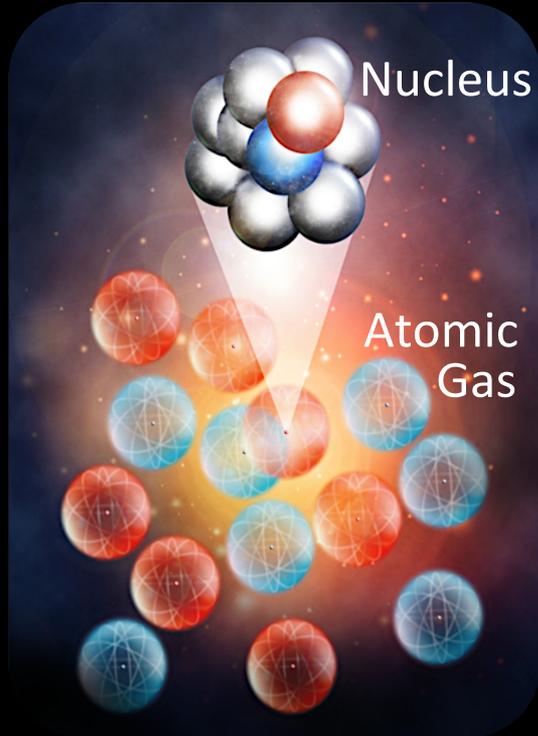
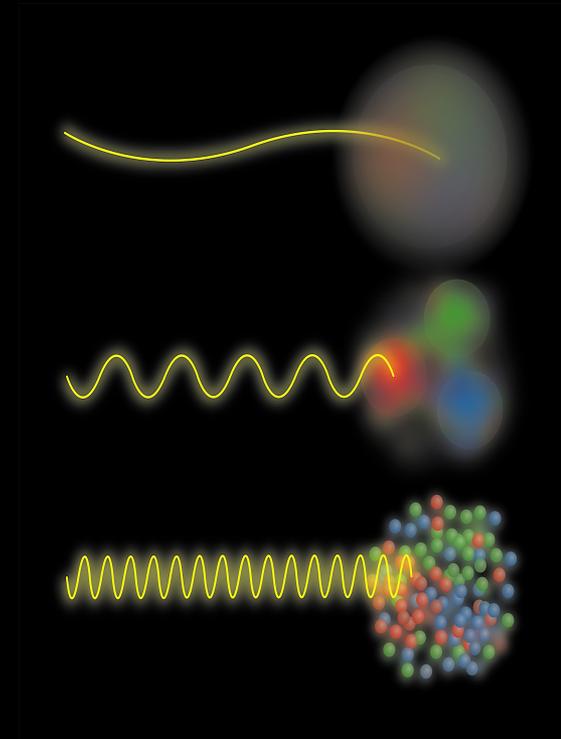
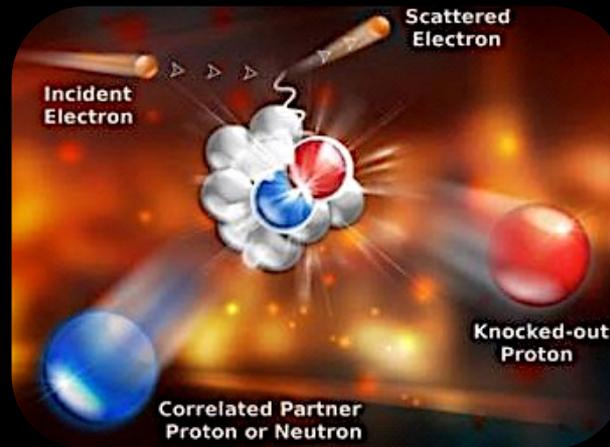


Contact Interactions and the Nuclear Symmetry Energy



Or Hen
Tel-Aviv (-> MIT)



Two-component interacting Fermi systems

The contact term

Please forget about nuclear physics
for a moment





A concept developed for a *dilute* two-component Fermi systems with a short-range interaction.

$$\text{dilute} \equiv r_{\text{eff}} \ll a, d$$

Scattering length

Distance between fermions



The Contact and Universal Relations



A concept developed for a dilute two-component Fermi systems with a short-range interaction.

$$\text{dilute} \equiv r_{\text{eff}} \ll a, d$$

Scattering length

Distance between fermions

These systems have a high-momentum tail:

$$n(k) = C / k^4 \quad \text{for } k > k_F$$

C is the contact term



A concept developed for a dilute two-component Fermi systems with a short-range interaction.

$$\text{dilute} \equiv r_{\text{eff}} \ll a, d$$

Scattering length \rightarrow a
Distance between fermions \rightarrow d

These systems have a high-momentum tail:

$$n(k) = C / k^4 \quad \text{for } k > k_F$$

\swarrow
C is the contact term

Tan's Contact term:

1. Measures the number of SRC different fermion pairs.
2. Determines the thermodynamics through a series of universal relations.

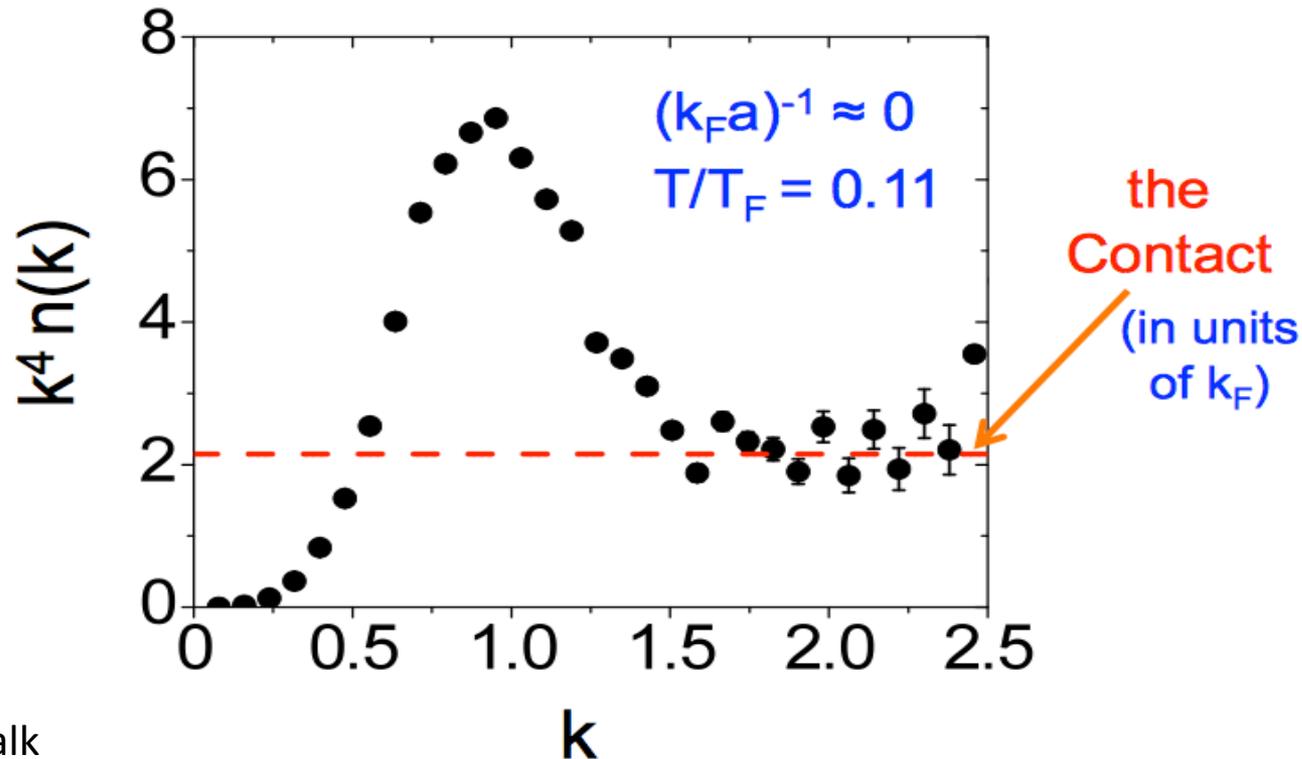


Experimental Validation



Two spin-state mixtures of ultra-cold ^{40}K and ^6Li atomic gas systems.

=> extracted the contact and verified the universal relations



Adapted from a talk
by Debbie Jin (JILA)

Kuhnle et al. PRL **105**, 070402 (2010)



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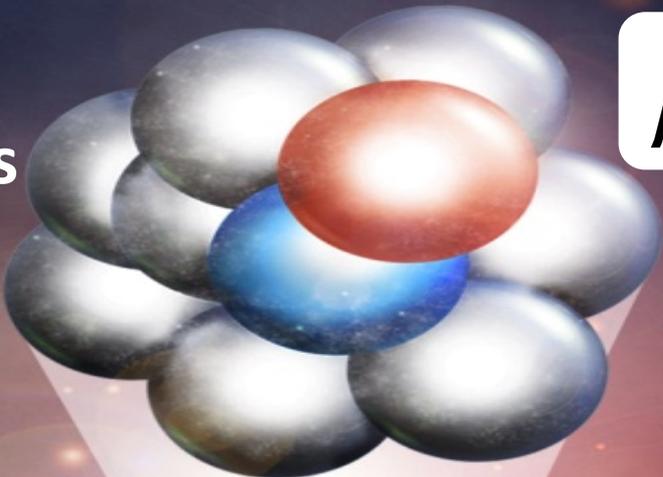
What About
a Nuclear
Contact ?

