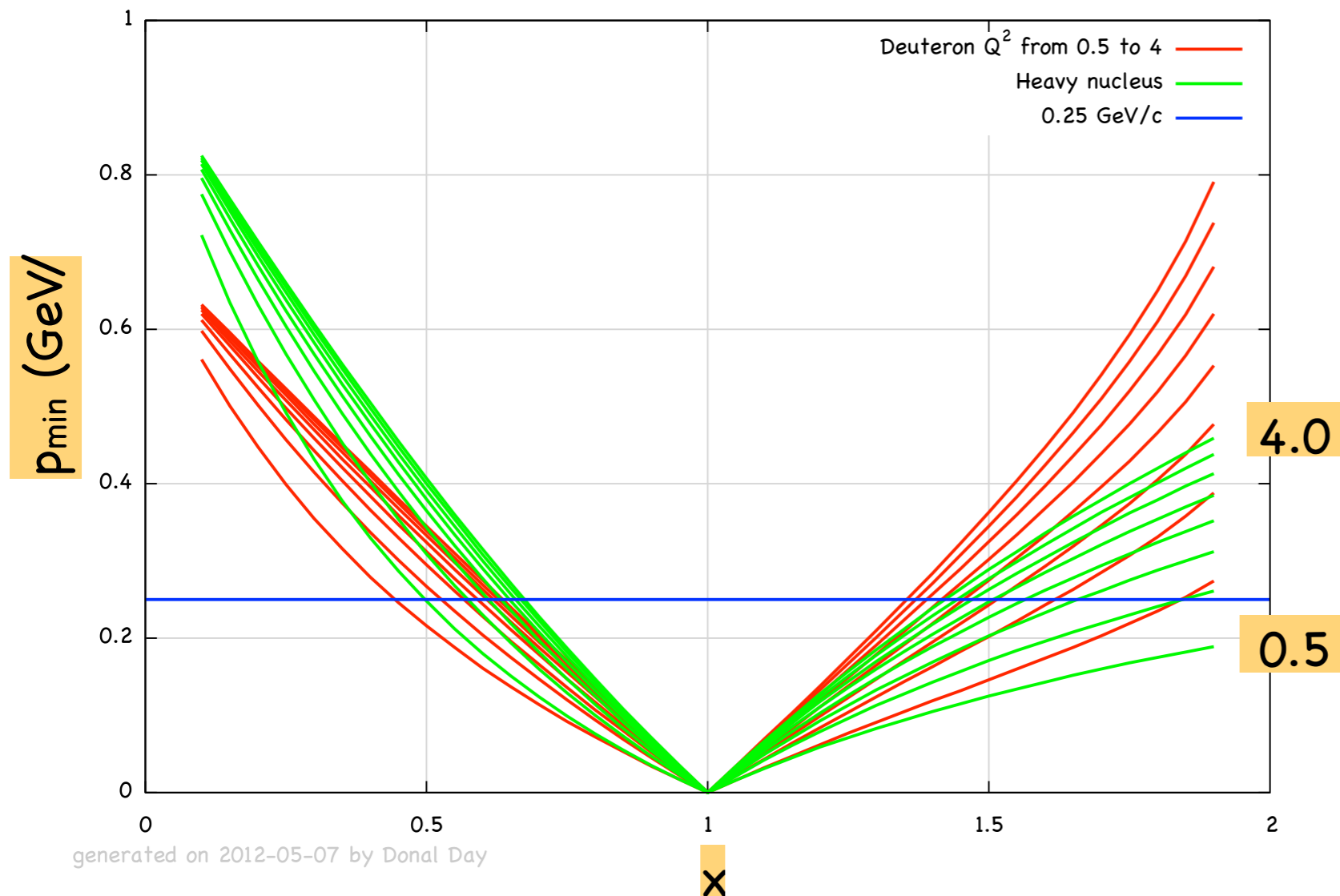
Experiment - Cavedon et al (1982)

Theory: Hartree-Fock orbitals with **adjusted**  
occupation numbers describe the **shape** of  
the  $3s^{1/2}$  orbit.

Occupation numbers  
**scaled down by a factor  $\sim 0.65$ .**



# Correlations: Where to look in inclusive $A(e,e')$



$x > 1$ , low  $\omega$  side of qep

Inelastic electron scattering from fluctuations in the nuclear charge distribution

Wieslaw Czyż and Kurt Gottfried  
Annals of Physics 21, 47 (1963)

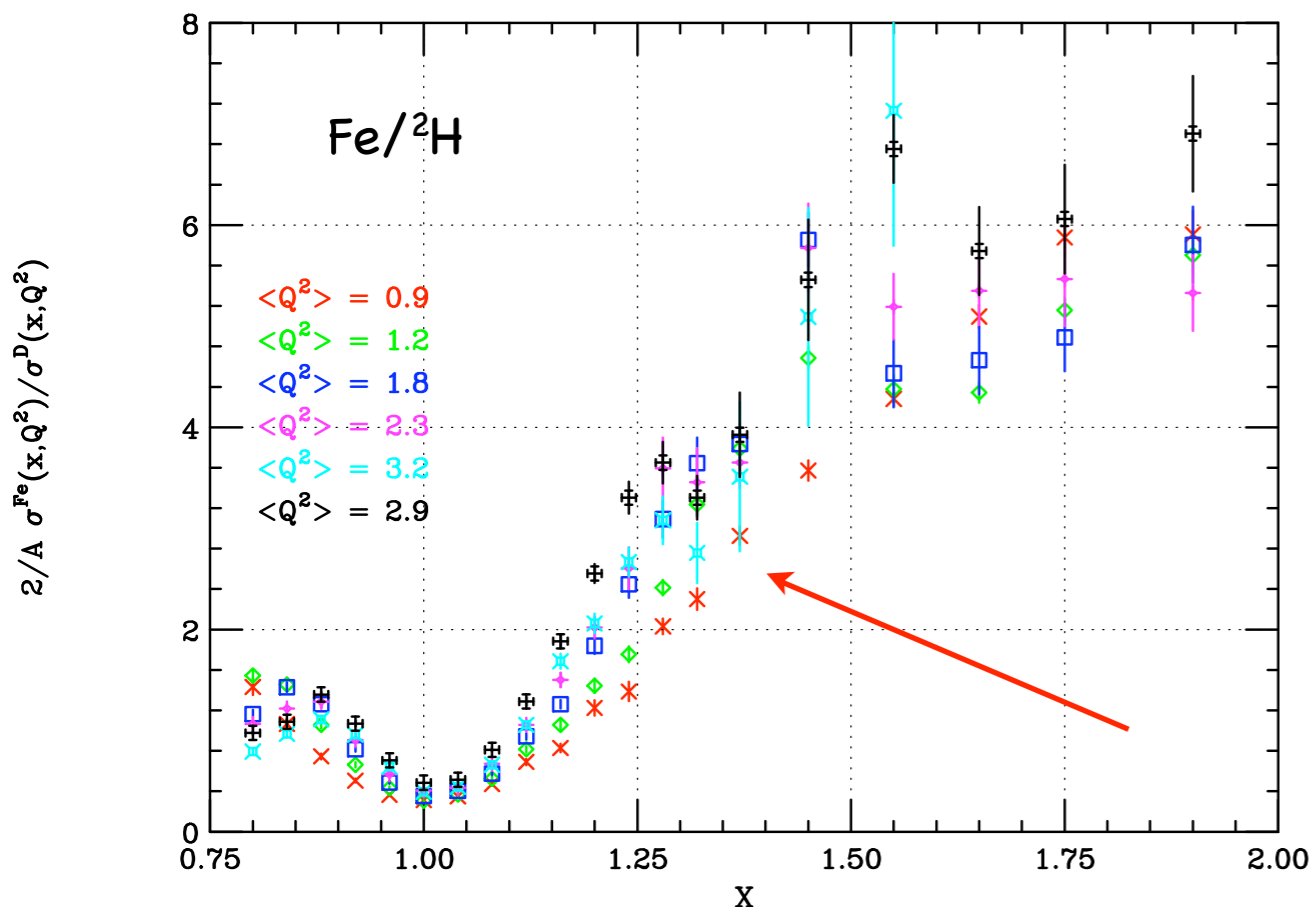
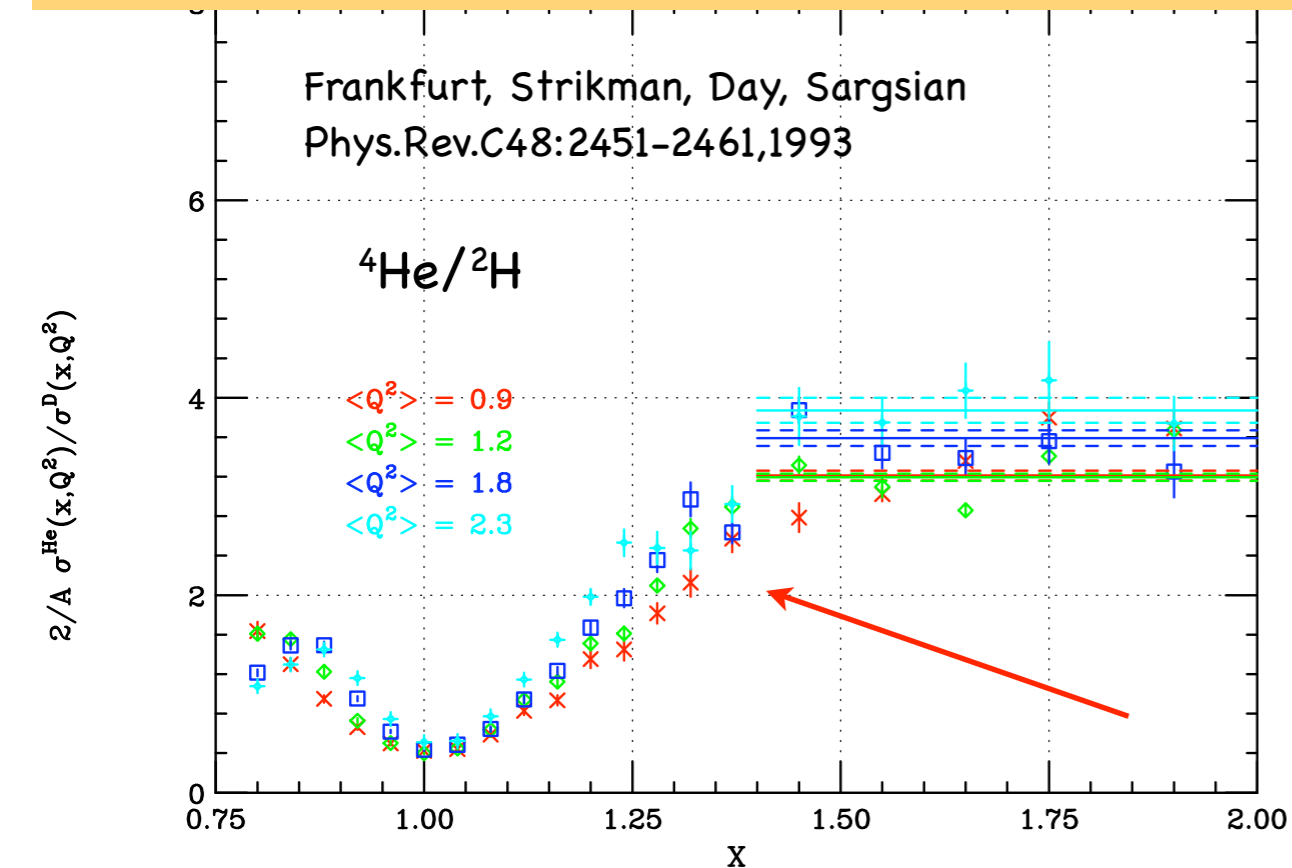
Appearance of plateaus is  $A$  dependent.

**Kinematics:** heavier recoil systems do not require as much energy to balance momentum of struck nucleon – hence  $p_{\min}$  for a given  $x$  and  $Q^2$  is smaller.

**Dynamics:** mean field part in heavy nuclei persist in  $x$  to larger values

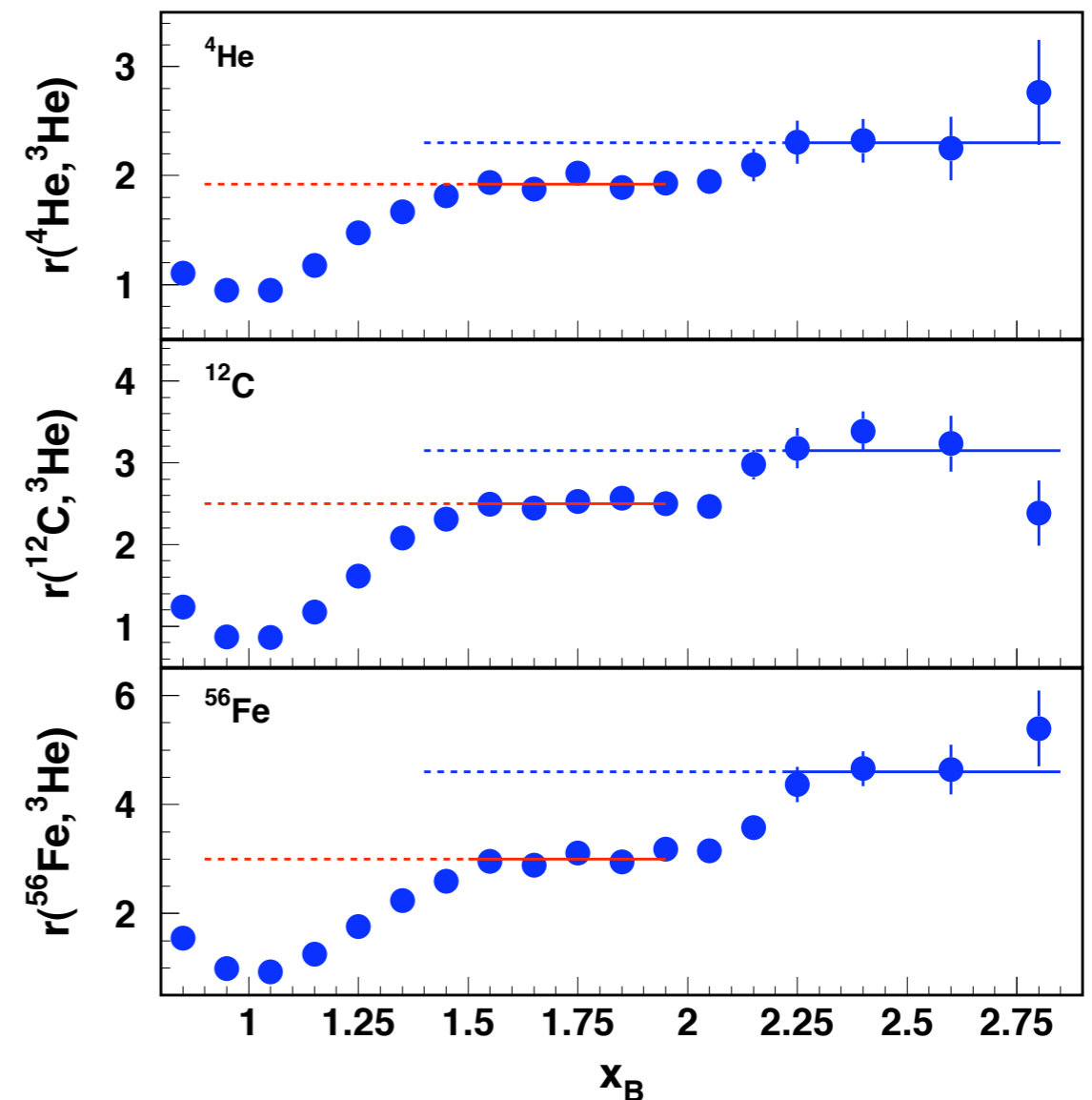
Have to go to higher  $x$  or  $Q^2$  to insure scattering is not from mean-field nucleon

# Ratios, SRC's and $Q^2$ scaling



$$\frac{2 \sigma_A}{A \sigma_D} = a_2(A); \quad (1.4 < x < 2.0)$$

$A(e, e'), 1.4 < Q^2 < 2.6$



$\alpha_{2N} \approx 20\%$   
 $\alpha_{3N} \approx 1\%$

CLAS data

Egiyan et al., PRL 96,  
082501, 2006

$a_j(A)$  is probability of finding a  $j$ -nucleon correlation

# Ratios and SRC

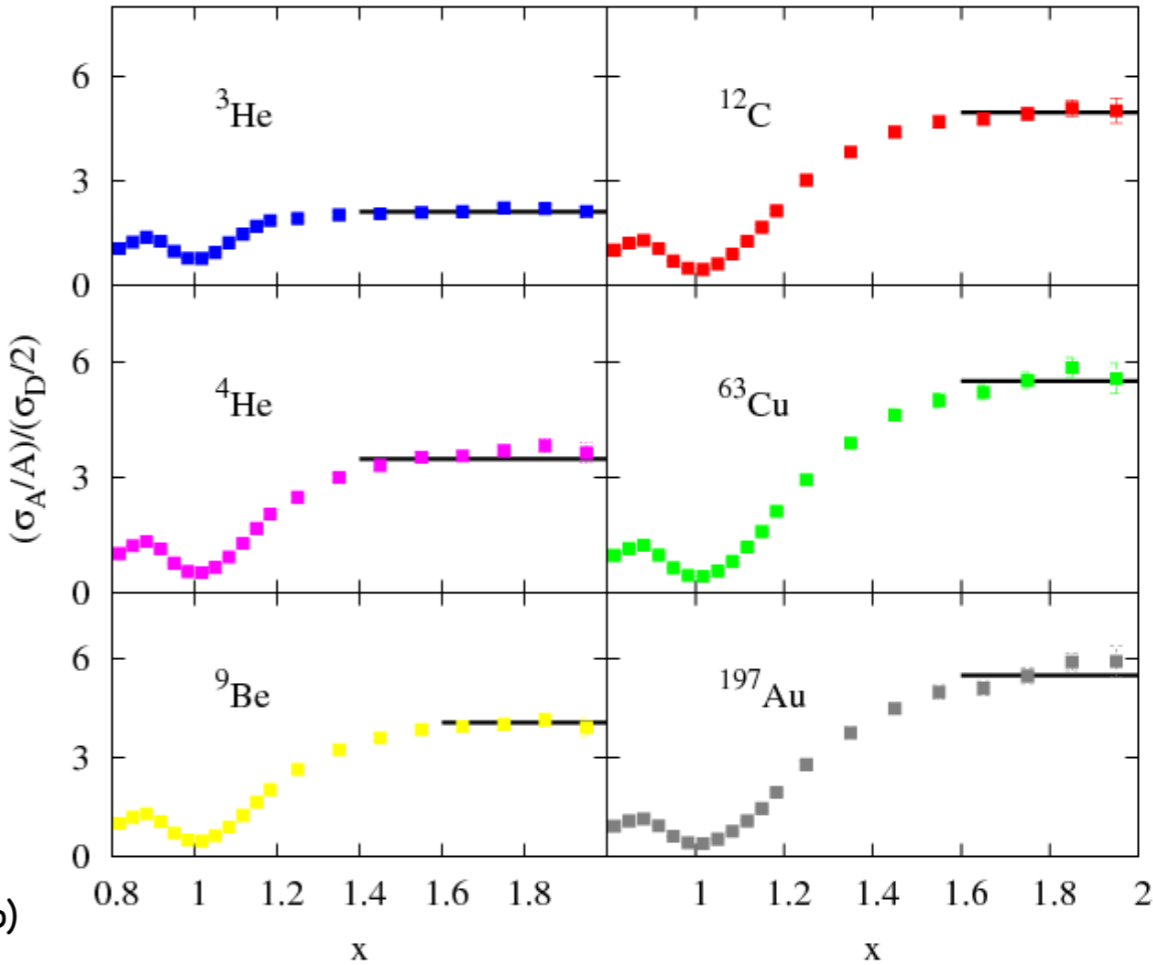
Dominance of np pairs in SRC region leads us to drop the isoscalar correction. We correct for COM motion of pair.

