Two- and Three-Nucleon Short-Range Correlations in Nuclear Momentum Distributions

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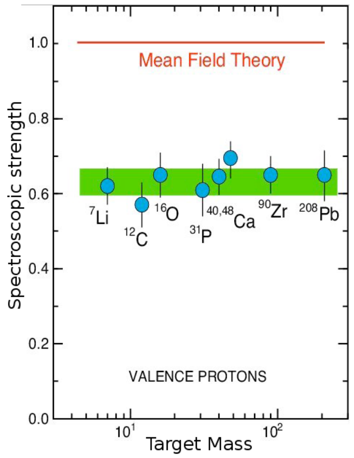
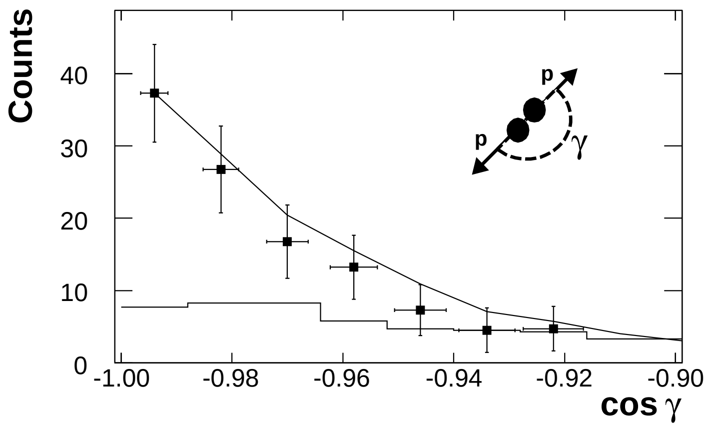
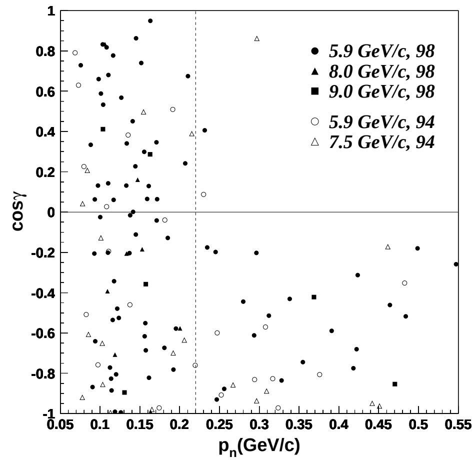
0. Comprehensive review of experimental results ☺



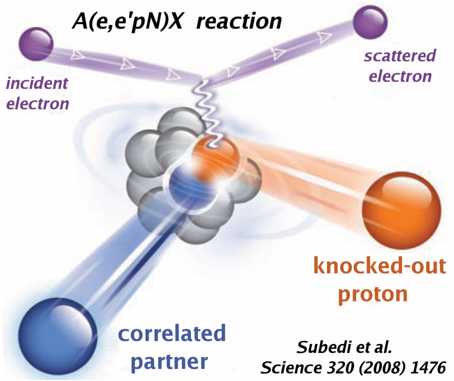
$A(e, e'p)X$

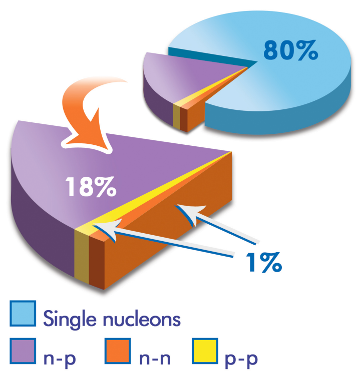


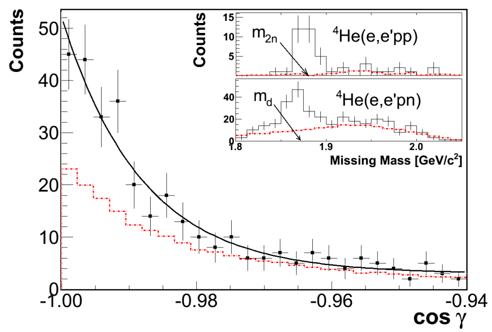
$A(p, 2p)$ Tang et al., PRL90 (2003)
theory: Ciofi et al., PRC53 (1996)

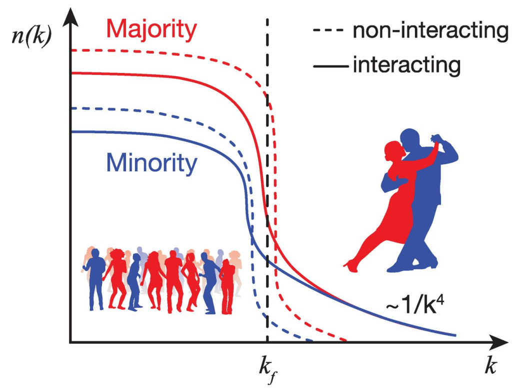


$A(p, ppn)$ Aclander et al., PLB (1999)
theory: Piasetzky et al., PLB (1999)









triple coincidence $A(e, e'pN)X$: **Carbon**
Subedi et al., **Science, 320** (2008)
Shneor et al., **PRL99** (2007)

Helium
Korover et al., **PRL 113** (2014)

heavier targets
Hen et al., **Science, 356** (2014)

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0. Nuclear Hamiltonian

- The non-relativistic nuclear many-body problem:

$$\hat{\mathbf{H}}\,\Psi_n = E_n\,\Psi_n\,, \qquad \hat{\mathbf{H}} = -\frac{\hbar^2}{2\,m}\sum_i \hat{\nabla}_i^2 + \frac{1}{2}\sum_{ij}\hat{v}_{ij} + \ldots$$

- *Exact* ground-state wave functions obtained by various methods are available for ***light nuclei*** ($A \leq 12$);
$\Longrightarrow$ calculations will be shown using ^2H, ^3He, ^4He WFs;
- Variational wave functions of nuclei can be obtained with approximated methods; usually difficult to use/generalize
$\Longrightarrow$ we developed an easy-to-use *cluster expansion* technique for the calculation of basic quantities of ***medium-heavy nuclei***, ^{12}C, ^{16}O, ^{40}Ca;
- SRCs implemented MC generator for nuclear configurations for ^{12}C to ^{238}U for the initialization of pA and AA collisions simulations
http://http://users.phys.psu.edu/~malvioli/eventgenerator/

0. Calculation of basic quantities

- one- and two-body densities:

$$\rho_N^{ST}(\boldsymbol{r}_1,\boldsymbol{r}'_1) = \int d\boldsymbol{r}_1 \sum\!\!\!\!\!\!\int \prod_{j=3}^{A} d\boldsymbol{r}_j \Psi_A^{o\dagger}(\boldsymbol{x}_1,...,\boldsymbol{x}_A)\,\hat{P}_{pN}^{ST}\,\Psi_A^{o}(\boldsymbol{x}'_1,\boldsymbol{x}_2,...,\boldsymbol{x}_A)$$

$$\rho_{pN}^{(2)}(\boldsymbol{r}_1,\boldsymbol{r}_2;\boldsymbol{r}'_1,\boldsymbol{r}'_2) = \sum\!\!\!\!\!\!\int \prod_{j=3}^{A} d\boldsymbol{r}_j \Psi_A^{o\dagger}(\boldsymbol{x}_1,...,\boldsymbol{x}_A)\,\hat{P}_{pN}\,\Psi_A^{o}(\boldsymbol{x}'_1,\boldsymbol{x}'_2,\boldsymbol{x}_3,...,\boldsymbol{x}_A)$$

- one- and two-body momentum distributions:

$$n_N^{ST}(k_1) = \frac{1}{(2\pi)^3}\int d\boldsymbol{r}_1 d\boldsymbol{r}'_1\, e^{-\boldsymbol{k}_1\cdot(\boldsymbol{r}_1-\boldsymbol{r}'_1)}\rho_N^{ST}(\boldsymbol{r}_1,\boldsymbol{r}'_1)$$

$$n_{pN}^{(2)}(\boldsymbol{k}_1,\boldsymbol{k}_2) = \frac{1}{(2\pi)^6}\int d\boldsymbol{r}_1 d\boldsymbol{r}'_1 d\boldsymbol{r}_2 d\boldsymbol{r}'_2\, e^{-\boldsymbol{k}_1\cdot(\boldsymbol{r}_1-\boldsymbol{r}'_1)}\,e^{-\boldsymbol{k}_2\cdot(\boldsymbol{r}_2-\boldsymbol{r}'_2)}\ .$$

$$\cdot\,\rho_{pN}^{(2)}(\boldsymbol{r}_1,\boldsymbol{r}_2;\boldsymbol{r}'_1,\boldsymbol{r}'_2) \quad\longleftrightarrow\quad n_{pN}^{(2)}(\boldsymbol{k}_{rel},\boldsymbol{K}_{CM})$$

0. Two-Body Momentum Distributions

$$\boldsymbol{k}_{rel} \equiv \boldsymbol{k} = \frac{1}{2}(\boldsymbol{k}_1 - \boldsymbol{k}_2) \qquad \boldsymbol{r} = \boldsymbol{r}_1 - \boldsymbol{r}_2 \qquad \boldsymbol{r}' = \boldsymbol{r}'_1 - \boldsymbol{r}'_2$$

$$\boldsymbol{K}_{CM} \equiv \boldsymbol{K} = \boldsymbol{k}_1 + \boldsymbol{k}_2 \qquad \boldsymbol{R} = \frac{1}{2}(\boldsymbol{r}_1 + \boldsymbol{r}_2) \qquad \boldsymbol{R}' = \frac{1}{2}(\boldsymbol{r}'_1 + \boldsymbol{r}'_2)$$

$$n^{(2)}(\boldsymbol{k},\boldsymbol{K}) = n^{(2)}(k_{rel}, K_{CM}, \Theta) = $$
$$= \frac{1}{(2\pi)^6}\int d\boldsymbol{r}\,d\boldsymbol{r}'\,d\boldsymbol{R}\,d\boldsymbol{R}'\, e^{-i\,\boldsymbol{K}\cdot(\boldsymbol{R}-\boldsymbol{R}')}\, e^{-i\,\boldsymbol{k}\cdot(\boldsymbol{r}-\boldsymbol{r}')}\rho^{(2)}(\boldsymbol{r},\boldsymbol{r}';\boldsymbol{R},\boldsymbol{R}')$$

$$n^{(2)}(\boldsymbol{k}) = \int d\boldsymbol{K}\, n(\boldsymbol{k},\boldsymbol{K}) = \frac{1}{(2\pi)^3}\int d\boldsymbol{r}\,d\boldsymbol{r}'\,d\boldsymbol{R}\, e^{-i\,\boldsymbol{k}\cdot(\boldsymbol{r}-\boldsymbol{r}')}\rho^{(2)}(\boldsymbol{r},\boldsymbol{r}';\boldsymbol{R},\boldsymbol{R})$$

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$$n^{(2)}(\boldsymbol{k},\boldsymbol{K}=0) = \frac{1}{(2\pi)^6}\int d\boldsymbol{r}\,d\boldsymbol{r}'\,d\boldsymbol{R}\,d\boldsymbol{R}'\, e^{-i\,\boldsymbol{k}\cdot(\boldsymbol{r}-\boldsymbol{r}')}\rho^{(2)}(\boldsymbol{r},\boldsymbol{r}';\boldsymbol{R},\boldsymbol{R}')$$

$\boldsymbol{K}_{CM} = 0$ corresponds to $\boldsymbol{k}_2 = -\boldsymbol{k}_1$, *i.e. back-to-back* nucleons

0. Using Realistic WFs of large nuclei: *Cluster Expansion*

- Cluster Expansion is a technique to reduce the computational effort in many many-body calculations; we use: $\Psi_o = \hat{F}\Phi_o = \prod_{ij}\sum_n \hat{f}^{(n)}_{ij}\Phi_o$
- Expectation value over Ψ_o of any one- or two-body operator $\hat{Q}$:

$$\frac{\langle\Psi_o|\hat{Q}|\Psi_o\rangle}{\langle\Psi_o|\Psi_o\rangle} = \frac{\langle\hat{F}^\dagger\hat{Q}\hat{F}\rangle}{\langle\hat{F}^2\rangle} = \frac{\langle\prod\hat{f}^\dagger\hat{Q}\hat{f}\rangle}{\langle\prod\hat{f}^2\rangle} = \frac{\langle\hat{Q}\prod(1+\hat{\eta})\rangle}{\langle\prod(1+\hat{\eta})\rangle} =$$

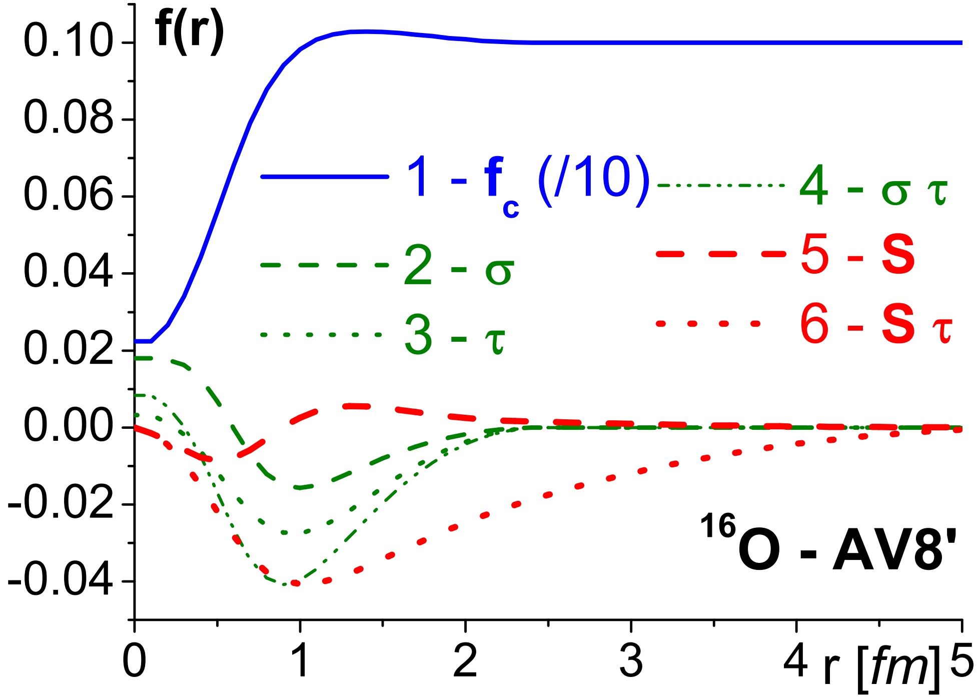
$$= \frac{\langle\hat{Q}\,(1+\sum\hat{\eta}+\sum\hat{\eta}\hat{\eta}+\ldots)\rangle}{\langle(1+\sum\hat{\eta}+\sum\hat{\eta}\hat{\eta}+\ldots)\rangle} \simeq \frac{\langle\hat{Q}\rangle+\langle\hat{Q}\sum\hat{\eta}\rangle}{1+\langle\sum\hat{\eta}\rangle} =$$

$$\simeq \left[\langle\hat{Q}\rangle+\langle\hat{Q}\sum\hat{\eta}\rangle\right]\left(1-\langle\sum\hat{\eta}\rangle+\ldots\right) \simeq \langle\hat{Q}\rangle+\langle\hat{Q}\sum\hat{\eta}\rangle_L$$

- $\langle\hat{\eta}\rangle = \langle\left[\hat{\mathbf{f}}^{\mathbf{2}}-\mathbf{1}\right]\rangle$ is the *small expansion parameter*; $\langle\hat{Q}\rangle \equiv \langle\Phi_o|\hat{Q}|\Phi_o\rangle$
- we end up with ***linked*** clusters; up to **4b diagrams needed for 2B density**, each involving the **square** of: $\hat{f} = \sum_n f_n(r_{ij})\hat{O}_n(ij)$

0. Using Realistic WFs of large nuclei: *Cluster Expansion*

correlation functions: *Central*, *Spin-Isospin*, *Tensor*



- $\langle \hat{\eta} \rangle = \langle \left[\hat{\mathbf{f}}^{\mathbf{2}} - \mathbf{1} \right] \rangle$ is the *small expansion parameter*
- we end up with ***linked*** clusters; up to **4b diagrams needed for 2B density**, each involving the **square** of: $\hat{f} = \sum_n f_n(r_{ij}) \hat{O}_n(ij)$

• at **first order** of the $\eta-$expansion, the **full correlated** `one-body mixed` density matrix expression is as follows:

$$\rho^{(1)}(\boldsymbol{r}_1,\boldsymbol{r}'_1) = \rho_o^{(1)}(\boldsymbol{r}_1,\boldsymbol{r}'_1) + \rho_H^{(1)}(\boldsymbol{r}_1,\boldsymbol{r}'_1) + \rho_S^{(1)}(\boldsymbol{r}_1,\boldsymbol{r}'_1)\,,$$

with

$$\rho_H^{(1)}(\boldsymbol{r}_1,\boldsymbol{r}'_1) = \int d\boldsymbol{r}_2\left[H_D(r_{12},r_{1'2})\,\rho_o^{(1)}(\boldsymbol{r}_1,\boldsymbol{r}'_1)\,\rho_o(\boldsymbol{r}_2) - H_E(r_{12},r_{1'2})\,\rho_o^{(1)}(\boldsymbol{r}_1,\boldsymbol{r}_2)\,\rho_o^{(1)}(\boldsymbol{r}_2,\boldsymbol{r}'_1)\right]$$

$$\rho_S^{(1)}(\boldsymbol{r}_1,\boldsymbol{r}'_1) = -\int d\boldsymbol{r}_2 d\boldsymbol{r}_3\rho_o^{(1)}(\boldsymbol{r}_1,\boldsymbol{r}_2)\left[H_D(r_{23})\rho_o^{(1)}(\boldsymbol{r}_2,\boldsymbol{r}'_1)\rho_o(\boldsymbol{r}_3) - H_E(r_{23})\rho_o^{(1)}(\boldsymbol{r}_2,\boldsymbol{r}_3)\rho_o^{(1)}(\boldsymbol{r}_3,\boldsymbol{r}'_1)\right]$$

and the functions H_D and H_E are defined as:

$$H_{D(E)}(r_{ij},r_{kl}) = \sum_{p,q=1}^{6} f^{(p)}(r_{ij})\,f^{(q)}(r_{kl})\,C_{D(E)}^{(p,q)}(r_{ij},r_{kl}) - C_{D(E)}^{(1,1)}(r_{ij},r_{kl})$$

with $C_{D(E)}^{(p,q)}(r_{ij},r_{kl})$ proper functions arising from spin-isospin traces;

(*Alvioli, Ciofi degli Atti, Morita,* ***PRC72*** (2005))

• at **first order** of the $\eta-$expansion, the **full correlated** `two-body mixed` density matrix expression is as follows:

