New Results on Short-Range Correlation Studies

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Abstract:

- Emergence of Short-Range Correlations in multi fermion systems.

- When Correlation between unlike components dominate

- Possible Implications: In Nuclear Physics

  In Atomic Physics

  QCD

Abstract:

- Protons are more energetic in neutron rich nuclei/nuclear matter

"conventional" nuclear theory claims the opposite

- Implications for nuclei and nuclear astrophysics

- This may be a <u>Universal Property</u> for any Asymmetric Two Component Fermi System

Interacting only through the <u>Unlike</u> components at <u>Short Distances</u>

Modeling High Momentum and Missing Energy Nuclear Spectral Functions

- All best models are nonrelativistic

- SRC model allows to derive relativistic spectral functions if we know how to treat 2N SRCs relativistically

- 3N SRCs

Emergence of Short-Range Correlations

- start with A-body Schroedinger equation interacting by two body potential only

$$\left[-\sum_i \frac{\nabla_i^2}{2m} + \frac{1}{2}\sum_{i,j} V(x_i - x_j)\right]\psi(x_1,\cdots,x_A) = E\psi(x_1,\cdots,x_A)$$

- Introducing

$$\psi(x_1,\cdots,x_A) = \int \Phi(k_1,\cdots k_A)e^{i\sum_i k_i x_i}\prod_i \frac{d^3k_i}{(2\pi)^{3/2}}$$

$$V(x_i - x_j) = \int U(q)e^{iq(x_i - x_j)}d^3q$$

$$\boxed{\left(\sum_i \frac{k_i^2}{2m} - E_b\right)\Phi(k_1,\cdots,k_A) = -\frac{1}{2}\sum_{i,j}\int U(q)\,\Phi(k_1,\cdots,k_i - q,\cdots k_j + q,\cdots,k_A)d^3q}$$

- Assume: system is dilute

- Assume: $U_{NN}(q) \sim \frac{1}{q^n}$ with $n > 1$

$$\left(\sum_i \frac{k_i^2}{2m} - E_b\right)\Phi(k_1,\cdots,k_A) = -\frac{1}{2}\sum_{i,j}\int U(q)\,\Phi(k_1,\cdots,k_i - q,\cdots k_j + q,\cdots,k_A)d^3q$$

- then the k dependence of the wave function for $k^2/2m_N \gg |E_B|$

$$\Phi^{(1)}(k_1,\cdots,k_c,\cdots,-k_c,\cdots,k_A) \approx \frac{U_{NN}(k_c)}{k_c^2} F_A(k_1,\cdots{}'\cdots{}',\cdots,k_A)$$

$$\Phi^{(2)}(\cdots k_c,\cdots) \sim \frac{1}{k_c^{2+n}}\int \frac{1}{q^n}dq$$

- For large k_c $\Phi^{(2)}(k_c) \ll \Phi^{(1)}(k_c)$ Amado, 1976

Frankfurt, Strikman 1981

- The same is true for relativistic equations as:
  Bethe-Salpeter or Weinberg Light Cone Equations

- From $\Phi^{(1)}(k_1,\cdots,k_c,\cdots,-k_c,\cdots,k_A) \approx \frac{U_{NN}(k_c)}{k_c^2} F_A(k_1,\cdots{}'\cdots{}',\cdots,k_A)$ follows

for large $k > k_{Fermi}$

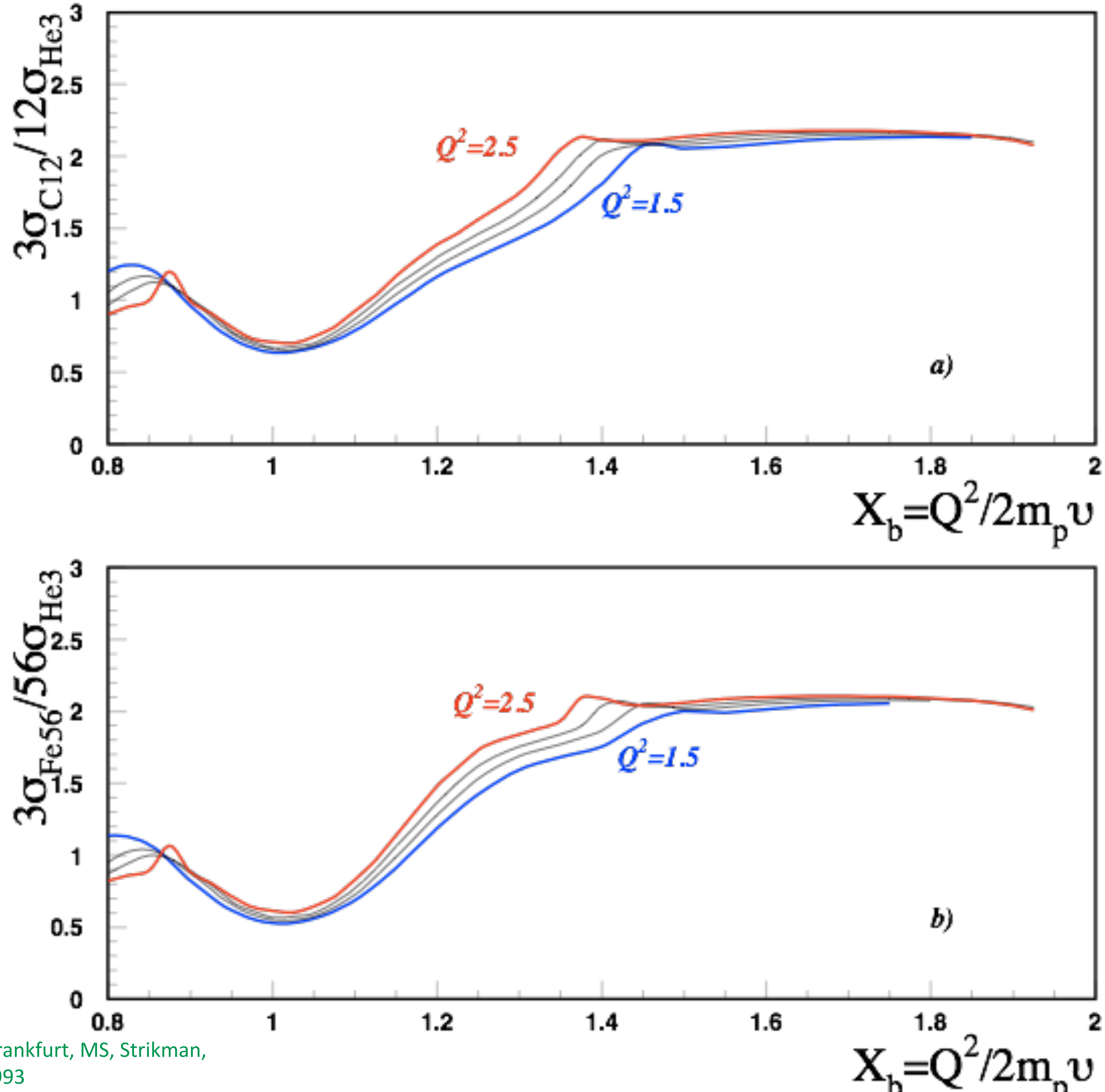
$$\boxed{n_A(k) \approx a_{NN}(A) n_{NN}(k)}$$

Frankfurt, Strikman Phys. Rep, 1988
Day, Frankfurt, Strikman, MS, Phys. Rev. C 1993

- Experimental observations

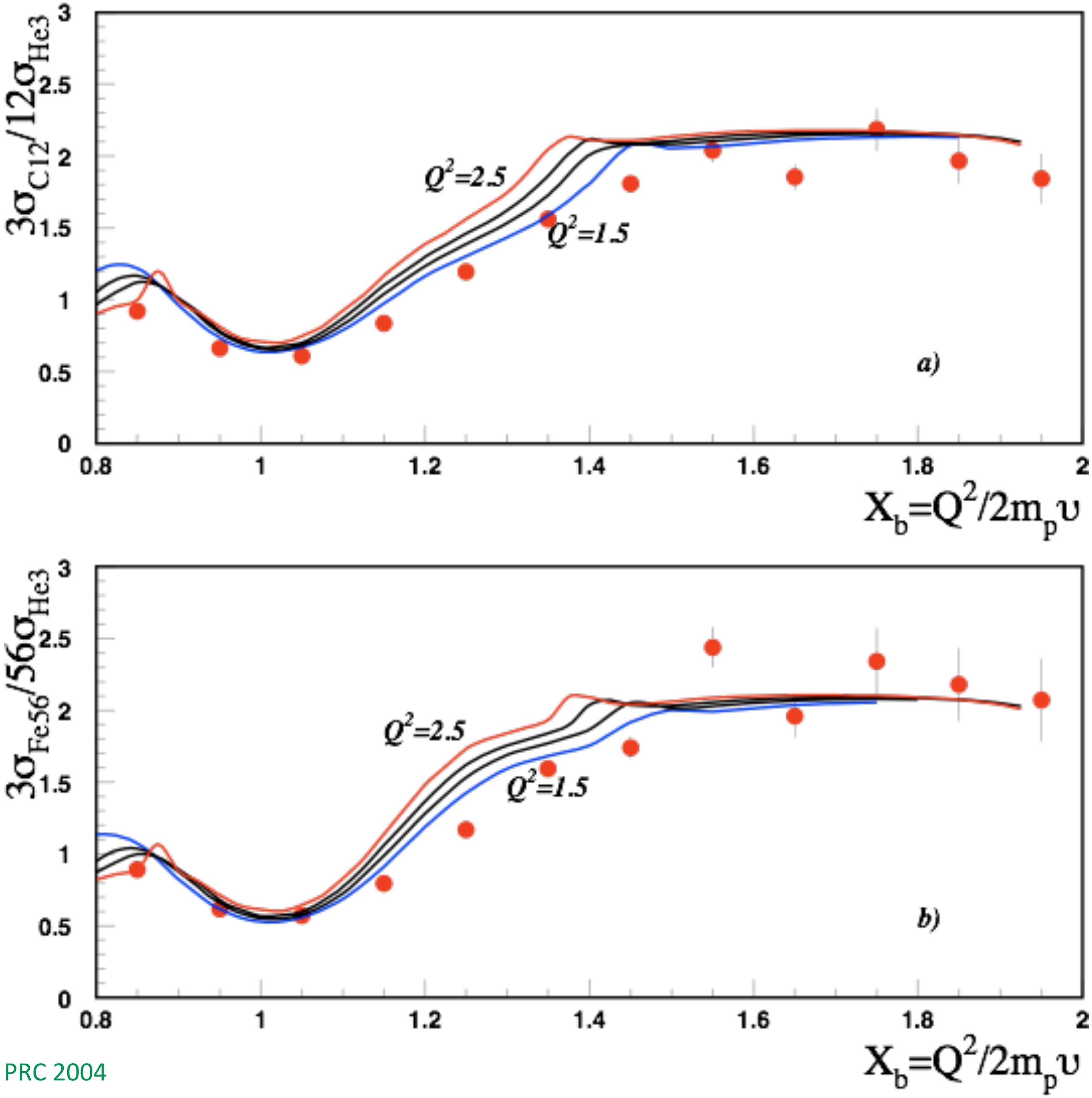
Egiyan et al, 2002, 2006
Fomin et al, 2011

$A(e,e')$



Day, Frankfurt, MS, Strikman,
PRC 1993

$A(e,e')$



Egiyan, et al PRC 2004

Meaning of the scaling values

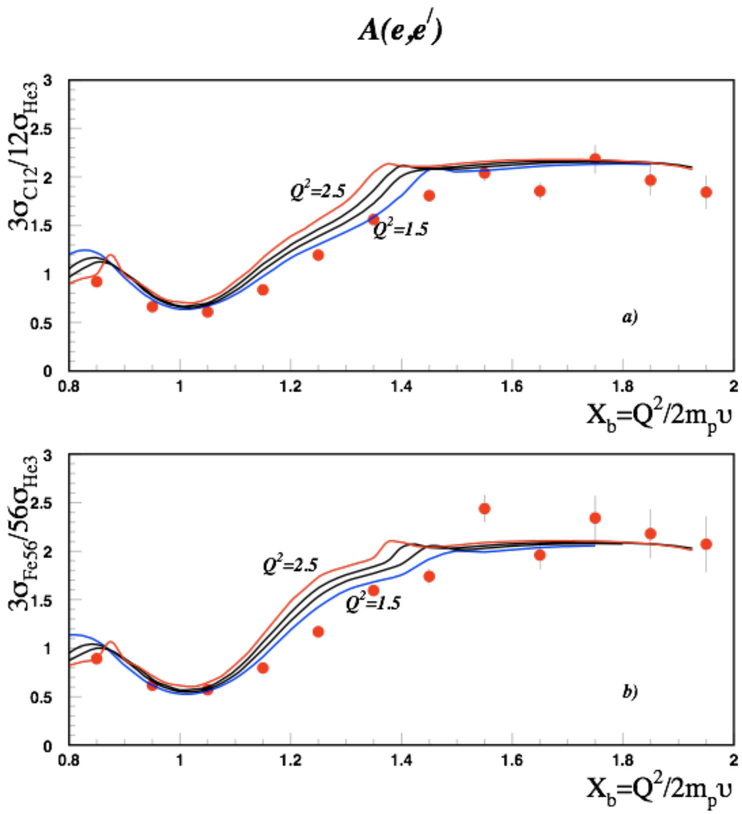
Day, Frankfurt, MS, Strikman, PRC 1993

Frankfurt, MS, Strikman, IJMP A 2008

Fomin et al PRL 2011

$$\boxed{R = \frac{A_2\sigma[A_1(e,e')X]}{A_1\sigma[A_2(e,e')X]}}$$

$$\text{For } 1 < x < 2 \; R \approx \frac{a_2(A_1)}{a_2(A_2)}$$



Egiyan, et al PRL 2006, PRC 2004

a2's as relative probability of 2N SRCs

Table 1: The results for $\boxed{a_2(A, y}$)

A	y	This Work	Frankfurt et al	Egiyan et al	Famin et al
^{3}He	0.33	2.07±0.08	1.7±0.3		2.13±0.04
^{4}He	0	3.51±0.03	3.3±0.5	3.38±0.2	3.60±0.10
^{9}Be	0.11	3.92±0.03			3.91±0.12
^{12}C	0	4.19±0.02	5.0±0.5	4.32±0.4	4.75±0.16
^{27}Al	0.037	4.50±0.12	5.3±0.6		
^{56}Fe	0.071	4.95±0.07	5.6±0.9	4.99±0.5	
^{64}Cu	0.094	5.02±0.04			5.21±0.20
^{197}Au	0.198	4.56±0.03	4.8±0.7		5.16±0.22