$R_{2n}$  : number of np pairs relative to the deuteron

A	$R_{2N}$ (E02-019)	SLAC	CLAS	$F_{CM}$	Ciofi/Simula
$^3\text{He}$	$1.93 \pm 0.10$	$1.8 \pm 0.3$	...	$1.10 \pm 0.05$	1.9
$^4\text{He}$	$3.02 \pm 0.17$	$2.8 \pm 0.4$	$2.80 \pm 0.28$	$1.19 \pm 0.06$	3.8
Be	$3.37 \pm 0.17$	...	...	$1.16 \pm 0.05$	
C	$4.00 \pm 0.24$	$4.2 \pm 0.5$	$3.50 \pm 0.35$	$1.19 \pm 0.06$	4.0
Cu(Fe)	$4.33 \pm 0.28$	$(4.3 \pm 0.8)$	$(3.90 \pm 0.37)$	$1.20 \pm 0.06$	4.5
Au	$4.26 \pm 0.29$	$4.0 \pm 0.6$	...	$1.21 \pm 0.06$	4.8 ( $^{208}\text{Pb}$ )
$\langle Q^2 \rangle$	$\sim 2.7 \text{ GeV}^2$	$\sim 1.2 \text{ GeV}^2$	$\sim 2 \text{ GeV}^2$		
$x_{\min}$	1.5	...	1.5		
$\alpha_{\min}$	1.275	1.25	1.22–1.26		

## Hall C

N. Fomin et al., Phys. Rev. Lett. 108, 092502 (2012)

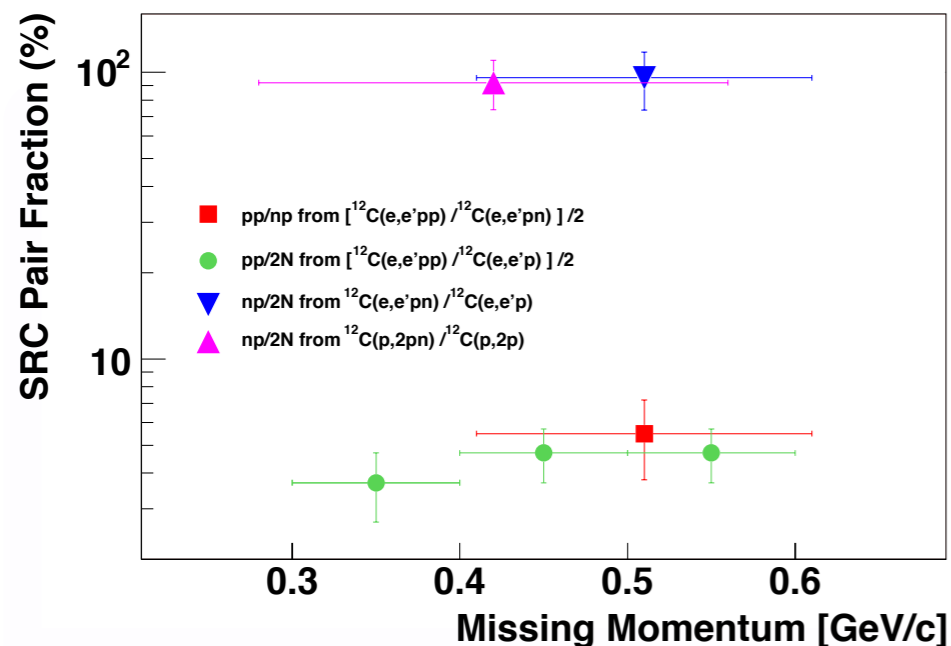
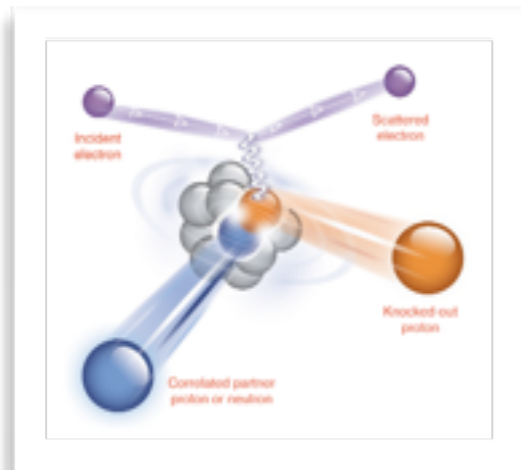


Evidence of 2N-SRC at  $x > 1.5$

# Theory and experiment display isospin dependence

## Two-nucleon knock-out experiment

R. Subedi et al, Science 320, 1476(2008)

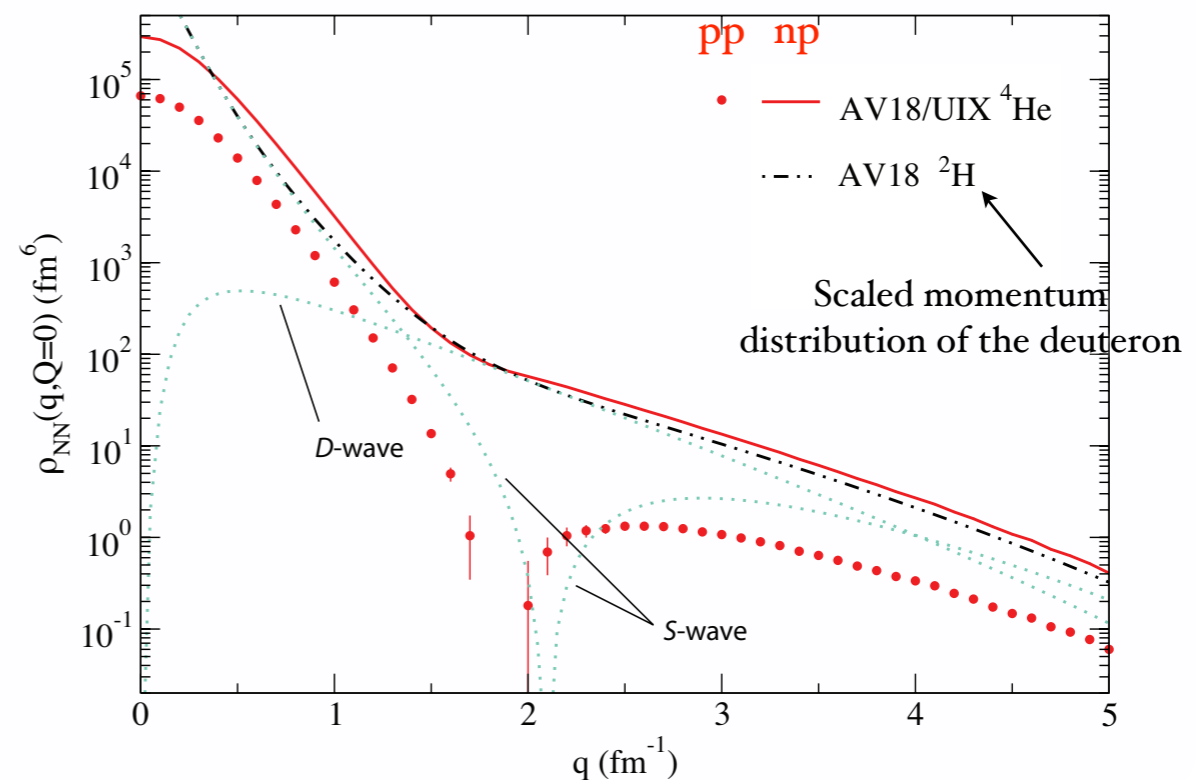


PRL 98, 132501 (2007) PHYSICAL REVIEW LETTERS week ending 30 MARCH 2007

### Tensor Forces and the Ground-State Structure of Nuclei

R. Schiavilla,<sup>1,2</sup> R. B. Wiringa,<sup>3</sup> Steven C. Pieper,<sup>3</sup> and J. Carlson<sup>4</sup>

<sup>1</sup>Jefferson Laboratory, Newport News, Virginia 23606, USA  
<sup>2</sup>Department of Physics, Old Dominion University, Norfolk, Virginia 23529, USA  
<sup>3</sup>Physics Division, Argonne National Laboratory, Argonne, Illinois 61801, USA  
<sup>4</sup>Theoretical Division, Los Alamos National Laboratory, Los Alamos, New Mexico 87545, USA  
 (Received 10 November 2006; published 27 March 2007)



**Data show large asymmetry between np, pp pairs:**

Qualitative agreement with calculations; effect of tensor force. Huge violation of **often assumed** isospin symmetry

# High momentum(!!) strength in proton knockout in (e,e'p)

## $^2\text{H}(ee'p)n$ Mainz

Boeglin et al, Phys. Rev. C 78, 054001 (2008)

Blomqvist et al, Phys Lett B, (1998), 33-38

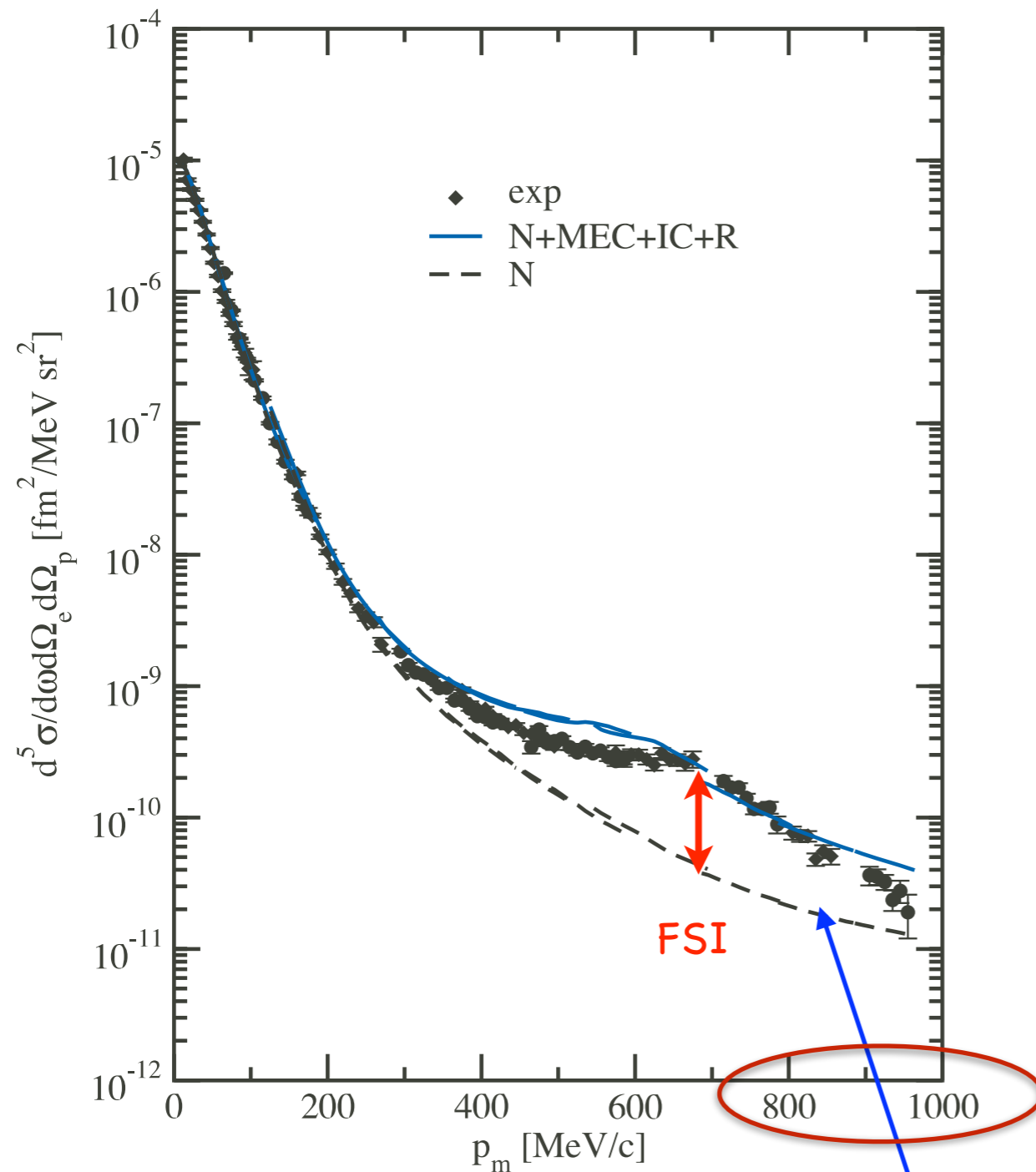
$$E = .855$$

$$\theta = 45$$

$$E' = .657$$

$$Q^2 = 0.33$$

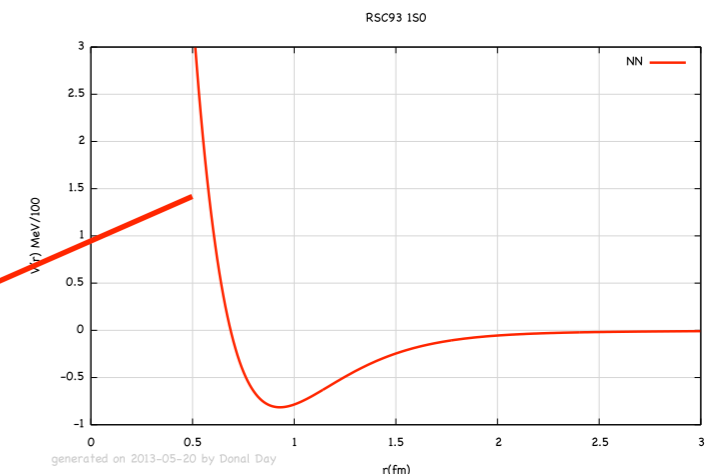
$$x = .88$$



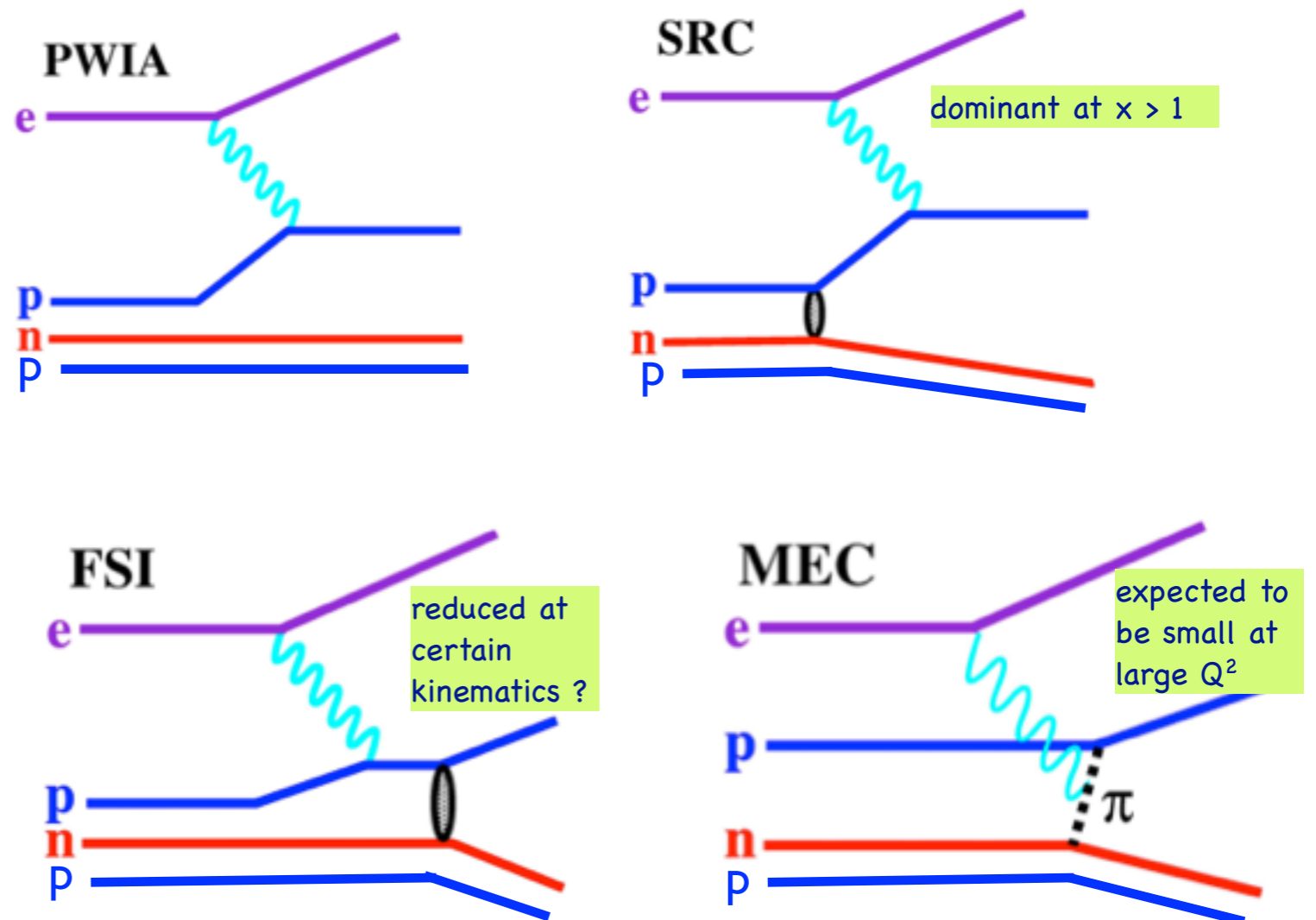
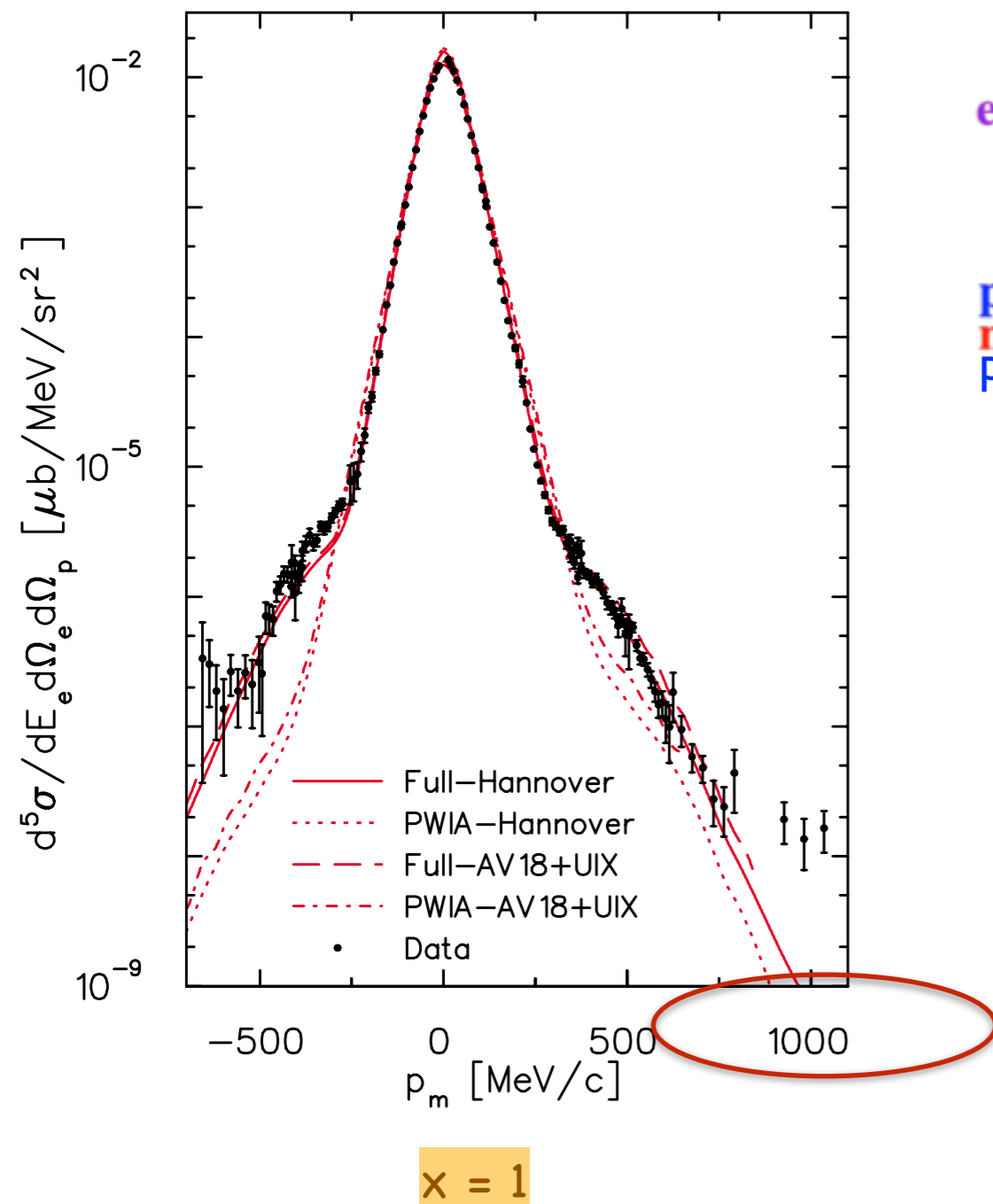
Not the best place to look for SRCs –  $\Delta$ s, MECs FSI dominate