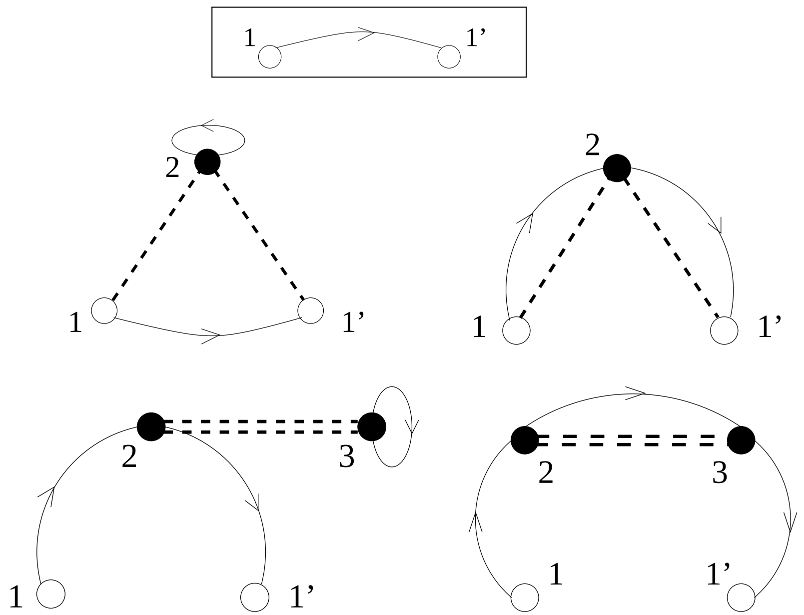
$$\rho^{(2)}(\boldsymbol{r}_1,\boldsymbol{r}_2;\boldsymbol{r}'_1,\boldsymbol{r}'_2) \;=\; \rho^{(2)}_{\mathbf{SM}}(\boldsymbol{r}_1,\boldsymbol{r}_2;\boldsymbol{r}'_1,\boldsymbol{r}'_2) + \rho^{(2)}_{\mathbf{2b}}(\boldsymbol{r}_1,\boldsymbol{r}_2;\boldsymbol{r}'_1,\boldsymbol{r}'_2) + \rho^{(2)}_{\mathbf{3b}}(\boldsymbol{r}_1,\boldsymbol{r}_2;\boldsymbol{r}'_1,\boldsymbol{r}'_2) + \rho^{(2)}_{\mathbf{4b}}(\boldsymbol{r}_1,\boldsymbol{r}_2;\boldsymbol{r}'_1,\boldsymbol{r}'_2)$$

with:

$$\begin{aligned}
\rho^{(2)}_{\mathbf{SM}}(\boldsymbol{r}_1,\boldsymbol{r}_2;\boldsymbol{r}'_1,\boldsymbol{r}'_2) &= C_D\,\rho_o(\boldsymbol{r}_1,\boldsymbol{r}'_1)\,\rho_o(\boldsymbol{r}_2,\boldsymbol{r}'_2) - C_E\,\rho_o(\boldsymbol{r}_1,\boldsymbol{r}'_2)\,\rho_o(\boldsymbol{r}_2,\boldsymbol{r}'_1)\\
\rho^{(2)}_{\mathbf{2b}}(\boldsymbol{r}_1,\boldsymbol{r}_2;\boldsymbol{r}'_1,\boldsymbol{r}'_2) &= \frac{1}{2}\,\hat{\eta}(r_{12},r_{1'2'})\,\rho_o(\boldsymbol{r}_1,\boldsymbol{r}'_1)\,\rho_o(\boldsymbol{r}_2,\boldsymbol{r}'_2) - \frac{1}{2}\,\hat{\eta}(r_{12},r_{1'2'})\,\rho_o(\boldsymbol{r}_1,\boldsymbol{r}'_2)\,\rho_o(\boldsymbol{r}_2,\boldsymbol{r}'_1)\\
\rho^{(2)}_{\mathbf{3b}}(\boldsymbol{r}_1,\boldsymbol{r}_2;\boldsymbol{r}'_1,\boldsymbol{r}'_2) &= \int d\boldsymbol{r_3}\,\hat{\eta}(r_{13},r_{1'3})\,[\,\rho_o(\boldsymbol{r}_1,\boldsymbol{r}'_1)\,\rho_o(\boldsymbol{r}_2,\boldsymbol{r}'_2)\,\rho_o(\boldsymbol{r}_3,\boldsymbol{r}_3) +\\
&\qquad - \rho_o(\boldsymbol{r}_1,\boldsymbol{r}'_1)\,\rho_o(\boldsymbol{r}_2,\boldsymbol{r}_3)\,\rho_o(\boldsymbol{r}_3,\boldsymbol{r}'_2) +\\
&\qquad + \rho_o(\boldsymbol{r}_1,\boldsymbol{r}_3)\,\rho_o(\boldsymbol{r}_2,\boldsymbol{r}'_1)\,\rho_o(\boldsymbol{r}_3,\boldsymbol{r}'_2) +\\
&\qquad - \rho_o(\boldsymbol{r}_1,\boldsymbol{r}_3)\,\rho_o(\boldsymbol{r}_2,\boldsymbol{r}'_2)\,\rho_o(\boldsymbol{r}_3,\boldsymbol{r}'_1) +\\
&\qquad + \rho_o(\boldsymbol{r}_1,\boldsymbol{r}'_2)\,\rho_o(\boldsymbol{r}_2,\boldsymbol{r}_3)\,\rho_o(\boldsymbol{r}_3,\boldsymbol{r}'_1) +\\
&\qquad - \rho_o(\boldsymbol{r}_1,\boldsymbol{r}'_2)\,\rho_o(\boldsymbol{r}_2,\boldsymbol{r}'_1)\,\rho_o(\boldsymbol{r}_3,\boldsymbol{r}_3)\,]\\
\rho^{(2)}_{\mathbf{4b}}(\boldsymbol{r}_1,\boldsymbol{r}_2;\boldsymbol{r}'_1,\boldsymbol{r}'_2) &= \frac{1}{4}\int d\boldsymbol{r_3}d\boldsymbol{r_4}\,\hat{\eta}(r_{34})\;\cdot\\
&\quad \cdot\sum_{\mathcal{P}\in\mathcal{C}}(-1)^{\mathcal{P}}\,[\,\rho_o(\boldsymbol{r}_1,\boldsymbol{r}_{\mathcal{P}1'})\,\rho_o(\boldsymbol{r}_2,\boldsymbol{r}_{\mathcal{P}2'})\,\rho_o(\boldsymbol{r}_3,\boldsymbol{r}_{\mathcal{P}3})\,\rho_o(\boldsymbol{r}_4,\boldsymbol{r}_{\mathcal{P}4})\,]
\end{aligned}$$

(*Alvioli, Ciofi degli Atti, Morita,* ***PRC72*** (2005))

(*Alvioli, Ciofi degli Atti, Morita,* ***PRL100*** (2008))

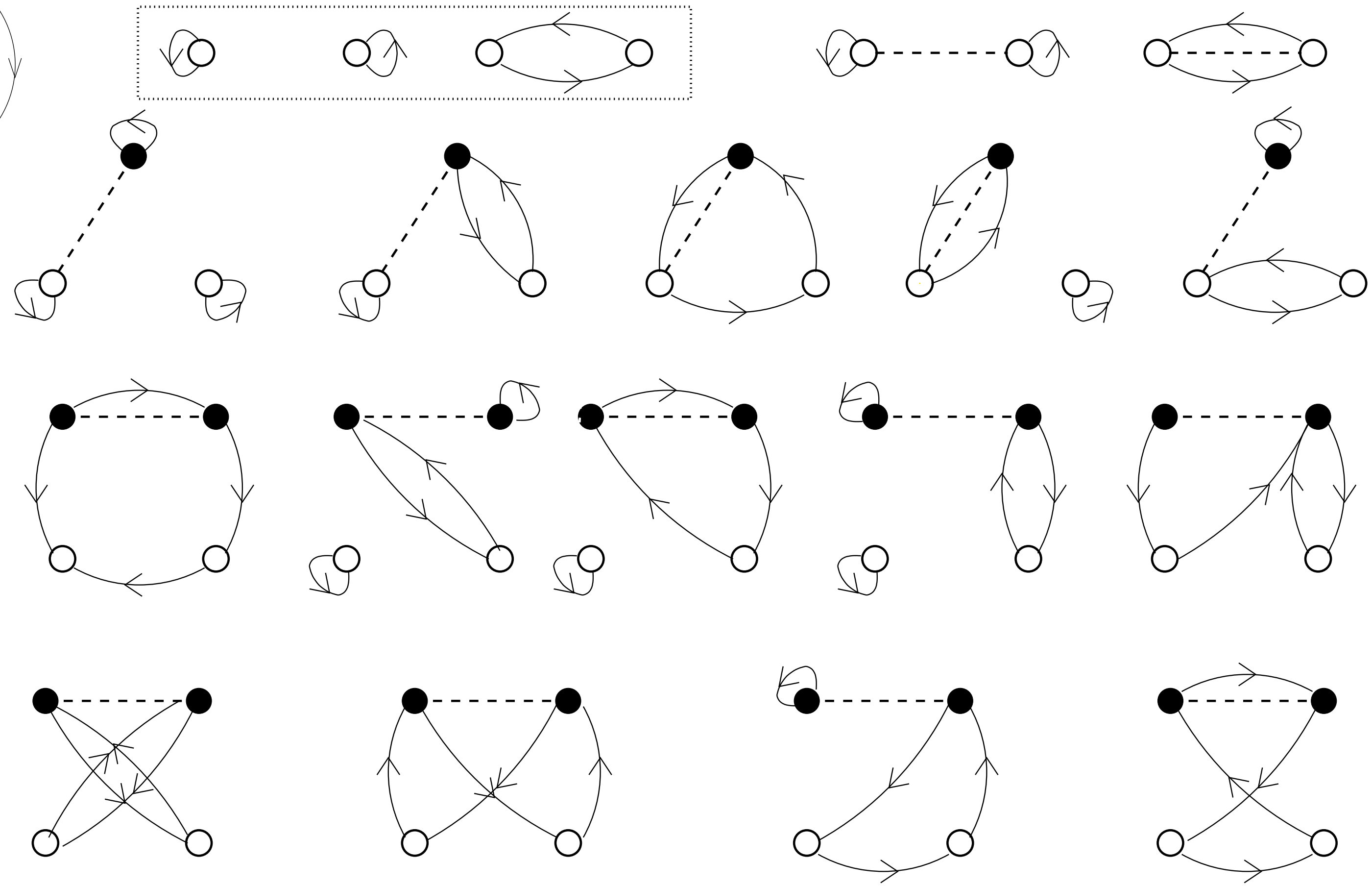


one-body, *non-diagonal*

$\longleftarrow \rho(\boldsymbol{r}_1, \boldsymbol{r}'_1)$ *diagrams*

two-body, *diagonal*

$\rho^{(2)}(\boldsymbol{r}_1, \boldsymbol{r}_2)$ *diagrams* $\longrightarrow$



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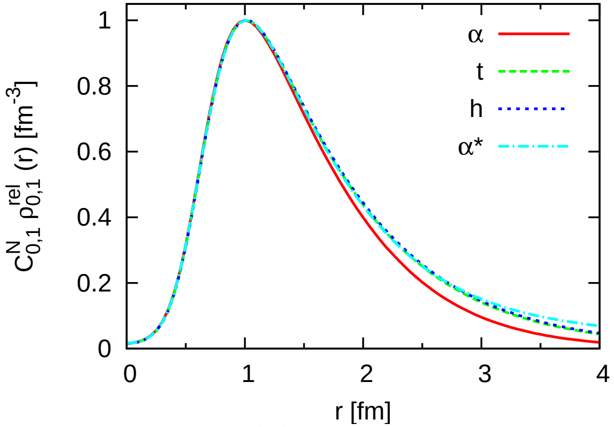
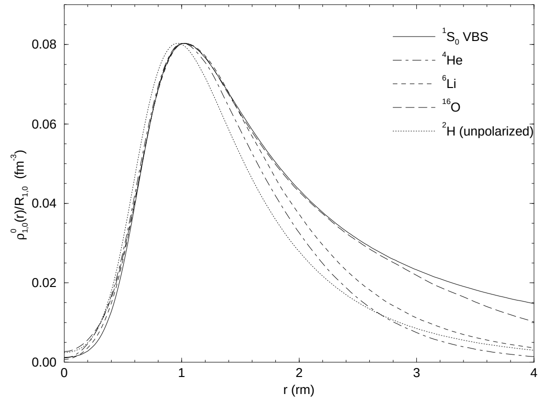
0. Correlations signatures in coordinate space densities

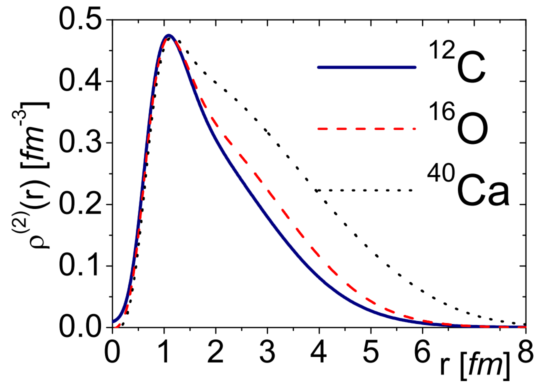
- ***realistic*** relative two-body density $\rho(r) = \int d\boldsymbol{R}\, \rho^{(2)}\Big(\boldsymbol{r} = \boldsymbol{r}_1 - \boldsymbol{r}_2, \boldsymbol{R} = (\boldsymbol{r}_1 + \boldsymbol{r}_2)/2\Big)$



Feldemeier *et al*,
Phys. Rev. **C84**, 054003 (2011)

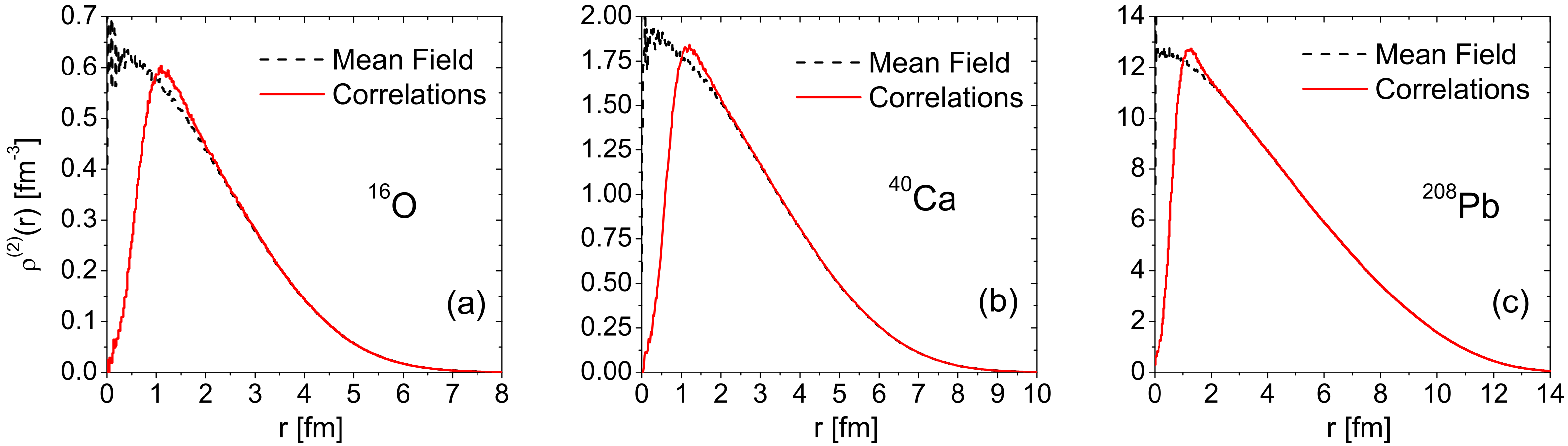


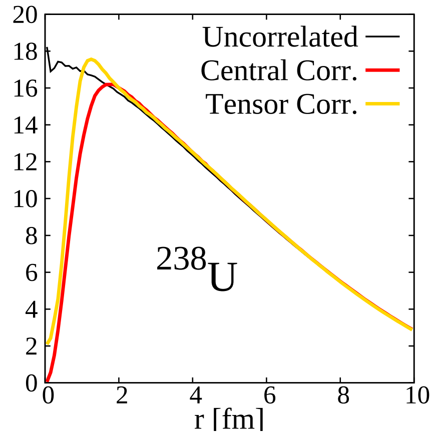
Forest *et al*,
Phys. Rev. **C54** (1996) 646-667



Alvioli *et al*, Phys. Rev. **C72**;
Phys. Rev. Lett. **100** (2008)

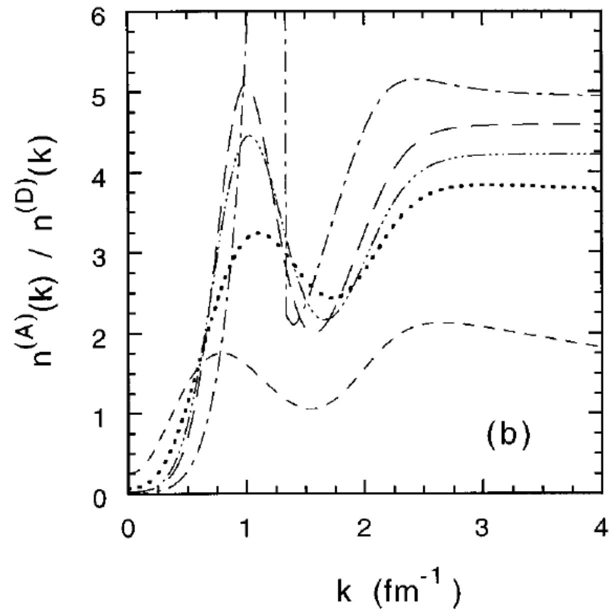
- ***MC algorithm*** for correlated configurations in pA and AA





M. Alvioli, H.-J. Drescher, M. Strikman, Phys. Lett. **B680** (2009) 225

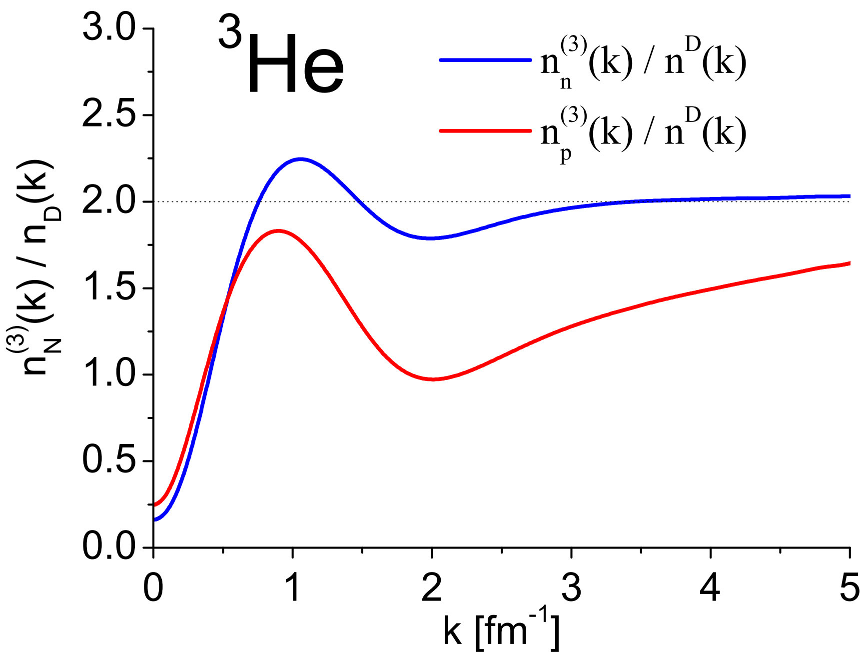
1. One-Body Momentum distributions & convolution model

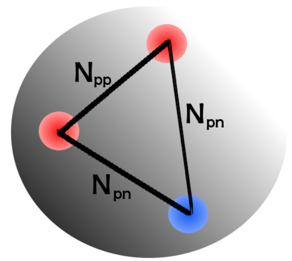


- The figure (TNC model) ***assumes*** deuteron-like high-momentum tail
- Many-body calculations actually show a ***rise*** of the ratio $n^A(k)/n^D(k)$
- Different potentials/methods provide (slightly) different high-momentum components

1. One-Body Momentum distributions: ^3He

- $n_N(k_1) = \frac{1}{(2\pi)^3} \int d\boldsymbol{r} d\boldsymbol{r'} \rho_N(\boldsymbol{r}, \boldsymbol{r'}) e^{-\boldsymbol{k}_1 \cdot (\boldsymbol{r} - \boldsymbol{r'})} = \int d\boldsymbol{k}_2\, n^{(2)}(\boldsymbol{k}_1, \boldsymbol{k}_2)$





$\longrightarrow n_p(k) \propto N_{pp}(k) + N_{pn}(k)$

$\longrightarrow n_n(k) \propto 2\, N_{pn}(k)$

M. Alvioli et al. ***PRC87*** (2013),
and
IntJModPhys E22 *(2013)*

- proton and neutron distributions reflect the different isospin pairs in ^3He;
- the neutron distribution is about twice the deuteron distribution.
- the proton one is larger than the deuteron's.

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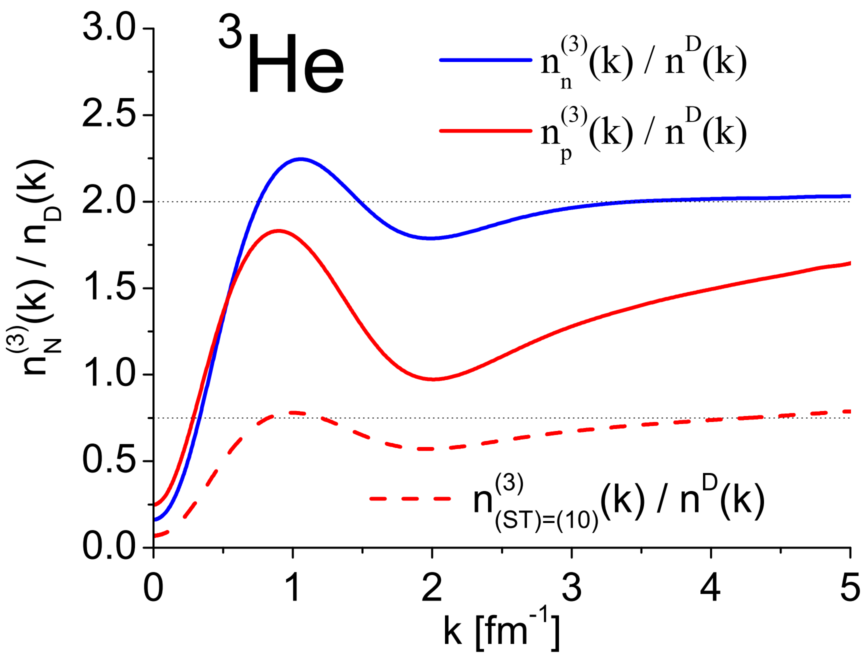

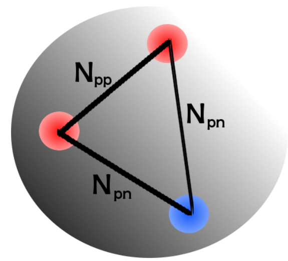

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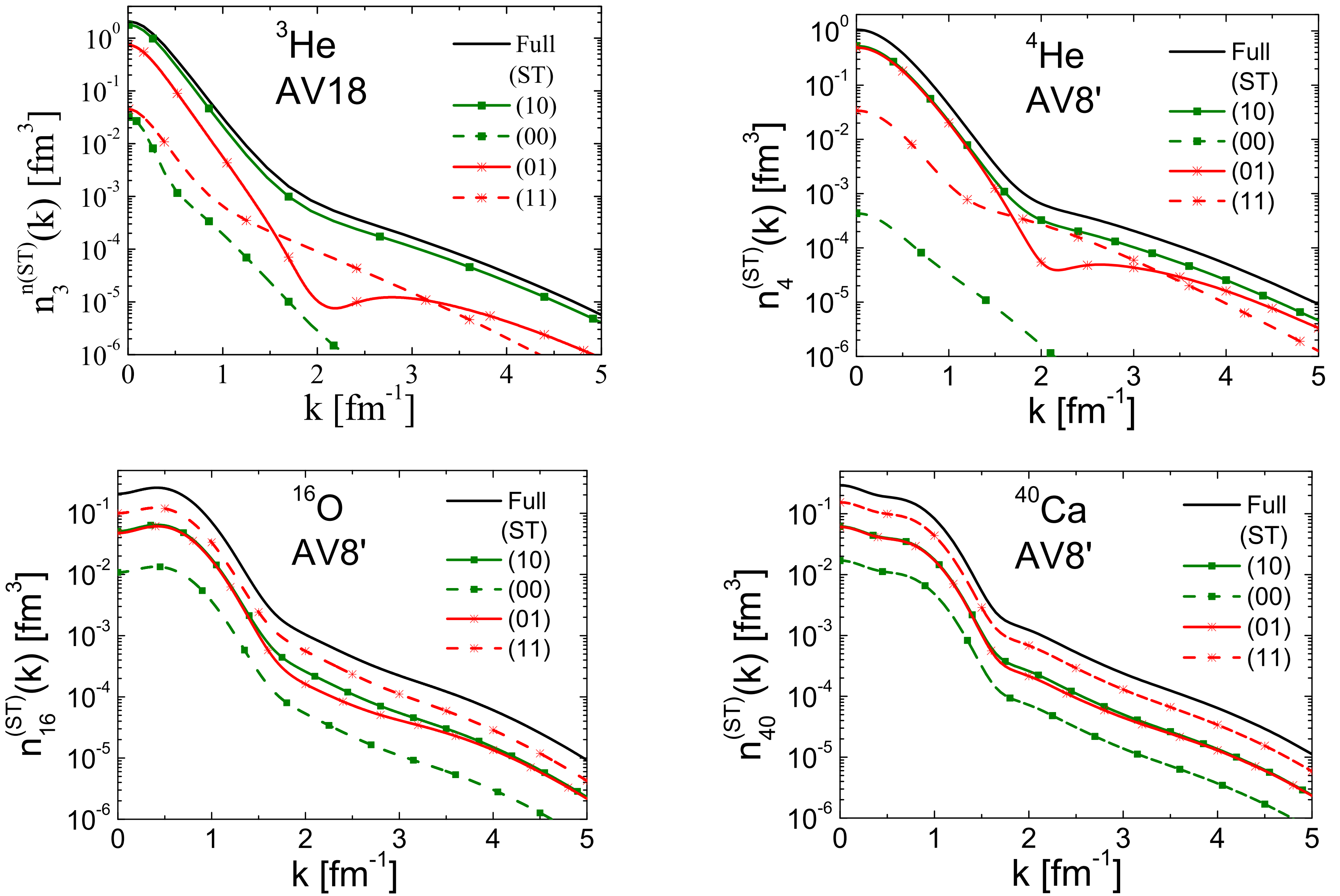
1. One-Body Mom distrs: Few- and Many-Body nuclei