L.Weinstein's talk

for large $k > k_{Fermi}$

$$n_A(k) \approx a_{NN}(A) n_{NN}(k)$$

- Isospin composition ?

$$P_{pn/pX} = 0.92^{+0.08}_{-0.18}$$

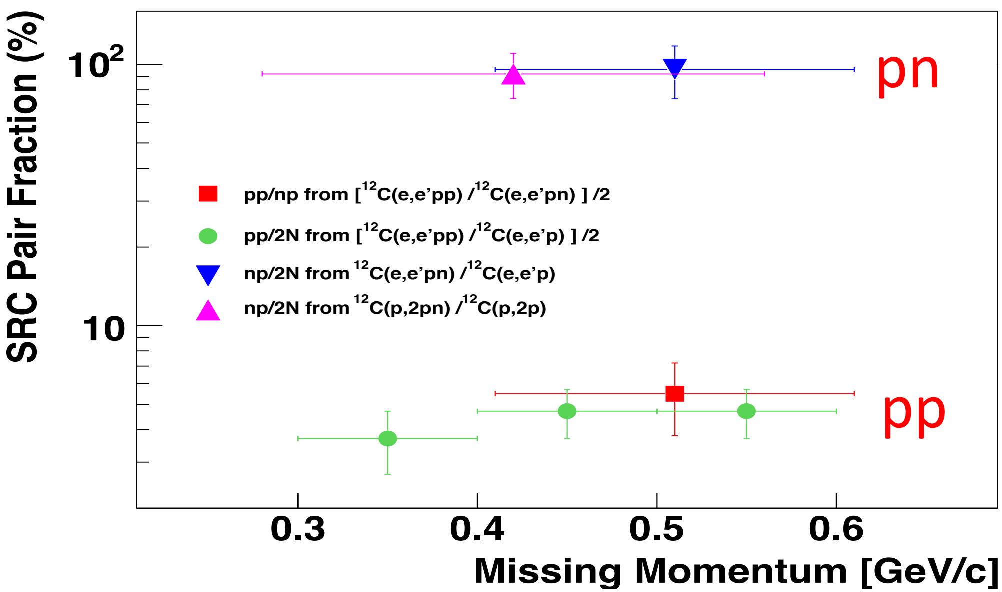
Theoretical analysis of BNL Data

$$\frac{P_{pp}}{P_{pn}} \leq \frac{1}{2}(1 - P_{pn/pX}) = 0.04^{+0.09}_{-0.04}.$$

E. Piasetzky, MS, L. Frankfurt, M. Strikman, J.Watson PRL , 2006

$$P_{pp/pn} = 0.056 \pm 0.018$$

Direct Measurement at JLab R.Subdei, et al Science , 2008



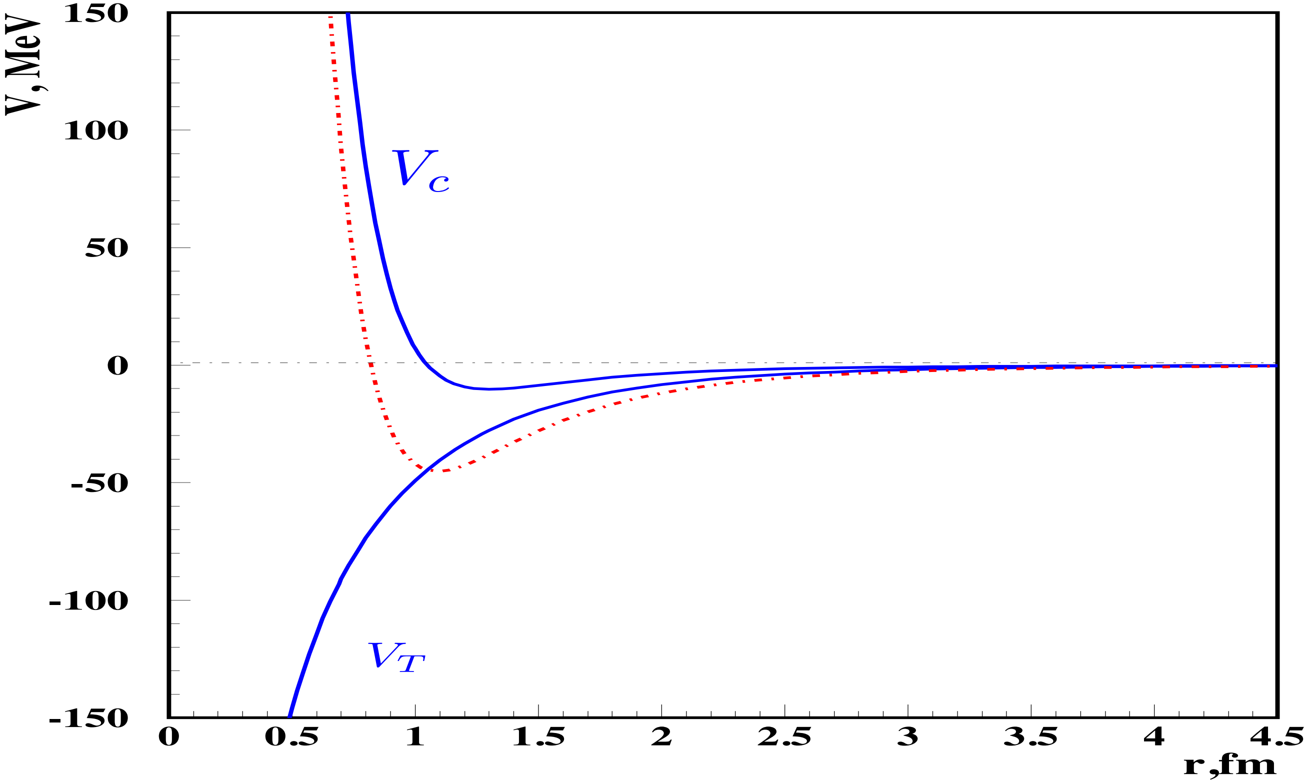
Factor of 20

Expected 4
(Wigner counting)

Theoretical Interpretation

$$\Phi^{(1)}(k_1,\cdots,k_c,\cdots,-k_c,\cdots,k_A) \approx \frac{U_{NN}(k_c)}{k_c^2} F_A(k_1,\cdots'\cdots',\cdots,k_A)$$

$$n_A(k) \approx a_{NN}(A) n_{NN}(k)$$

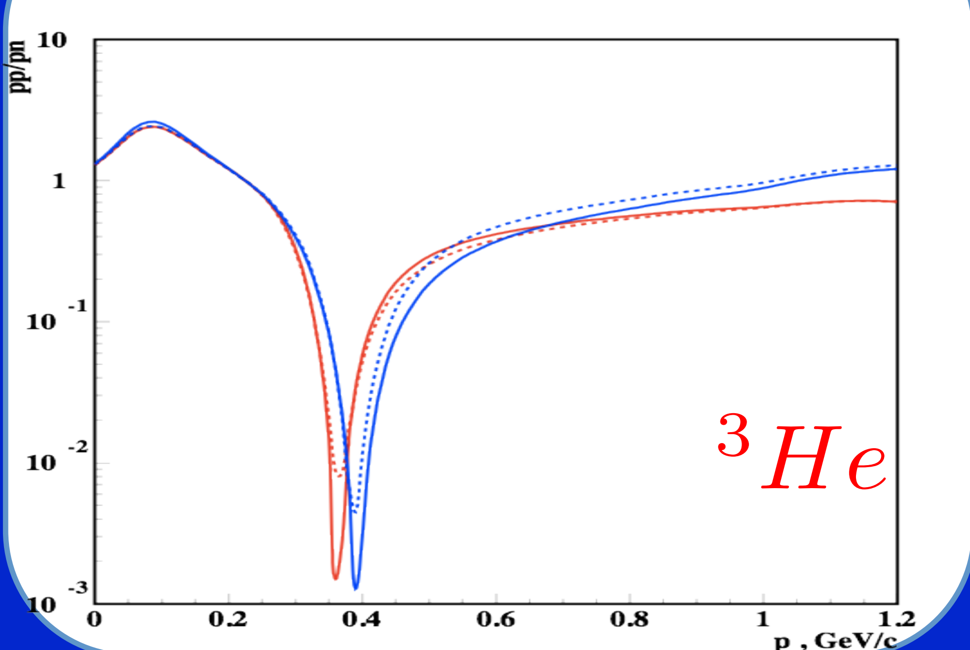


Explanation lies in the dominance of the <u>tensor</u> part in the NN interaction

$$V_{NN}(r) \approx V_c(r) + V_t(r)\cdot S_{12}(r) + V_{LS}\cdot \vec{L}\vec{S}$$

$$S_{12} = 3(\sigma_1\cdot\hat{r})(\sigma_2\cdot\hat{r}) - \sigma_1\sigma_2$$

M.S, Abrahamyan, Frankfurt,Strikman PRC,2005



$$S_{12}|pp\rangle = 0$$

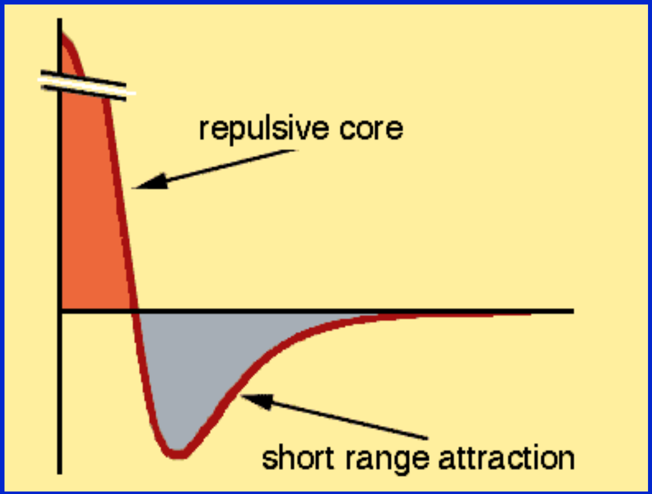
$$S_{12}|nn\rangle = 0$$

Isospin 1 states

$$S_{12}|pn\rangle = 0$$

$$S_{12}|pn\rangle \neq 0$$

Isospin 0 states



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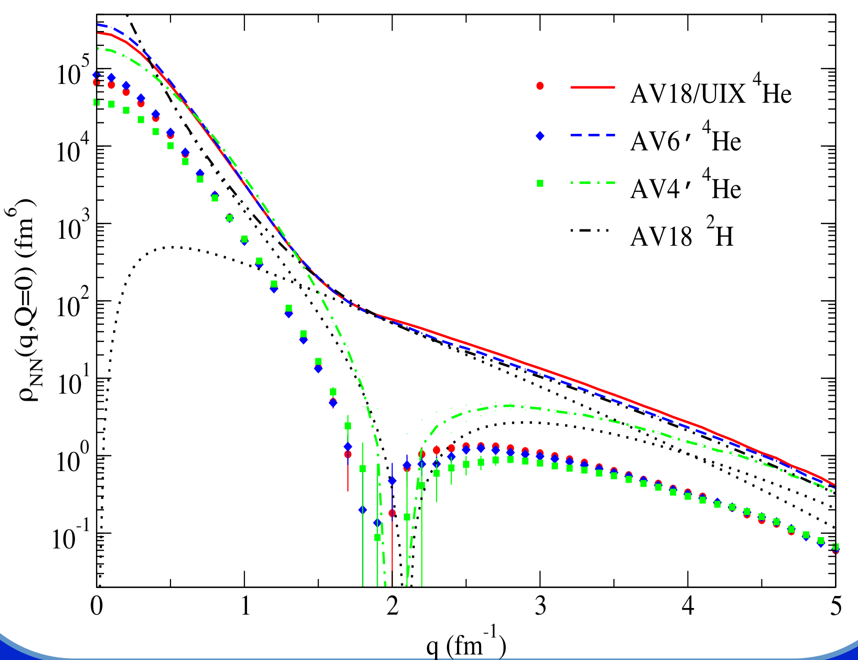
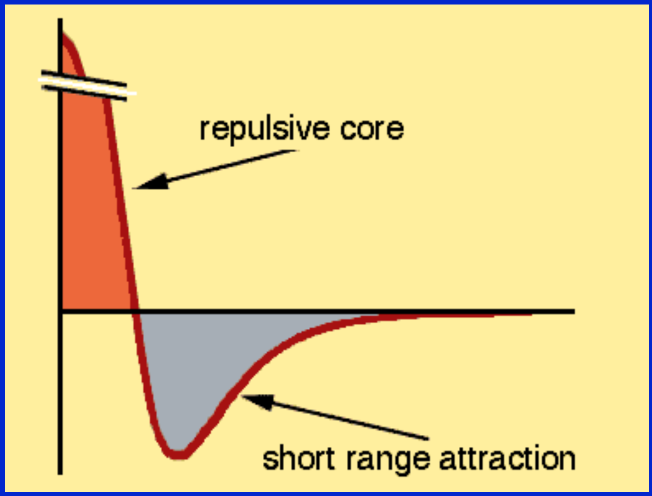
Isospin 0 states

Sciavilla, Wiringa, Pieper, Carlson PRL,2007





- *Dominance of pn short range correlations as compared to pp and nn SRCS*

2006-2008s

- *Dominance of NN Tensor as compared to the NN Central Forces at <= 1fm*

- Two New Properties of High Momentum Component

- Energetic Protons in Neutron Rich Nuclei at $p > k_F$

$$\boxed{n^A(p) \sim a_{NN}(A) \cdot n_{NN}(p)} \qquad (1)$$

- Dominance of pn Correlations
  (neglecting pp and nn SRCs)

$$n_{NN}(p) \approx n_{pn}(p) \approx n_{(d)}(p) \qquad (2)$$

$$\boxed{n^A(p) \sim a_{pn}(A) \cdot n_d(p)}$$

$$a_2(A) \equiv a_{NN}(A) \approx a_{pn}(A)$$

- Define momentum distribution of proton & neutron

$$n^A(p) = \frac{Z}{A} n_p^A(p) + \frac{A-Z}{A} n_n^A(p) \qquad (3)$$

$$\int n_{p/n}^A(p) d^3p = 1$$

- Define

$$I_p = \frac{Z}{A} \int_{k_F}^{600} n_p^A(p) d^3p \qquad I_n = \frac{A-Z}{A} \int_{k_F}^{600} n_n^A(p) d^3p$$