Adapted from a talk
by Debbie Jin (JILA)

PHYSICS SCIENCE CENTER 105, 070402 (2010)

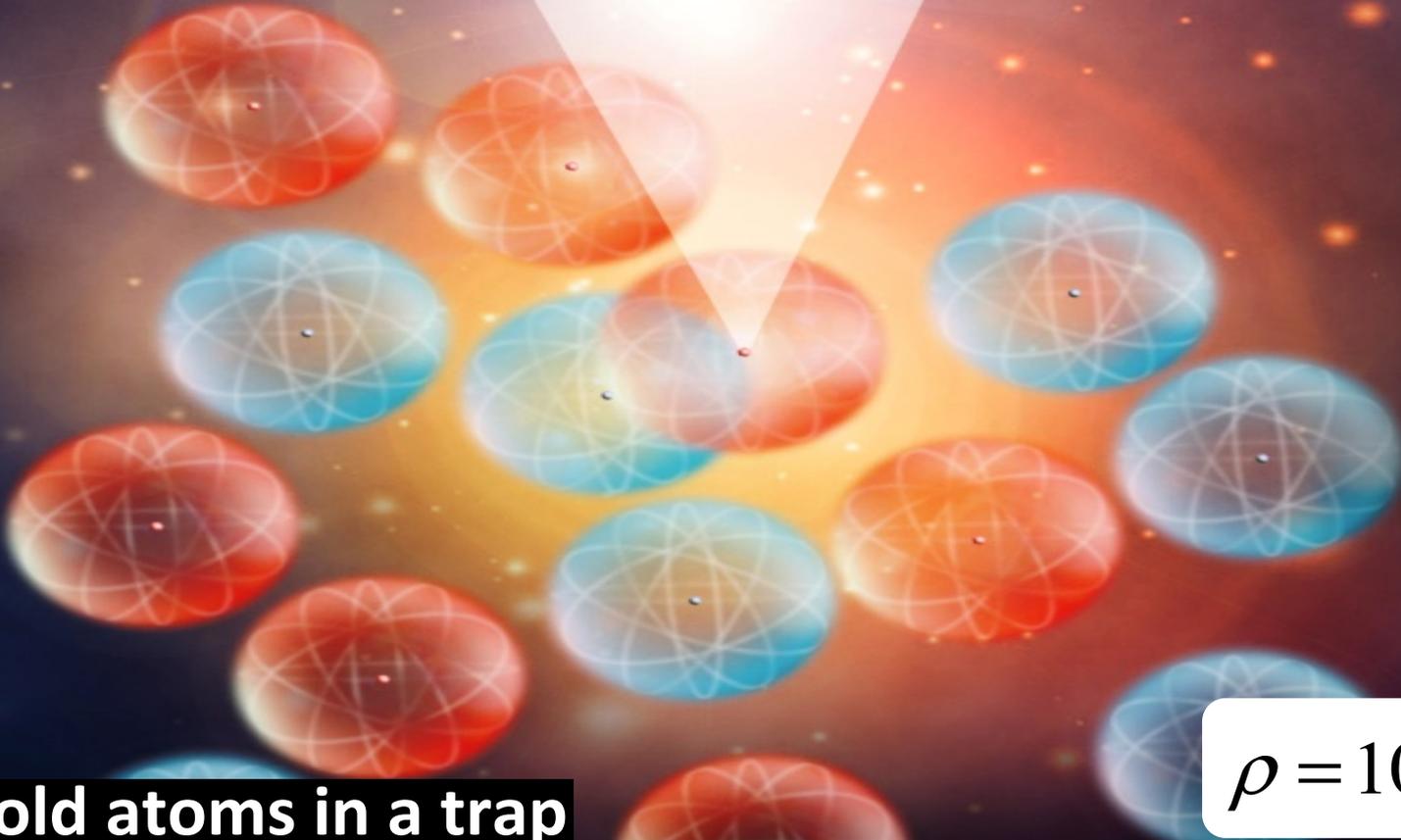
$$\rho = 10^{44} \text{ m}^{-3}$$

Nucleons in a nucleus



Ultra-cold atoms in a trap

$$\rho = 10^{21} \text{ m}^{-3}$$





$$\sigma_1 \approx 1 \text{ person/m}^2$$



$$\sigma_1 \approx 1 \text{ person/m}^2$$



$$\sigma_2 \approx 1 \text{ person/km}^2$$

$$\frac{\sigma_1}{\sigma_2} \approx 10^6$$



A Nuclear Contact?



Are nuclei dilute? (i.e. $r_{\text{eff}} \ll a, d$)

$$d = \left(\frac{\rho}{2} \right)^{-1/3} \approx 2.3 \text{ fm}$$

$$r_{\text{eff}} \approx \frac{\hbar}{2 \cdot m_{\pi} \cdot c} \approx 0.7 \text{ fm} \text{ [Tensor force]}$$

$$a(^3S_1) = 5.42 \text{ fm}$$

[The high-momentum tail is predominantly

$$^3S_1 (^3D_1)]$$



A Nuclear Contact?



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$$a(^3S_1) = 5.42 \text{ fm}$$

$$r_{\text{eff}} (0.7 \text{ fm}) < d(2.3 \text{ fm}), a(5.4 \text{ fm})$$



A Nuclear Contact?



Is there $1/k^4$ scaling regardless?

$$1.5k_F < k < 3k_F$$

$$n_A(k) = a_2(A/d) \cdot n_d(k)$$

Constant

Deuteron
Momentum
Distribution



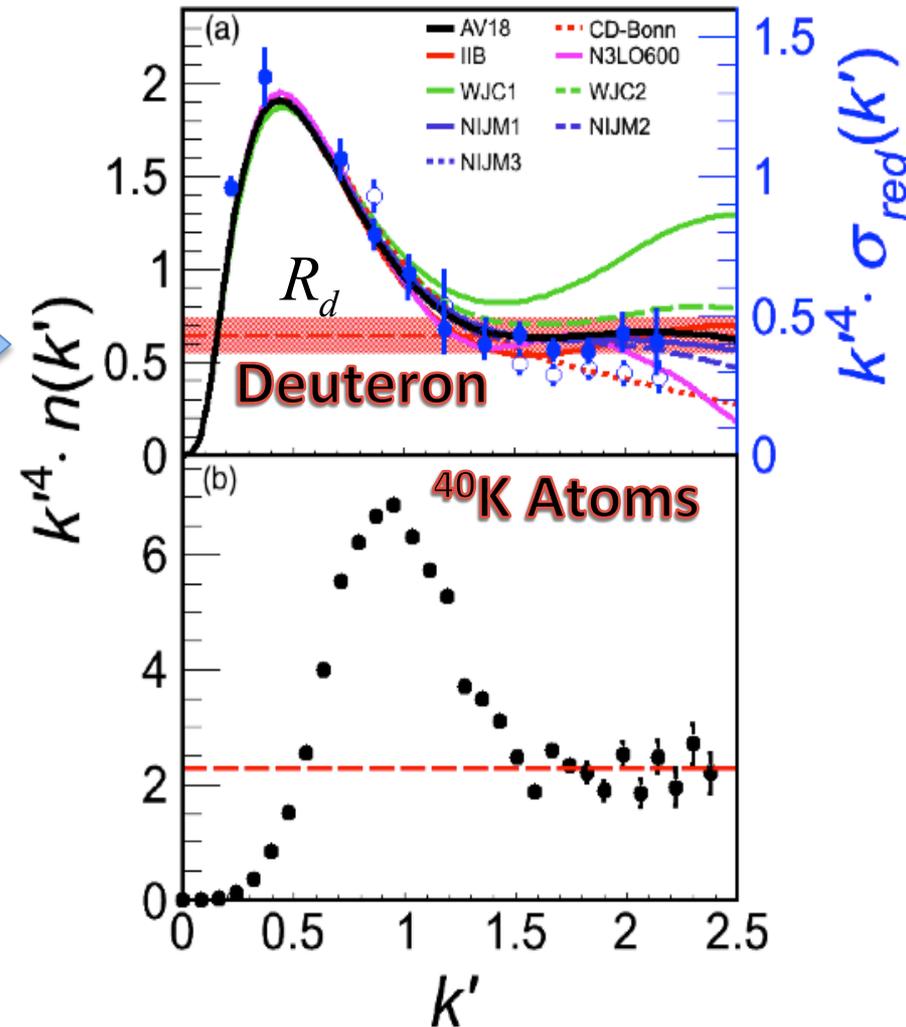
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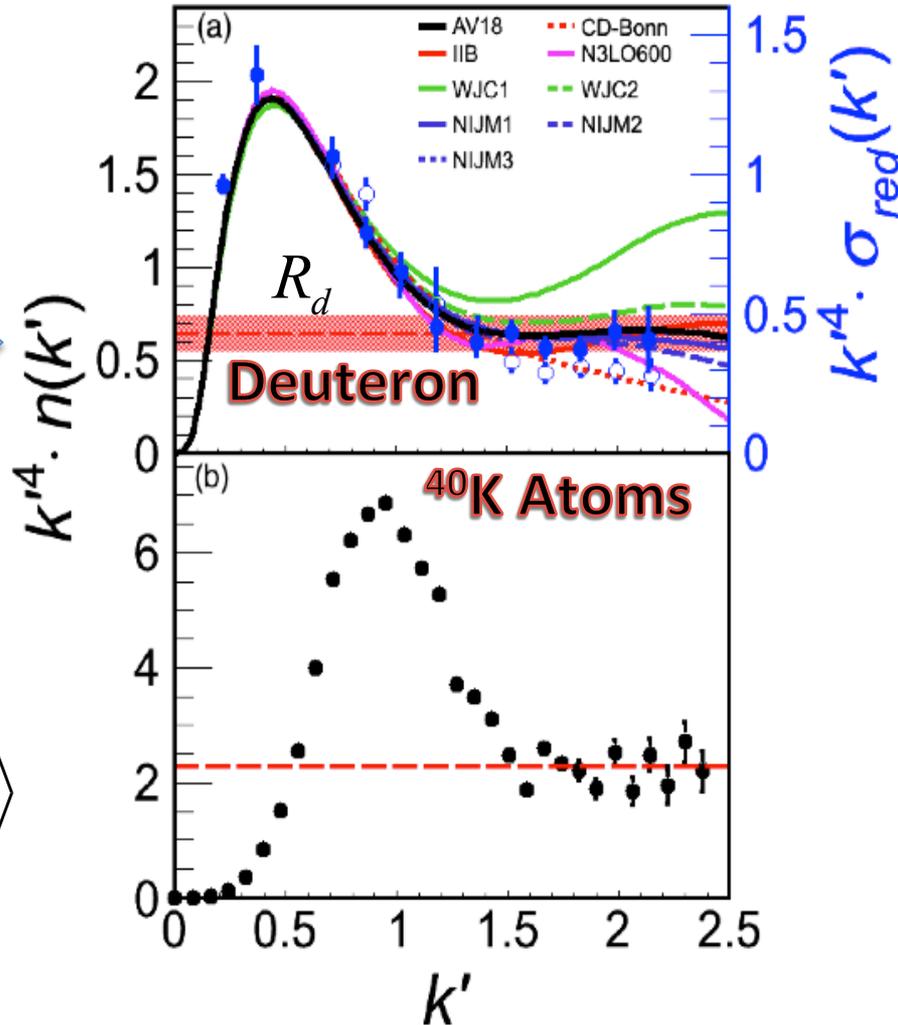


Why $1/k^4$?

