Short-Range Central and Tensor Correlations

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EMMI workshop
“Cold dense nuclear matter, from short-range correlations to neutron stars”
GSI, Darmstadt
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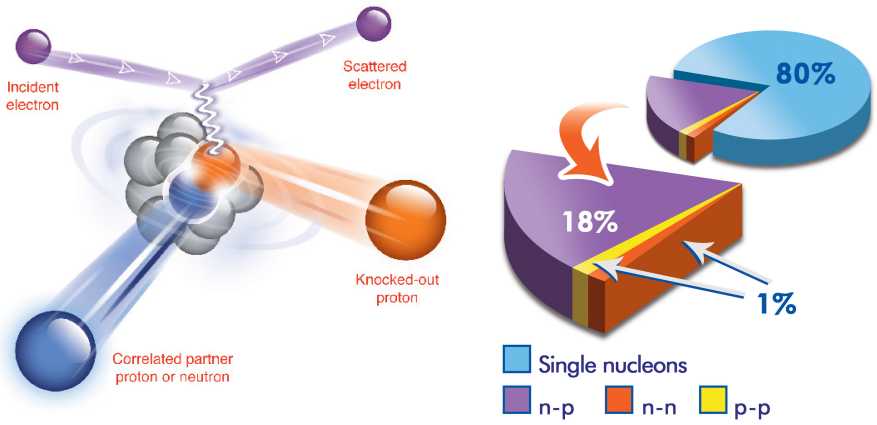
Motivation

Observations

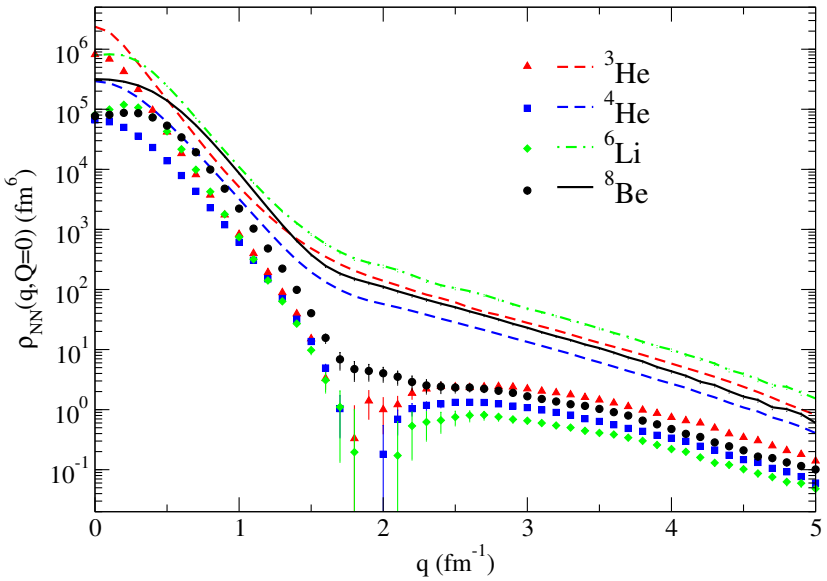
- JLab experiments found that a knocked out high-momentum proton is accompanied by a second nucleon with opposite momentum
- Cross sections for $(e, e'pn)$ and $(e, e'pp)$ reactions show strong dominance of *pn*- over *pp*-pairs



Subedi *et al.*, Science **320**, 1476 (2008)

Theoretical interpretation

- *ab initio* calculations with Argonne interactions show high-momentum components
- dominance of *pn*- over *pp*-pairs due to the tensor force



Wiringa, Schiavilla, Pieper, Carlson, PRC **85**, 021001(R) (2008)

Alvioli *et al.*, Int. J. Mod. Phys. E **22**, 1330021 (2013)

Outline

Nucleon-nucleon interaction

- Properties
- Realistic interactions

Correlations in light nuclei

- One- and two-body densities with AV8' interacton

Unitary transformations

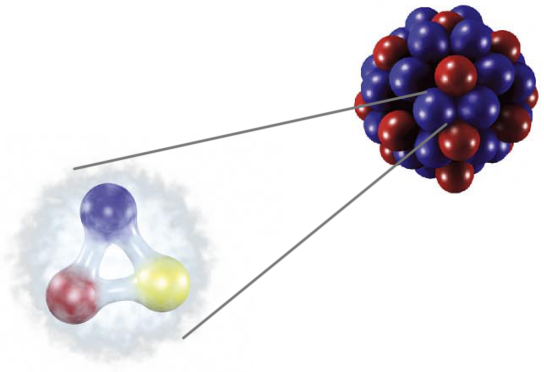
- Similarity Renormalization Group
- What happens to short-range physics ?

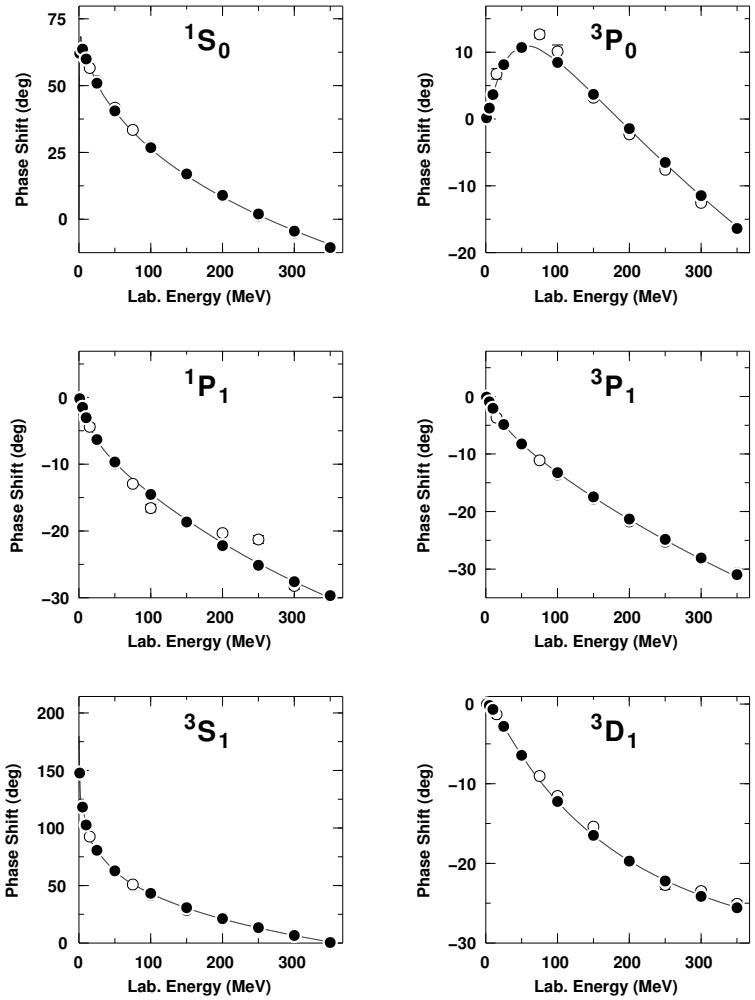
NCSM+SRG calculations

- compare AV18 and N3LO results
- ^{4}He two-body densities
- ^{6}He,^{9}Be,^{12}C two-body densities

