Effects of SRC on Nuclear Symmetry Energy

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NSF

Office of Science
U.S. DEPARTMENT OF ENERGY

国家自然科学基金委员会
National Natural Science Foundation of China

Outline

1. **Brief overview of the current status in determining the density dependence of $E_{sym}(\rho)$ from theories and experiments**

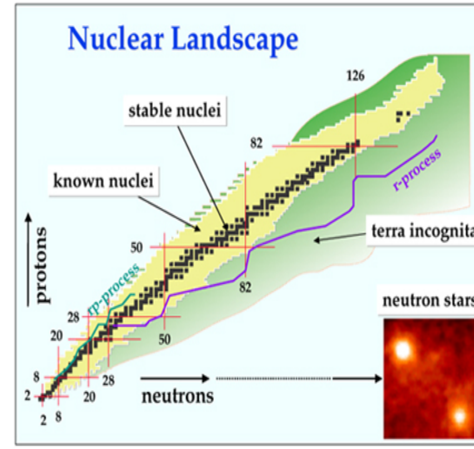
2. **Characteristics of single-nucleon momentum distribution n(k) with a high-momentum tail (HMT) due to SRC in n-rich matter**

3. **Effects of SRC on nuclear $E_{sym}(\rho)$ within a nonlinear RMF model**

4. **Effects of SRC-modified $E_{sym}(\rho)$ on heavy-ion collisions**

What is the Equation of State of neutron-rich nuclear matter?

$$E_{sym}(\rho) = \frac{1}{2}\frac{\partial^2 E}{\partial \delta^2} \approx E(\rho)_{\text{pure neutron matter}} - E(\rho)_{\text{symmetric nuclear matter}}$$



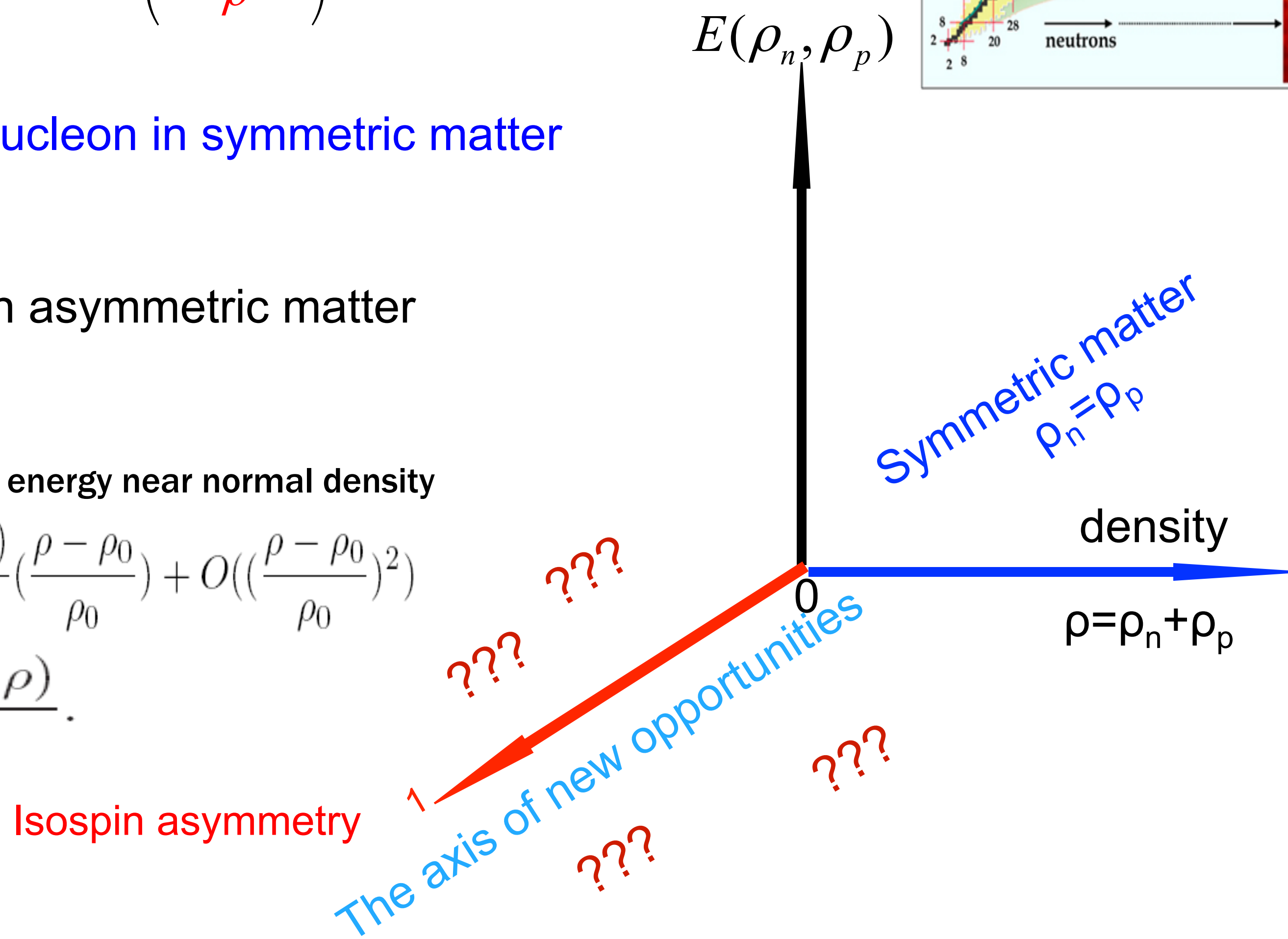
symmetry energy

Isospin asymmetry δ

$$E(\rho_n,\rho_p) = E_0(\rho_n=\rho_p) + E_{sym}(\rho)\left(\frac{\rho_n-\rho_p}{\rho}\right)^2 + o(\delta^4)$$

Energy per nucleon in symmetric matter

Energy per nucleon in asymmetric matter



Characterization of symmetry energy near normal density

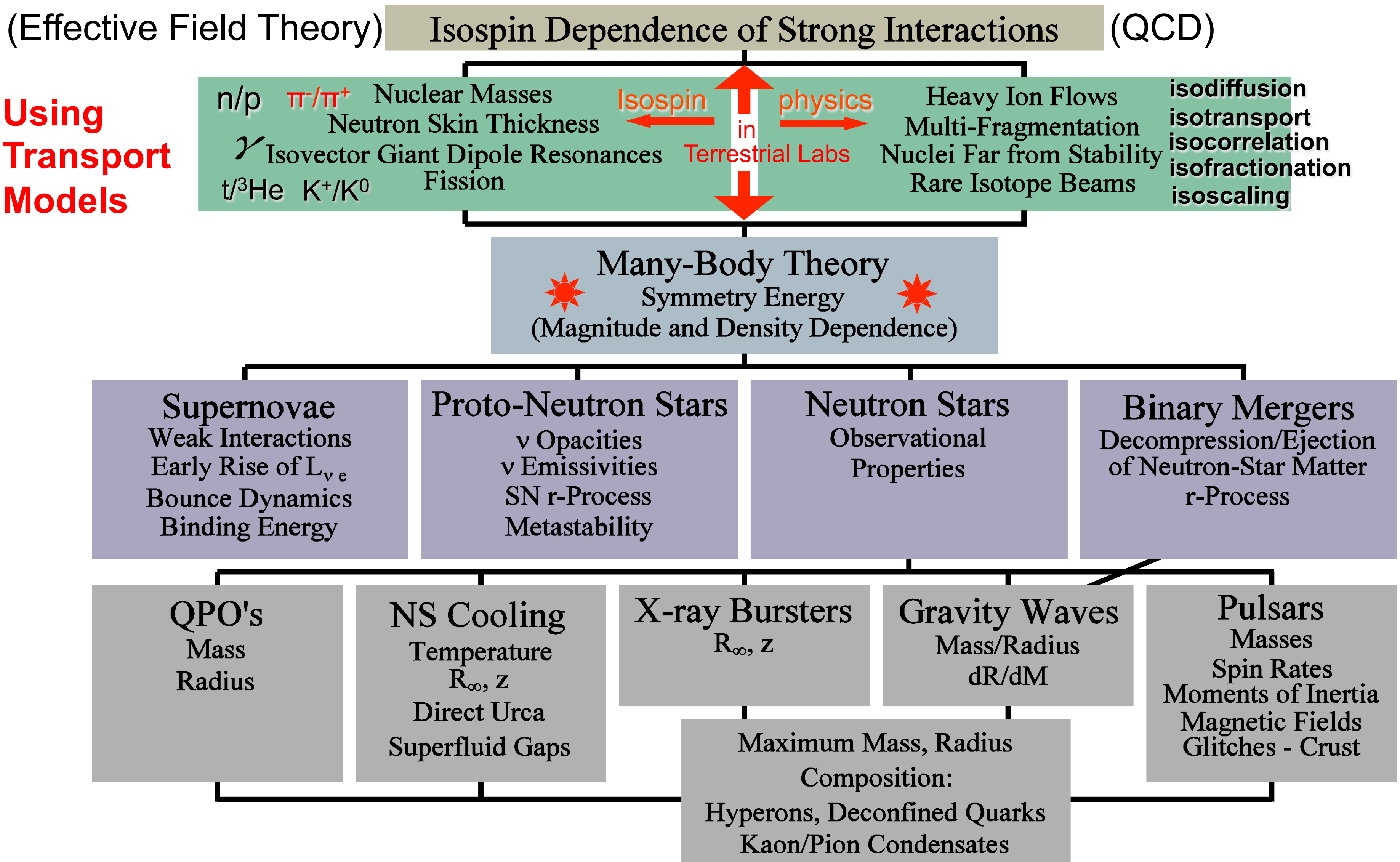
$$E_{sym}(\rho) = E_{sym}(\rho_0) + \frac{L(\rho_0)}{3}\left(\frac{\rho-\rho_0}{\rho_0}\right) + O\left(\left(\frac{\rho-\rho_0}{\rho_0}\right)^2\right)$$

$$L(\rho) = 3\rho\frac{dE_{\text{sym}}(\rho)}{d\rho}.$$

The multifaceted influence of the isospin dependence of strong interaction and symmetry energy in nuclear physics and astrophysics

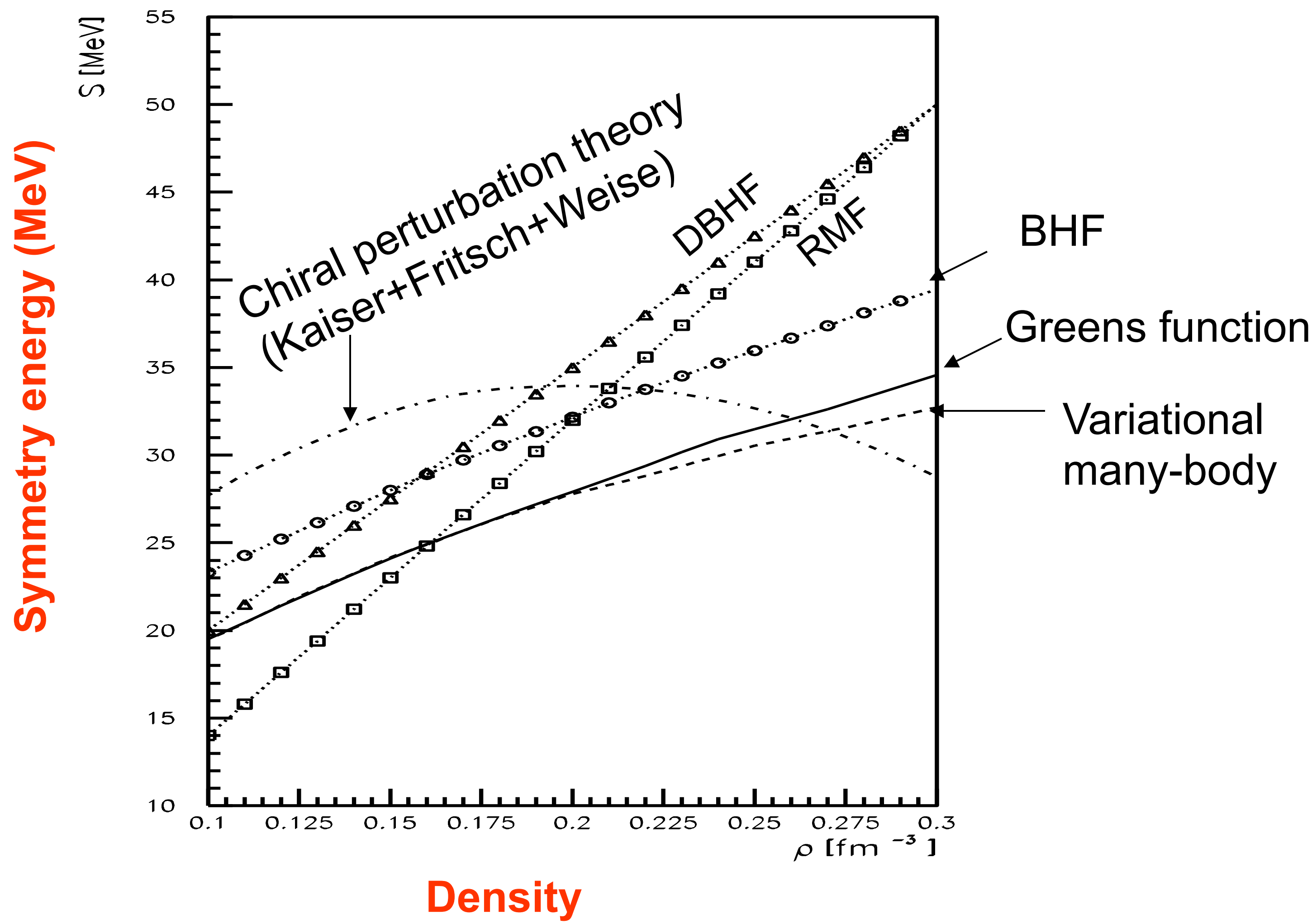
J.M. Lattimer and M. Prakash, *Science Vol. 304 (2004) 536-542.*

A.W. Steiner, M. Prakash, J.M. Lattimer and P.J. Ellis, *Phys. Rep. 411, 325 (2005).*