Effect of the one pion exchange (OPE) contribution to the tensor potential acting in second order

$$(-B - H_0)|\Psi_D\rangle = V_T|\Psi_S\rangle$$

$$V_{00} = V_T(-B - H_0)^{-1}V_T$$





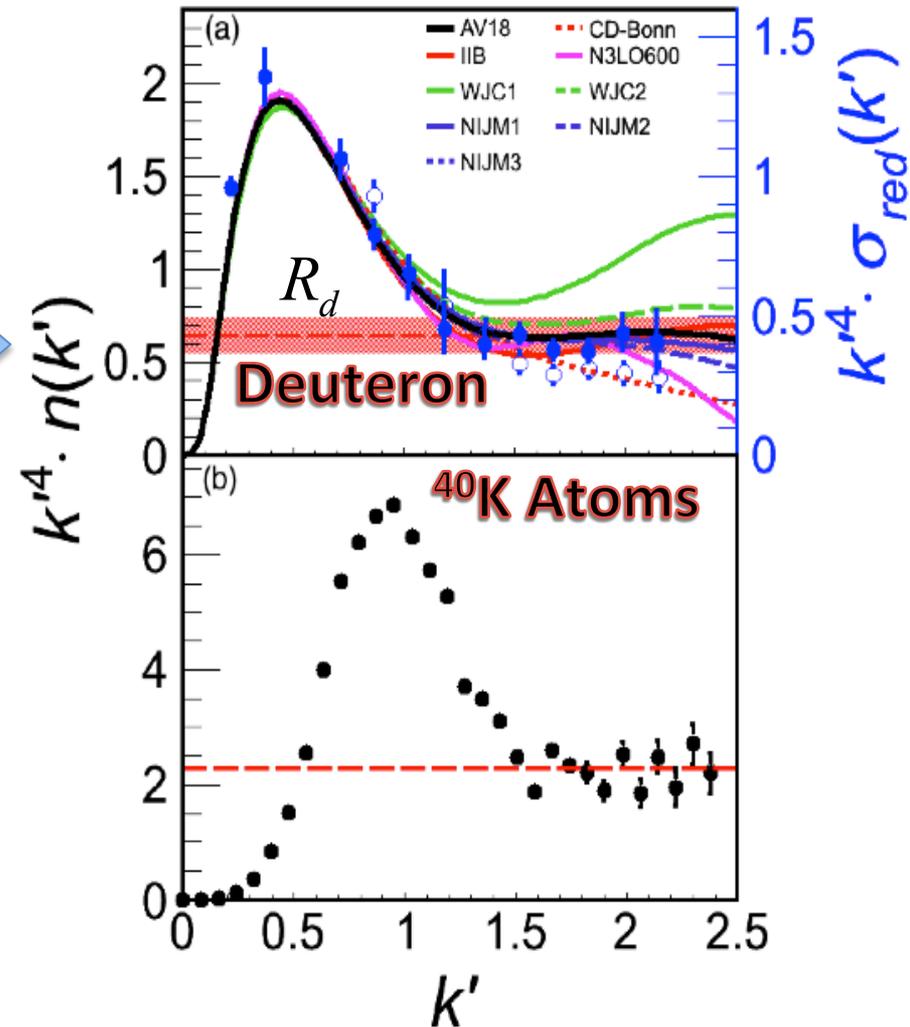
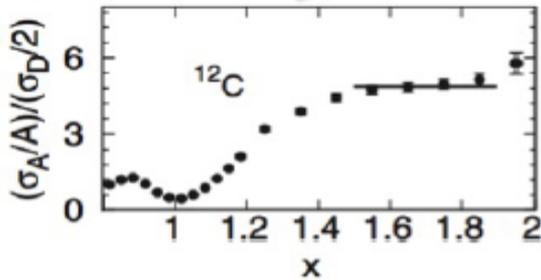
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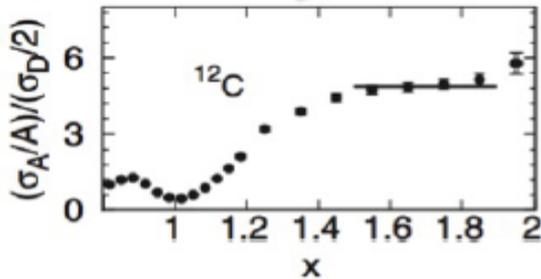
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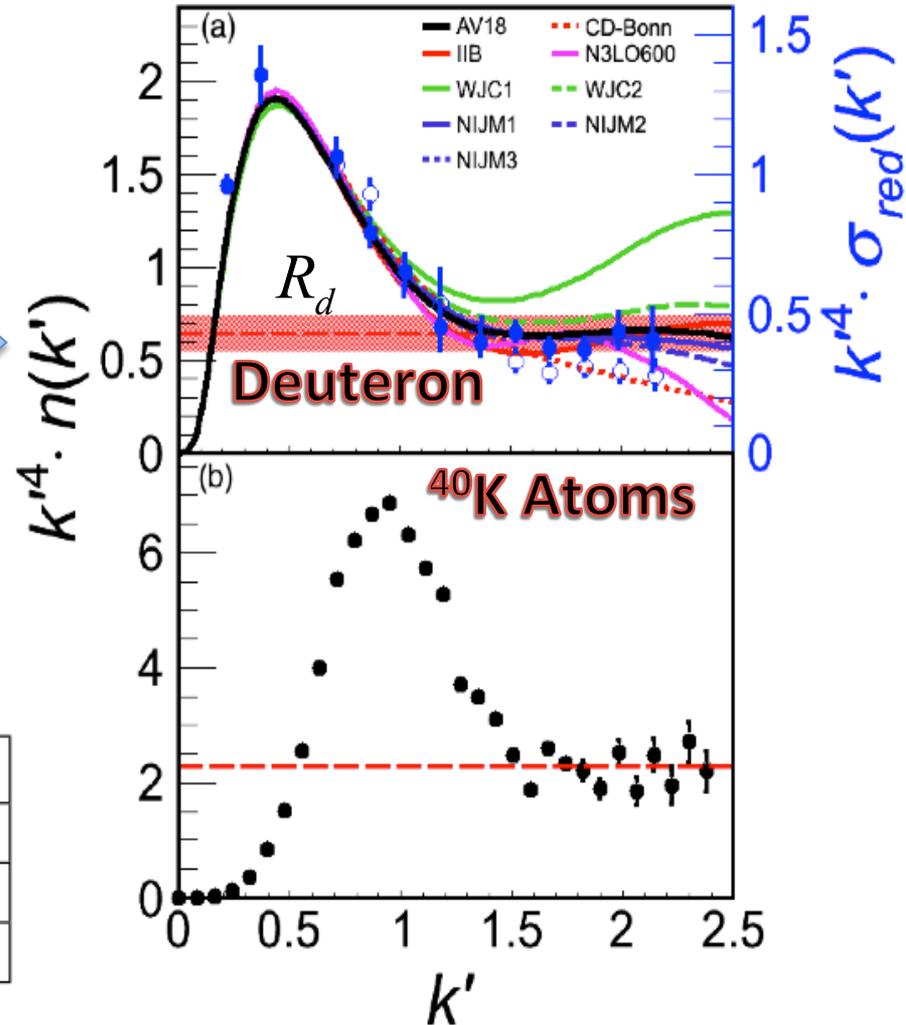
$$1.5k_F < k < 3k_F$$

$$n_A(k) = a_2(A/d) \cdot n_d(k)$$



$$\frac{C}{k_F \cdot A} = a_2(A) \cdot R_d$$

Nucleus	$a_2(A)$	$\frac{C}{k_F A}$
^{12}C	4.75 ± 0.16	3.04 ± 0.49
^{56}Fe	5.21 ± 0.20	3.33 ± 0.54
^{197}Au	5.16 ± 0.22	3.30 ± 0.53