Summary

Nucleon-nucleon interaction

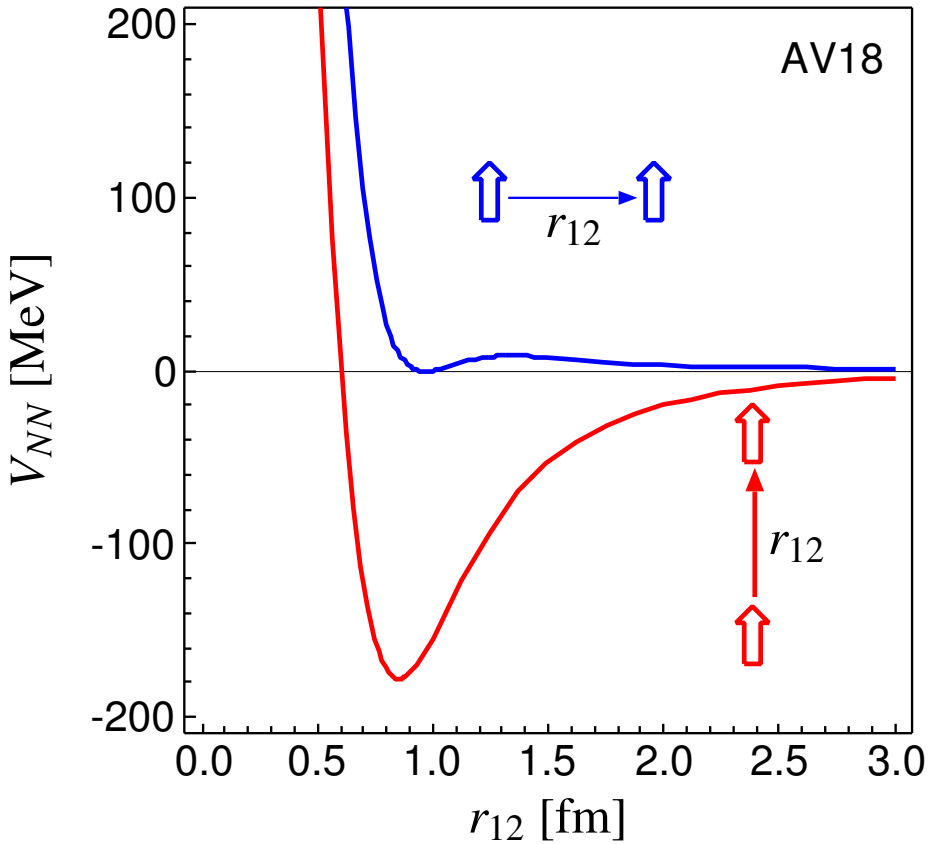




- Nucleons are not point-like, complicated quark and gluon sub-structure
- Nucleon-nucleon (NN) interaction: residual interaction
- Calculation within QCD not possible yet → construct **realistic NN potentials** describing two-nucleon properties (scattering, deuteron) with high accuracy
- Different potentials available, but same general features ...

NN interaction — Short-range and tensor correlations

$S = 1, T = 0$

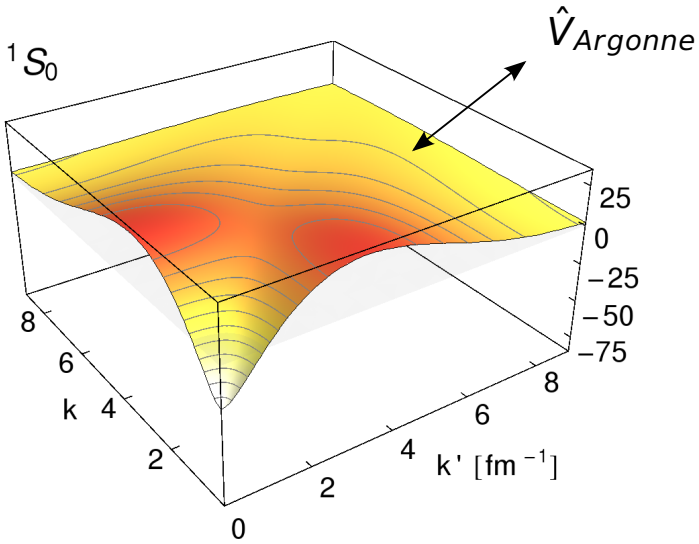


- repulsive core:
  nucleons can not get closer than ≈ 0.5 fm→ **central correlations**
- strong dependence on the orientation of the spins due to the tensor force (mainly from π-exchange) → **tensor correlations**
- the nuclear force will induce **strong short-range correlations** in the nuclear wave function