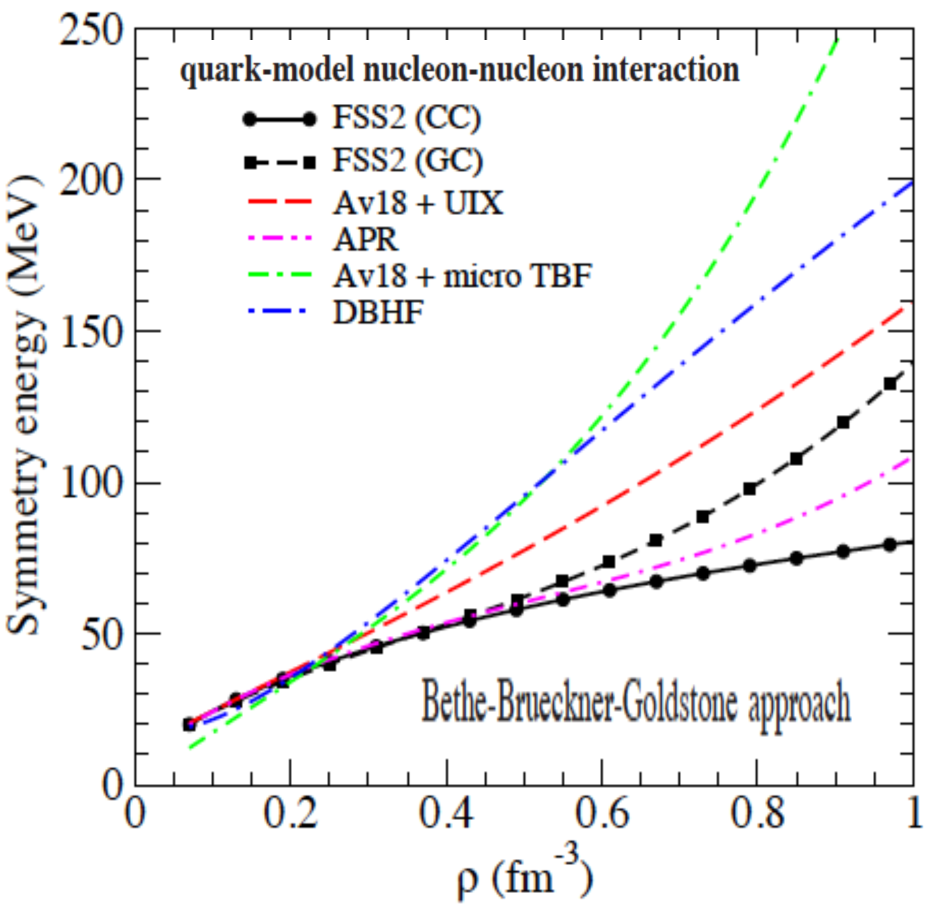


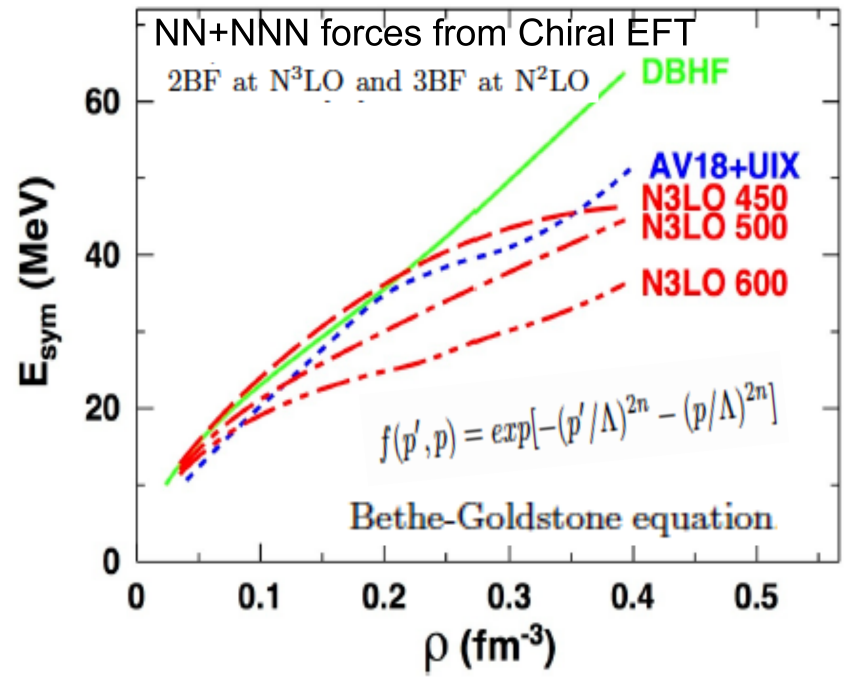
E_{sym} (ρ) predicted by microscopic and/or phenomenological many-body theories —— in the old days

A.E. L. Dieperink et al., Phys. Rev. C68 (2003) 064307



Examples of the latest predictions using microscopic many-body theories





Francesca Sammarruca, Phys. Rev. C 90, 064312 (2014)

arXiv:1507.07288v1

K. Fukukawa,[1,2] M. Baldo,[1] G. F. Burgio,[1] L. Lo Monaco,[3] and H.-J. Schulze[1]

Promising lab Probes of the $E_{sym}(\rho)$ from low to high densities

At sub-saturation densities

- Global nucleon optical potentials from n/p-nucleus and (p,n) reactions
- **Thickness of n-skin in ^{208}Pb measured using various approaches and sizes of n-skins of unstable nuclei from total reaction cross sections**
- **n/p ratio of FAST, pre-equilibrium nucleons**
- **Isospin fractionation and isoscaling in nuclear multifragmentation**
- **Isospin diffusion/transport**
- **Neutron-proton differential flow**
- **Neutron-proton correlation functions at low relative momenta**
- **t/^{3}He ratio and their differential flow**
- **Pygmy dipole resonances**

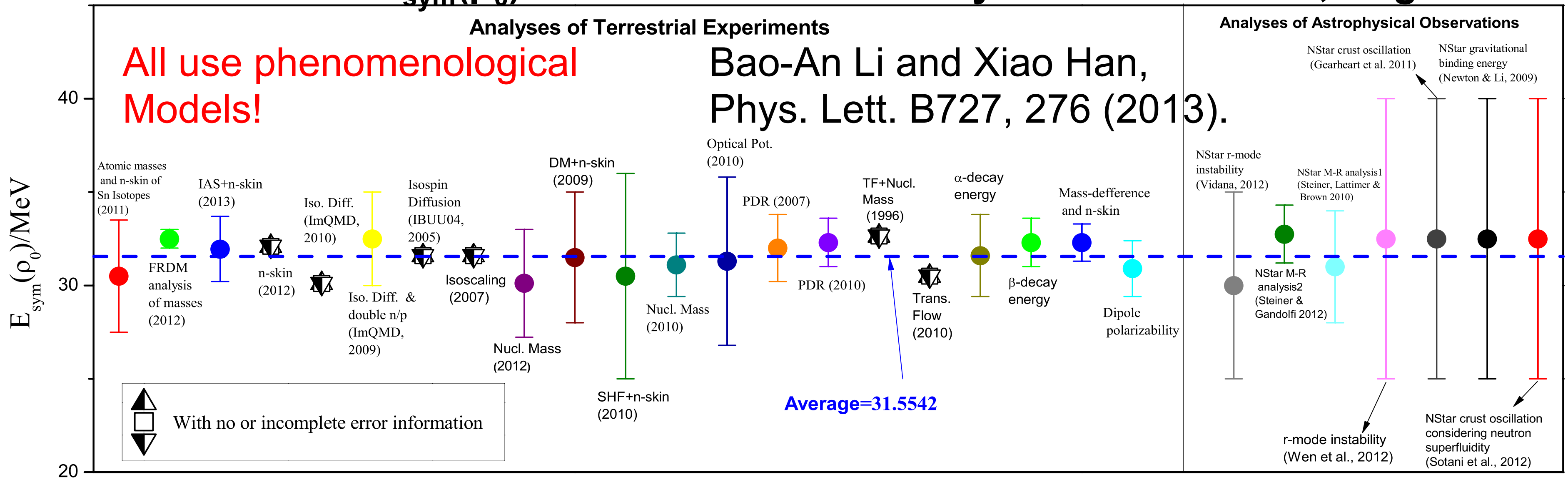
Towards supra-saturation densities

- **π^-/π^+ ratio, K^+/K^0 ?**
- **Neutron-proton differential transverse flow**
- **n/p ratio of squeezed-out nucleons perpendicular to the reaction plane**
- **Nucleon elliptical flow at high transverse momentum**
- **t-^{3}He differential and difference transverse flow**

(1) Correlations of multi-observable are important
(2) Detecting neutrons simultaneously with charged particles is critical

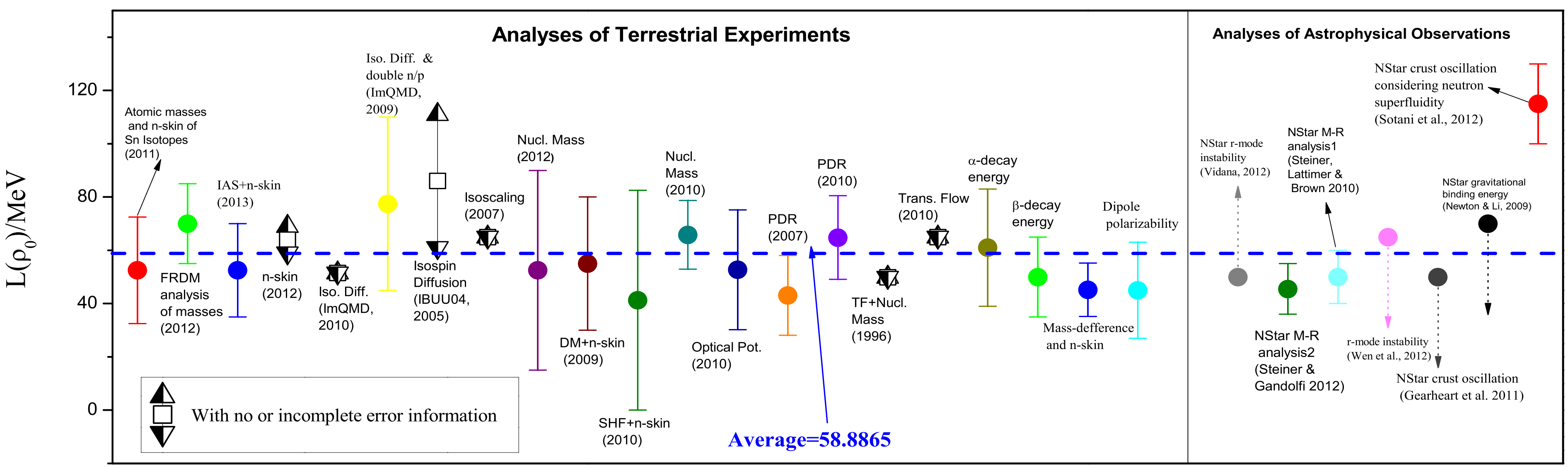
B.A. Li, L.W. Chen and C.M. Ko, ***Physics Reports 464, 113 (2008)***

Constraints on $E_{sym}(\rho_0)$ and L based on 29 analyses of some data, Aug. 2013



	$E_{sym}(\rho_0)$	Slope L
2013 average of the means	**31.55**	**58.89**
2013 average of "error bars"	**2.66**	**16.00**

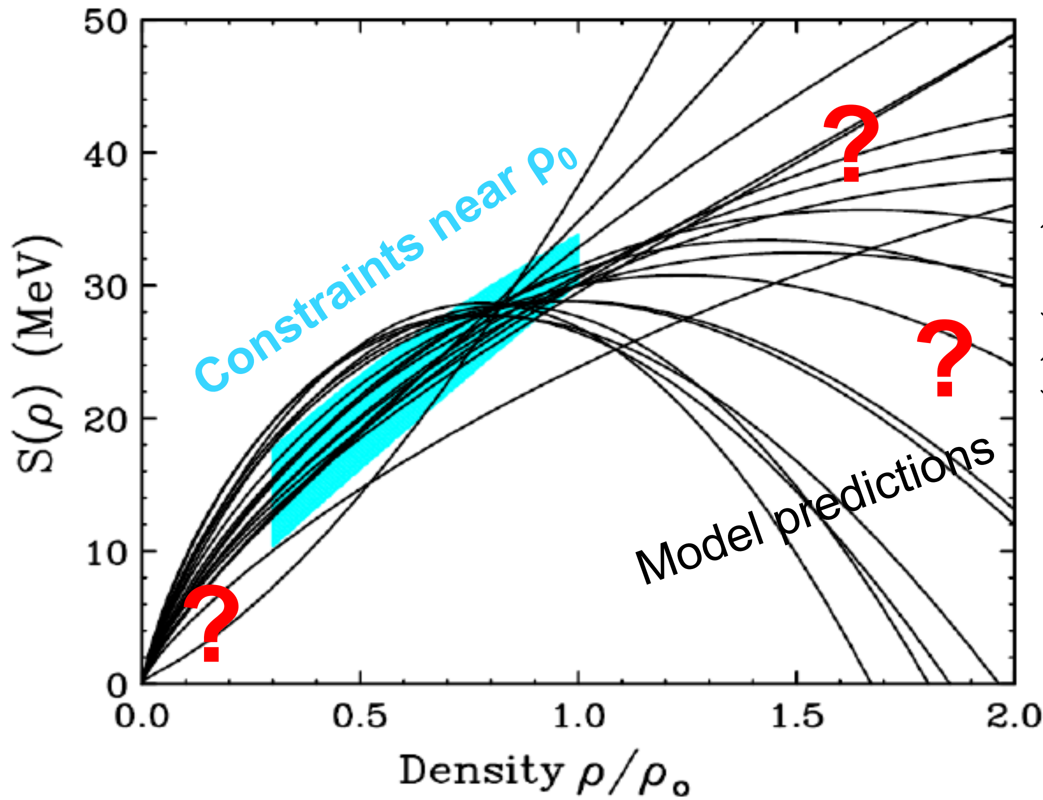
$L \approx 2\, E_{sym}(\rho_0)$

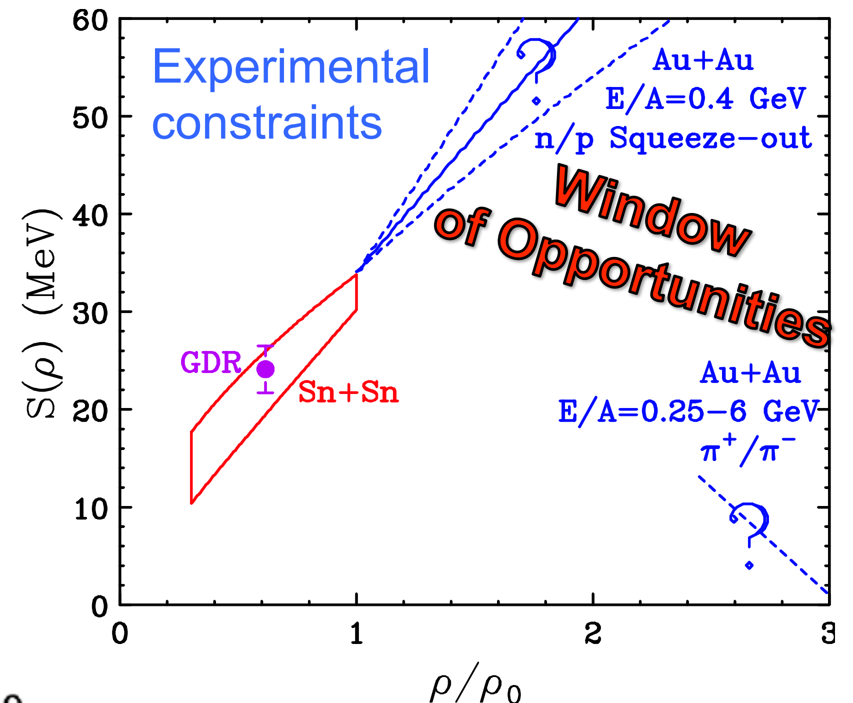


Why is the symmetry energy so uncertain especially at high densities?
Where does the symmetry energy come from?