M. Alvioli et al., ***PRC87*** (2013); ***IntJModPhys E22*** (2013)

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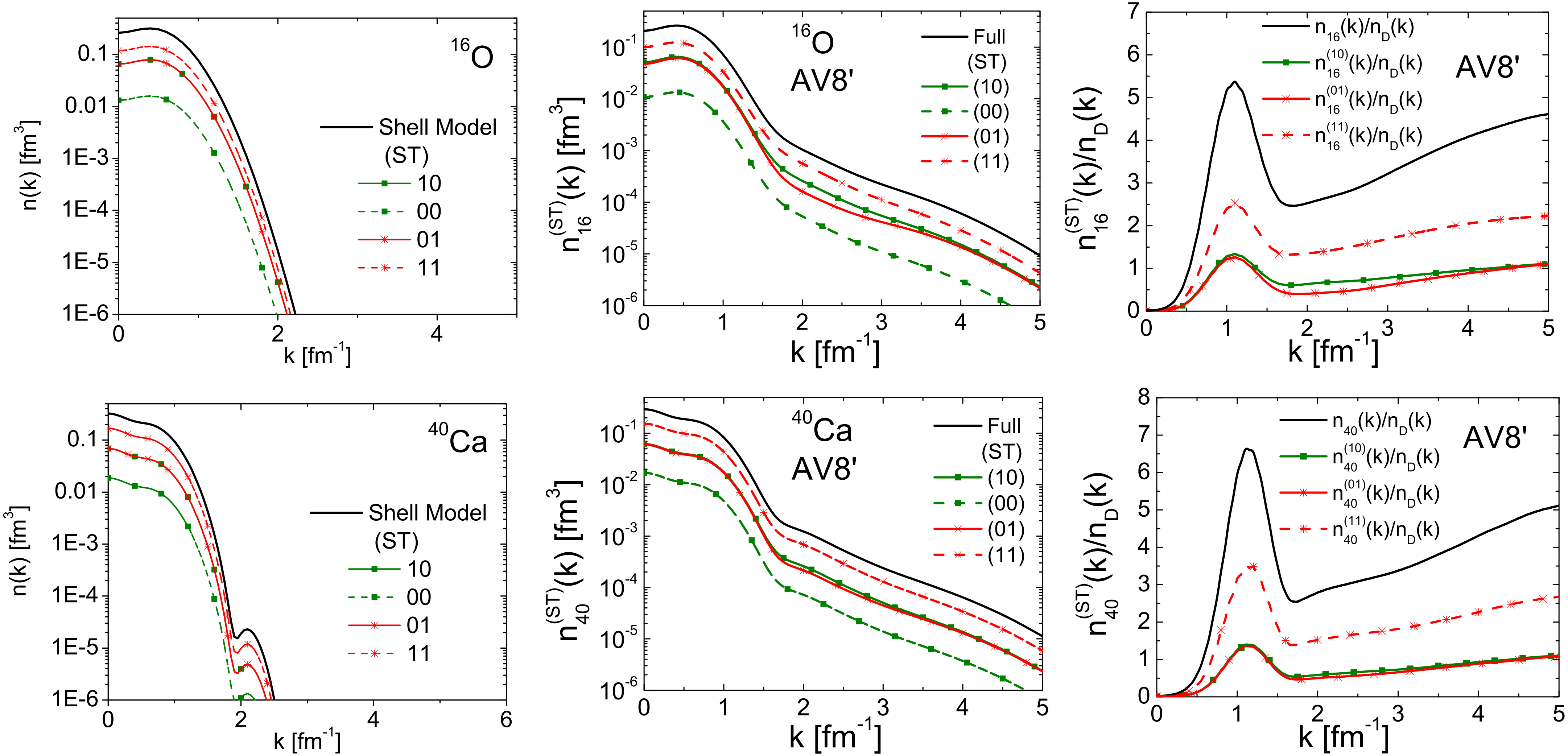
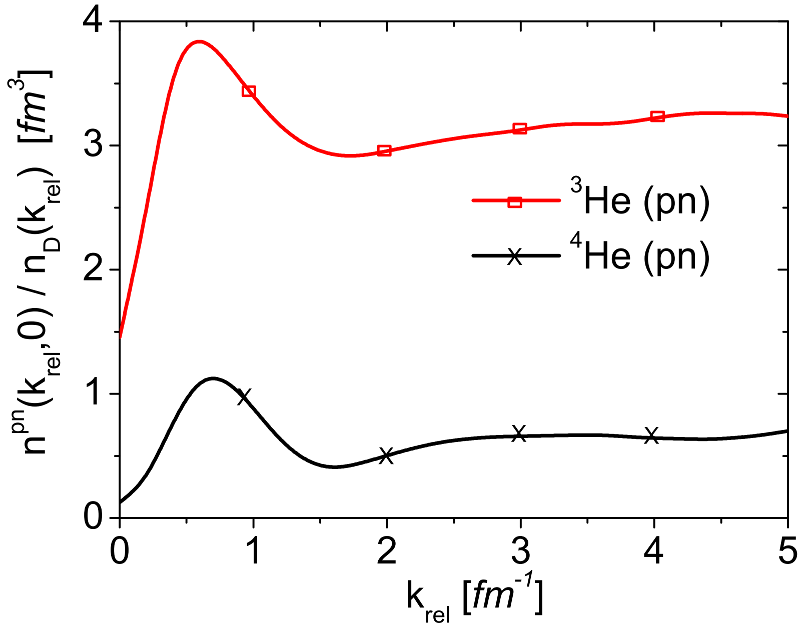


M. Alvioli et al., ***PRC87*** (2013); ***IntJModPhys E22*** (2013)

2. Two-Body Distributions: a closer look to deuteron scaling



M. Alvioli, C.Ciofi degli Atti, L.P. Kaptari, C.B. Mezzetti, H. Morita, S. Scopetta; *Phys. Rev.* ***C85*** *(2012) 021001*

- Should a nucleus' $n^{pn}(k_{rel}, K_{CM} = 0)$ scale to ^2H's $n_D(k_{rel})$?

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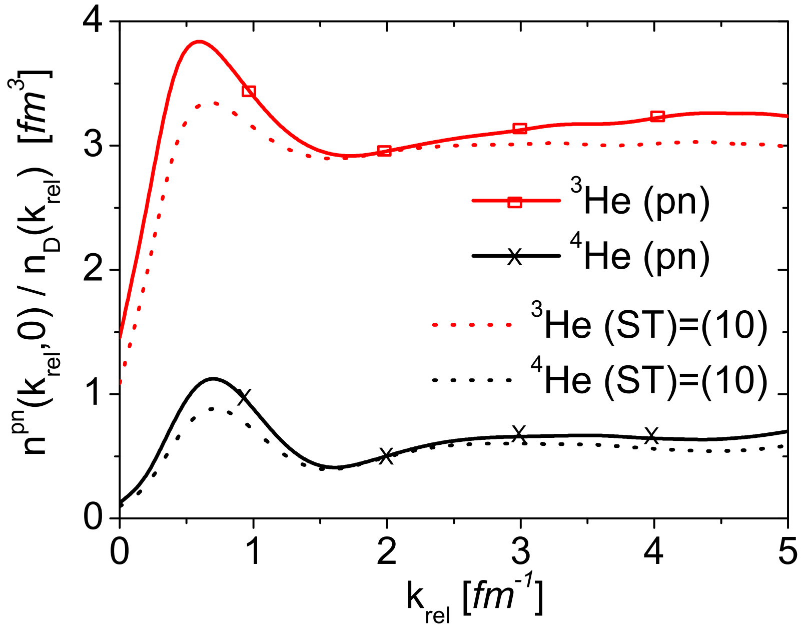

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- Including only pairs with deuteron-like quantum numbers (ST)=(10) we find exact scaling!

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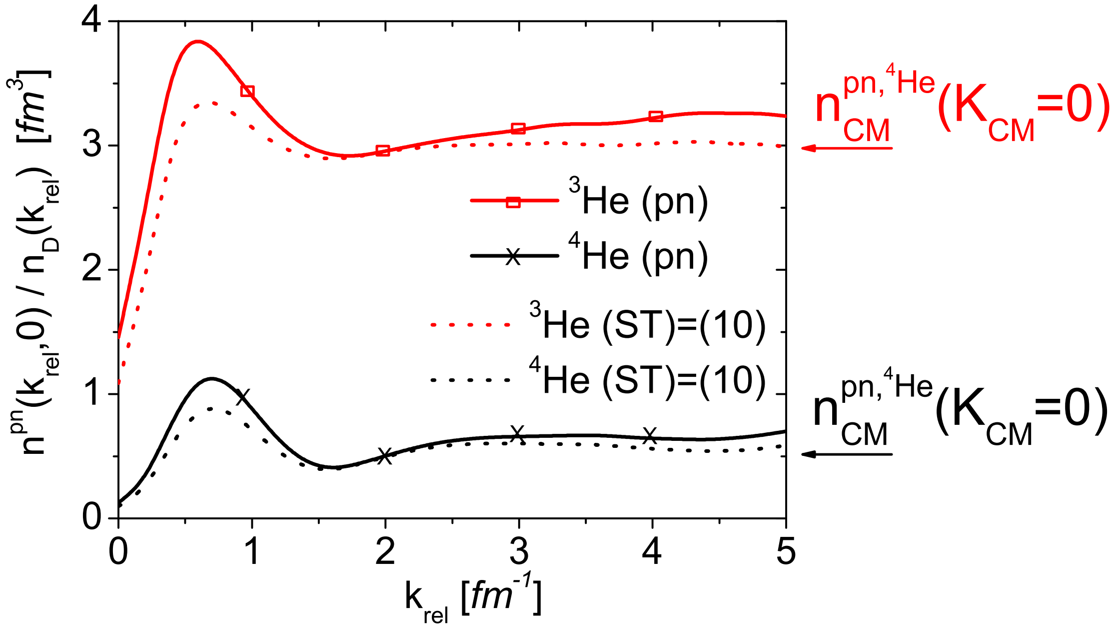

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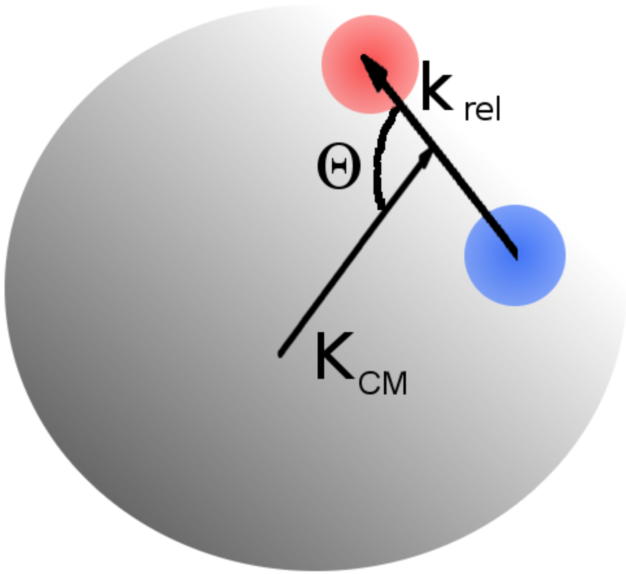
- Should a nucleus' $n^{pn}(k_{rel}, K_{CM} = 0)$ scale to ^2H's $n_D(k_{rel})$?
- Including only pairs with deuteron-like quantum numbers (ST)=(10) we find exact scaling!
- $n(\boldsymbol{k}_{rel}, 0)/n^D(\boldsymbol{k}_{rel}) \simeq n^D(k_{rel})n_{CM}(0)/n^D(k_{rel}) = n_{CM}(K_{CM} = 0)$!

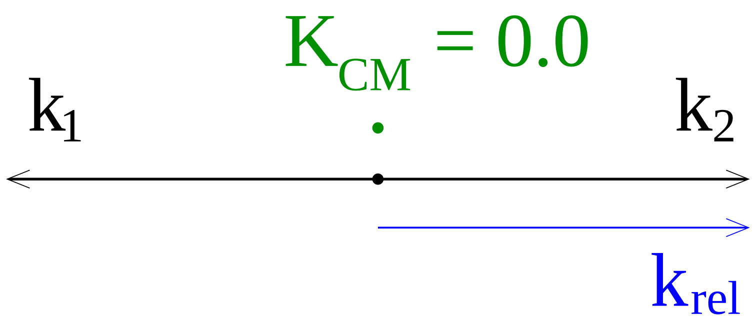
2. Motion of a (correlated) pair in the nucleus

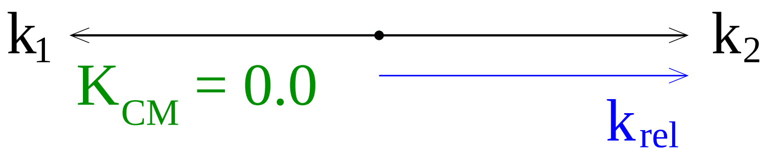
- Transform $\rho^{(2)}_{NN}(\boldsymbol{r}_1, \boldsymbol{r}_2; \boldsymbol{r}'_1; \boldsymbol{r}'_2)$ to momentum space:



- $n(\boldsymbol{k}_{rel}, \boldsymbol{K}_{CM}) = n(k_{rel}, K_{CM}, \Theta)$
- Back-to-back nucleons at K_{CM}=0
- We can select any orientation of the two momenta $\boldsymbol{k}_1$, $\boldsymbol{k}_2 \longleftrightarrow \boldsymbol{k}_{rel}$, $\boldsymbol{K}_{CM}$

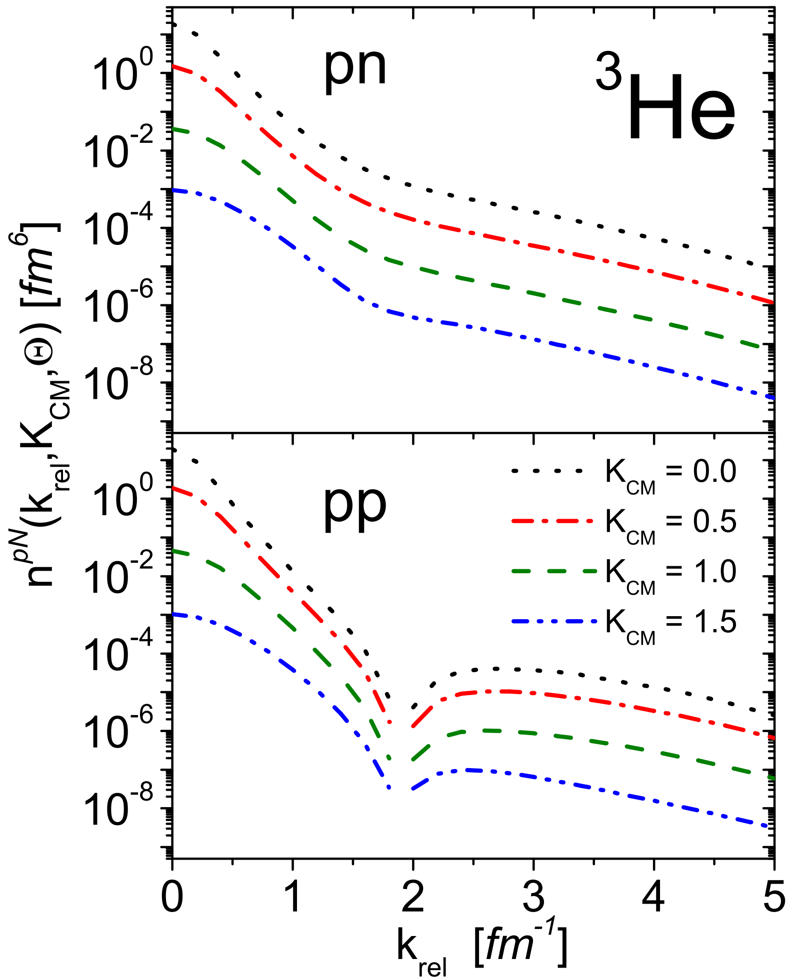
- We discuss: ***parallel*** and ***perpendicular***