NN interaction — Argonne V18

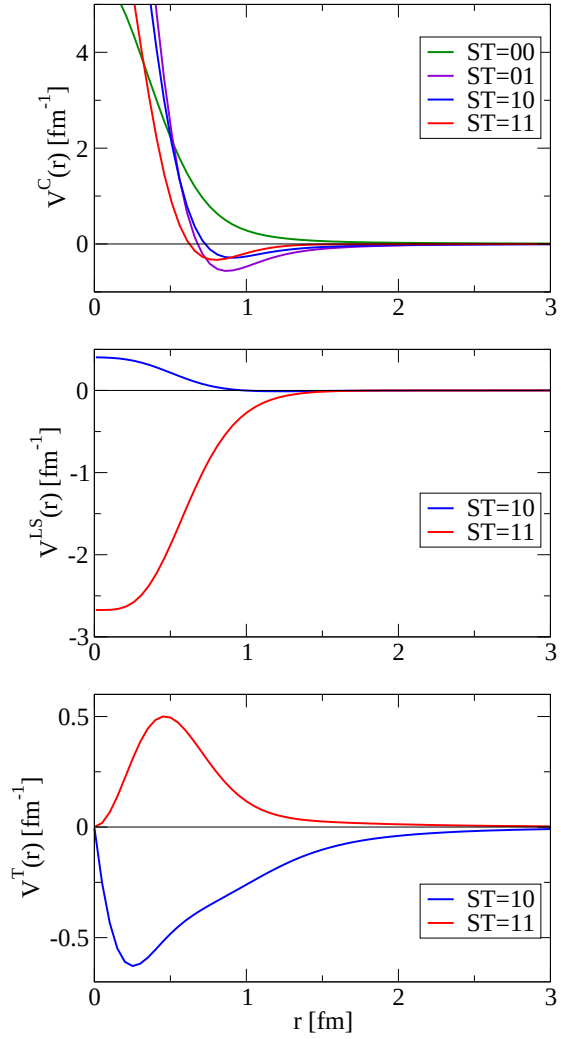
Argonne V18

- π-exchange, phenomenological short-range
- as local as possible
- fitted to phase shifts up to 350 MeV, but describes elastic phase shifts up to 1 GeV



$$\langle k(LS)J;T|\hat{V}_{Argonne}|k'(L'S)J;T\rangle$$

$$\begin{aligned}
\hat{V}_{Argonne} &= \sum_{S,T} V^C_{ST}(\hat{r})\,\hat{\Pi}_{ST} \\
&+ \sum_{S,T} V^{L2}_{ST}(\hat{r})\hat{\boldsymbol{L}}^2\,\hat{\Pi}_{ST} \\
&+ \sum_{T} V^{LS}_{1T}(\hat{r})\hat{\boldsymbol{L}}\hat{\boldsymbol{S}}\,\hat{\Pi}_{1T} \\
&+ \sum_{T} V^{T}_{1T}(\hat{r})\hat{S}_{12}\,\hat{\Pi}_{1T} \\
&+ \sum_{T} V^{TLL}_{1T}(\hat{r})s_{12}(\hat{\boldsymbol{L}},\hat{\boldsymbol{L}})\,\hat{\Pi}_{1T} \\
&+ \ldots
\end{aligned}$$

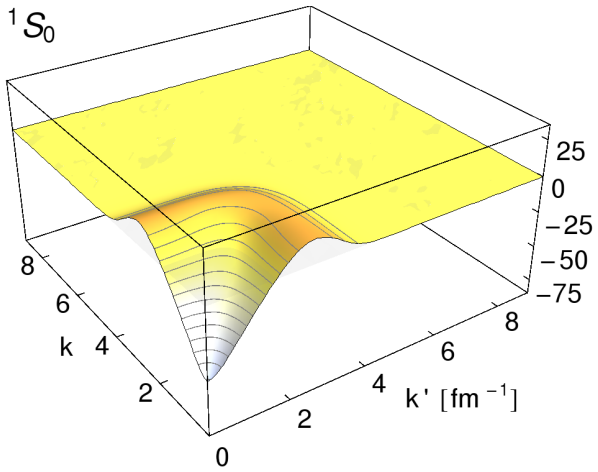


Wiringa, Stoks, Schiavilla, PRC **51**, 38 (1995)

NN interaction — Chiral effective field theory

N^{3}LO

- potential derived using chiral EFT
- includes full π dynamics
- short-range behavior given by contact-terms
- power counting
- regulated by cut-off (500 MeV)



$$\langle k(LS)J;T|\hat{V}_{N3LO}|k'(L'S)J;T\rangle$$

Entem, Machleidt, PRC **68**, 041001 (2003)

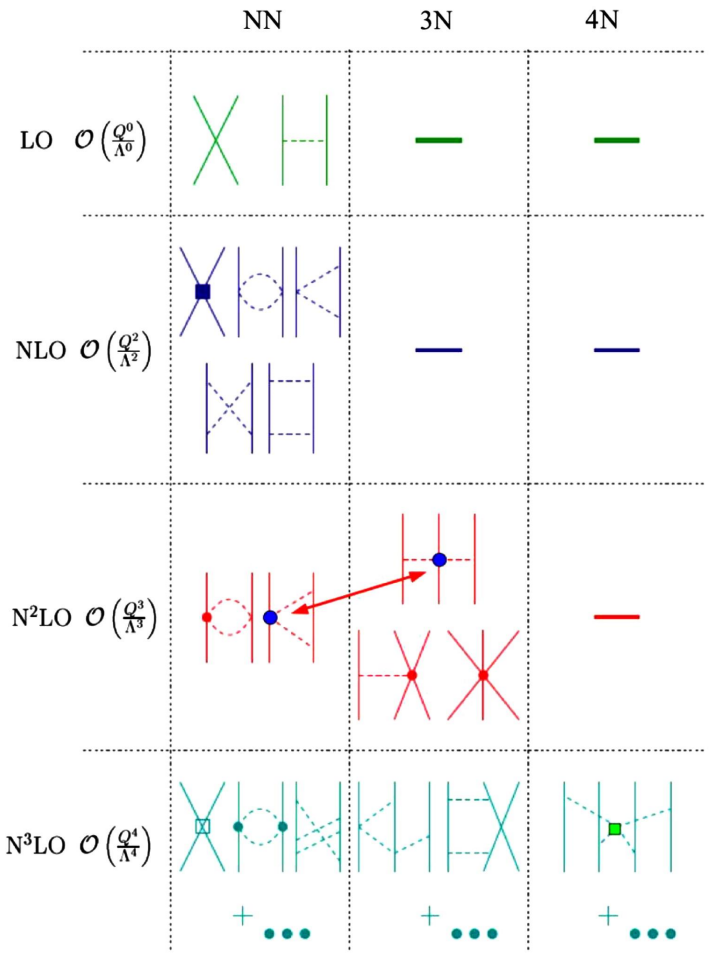


Figure taken from:

Bogner, Furnstahl, Schwenk, Part. Nucl. Phys. **65**, 94 (2010)