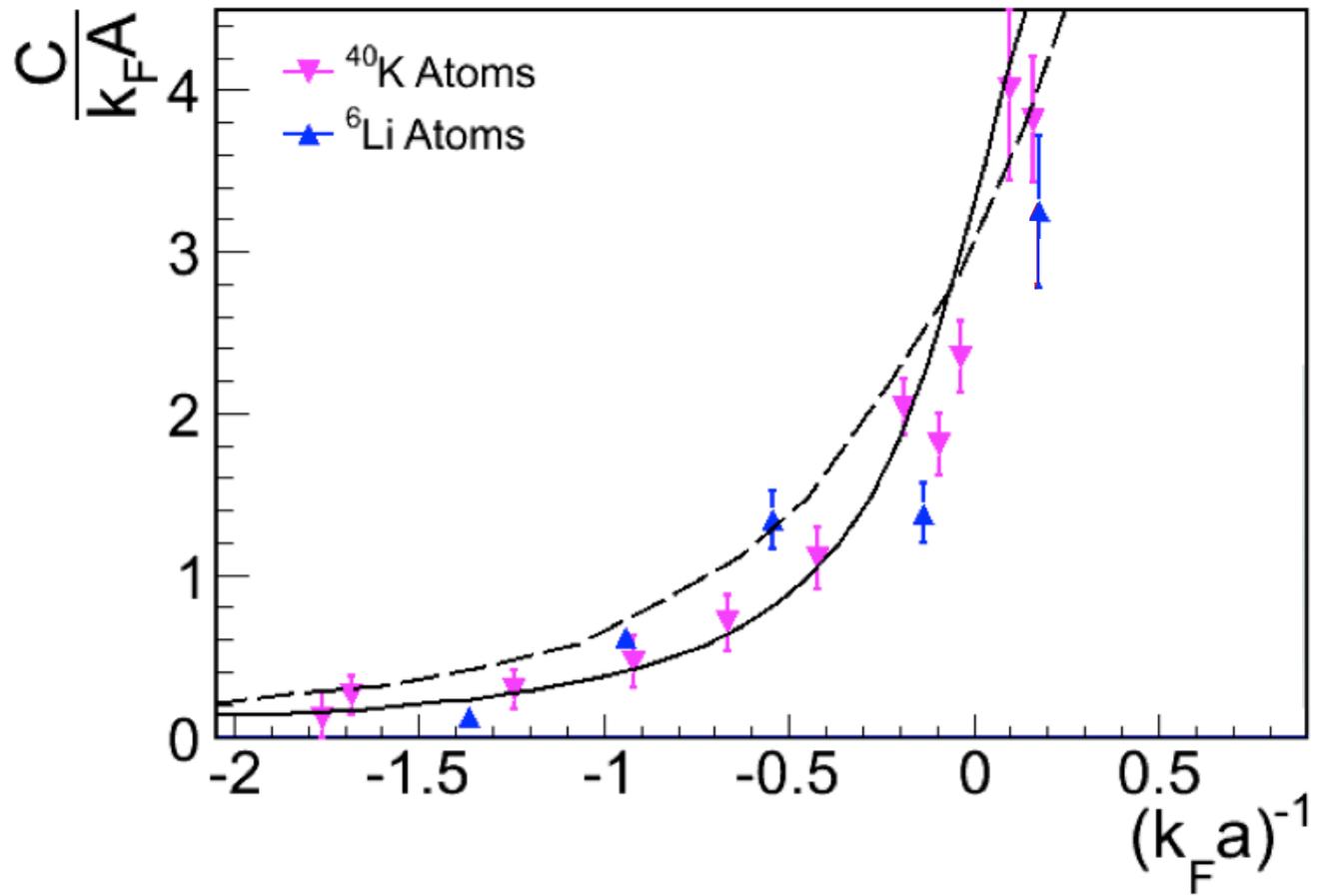




Comparing with atomic systems



Finding the same *dimensionless* interaction strength



Stewart et al. Phys. Rev. Lett. **104**, 235301 (2010)

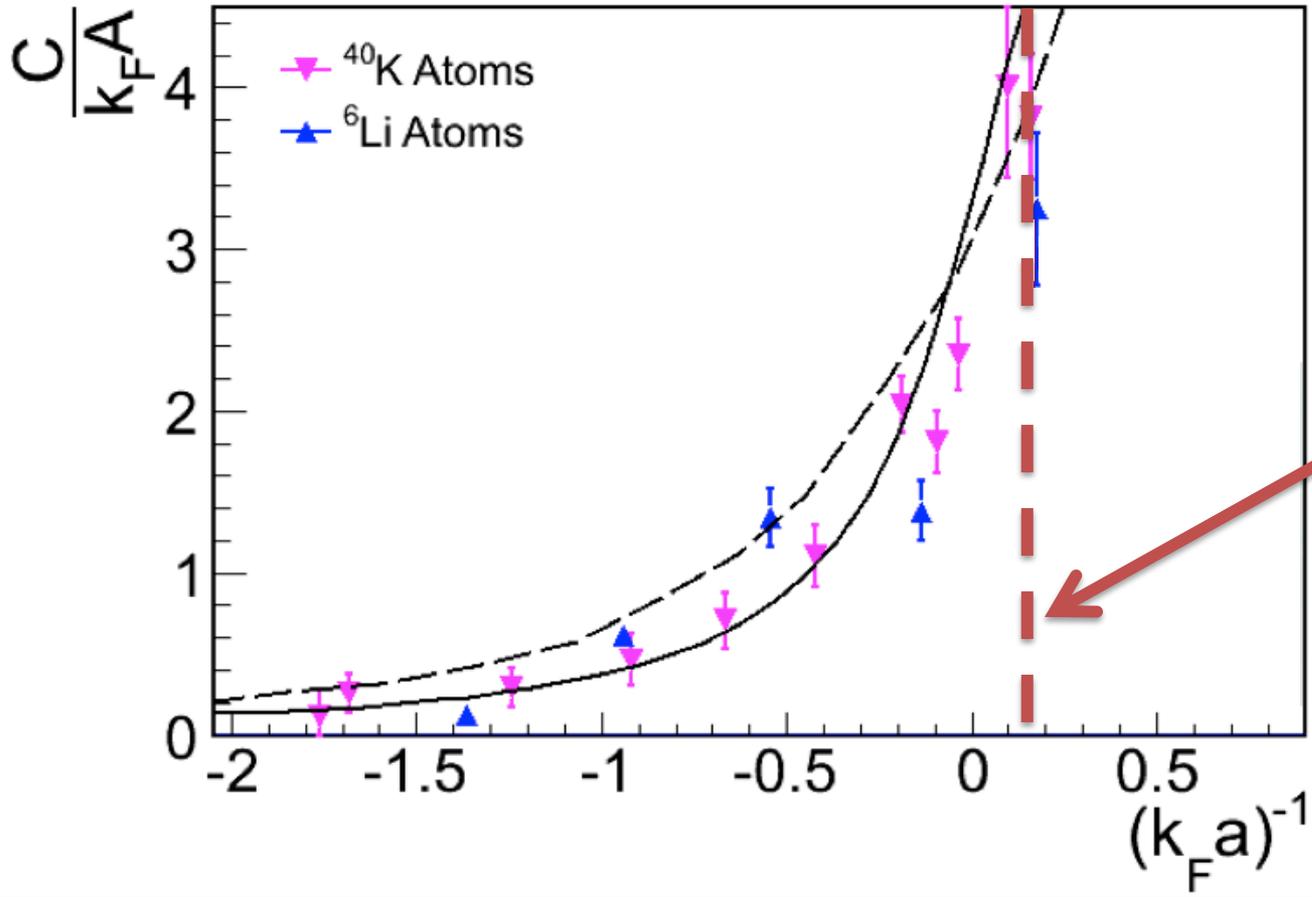
Kuhnle et al. Phys. Rev. Lett. **105**, 070402 (2010)



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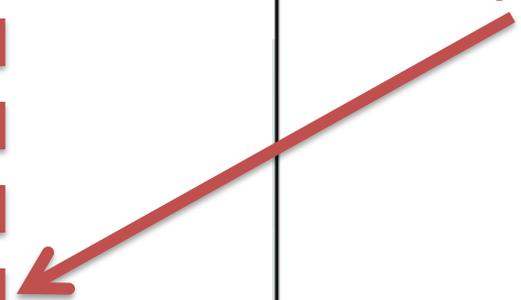


For Nuclei:

$$k_F \approx 1.27 \text{ fm}^{-1}$$

$$a \approx 5.4 \text{ fm}$$

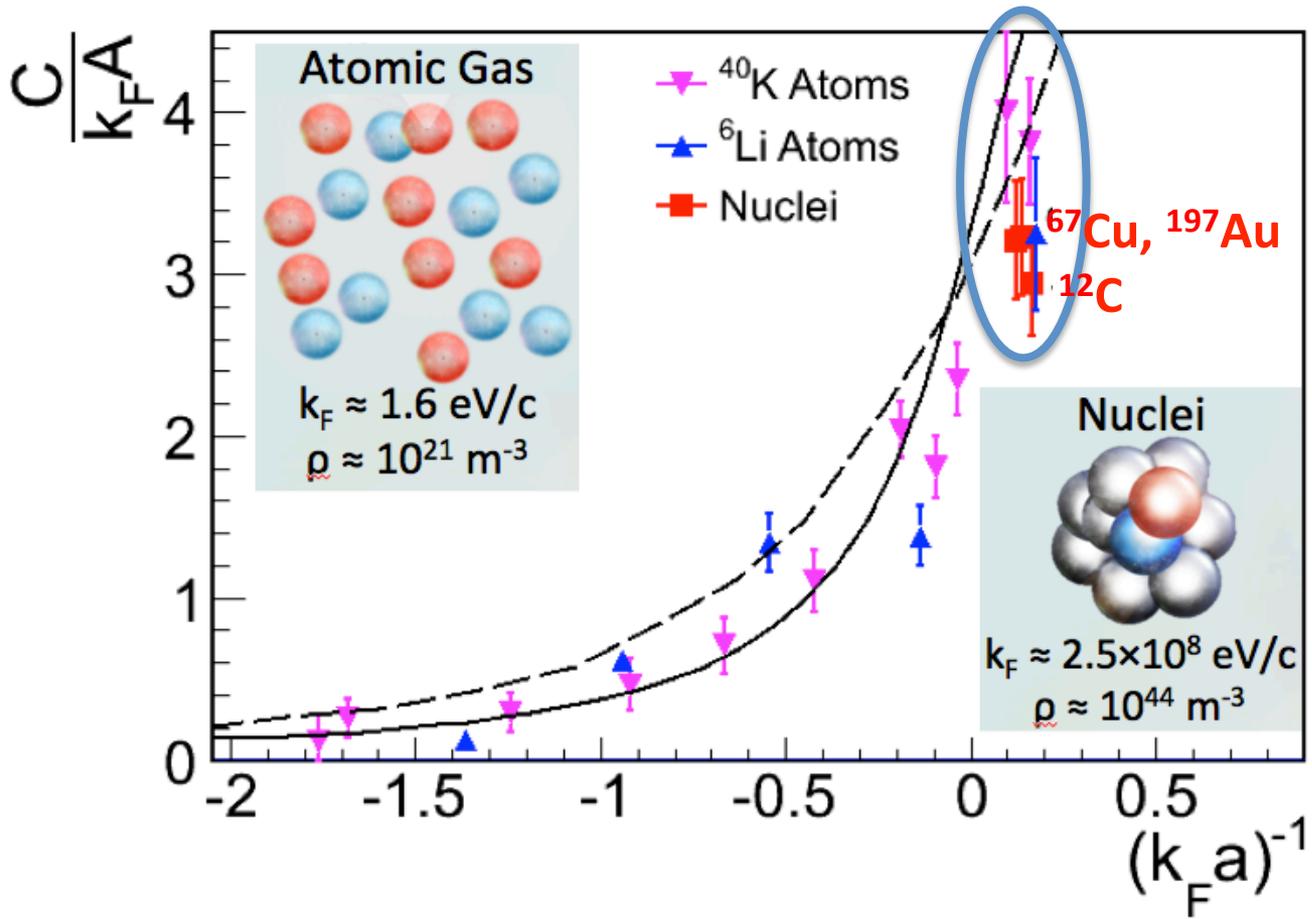
$$\Rightarrow (k_F a)^{-1} \approx 0.15$$





Comparing with atomic systems

Equal contacts for equal interactions strength!