Among the major challenges:

1) Quantify theoretical uncertainties of E_{sym} around the saturation density
2) Understand effects of spin-isospin dependence of 3BF, tensor force and isospin-dependence of short-range correlations
3) Understand effects of pairing and clustering at low densities
4) Connect observables with isovector interactions at various levels





M. B. Tsang et al., Phys. Rev. C86, 015803 (2012)

Among the promising observables of high-density symmetry energy:

- π^-/π^+, neutron-proton differential flow in heavy-ion collisions
- Radii of neutron stars
- Neutrino flux of supernova explosions
- Strain amplitude and frequency of gravitational waves from spiraling neutron star binaries and/or oscillations/rotations of deformed pulsars

A major scientific motivation of

(1) Rare isotope beam facilities around the world
(2) Neutron Star Interior Composition Explorer (NICER of NASA) and various x-ray satellite (Chandra, LOFT, XMM-Newton, etc)
(3) Various gravitational wave detectors

EPJA, Vol. 50, No. 2 (2014)

Tensor force induced high-momentum tail in single-nucleon momentum distribution and the strong isospin dependence of short-range correlations

Theory of Nuclear matter
H.A. Bethe, ARNPS 21, 93 (1971)

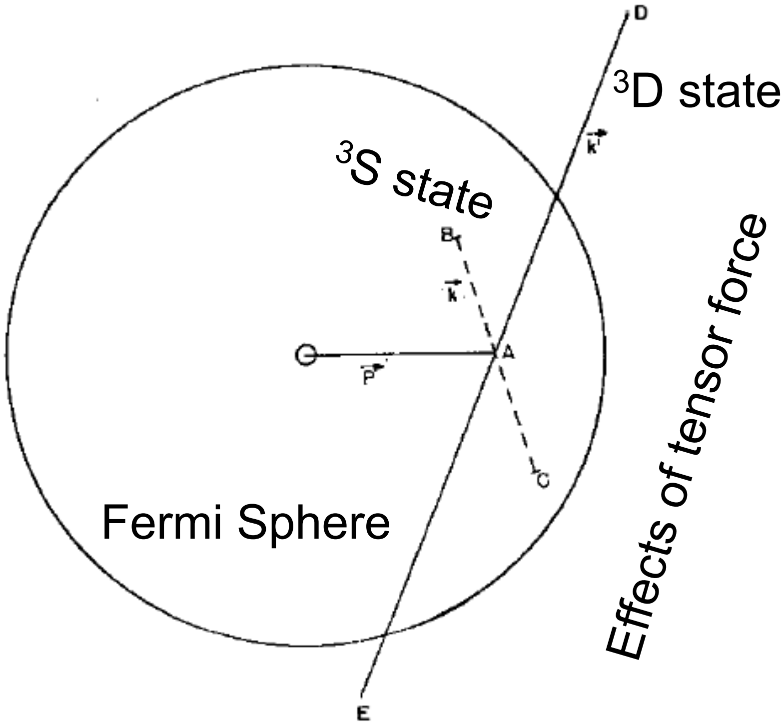
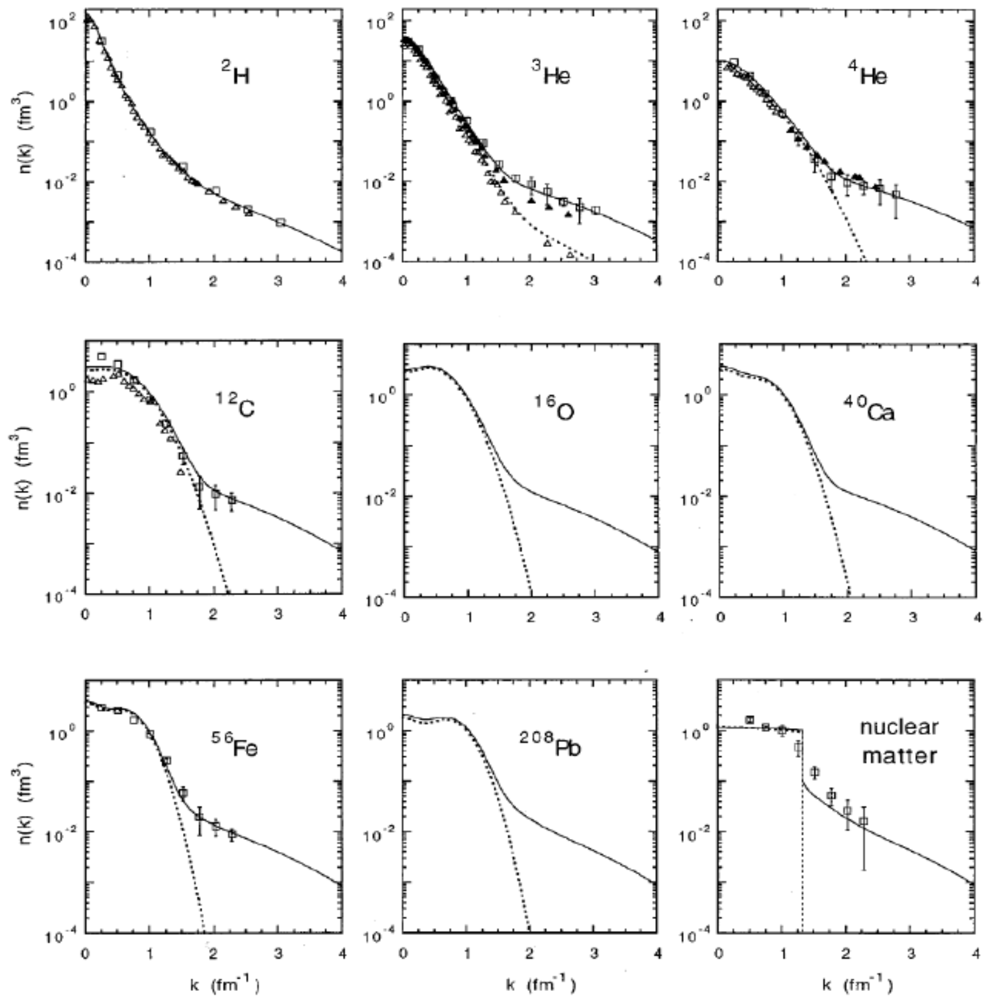


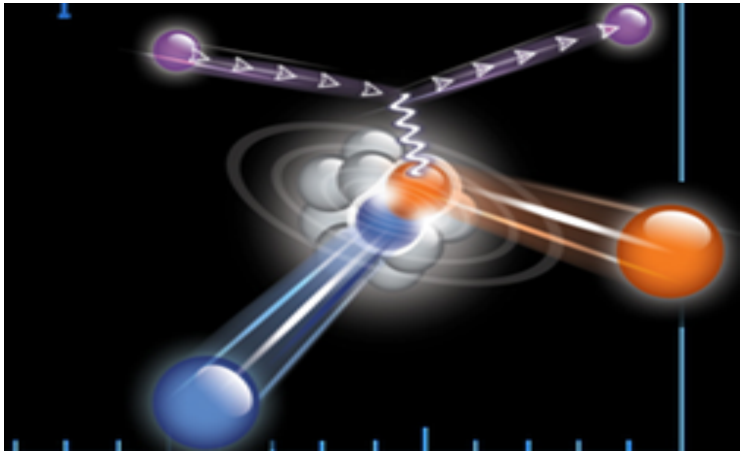
FIGURE 10. Two nucleons are initially in states B and C, having average momentum $\boldsymbol{P}$ and relative momentum $\boldsymbol{k}$. When they interact they are shifted to states D and E outside the Fermi sphere, with relative momentum $\boldsymbol{k}'$. If they are initially in a ^3S state and interact by tensor force, then they are in a $^3D_1$ state in DE.



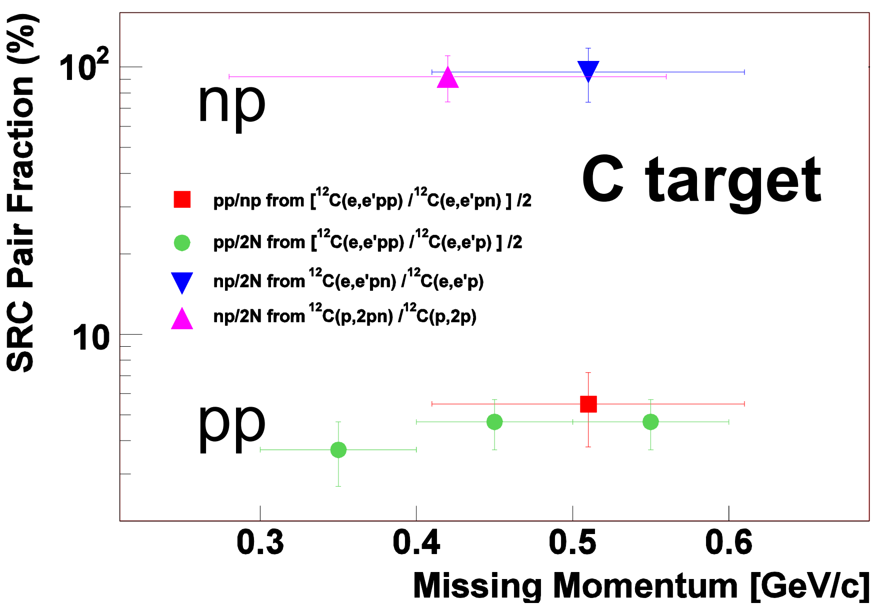
Latest review:
Claudio Ciofi degli Atti,
Physics Reports 590, 1 (2015)

S. Fantoni and V. R. Pandharipande, Nucl. Phys. A **427**, 473 (1984).
C. Ciofi degli Atti and S. Simula, Phys. Rev. C **53**, 1689 (1996).

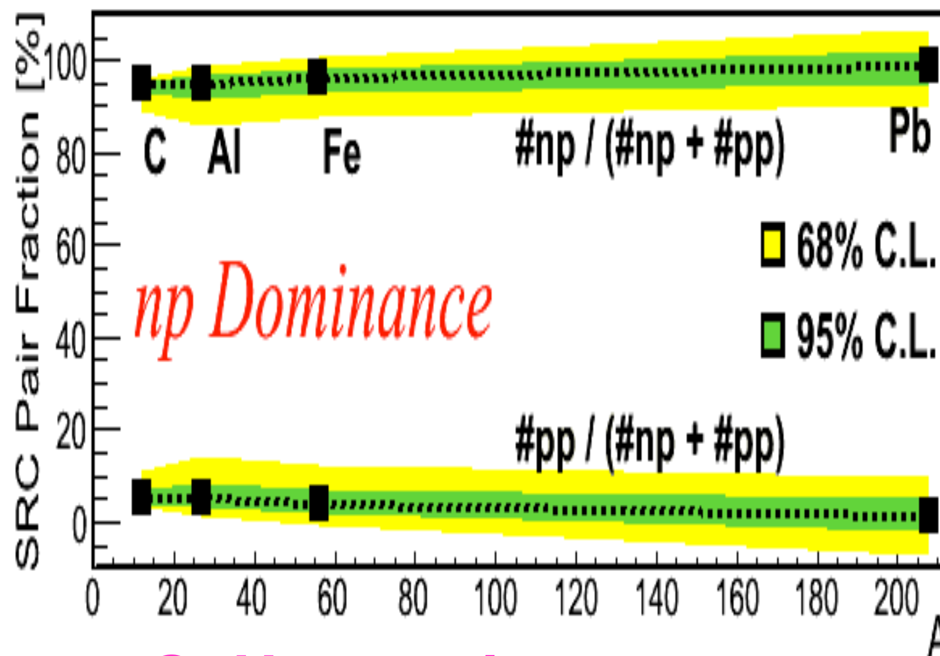
Experimental findings on the isospin-dependence of high-momentum tails



1. About 25% nucleons in the high-momentum tail in symmetric nuclear matter
2. pp/np≈1/18 from light to heavy nuclei indicates 1-2% high momentum tail in pure neutron matter
3. The high momentum tail goes like $1/k^4$