- and observe that in the limit of no pp and nn SRCs

$$I_p = I_n$$

- Neglecting CM motion of SRCs

$$\frac{Z}{A} n_p^A(p) \approx \frac{A-Z}{A} n_n^A(p)$$

First Property: Approximate Scaling Relation

-if contributions by pp and nn SRCs are neglected and
the pn SRC is assumed at rest

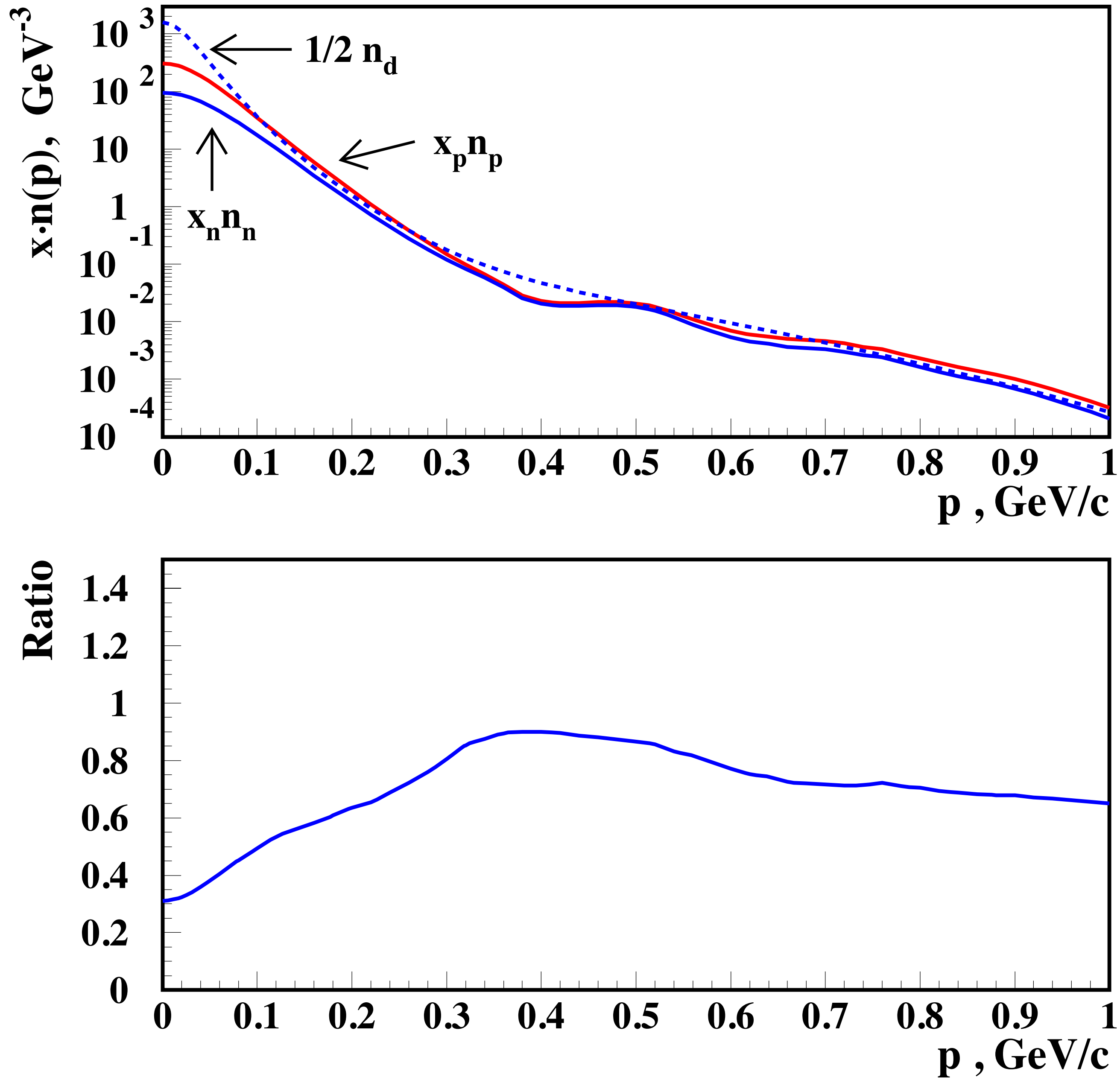
MS,arXiv:1210.3280
Phys. Rev. C 2014

- for $\sim k_F - 600$ MeV/c region:

$$\boxed{x_p \cdot n_p^A(p) \approx x_n \cdot n_n^A(p)}$$

where $x_p = \frac{Z}{A}$ and $x_n = \frac{A-Z}{A}$.

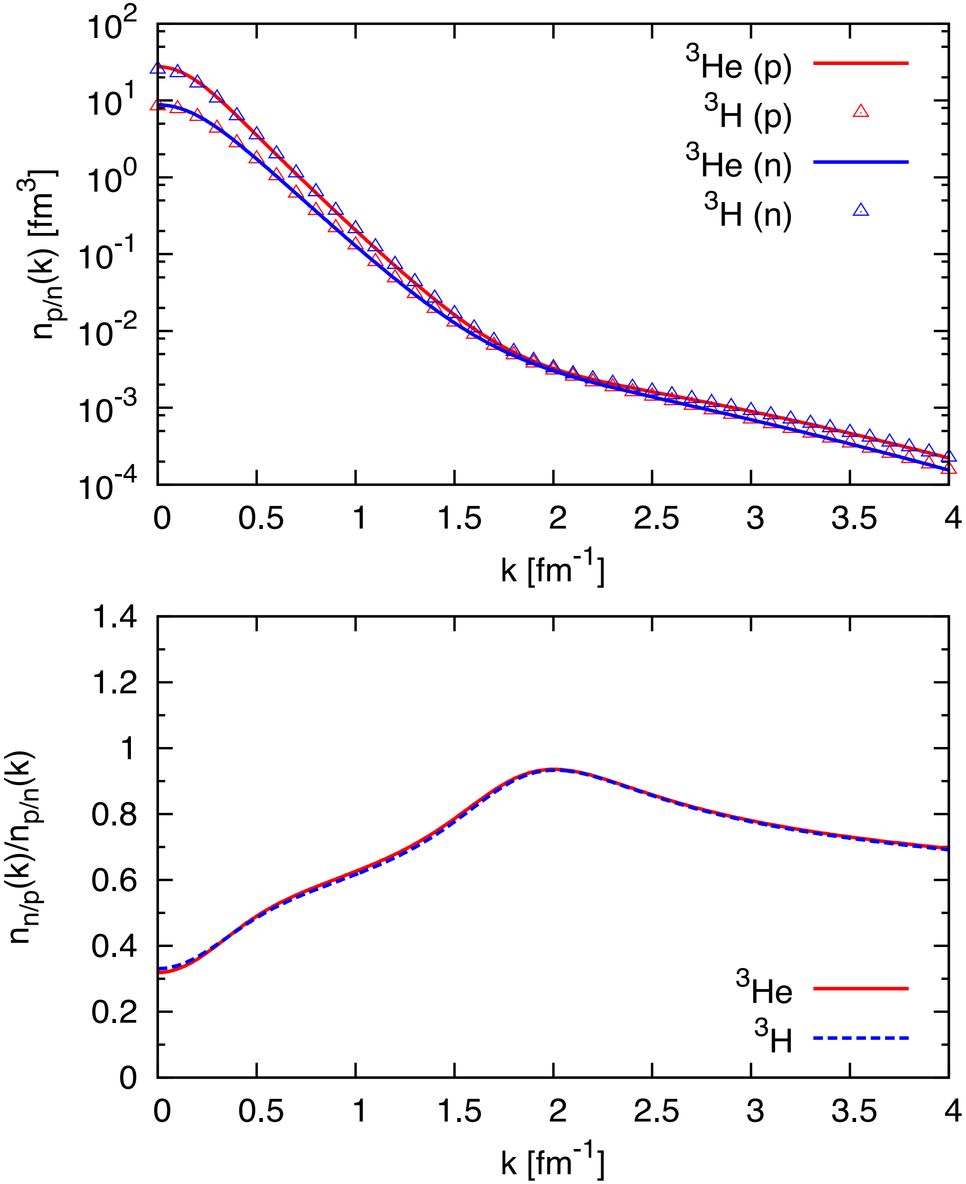
Realistic 3He Wave Function: Faddeev Equation



MS,PRC 2014

Realistic 3He Wave Function: Correlated Gaussian Basis

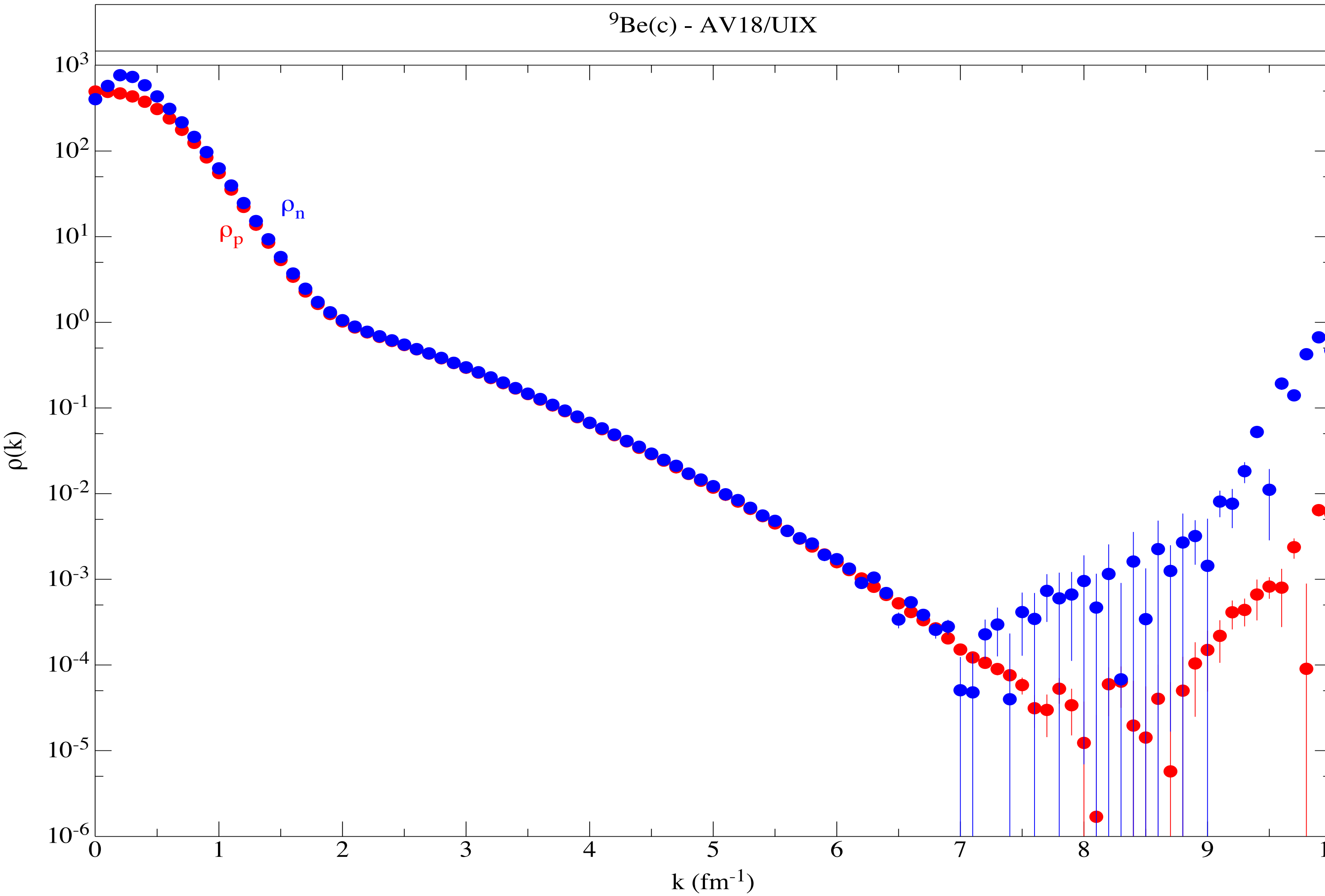
T.Neff & W. Horiuchi



April 2013

Be9 Variational Monte Carlo Calculation:

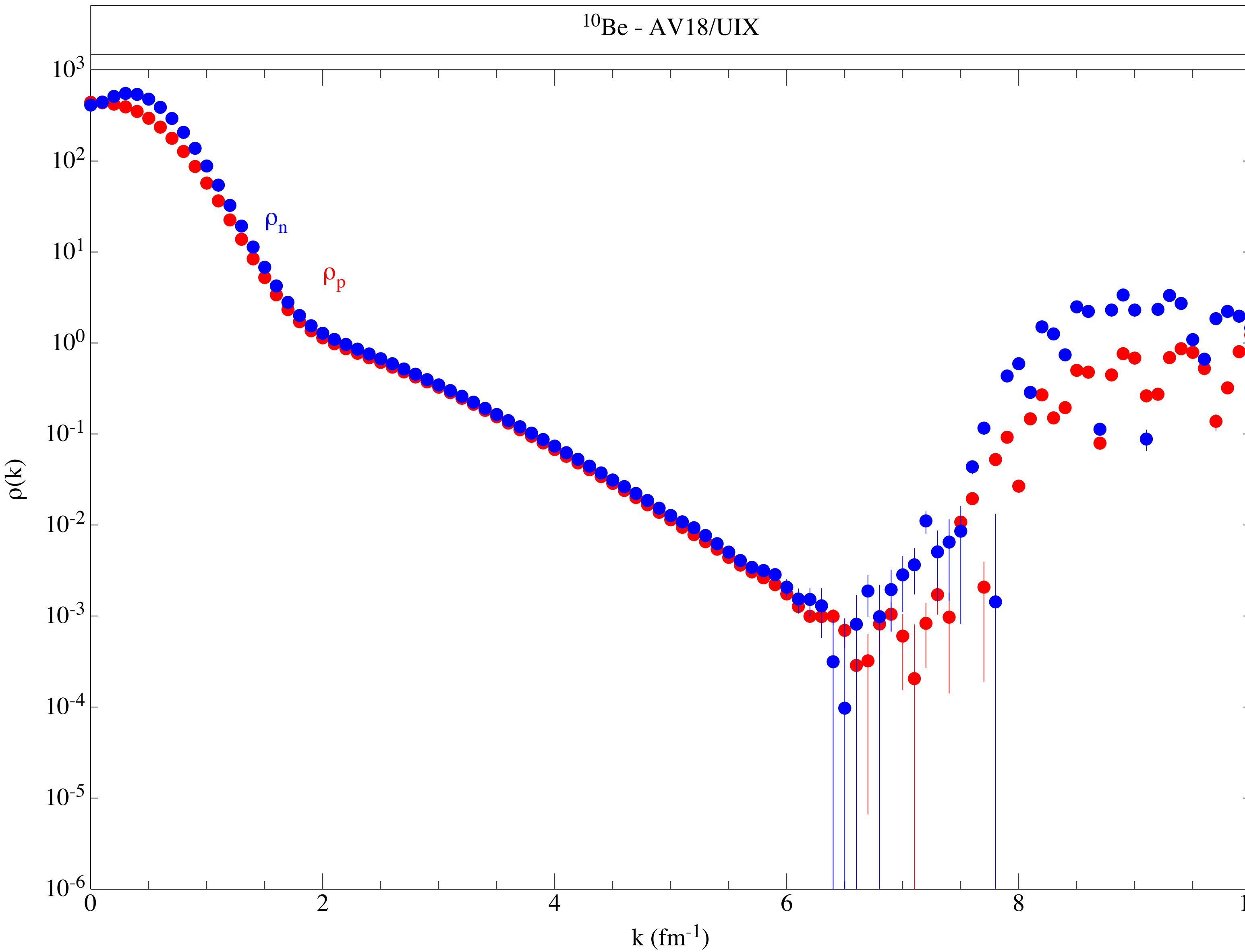
Robert Wiringa 2013 http://www.phy.anl.gov/theory/research/momenta/



Tanks to S. Pastore

B10 Variational Monte Carlo Calculation:

Robert Wiringa



Second Property:

MS,arXiv:1210.3280
Phys. Rev. C 2014

Using Definition: $n^A(p) = \frac{Z}{A} n_p^A(p) + \frac{A-Z}{A} n_n^A(p)$

Approximations: $n^A(p) \sim a_{NN}(A) \cdot n_{NN}(p)$

$$n_{NN}(p) \approx n_{pn}(p) \approx n_{(d)}(p)$$

And: $I_p = I_n$ $\quad I_p + I_n = 2I_N = a_{pn}(A) \int\limits_0^{600} n_d(p) d^3p$

One Obtains:

$$x_p \cdot n_p^A(p) \approx x_n \cdot n_n^A(p) \approx \frac{1}{2} a_{NN}(A, y) n_d(p)$$

where $y = |1 - 2x_p| = |x_n - x_p|$

- $a_{NN}(A,0)$ corresponds to the probability of pn SRC in symmetric nuclei
- $a_{NN}(A,1) = 0$ according to our approximation of neglecting pp/nn SRCs

Second Property: Fractional Dependence of High Momentum Component

$$a_{NN}(A,y) \approx a_{NN}(A,0) \cdot f(y)$$ with $f(0) = 1$ and $f(1) = 0$

$$f(|x_p - x_n|) = 1 - \sum_{j=1}^{n} b_i |x_p - x_x|^i \text{ with } \sum_{j=1}^{n} b_i = 0$$

In the limit $\sum_{j=1}^{n} b_i |x_p - x_x|^i \ll 1$ Momentum distributions of p & n are inverse proportional to their fractions

$$\boxed{n^A_{p/n}(p) \approx \frac{1}{2x_{p/n}} a_2(A,y) \cdot n_d(p)}$$

Observations: High Momentum Fractions

MS,arXiv:1210.3280
Phys. Rev. C 2014

$$P_{p/n}(A,y) = \frac{1}{2x_{p/n}} a_2(A,y) \int\limits_{k_F}^{\infty} n_d(p) d^3p$$

A	Pp(%)	Pn(%)
12	20	20
27	23	22
56	27	23
197	31	20

Requires dominance of pn SRCs in heavy neutron reach nuclei

O. Hen, M.S. L. Weinstein, et.al.
Science, 2014

Is the total kinetic energy inversion possible?

MS,arXiv:1210.3280
Phys. Rev. C 2014

Checking for He3

Energetic Neutron

$$E^p_{kin} = 14 \text{ MeV (p= 157 MeV/c)}$$

$$E^n_{kin} = 19 \text{ MeV (p= 182 MeV/c)}$$

Energetic Neutron
(Neff & Horiuchi)

$$E^p_{kin} = 13.97 \text{ MeV}$$

$$E^n_{kin} = 18.74 \text{ MeV}$$

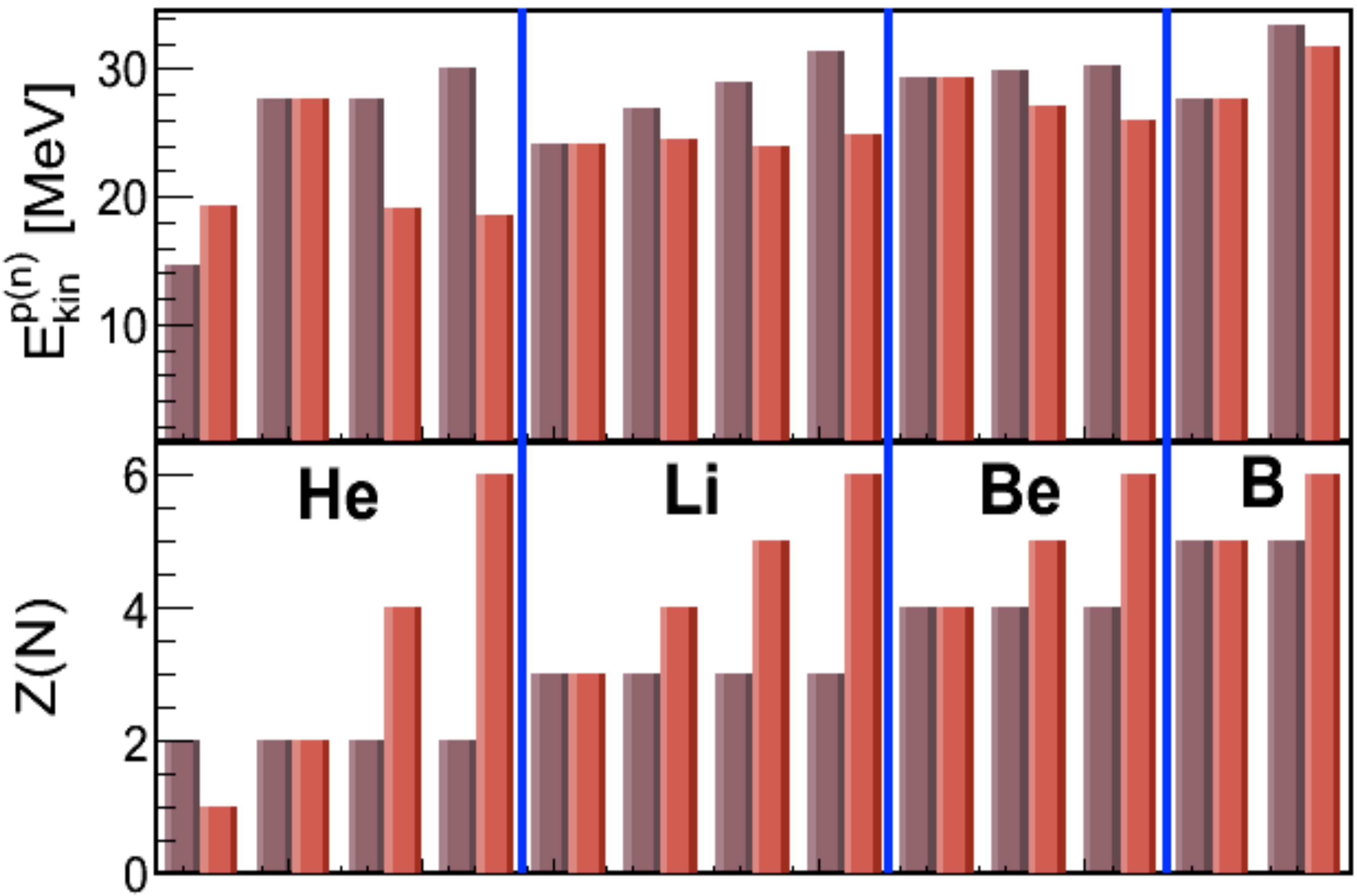
VMC Estimates: Robert Wiringa

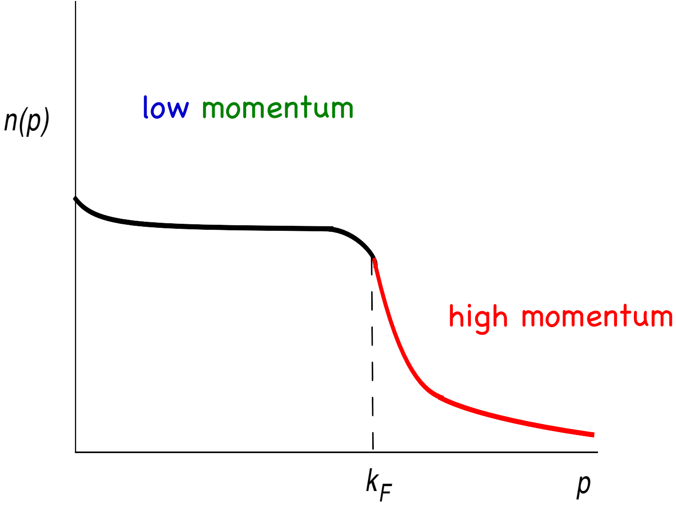
MS,arXiv:1210.3280
Phys. Rev. C 2014

Table 1: Kinetic energies (in MeV) of proton and neutron

A	y	E^p_{kin}	E^n_{kin}	$E^p_{kin} - E^n_{kin}$
^{8}He	0.50	30.13	18.60	11.53
^{6}He	0.33	27.66	19.06	8.60
^{9}Li	0.33	31.39	24.91	6.48
^{3}He	0.33	14.71	19.35	-4.64
^{3}H	0.33	19.61	14.96	4.65
^{8}Li	0.25	28.95	23.98	4.97
^{10}Be	0.2	30.20	25.95	4.25
^{7}Li	0.14	26.88	24.54	2.34
^{9}Be	0.11	29.82	27.09	2.73
^{11}B	0.09	33.40	31.75	1.65

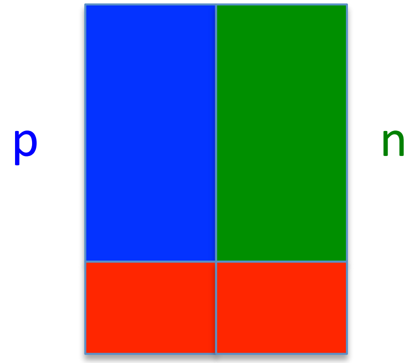
VMC Estimates: Robert Wiringa/Or Hen





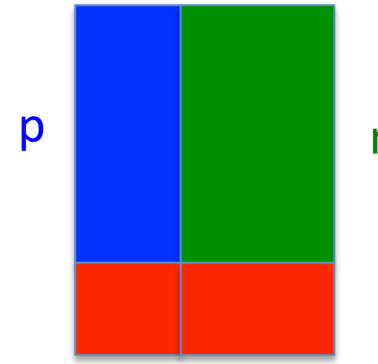
$$k_F = (3\pi^2 \rho_N)^{\frac{1}{3}}$$

Symmetric Nuclei



$$K_p = K_n$$

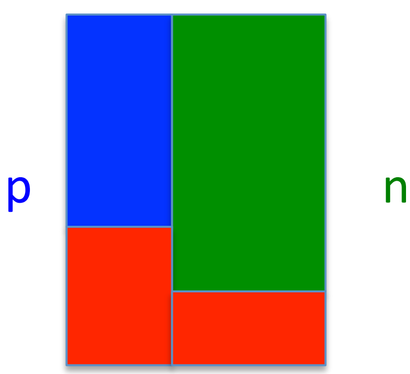
Asymmetric Nuclei



Conventional Theory: $K_n > K_p$

(Shell Model, HO Ab Initio)

Asymmetric Nuclei



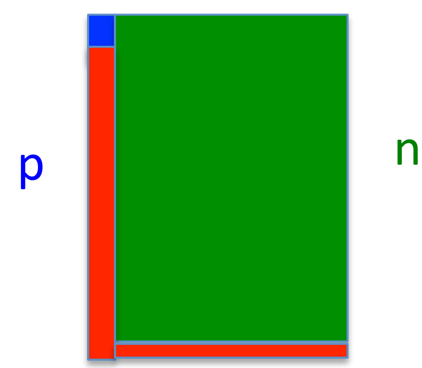
New Predictions

1. **Per nucleon, more protons are in high momentum tail**

2. **Kin Energy Inversion**

$$K_p > K_n \quad ?$$

Neutron Stars



Protons my completely populate the high momentum tail

-New Properties of High Momentum Distribution of Nucleons in Asymmetric Nuclei

MS,arXiv:1210.3280
Phys. Rev. C 2014

- Protons are more Energetic in Neutron Rich High Density Nuclear Matter

M. McGauley, MS
arXiv:1102.3973

- First Experimental Indication

O. Hen, M.S. L. Weinstein, et.al.
Science, 2014, "accepted"

- Confirmed by VMC calculations for A<12

R.B. Wiringa et al,
Phys. Rev. C 2014

- For Nuclear Matter

W. Dickhoff et al
Phys. Rev. C 2014

- For Medium/Heavy Nuclei

M. Vanhalst. W. Cosyn
J. Ryckebusch, arXiv 2014

Implications: Energetic Protons in neutron rich Nuclei

Implications: Protons are more modified in neutron rich nuclei

u-quarks are more modified then d-quarks in Large A Nuclei

- Flavor Dependence of EMC effect
- Different explanation of NuTev Anomaly
- Can be checked in neutrino-nuclei or in pvDIS processes

Are the observed effects universal for any two-component asymmetric/imbalanced Fermi Systems?