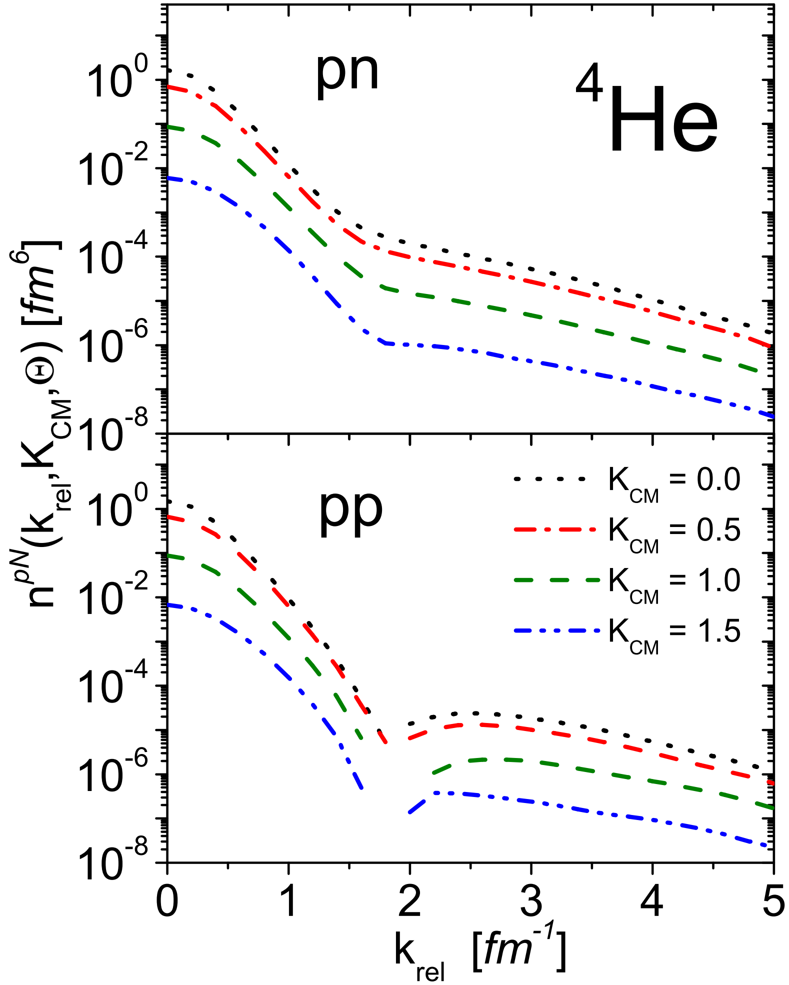




2. Two-Body momentum Distributions of Few-Body Nuclei

Pisa AV18 +UIX





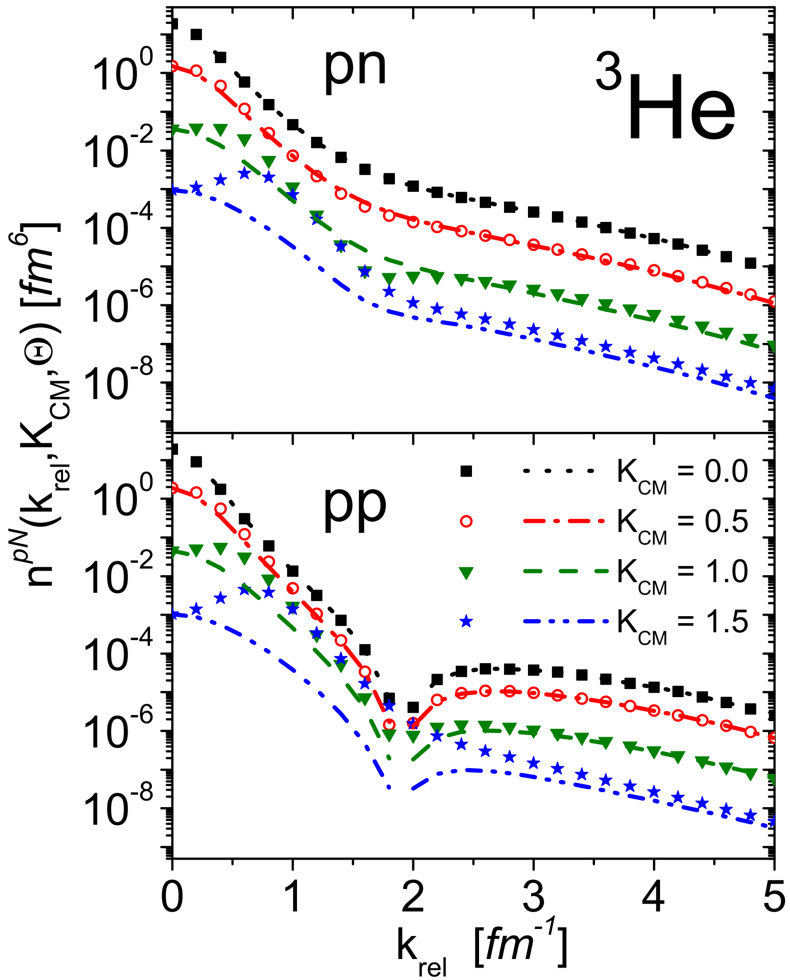
ATMS AV8′

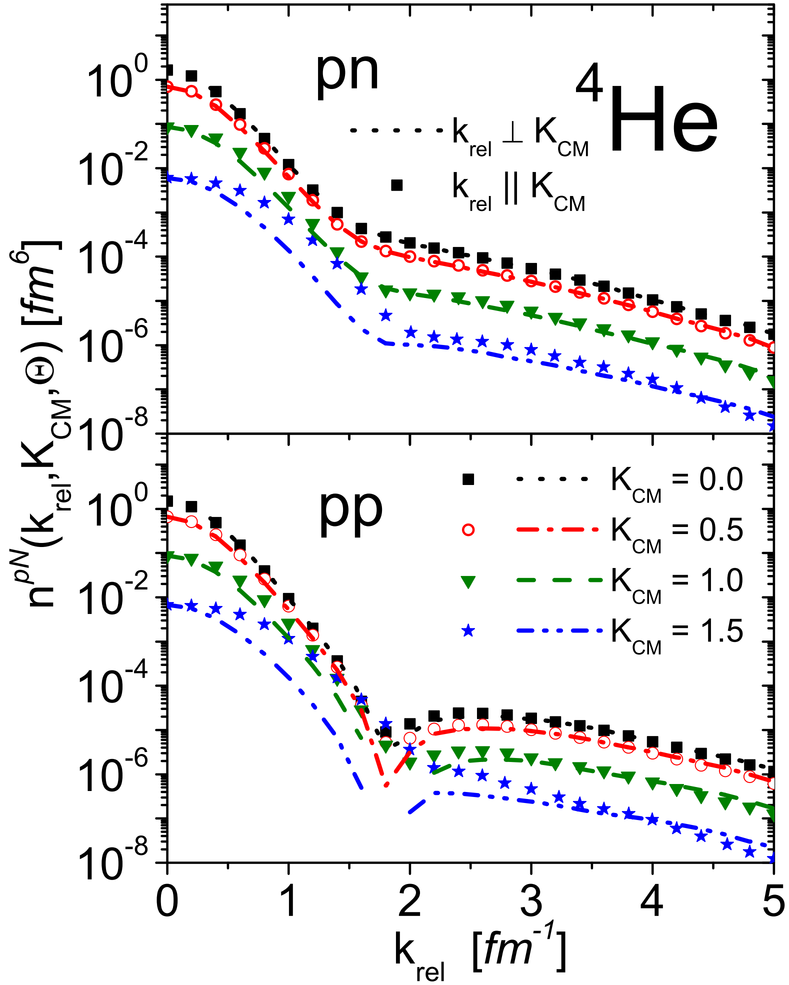
here $\boldsymbol{k}_{rel}$ is perpendicular to $\boldsymbol{K}_{CM}$

M. Alvioli, C.Ciofi degli Atti, L.P. Kaptari, C.B. Mezzetti, H. Morita, S. Scopetta; *Phys. Rev.* ***C85*** *(2012) 021001*

2. Two-Body momentum Distributions of Few-Body Nuclei

Pisa
AV18
+UIX





ATMS
AV8′

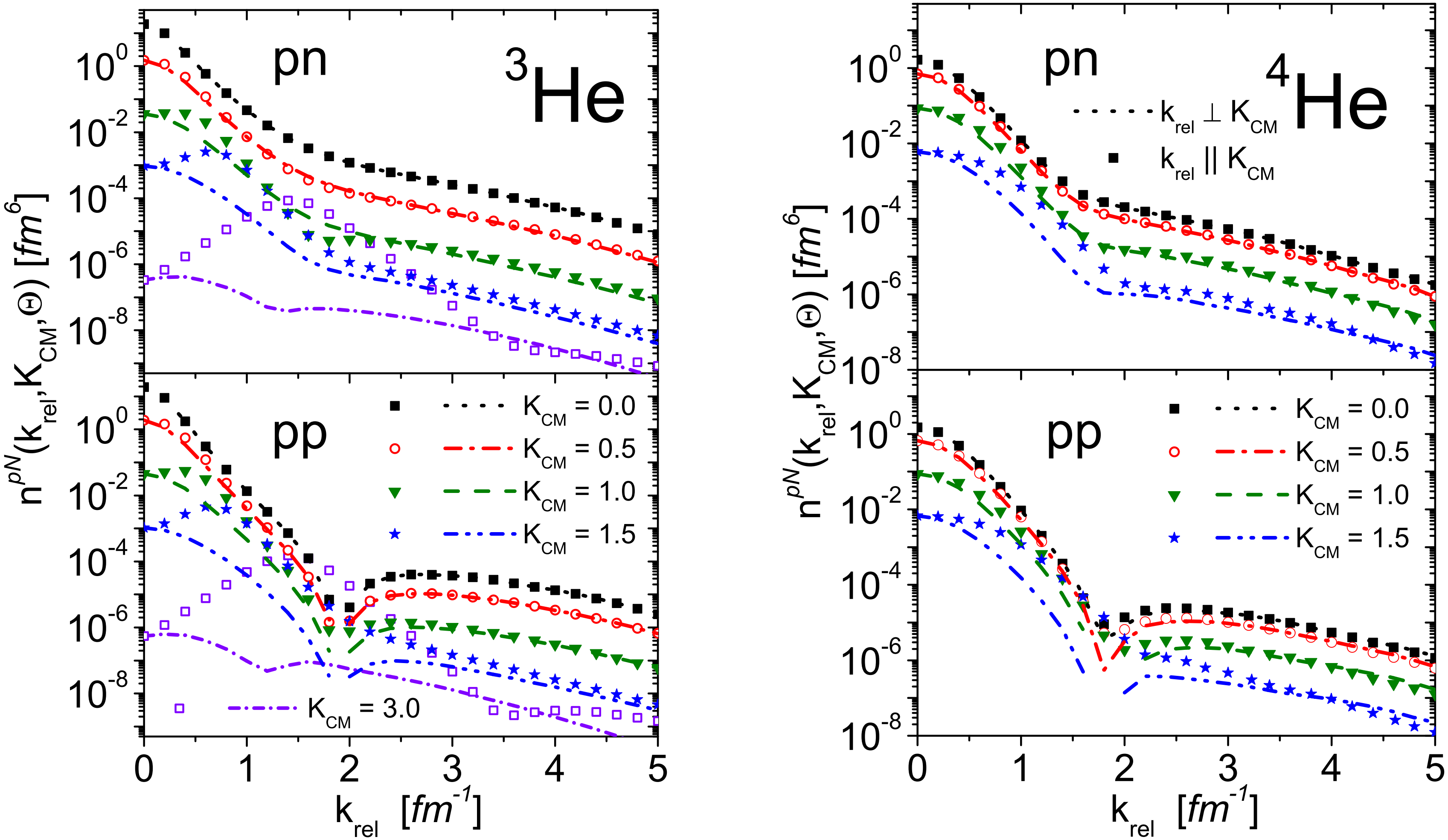
$n^{pN}(k_{rel}, K_{CM}, \Theta)$ is *angle independent* for **large** k_{rel} and **small** K_{CM}

M. Alvioli, C.Ciofi degli Atti, L.P. Kaptari, C.B. Mezzetti, H. Morita, S. Scopetta; *Phys. Rev.* ***C85*** *(2012) 021001*

2. Two-Body momentum Distributions of Few-Body Nuclei

Pisa
AV18
+UIX

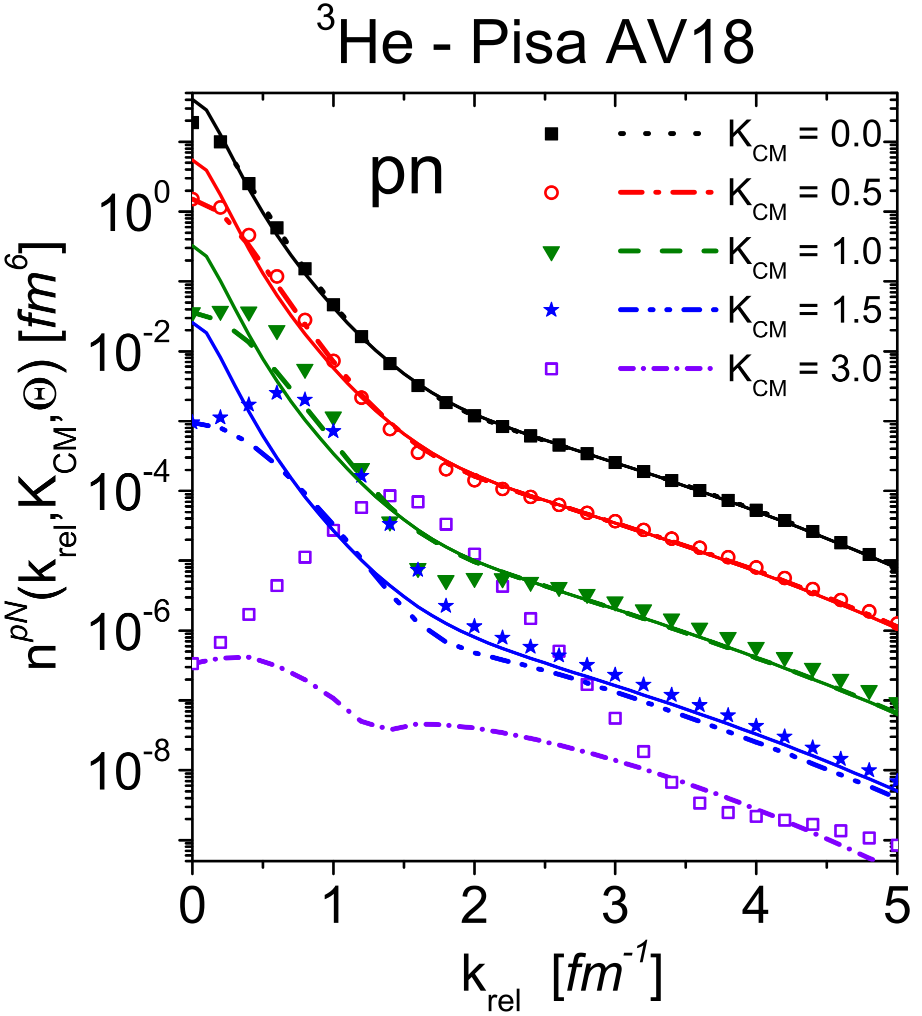
ATMS
AV8′

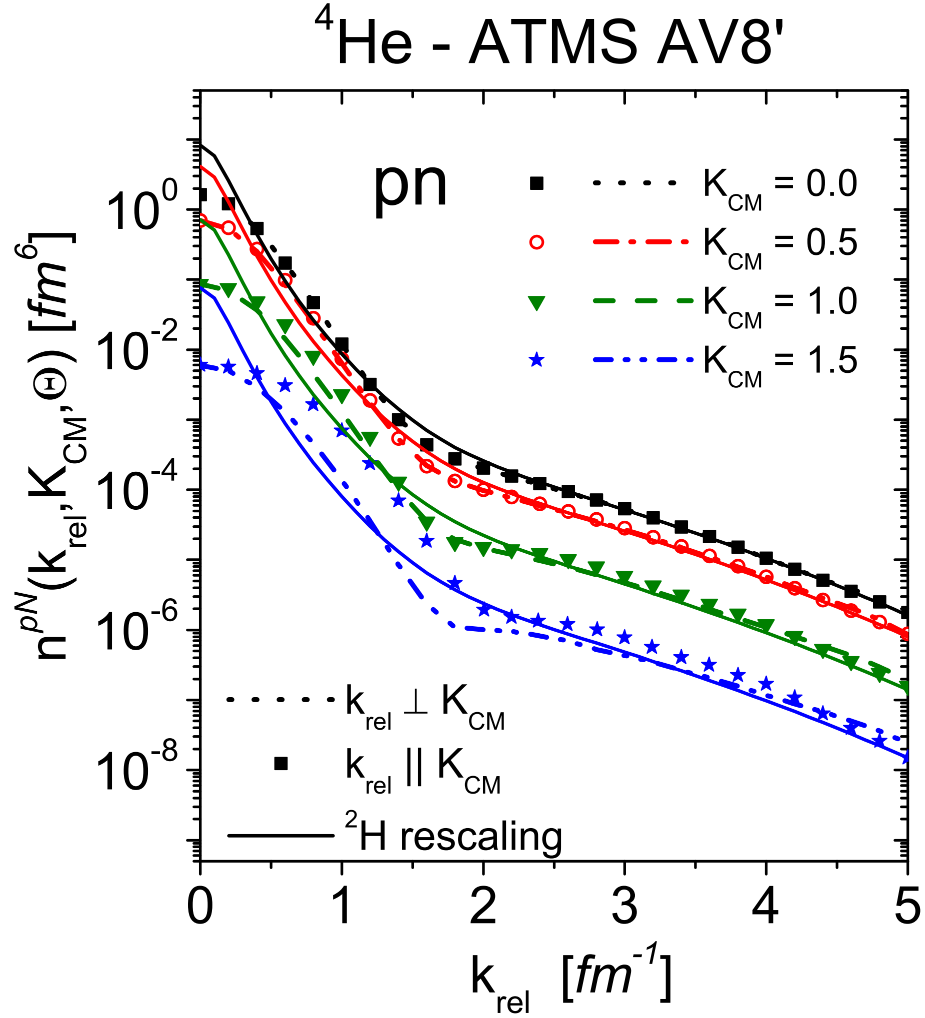


three-body correlations must be in the ***large*** K_{CM} *region*

M. Alvioli, C.Ciofi degli Atti, L.P. Kaptari, C.B. Mezzetti, H. Morita, S. Scopetta; *Phys. Rev.* ***C85*** *(2012) 021001*

2. Two-Body momentum Distributions of Few-Body Nuclei



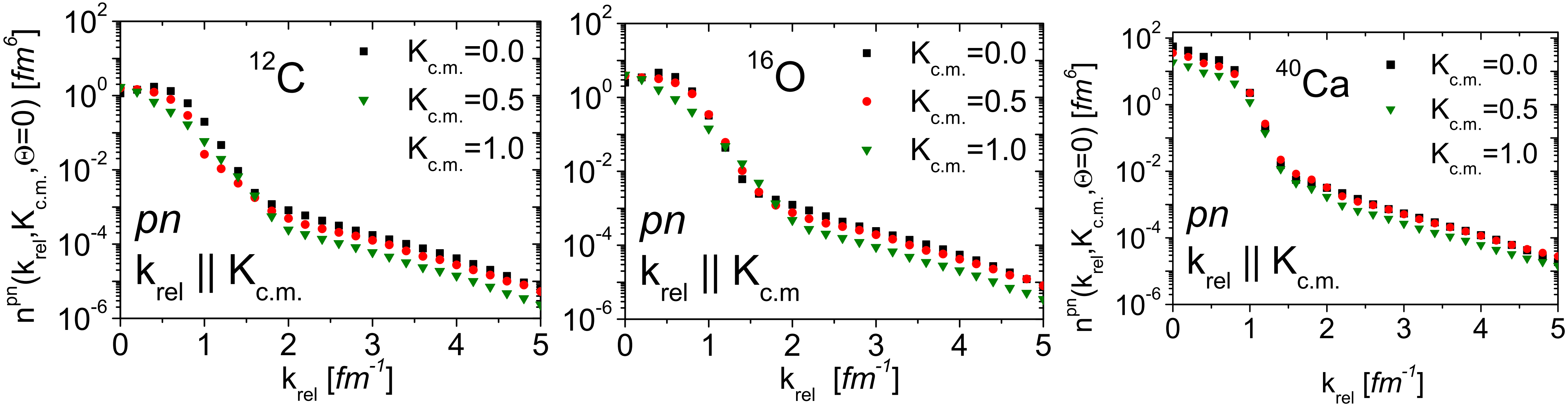


solid curves, the TNC model: rescaling of the deuteron by $n^A_{CM}(K_{CM})$!

M. Alvioli, C.Ciofi degli Atti, L.P. Kaptari, C.B. Mezzetti, H. Morita, S. Scopetta; *Phys. Rev.* ***C85*** *(2012) 021001*

2. Two-Body momentum Distributions of Many-Body Nuclei

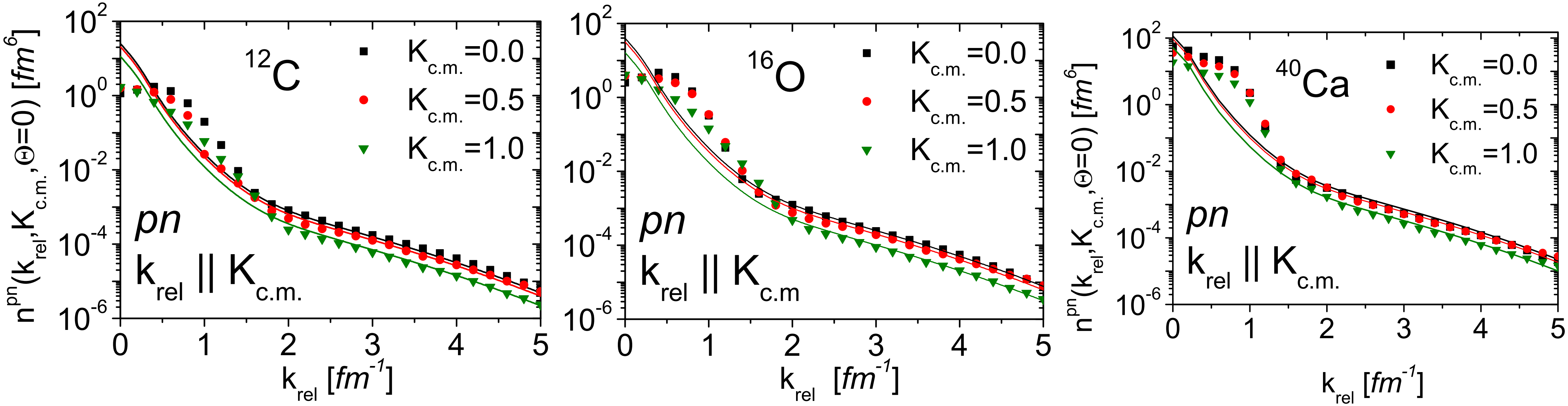
cluster expansion $\rho(\boldsymbol{r}_1, \boldsymbol{r}_2; \boldsymbol{r}'_1, \boldsymbol{r}'_2), \Longrightarrow n^{pn}(k_{rel}, K_{CM}, \Theta)$



M. Alvioli, C.Ciofi degli Atti, H. Morita, *PRL* ***100*** *(2008) 162503*
and M. Alvioli, C.Ciofi degli Atti, H. Morita, ***to appear***

2. Two-Body momentum Distributions of Many-Body Nuclei

$$\text{cluster expansion } \rho(\boldsymbol{r}_1, \boldsymbol{r}_2; \boldsymbol{r}'_1, \boldsymbol{r}'_2), \Longrightarrow n^{pn}(k_{rel}, K_{CM}, \Theta)$$

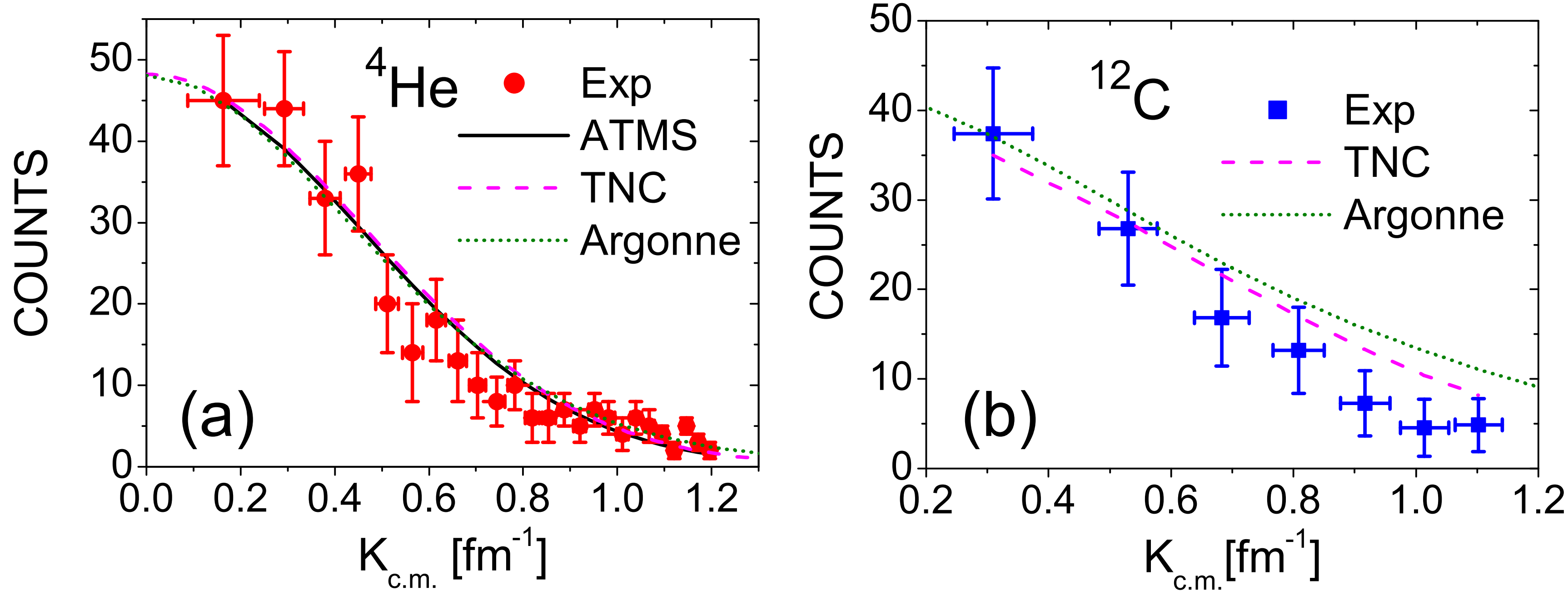


M. Alvioli, C.Ciofi degli Atti, H. Morita, *PRL* ***100*** *(2008) 162503*

and M. Alvioli, C.Ciofi degli Atti, H. Morita, ***to appear***

- symbols are the rescaled deuteron with α_{CM} gaussian parameters
- same behaviour & conclusions as in few-body ($K_{CM} > 1$ not shown)
- ***universality of NN correlations***
- we can *update* the TNC model with many-body quantities