For Nuclei:

$$k_F \approx 1.27 \text{ fm}^{-1}$$

$$a \approx 5.4 \text{ fm}$$

$$\Rightarrow (k_F a)^{-1} \approx 0.15$$

Nucleus	$\frac{C}{k_F A}$
^{12}C	3.04 ± 0.49
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$$\frac{C}{k_F \cdot A} = a_2(A) \cdot R_d$$

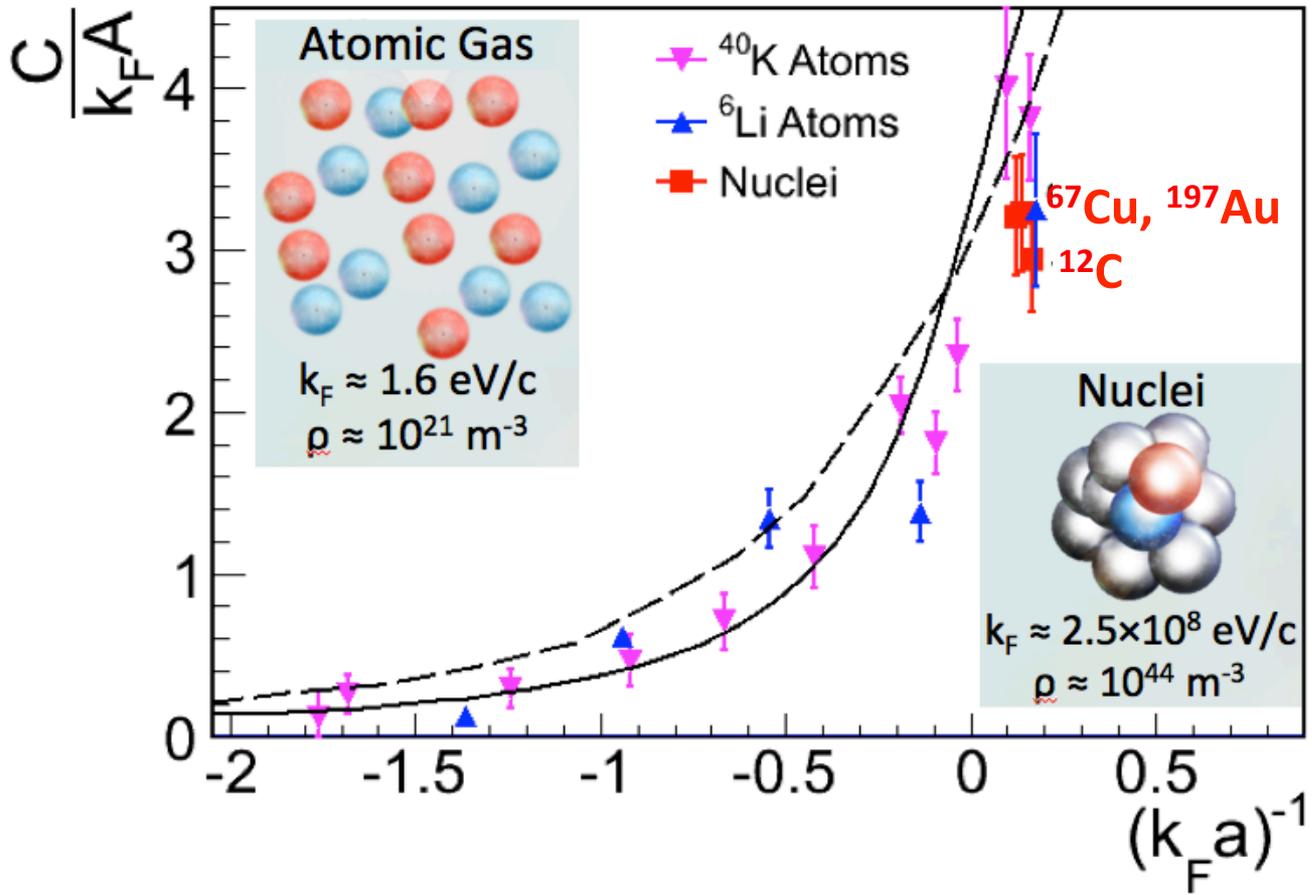
O. Hen et al. Phys. Rev. C, In-Print (2015)
 Stewart et al. Phys. Rev. Lett. **104**, 235301 (2010)
 Kuhnle et al. Phys. Rev. Lett. **105**, 070402 (2010)



Comparing with atomic systems



At unitary (i.e. $(k_F a)^{-1} \approx 0$) the SRC probability is $\sim 20\%$ for both systems



O. Hen et al. Phys. Rev. C, In-Print (2015)
Stewart et al. Phys. Rev. Lett. **104**, 235301 (2010)
Kuhnle et al. Phys. Rev. Lett. **105**, 070402 (2010)



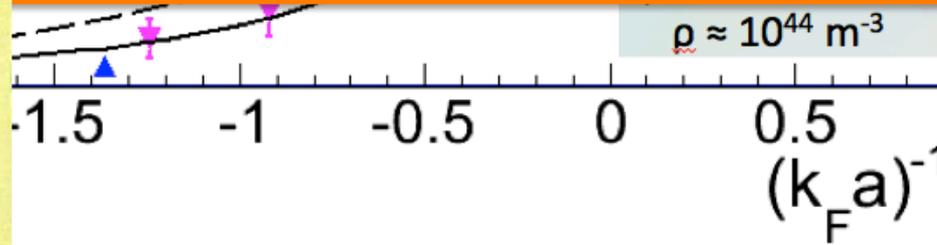
Comparing with atomic systems



At unitary (i.e. $(k_F a)^{-1} \sim 0$) the SRC

Can approximate universal relations (similar to Tan's) be extracted for the nuclear physics case?

$$\frac{C}{k_F a}$$



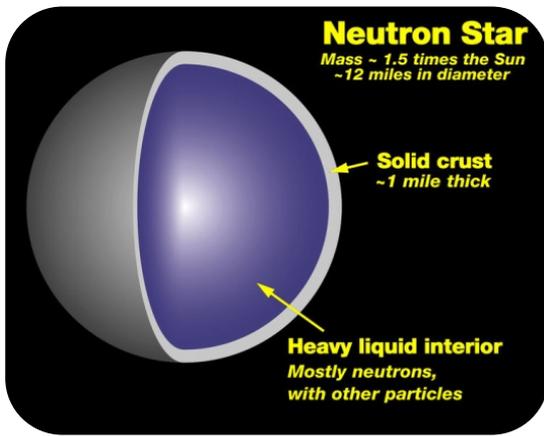
DAMMIT. I SHOULD KNOW THIS...



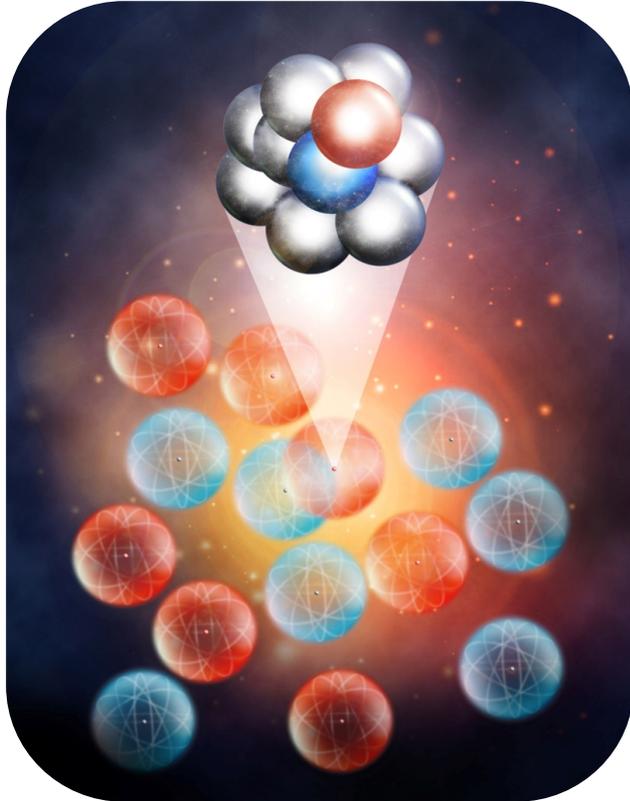
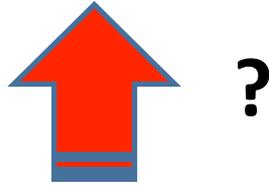
Hen?

Rev. C, In-Print (2015)
Rev. Lett. **104**, 235301 (2010)
Rev. Lett. **105**, 070402 (2010)





$$(2-3) \cdot \rho_0$$



$$\rho_0$$

$$10^{-25} \cdot \rho_0$$

Nuclear Symmetry Energy

Energy of *asymmetric* nuclear matter:

$$E(\rho_n, \rho_p) = E_0(\rho_n = \rho_p) + E_{\text{sym}}(\rho) \left(\frac{\rho_n - \rho_p}{\rho} \right)^2 + O(\delta^4)$$

Energy of *symmetric*
nuclear matter

symmetry energy

Isospin asymmetry (δ)

Nuclear Symmetry Energy

Energy of *asymmetric* nuclear matter:

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symmetry energy

$$E_{sym}(\rho) \approx E(\rho)_{\text{PNM}} - E(\rho)_{\text{SNM}}$$

Relates to the energy change when replacing n with p

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$$E(\rho_n, \rho_p) = E_0(\rho_n = \rho_p) + E_{sym}(\rho) \left(\frac{\rho_n - \rho_p}{\rho} \right)^2 + \mathcal{O}(\delta^4)$$

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$$E_{sym}(\rho) \approx E(\rho)_{\text{PNM}} - E(\rho)_{\text{SNM}}$$

Relates to the energy change when replacing n with p

- equation-of-state of neutron stars
- heavy-ion collisions
- r-process nucleosynthesis
- core-collapse supernovae
- more...