Solving the nuclear many-body problem

Correlated Gaussian Method

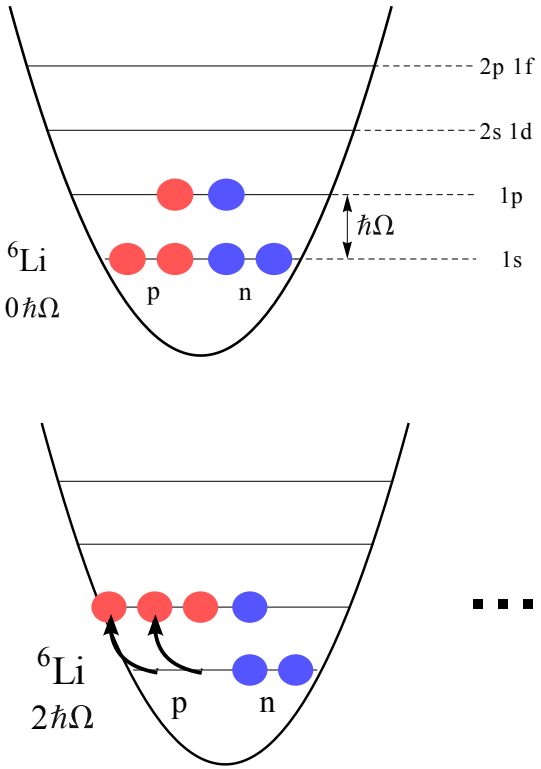
- Correlated Gaussian basis
- uses Jacobi coordinates
- exact results for $A \leq 4$

Suzuki, Horiuchi, Orabi, Arai, Few-Body Syst. **42**, 33 (2008)

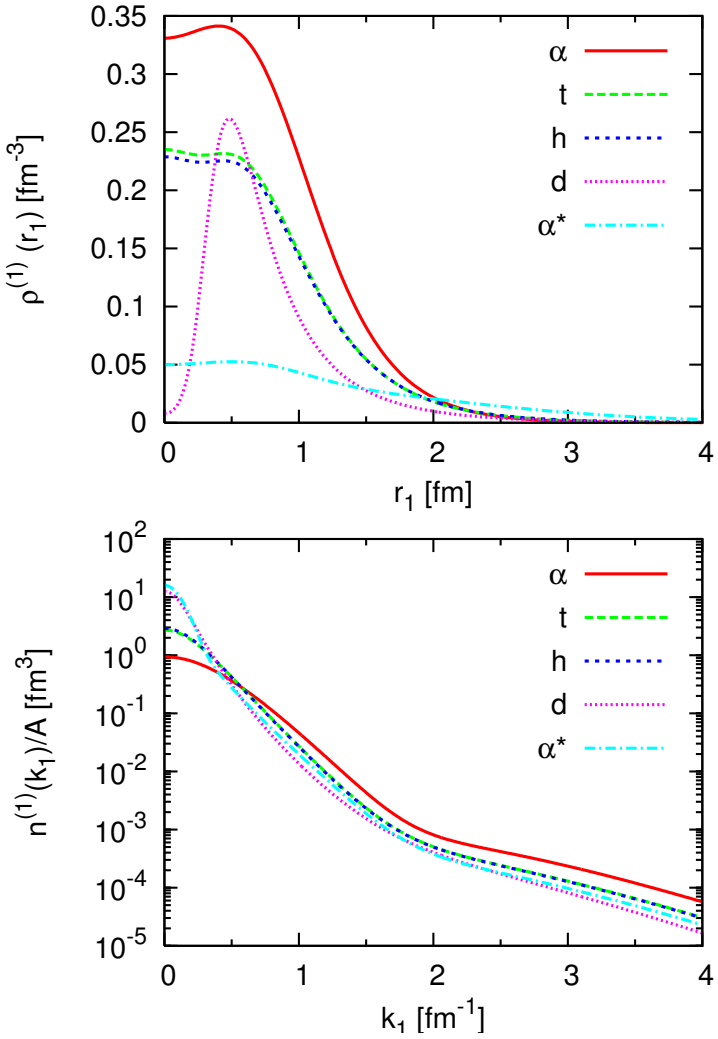
No-Core Shell Model (NCSM)

- Diagonalization of Hamiltonian in harmonic oscillator basis
- Truncation scheme
  - $0\hbar\Omega$ configuration:
    lowest single particle orbits filled
  - $N\hbar\Omega$ configuration:
    N oscillator quanta above $0\hbar\Omega$ configuration
  - Truncation: use only configurations with $N \leq N_{max}$
- Check for convergence
- Model space sizes grow rapidly with A and N_{max}

Navrátil, Kamuntavičius, Barrett, Phys. Rev. C, **61**, 044001 (2000)



One-body densities for A=2,3,4 nuclei



$$\rho^{(1)}(\boldsymbol{r}_1) = \langle\Psi|\sum_{i=1}^{A}\delta^3(\hat{\boldsymbol{r}}_i - \boldsymbol{r}_1)|\Psi\rangle$$

$$n^{(1)}(\boldsymbol{k}_1) = \langle\Psi|\sum_{i=1}^{A}\delta^3(\hat{\boldsymbol{k}}_i - \boldsymbol{k}_1)|\Psi\rangle$$

- one-body densities calculated from **exact many-body state** $|\Psi\rangle$ (Correlated Gaussian Method) for AV8' interaction
- coordinate space densities reflect different sizes and densities of ^{2}H, ^{3}H, ^{3}He, ^{4}He and the 0_2^+ state in ^{4}He
- similar high-momentum tails in the one-body momentum distributions

Feldmeier, Horiuchi, Neff, Suzuki, PRC **84**, 054003 (2011)

Two-body densities

number of pairs in given spin-, isospin channels

$$\rho^{(2)}_{SM_S,TM_T}(\boldsymbol{r}_1,\boldsymbol{r}_2) = \langle\Psi|\sum_{i<j}^{A}\hat{P}^{SM_S}_{ij}\hat{P}^{TM_T}_{ij}\,\delta^3(\hat{\boldsymbol{r}}_i-\boldsymbol{r}_1)\delta^3(\hat{\boldsymbol{r}}_j-\boldsymbol{r}_2)\,|\Psi\rangle$$

$$n^{(2)}_{SM_S,TM_T}(\boldsymbol{k}_1,\boldsymbol{k}_2) = \langle\Psi|\sum_{i<j}^{A}\hat{P}^{SM_S}_{ij}\hat{P}^{TM_T}_{ij}\,\delta^3(\hat{\boldsymbol{k}}_i-\boldsymbol{k}_1)\delta^3(\hat{\boldsymbol{k}}_j-\boldsymbol{k}_2)\,|\Psi\rangle$$

integrated over center-of-mass position $\boldsymbol{R} = \frac{1}{2}(\boldsymbol{r}_1+\boldsymbol{r}_2)$ or the total momentum of the nucleon pair $\boldsymbol{K} = \boldsymbol{k}_1+\boldsymbol{k}_2$:

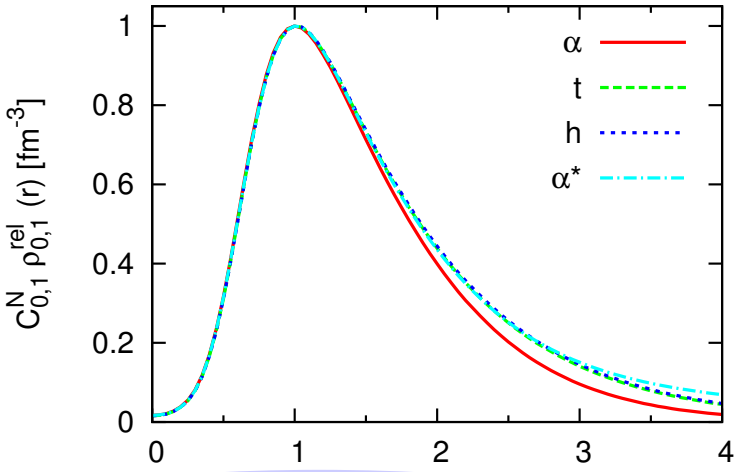
$$\rho^{\mathrm{rel}}_{SM_S,TM_T}(\boldsymbol{r}) = \langle\Psi|\sum_{i<j}^{A}\hat{P}^{SM_S}_{ij}\hat{P}^{TM_T}_{ij}\,\delta^3(\hat{\boldsymbol{r}}_i-\hat{\boldsymbol{r}}_j-\boldsymbol{r})\,|\Psi\rangle$$

$$n^{\mathrm{rel}}_{SM_S,TM_T}(\boldsymbol{k}) = \langle\Psi|\sum_{i<j}^{A}\hat{P}^{SM_S}_{ij}\hat{P}^{TM_T}_{ij}\,\delta^3\big(\frac{1}{2}(\hat{\boldsymbol{k}}_i-\hat{\boldsymbol{k}}_j)-\boldsymbol{k}\big)\,|\Psi\rangle$$

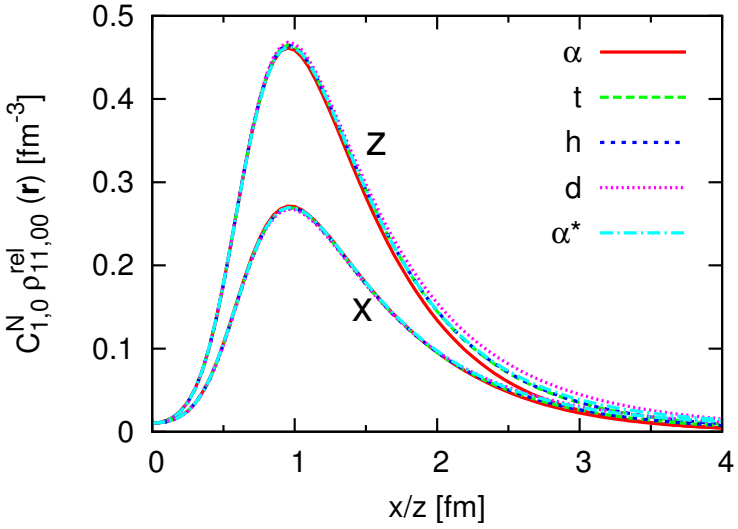
$|\Psi\rangle$ many-body state, eigenstate of Hamiltonian