- In Atomic Physics

- In QCD

Momentum Sharing in Atoms

- Start with Two Component Asymmetric Degenerate Fermi Gas

- Asymmetric: $\rho_1 << \rho_2$

- Switch on the short-range interaction between two-component

- While interaction within each components is weak

- Spectrum of the small component gas will strongly deform

Cold Atoms

Trapped 2 component gas
diffusion

Momentum Sharing in QCD

- *Way of probing qq short range correlations*

- *In analogy of:* $x_p \cdot n_p^A(p) \approx x_n \cdot n_n^A(p)$

- *For valence quarks: Weinberg type LC equations for nucleon*

- *Observation of* $f_u(x) \approx f_d(x) \text{ at } x > x_c$

- *Observation of* $\frac{F_{2n}(x,Q^2)}{F_{2p}(x,Q^2} \to 1 \text{ at } x > x_c$

- *Predictions:*

Modeling Partonic Distributions:

$$f_q(x_B,Q^2) = \frac{1}{2}\sum_{i=1}^{3}\int\prod_{j=1}^{3}\frac{dx_j}{x_j}\frac{d^k_\perp}{2(2\pi)^3} \mid \langle\psi(x_1,x_2,x_3)\mid q(x_i)\rangle\mid^2 2(2\pi)^3\delta(x_B-x_i)\delta(1-\sum x_i)\delta(\sum k_{\perp,i})$$

$$\psi^{i^3_N,h_N} = \frac{1}{\sqrt{2}}\Big\{\Phi_{0,0}(k_1,k_2,k_3)(\chi^{(23)}_{0,0}\chi^{(1)}_{\frac{1}{2},h_N})\cdot(\tau^{(23)}_{0,0}\tau^{(1)}_{\frac{1}{2},i^3_N}) + \Phi_{1,1}(k_1,k_2,k_3)\times$$

$$\sum_{i^3_{23}=-1}^{1}\sum_{h^3_{23}=-1}^{1}\langle 1,h_{23};\frac{1}{2},h_N-h_{23}\mid\frac{1}{2},h_N\rangle\langle 1,i^3_{23};\frac{1}{2},i^3_N-i^3_{23}\mid\frac{1}{2},i^3_N\rangle(\chi^{(23)}_{1,h_{23}}\chi^{(1)}_{\frac{1}{2},h_N-h_{23}})\cdot(\tau^{(23)}_{1,i^3_{23}}\tau^{(1)}_{\frac{1}{2},i^3_N-i^3_{23}})\Big\}$$

Single (Mean Field) Quark Distributions

$$\mid\langle\psi_p(x_1,x_2,x_3)\mid u(x_i)\rangle\mid^2 = 2N_u\left(\mid\Phi_{0,0}(k_1,k_2,k_3)\mid^2 + \mid\Phi_{1,1}(k_1,k_2,k_3)\mid^2\right)$$

$$\mid\langle\psi_p(x_1,x_2,x_3)\mid d(x_i)\rangle\mid^2 = N_d\left(\mid\Phi_{0,0}(k_1,k_2,k_3)\mid^2 + \mid\Phi_{1,1}(k_1,k_2,k_3)\mid^2\right)$$

$$N_d < N_u$$

Modeling Partonic Distributions:

Single (Mean Field) Quark Distributions

$$\mid \langle \psi_p(x_1, x_2, x_3) \mid u(x_i) \rangle \mid^2 = 2N_u \left(\mid \Phi_{0,0}(k_1, k_2, k_3) \mid^2 + \mid \Phi_{1,1}(k_1, k_2, k_3) \mid^2 \right)$$

$$\mid \langle \psi_p(x_1, x_2, x_3) \mid d(x_i) \rangle \mid^2 = N_d \left(\mid \Phi_{0,0}(k_1, k_2, k_3) \mid^2 + \mid \Phi_{1,1}(k_1, k_2, k_3) \mid^2 \right)$$

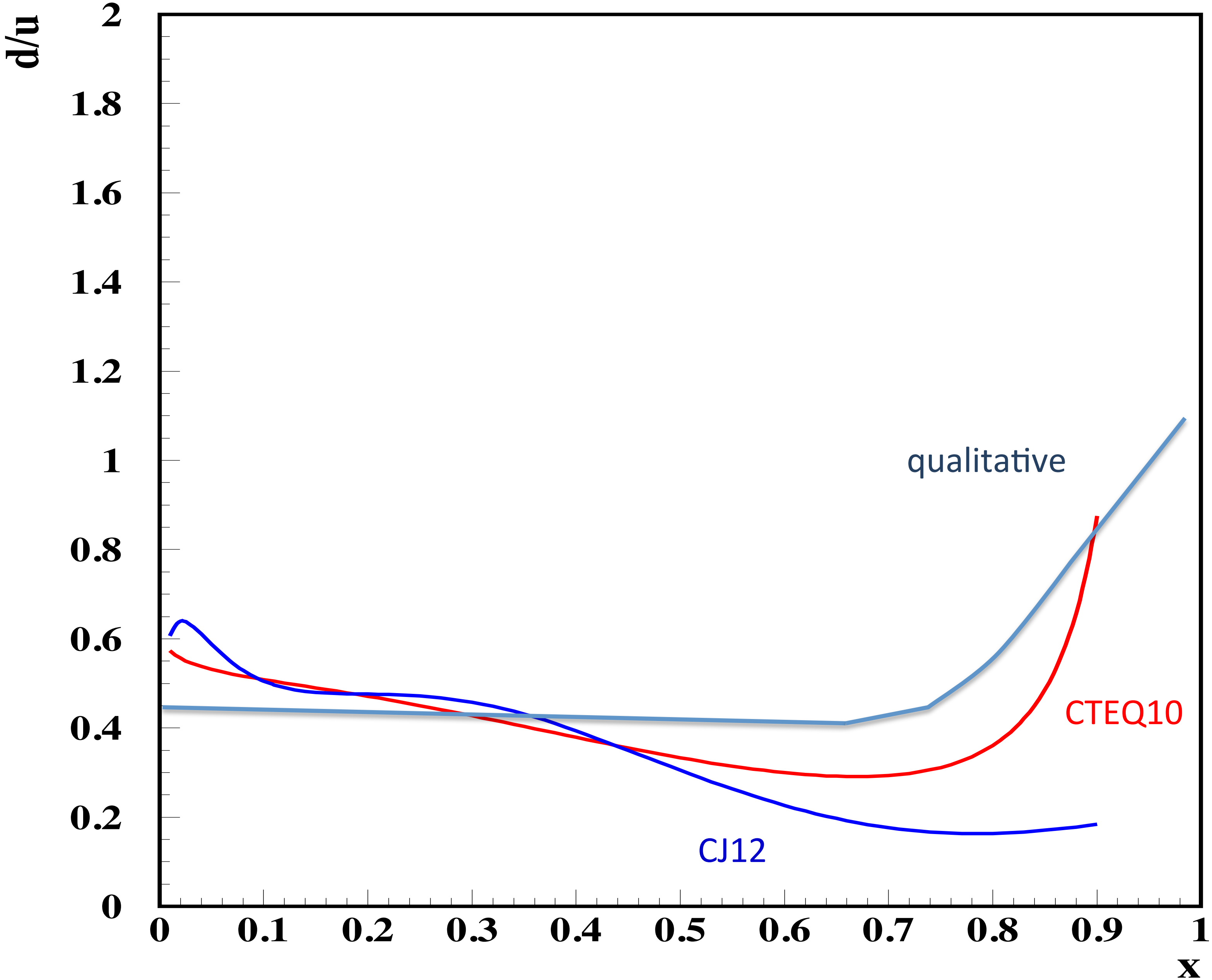
$$N_d < N_u$$

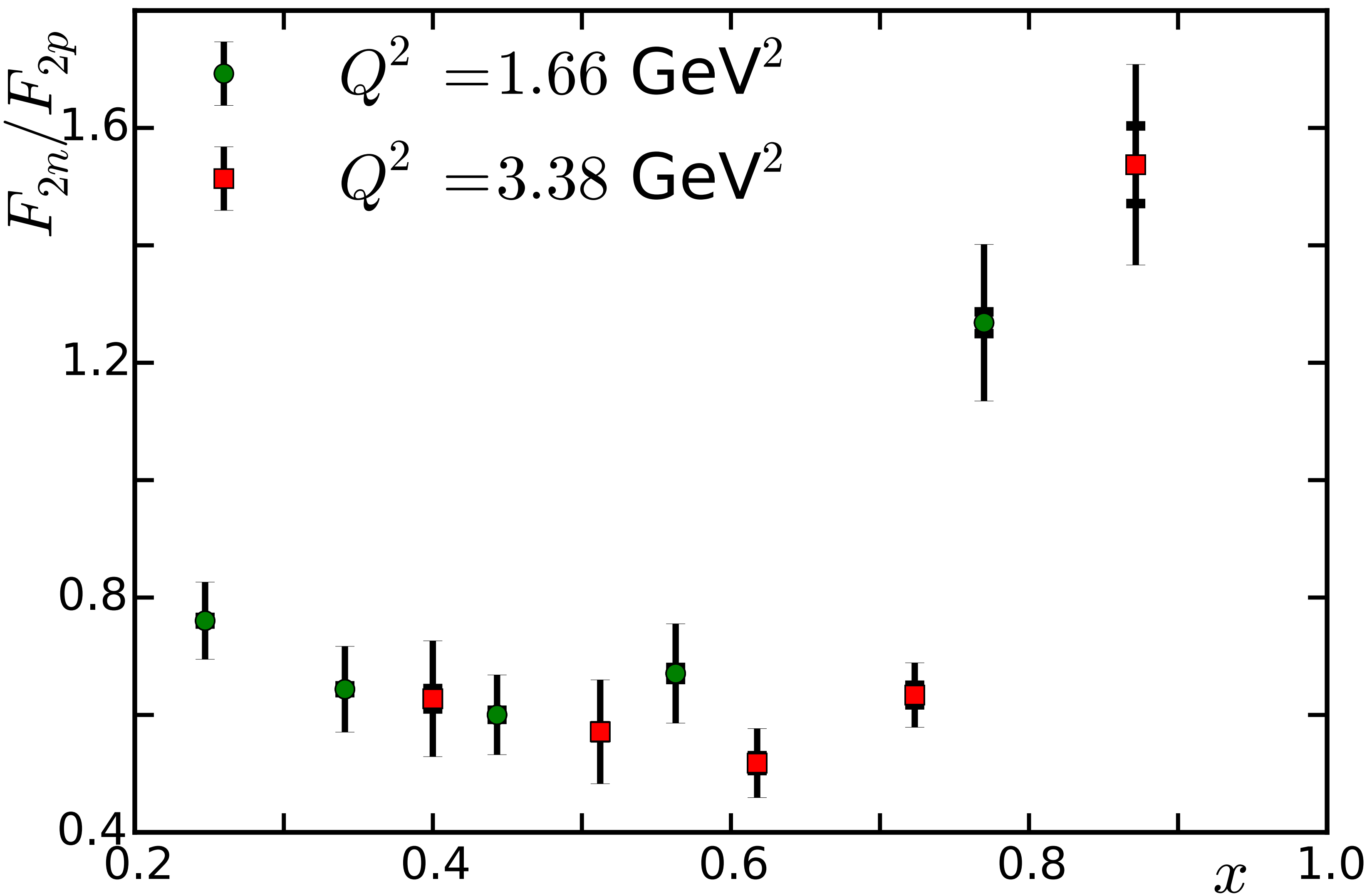
Quark Distribution in "good" qq -pair

$$\mid \langle \psi_p(x_1, x_2, x_3) \mid u(x_i) \rangle \mid^2 = N_u^{src} \mid \Phi_{0,0}^{src}(k_1, k_2, k_3) \mid^2$$

$$\mid \langle \psi_p(x_1, x_2, x_3) \mid d(x_i) \rangle \mid^2 = N_d^{src} \mid \Phi_{0,0}^{src}(k_1, k_2, k_3) \mid^2$$

Modeling Partonic Distributions:





Wim Coyn, MS arxiv 2015

Conclusions and Outlook

- High Momentum Sharing can be a Universal Feature of Fermi Systems

- In Nuclear Physics it predicts protons to be more energetic than neutrons

- In QCD it can be used to probe signatures of qq correlations

- Predicts rise of F2n/F2p at x> xc

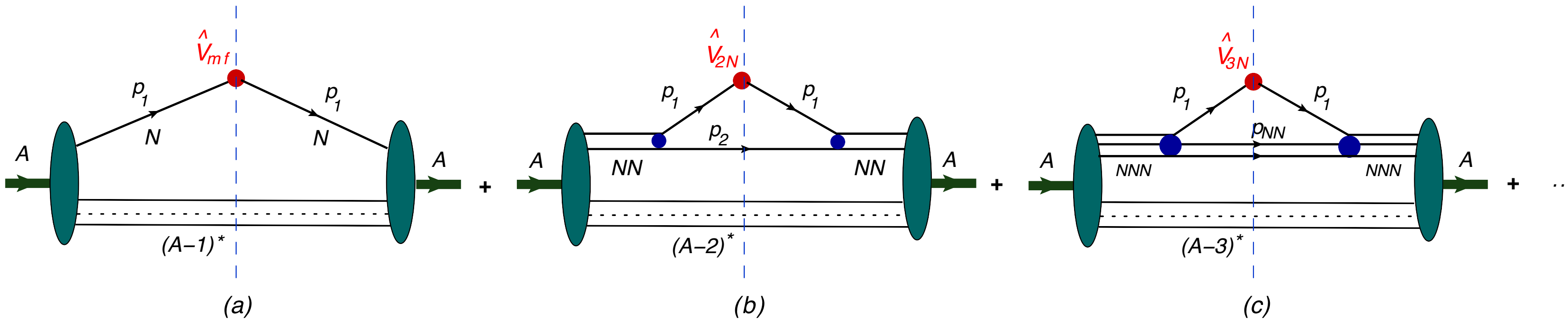
- xc will defines the threshold of qq-correlations