2. Two-body mom distrs results - comparison with data

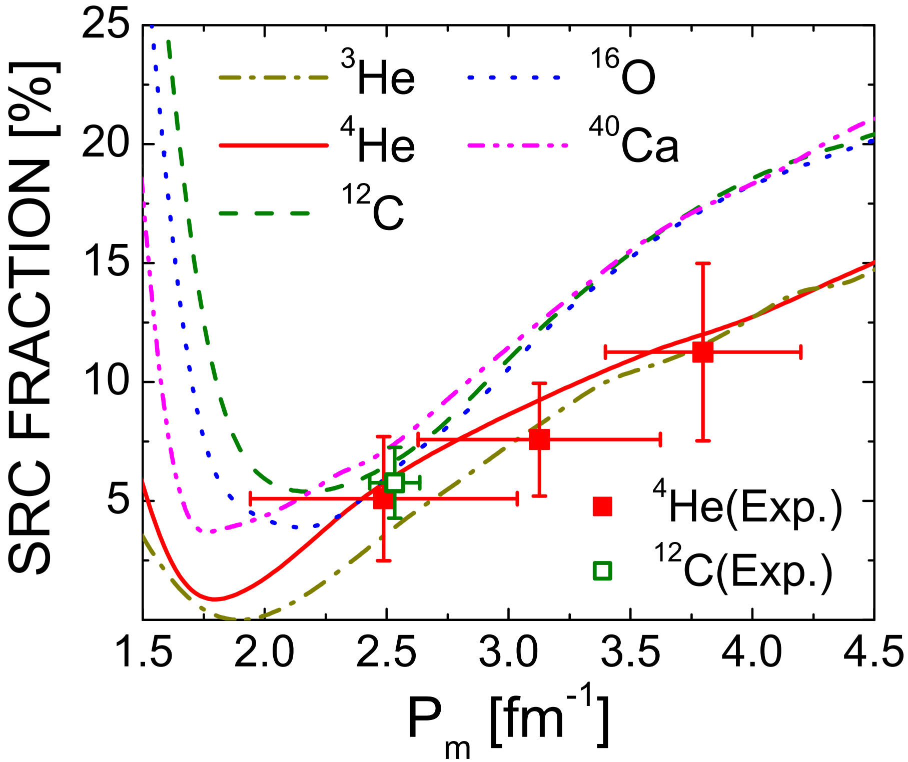


$$n^{(2)}(K_{CM})\ (k_{rel}\ \text{integrated})$$

M. Alvioli, C.Ciofi degli Atti, H. Morita, ***to appear***

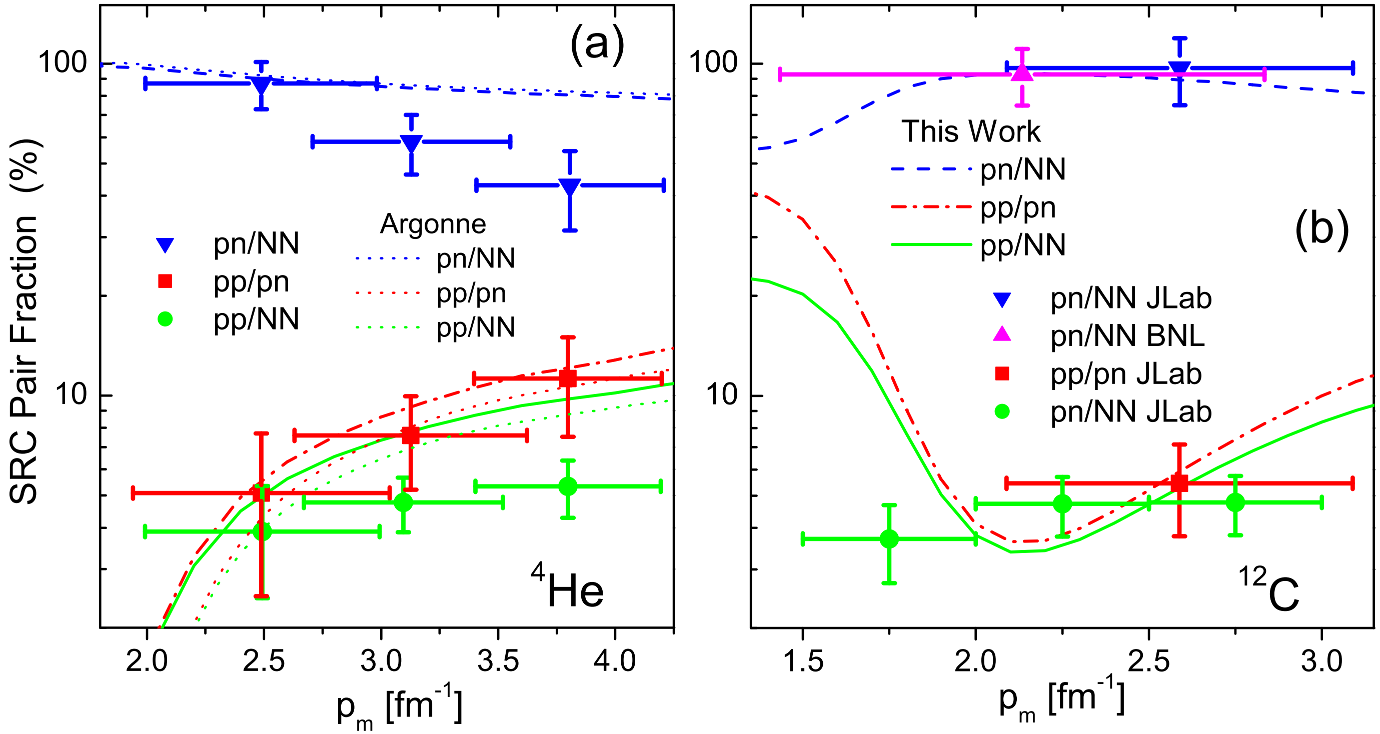
(***Preliminary Results***)

2. Two-body mom distrs results - comparison with data



$$n^{\mathbf{pp}}(k_{rel}, K_{CM} = 0) \,/\, n^{\mathbf{pn}}(k_{rel}, K_{CM} = 0)$$

M. Alvioli, C.Ciofi degli Atti, H. Morita, ***PRL 100*** (2008)
and M. Alvioli, C.Ciofi degli Atti, H. Morita, ***to appear***
(***Preliminary Results***)



$$\text{pp/pn: } n^{\mathbf{pp}}(k_{rel}, K_{CM} = 0)\, /\, n^{\mathbf{pn}}(k_{rel}, K_{CM} = 0)$$

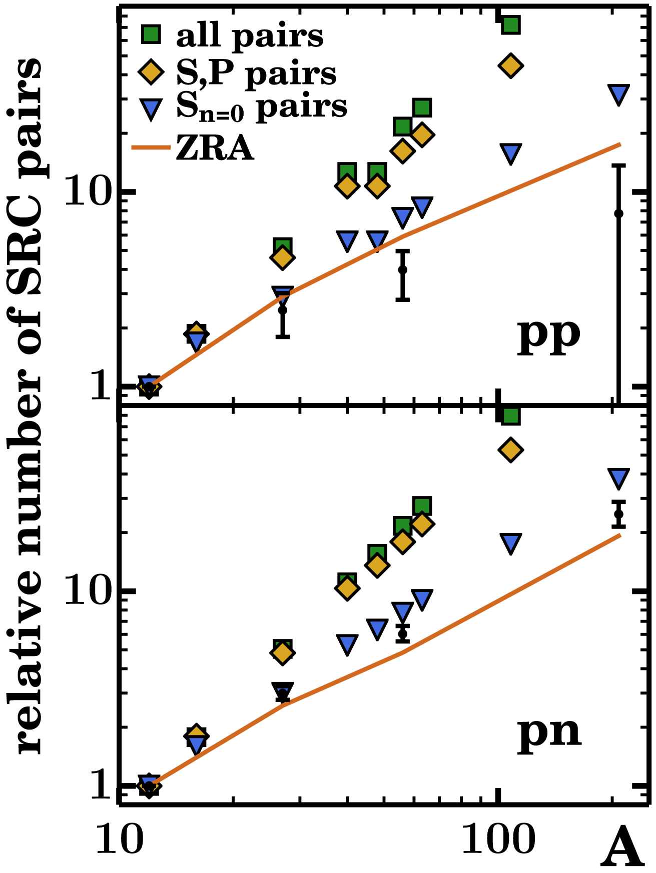
$$\text{pp/NN: } n^{\mathbf{pp}}(k_{rel}, K_{CM} = 0)\, /(\,2\, n^{\mathbf{pp}}(k_{rel}, K_{CM} = 0) + n^{\mathbf{pn}}(k_{rel}, K_{CM} = 0))$$

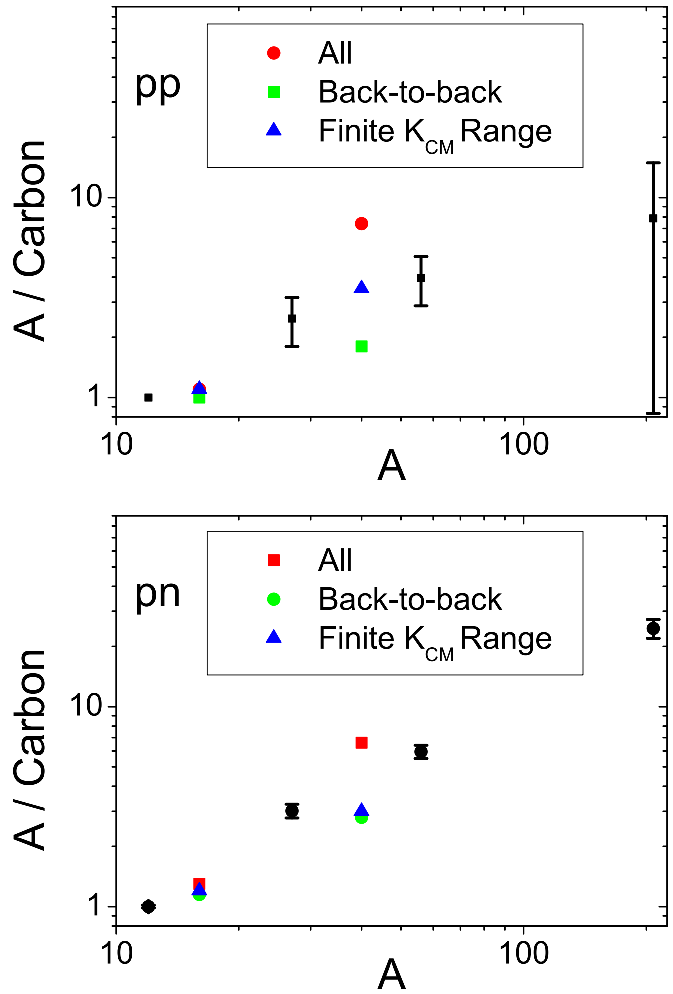
$$\text{pn/NN: } n^{\mathbf{pn}}(k_{rel}, K_{CM} = 0)\, /(\,2\, n^{\mathbf{pp}}(k_{rel}, K_{CM} = 0) + n^{\mathbf{pn}}(k_{rel}, K_{CM} = 0))$$

M. Alvioli, C.Ciofi degli Atti, H. Morita, ***to appear***

(***Preliminary Results***)

2. Two-body mom distrs results - comparison with data





Left: *C. Colle et al.*, ***PRC92*** *2015*

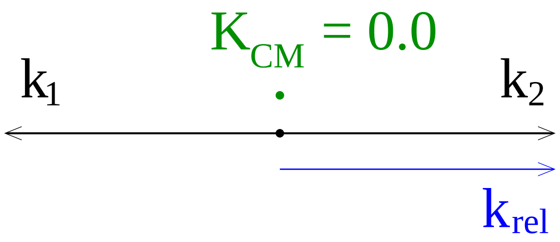
Right: *M. Alvioli, C.Ciofi degli Atti, H. Morita*, ***to appear***

(***Preliminary Results***)

ALL: $\int_{k_-}^{k^+} d\boldsymbol{k}_{rel} n^{(2)}(\boldsymbol{k}_{rel})$; B-T-B: $\int_{k_-}^{k^+} d\boldsymbol{k}_{rel} n^{(2)}(\boldsymbol{k}_{rel}, K_{CM}=0)$; Finite K_{CM}: $\int_{k_-}^{k^+}\int_{K_-}^{K^+} d\boldsymbol{k}_{rel} n^{(2)}(\boldsymbol{k}_{rel}, K_{CM})$

3. Two-Body Distrs: defining the 2B & 3B correlation region

parallel



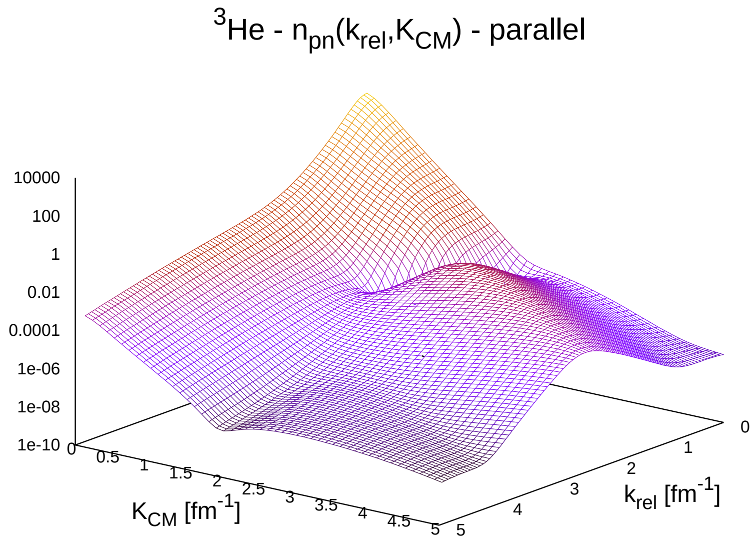
AV18 + UIX

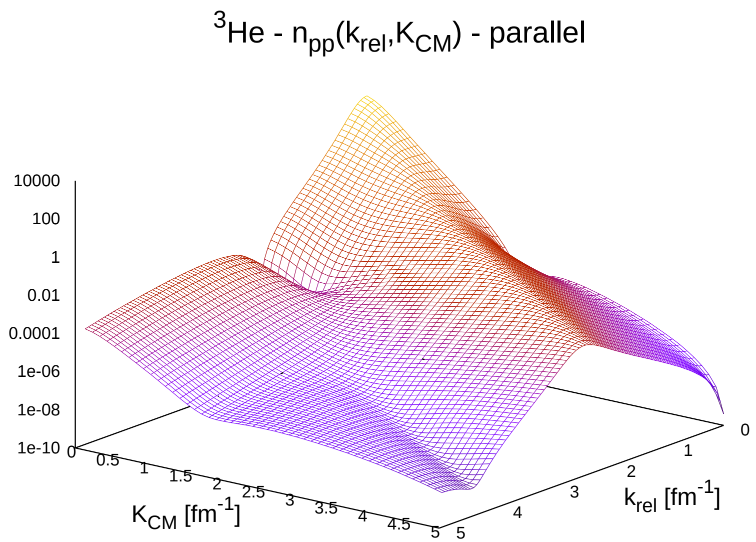
^3He wave function

from

Nogga *et al.*,

PRC ***67*** *(2003)*

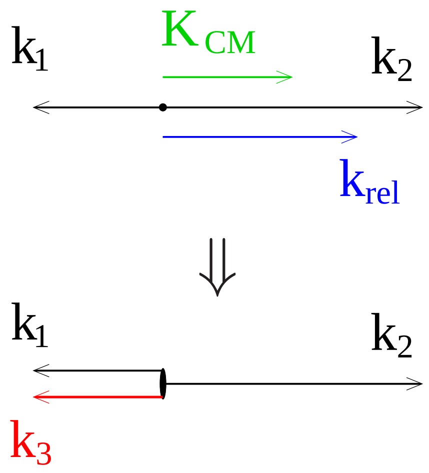




add a third nucleon:

$$k_1 + k_3 = k_2$$

$$|\boldsymbol{k}_1| = |\boldsymbol{k}_3|$$

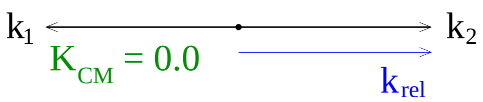


$\Downarrow$

$$\boxed{K_{CM} = \frac{2}{3} k_{rel}}$$

3. Two-Body Distrs: defining the 2B & 3B correlation region

perpendicular



AV18 + UIX

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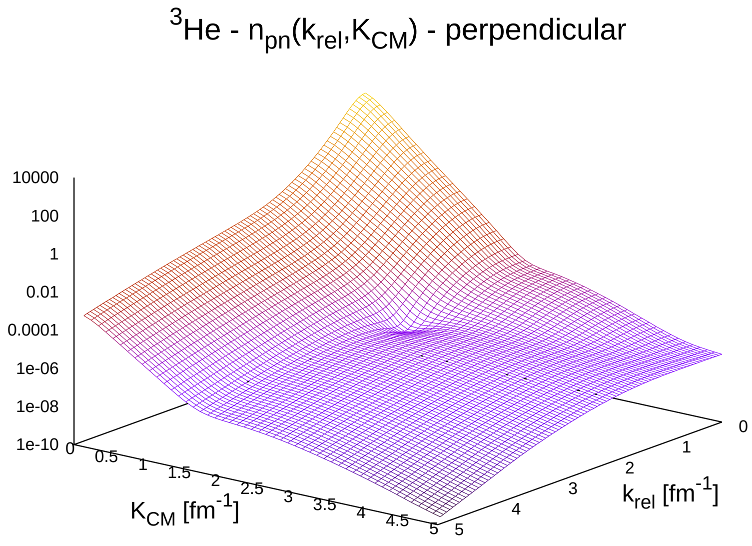
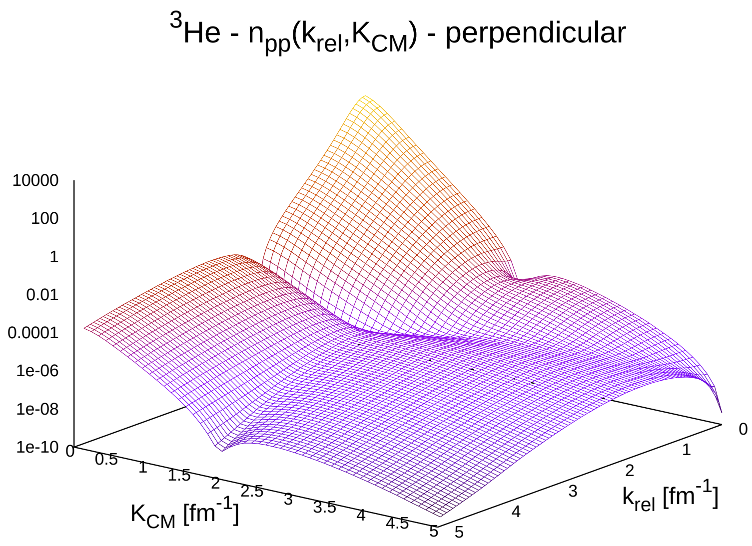
PRC ***67*** *(2003)*

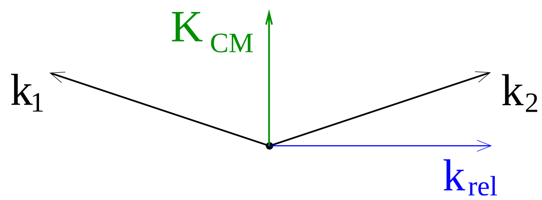




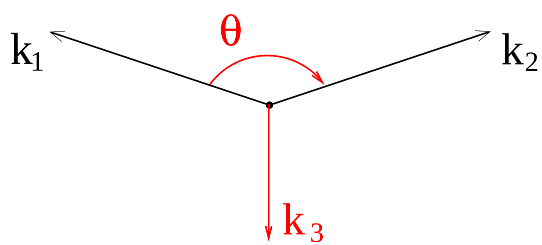
add a third nucleon:

$$\boldsymbol{k}_1 + \boldsymbol{k}_2 + \boldsymbol{k}_3 = 0$$

$$|\boldsymbol{k}_3| = |\boldsymbol{K}_{CM}|$$



$\Downarrow$

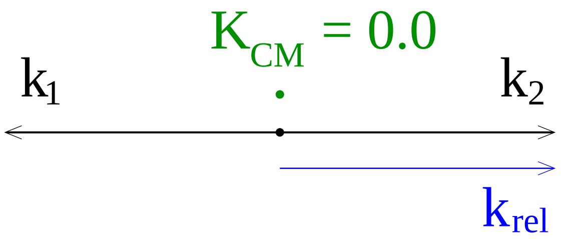


$\Downarrow$

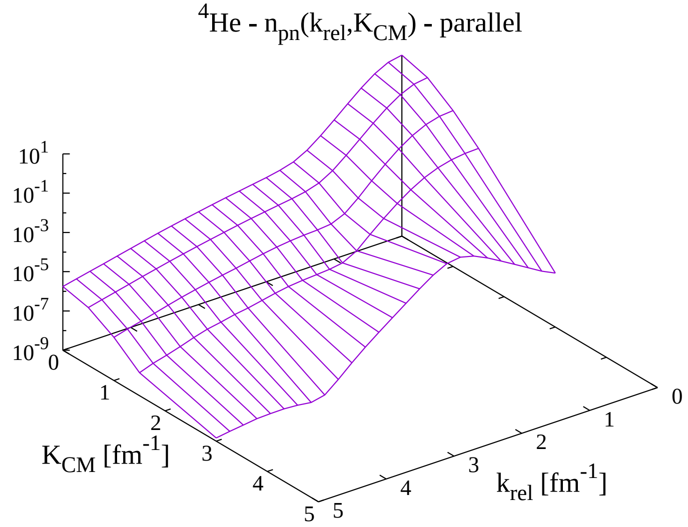
$$\boxed{K_{CM} = \frac{2}{\tan\theta/2} k_{rel}}$$