large IC+MEC

hard core



# High momentum strength in $A(e,e'p)$ $^3\text{He}(e,e'p)d$ E89-044, Hall A



## E89-044: $^3\text{He}(e,e'p)d$

Measured far into high momentum tail:

Cross section is  $\sim 5-10\times$  expectation

High momentum pair can come from SRC (initial state)

OR

Final State Interactions (FSI) and Meson Exchange Contributions (MEC),  $\Delta$ 's

M. M. Rvachev et al. PRL 94, 192302 (2005)

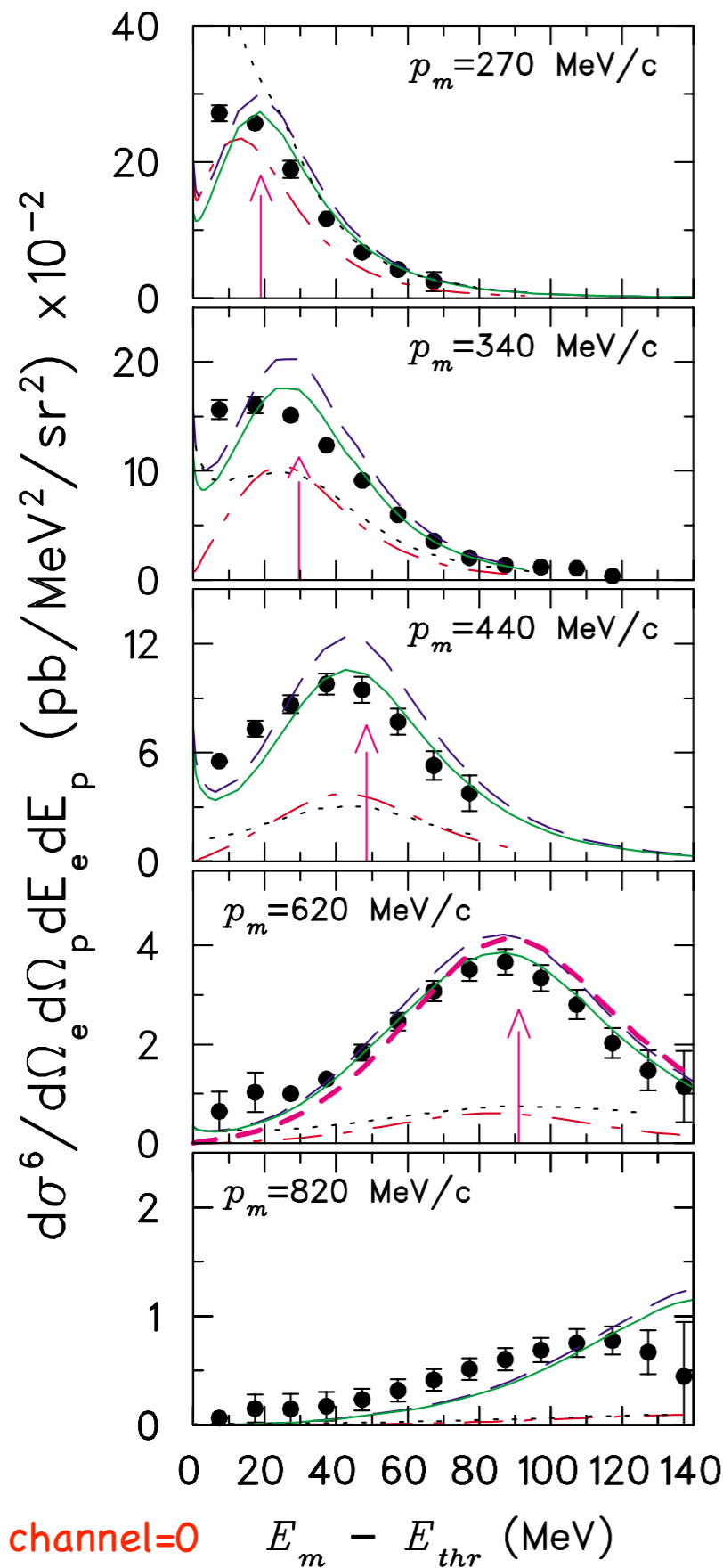
# High momentum strength in $A(e,e'p)$

$^3\text{He}(e,e'p)pn$  E89-044, Hall A

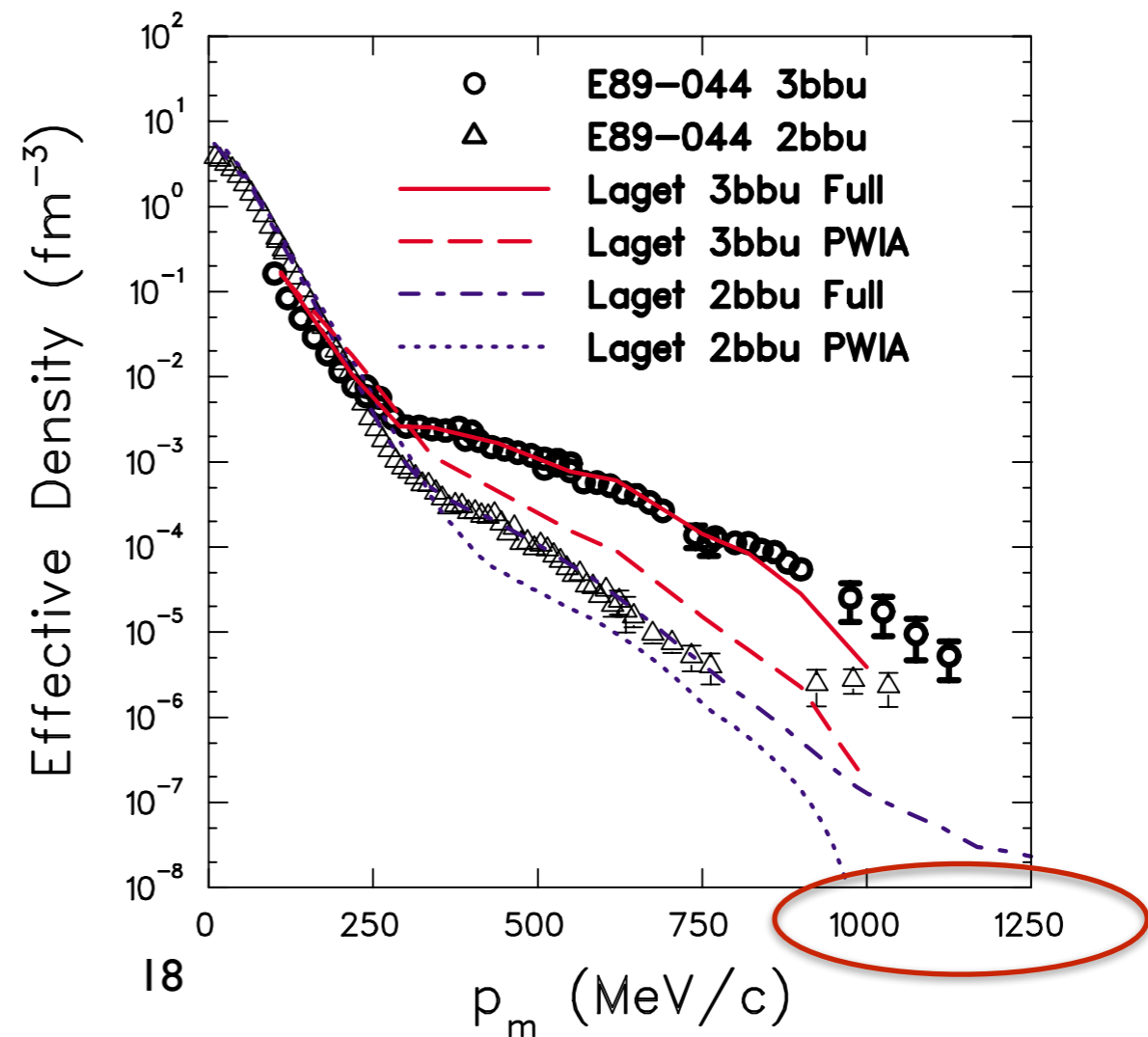
$^3\text{He}(e,e'p)np$  F. Benmokhtar et al. , PRL 94, 082305 (2005)

Arrows indicate expected location of correlated pair

- dotted line PWIA
- dash-dot: Laget (PWIA)
- FSI (long dashed line) to full calculation (solid line), including meson-exchange current and final-state interactions: Laget
- In the 620 MeV/c panel
  - short dashed curve is a calculation with PWIA + FSI only within the correlated pair.

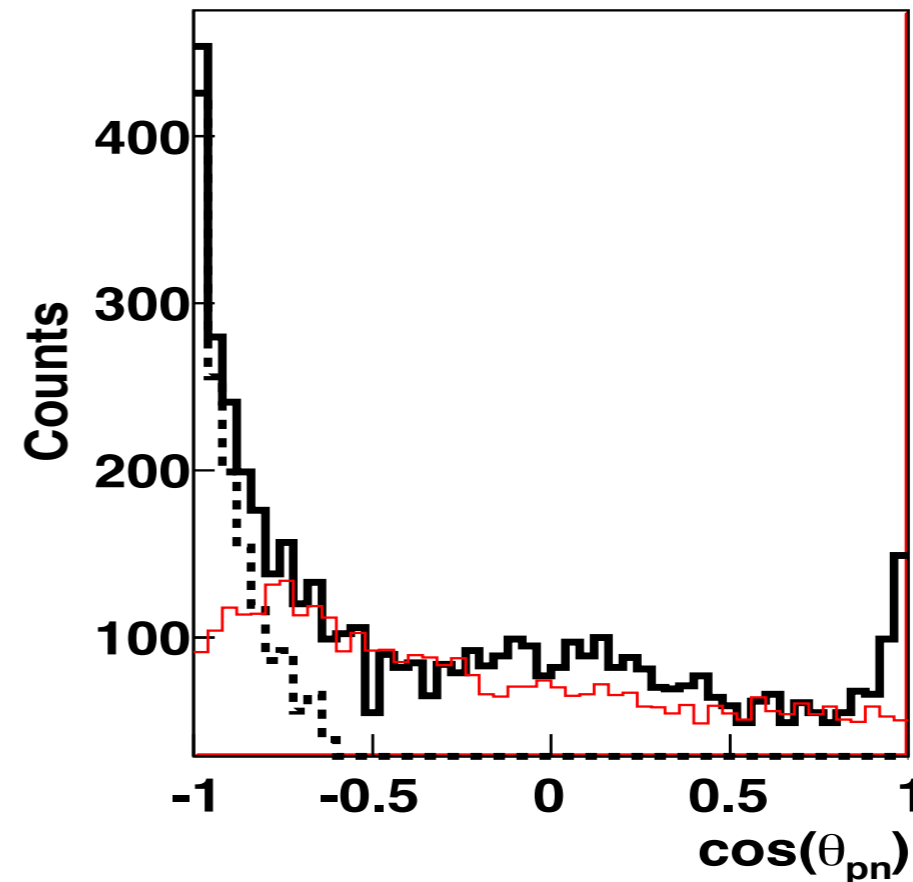
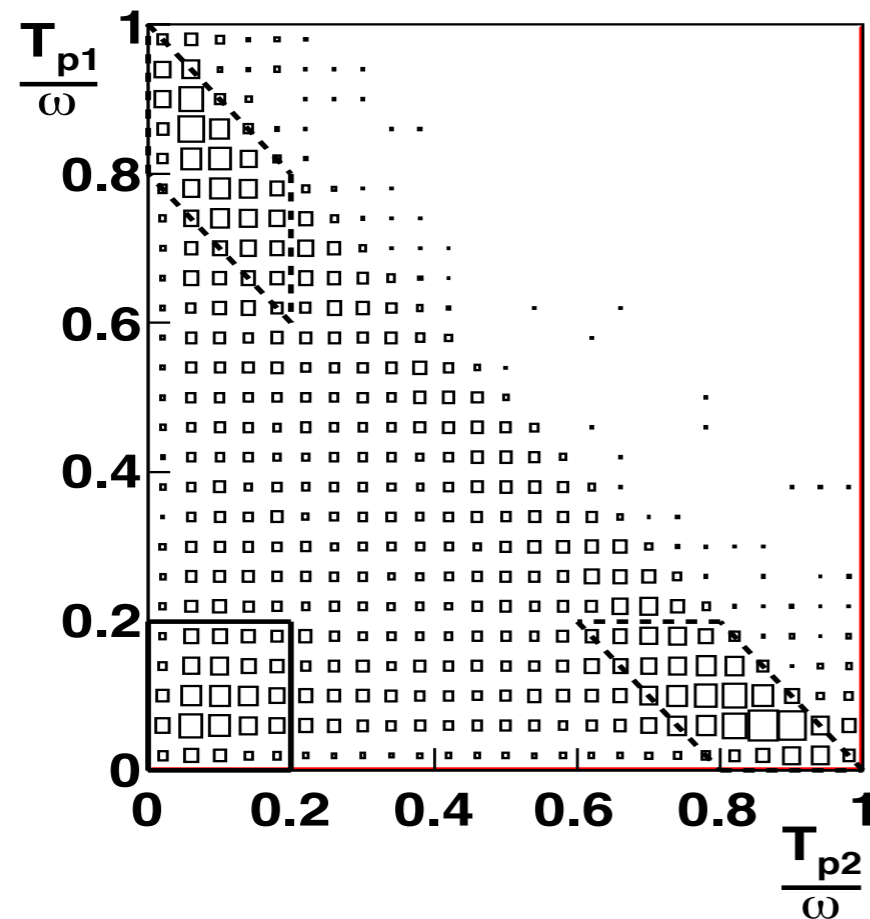