Two-body densities in coordinate space for A=2,3,4

$S = 0, T = 1$



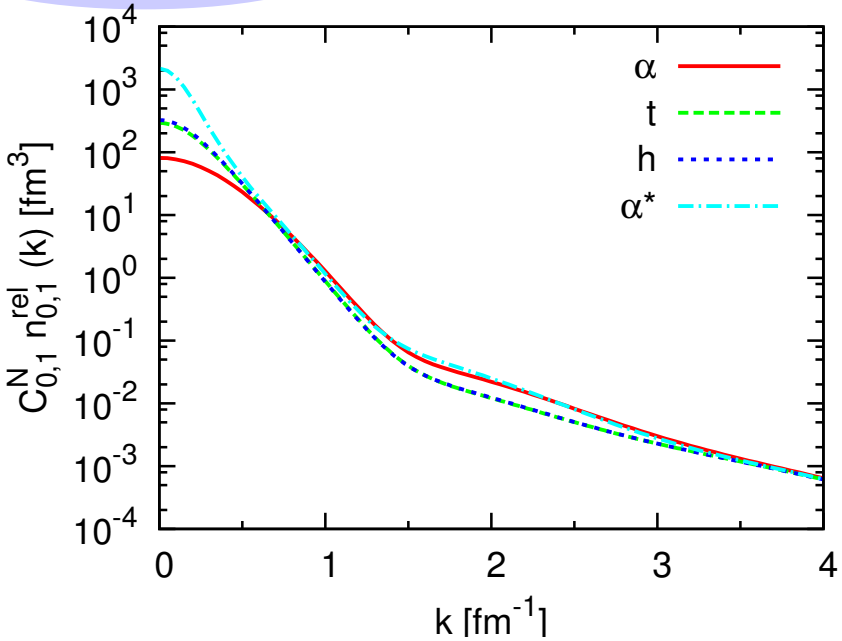
$S = 1, M_S = 1, T = 0$



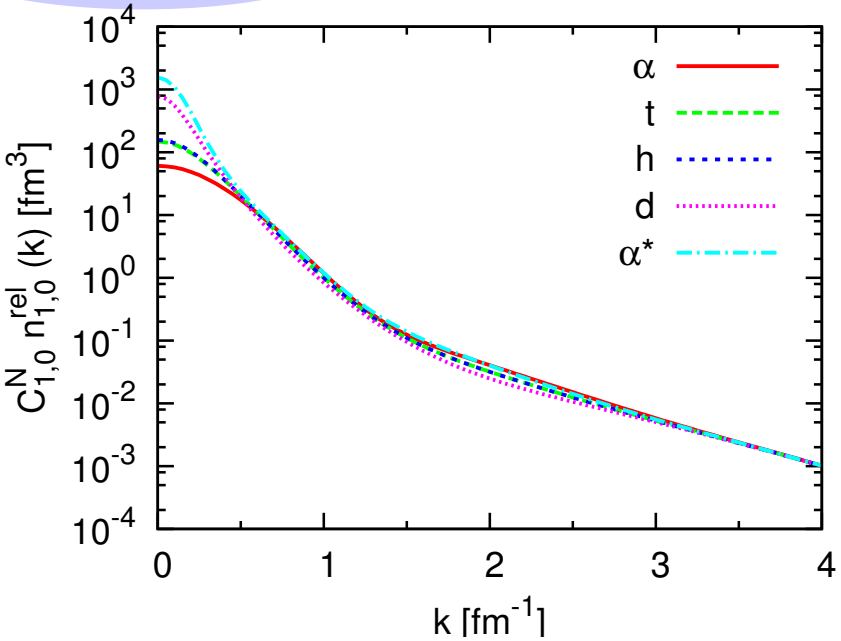
- two-body densities calculated from **exact many-body state** (Correlated Gaussian Method) for AV8' interaction
- coordinate space two-body densities reflect correlation hole and tensor correlations
- → normalize two-body density in coordinate space at r=1.0 fm
- → normalized two-body densities in coordinate space are **identical at short distances** for all nuclei
- also true for angular dependence in the $S = 1$, $T = 0$ channel (deuteron like)

Two-body densities in momentum space for A=2,3,4

$S = 0, T = 1$



$S = 1, T = 0$



- use **normalization factors fixed in coordinate space**
- two-body densities in momentum space agree for momenta $k > 3\text{fm}^{-1}$
- moderate nucleus dependence in momentum region $1.5\text{fm}^{-1} < k < 3\text{fm}^{-1}$

Feldmeier, Horiuchi, Neff, Suzuki, PRC **84**, 054003 (2011)

Unitary Transformations

- Many-body problem very hard to solve with bare interaction
- Universality of SRC suggests to use unitary transformations to obtain a "soft" realistic interaction

$$\hat{H}_{eff} = \hat{U}^\dagger \hat{H} \hat{U}$$

- The transformation is done in N-body approximation

$$\hat{H}_{eff} = \hat{T} + \hat{V}_{eff}^{[2]} + \ldots + \hat{V}_{eff}^{[N]}$$

  and is therefore unitary only up to the N-body level
- Deuteron binding energy and NN phase shifts are conserved

- Not only the Hamiltonian, **all operators have to be transformed**

$$\hat{B}_{eff} = \hat{U}^\dagger \hat{B} \hat{U}$$

- SRG operator evolution studied for Deuteron

  Anderson, Bogner, Furnstahl, Perry, PRC **82**, 054001 (2010)
- SRG operator evolution for radius and Gaussian two-body operator on 3-body level

  Schuster, Quaglioni, Johnson, Jurgenson, Navrátil, PRC **90**,011301(R) (2014)

Similarity Renormalization Group

- Evolve Hamiltonian and unitary transformation matrix (momentum space or HO basis)

$$\frac{d\hat{H}_\alpha}{d\alpha} = \left[\hat{\eta}_\alpha, \hat{H}_\alpha\right], \qquad \frac{d\hat{U}_\alpha}{d\alpha} = -\hat{U}_\alpha \hat{\eta}_\alpha$$

- Intrinsic kinetic energy as meta-generator

$$\hat{\eta}_\alpha = (2\mu)^2 \left[\hat{T}_{\text{int}}, \hat{H}_\alpha\right]$$

- Evolution is done on the 2-body level – α-dependence can be used to investigate missing higher-order contributions
- We will use $\hat{U}_\alpha$ to calculate effective operators
- Hamiltonian evolution can now also be done on the 3-body level

  (Jurgenson, Roth, Hebeler, Maris, . . .)

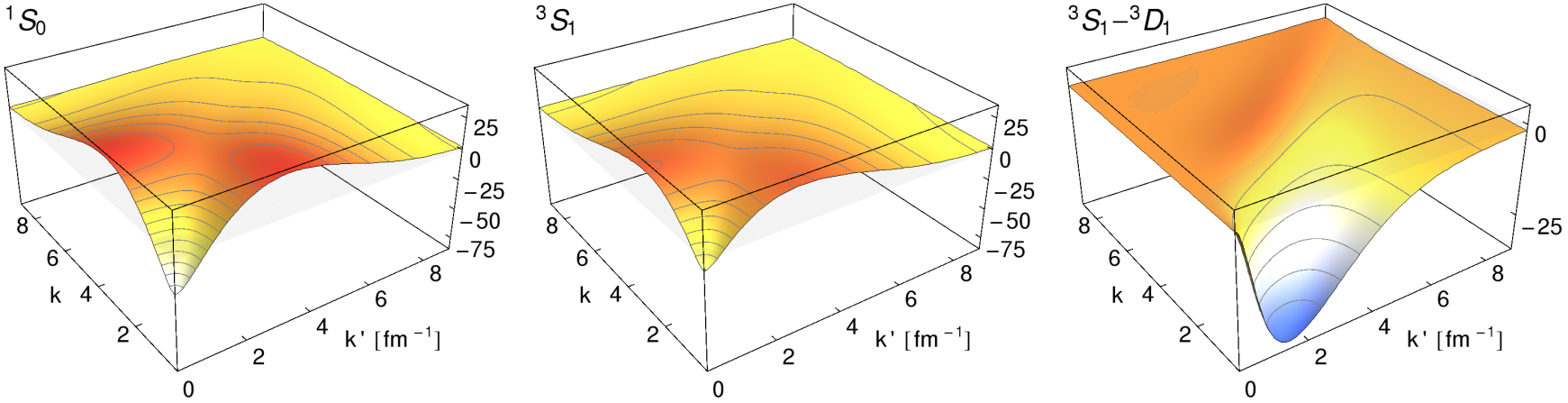
Bogner, Furnstahl, Perry, Phys. Rev. C, **75**, 061001 (2007)

Roth, Neff, Feldmeier, Prog. Part. Nucl. Phys. **65**, 50 (2010)