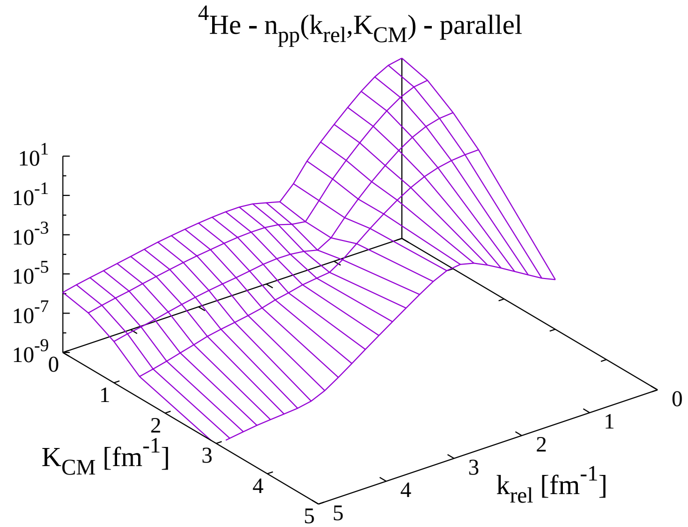
3. Two-Body Distrs: defining the 2B & 3B correlation region

parallel



AV8′ ATMS
^4He wave function
*Alvioli, Ciofi Morita PRC**85** (2012)*

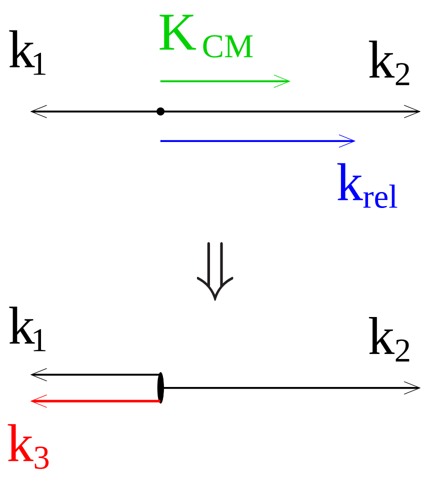




add a third nucleon:

$$k_1 + k_3 = k_2$$

$$|\boldsymbol{k}_1| = |\boldsymbol{k}_3|$$

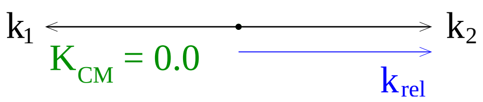


$\Downarrow$

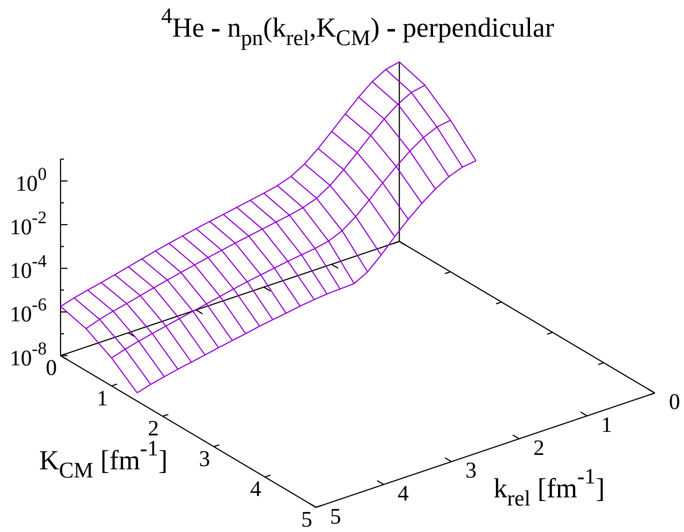
$$\boxed{K_{CM} = \frac{2}{3} k_{rel}}$$

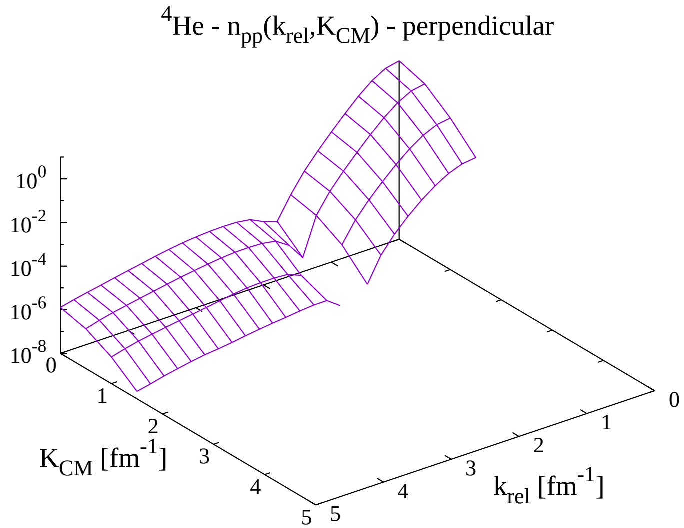
3. Two-Body Distrs: defining the 2B & 3B correlation region

perpendicular



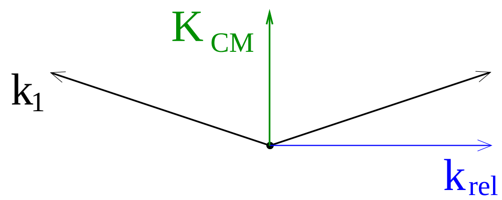
AV8′ ATMS
^4He wave function
Alvioli, Ciofi
Morita
*PRC**85** (2012)*



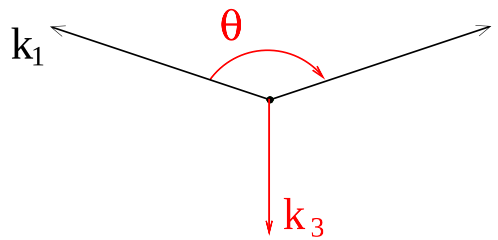


add a third nucleon:

$$\boldsymbol{k}_1 + \boldsymbol{k}_2 + \boldsymbol{k}_3 = 0$$
$$|\boldsymbol{k}_3| = |\boldsymbol{K}_{CM}|$$



$\Downarrow$



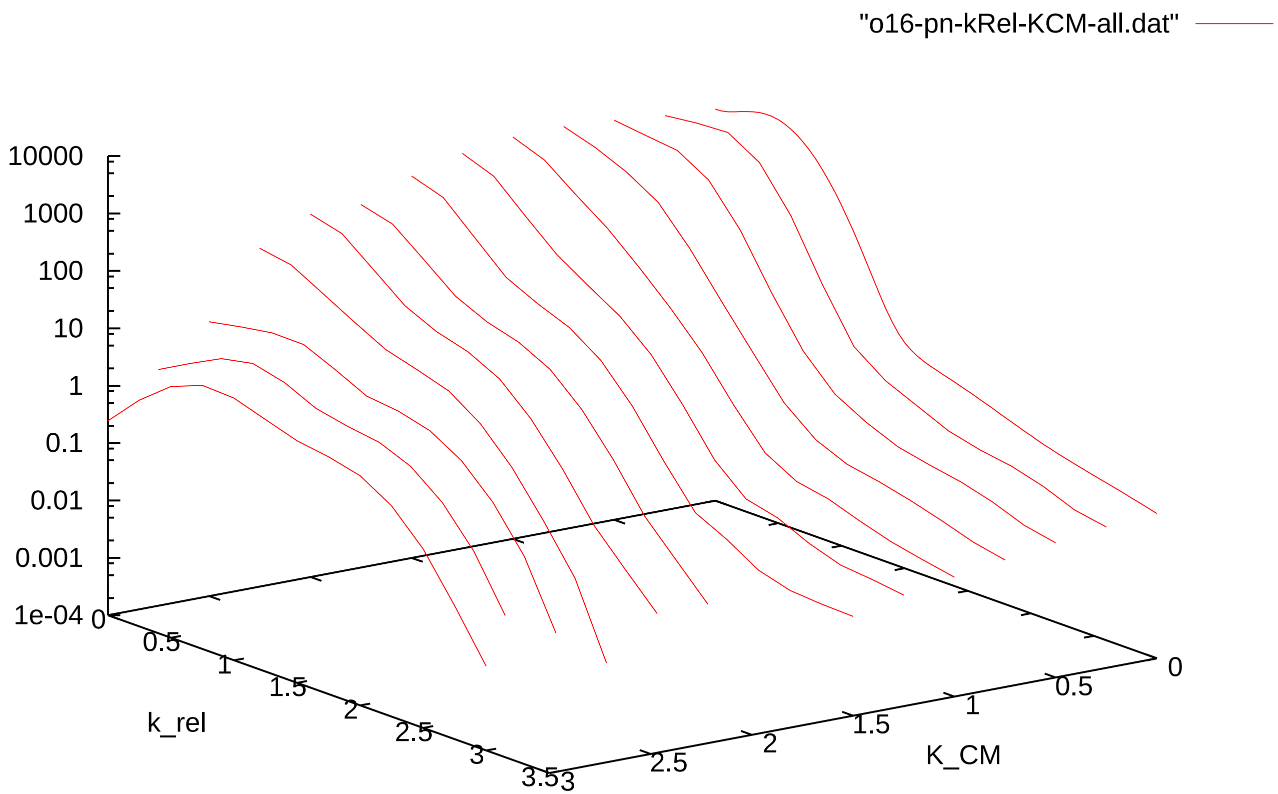
$\Downarrow$

$$\boxed{K_{CM} = \frac{2}{\tan\theta/2} k_{rel}}$$

3. Defining the 2B & 3B correlation region in k_{rel}, K_{CM}

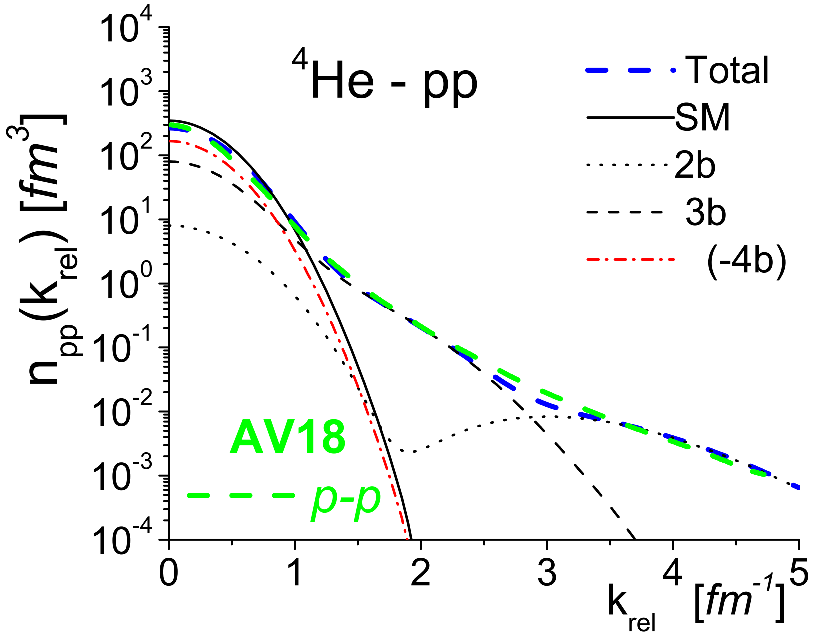
$$n(k_{rel}, K_{CM}, \Theta) \text{ for } {}^{16}O$$

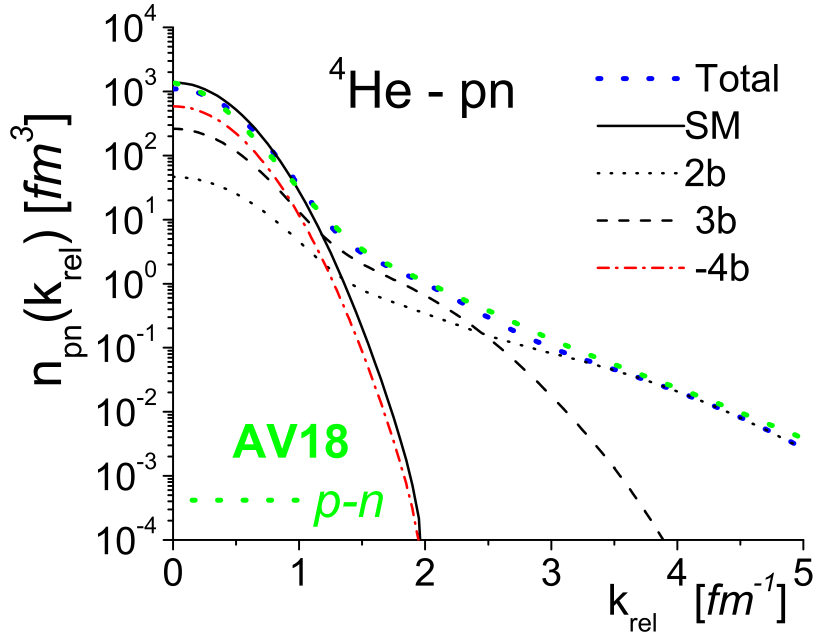
$$K_{CM} \text{ parallel to } k_{rel}, \text{ pn}$$



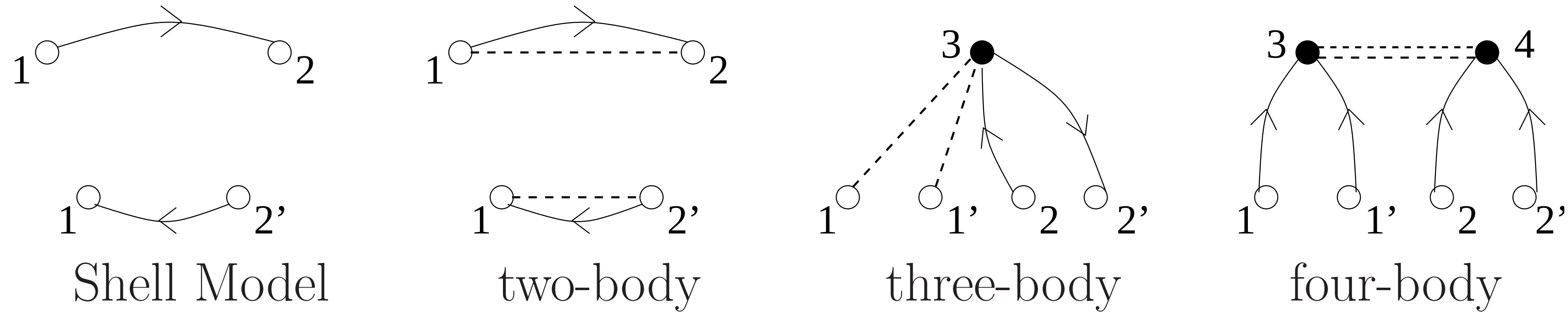
example: many-body contributions in ^{4}He 2BMD

$$n_{pN}(k_{rel}) = \int d\boldsymbol{K}_{CM}\; n_{pN}(\boldsymbol{k}_{rel}, \boldsymbol{K}_{CM})$$

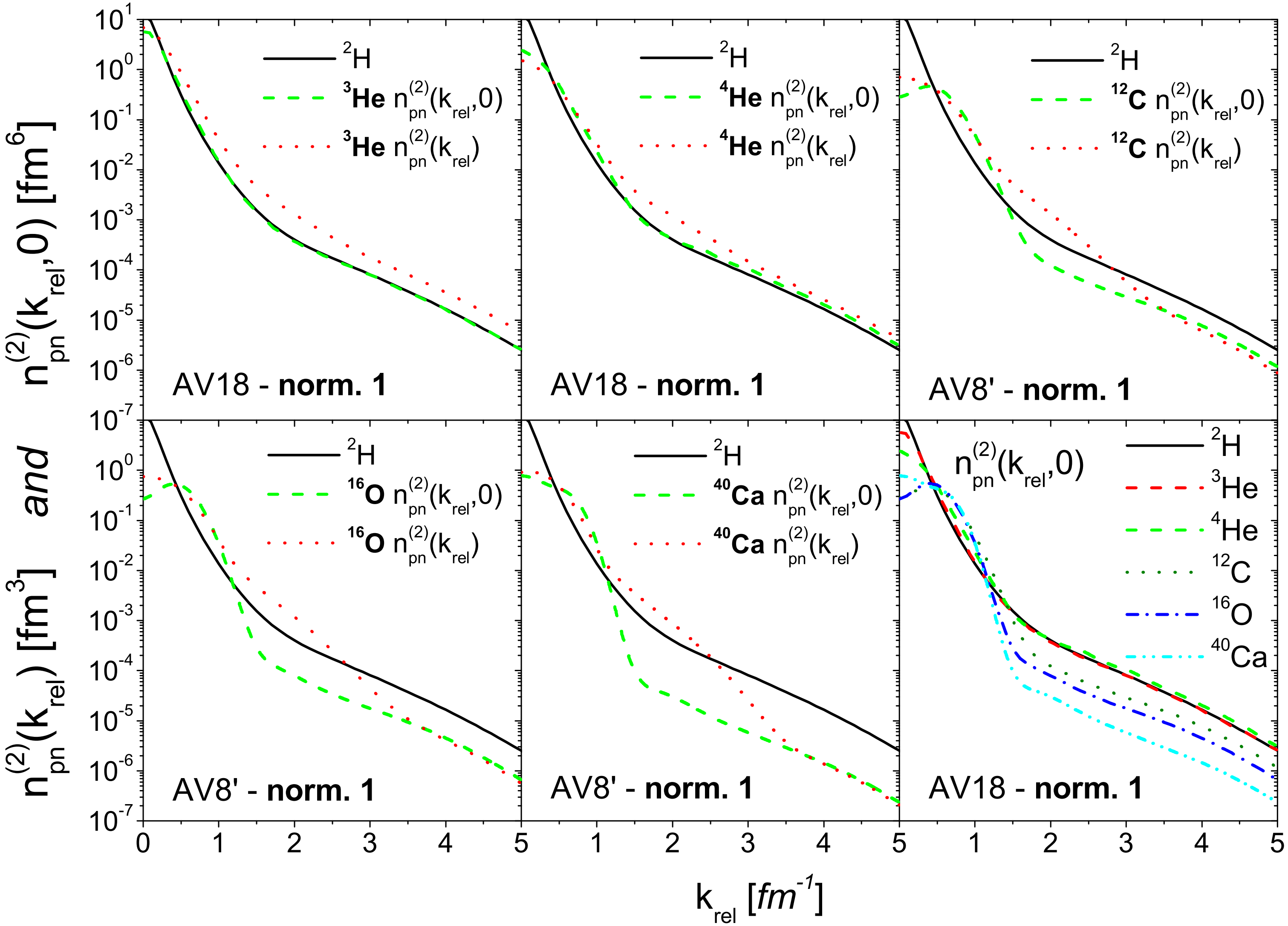


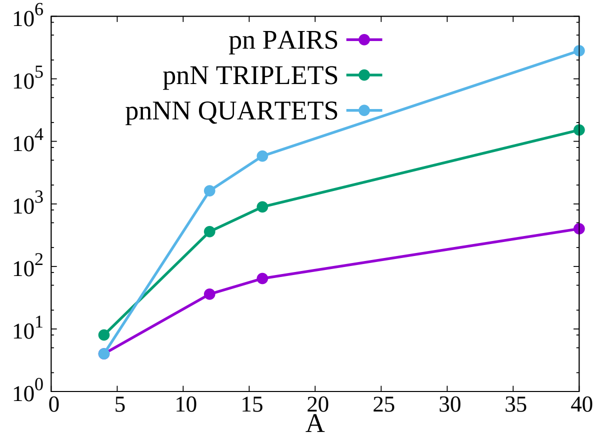


(**AV18**: *Schiavilla at al.* **PRL98** (2007))