Thomson Research Fronts 2013

RESEARCH FRONTS 2013

ASTRONOMY AND ASTROPHYSICS

RANK	RESEARCH FRONTS	CORE PAPERS	CITATIONS	MEAN YEAR OF CORE PAPERS
1	Galileon cosmology	34	1,584	2010.7
2	Probing extreme redshift galaxies in the Hubble Ultra Deep Field	31	2,415	2010.3
3	Sterile neutrinos at the eV scale	41	2,472	2010.2
4	Herschel Space Observatory and initial performance	9	1,456	2010.2
5	Kepler Mission and the search for extra-solar planets	47	4,211	2010.0
6	Neutron star observations and nuclear symmetry energy	18	1,536	2009.9
7	Evolution of massive early-type galaxies	18	1,724	2009.6
8	Gamma-ray sources detected by the Fermi Large Area Telescope	8	1,531	2009.5
9	Data from Hinode (Solar-B) Solar Optical Telescope and Solar Dynamics Observatory (SDO)	24	3,023	2009.4
10	Supernova Type Ia light curves and dark energy	19	5,920	2009.2



Source: Thomson Reuters Essential Science Indicators

Nuclear Symmetry Energy

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np-SRC exist in SNM but not in PNM

$$\Rightarrow E_{sym}(\rho) = E_{PNM}(\rho) - E_{SNM}(\rho)$$

Could change drastically

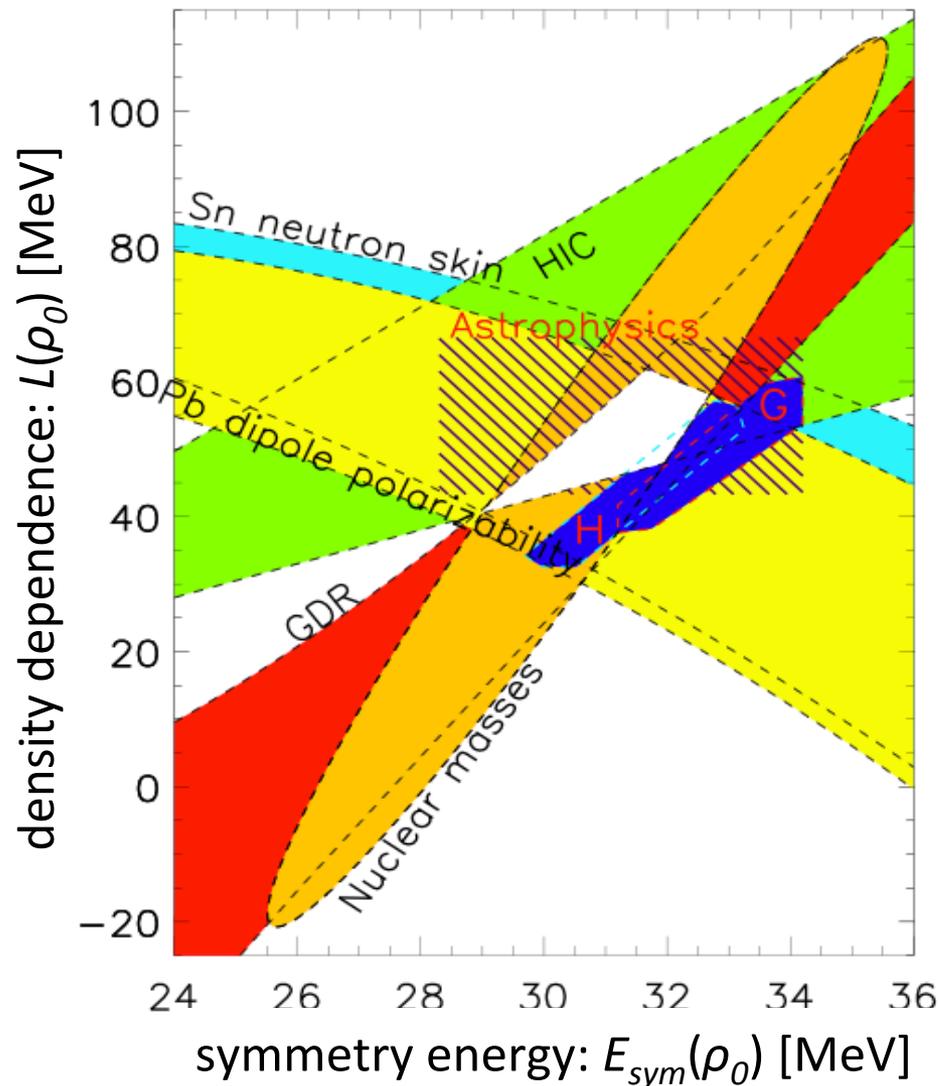
Relates to the energy change when replacing n with p

[SNM: Symmetric Nuclear Matter, PNM: Pure Neutron Matter]

- neutron stars
- heavy-ion collisions
- core-collapse supernovae
- more...



Symmetry Energy @ Saturation Density



Global analysis
of world data:

$$28.9 \leq E_{sym}(\rho_0) \leq 34.1$$

$$42.4 \leq L(\rho_0) \leq 74.4$$



Constraining the Symmetry Energy



$$E_{sym}(\rho) = E_{sym}^{kin}(\rho_0) \cdot \left(\frac{\rho}{\rho_0}\right)^\alpha + E_{sym}^{pot}(\rho_0) \cdot \left(\frac{\rho}{\rho_0}\right)^{\gamma_i}$$

$E_{sym}(\rho)$ requires separate knowledge of the kinetic and potential parts.

Fermi-Gas Model: a common approximation for the kinetic term

M.B. Tsang et al., Phys. Rev. Lett **102**, 122701 (2009)

A.W. Steiner, J.M. Lattimer, and E.F. Brown, Astrophys. J. **722**, 33 (2010).

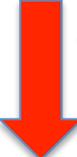


Constraining the Symmetry Energy



[Fermi-Gas Picture]

$$E_{sym}(\rho) = E_{sym}^{kin}(\rho_0) \cdot \left(\frac{\rho}{\rho_0}\right)^\alpha + E_{sym}^{pot}(\rho_0) \cdot \left(\frac{\rho}{\rho_0}\right)^{\gamma_i}$$

 Fermi-Gas Model

$$E_{sym}^{kin}(\rho) = \frac{1}{3} E_F(\rho_0) \cdot \left(\frac{\rho}{\rho_0}\right)^{2/3}$$



Constraining the Symmetry Energy



[Fermi-Gas Picture]

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**Fermi-Gas
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- ✓ $\alpha = \frac{2}{3}$
- ✓ $E_{sym}^{kin}(\rho_0) = 12.5 \text{ MeV}$



Constraining the Symmetry Energy



[Fermi-Gas Picture]

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Constraining the Symmetry Energy



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Fermi-Gas Model

$$E_{sym}^{kin}(\rho) = \frac{1}{3} E_F(\rho_0) \cdot \left(\frac{\rho}{\rho_0}\right)^{2/3}$$

Only unknown is γ_i
probed in HI collision
measurements and
neutron stars
observations

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Constraining the Symmetry Energy

[With SRCs]

$$E_{sym}(\rho) = E_{sym}^{kin}(\rho_0) \cdot \left(\frac{\rho}{\rho_0}\right)^\alpha + E_{sym}^{pot}(\rho_0) \cdot \left(\frac{\rho}{\rho_0}\right)^{\gamma_i}$$

Fermi-Gas Model + Correlations

$$E_{sym}^{kin}(\rho) = \frac{1}{3} E_F(\rho_0) \cdot \left(\frac{\rho}{\rho_0}\right)^{2/3} + \Delta_{SRC}(\rho)$$

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probed in HI collision measurements and neutron stars observations

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Constraining the Symmetry Energy

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Fermi-Gas Model

+ Correlations

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Constraining the Symmetry Energy



[With SRCs]

Adding np-SRCs breaks the Fermi-Gas picture

Fermi-Gas Model

+ Correlations

=> Need a correlated Fermi-Gas Model

~~$\alpha = \frac{2}{3}$~~

~~$E_{sym}^{kin}(\rho_0) = 12.5 \text{ MeV}$~~

~~$E_{sym}^{pot}(\rho_0) = E_{sym}(\rho_0) - E_{sym}^{kin}(\rho_0) \approx 12.5 \text{ MeV}$~~

~~Only unknown is γ_i
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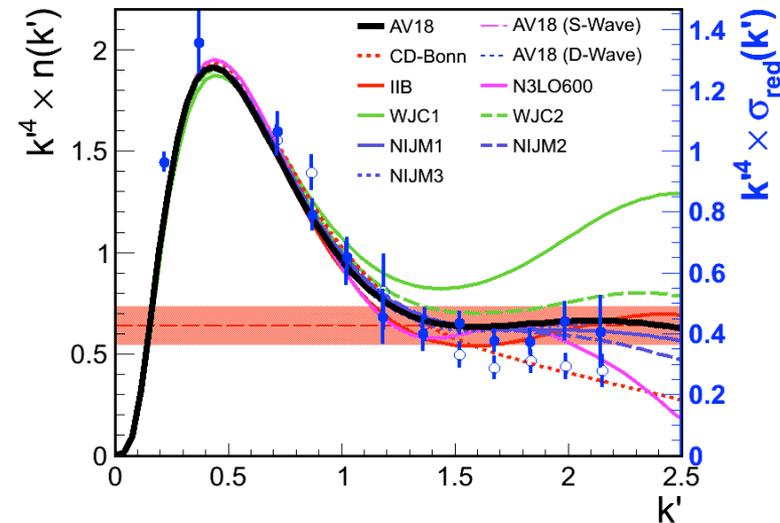
Correlated Fermi-Gas Model (CFG)