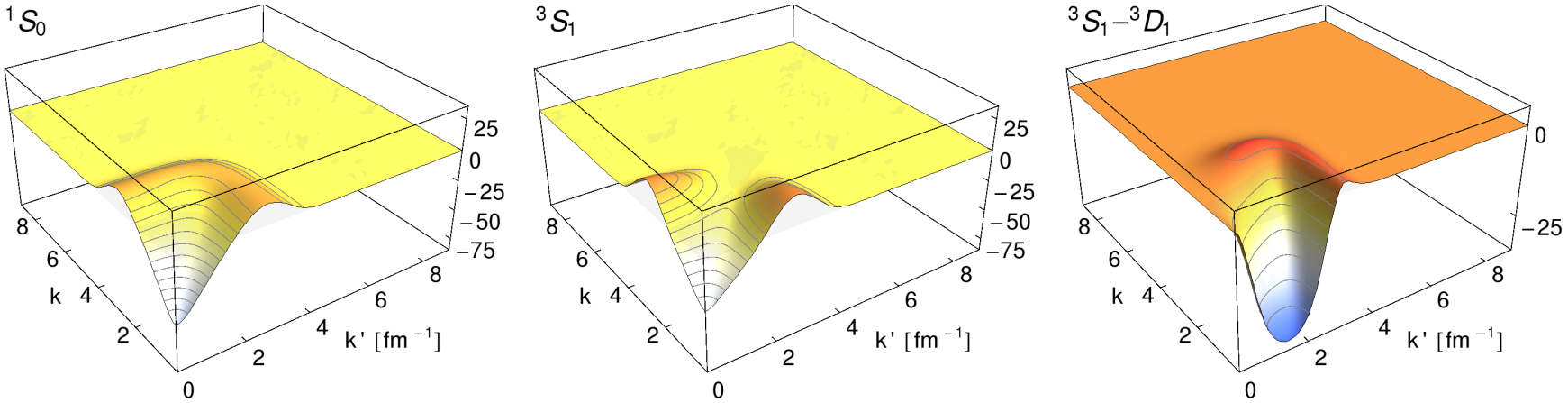
Similarity Renormalization Group

Evolution of the potential

AV8'



N^{3}LO

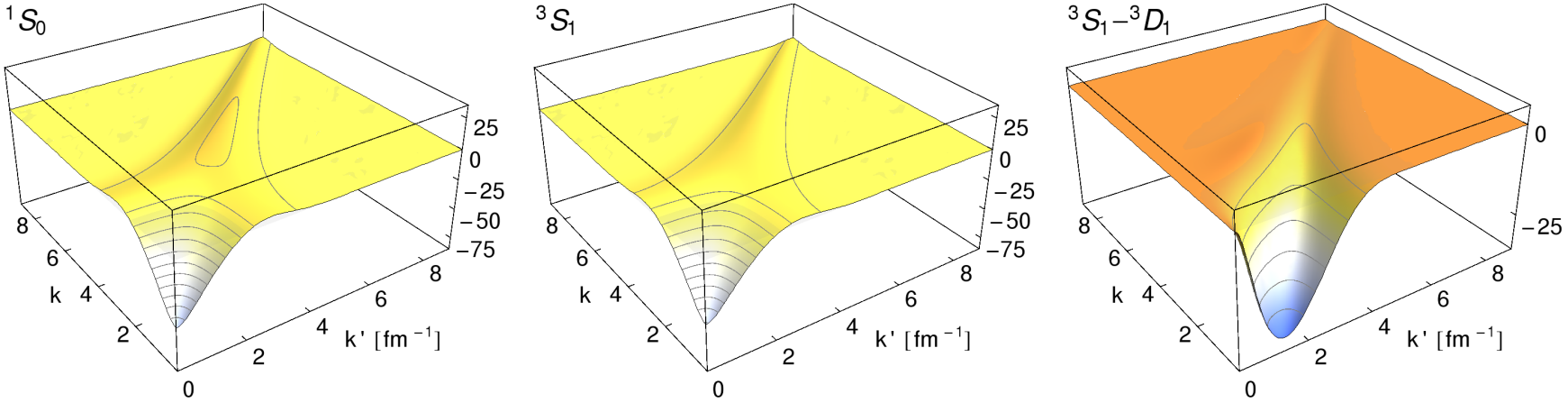


$V(k,k') - \alpha = 0.00$ (bare)

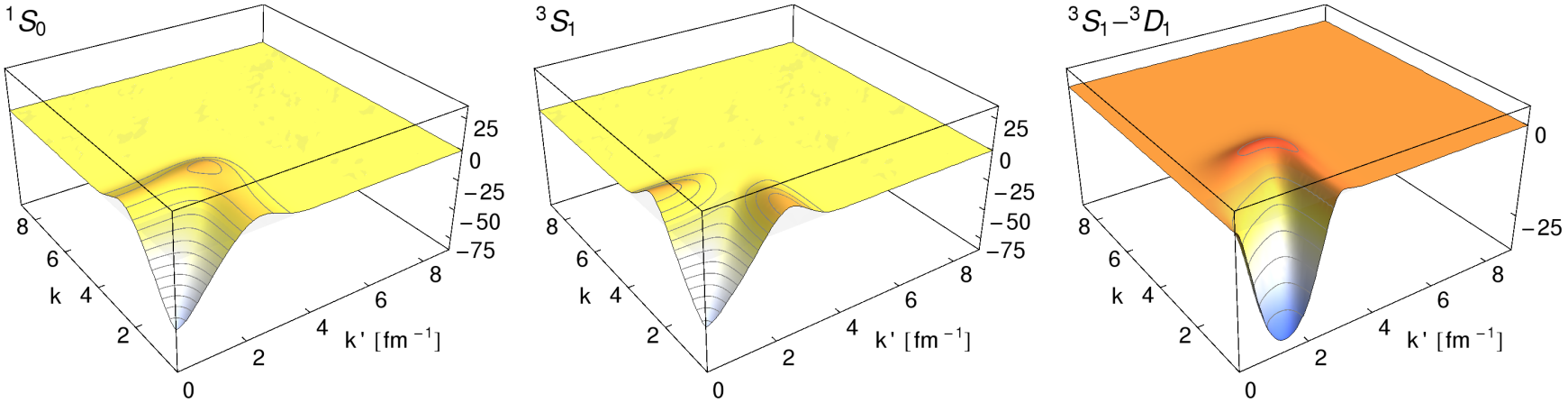
Similarity Renormalization Group

Evolution of the potential

AV8'