3bbu channel=0



# Tensor Correlations Measured in $^3\text{He}(e,e',pp)n$

H. Baghdasaryan, L. B. Weinstein, et al. PRL 105, 222501 (2010)



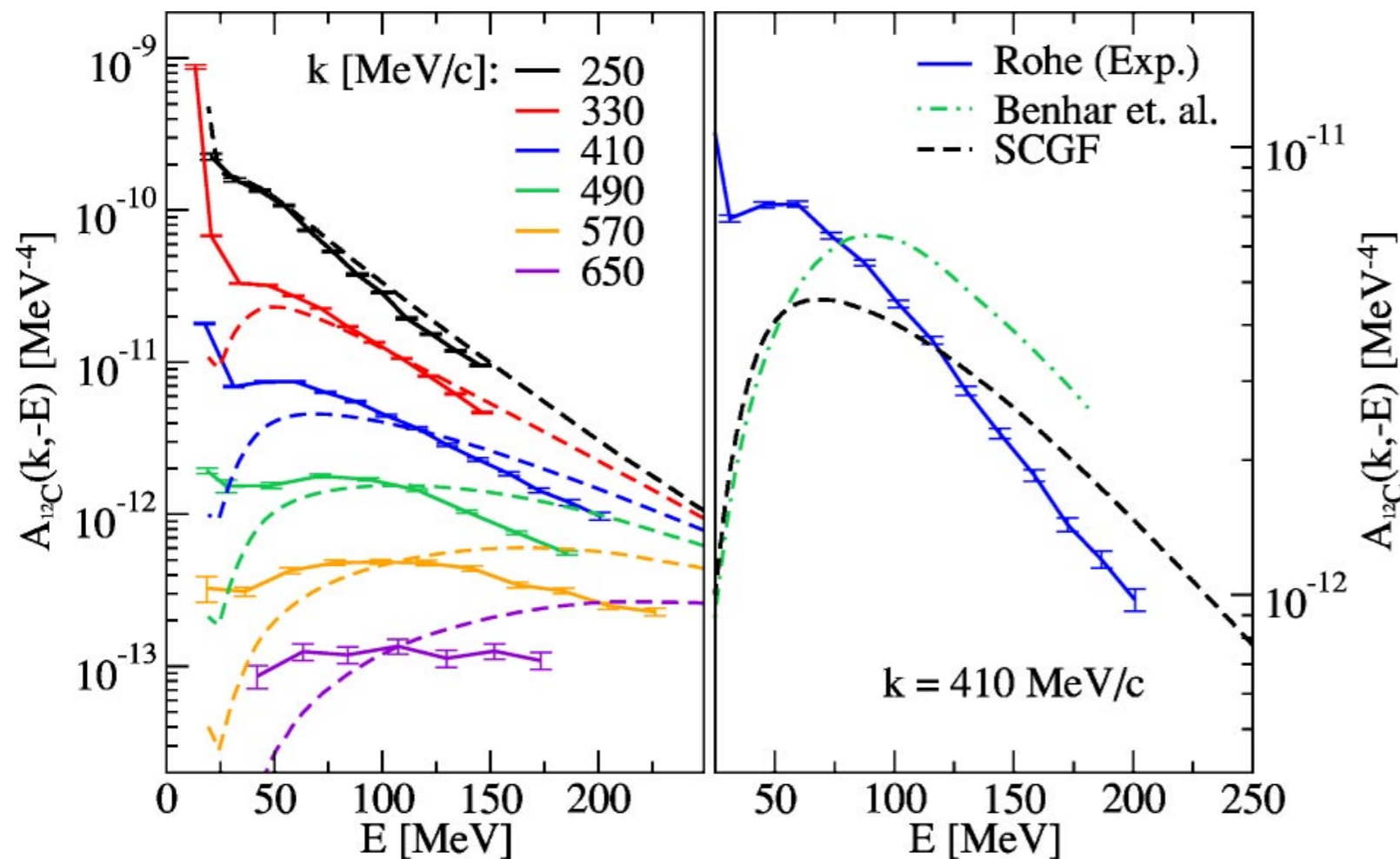
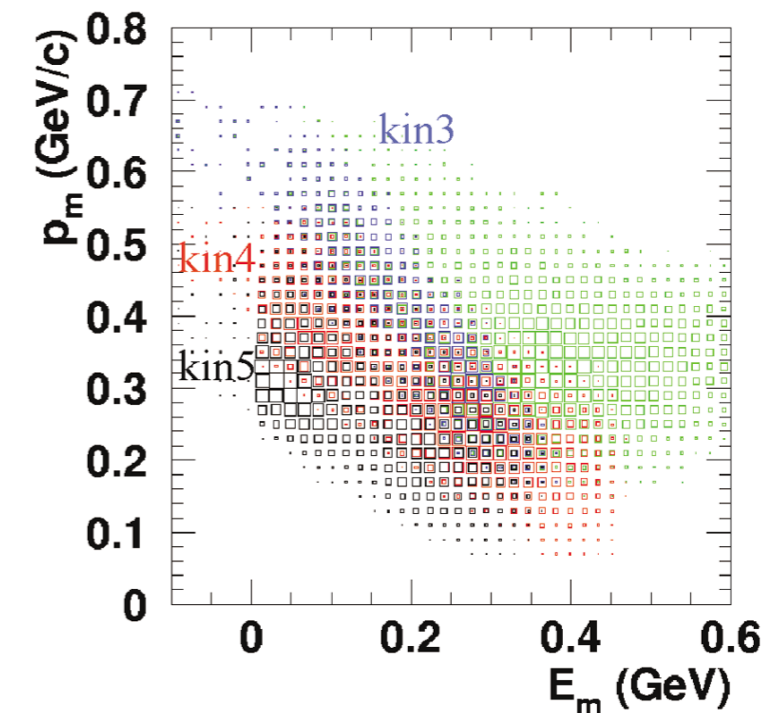
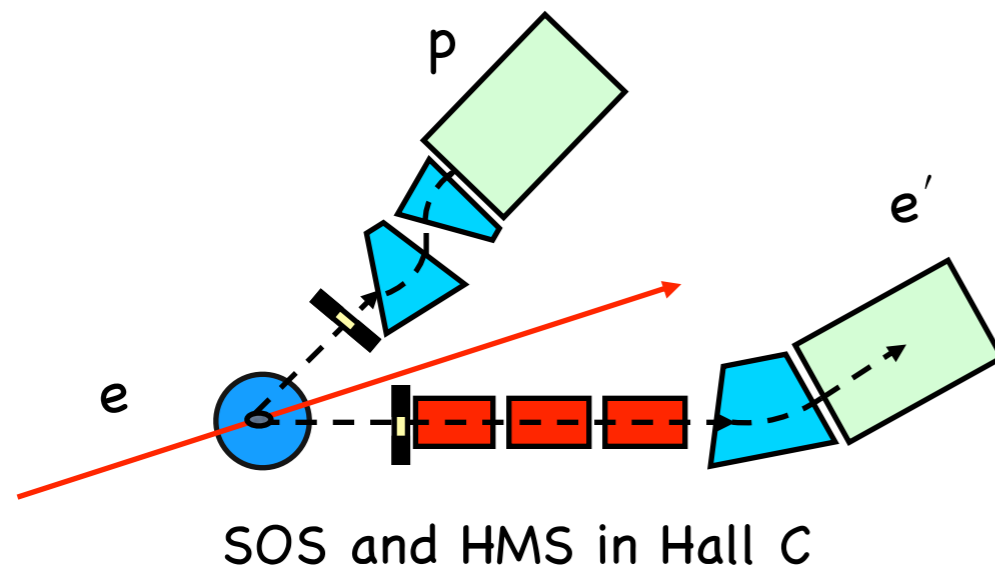
Events with one leading nucleon and a spectator correlated NN pair

- The spectator nucleons each have less than 20% of the transferred energy
- Leading nucleon's momentum perpendicular to  $\vec{q}$  be less than 0.3 GeV/c.
- The ratio of pp to pn pair cross sections for  $0.3 < p_{\text{rel}} < 0.5$  GeV/c is very small at low  $p_{\text{tot}}$  and rises to approximately 0.5 at large  $p_{\text{tot}}$ .

The pp pairs at low  $p_{\text{tot}}$  are in an s-state, this ratio shows the dominance of tensor over central correlations.

# E97-006 Correlated Spectral Function and $^{12}\text{C}(e,e'p)$ Reaction Mechanism

D. Rohe, et al. Phys. Rev Lett. 93 182501



Data suggests more strength at smaller  $E$  – accessible at large  $x$

Frick et al. PRC 70, 024309 (2004)

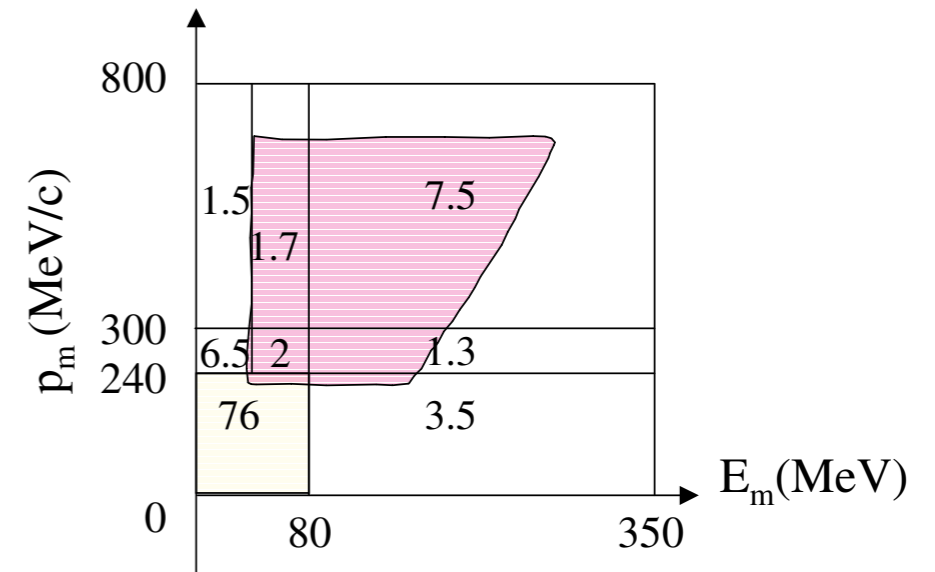
Self consistent Green's Function (SCGF)

Integrated strength in the covered  $E_m$ - $p_m$  region:

$$Z_c = 4\pi \int_{130}^{670} dp p_m^2 \int dE_m S(E_m, p_m)$$

"correlated strength" in the chosen  $E_m$ - $p_m$  region:

Rohe et al.,  
Phys. Rev. Lett. 93, 182501 (2004)



In terms of # of protons in  $^{12}\text{C}$

$^{12}\text{C}$	exp.	CBF theory	G.F. 2.order	self-consistent G.F.
experimental area	0.61	0.64 $\approx$ 10 %	0.46	0.61
in total (correlated part)		22 %	12%	$\approx$ 20%

contribution from FSI: -4 %

- $\approx$  10% of the protons in  $^{12}\text{C}$  at high  $p_m$ ,  $E_m$  found
- first time directly measured

comparing to theory leads to conclusion that

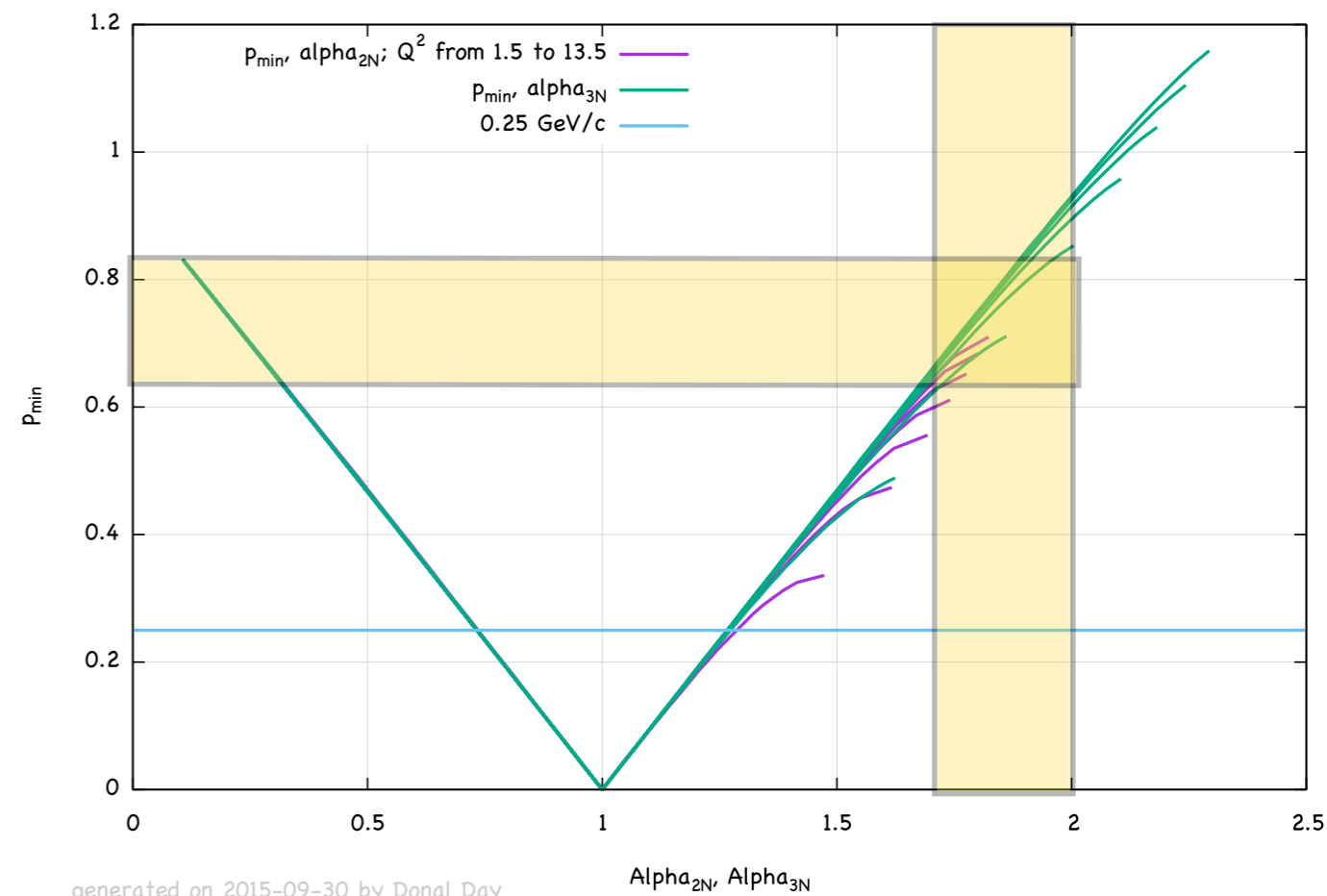
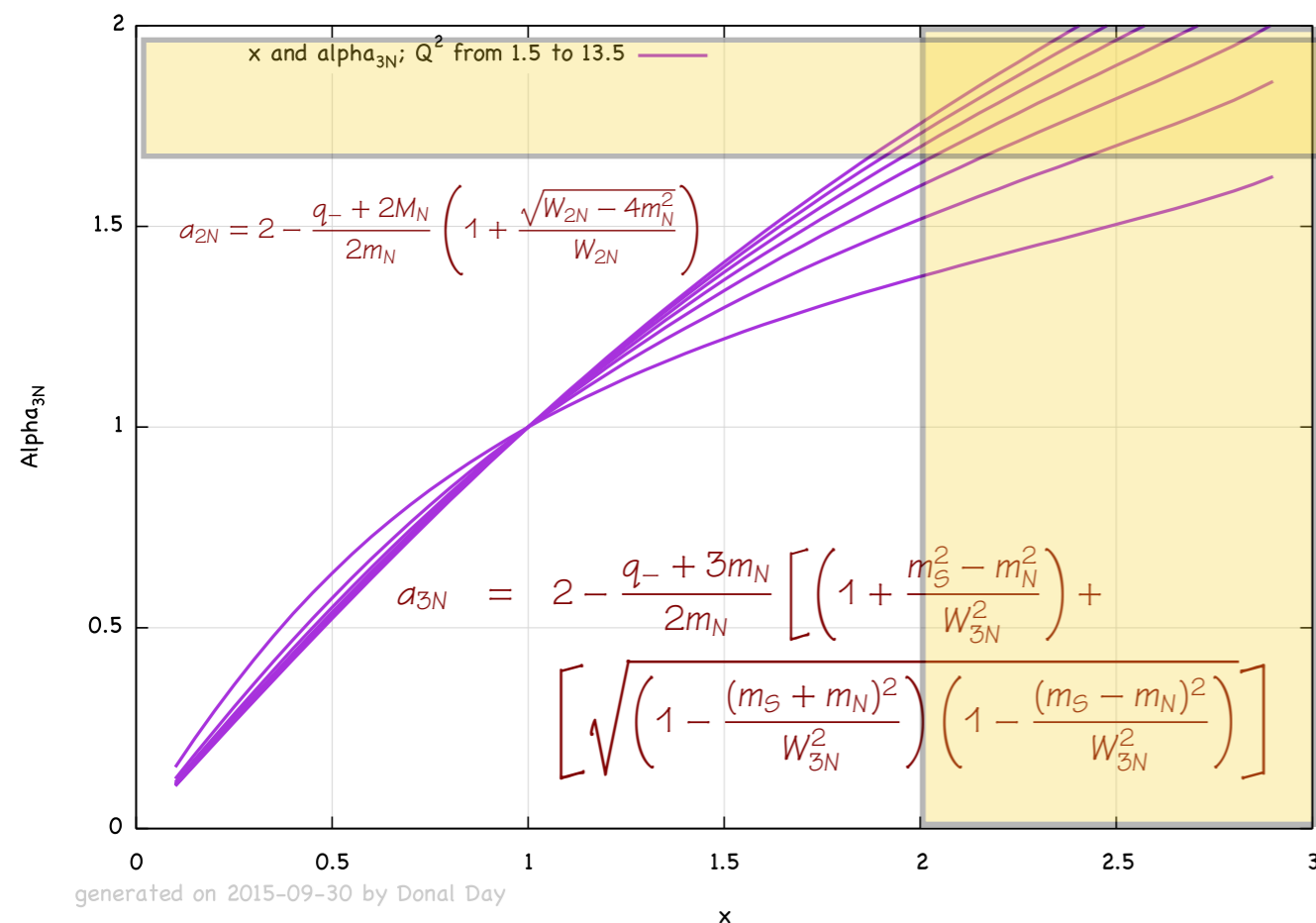
$\approx$  20% of the protons in Carbon are beyond the IPSM region

# 3N Correlations

## 2N SRC (3N SRC)

- $p > k_F$  i.e. its momentum exceeds characteristic nuclear Fermi momentum, ( $k_F \approx 250 \text{ MeV/c}$ )
- balanced by the momentum of a (two) correlated nucleon(s)
- In both cases the center of mass momentum of the SRC,  $p_{cm} \approx k_F$

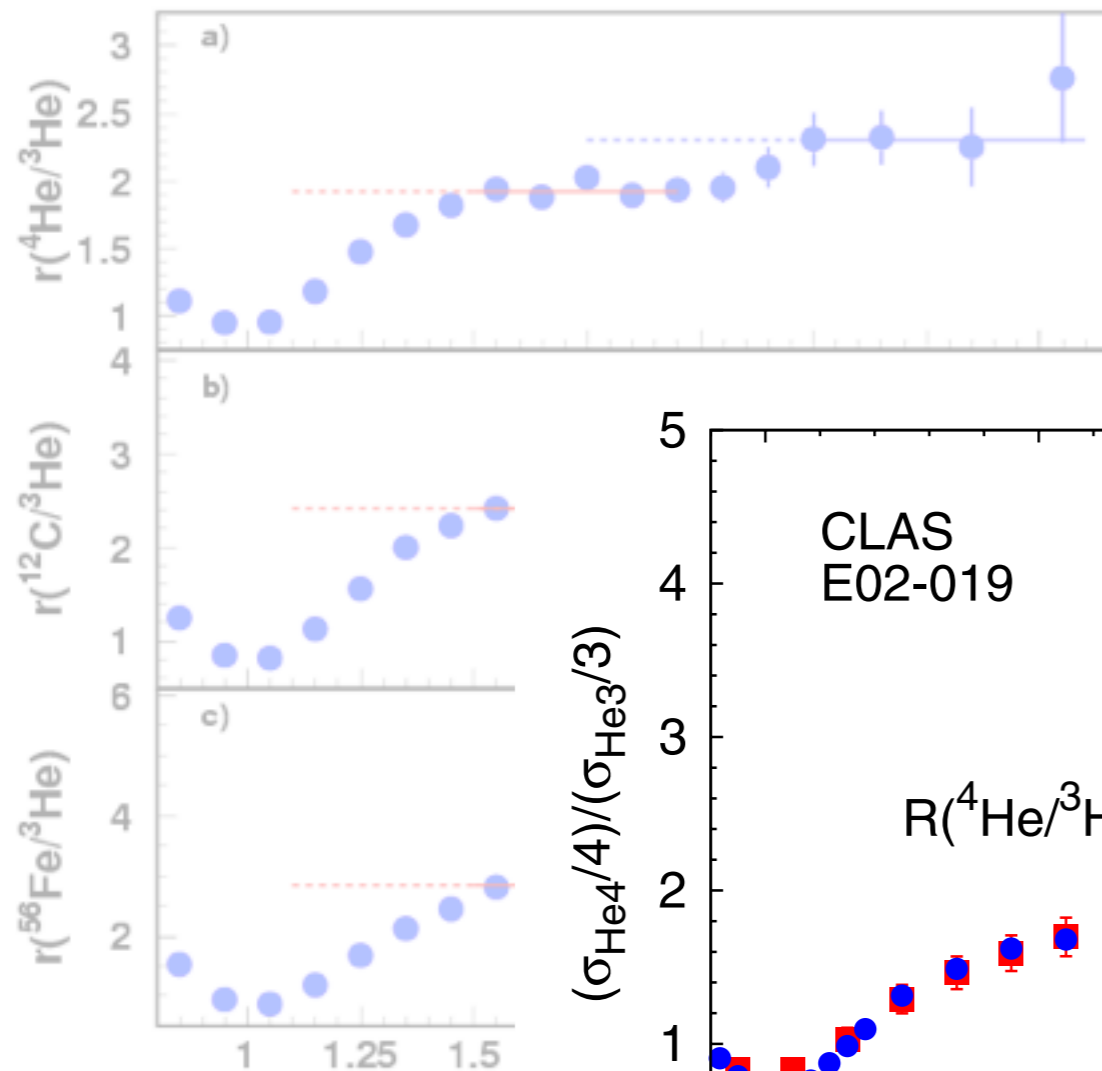
Where to look:  $p \gtrsim 600 \text{ MeV/c}$ ,  $Q^2 > 4$ ,  $x = 2 - 3$  Misak's talk