Modeling High Momentum and Missing Energy Nuclear Spectral Functions

- All best models are nonrelativistic

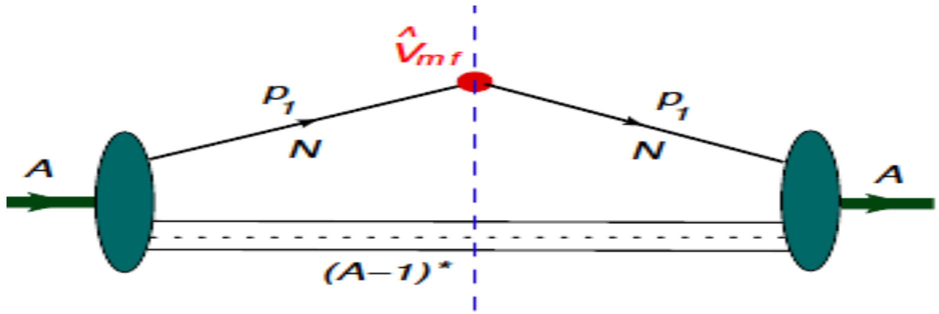
- SRC model allows to derive relativistic spectral functions if we know how to treat 2N SRCs relativistically

Diagrammatic Method of Calculation of Nuclear Spectral Function



- Nonrelativistic approximation
- Virtual Nucleon Approximation
- Light-Cone Approximation

Mean Field Approximation



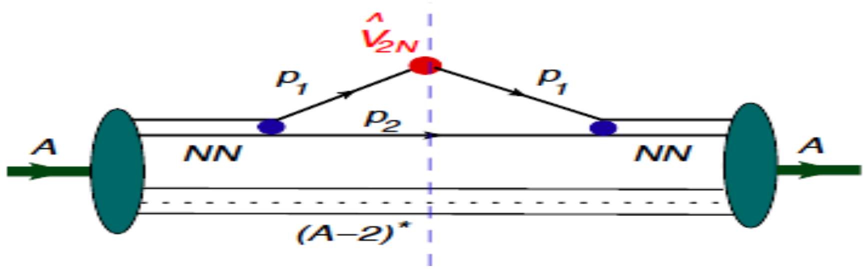
$$S_A^{MF} = -Im\int \chi_A^\dagger \Gamma^\dagger_{A,N,A-1}\frac{\not{p}_1 + m}{p_1^2 - m^2 + i\varepsilon}\hat{V}^{MF}\frac{\not{p}_1 + m}{p_1^2 - m^2 + i\times\varepsilon}\left[\frac{G_{A-1}(p_{A-1})}{p_{A-1}^2 - M_{A-1}^2 + i\varepsilon}\right]^{on}\Gamma_{A,N,A-1}\chi_A\frac{d^4p_{A-1}}{i(2\pi)^4}$$

$$\hat{V}^{MF} = ia^\dagger(p_1,s_1)\delta^3(p_1+p_{A-1})\delta(E_m-E_\alpha)a(p_1,s_1)$$

$$\psi_{N/A}(p_1,s_1,s_A,E_\alpha) = \frac{\bar{u}(p_1,s_1)\Psi^\dagger_{A-1}(p_{A-1},s_{A-1},E_\alpha)\Gamma_{A,N,A-1}\chi_A}{(M_{A-1}^2 - p_{A-1}^2)\sqrt{(2\pi)^3 2E_{A-1}}}$$

$$S_A^{MF}(p_1,E_m) = \sum_\alpha \sum_{s_1,s_{A-1}} \mid \psi_{N/A}(p_1,s_1,s_A,E_\alpha)\mid^2 \delta(E_m-E_\alpha)$$

2N SRC model



$$S_A^{2N} = Im \int \chi_A^\dagger \Gamma^\dagger_{A,NN,A-2} \frac{G(p_{NN})}{p_{NN}^2 - M_{NN}^2 + i\varepsilon} \Gamma^\dagger_{NN\to NN} \frac{\not{p}_1 + m}{p_1^2 - m^2 + i\varepsilon} \hat{V}^{2N} \frac{\not{p}_1 + m}{p_1^2 - m^2 + i\varepsilon} \left[\frac{\not{p}_2 + m}{p_2^2 - m^2 + i\varepsilon}\right]^{on} \times$$

$$\Gamma_{NN\to NN} \frac{G(p_{NN})}{p_{NN}^2 - M_{NN}^2 + i\varepsilon} \left[\frac{G_{A-2}(p_{A-2})}{p_{A-2}^2 - M_{A-2}^2 + i\varepsilon}\right]^{on} \Gamma_{A,NN,A-2} \chi_A \frac{d^4 p_2}{i(2\pi)^4} \frac{d^4 p_{A-2}}{i(2\pi)^4}, \quad (1$$

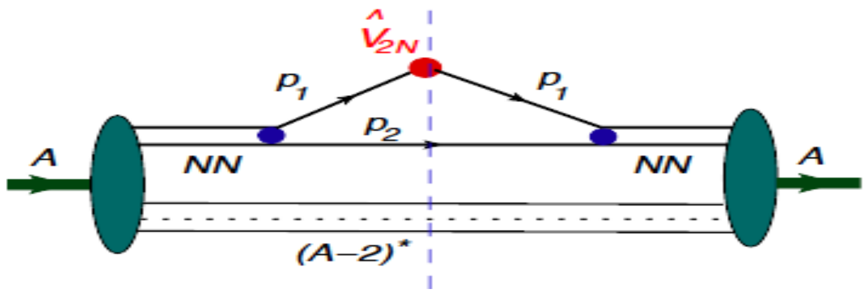
- Virtual Nucleon Approximation

$$\hat{V}_{2N} = ia(p_1, s_1)^\dagger \delta^3(p_1 + p_2 + p_{A-2}) \delta(E_m - E_\alpha^{2N}) a(p_1, s_1)$$

$$E_m^{2N} = E_{thr}^{(2)} + T_{A-2} + T_2 - T_{A-1} = E_{thr}^{(2)} + \frac{p_{A-2}^2}{2M_{A-2}} + T_2 - \frac{p_1^2}{2M_{A-1}}$$

$$\begin{aligned}
\psi_{NN}(p_1, s_1, p_2, s_2) &= \frac{\bar{u}(p_1, s_1)\bar{u}(p_2, s_2)\Gamma^{NN\to NN}\psi_{NN}(p_{NN}, s_{NN})}{(m_N^2 - p_1^2)\sqrt{2E_2(2\pi)^3}} \\
\psi_{CM}(p_{NN}, s_{NN}, p_{A-2}, s_{A-2}) &= \frac{\bar{\psi}(p_{NN}, s_{NN})\bar{\psi}_{A-2}(p_{A-2}, s_{A-2})\Gamma^{A,NN,A-2}\chi_A}{(M_{A-2}^2 - p_1^2)\sqrt{2E_{A-2}(2\pi)^3}}
\end{aligned} \quad (1)$$

2N SRC model Virtual Nucleon Approximation



$$S^N_{NN/A}(p_1, E_m) = \int n_{CM}(p_{NN}) n_{NN}(p_{rel}) \delta(E_m - E_m^{2N}) d^3 p_{NN}$$

$$n_{CM}(p_{NN}) = N_0(A) e^{-\alpha(A) p^2_{NN}}$$

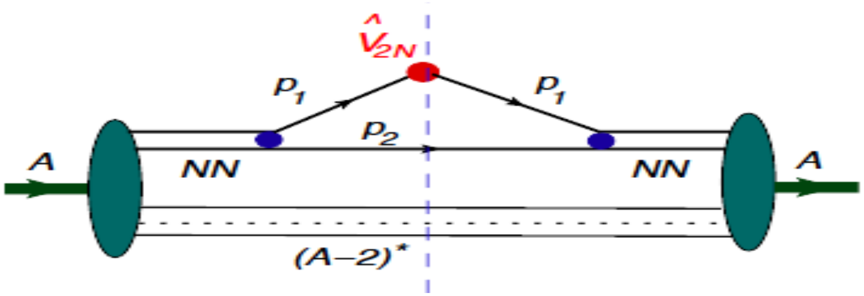
$$n^N_{NN}(p_{rel}) = \frac{a_2(A)}{(2x_N)^\gamma} \frac{n_d(p_{rel})}{\frac{m_N - E_m - T_{A-1}}{M_A/A}}$$

O.Artiles & M.S. 2015

$$n^N_{NN}(p_{rel}) = a_2(A) n_d(p_{rel})$$

Ciofi & Simula PRC 1996

2N SRC model Non Relativistic Approximation



$$S^N_{NN/A}(p_1, E_m) = \int n_{CM}(p_{NN}) n_{NN}(p_{rel}) \delta(E_m - E^{2N}_m) d^3 p_{NN}$$

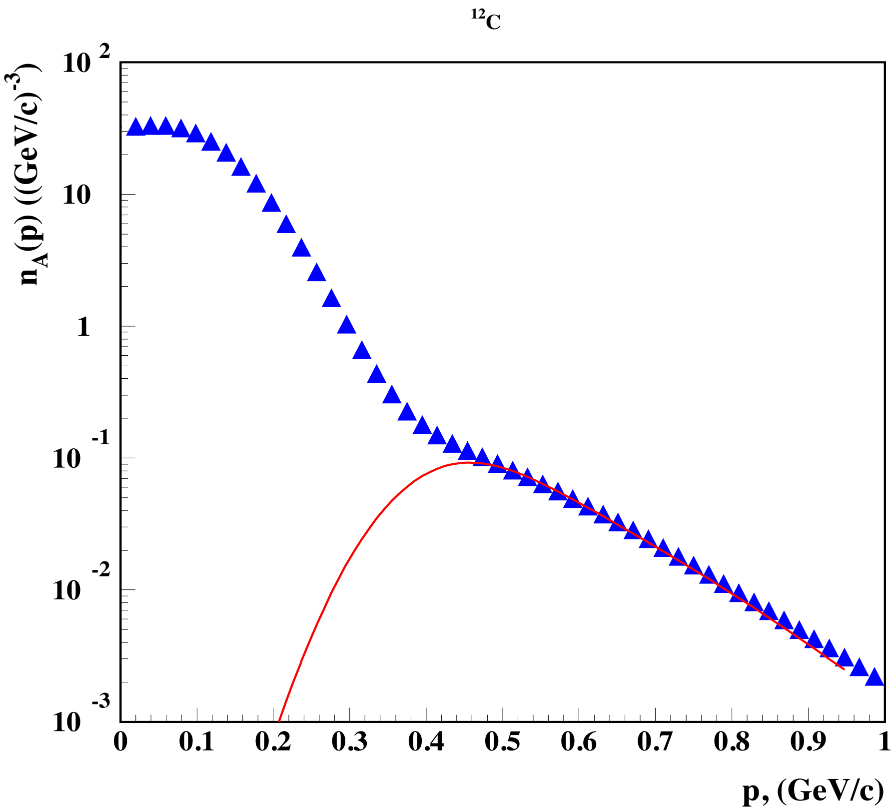
$$n_{CM}(p_{NN}) = N_0(A) e^{-\alpha(A) p^2_{NN}}$$

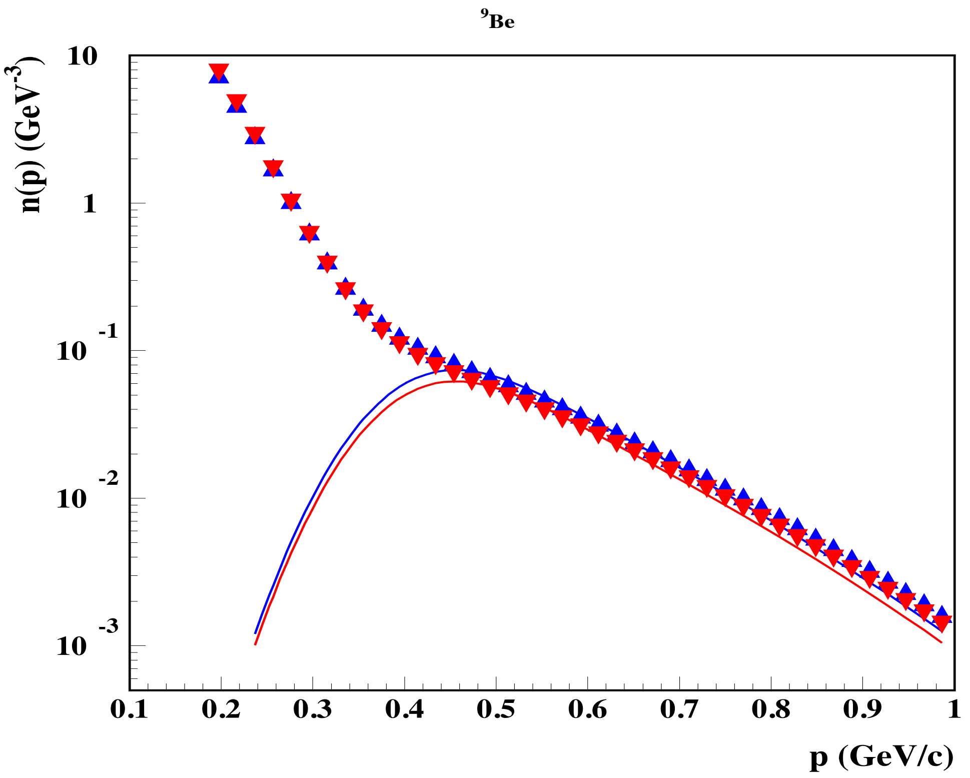
O.Artiles & M.S. 2015

$$n^N_{NN}(p_{rel}) = \frac{a_2(A)}{(2x_N)^\gamma} n_d(p_{rel})$$

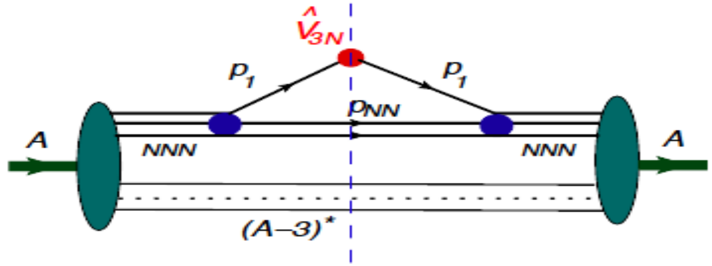
Compare with NonRelativistic Ab-Initio Microscopic Calculations, VMC, Ciofi, Ryckebusch talks