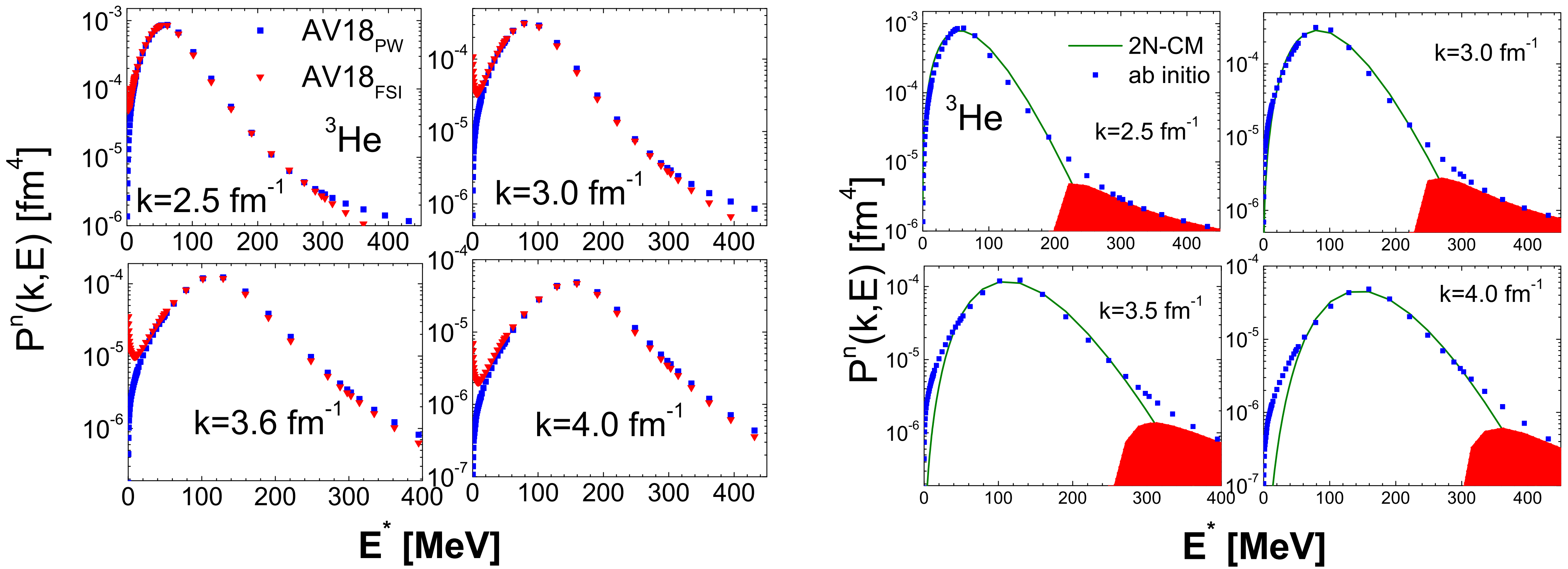


3. Defining the 2B & 3B correlation region in k_{rel}, K_{CM}





3. More on three-body correlations?



C. Ciofi degli Atti, ***Phys. Rep. 590 (2015)***

- Left: PW *v.s.* FSI $\longrightarrow$ FSI not relevant at high k and $E^\star$
- Right: exact *v.s.* TNC model $\longrightarrow$ thre-body correlations relevant only at high k and $E^\star$

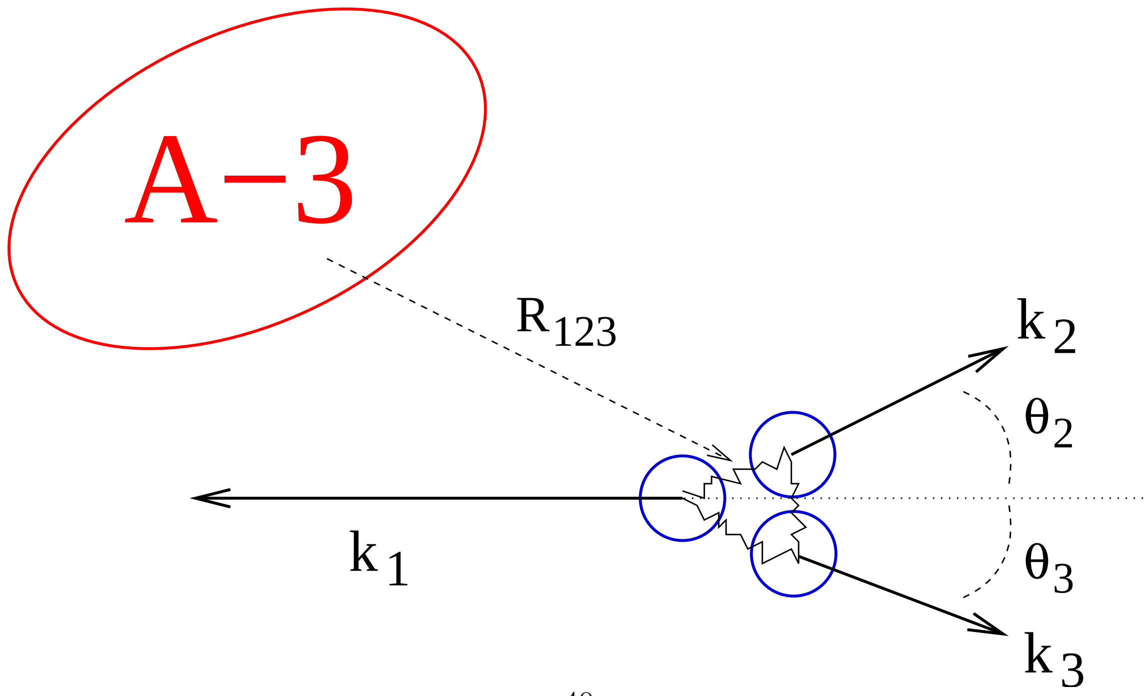
3. More on three-body correlations?

We can easily ☺ evaluate within the cluster expansion the three-body density

$$\rho^{(3)}(\boldsymbol{r}_1, \boldsymbol{r}_2, \boldsymbol{r}_3; \boldsymbol{r}'_1, \boldsymbol{r}'_2, \boldsymbol{r}'_3)$$

and calculate, for given values of $\boldsymbol{k}_1$, $\boldsymbol{k}_2$ and $\boldsymbol{k}_3$:

$$n(\boldsymbol{k}_1, \boldsymbol{k}_2, \boldsymbol{k}_3) = \frac{1}{(2\pi)^9} \int \prod_{i=1}^{3} d\boldsymbol{r}_i d\boldsymbol{r}'_i \, e^{i\sum_{j=1}^{3} \boldsymbol{k}_j \cdot (\boldsymbol{r}_j - \boldsymbol{r}'_j)} \rho^{(3)}(\boldsymbol{r}_1, \boldsymbol{r}_2, \boldsymbol{r}_3; \boldsymbol{r}'_1, \boldsymbol{r}'_2, \boldsymbol{r}'_3)$$



• at **first order** of the $\eta-$expansion, the **full correlated** `three-body` `mixed` density matrix expression is as follows:

$$\rho^{(3)}(\boldsymbol{r}_1,\boldsymbol{r}_2,\boldsymbol{r}_3;\boldsymbol{r}'_1,\boldsymbol{r}'_2,\boldsymbol{r}'_3) \;=\; \rho^{(3)}_{\mathbf{SM}}(\boldsymbol{r}_1,\dots) + \rho^{(3)}_{\mathbf{3b}}(\boldsymbol{r}_1,\dots) + \rho^{(3)}_{\mathbf{4b}}(\boldsymbol{r}_1,\dots) + \rho^{(3)}_{\mathbf{5b}}(\boldsymbol{r}_1,\dots)$$

with:

$$\begin{aligned}\rho^{(3)}_{\mathbf{SM}}(\boldsymbol{r}_1,\boldsymbol{r}_2,\boldsymbol{r}_3;\boldsymbol{r}'_1,\boldsymbol{r}'_2,\boldsymbol{r}'_3) \;=&\\ &\rho_o(\boldsymbol{r}_1,\boldsymbol{r}'_1)\,\rho_o(\boldsymbol{r}_2,\boldsymbol{r}'_2)\,\rho_o(\boldsymbol{r}_3,\boldsymbol{r}'_3)\;+\\ &-\rho_o(\boldsymbol{r}_1,\boldsymbol{r}'_1)\,\rho_o(\boldsymbol{r}_2,\boldsymbol{r}'_3)\,\rho_o(\boldsymbol{r}_3,\boldsymbol{r}'_2)\;+\\ &+\rho_o(\boldsymbol{r}_1,\boldsymbol{r}'_3)\,\rho_o(\boldsymbol{r}_2,\boldsymbol{r}'_1)\,\rho_o(\boldsymbol{r}_3,\boldsymbol{r}'_2)\;+\\ &-\rho_o(\boldsymbol{r}_1,\boldsymbol{r}'_3)\,\rho_o(\boldsymbol{r}_2,\boldsymbol{r}'_2)\,\rho_o(\boldsymbol{r}_3,\boldsymbol{r}'_1)\;+\\ &+\rho_o(\boldsymbol{r}_1,\boldsymbol{r}'_2)\,\rho_o(\boldsymbol{r}_2,\boldsymbol{r}'_3)\,\rho_o(\boldsymbol{r}_3,\boldsymbol{r}'_1)\;+\\ &-\rho_o(\boldsymbol{r}_1,\boldsymbol{r}'_2)\,\rho_o(\boldsymbol{r}_2,\boldsymbol{r}'_1)\,\rho_o(\boldsymbol{r}_3,\boldsymbol{r}'_3)\end{aligned}$$

$$\begin{aligned}\rho^{(3)}_{\mathbf{3b}}(\boldsymbol{r}_1,\boldsymbol{r}_2,\boldsymbol{r}_3;\boldsymbol{r}'_1,\boldsymbol{r}'_2,\boldsymbol{r}'_3) \;=&\\ &\left[\hat{\eta}(r_{12},r_{1'2'}) + \hat{\eta}(r_{13},r_{1'3'}) + \hat{\eta}(r_{23},r_{2'3'})\right]\cdot\\ &\cdot\left[\,\rho_o(\boldsymbol{r}_1,\boldsymbol{r}'_1)\,\rho_o(\boldsymbol{r}_2,\boldsymbol{r}'_2)\,\rho_o(\boldsymbol{r}_3,\boldsymbol{r}'_3)+\right.\\ &-\rho_o(\boldsymbol{r}_1,\boldsymbol{r}'_1)\,\rho_o(\boldsymbol{r}_2,\boldsymbol{r}'_3)\,\rho_o(\boldsymbol{r}_3,\boldsymbol{r}'_2)+\\ &+\rho_o(\boldsymbol{r}_1,\boldsymbol{r}'_3)\,\rho_o(\boldsymbol{r}_2,\boldsymbol{r}'_1)\,\rho_o(\boldsymbol{r}_3,\boldsymbol{r}'_2)+\\ &-\rho_o(\boldsymbol{r}_1,\boldsymbol{r}'_3)\,\rho_o(\boldsymbol{r}_2,\boldsymbol{r}'_2)\,\rho_o(\boldsymbol{r}_3,\boldsymbol{r}'_1)+\\ &+\rho_o(\boldsymbol{r}_1,\boldsymbol{r}'_2)\,\rho_o(\boldsymbol{r}_2,\boldsymbol{r}'_3)\,\rho_o(\boldsymbol{r}_3,\boldsymbol{r}'_1)+\\ &\left.-\rho_o(\boldsymbol{r}_1,\boldsymbol{r}'_2)\,\rho_o(\boldsymbol{r}_2,\boldsymbol{r}'_1)\,\rho_o(\boldsymbol{r}_3,\boldsymbol{r}'_3)\,\right]\end{aligned}$$

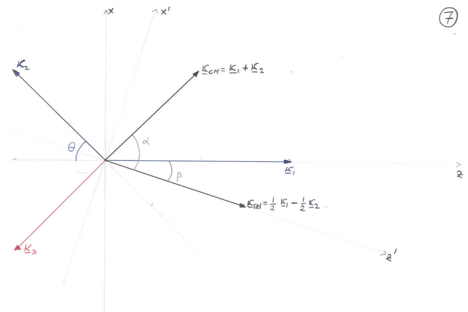
$$\begin{aligned}\rho^{(3)}_{\mathbf{4b}}(\boldsymbol{r}_1,\boldsymbol{r}_2,\boldsymbol{r}_3;\boldsymbol{r}'_1,\boldsymbol{r}'_2,\boldsymbol{r}'_3) \;=&\; \int d\boldsymbol{r}_{\mathbf{4}}\left[\hat{\eta}(r_{14},r_{1'4}) + \hat{\eta}(r_{24},r_{1'4}) + \hat{\eta}(r_{34},r_{3'4})\right]\cdot\\ &\cdot\sum_{\mathcal{P}\in\mathcal{C}}(-1)^{\mathcal{P}}\left[\,\rho_o(\boldsymbol{r}_1,\boldsymbol{r}_{\mathcal{P}1'})\,\rho_o(\boldsymbol{r}_2,\boldsymbol{r}_{\mathcal{P}2'})\,\rho_o(\boldsymbol{r}_3,\boldsymbol{r}_{\mathcal{P}3'})\,\rho_o(\boldsymbol{r}_4,\boldsymbol{r}_{\mathcal{P}4})\,\right] \qquad \text{20 terms}\end{aligned}$$

$$\begin{aligned}\rho^{(3)}_{\mathbf{5b}}(\boldsymbol{r}_1,\boldsymbol{r}_2,\boldsymbol{r}_3;\boldsymbol{r}'_1,\boldsymbol{r}'_2,\boldsymbol{r}'_3) \;=&\; \frac{1}{2}\int d\boldsymbol{r}_{\mathbf{4}}d\boldsymbol{r}_{\mathbf{5}}\,\hat{\eta}(r_{45},r_{45})\;\cdot\\ &\cdot\sum_{\mathcal{P}\in\mathcal{C}}(-1)^{\mathcal{P}}\left[\,\rho_o(\boldsymbol{r}_1,\boldsymbol{r}_{\mathcal{P}1'})\,\rho_o(\boldsymbol{r}_2,\boldsymbol{r}_{\mathcal{P}2'})\,\rho_o(\boldsymbol{r}_3,\boldsymbol{r}_{\mathcal{P}3'})\,\rho_o(\boldsymbol{r}_4,\boldsymbol{r}_{\mathcal{P}4})\,\rho_o(\boldsymbol{r}_5,\boldsymbol{r}_{\mathcal{P}5})\,\right] \qquad \text{108 terms}\end{aligned}$$

3. More on three-body correlations?

for given values of $\boldsymbol{k}_1$, $\boldsymbol{k}_2$ and $\boldsymbol{k}_3$:

$$n(\boldsymbol{k}_1,\boldsymbol{k}_2,\boldsymbol{k}_3) = \frac{1}{(2\pi)^9}\int \prod_{i=1}^{3} d\boldsymbol{r}_i d\boldsymbol{r}'_i \, e^{i\sum_{j=1}^{3}\boldsymbol{k}_j\cdot(\boldsymbol{r}_j-\boldsymbol{r}'_j)}\rho^{(3)}(\boldsymbol{r}_1,\boldsymbol{r}_2,\boldsymbol{r}_3;\boldsymbol{r}'_1,\boldsymbol{r}'_2,\boldsymbol{r}'_3)$$



or given values of $\boldsymbol{k}_{rel}$, $\boldsymbol{K}_{CM}$ and $\boldsymbol{k}$:

$$n(\boldsymbol{k}_{rel},\boldsymbol{K}_{CM},\boldsymbol{k}) = \frac{1}{(2\pi)^9}\int d\boldsymbol{x}d\boldsymbol{x}'d\boldsymbol{y}d\boldsymbol{y}'d\boldsymbol{z}d\boldsymbol{z}' \cdot$$

$$\cdot\, e^{i\boldsymbol{k}_{rel}\cdot(\boldsymbol{x}-\boldsymbol{x}')+i\boldsymbol{K}_{CM}\cdot(\boldsymbol{y}-\boldsymbol{y}')+i\boldsymbol{k}\cdot(\boldsymbol{z}-\boldsymbol{z}')}\rho^{(3)}(\boldsymbol{r}_1,\boldsymbol{r}_2,\boldsymbol{r}_3;\boldsymbol{r}'_1,\boldsymbol{r}'_2,\boldsymbol{r}'_3)$$

with

$$\begin{aligned}\boldsymbol{k}_{rel} &= (\boldsymbol{k}_1 - \boldsymbol{k}_2)/2 \\ \boldsymbol{K}_{CM} &= \boldsymbol{k}_1 + \boldsymbol{k}_2 \\ \boldsymbol{k} &= \boldsymbol{k}_1 + \boldsymbol{k}_2 + \boldsymbol{k}_3\end{aligned} \qquad \begin{aligned}\boldsymbol{x} &= \boldsymbol{r}_1/2 + \boldsymbol{r}_2/2 - \boldsymbol{r}_3 \\ \boldsymbol{y} &= \boldsymbol{r}_1 - \boldsymbol{r}_2 \\ \boldsymbol{z} &= 3\,\boldsymbol{r}_3/2\end{aligned}$$

Summary

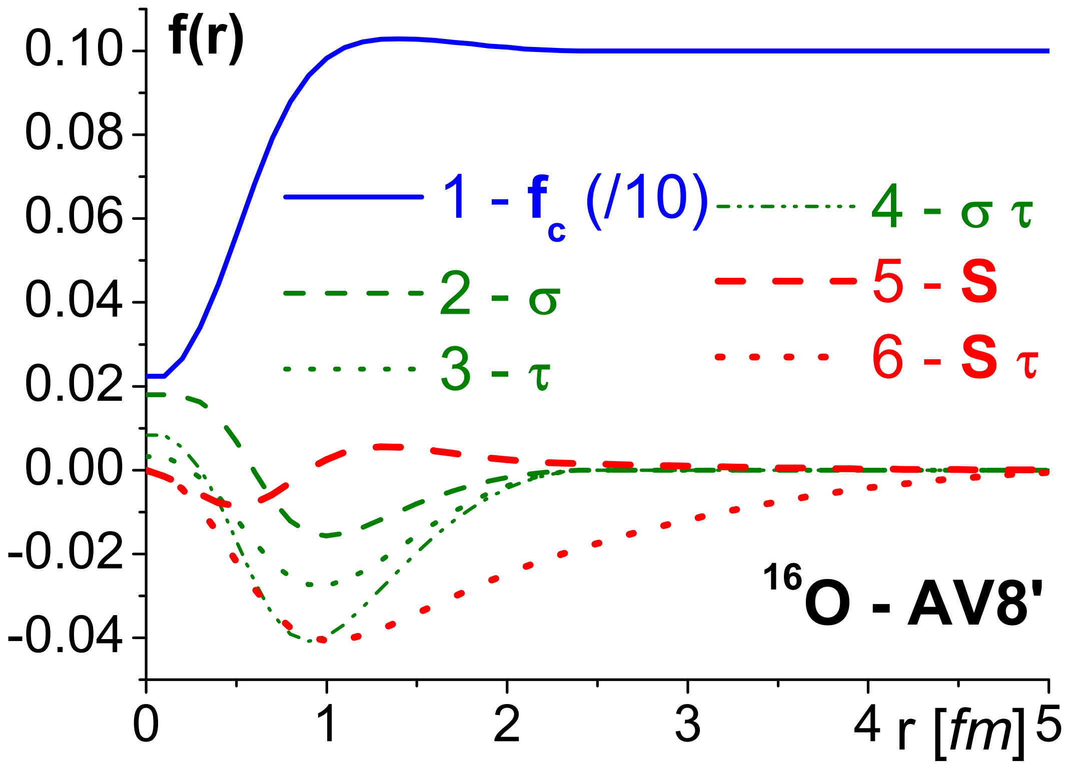
- Reliable calculations can be performed with realistic wave functions embodying full *short range* structure & *high momentum* components
- Few-body nuclei with exact wave functions; many-body within cluster expansion approximation: any one- and two-body quantity can be calculated
- *Universality of correlations*: I) rise of the nucleus-to-deuteron ratio of one-body $n(k)$ understood by analysing the ST quantum number of NN pairs $\longrightarrow$ update of the TNC factorization model
- *Universality of correlations*: II) scaling of $K_{CM} = 0$ two-body momentum distributions to the deuteron one; exact scaling if appropriate (ST)=(10) quantum numbers for the pair are selected $\longrightarrow$ update of the TNC factorization model
- three-body momentum distributions can be calculated within the same frameworks, and comparable accuracy, to investigate three-nucleons effects for selected configurations, *i.e.* $k_1 + k_2 + k_3 = 0$ and *high* $k_1 \longrightarrow$ beyond TNC factorization model

Additional Slides

Ground state energy: ^{16}O - Argonne $V8'$

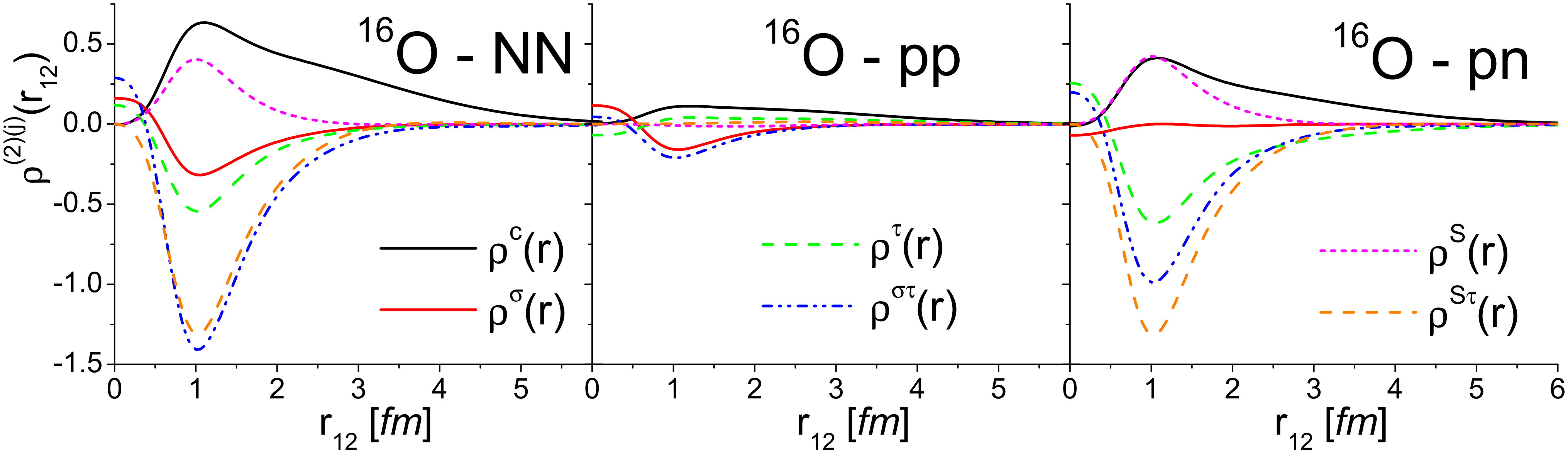
	$< V_c >$	$< V_\sigma >$	$< V_\tau >$	$< V_{\sigma\tau} >$	$< V_S >$	$< V_{S\tau} >$	$< \mathbf{V} >$	$< \mathbf{T} >$	**E**	**E/A** MeV
$\eta - exp$	0.19	-35.88	-9.47	-171.32	-0.003	-172.89	-389.40	323.50	**-65.90**	-4.12
FHNC	0.694	-40.13	-10.61	-180.00	-0.07	-160.32	-390.30	325.18	**-65.12**	-4.07

correlation functions: *Central*, *Spin-Isospin*, *Tensor*



Potential energy: *pn* and *pp* contributions

$$\langle V \rangle_{pN} = \sum_{i<j} V_{ij} = \sum_j \int d\boldsymbol{r}_{12}\, v^{(j)}_{pN}(r_{12})\, \rho^{(2)(j)}_{pN}(r_{12})$$