# $A(e,e')$ cross section ratios and SRC

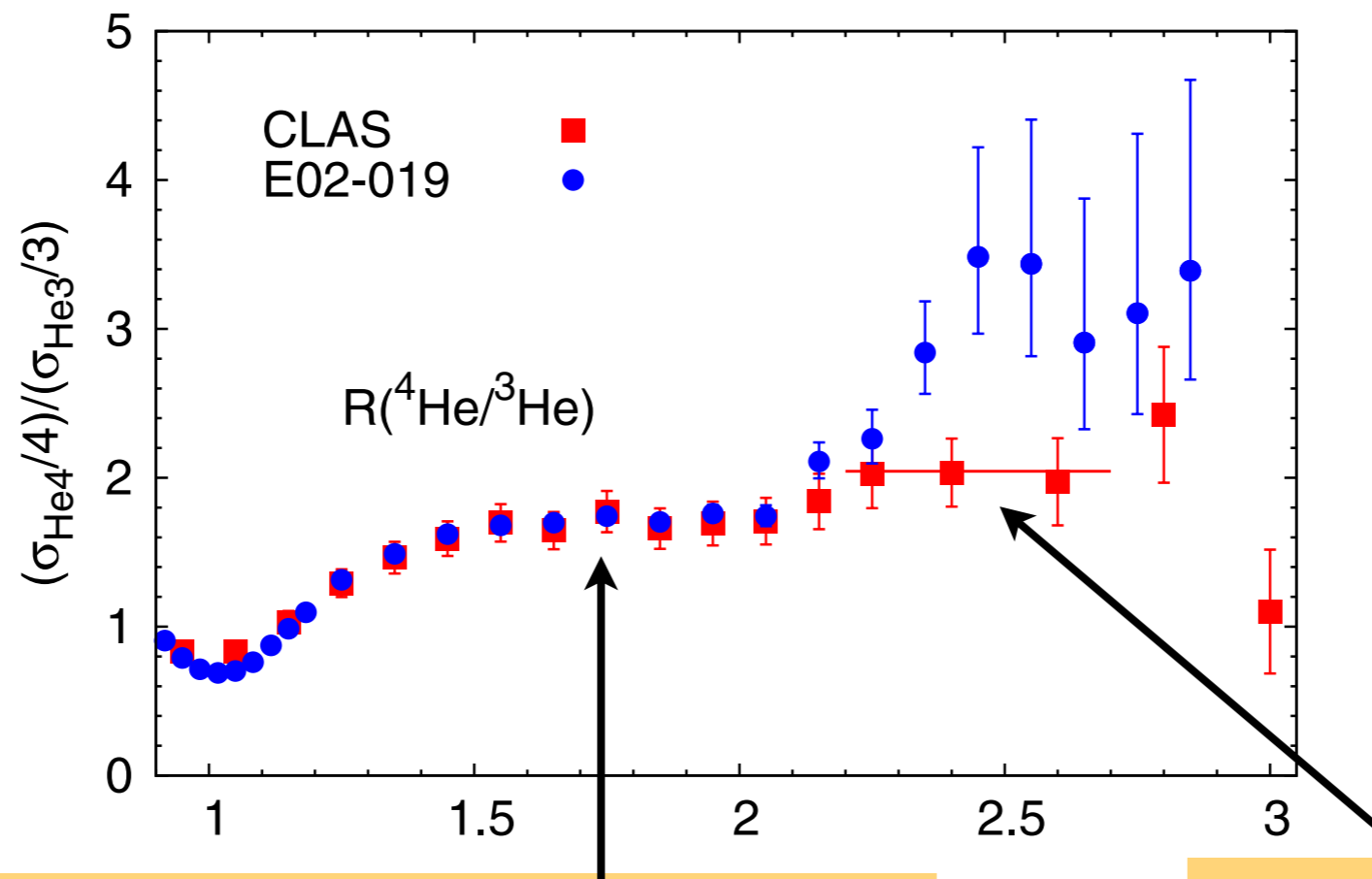
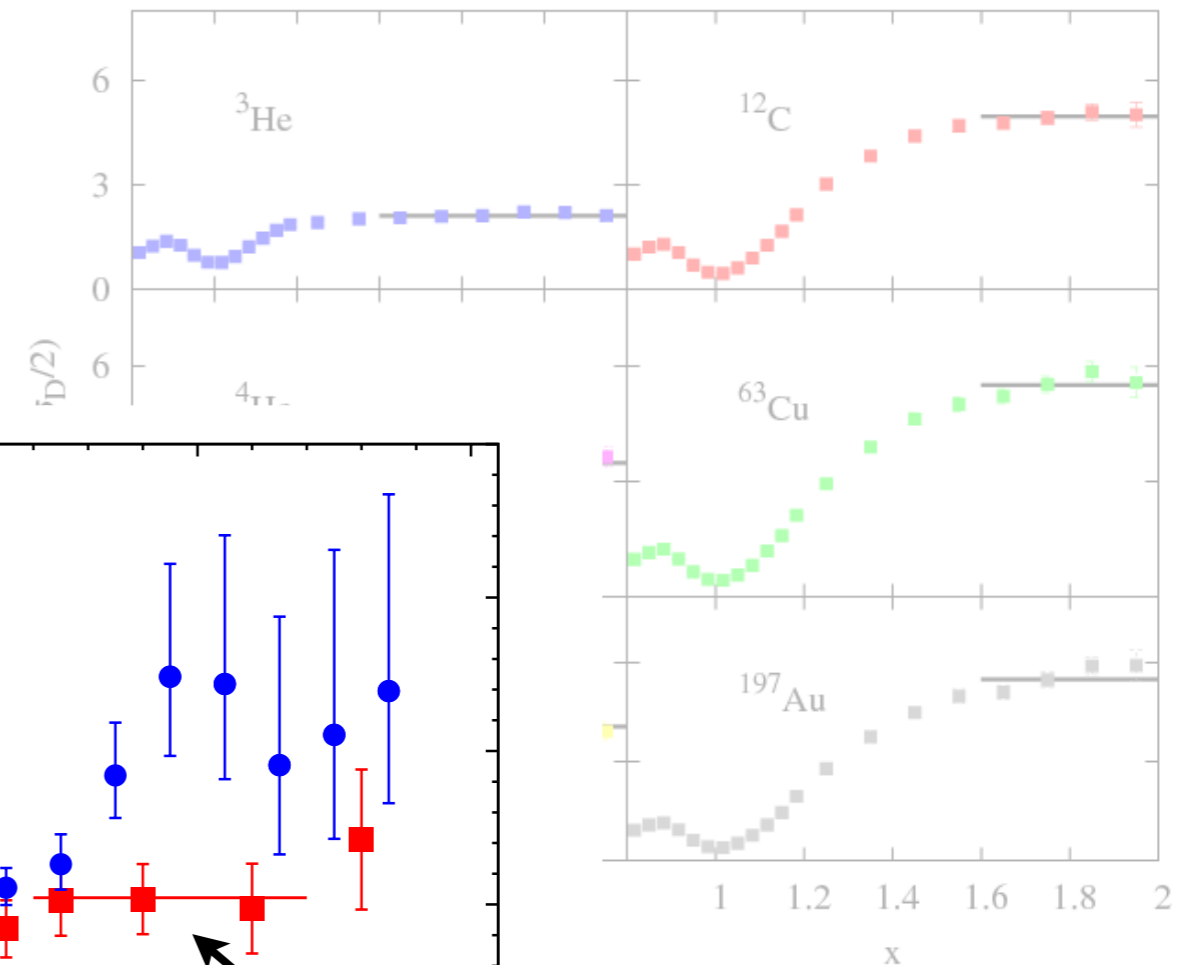
## Hall B

K. S. Egiyan et al., Phys. Rev. Lett. 96, 082501 (2006)



## Hall C

N. Fomin et al., Phys. Rev. Lett. 108, 092502 (2012)



Good agreement in the 2N-SRC region

but potential difference in the 3N-SRC region

# Data from SLAC

Rock et al, PRC 26, 1593 (1982)

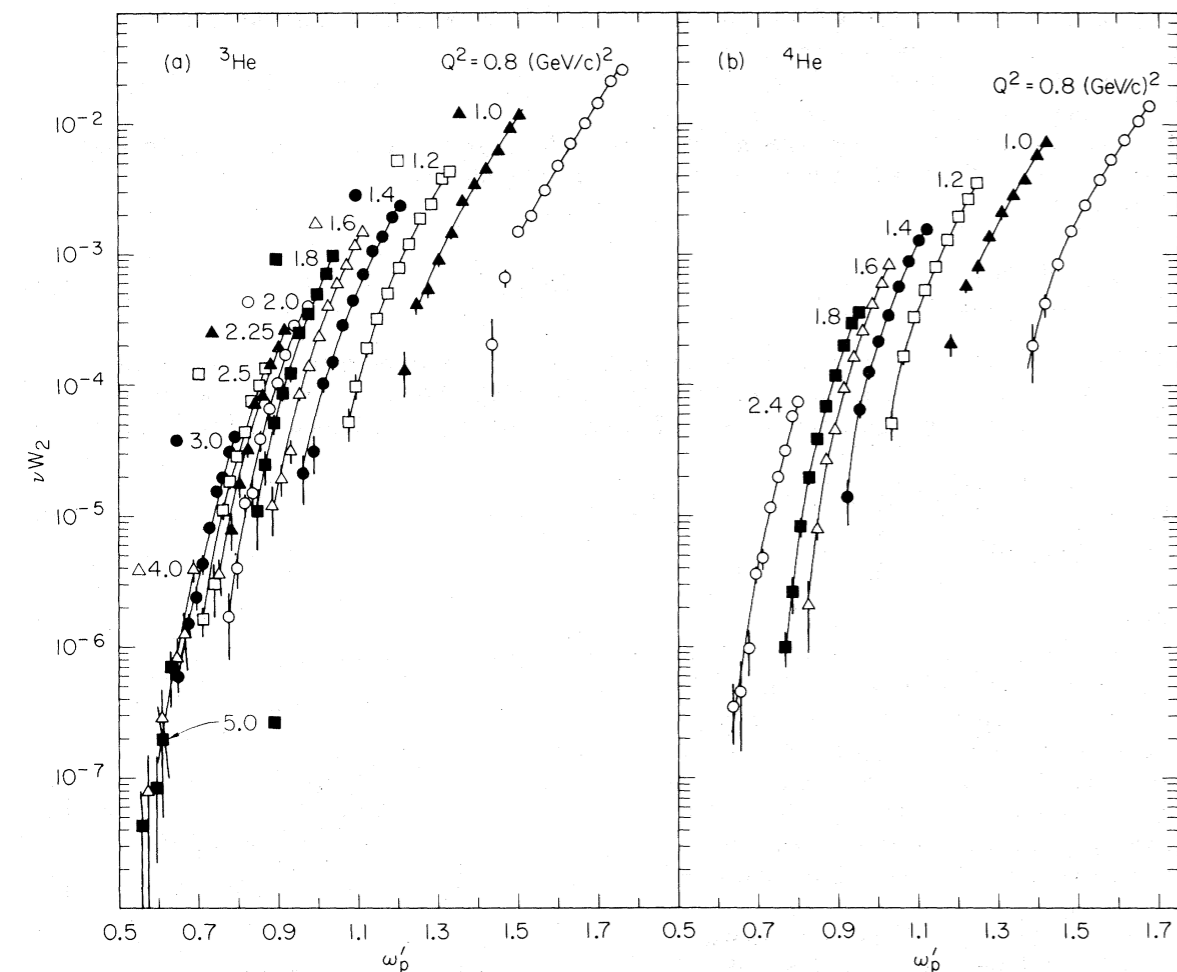
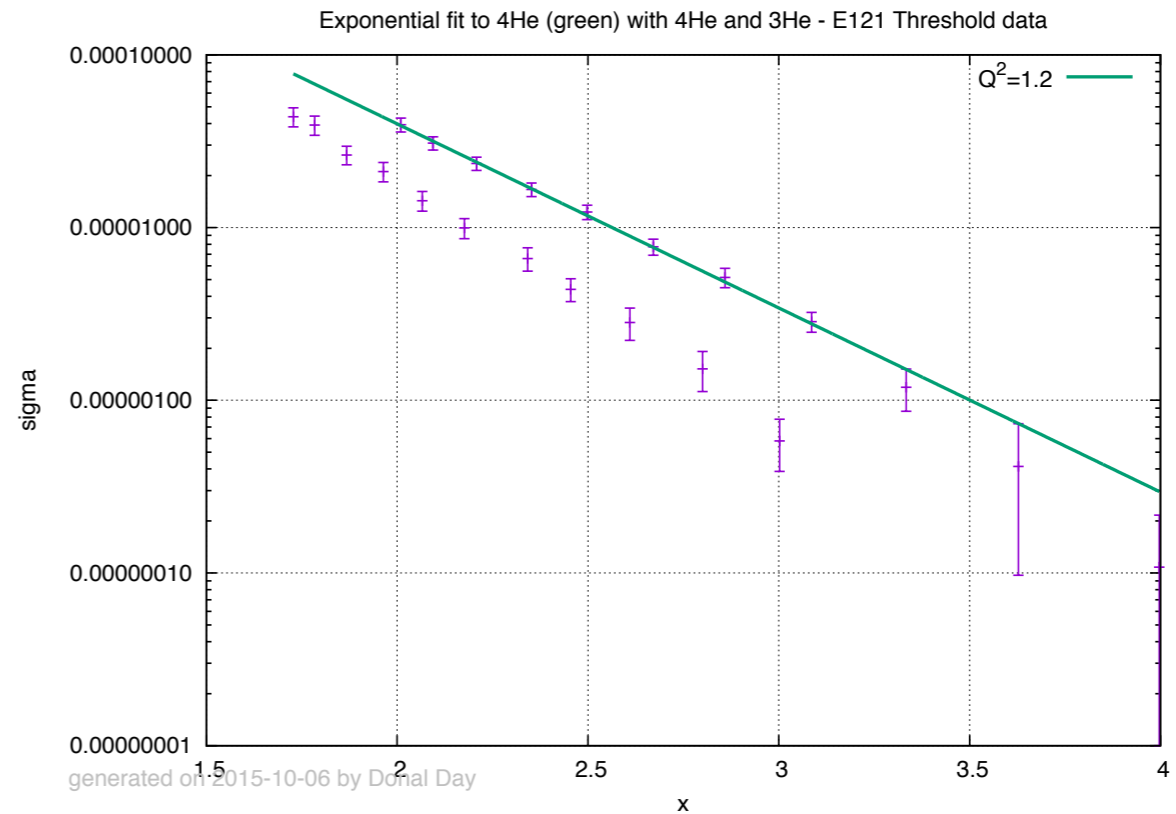
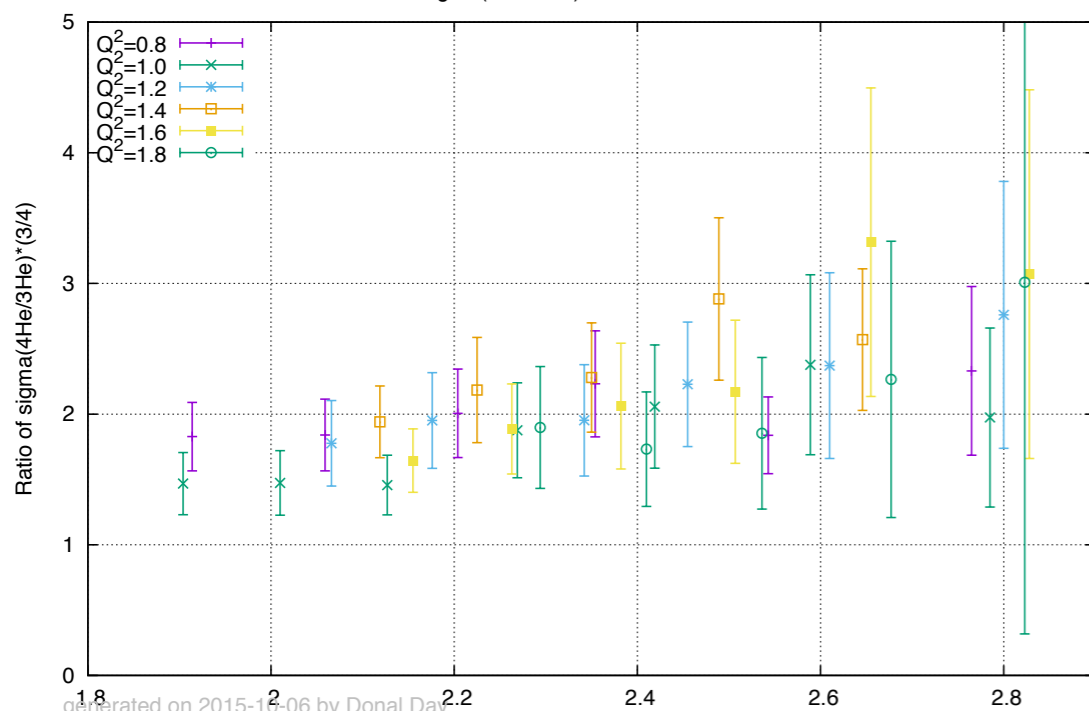
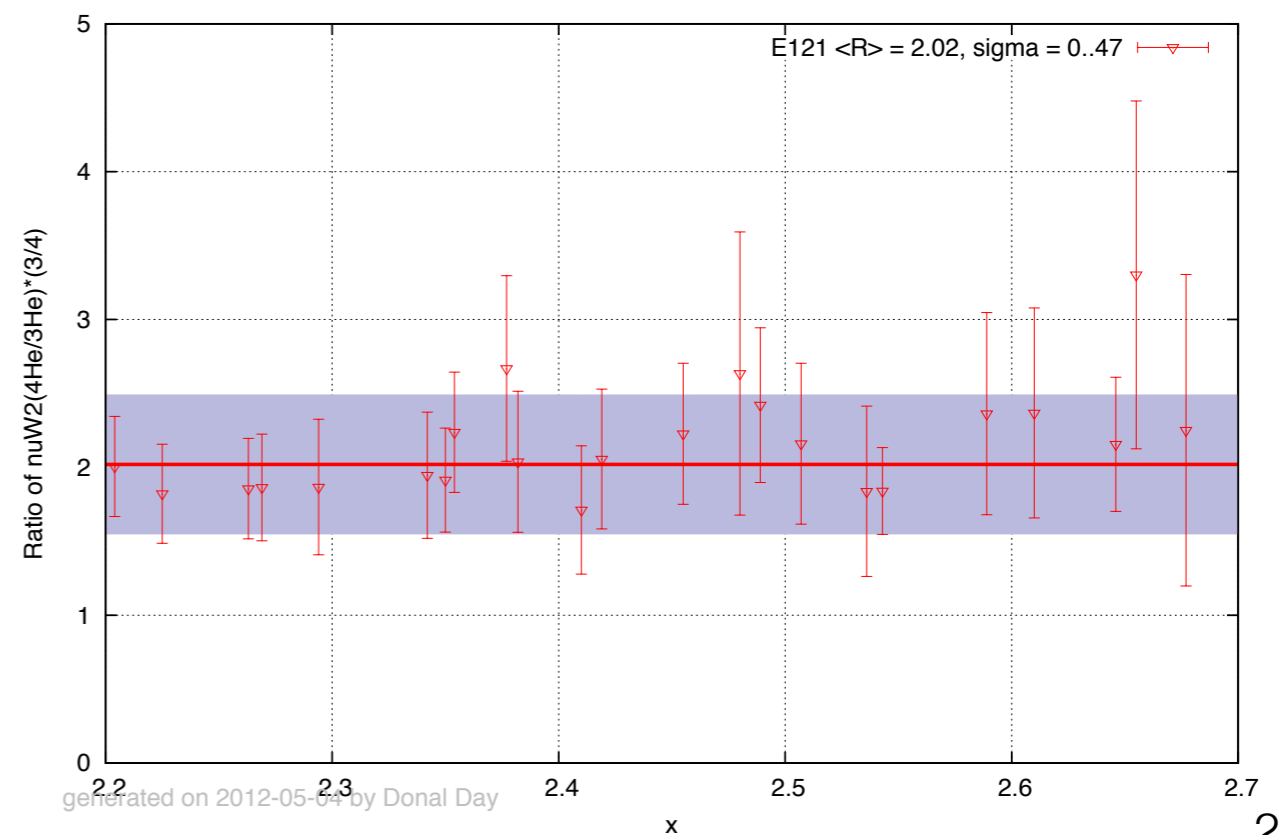