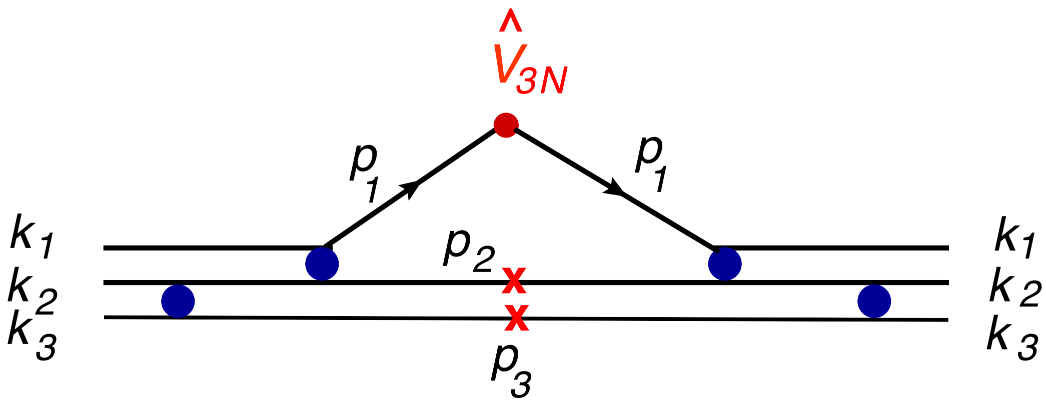
2N SRC model Non Relativistic Approximation





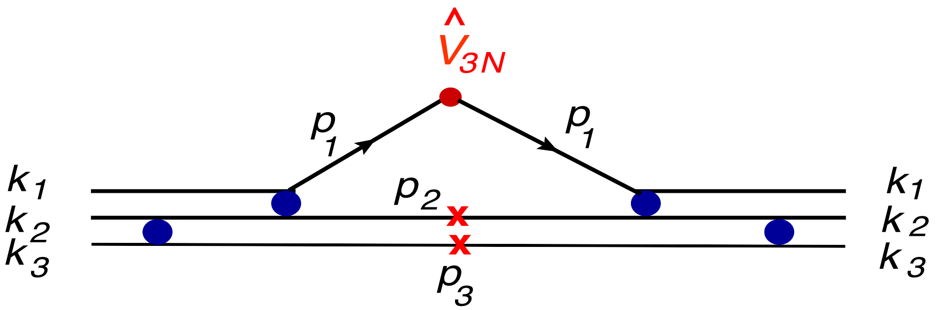
3N SRC model





$$
\begin{aligned}
S_A^{3N}(p_1, E_m) \quad = \quad & \int \bar{u}(k_1)\bar{u}(k_2)\bar{u}(k_3)\Gamma^{\dagger}_{NN\to NN} \frac{\sum\limits_{s_{2'}} u(p_{2'}, s_{2'})\bar{u}(p_{2'}, s_{2'})}{p_{2'}^2 - m^2} \Gamma^{\dagger}_{NN\to NN} \frac{u(p_1, s_1)}{p_1^2 - m^2} \\
& \delta^3(p_1 + p_2 + p_3)\delta(E_m - E^{3N}) \frac{\bar{u}(p_1, s_1)}{p_1^2 - m^2} \frac{\sum\limits_{s_2} u(p_2, s_2)\bar{u}(p_2, s_2)}{2E_2} \Gamma_{NN\to NN} \\
& \frac{\sum\limits_{\tilde{s}_{2'}} u(p_{2'}, \tilde{s}_{2'})\bar{u}(p_{2'}, \tilde{s}_{2'})}{p_{2'}^2 - m^2} \frac{\sum\limits_{s_3} u(p_3, s_3)\bar{u}(p_3, s_3)}{2E_3} \Gamma_{NN\to NN} u(k_1)u(k_2)u(k_3) \frac{d^3p_3}{(2\pi)^3} \frac{d^3p_2}{(2\pi)^3}.
\end{aligned}
\tag{1}
$$

3N SRC model: Virtual Nucleon Approximation



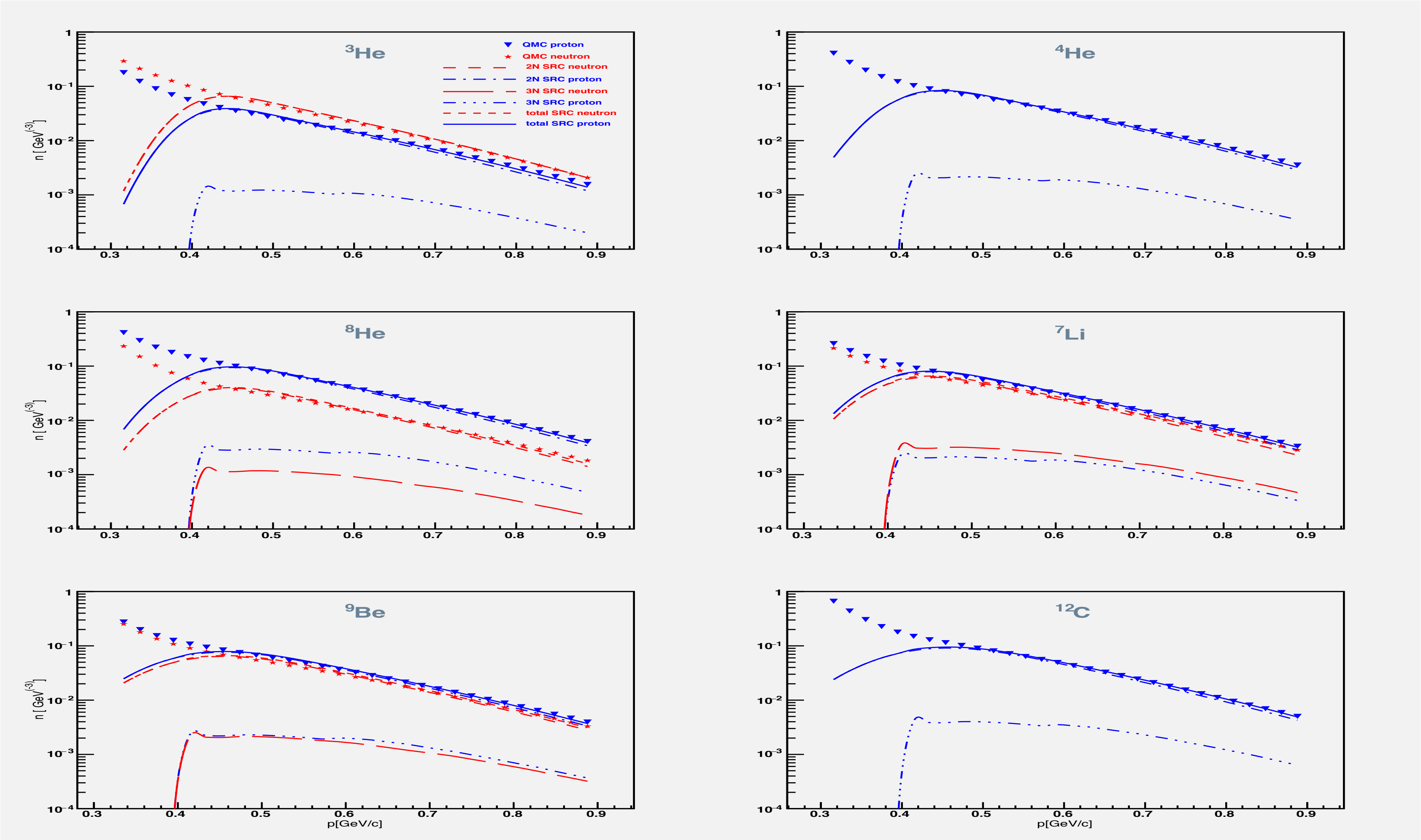
$$\hat{V}_{3N} = ia^{\dagger}(p_1, s_1)\delta^3(p_1 + p_2 + p_3)\delta(E_m - E^{3N})a(p_1, s_1)$$

$$E^{3N} = E_{thr}^{(3)} + T_3 + T_2 - \frac{p_1^2}{2M_{A-1}}$$

$$S_A^{3N}(p_1, E_m) = N_{3N}\int \frac{n_d(p_{2',3})n_d(p_{12})}{\frac{m_N - E_m - T_{A-1}}{M_A/A}}\delta(E_m - E^{3N})d^3p_3$$

$$N_{3N} \sim a_2(A)^2$$

3N SRC model: Non Relativistic Approximation

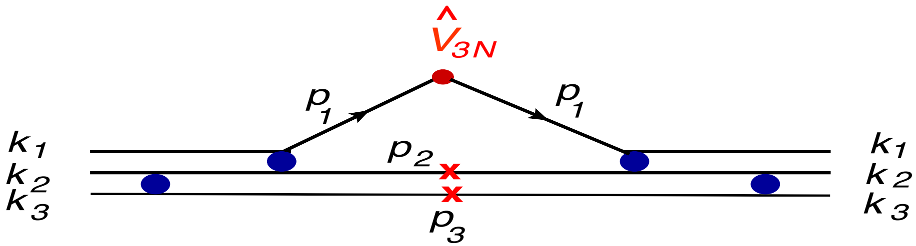


Momentum distribution is not the best quantity

3N SRC:

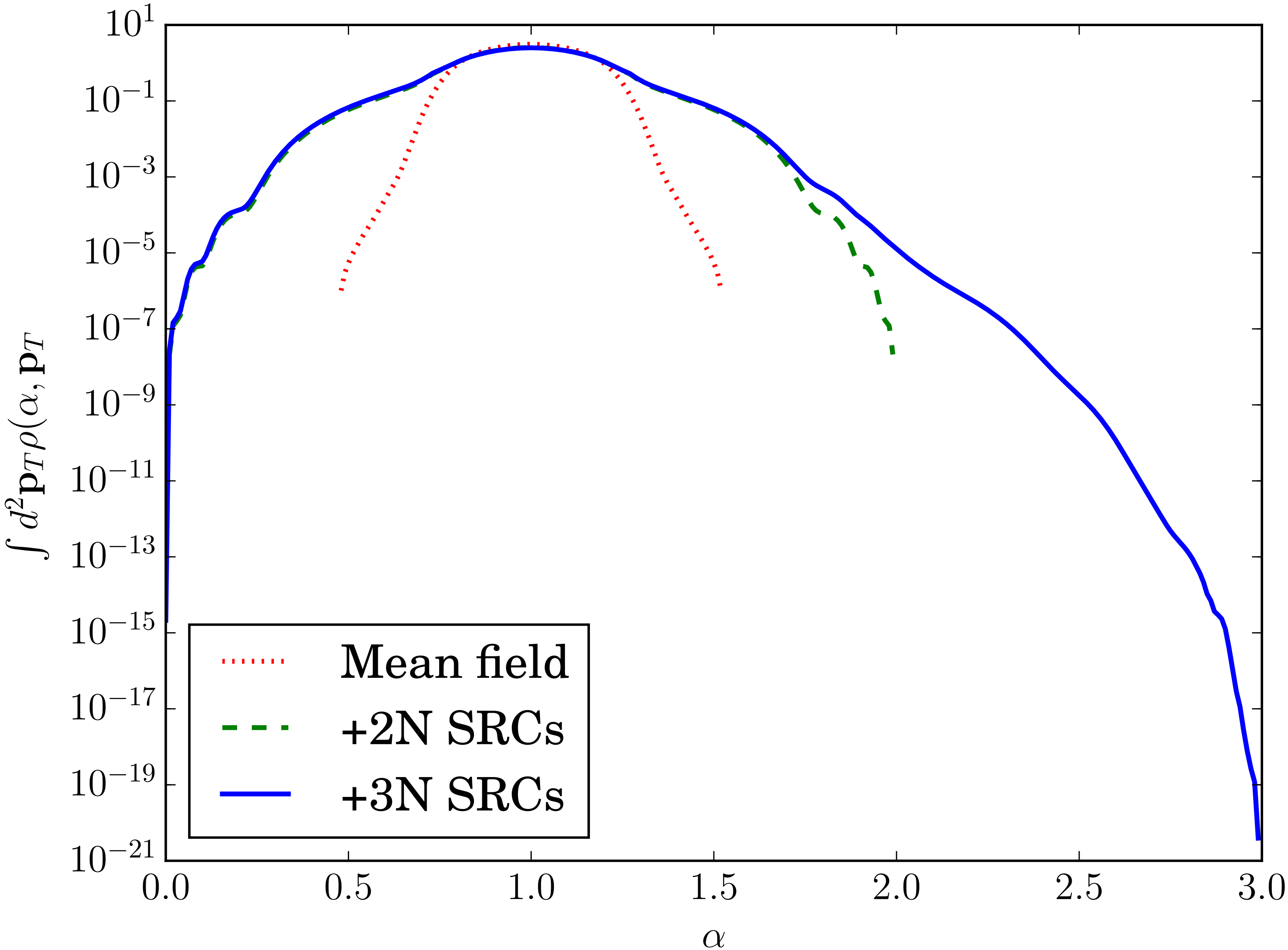
$$\alpha = \frac{A(E_k + k_z)}{E_A + p_{Az}}$$ Light-Cone Momentum Fraction Distribution

$$j - 1 < \alpha < j$$ for jxN SRC



A.Freese, M.S., M.Strikman 2015

$$\rho_3(\alpha, \mathbf{p}_T) = \mathcal{N}_{3N} \int d\alpha_3 d^2\mathbf{p}_{3T} \frac{1}{\alpha_3(3-\alpha-\alpha_3)} \left\{ \frac{3-\alpha_3}{2(2-\alpha_3)} \right\}^2 |\psi_d(k_{12})|^2 \, |\psi_d(k_{23})|^2$$



10^{1}
10^{-1}
10^{-3}
10^{-5}
10^{-7}
10^{-9}
10^{-11}
10^{-13}
10^{-15}
10^{-17}
10^{-19}
10^{-21}
$\int d^2\mathbf{p}_T \rho(\alpha, \mathbf{p}_T$
0.0
0.5
1.0
1.5
2.0
2.5
3.0
α
Mean field
+2N SRCs
+3N SRCs

Probing SRCs in Inclusive Scattering:

$$\frac{2\sigma(eA \to e'X)}{A\sigma(ed \to e'X)} = \frac{\rho_A(\alpha_{2N})}{\rho_d(\alpha_{2N})} = a_2(A)$$ For $1 < \alpha_{2N} < 2$

$$q + 2m = p_f + p_s$$

$$\alpha_{2N} = 2 - \frac{q_- + 2m_N}{2m_N}\left(1 + \frac{\sqrt{W_{2N}^2 - 4m_N^2}}{W_{2N}^2}\right)$$

$$\frac{3\sigma(eA \to e'X)}{A\sigma(e^3He \to e'X)} = \frac{\rho_A(\alpha_{3N})}{\rho_{^3He}(\alpha_{3N})} = a_3(A)$$ For $2 < \alpha_{3N} < 3$