A	$\langle V\rangle_{pp} (= \langle V\rangle_{nn})$	$\langle V\rangle_{pn}$
16	8%	83%
40	9%	82%

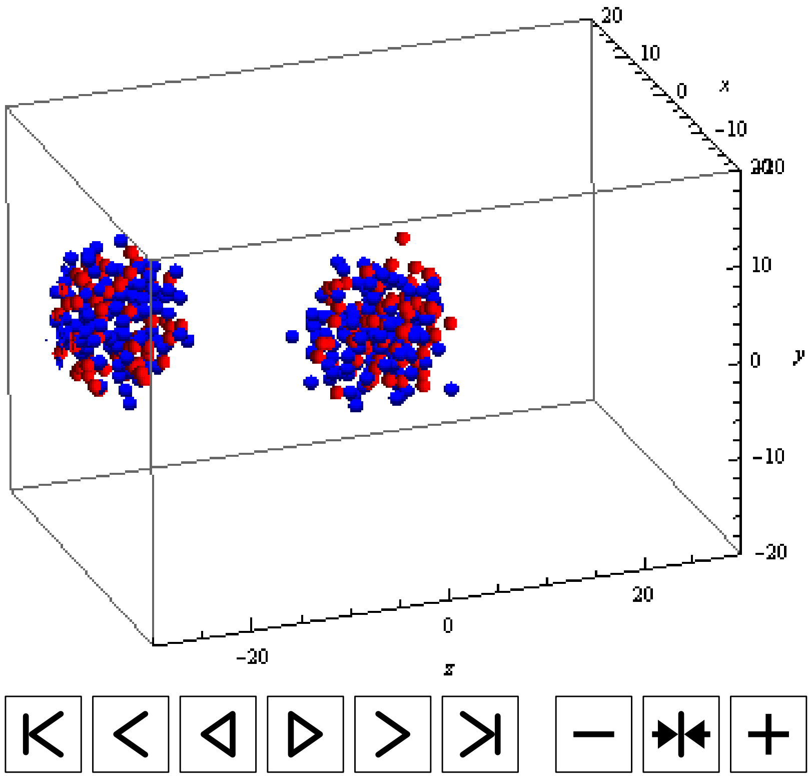
mostly pn pairs

switching off $\Rightarrow$ correlations

A	$\langle V\rangle_{pp} (= \langle V\rangle_{nn})$	$\langle V\rangle_{pn}$
16	23%	53%
40	24%	51%

$\Rightarrow$ proportional to # of pairs

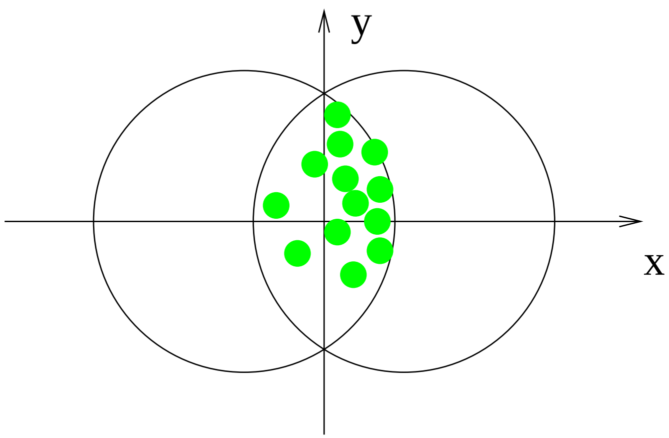
4. Monte Carlo Glauber description - Fluctuations



- Fluctuations effects on geometry investigated through participant matter distribution moments and their dispersion

$$\epsilon_n = -\frac{\langle w(r)\ \cos n(\phi - \psi_n)\rangle}{\langle w(r)\rangle}$$

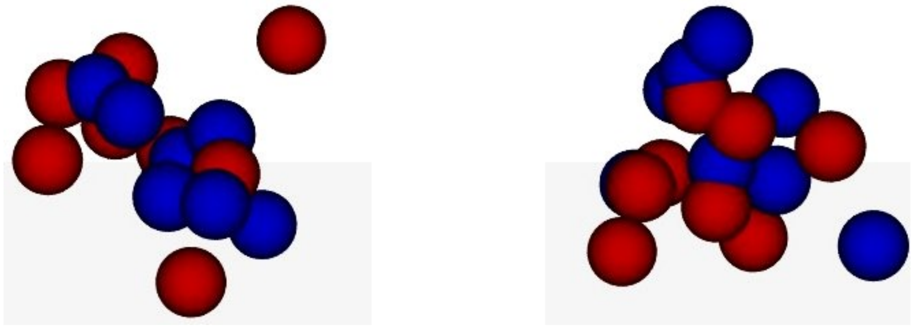
$$\Delta\epsilon_n = \sqrt{\frac{\sum(\epsilon_n^i - \langle\epsilon_i\rangle)^2}{N}}$$



$\longrightarrow$ participant nucleons • in transverse plane

4. A Monte Carlo generator for nucleon configurations

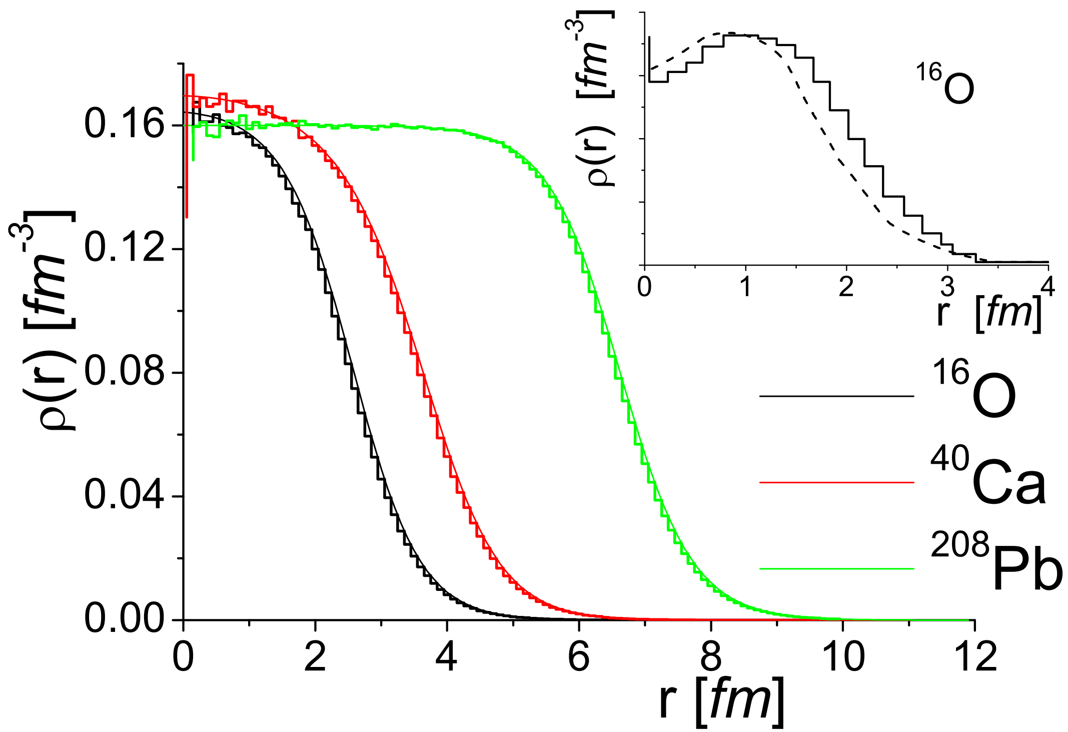
- Configurations generated according to the independent particle model contain *overlapping nucleons*



- Ad-hoc *hard-core* rejection methods avoids overlapping nucleons but is not linked to realistic correlations and do not reproduce two-body density

- We developed a Metropolis code which includes **realistic NN correlations functions** in a way which is consistent with the input one-body density

- We also have a two-body density close to the one obtained in microscopic calculations of w.f.



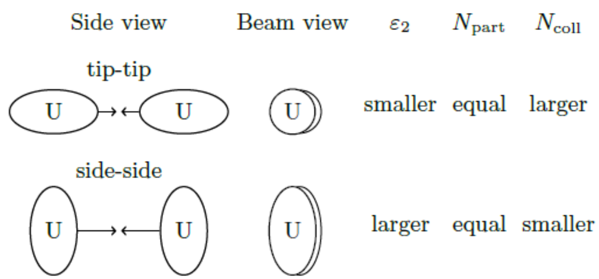
4. Latest updates of nuclear configurations - I

- ***Nucleus deformation*** – for ^{238}U we use a modified WS profile:

$$\rho(r) = \frac{\rho_0}{1 + e^{(r-R_0)/a}} \quad \longrightarrow \quad \rho(r,\theta) = \frac{\rho_0}{1 + e^{(r-R_0-R_0\beta_2 Y_{20}(\theta)-R_0\beta_4 Y_{40}(\theta))/a}}$$

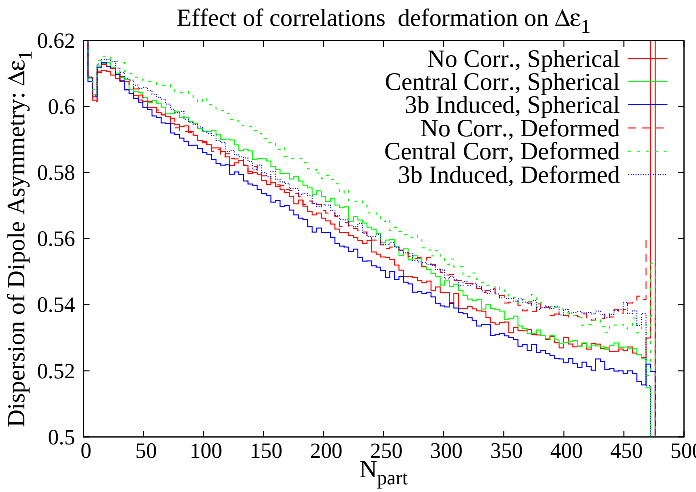
$$Y_{20}(\theta) = \frac{1}{4\,r^2}\sqrt{\frac{5}{\pi}}\left(2z^2 - x^2 - y^2\right)$$

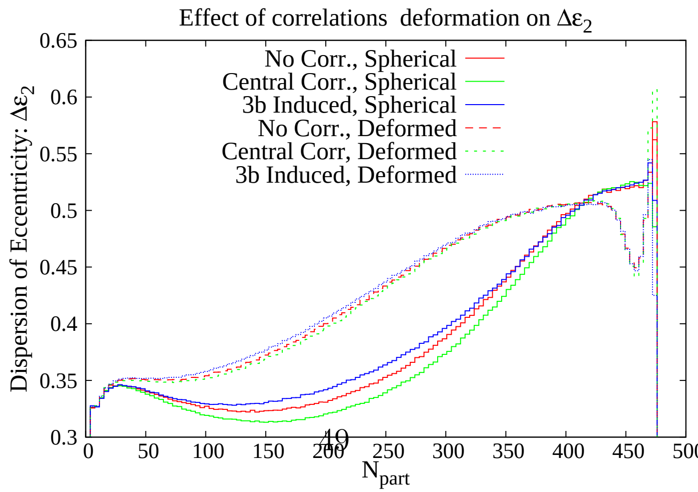
$$Y_{40}(\theta) = \frac{1}{16\,r^4}\sqrt{\frac{9}{\pi}}\left(35z^4 - 30z^2r^2 + 3r^4\right)$$

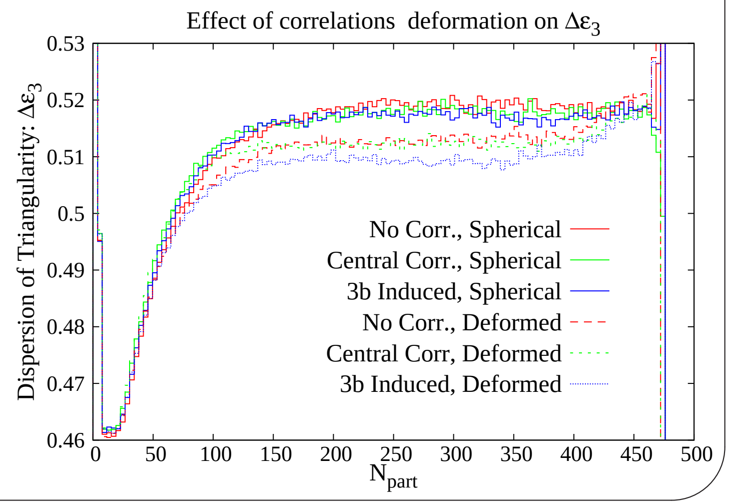


(*P. Filip, R. Lednicky, H. Masui, N. Xu Phys. Lett.* ***C80*** *(2009)*)

- deformation effect on dispersion of moments (*unpublished*):







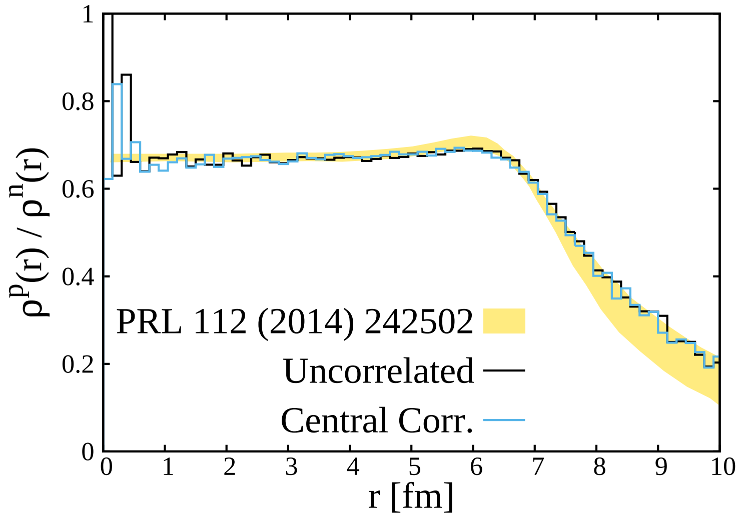
4. Latest updates of nuclear configurations - II

- ***Neutron skin*** – **p/n** profiles for ^{208}Pb:

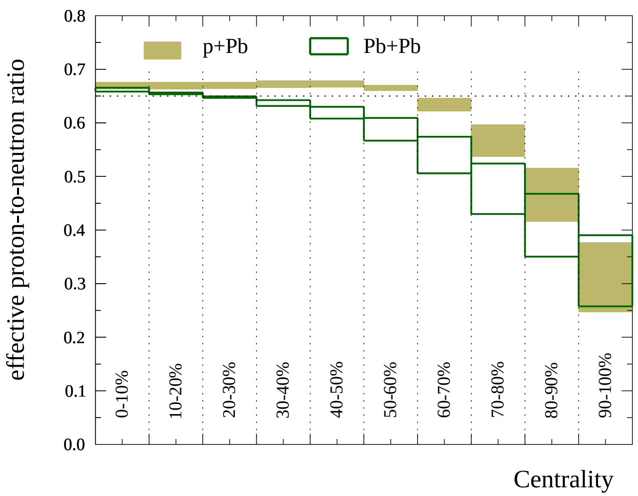
$$\rho(r) = \rho_0^{(p,n)} \; / \; \left(1 + e^{(r - R_0^{p,n})/a^{p,n}}\right)$$

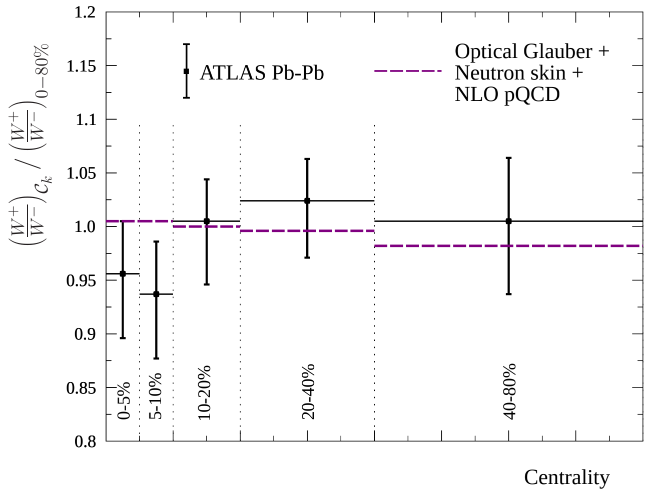
$$(\rho_0^p, R_0^p, a_0^p) \;=\; (\text{"82"}, 6.680 fm, 0.447 fm)$$
$$(\rho_0^n, R_0^n, a_0^n) \;=\; (\text{"126"}, 6.700 fm, 0.550 fm)$$

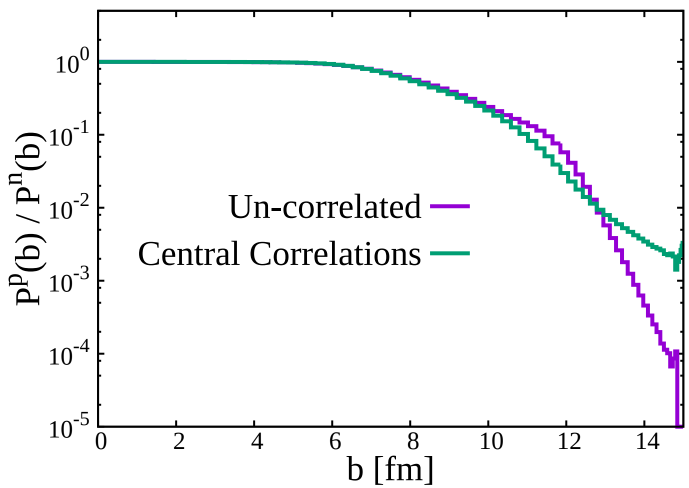
(*C.M. Tarbert et al., Phys. Rev. Lett.* ***112*** *(2014)*)



- additional tool for determination of centrality:





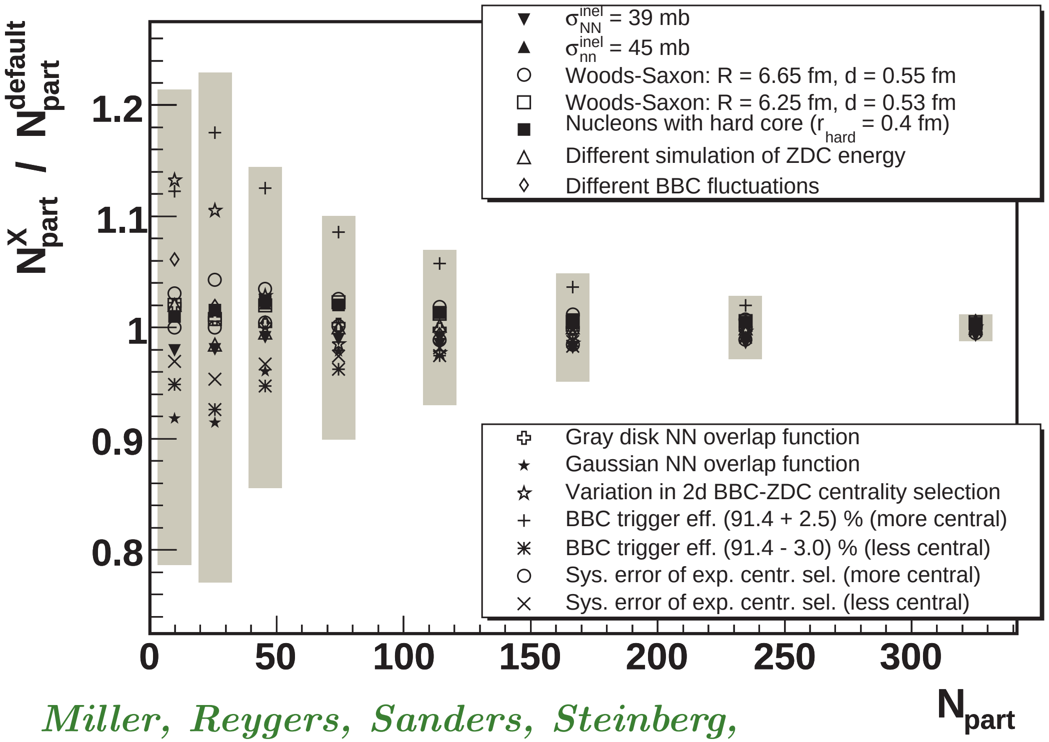


*H. Paukkunen, PL****B745*** *(2015)*

Alvioli, Strikman (unpub.)

- The smearing of impact parameter is expected to reduce the p/n difference

4. Monte Carlo Glauber (MCG) description: fluctuations



Miller, Reygers, Sanders, Steinberg, Ann. Rev. Nucl. Part. Sci. 57 (2007)

effects of different sources of fluctuations and parameter dependencies within MGC and detector simulation

We will focus on initial fluctuations due to:

- inclusion of NN correlations in preparing nuclear configurations
- avoid black-disk approximation for NN scattering ($r_{ij} < \sqrt{\sigma^{in}_{NN}/\pi}$)

.. and apply these methods in:

- ***spectator nucleons*** excitation and emission for studies of centrality
- fluctuations effects on ecentricity and triangularity of ***participant*** nucleons distribution