R. Subedi et al., Science 320, 1476 (2008)



O. Hen et al., Science 346, 614 (2014)

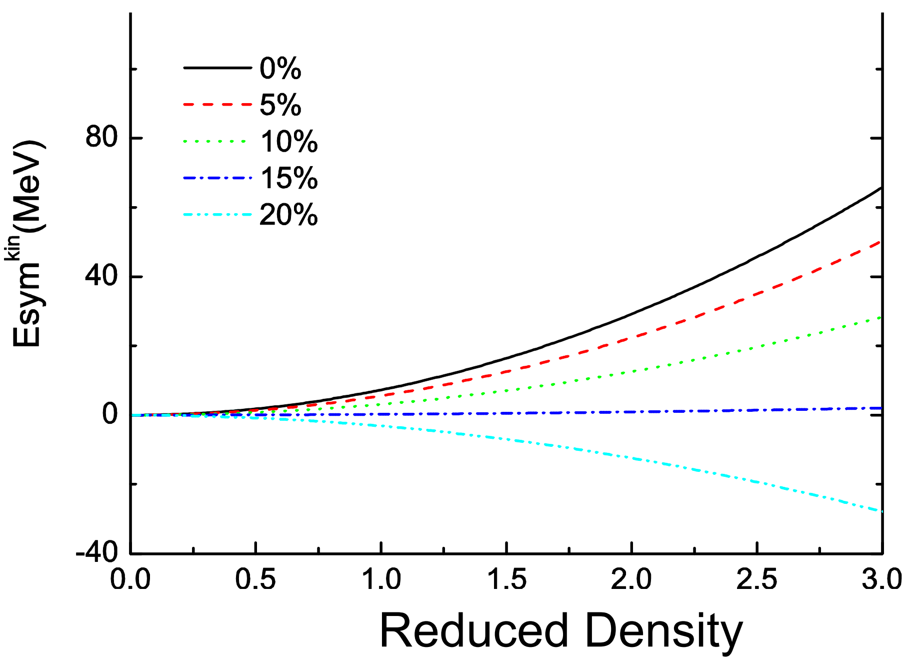
Effects of isospin-dependent SRC on the kinetic symmetry energy of quasi-nucleons

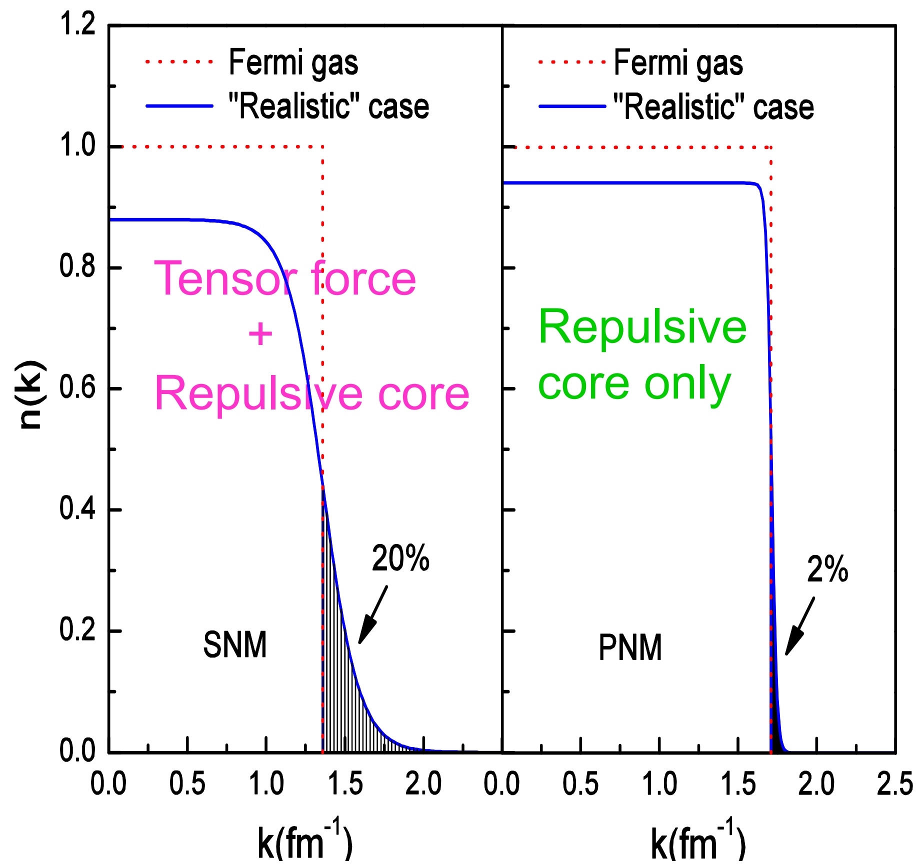
Chang Xu and Bao-An Li, arXiv:1104.2075
Chang Xu, Ang Li and Bao-An Li,
JPCS 420, 012190 (2013).

$$E_{kin} = \alpha \int_0^{\infty} \frac{\hbar^2 k^2}{2m} n(k) k^2 dk,$$

While the Fermi momentum for PNM is higher than that for SNM at the same density in the mean-field models, if more than 15% nucleons are in the high-momentum tail of SNM due to the tensor force for n-p T=0 channel, the kinetic symmetry energy becomes negative

$$E_{sym}^{kin} = E_{PNM}^{kin} - E_{SNM}^{kin} < 0$$





Confirmation by Microscopic Many-Body Theories

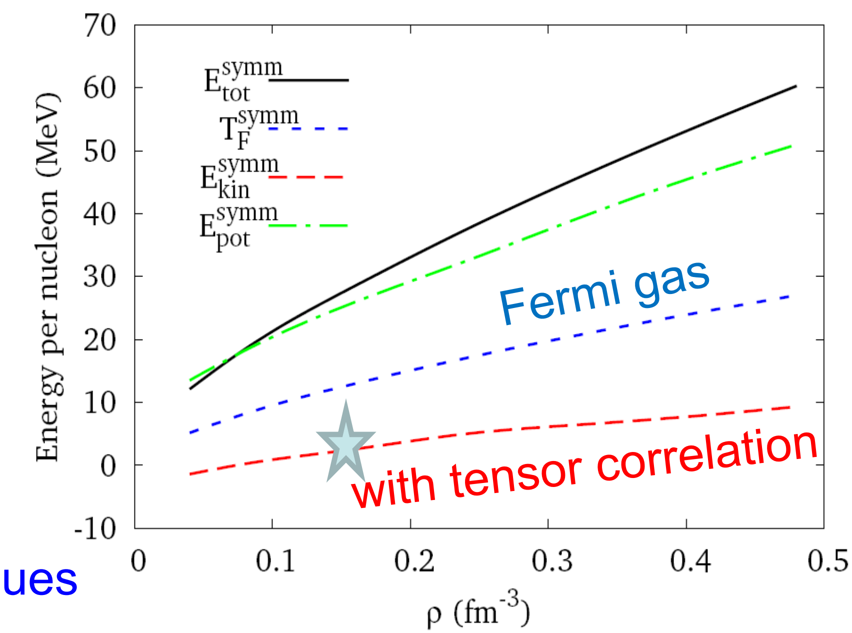
1. Isaac Vidana, Artur Polls, Constanca Providencia
PRC84, 062801(R) (2011)

Brueckner--Hartree--Fock approach using the Argonne V18 potential plus the Urbana IX three-body force

2. Arianna Carbone, Artur Polls, Arnau Rios, EPL 97, 22001 (2012)
A. Carbone, A. Polls, C. Providência, A. Rios, I. Vidaña, EPJA 50, 13 (2014)
Self-Consistent Green's Function Approach with Argonne Av18, CDBonn, Nij1, N3LO interactions

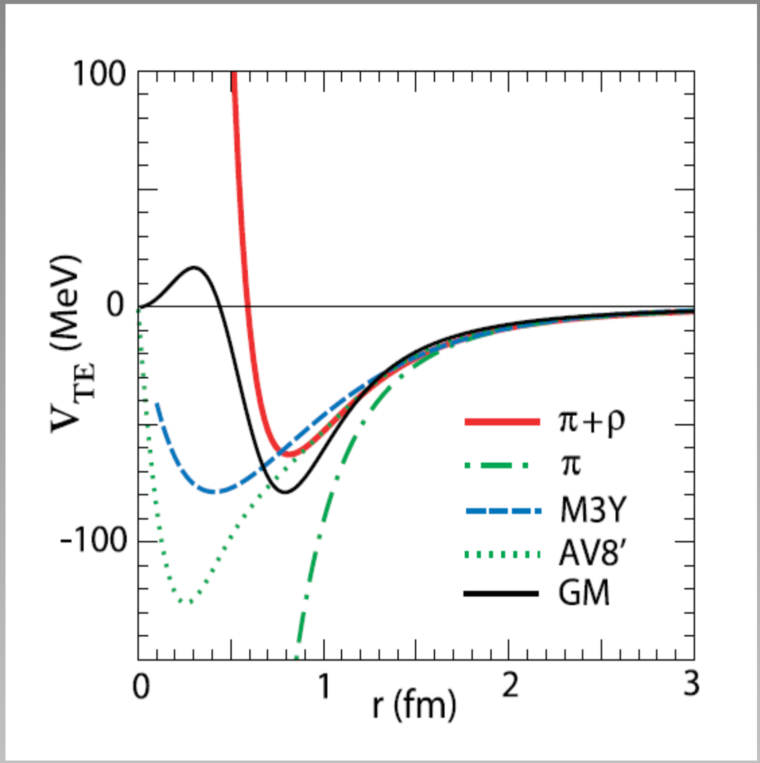
3. Alessandro Lovato, Omar Benhar et al.,
extracted from results already published in
Phys. Rev. C83:054003,2011
Using Argonne V'_6 interaction
Fermi-Hyper-Netted-Chain (FHNC)
Single Operator Chains (SOC)

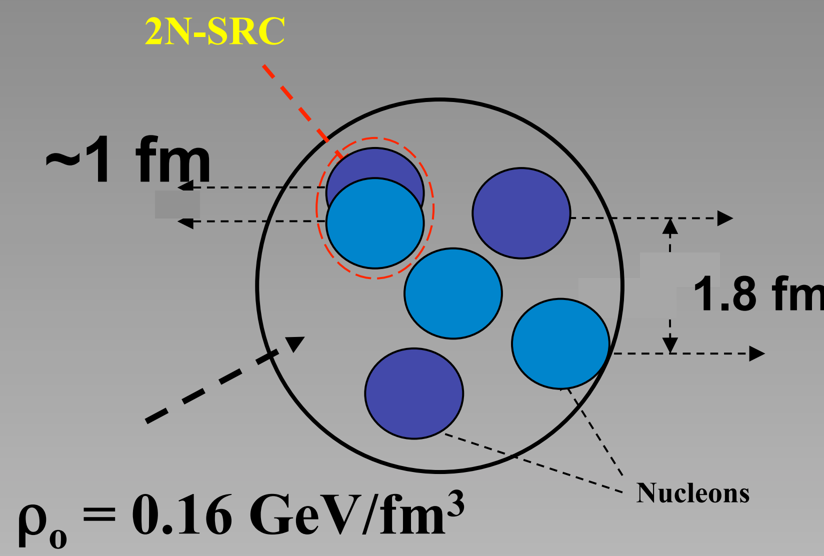
4. A. Rios, A. Polls, W. H. Dickhoff
PRC 89, 044303 (2014).
Ladder Self-Consistent Green Function



They all included the tensor force and many-body correlations using different techniques

Tensor force is very uncertain where the SRC is important!





T. Otsuka et al.,
PRL 95, 232502 (2005)

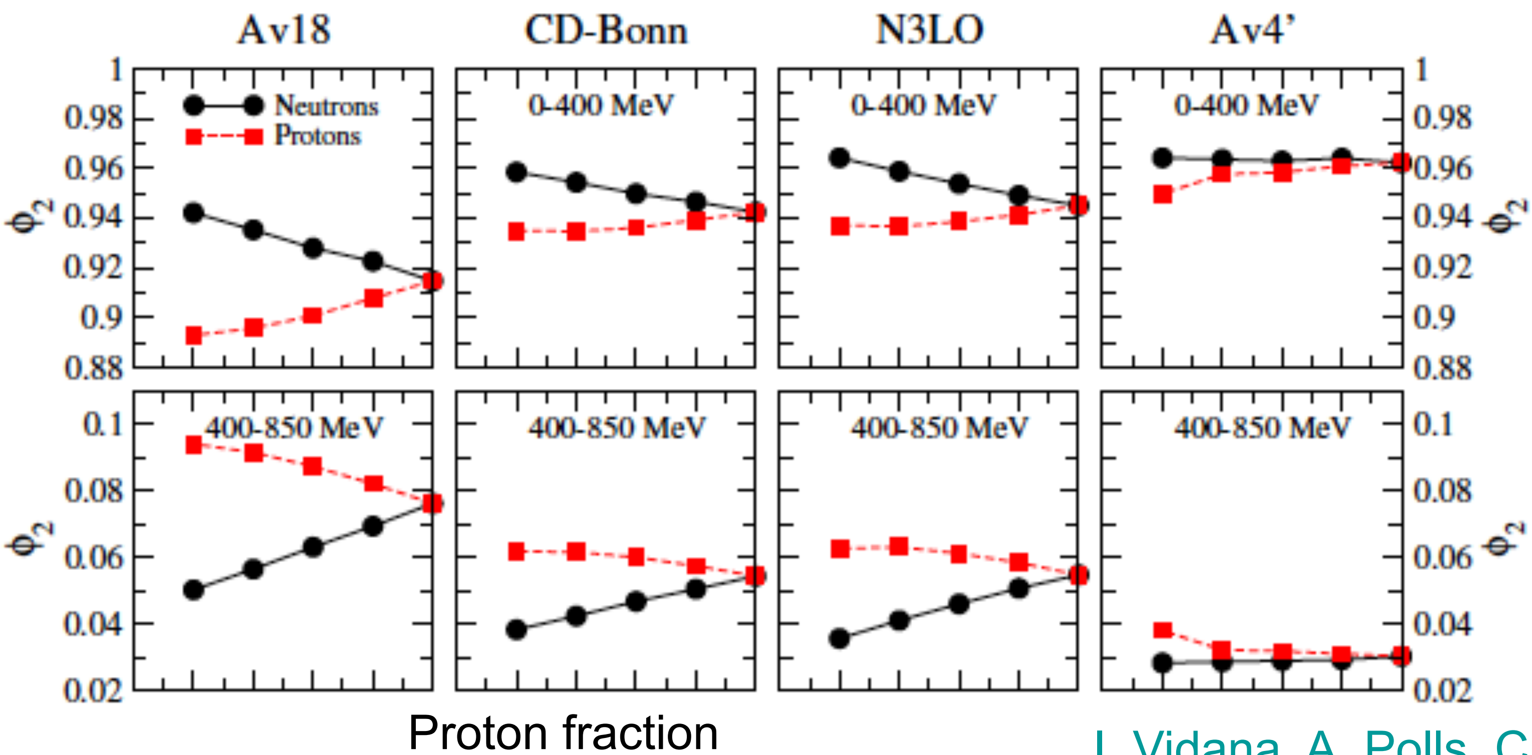
A pair of nucleons with large relative momentum but small CM momentum.