FIG. 7. The inelastic structure function  $\nu W_2$  as a function of the scaling variable  $\omega'_p = 1 + (M_p^2 + 2M_p\nu - Q^2)/Q^2$  for several different values of  $Q^2$ . The curves at fixed  $Q^2$  are to guide the eye and they seem to approach a common limit, (a)  $^3\text{He}$  and (b)  $^4\text{He}$ .

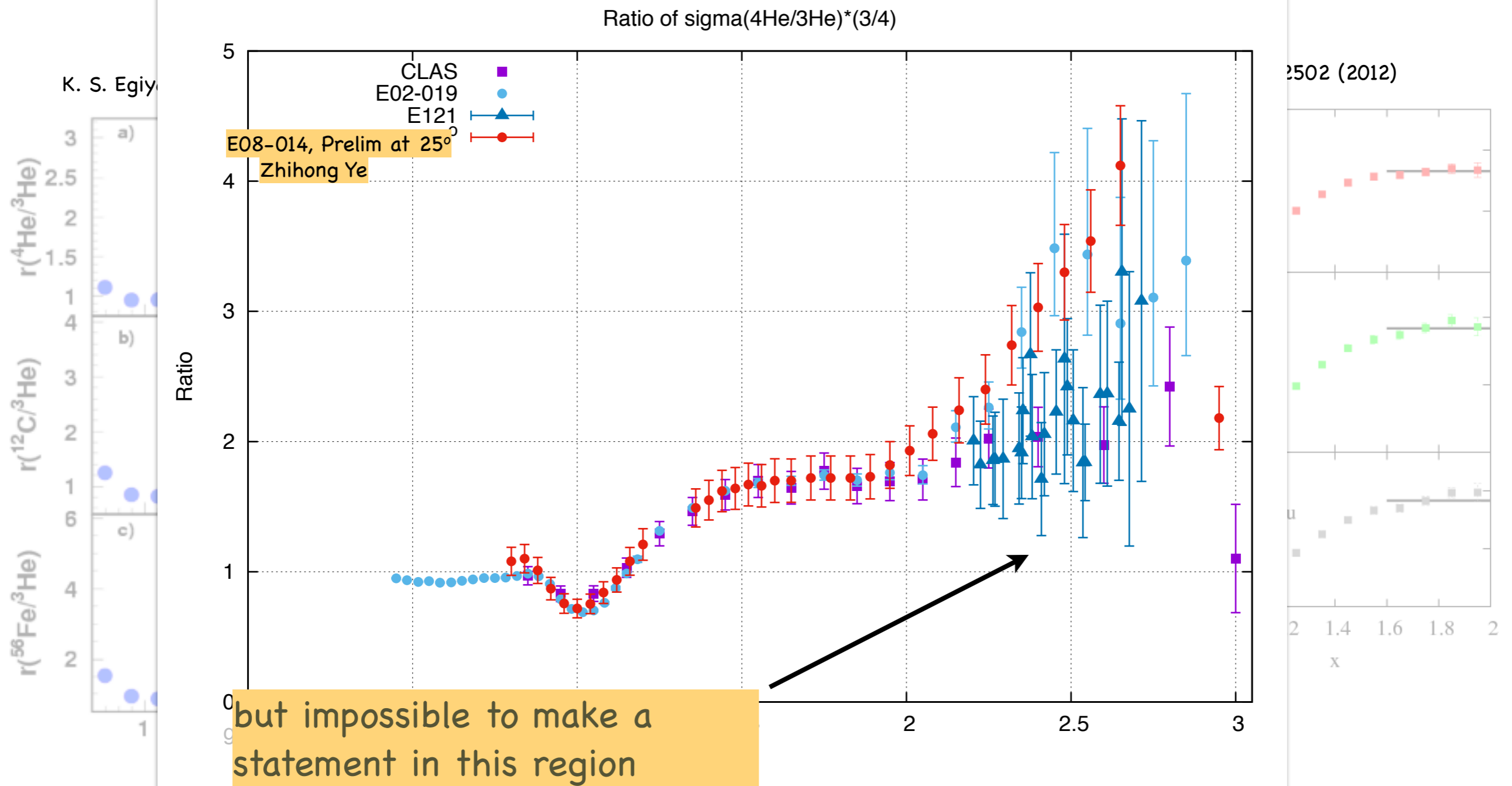
Ratio of  $\sigma(4\text{He}/3\text{He})^{*3/4}$  - E121 Threshold data



E121 threshold data  $\nu W_2$



# $A(e,e')$ cross section ratios and SRC

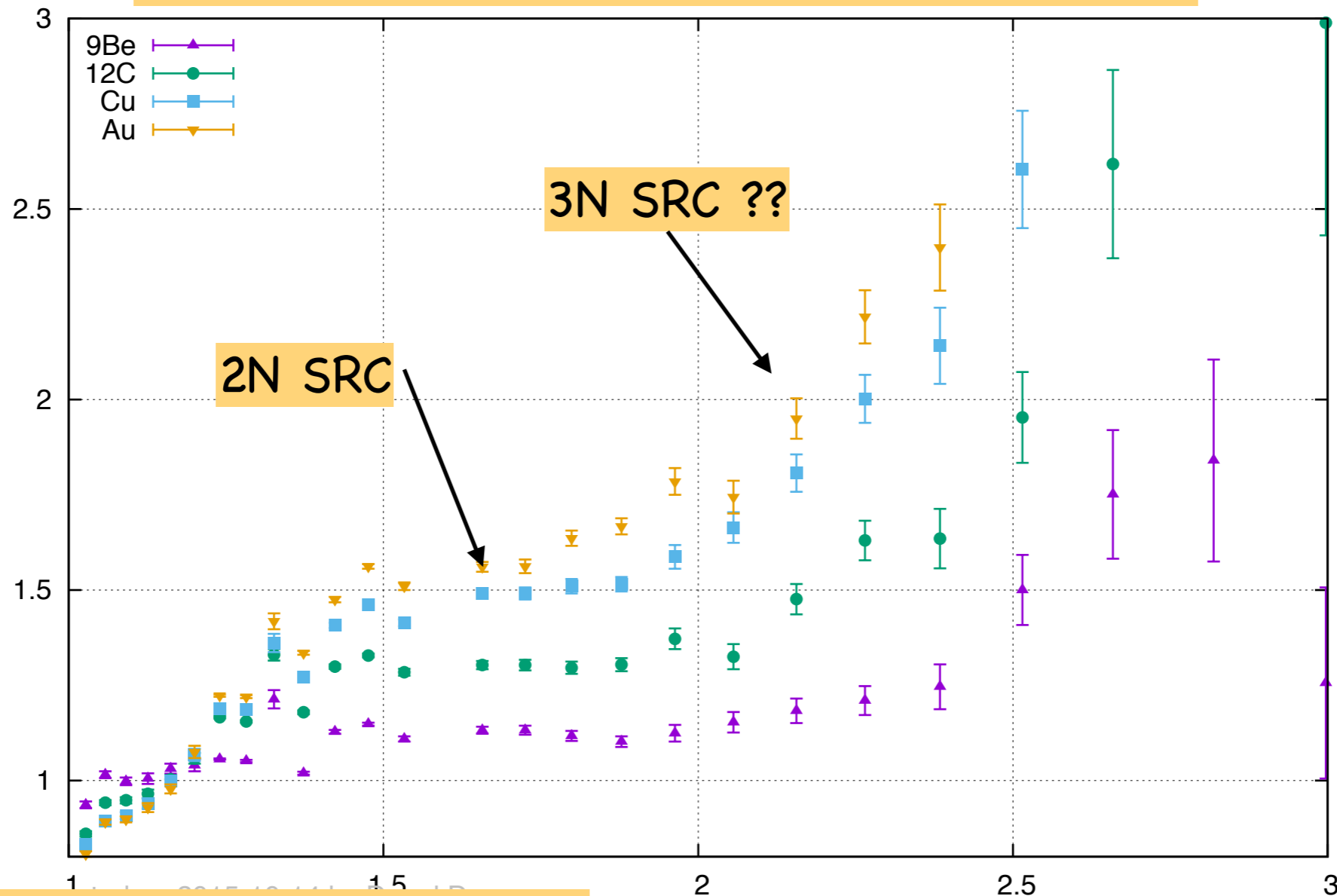


More data at larger  $Q^2$  needed,  $x > 1$  at 12 GeV, E12-06-105, , see Arrington, Fomin talks.

# $A(e,e')$ cross section ratios and SRC

Ratio of  $^9\text{Be}$ ,  $^{12}\text{C}$ ,  $\text{Cu}$ ,  $\text{Au}$  to  $^4\text{He}$  @  $Q^2 = 2.7$

Ratio to  $^4\text{He}$



From E02019, Fomin thesis data

×

Acceptance effects, windows

# E12-15-005: Measurements of Quasi-Elastic and Elastic Deuteron Tensor Asymmetries

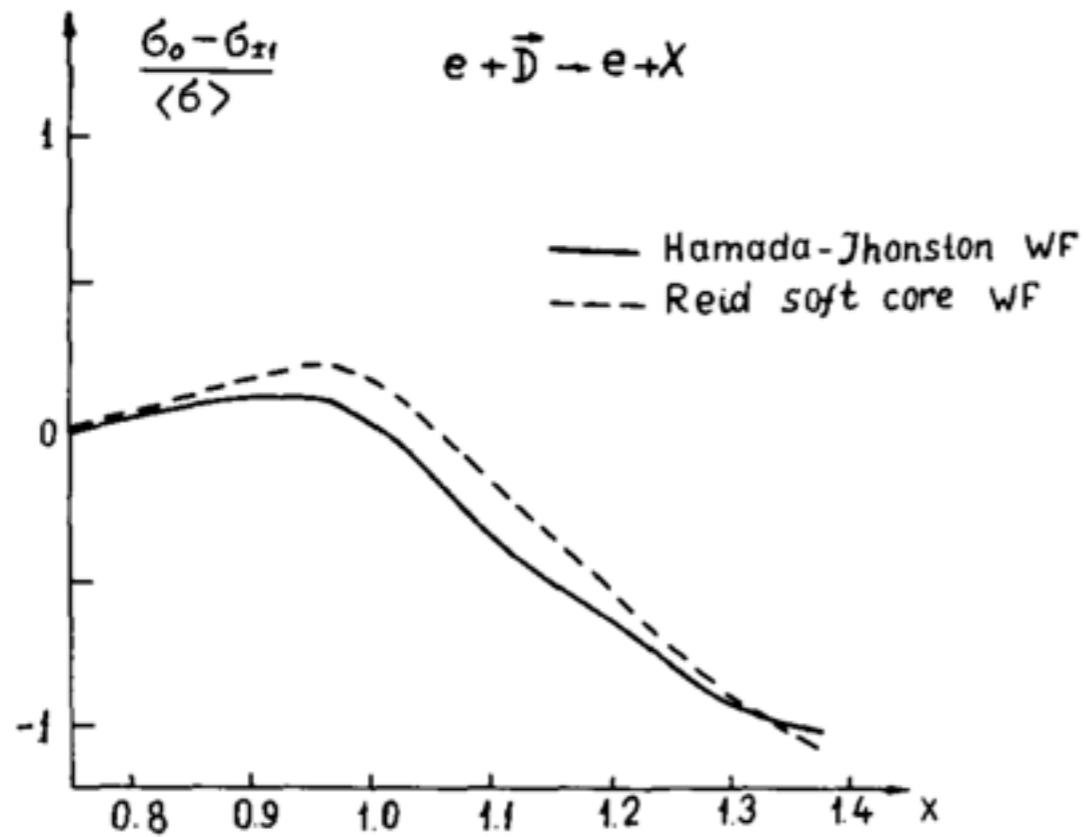
Probe short-range repulsion and tensor force in nucleon- nucleon interaction through tensor asymmetries from quasi-elastic and elastic deuteron scattering

Tensor asymmetries are predicted to be sensitive to the D state probability as well as relativistic effects.

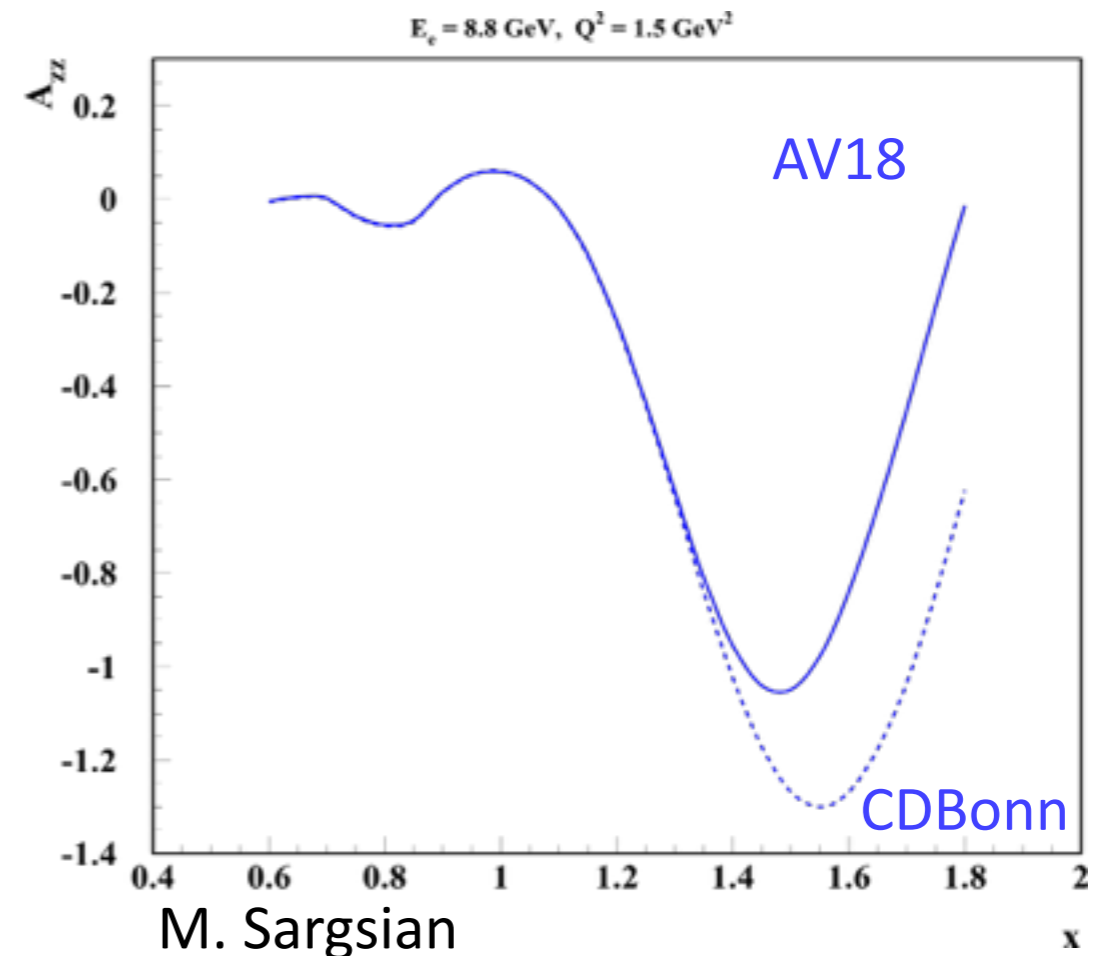
Conditionally Approved (44 days) on improving the tensor polarization and its measurement