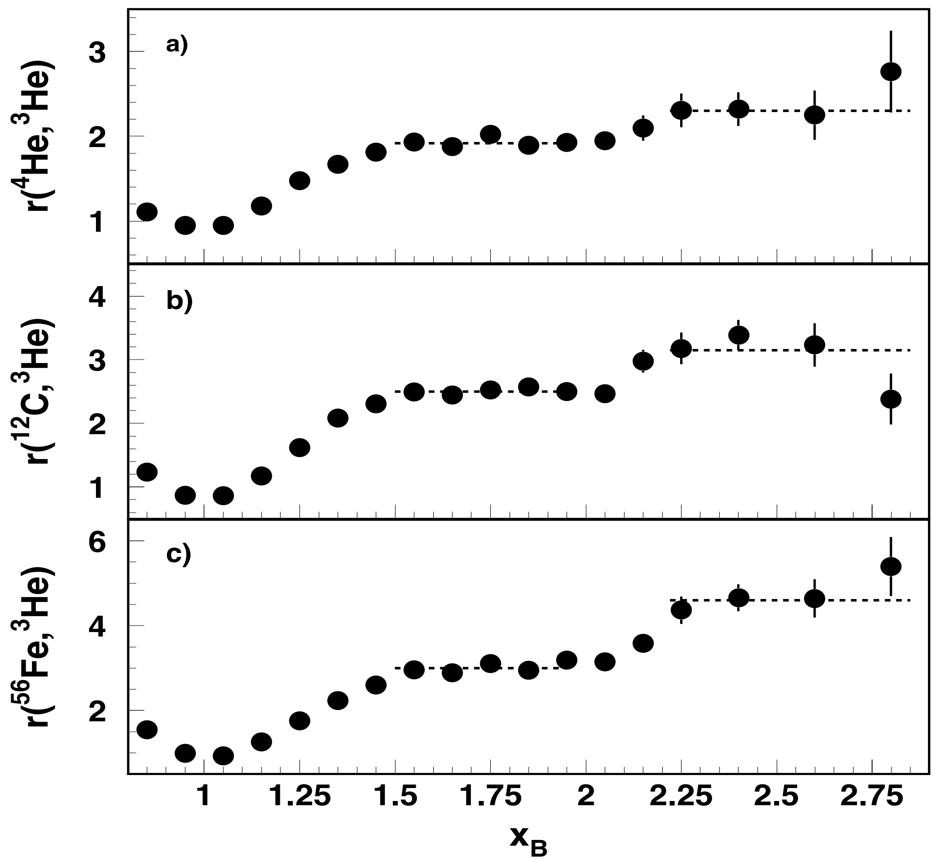
$$q + 3m = p_f + p_s$$

$$\alpha_{3N} = 3 - \frac{q_- + 3m_N}{2m_N}\left[1 + \frac{m_S^2 - m_N^2}{W_{3N}^2} + \sqrt{\left(1 - \frac{(m_S + m_n)^2}{W_{3N}^2}\right)\left(1 - \frac{(m_S - m_n)^2}{W_{3N}^2}\right)}\right]$$

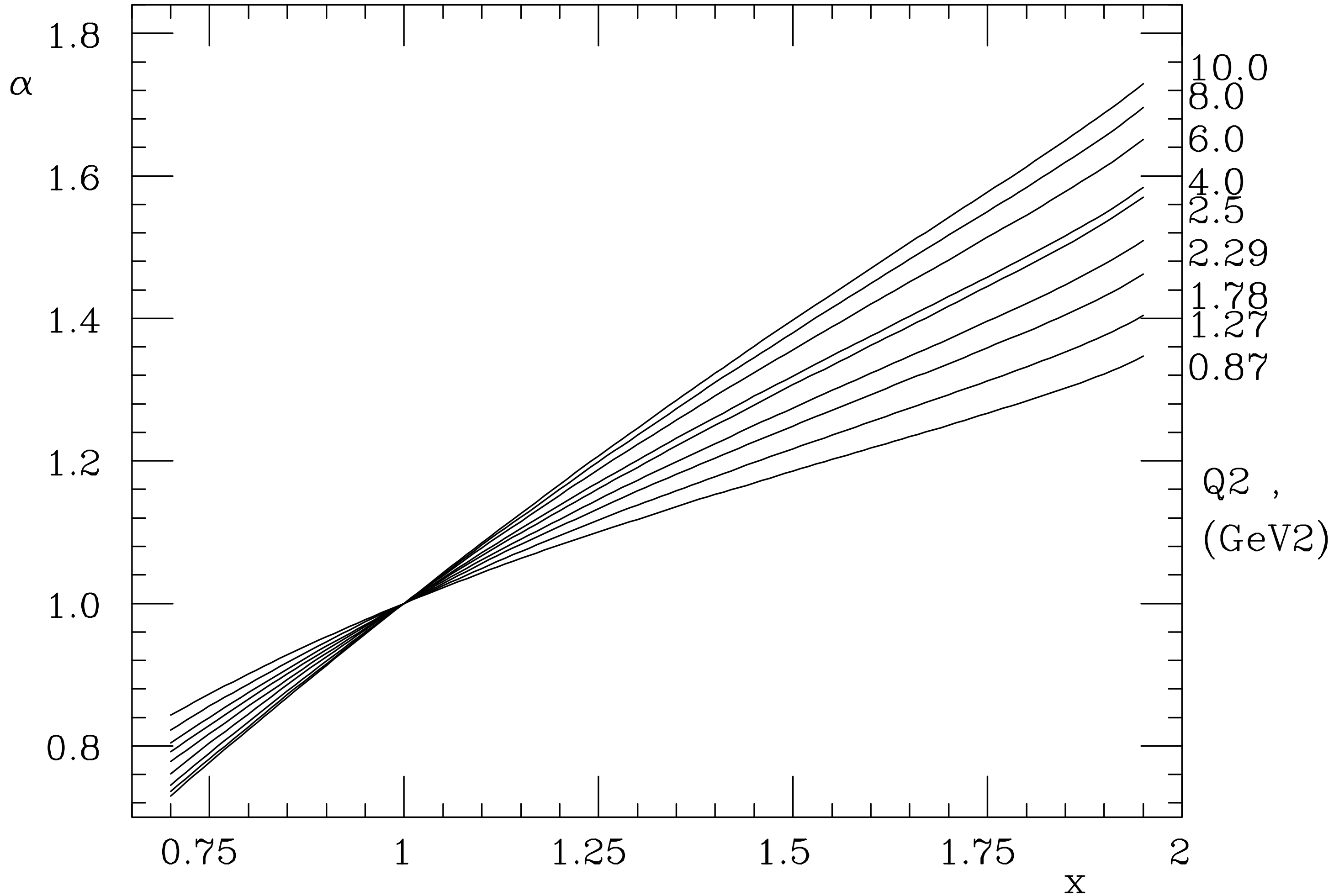
Probing SRCs in Inclusive Scattering:

in $Q^2 \to \infty$ $\quad \alpha_{2N} = \alpha_{3N} = x = \dfrac{Q^2}{2mq_0}$

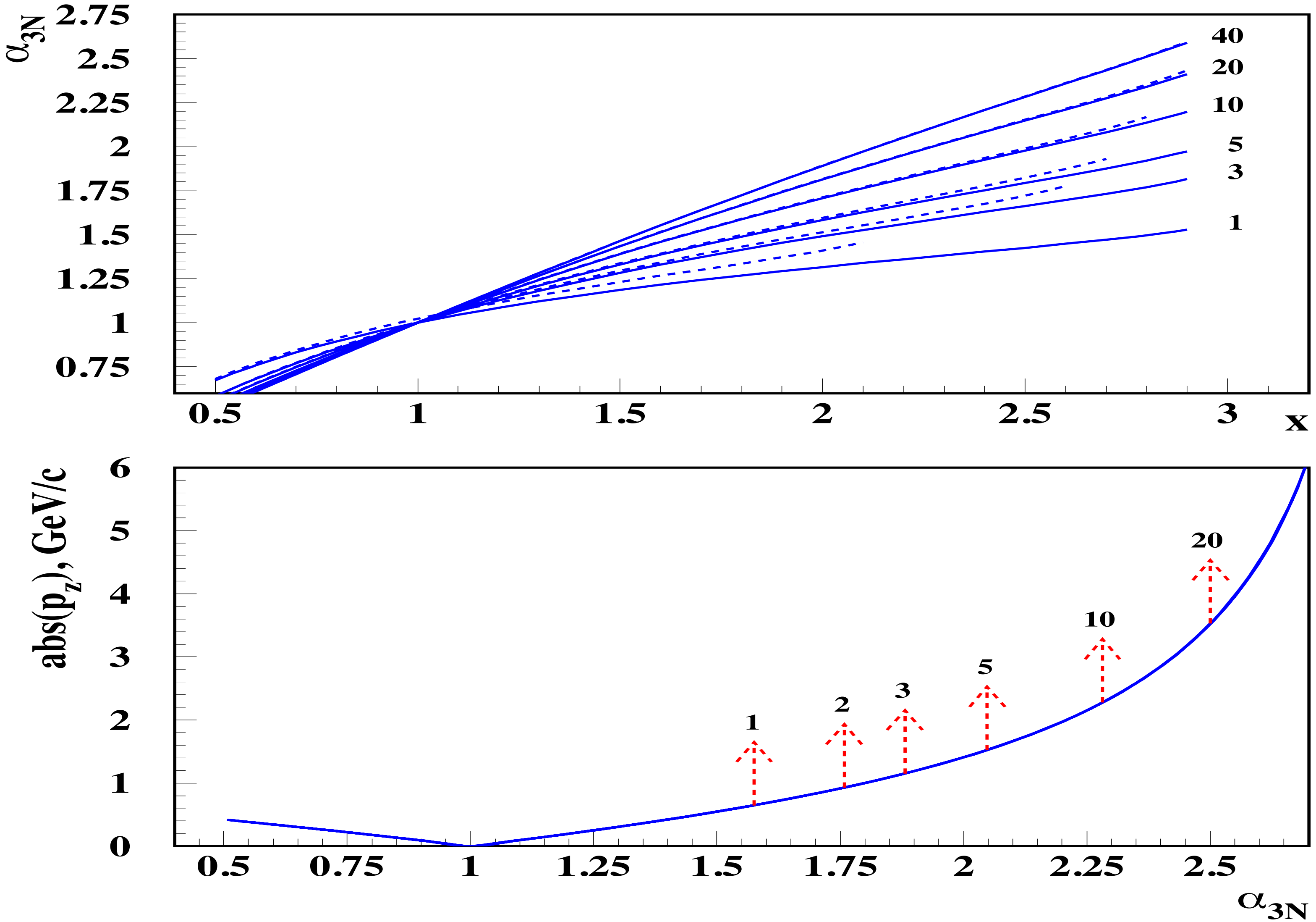
$\dfrac{3\sigma(e+A\to e'X)}{A\sigma(e+{}^3He\to e'X)}$ scales as a function x at $x > 1$



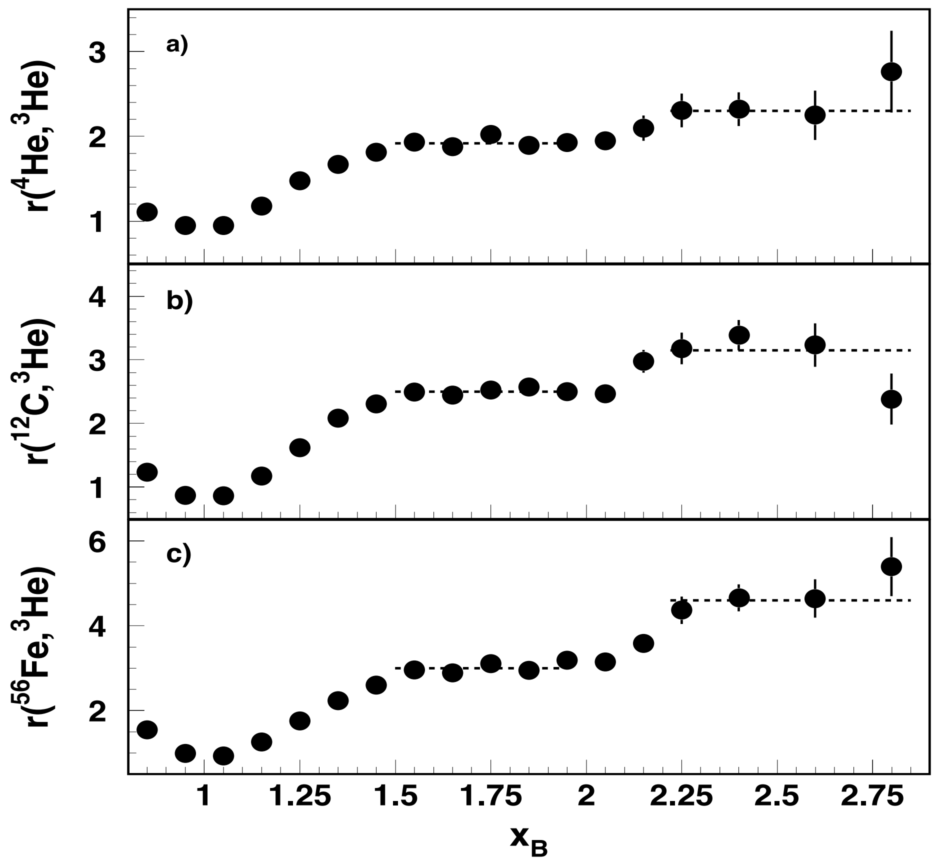
For finite Q2 - 2N SRCs



For finite Q2 - for 3N SRCs



$$R_3 = \frac{3\sigma(e+A\rightarrow e'X)}{A\sigma(e+{}^3He\rightarrow e'X)} = \frac{a_3(A)}{a_3({}^3He)} \sim \frac{a_2(A)^2}{a_2({}^3He)^2}$$



A	R_3^{exp}	R_3^{pred}
4	$2.33 \pm 0.12 \pm 0.04$	2.8
12	$3.18 \pm 0.14 \pm 0.19$	4
56	$4.63 \pm 0.19 \pm 0.27$	5.7

$R_3(A)/R_3({}^4He)$

12	1.3	1.4
56	1.9	2.0

Summary

- Way of including Relativistic Effects in

- 3N SRCs require larger Q2

- Proportionality of $a_2(A)^2$