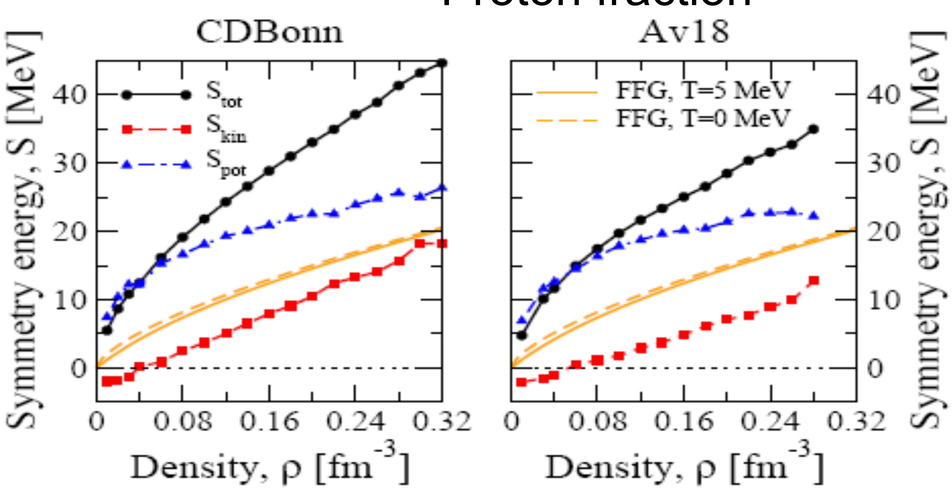
Eli Piasetzky

A. Rios, A. Polls and W. Dickhoff, PRC89, 044303 (2014),

Self-consistent Greens Function approach

$$\phi_2(k_i,k_f)=\frac{1}{\pi^2\rho_\tau}\int_{k_i}^{k_f}\mathrm{d}k\,k^2 n_\tau(k)\,,$$





I. Vidana, A. Polls, C. Providencia

PRC84, 062801(R) (2011)

V18 potential plus the Urbana IX three-body force.

BHF	E_{NM}	E_{SM}	E_{sym}	L
$\langle T\rangle$	53.321	54.294	−0.973	14.896
$\langle V\rangle$	−34.251	−69.524	35.273	51.604
Total	19.070	−15.230	34.300	66.500

The kinetic symmetry energy from Free Fermi Gas model is widely used !

(1) In extracting the symmetry energy from astrophysical observations:

in Equation 4. The symmetry energy is parameterized as

$$S_2(u) = S_k u^{2/3} + S_p u^\gamma . \qquad u=\rho/\rho_0 \qquad (24)$$

The kinetic part of the symmetry energy, $S_k \simeq 13$ MeV, was held fixed, but the compressibility and skewness coefficients, K_o and K'_o, and the terms describing the potential part of the symmetry energy, S_p and γ, were taken as parameters.

(2) Transport models simulate the motions of quasi-particles, how the total symmetry energy is split up affects the isospin dynamics and all isovector observables in heavy-ion collisions.

$$\mathrm{S}(\rho) = \frac{C_{s,k}}{2}\left(\frac{\rho}{\rho_0}\right)^{2/3} + \frac{C_{s,p}}{2}\left(\frac{\rho}{\rho_0}\right)^{\gamma_i}$$

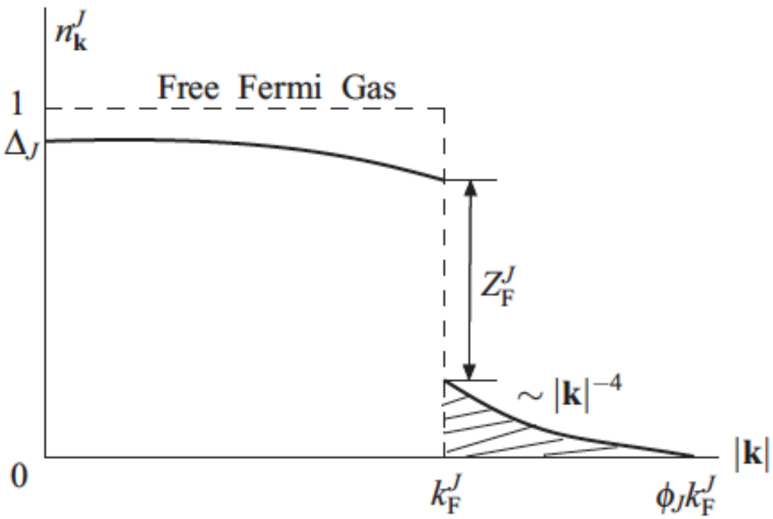
(3) In nuclear energy density functional theories,
e.g., Realtivistic Mean-Field Models, Gogny or Skyrme Hartree-Fock, etc,

NO high-momentum tail is considered in the kinetic energy density

$$\mathcal{E}^{\mathrm{kin}}(\rho_n, \rho_p) = \frac{3\hbar^2(3\pi^2)^{2/3}}{10m}\left(\rho_n^{5/3} + \rho_p^{5/3}\right)$$

Phenomenological nucleon momentum distribution n(k) including SRC effects guided by microscopic theories and experimental findings

O. Hen et al., Science 346, 614 (2015).
O. Hen, B.A. Li, W.J. Guo, L.B. Weinstein, and E. Piasetzky, PRC 91, 025803 (2015).
B.J. Cai and B.A. Li, PRC92, 011601(R) (2015).



$$n^J_{\mathbf{k}}(\rho,\delta) = \begin{cases} \Delta_J + \beta_J I\left(|\mathbf{k}|/k^J_F\right), & 0 < |\mathbf{k}| < k^J_F, \\ C_J\left(k^J_F/|\mathbf{k}|\right)^4, & k^J_F < |\mathbf{k}| < \phi_J k^J_F. \end{cases}$$

All parameters are assumed to have a linear dependence on isospin asymmetry as indicated by SCGF and BHF calculations

$$Y_J = Y_0(1 + Y^J_1 \delta)$$

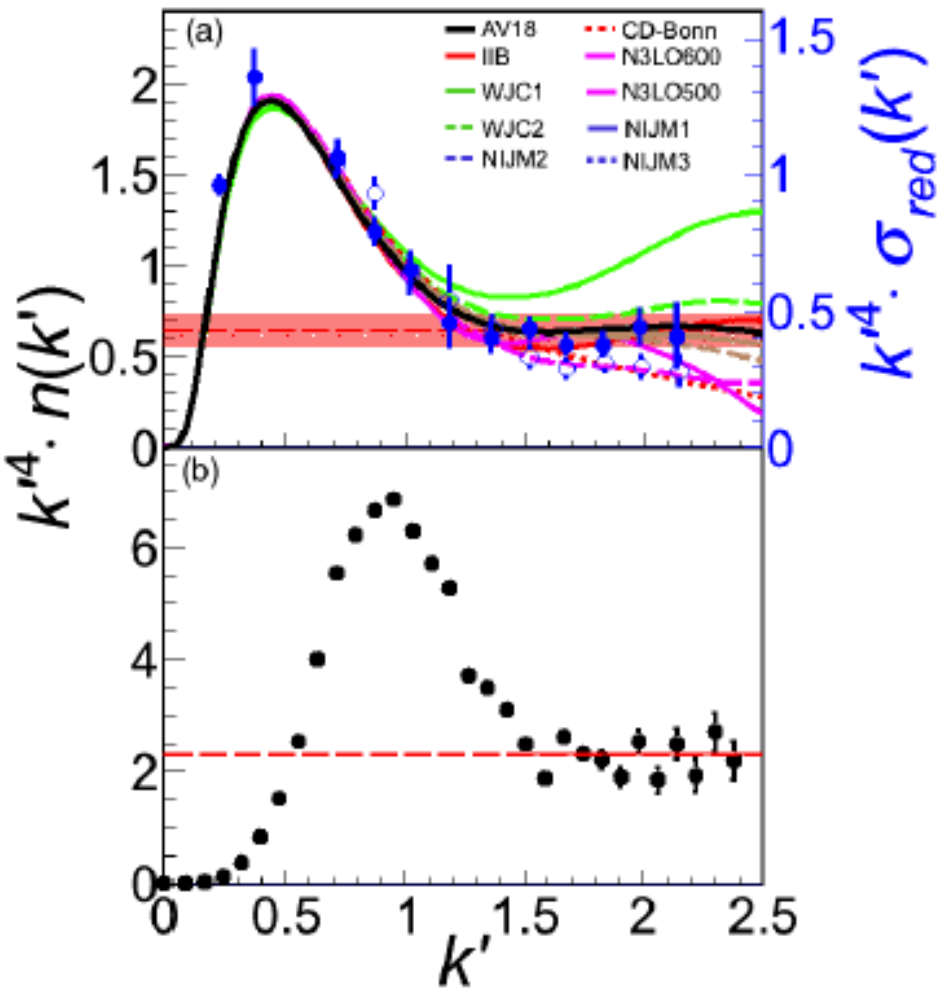
Fraction of high-k nucleons

$$x^{\mathrm{HMT}}_J = 3C_J\left(1 - \frac{1}{\phi_J}\right)$$

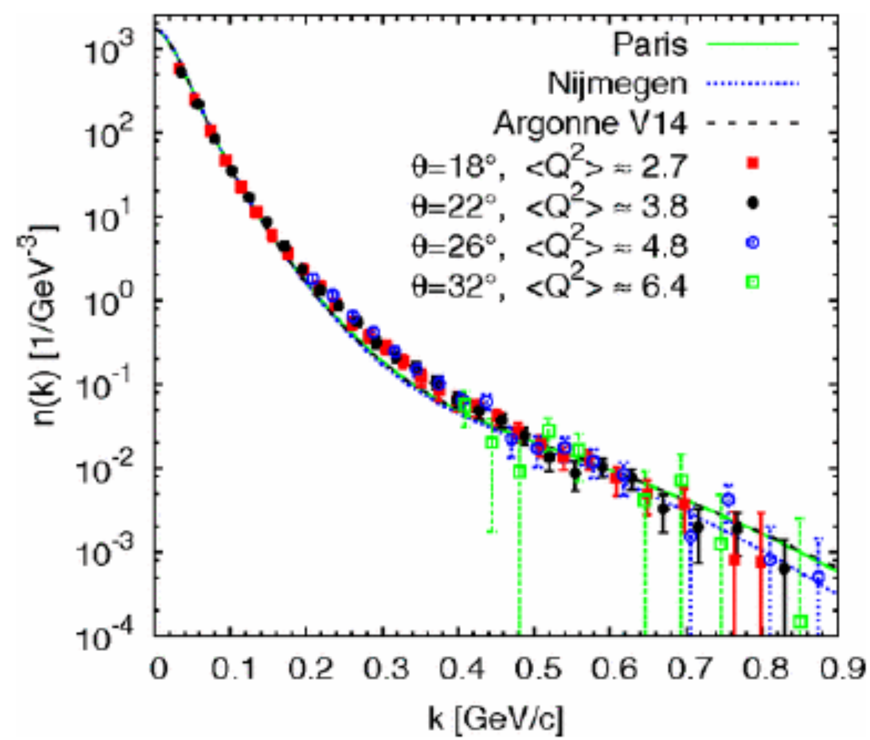
All parameters are fixed by
(1) Jlab data: HMT in SNM=25%, 1.5% in PNM,
(2) Contact C for SNM from deuteron wavefunction
(3) Contact C in PNM from microscopic theories

The high-momentum tail in deuteron scales as $1/K^4$

O. Hen, L. B. Weinstein, E. Piasetzky, G. A. Miller, M. M. Sargsian and Y. Sagi, PRC (2015) in press.



Stewart et al. PRL **104**, 235301 (2010)
Kuhnle et al. PRL **105**, 070402 (2010)

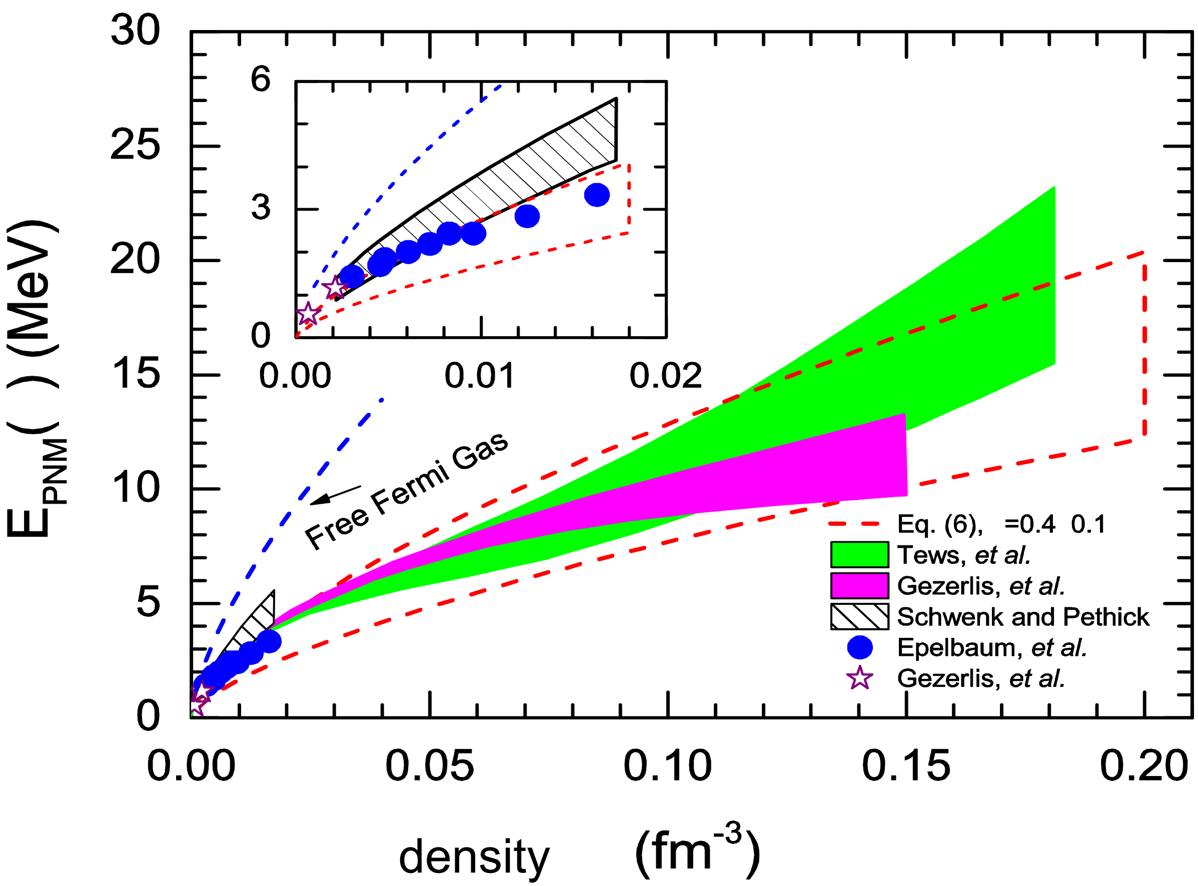


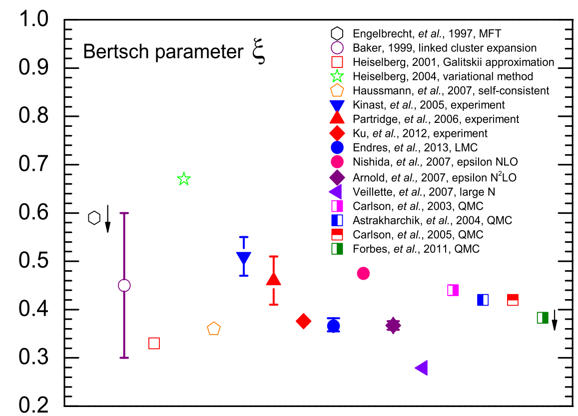
EOS and contact C of pure neutron matter