E.Long (UNH), DD, D. Keller, K. Slifer, P. Solvignon, D. Higinbotham

$A_{zz} = \frac{2}{fP_{zz}} \left( \frac{\sigma_p - \sigma_u}{\sigma_u} \right)$ , is sensitive to WF admixtures



$$A_{zz} \propto \frac{\frac{1}{2}w^2(k) - u(k)w(k)\sqrt{2}}{u^2(k) + w^2(k)}$$



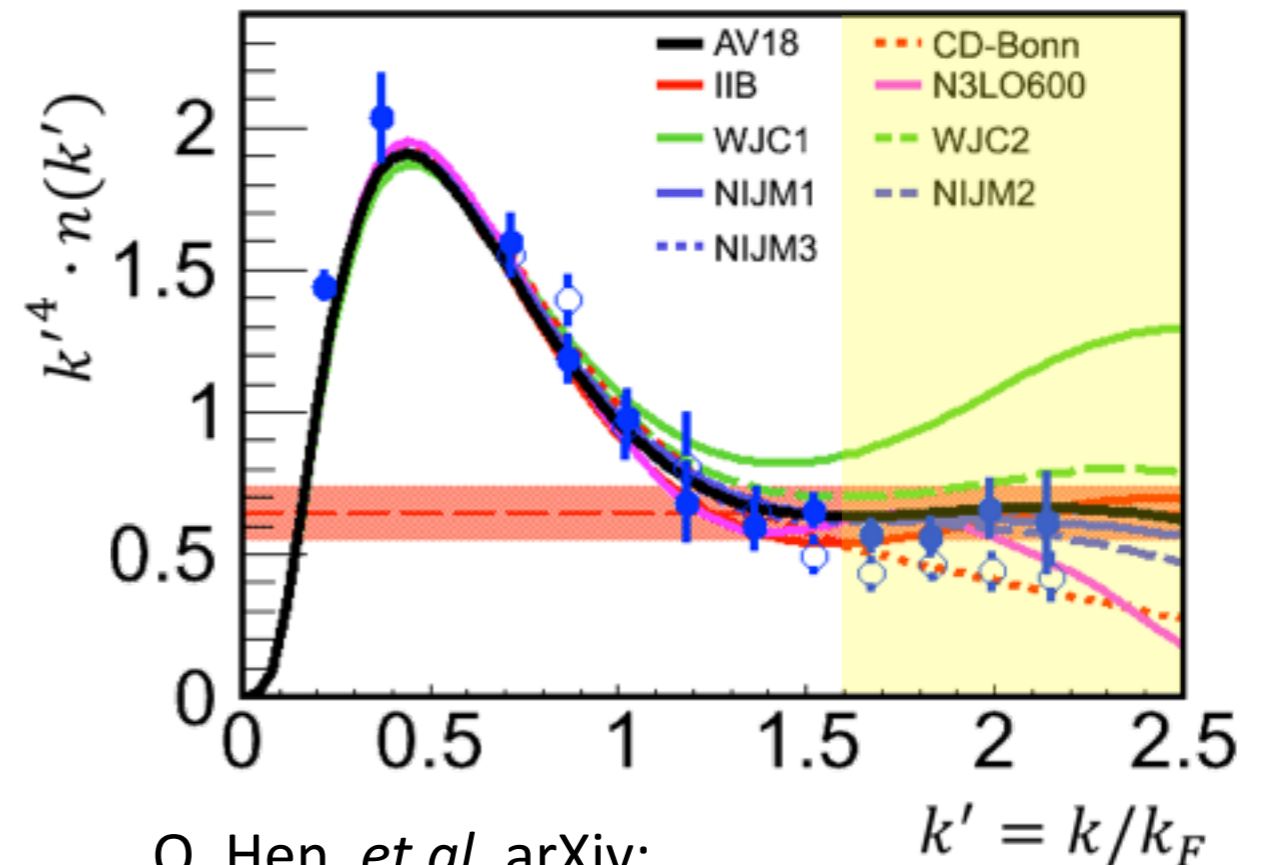
$A_{zz}$  can be used to discriminate between hard and soft wave functions. In impulse approximation  $A_{zz}$  is directly related to the S- and D-states which have very different  $r$  and  $p$  behavior.

Modern calculations indicate a large separation of hard and soft WFs begins above the quasi elastic peak at  $x > 1.4$

# Deuteron Wave function

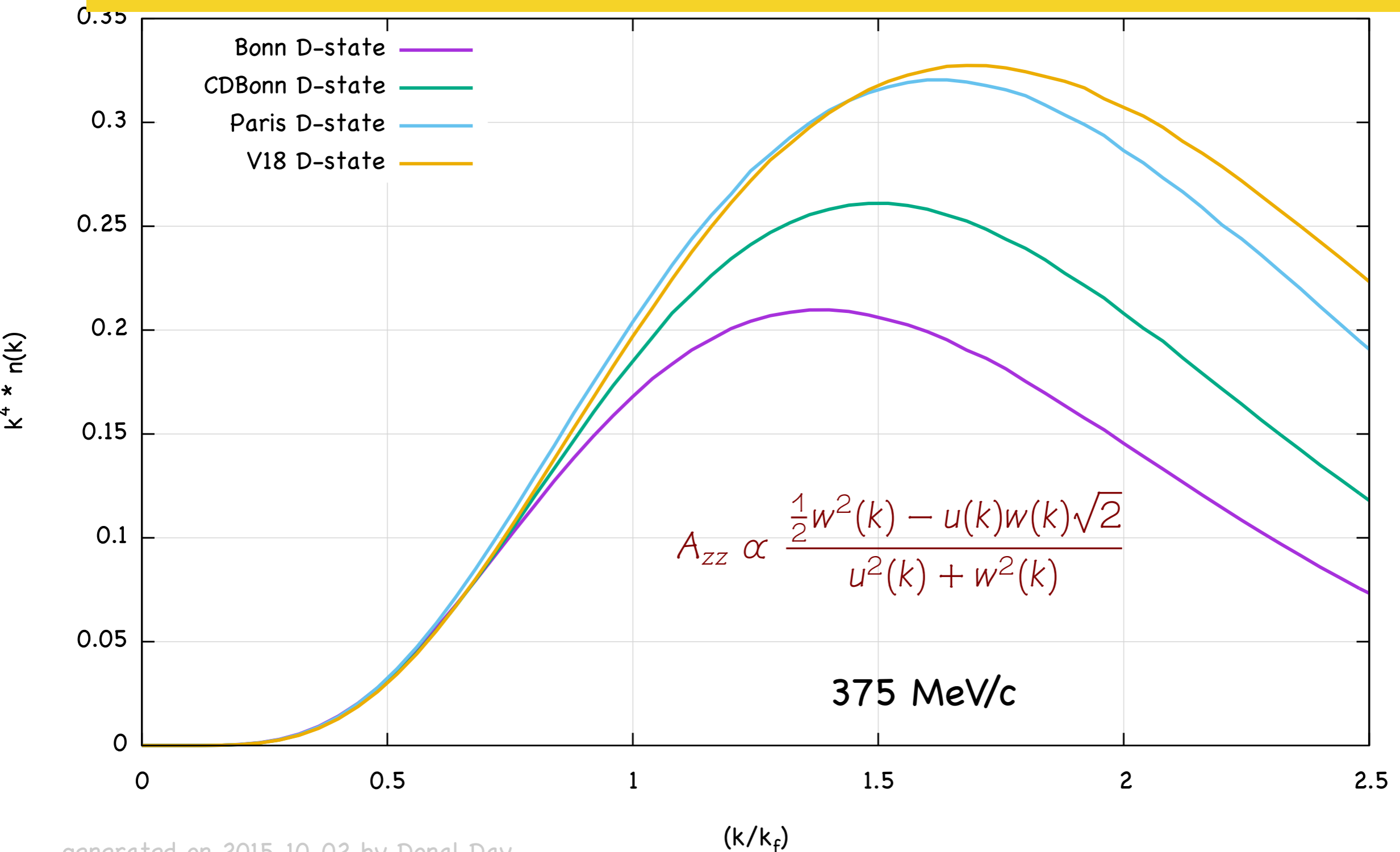
Is the deuteron wave function hard or soft?

- Hard like AV18 or softer like CD Bonn
- Unpolarized deuterons need to be probed at  $k > 500 \text{ MeV/c}$  to distinguish between hard and soft WFs
  - Difficult, absolute cross sections
- At present no unambiguous evidence for hard/soft.
- Tensor polarization exposes the D-state, allowing hard and soft WFs to be distinguished at lower momenta

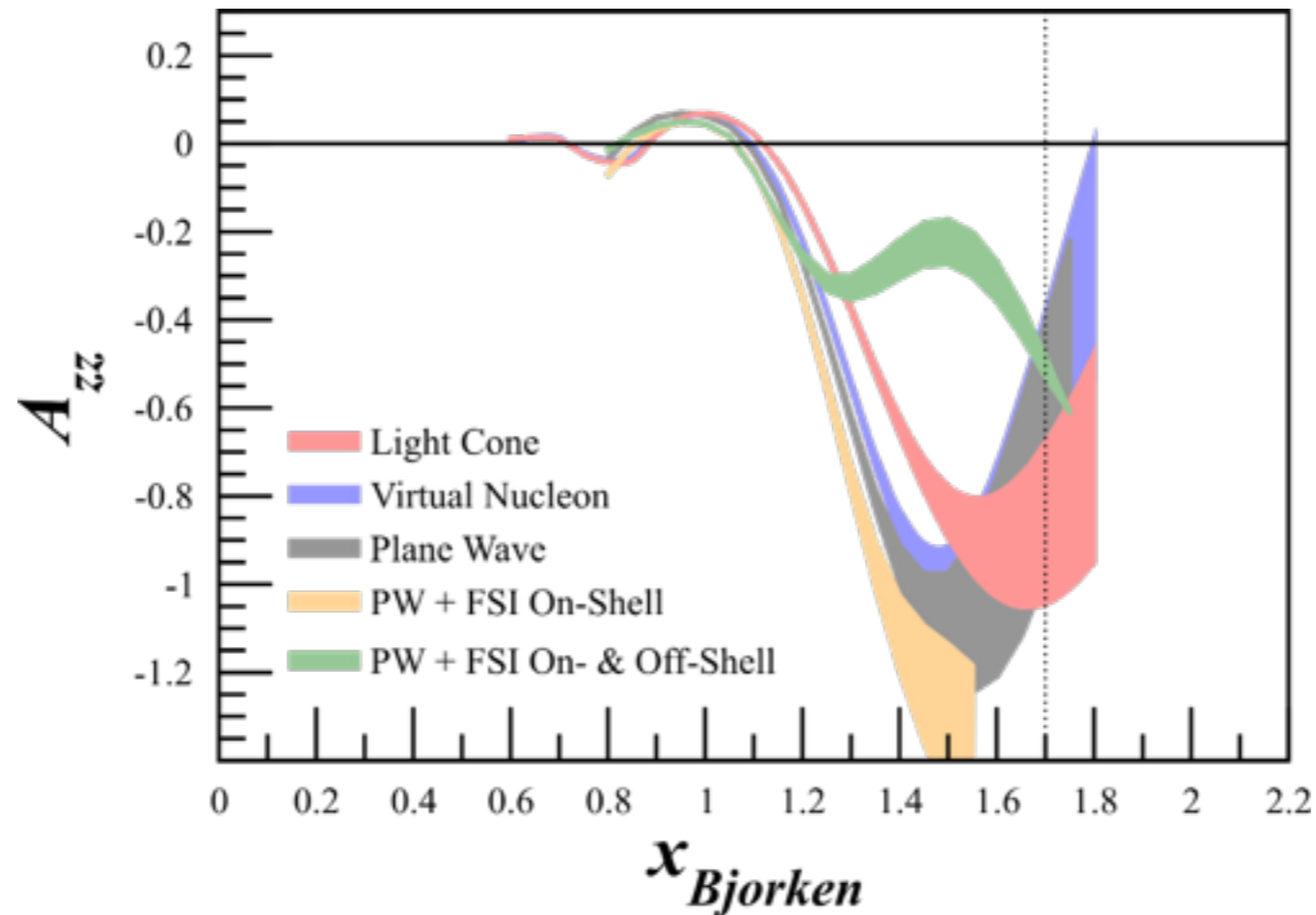


O. Hen, *et al*, arXiv:

# Deuteron D-states differ at large k



# Measurement of Quasi Elastic $A_{zz}$



Sensitive to effects that are very difficult to measure with unpolarized deuterons

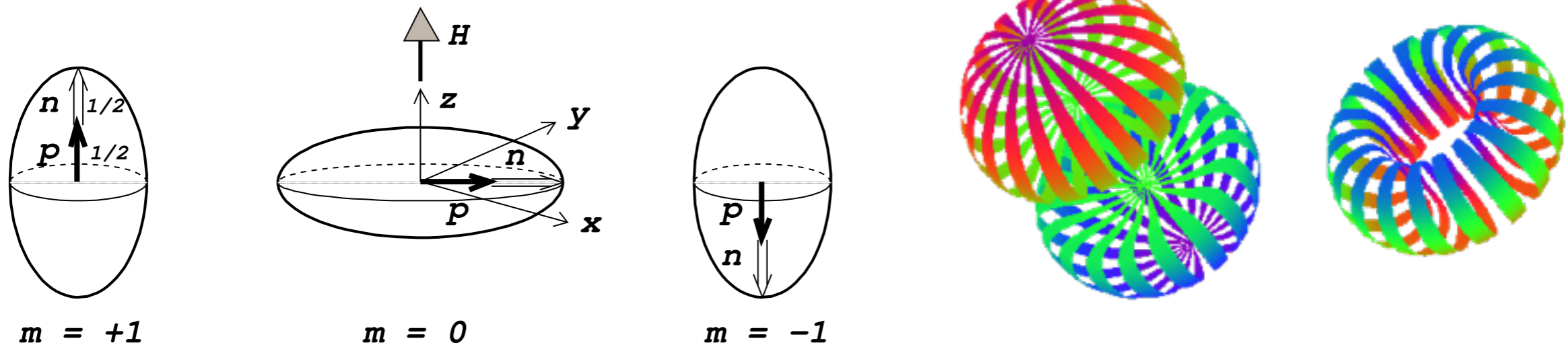
Huge 10-120% asymmetry

Measuring  $A_{zz}$  over a range in  $x$  and  $Q^2$  provides insight to

- Nature of NN Forces
- Hard/Soft wf
- Relativistic NN Dynamics
- On-Shell/Off-Shell Effect FSI

Decades of theoretical interest that we can only now probe with a high-luminosity tensor-polarized target

# Vector and Tensor Polarization of the Deuteron

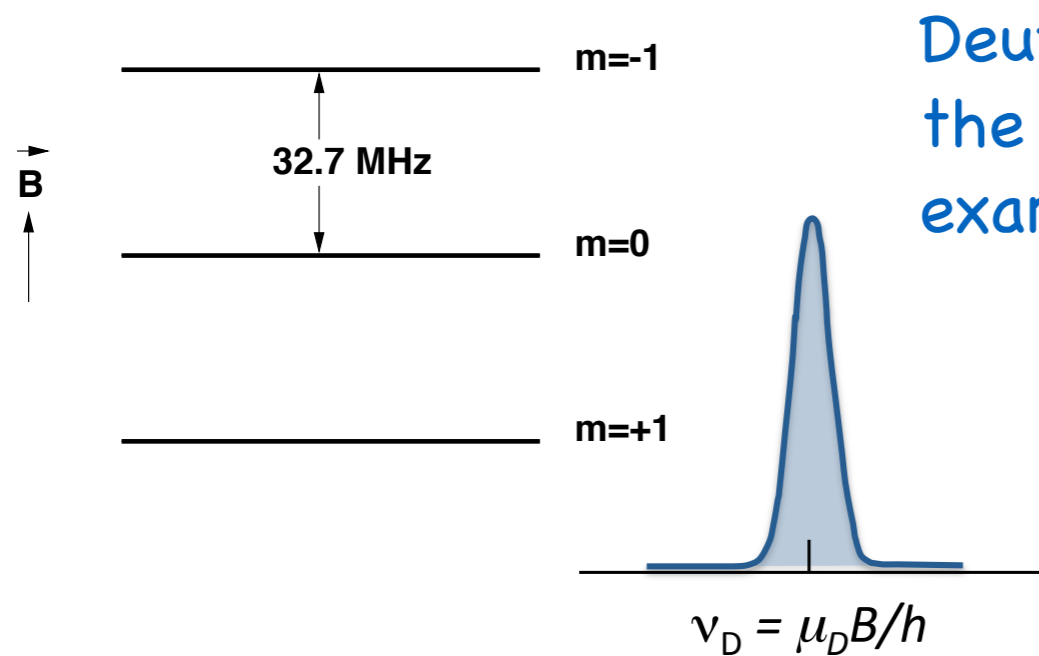


Deuteron in a magnetic field  $H$ , spin projection on  $Z$  can only take values  $m_d = +1, 0, -1$ .