[Fermi-Gas with an SRC tail]

$$n_{CFG}(k) = \begin{cases} A_0 & , & A_1 & k < k_F \\ C_\infty / k^4 & , & 0 & k_F < k < \lambda k_F^0 \\ 0 & , & 0 & 0 < k > \lambda k_F^0 \end{cases}$$

C/k^4 is a good parameterization of the high-momentum tail:



O. Hen et al., Phys. Rev. C 91, 025803 (2015).





Correlated Fermi-Gas Model (CFG)



[Fermi-Gas with an SRC tail]

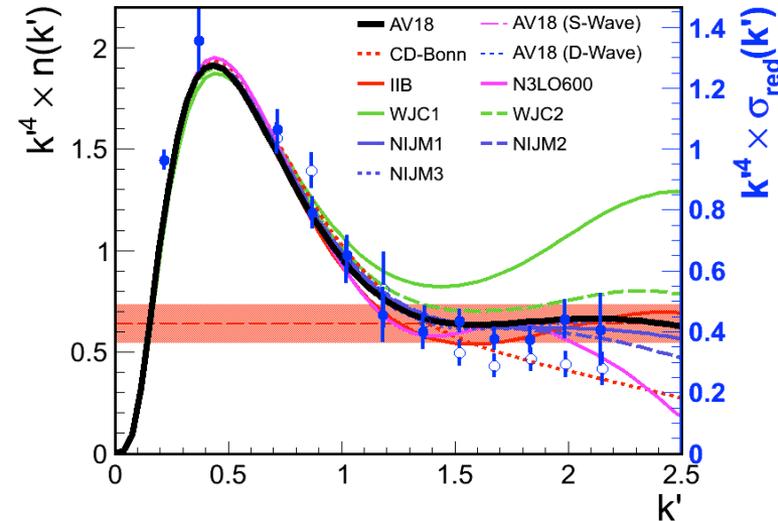
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SNM PNM

SNM Model:

- Depleted Fermi Distribution (A_0)
- High-Momentum tail (C/k^4)
- Momentum cutoff (λ)

C/k^4 is a good parameterization of the high-momentum tail:





Correlated Fermi-Gas Model (CFG)



[Fermi-Gas with an SRC tail]

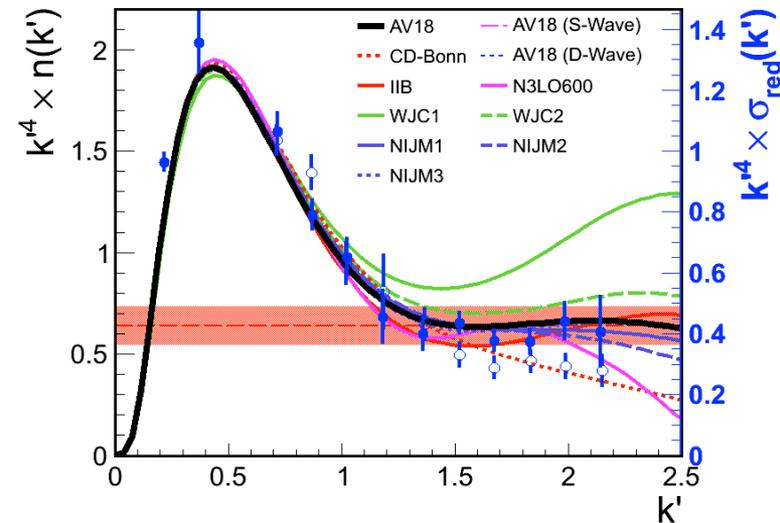
$$n_{CFG}(k) = \begin{cases} A_0 & , & A_1 & k < k_F \\ C_\infty / k^4 & , & 0 & k_F < k < \lambda k_F^0 \\ 0 & , & 0 & k > \lambda k_F^0 \end{cases}$$

SNM PNM

PNM Model:

- Free Fermi Gas

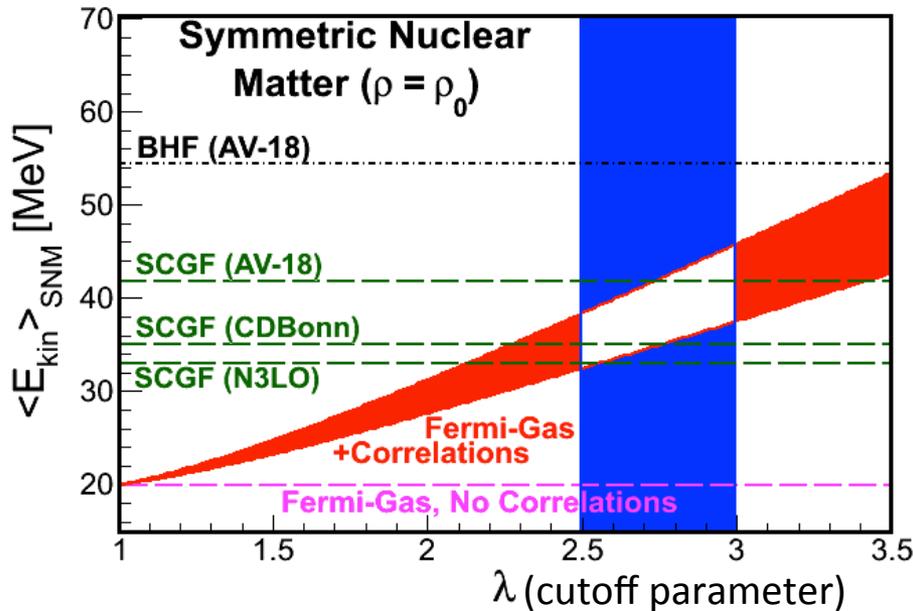
C/k^4 is a good parameterization of the high-momentum tail:



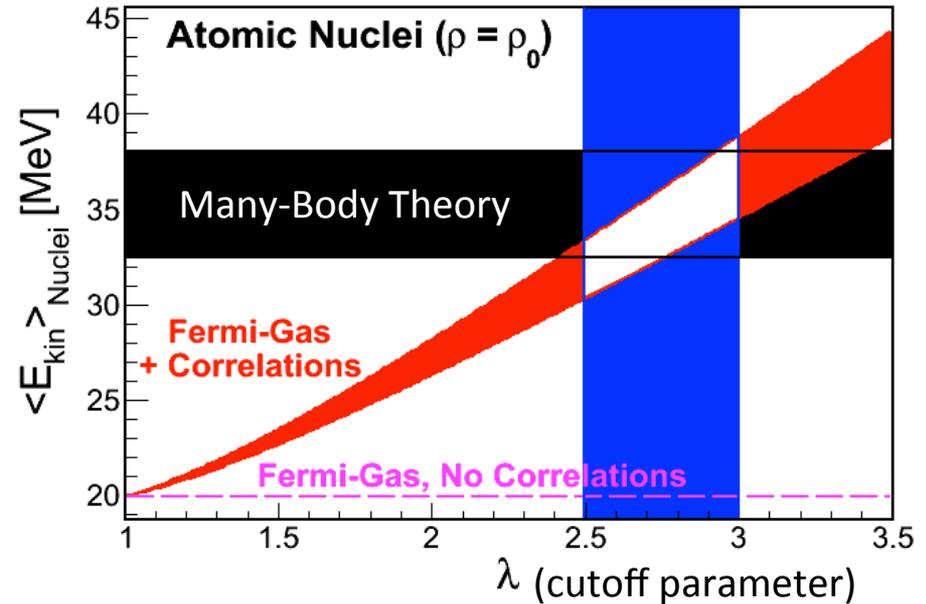
Benchmark Against Microscopic Calculations

$$E_{kin} = \frac{4\pi}{(2\pi)^3} \int_0^\infty \frac{\hbar^2 k^2}{2m} n(k) k^2 dk.$$