$$E_{\mathrm{PNM}}(\rho) \simeq \frac{3}{5}\frac{(k_{\mathrm{F}}^{\mathrm{PNM}})^2}{2M}\left[\xi - \frac{\zeta}{k_{\mathrm{F}}^{\mathrm{PNM}}a_{\mathrm{nn}}} - \frac{5\nu}{3(k_{\mathrm{F}}^{\mathrm{PNM}}a_{\mathrm{nn}})^2}\right]$$

$$C_{\mathrm{n}}^{\mathrm{PNM}} \approx 2\zeta/5\pi + 4\nu/(3\pi k_{\mathrm{F}}^{\mathrm{PNM}} a_{\mathrm{nn}}(^1\mathrm{S}_0)) \approx 0.12.$$

$$a_{\mathrm{nn}}(^1\mathrm{S}_0) = -18.8 \pm 0.3\ \mathrm{fm},$$





B.J. Cai and B.A. Li, PRC92, 011601(R) (2015).

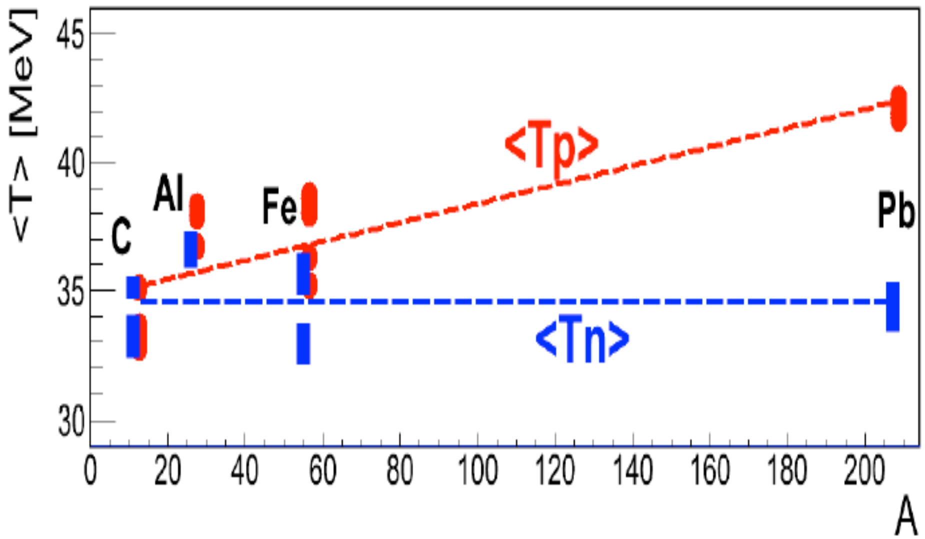
The contact C of PNM is derived from its EOS using the adiabatic sweep theorem

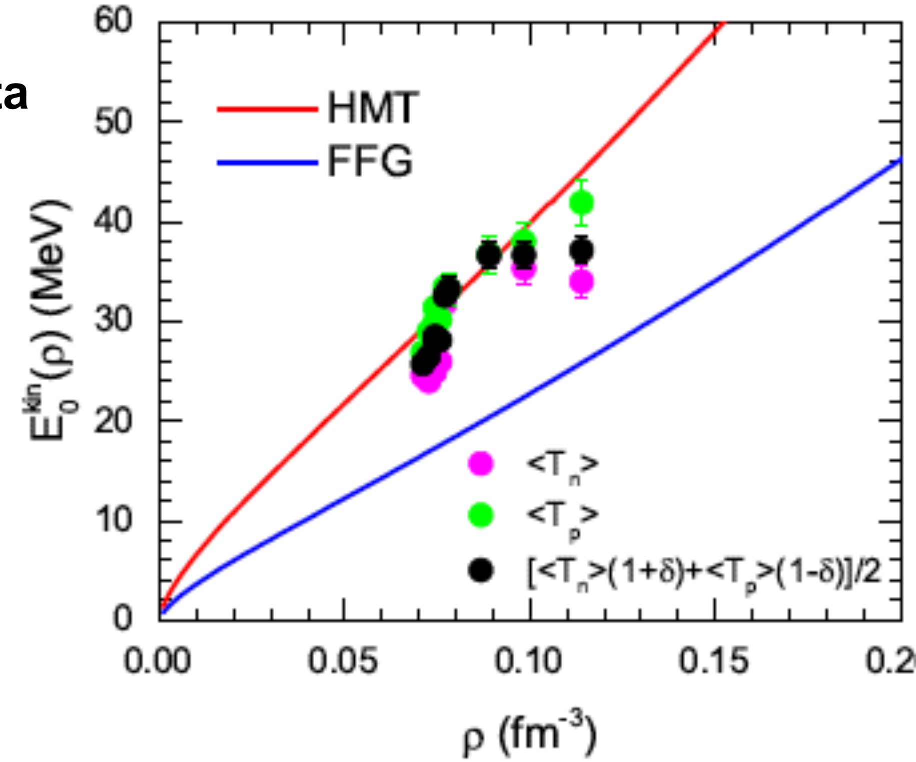
$$\frac{\hbar^2\Omega C}{4\pi m} = \frac{\mathrm{d}E}{\mathrm{d}(-1/a)},$$

S. Tan, Annals of Physics 323 (2008) 2971-2986

Testing the model parameters in the n(k)

Average kinetic energy extracted from data mining at Jlab using the proton-neutron dominance model





O. Hen et al., Science 346, 614 (2014)

Bao-Jun Cai and Bao-An LI arXiv:1509.09290

Empirical relation, average density in finite nuclei:

$$\rho_A \simeq \frac{\rho_0}{1+\alpha/A^{1/3}}$$

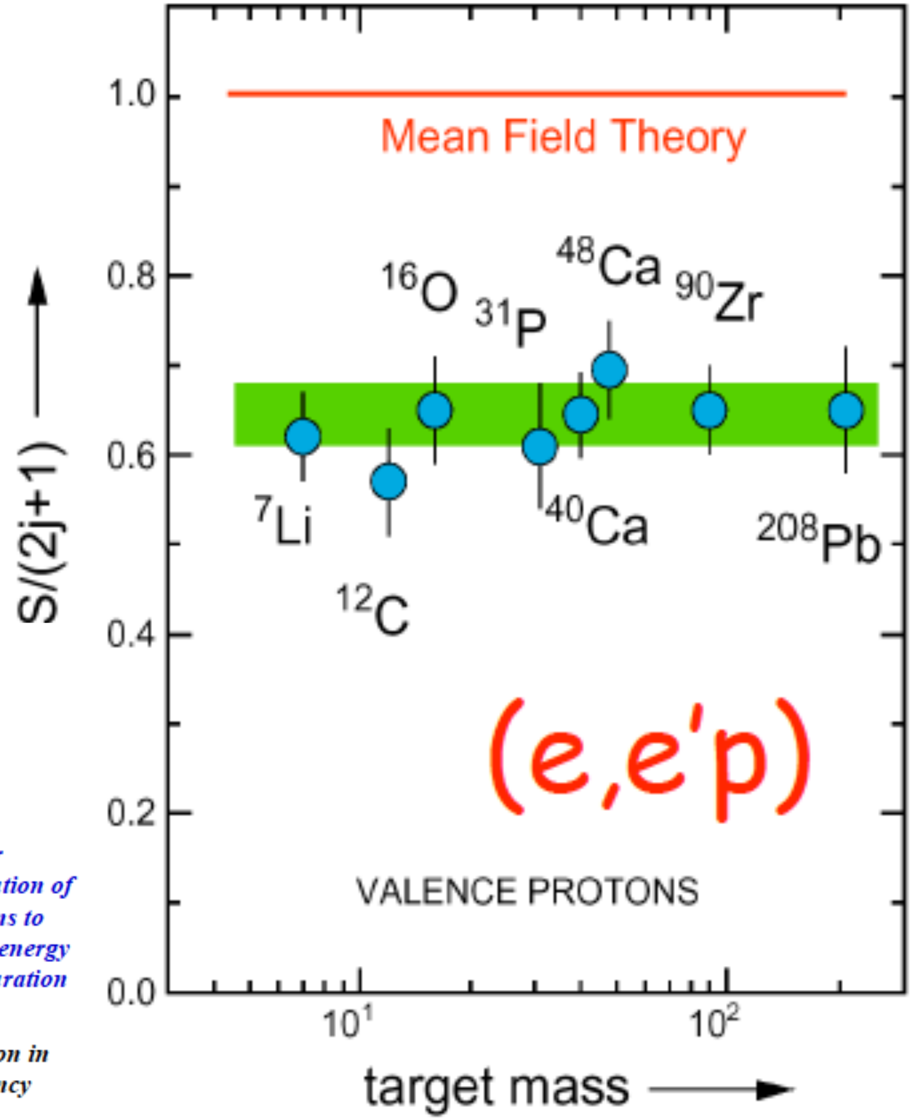
Ratio of volume/surface symmetry E

The FFG model underpredicts the average kinetic data by about 30-40% due to the same reason that mean-field models overshoot the removal probability of valence protons (spectroscopic factors)

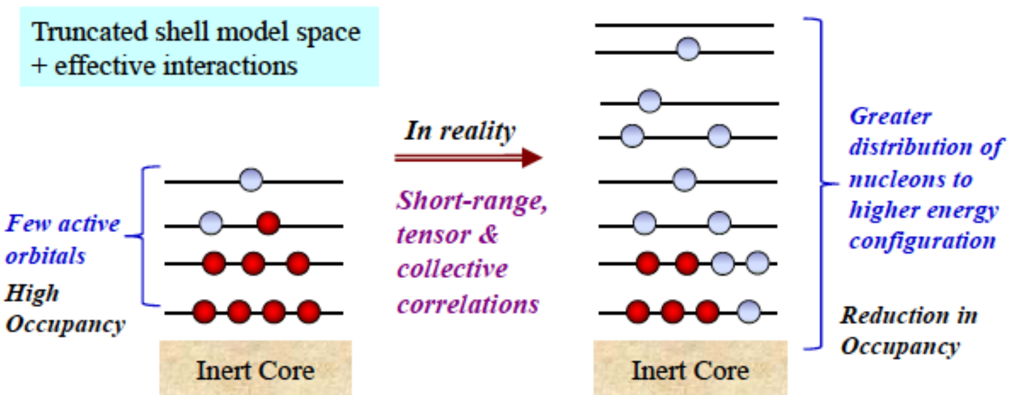
Removal probability for valence protons from NIKHEF data

L. Lapikás, Nucl. Phys. A553,297c (1993)

$S \approx 0.65$ for valence protons
Reduction $\Rightarrow$ both SRC and LRC



Nucleon Correlations

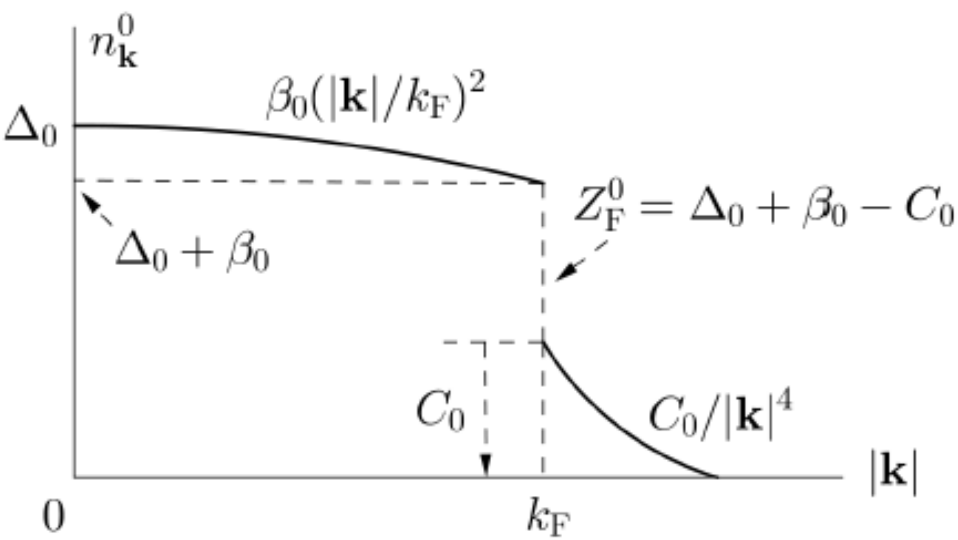


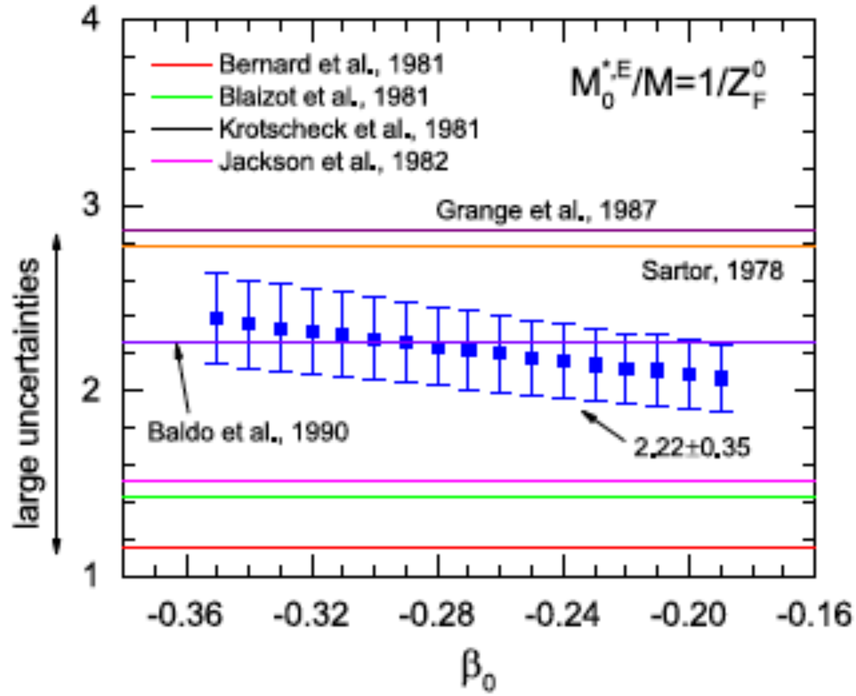
Taken from W. Dickhoff and Jenny Lee

Constraining the Migdal jump (occupation renormalization function)