If  $N_+$ ,  $N_0$ , and  $N_-$  are the relative numbers in the substates  $m_d = +1, 0, -1$ , ( $N_+ + N_0 + N_- = 1$ ), then vector  $p_z$  and tensor  $p_{zz}$  polarizations of a deuteron target are:

$$\text{Vector: } p_z = N_+ - N_-$$

$$\text{Tensor (Alignment) } p_{zz} = N_+ + N_- - 2N_0 = 1 - 3N_0.$$



Deuteron has quadrupole moment that interacts with the electric field gradient in the lattice (in  $\text{ND}_3$  for example)

$\nu_D$  = deut Lamour freq. (6.54 MHz/Tesla)

$\nu_Q$  =  $\text{ND}_3$  quadrupole freq (335.6 kHz)

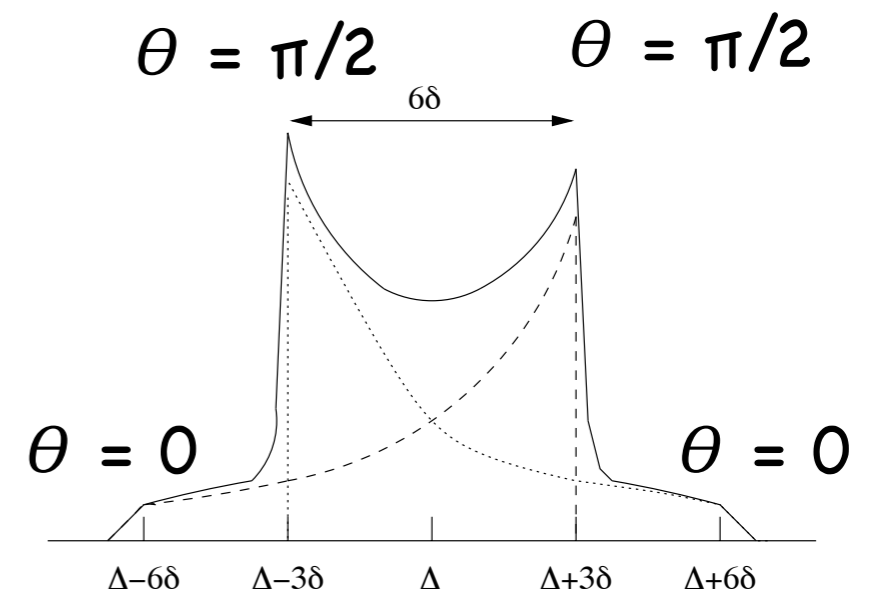
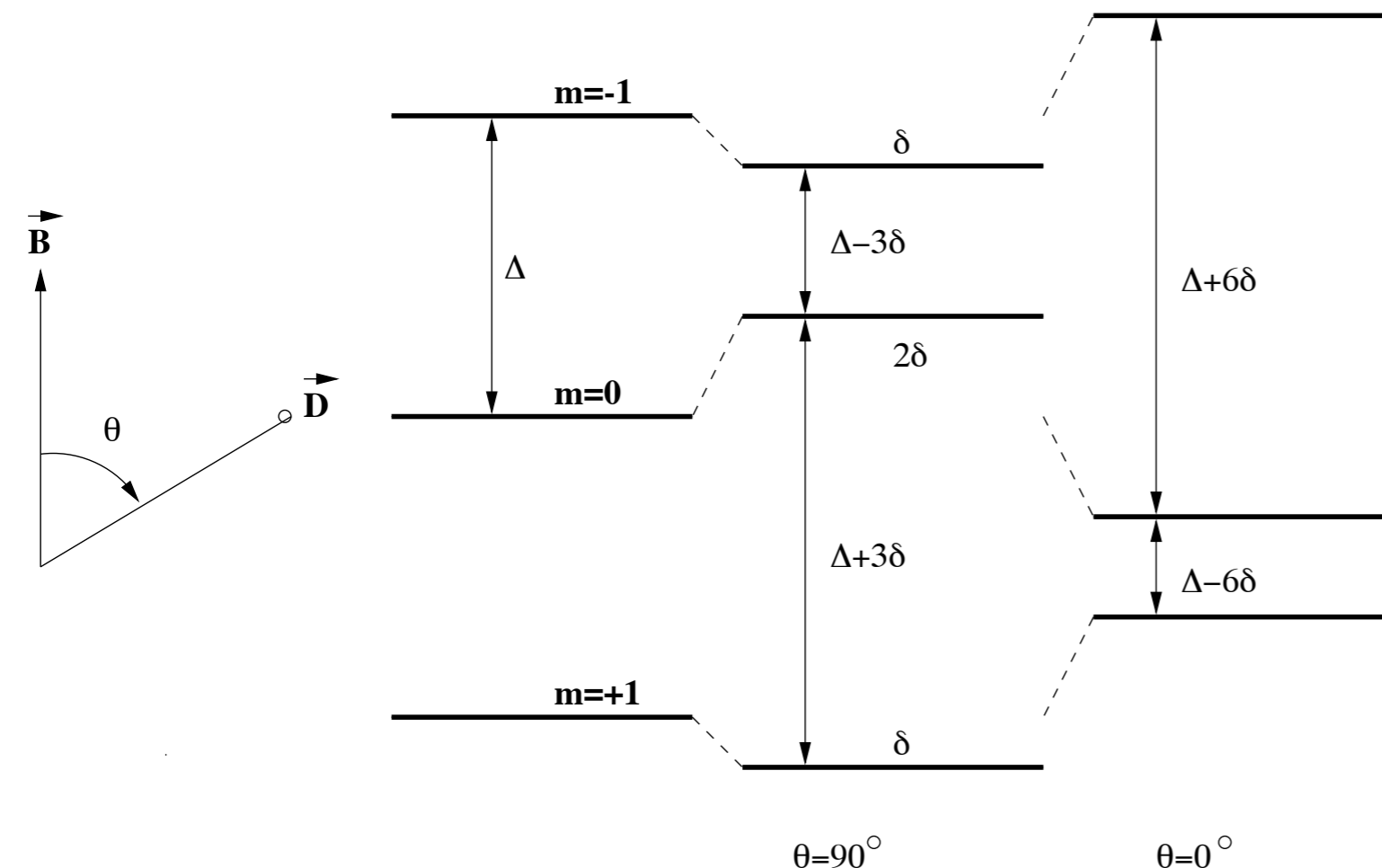
$$= \frac{1}{8} \frac{e^2 q Q}{h}$$

$eQ$  = deuteron quadrupole moment

$eq$  = electric field gradient

$\theta$  = angle between  $eq$  and  $B$

$$E_m = -h\nu_D m + h\nu_Q [3 \cos^2 \theta - 1] [3m^2 - 1(1+1)]$$



NMR line shape

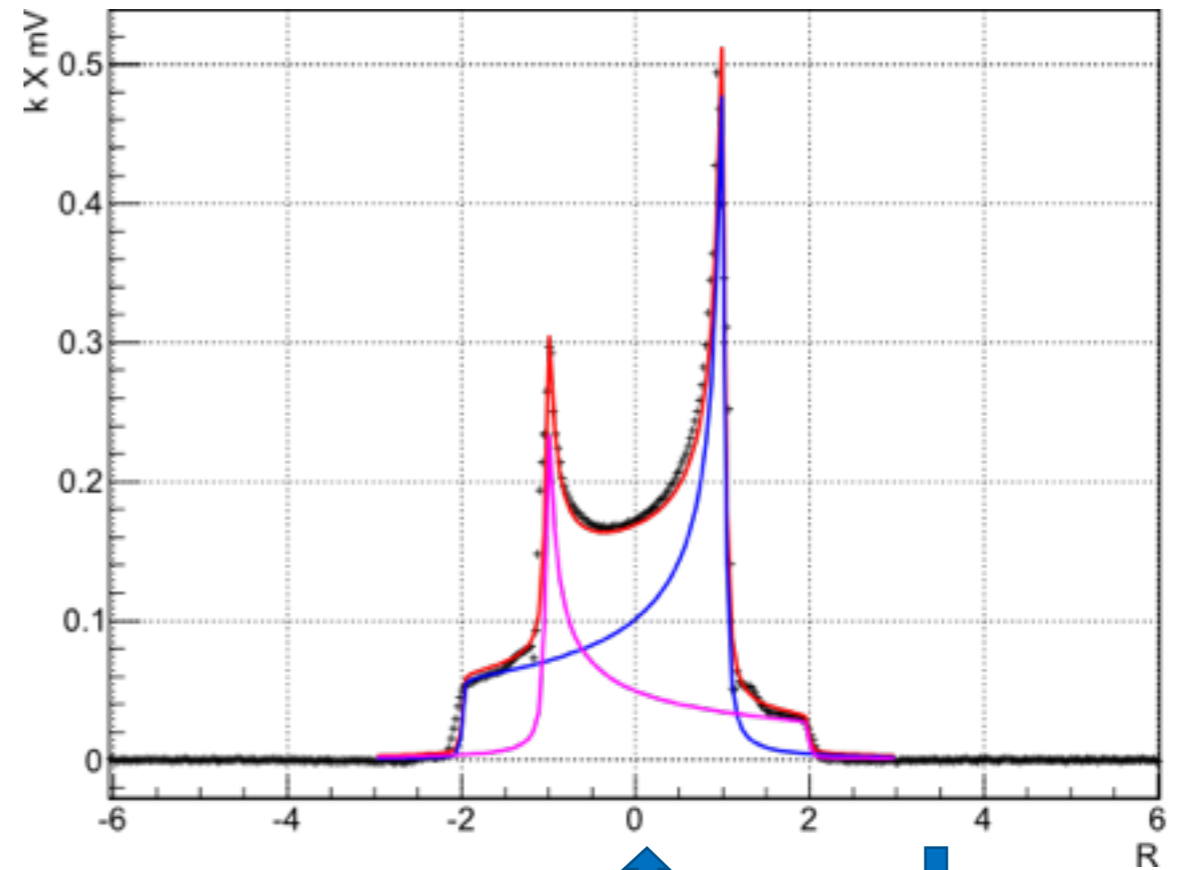
# Dynamic Nuclear Polarization

$$P_{zz} = 2 - \sqrt{4 - 3P_z^2}$$

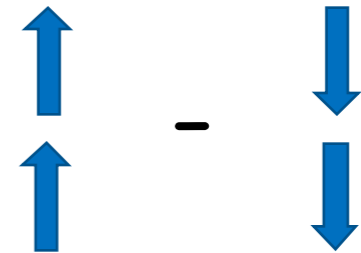
50% Vector  $P_z \Rightarrow 20\%$  Tensor



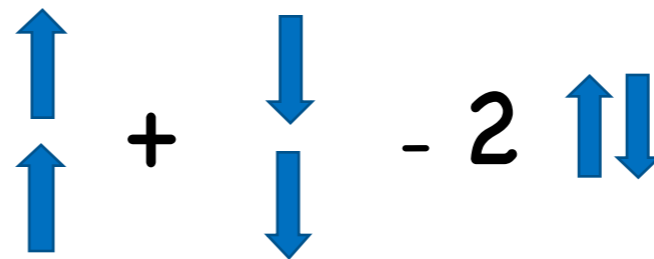
5 Tesla at 1K  
3cm target length



Vector  $P_z = p_+ - p_-$



Tensor  $P_{zz} = (p_+ + p_-) - 2p_0$



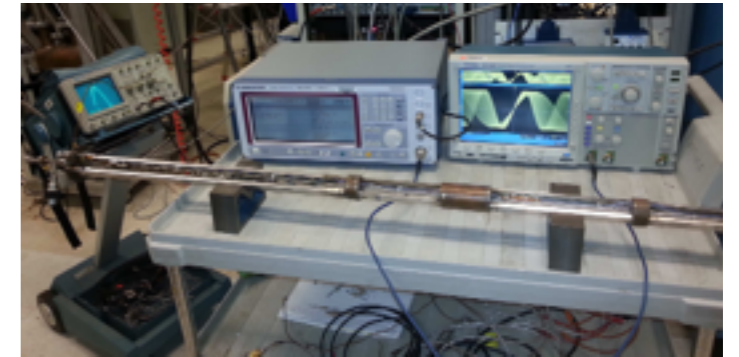
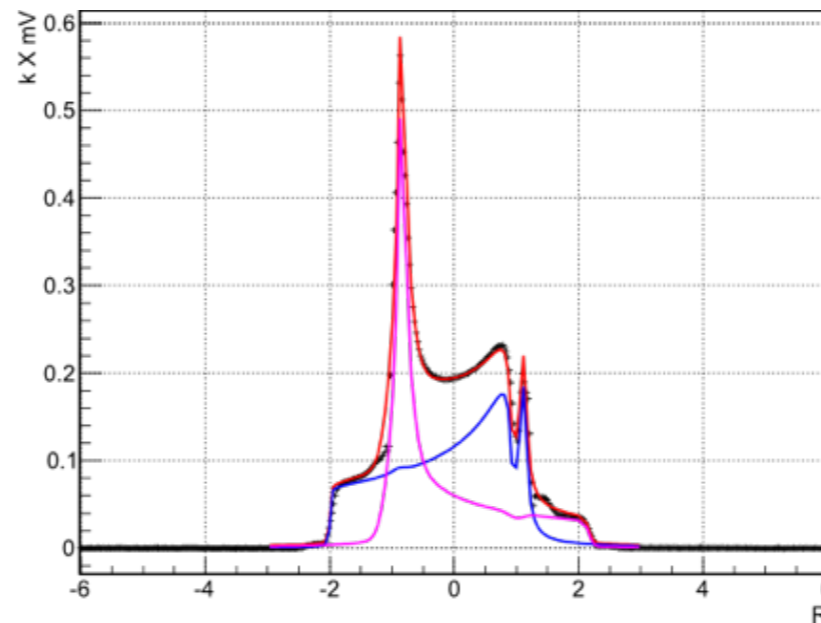
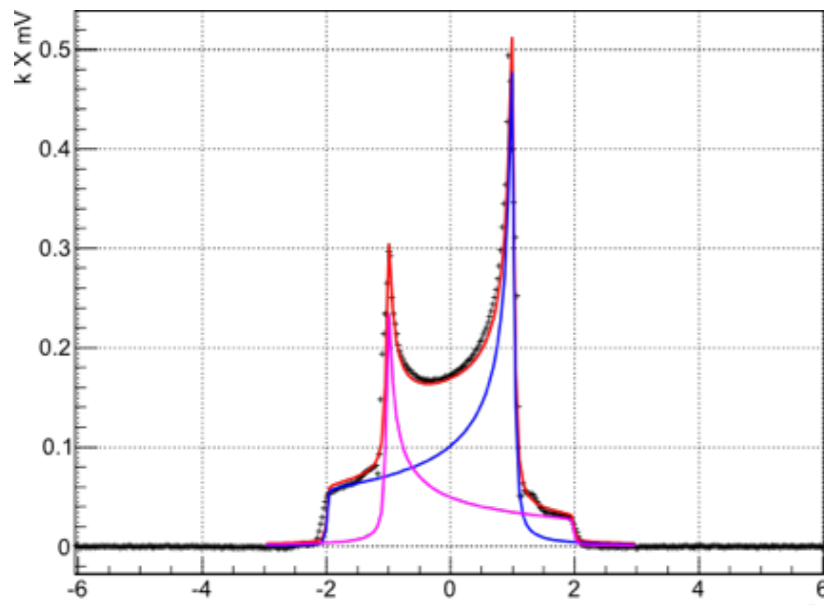
Positive  $P_{zz}$ : fill up the first two and minimize the  $m_0$ -state

# Tensor Polarization Progress

At UVA progress measuring  $P_{zz}$  through NMR line-shape analysis advancing (Dustin Keller)

Solid state NMR  $P_{zz}$  can be confirmed with elastic ( $T_{20}$ )

Enhancement through RF hole burning



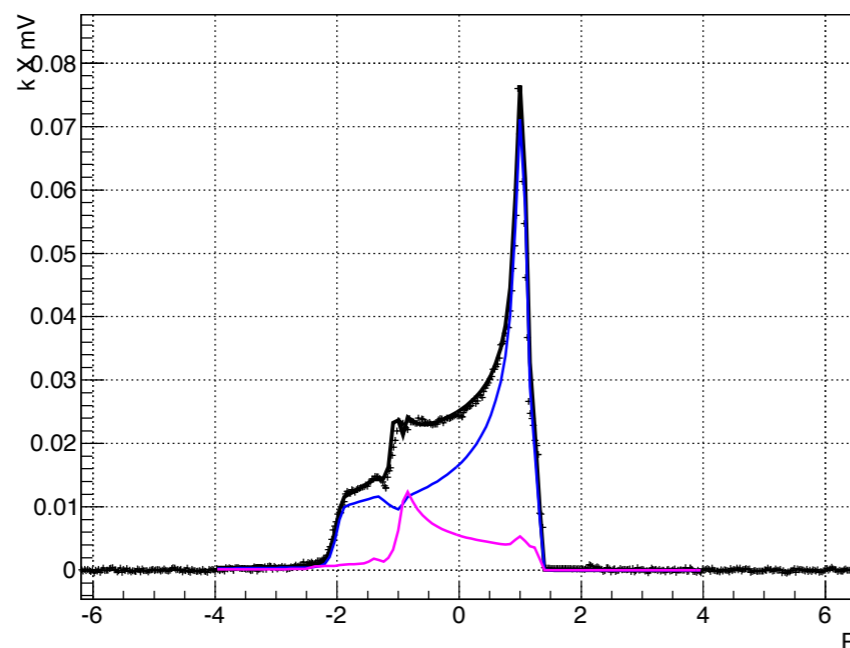
D Keller, PoS(PSTP 2013) 010

D Keller, HiX Workshop (2014)

D Keller, J.Phys.:Conf.Ser. 543, 012015

UVA Tensor Enhancement on Butanol (

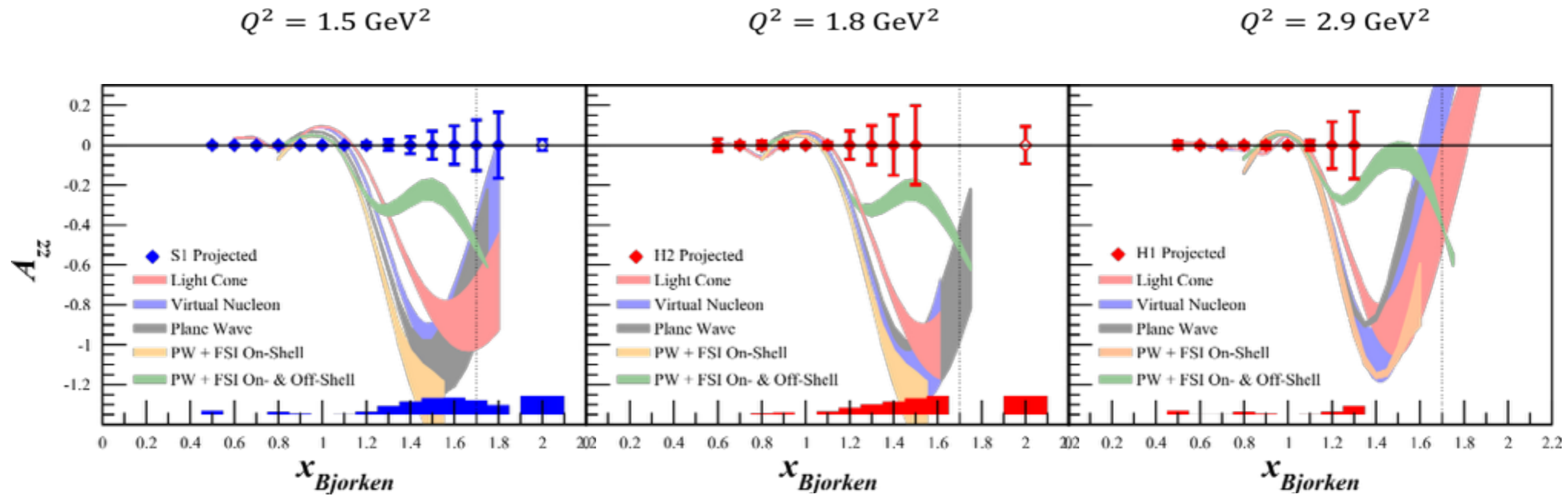
D Keller, PSTP 2015, to be published



$$P = 0.503 \rightarrow 0.447$$

$$P_{zz} = 0.196 \rightarrow 0.325$$

# $A_{zz}$ for $Q^2 > 1 \text{ GeV}^2$ with $P_{zz} = 30\%$



LL Frankfurt, *et al*, PRC **48** 2451 (1993)

Solid = Quasi-elastic

Open = Elastic

# Summary

- 2N SRC and their isospin dependence (anticipated by our understanding of the NN interaction, is now firmly established in multiple observables, experiments projectiles, final states and nuclei
- Relation of SRC to EMC established – only lacking are calculations that exposes the underlying connection
- Refined theory and calculation are needed incorporating SRC, FSI, and off-shell behavior will advance understanding
- SRC demand high densities (momenta, virtuality) and, if these rare fluctuations can be captured, they should expose, potentially large, medium modifications
- 3N SRC are as yet unseen in inclusive electron scattering – some sleuthing underway
- Approved experiments across labs with different focuses over next 5–7 years will reveal much
- Next big opportunity in inclusive scattering (in my view) is the transition from QES to DIS at  $x > 1$  at very large momenta transfer
- Tensor polarized targets advances will allow exposure of deuteron wf through asymmetry measurements
  - Opportunities exist for experiments with electrons and photons on tensor polarized deuterons