Average kinetic energy – SNM



Average kinetic energy - Nuclei



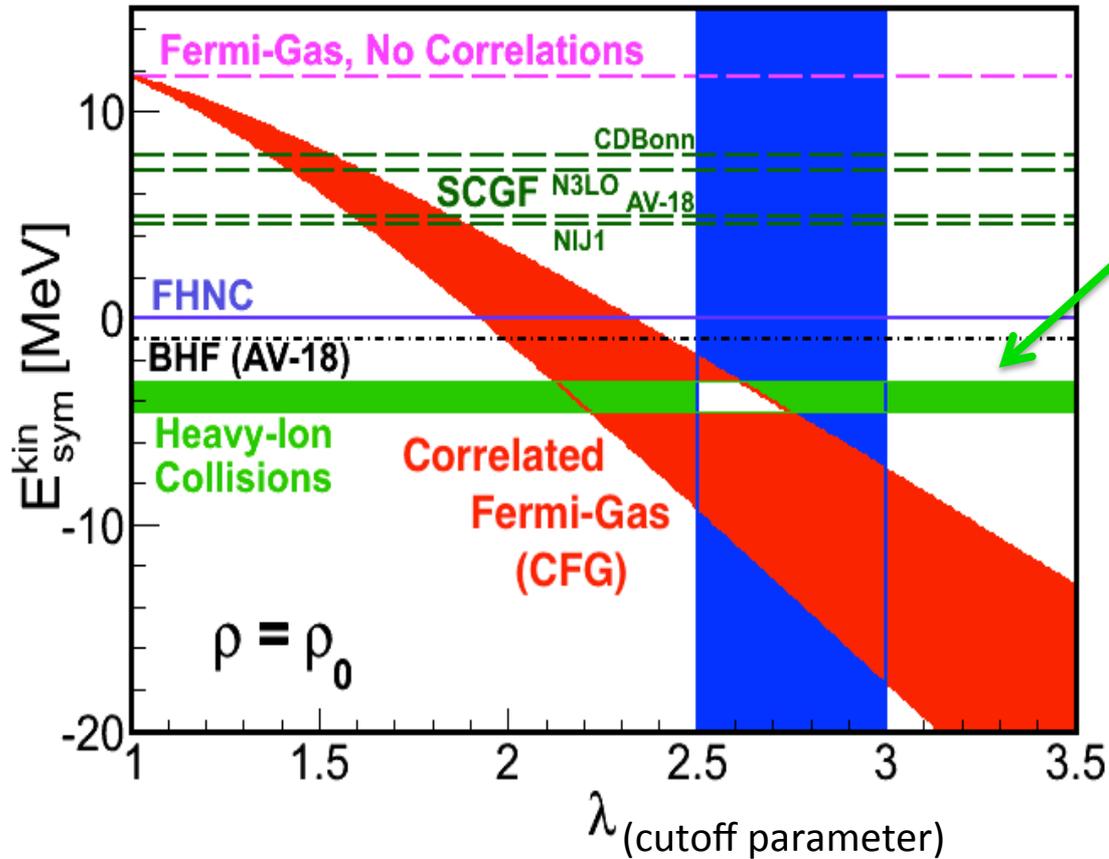
$$a_2(A) = 5.0 \pm 0.3$$

$$a_2(\infty) = 7.0 \pm 1.0$$



Extracting the Kinetic Symmetry Energy

Kinetic symmetry energy

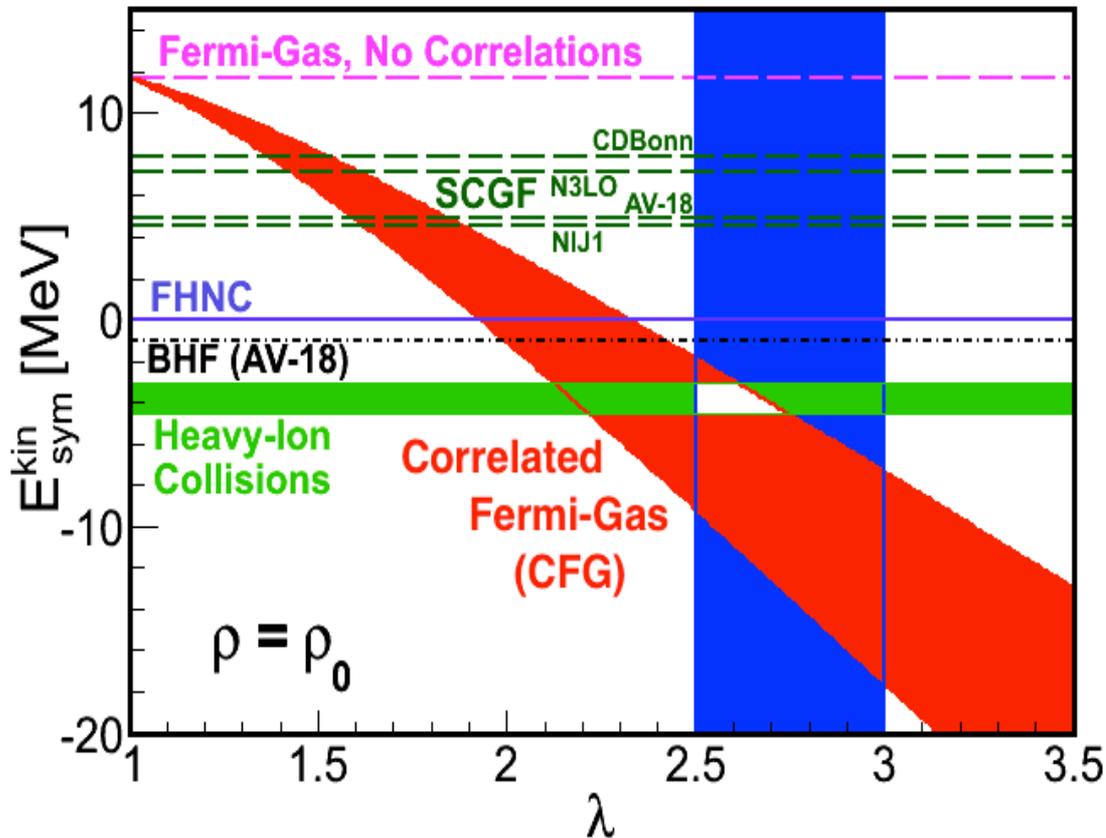


Transport calculation of $^{124}\text{Sn}+^{124}\text{Sn}$ and $^{112}\text{Sn}+^{112}\text{Sn}$ collisions also yield reduced kinetic symmetry energy



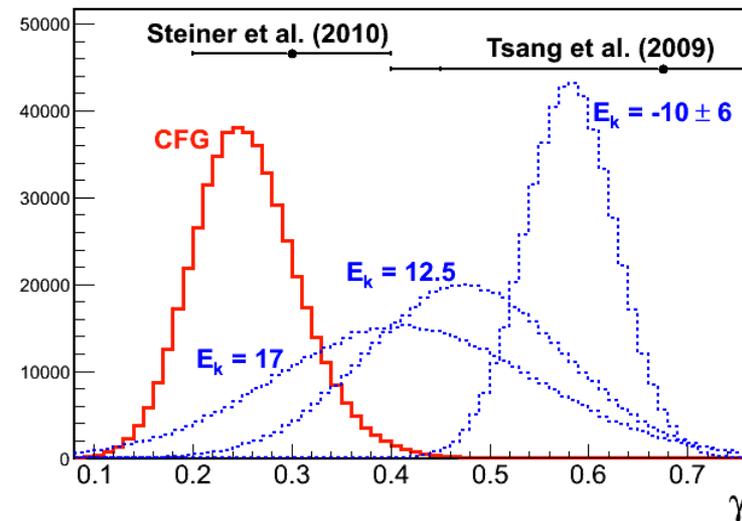
Extracting the Kinetic Symmetry Energy

Kinetic symmetry energy



SRCs reduce the kinetic symmetry energy

[Enhance the potential symmetry energy and alter its density dependence]



O. Hen et al., Phys. Rev. C 91, 025803 (2015).

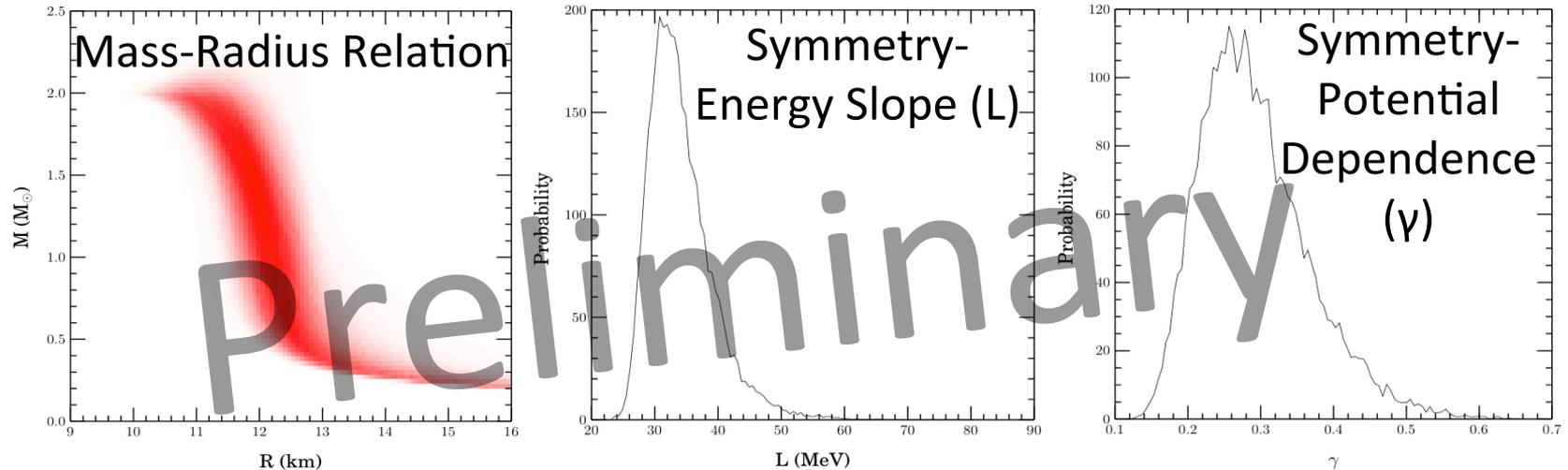
Next Step – Incorporating CFG model

into:

- **neutron stars equation-of-state fits**
- **Transport models for HI collision analysis**

Next (ongoing) Step – Incorporating CFG model into:

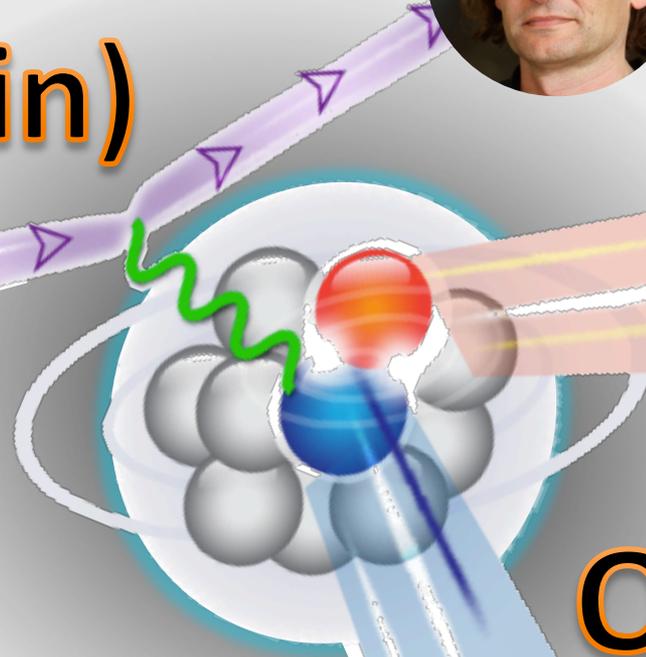
- neutron stars equation-of-state fits
- Transport models for HI collision analysis





Thank You!

(again)



Questions?