$$Z_F^J = n^J_{k_F^J - 0} - n^J_{k_F^J + 0} = M/M_E^{J,*}$$

In symmetric nuclear matter



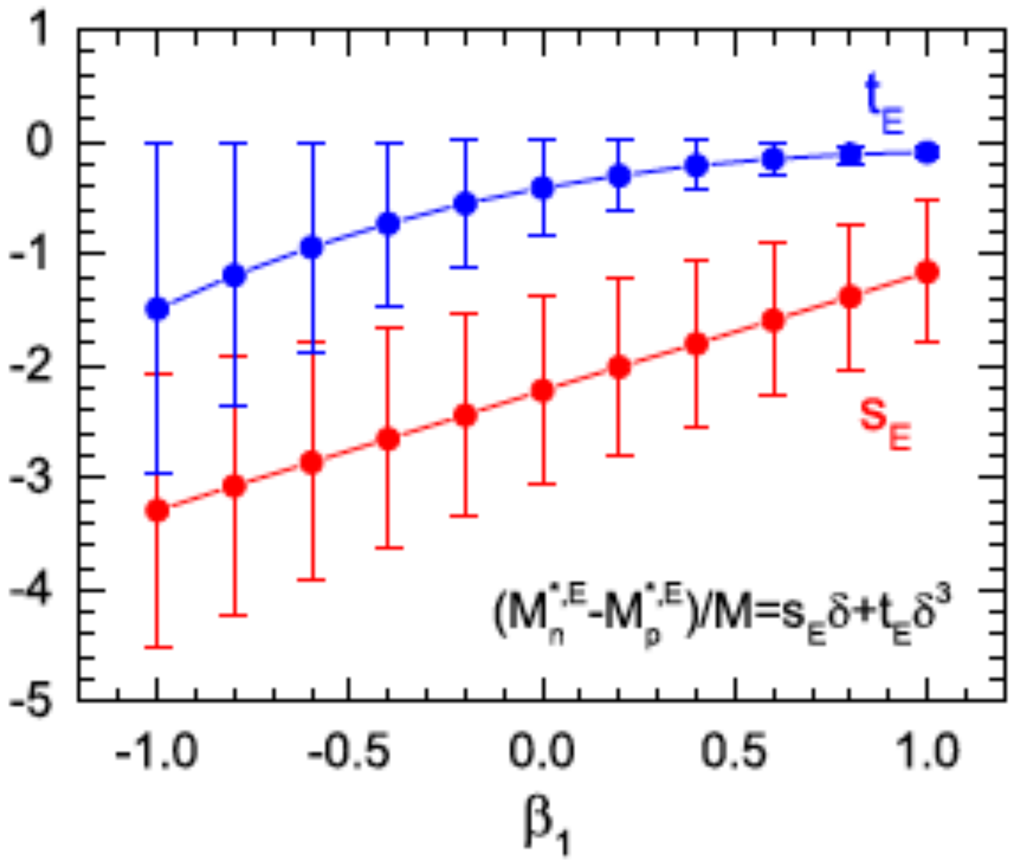


$$\beta_0 = (5/3)[1 - \Delta_0 - 3C_0(1 - \phi_0^{-1})]$$

$$-0.27 \pm 0.08.$$

Bao-Jun Cai and Bao-An Li (2015).

Constraining the neutron-proton effective E-mass splitting



Need more isospin-dependent information from experiments!

$$\beta_J = \beta_0(1 + \beta_1 \tau_3^J \delta)$$

$$n_{\mathbf{k}}^J(\rho,\delta) = \begin{cases} \Delta_J + \beta_J I\left(|\mathbf{k}|/k_F^J\right), & 0 < |\mathbf{k}| < k_F^J, \\ C_J\left(k_F^J/|\mathbf{k}|\right)^4, & k_F^J < |\mathbf{k}| < \phi_J k_F^J. \end{cases}$$

Incorporating the high-momentum tail in energy density functionals,

$$\int_0^{k_{\mathrm{F}}^J} f \mathrm{dk}\,(\mathrm{FFG}) \longrightarrow \int_0^{\phi_J k_{\mathrm{F}}^J} n_{\mathbf{k}}^J f \mathrm{dk}\,(\mathrm{HMT}),$$

e.g, nonlinear Relativistic Mean-Field Model

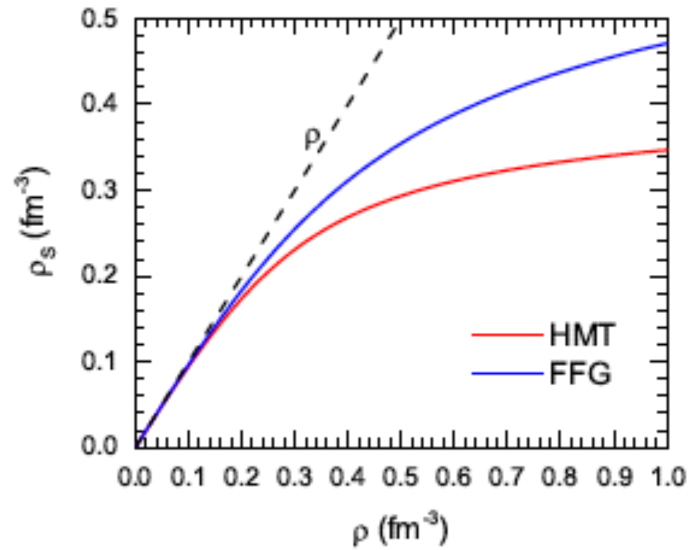
$$E_{\mathrm{sym}}^{\mathrm{kin}}(\mathrm{FFG}) \; = \; k_{\mathrm{F}}^2/6E_{\mathrm{F}}^* \; ; \qquad E_{\mathrm{sym}}^{\mathrm{pot}}(\rho) = \frac{g_\rho^2 \rho}{2Q_\rho} \text{ with } Q_\rho = m_\rho^2 + \Lambda_{\mathrm{V}} g_\rho^2 g_\omega^2 \overline{\omega}_0^2.$$

$$E_{\mathrm{F}}^* = (M_0^{*,2} + k_{\mathrm{F}}^2)^{1/2}$$

Scalar density:

$$\rho_{\mathrm{S},J} = \frac{2}{(2\pi)^3} \int_0^{\phi_J k_{\mathrm{F}}^J} n_{\mathbf{k}}^J \mathrm{dk} \frac{M_J^*}{\sqrt{|\mathrm{k}|^2 + M_J^{*2}}}$$

Affecting the Dirac mass through the sigma field



Energy density:

$$\begin{aligned} \varepsilon =& \langle \mathcal{T}^{00} \rangle \\ =& \varepsilon_{\mathrm{n}}^{\mathrm{kin}} + \varepsilon_{\mathrm{p}}^{\mathrm{kin}} + \frac{1}{2}\left[m_\sigma^2 \overline{\sigma}^2 + m_\omega^2 \overline{\omega}_0^2 + m_\rho^2 \left(\overline{\rho}_0^{(3)} \right)^2 \right] \\ &+ \frac{1}{3} b_\sigma (g_\sigma \overline{\sigma})^3 + \frac{1}{4} c_\sigma (g_\sigma \overline{\sigma})^4 + \frac{3}{4} c_\omega (g_\omega \overline{\omega}_0)^4 \\ &+ \frac{3}{2} \left(g_\rho \overline{\rho}_0^{(3)} \right)^2 \Lambda_{\mathrm{V}} (g_\omega \overline{\omega}_0)^2, \end{aligned} \qquad (13)$$

Assumptions, advantages and drawbacks of the hybrid approach:

(1) The nucleon momentum distribution n(k) in the ground state of nuclear matter Is NOT a step function but the n(k) with a high-momentum tail (HMT) constrained by the Jlab data and many microscopic many-body theories.

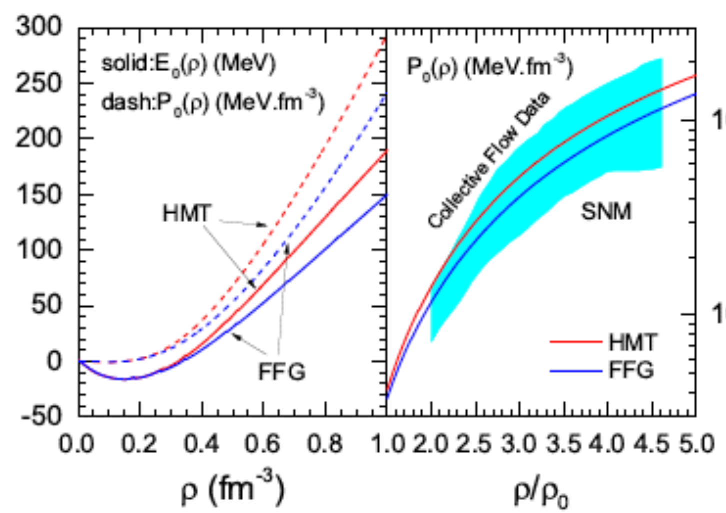
(2) Partially because the tensor force is very uncertain where the SRC is important, NOT all microscopic models predict the same size of HMT as observed at Jlab

(3) Replacing the step function with the n(k) with HMT in energy density functionals such as RMF, which do NOT consider any SRC and can NOT generate the HMT by themselves, allows us to see clearly effects of the SRC-modified n(k).

(4) All model parameters are self-consistently readjusted to reproduce identically ALL empirically properties of nuclear matter at both saturation density and high densities from heavy-ion collisions

Drawbacks: The n(k) with HMT is not generated by the RMF itself in an *ab-initio* way

EOS of symmetric nuclear matter



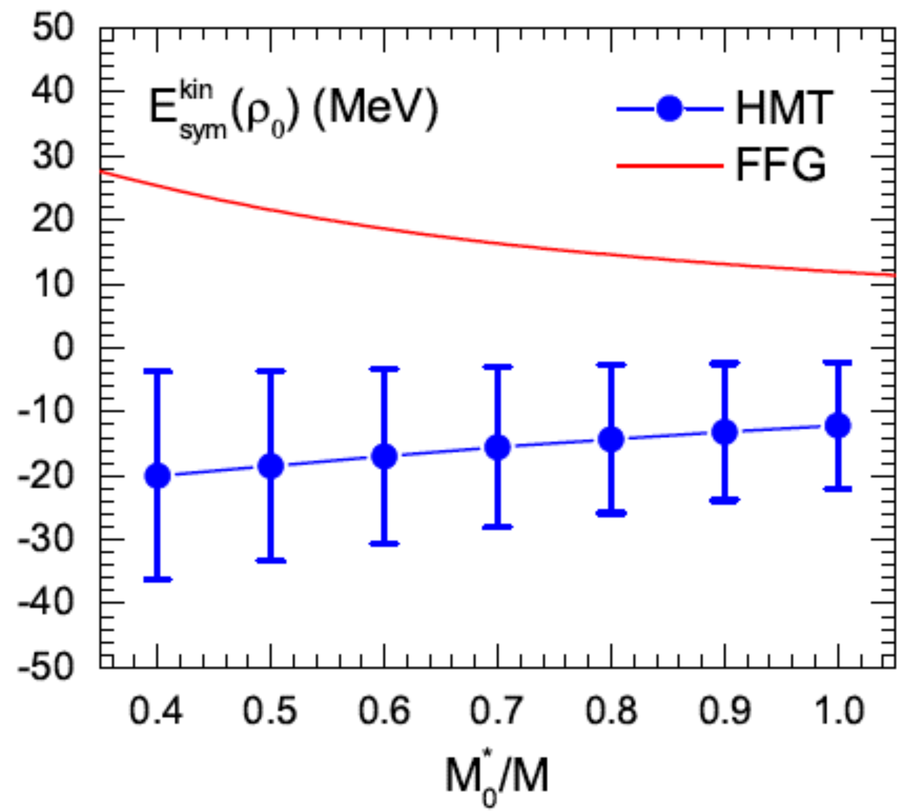
Model parameters are readjusted to reproduce identical empirical properties of asymmetric nuclear matter at saturation density

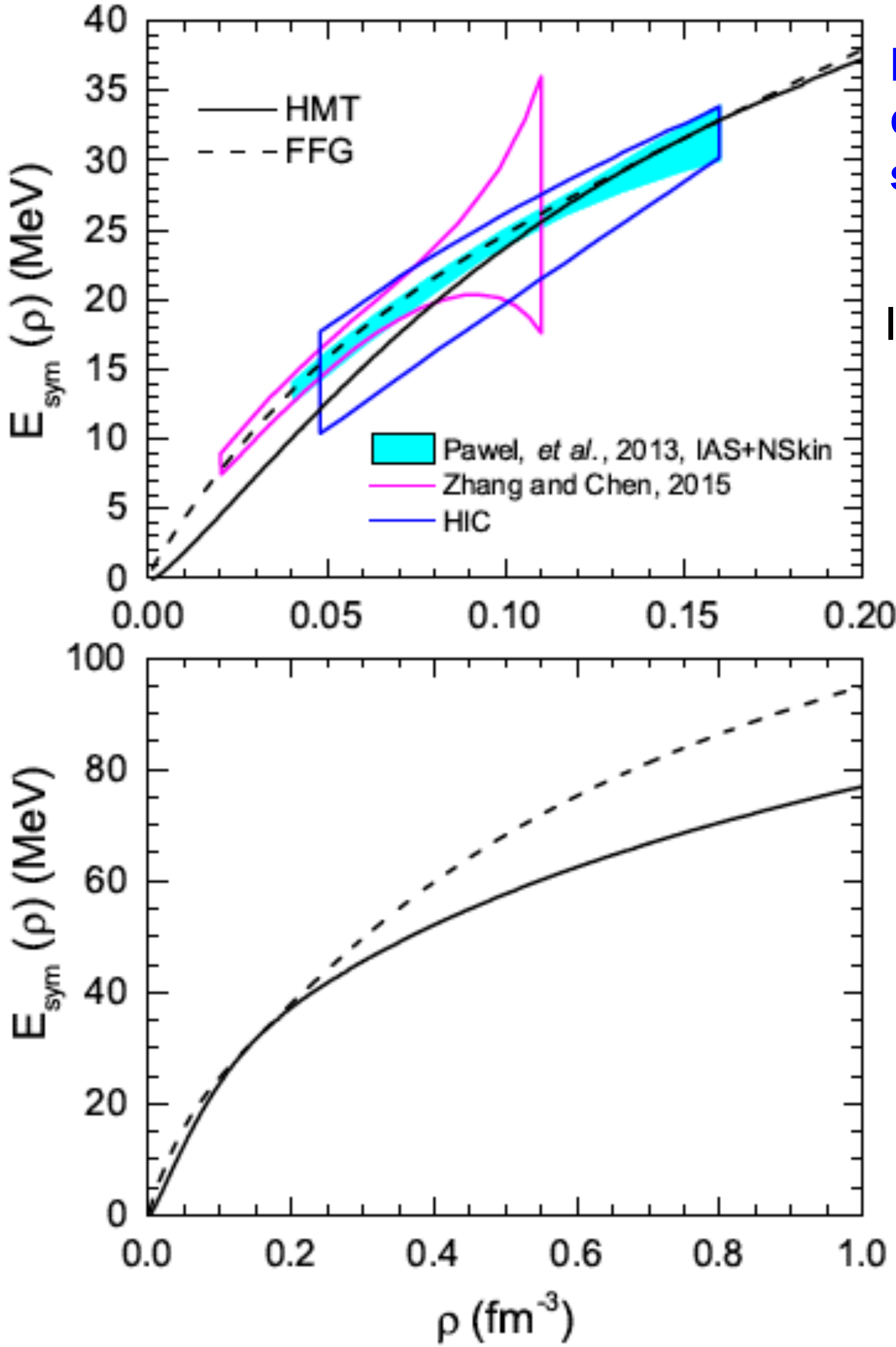
Quantity	this work	Coupling	FFG	HMT
ρ_0 (fm^{-3})	0.15	g_σ	10.9310	10.8626
$E_0(\rho_0)$ (MeV)	−16.0	g_ω	14.5947	12.9185
M_0^*/M	0.6	b_σ	0.0007473	0.002119
K_0 (MeV)	230.0	c_σ	0.003882	−0.0005139
$E_{\rm sym}(\rho_0)$ (MeV)	31.6	g_ρ	5.9163	7.8712
L (MeV)	58.9	Λ_V	0.2736	0.03740

Consequences:

(1) Kinetic symmetry energy becomes negative!

(2) Isovector parameters are changed, leading to differences in potential symmetry energy and isovector observables!





E_{sym} is softened at BOTH low and high densities while its value and slope L at saturation densities are kept the same!

Incompressibility of asymmetric matter

$$K(\delta) \approx K_0 + K_{\mathrm{sat},2}\delta^2 + \mathcal{O}(\delta^4)$$

E_{sym} becomes more concave→

$$K_{\mathrm{sat},2}^{\mathrm{FFG}} \approx -174\,\mathrm{MeV} \text{ and } K_{\mathrm{sat},2}^{\mathrm{HMT}} \approx -470\,\mathrm{MeV}$$

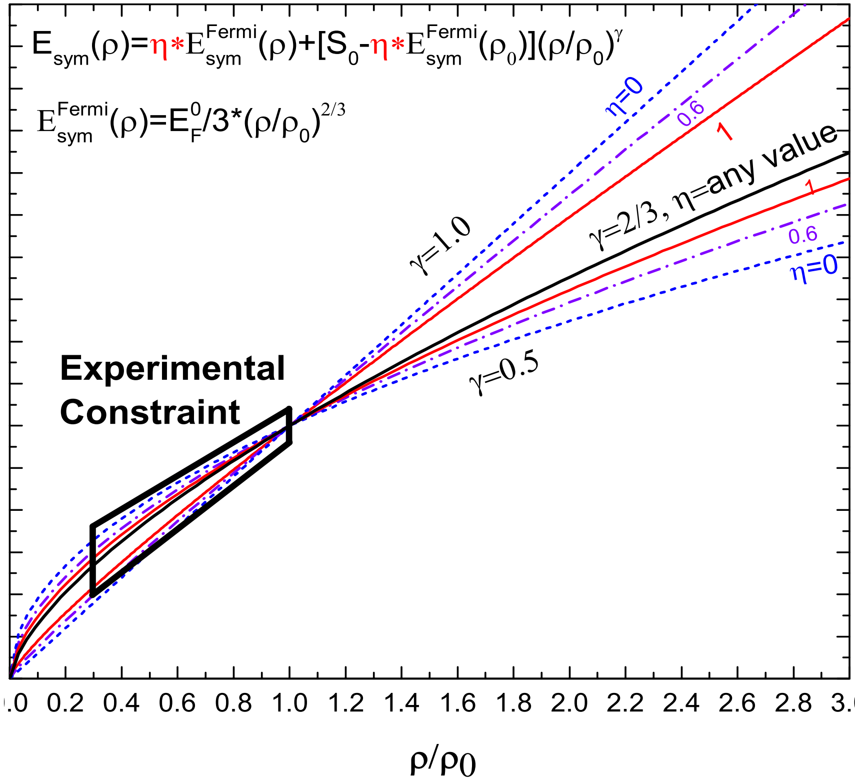
With SRC, the $K_{sat,2}$ agrees better with data from isospin-dependence of giant resonances and other experiments

Data: $K_{\mathrm{sat},2} = -550 \pm 100\ \mathrm{MeV}$

Effects on neutron-skins of nuclei and radii of neutron stars are expected

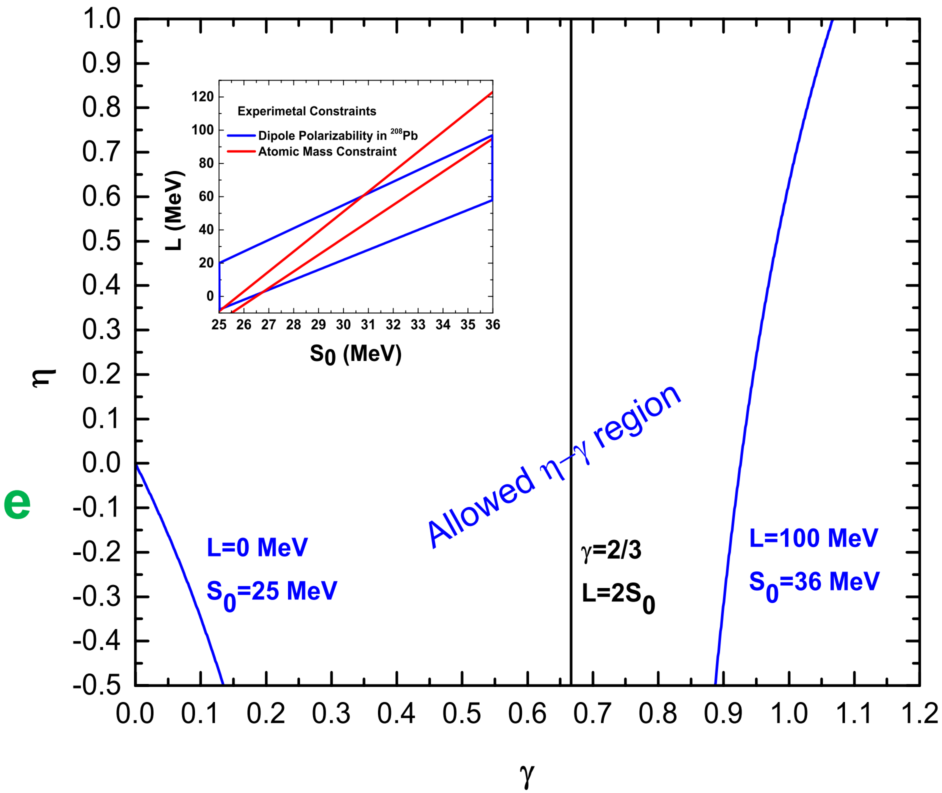
Bao-Jun Cai and Bao-An LI
arXiv:1509.09290

Incorporating the SRC-modified $E_{sym}^{Kinetic}$ in simulating heavy-ion collisions



$$L = 3\rho_0 \frac{\partial E_{sym}}{\partial \rho}\Big|_{\rho=\rho_0} = 3(2^{2/3}-1)\frac{3}{5}E_F^{(0)}(2/3-\gamma)\eta + 3\gamma S_0$$

There are big rooms to modify the sharing of kinetic and potential symmetry energy

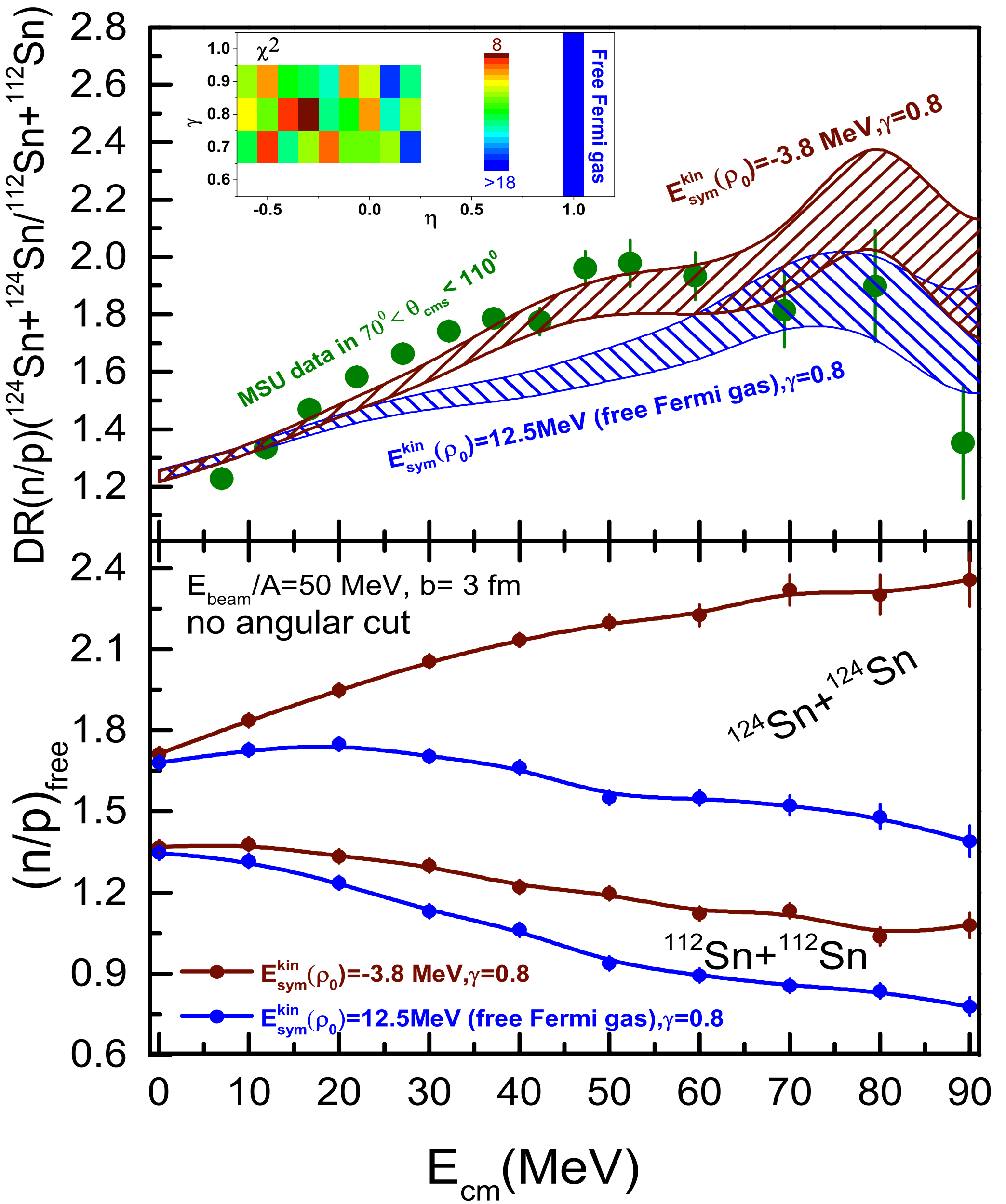


Existing constraints DO NOT exclude negative kinetic symmetry energy

Bao-An Li, Wen-Jun Guo, Zhaozhong Shi
Phys. Rev. C 91, 044601 (2015)

Effects of reduced kinetic symmetry energy on heavy-ion collisions

Or Hen, Bao-An Li, Wen-Jun Guo,
L.B. Weinstein, Eli Piasetzky
Phys. Rev. C 91, 025803 (2015)



$$E^{pot}_{sym}(\rho) = [E_{sym}(\rho_0) - \eta \cdot E^{kin}_{sym}(\rho_0)|_{\rm FG}] \cdot (\rho/\rho_0)^{\gamma}.$$

$$V^{n/p}_{\rm sym}(\rho, \delta) = [E_{sym}(\rho_0) - \eta \cdot E^{kin}_{sym}(\rho_0)|_{\rm FG}](\rho/\rho_0)^{\gamma} \times [\pm 2\delta + (\gamma - 1)\delta^2]. \qquad (10)$$

Summary

- E_{sym} at high-densities remain very uncertain
  while it has many ramifications in nuclear physics and astrophysics
- SRC has significant effects on the density dependence of E_{sym},
  it reduces the kinetic E_{sym} to negative values if more than 15% nucleons are in the high-momentum tail in symmetric nuclear matter
- SRC-modified symmetry energy has significant effects in heavy-ion reactions,
  it may also affect the n-skins of heavy nuclei and radii of neutron stars
- Some observables are (and some are not) sensitive to the individual forms and sizes of the kinetic and potential symmetry energy. But It is important to known where the symmetry energy comes from to better understand why the symmetry energy is so uncertain especially at high densities