N^{3}LO

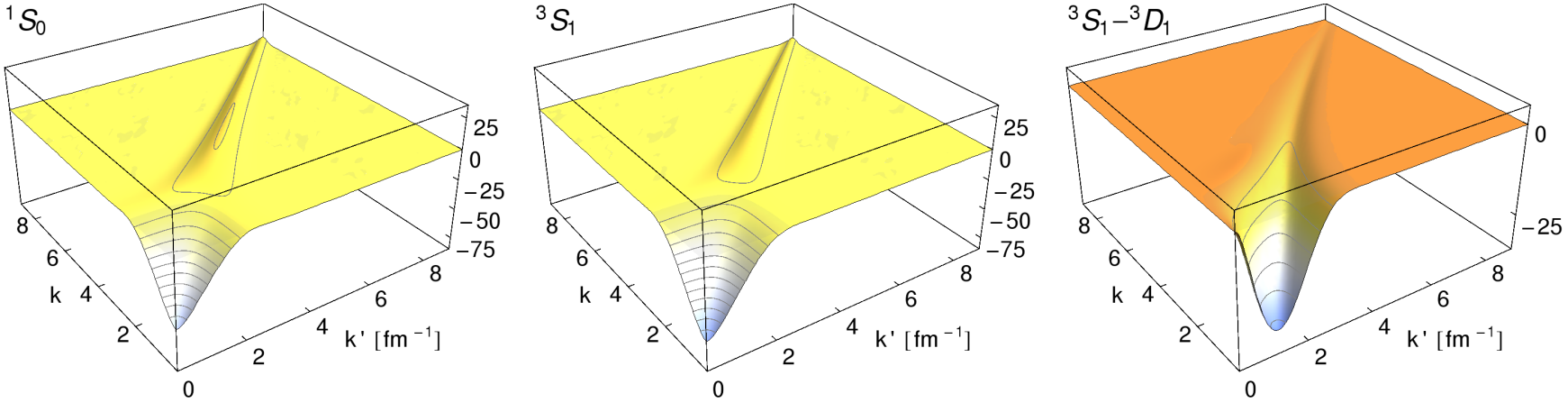


$V(k,k') - \alpha = 0.01\text{fm}^4$

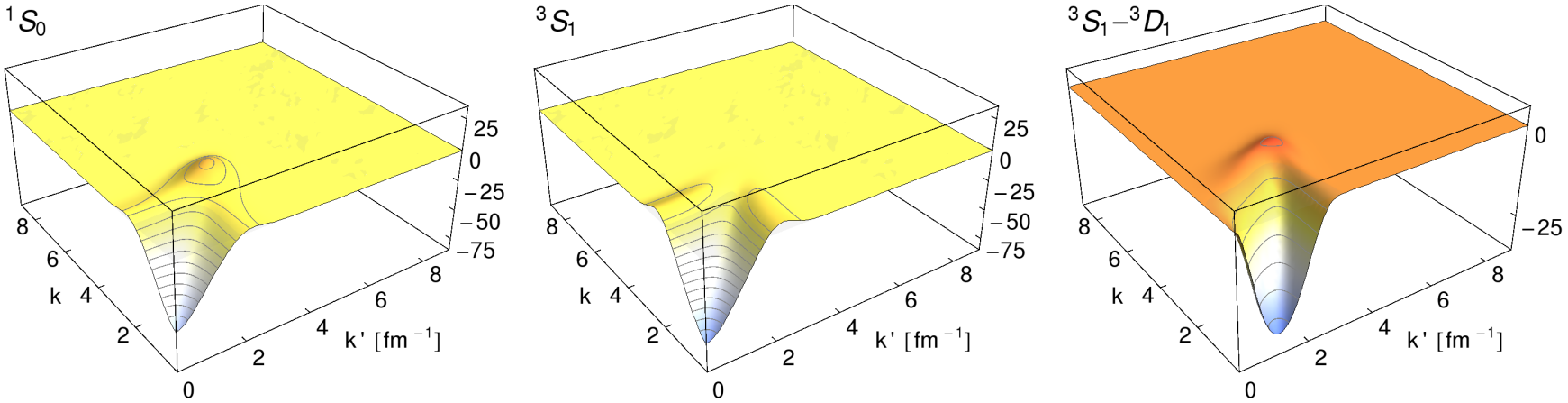
Similarity Renormalization Group

Evolution of the potential

AV8'



N^{3}LO

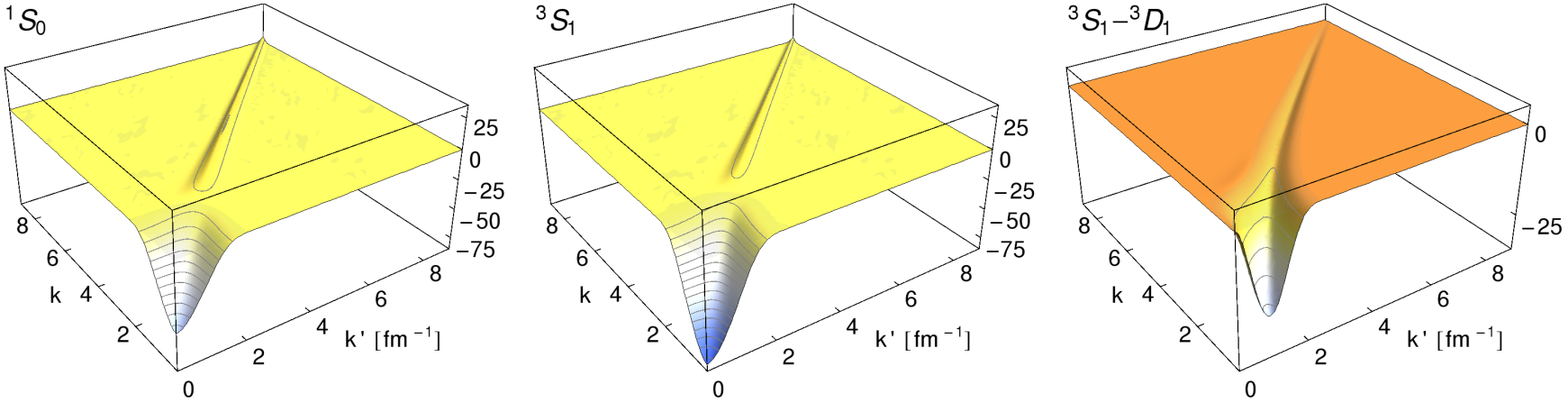


$V(k,k') - \alpha = 0.04\text{fm}^4$

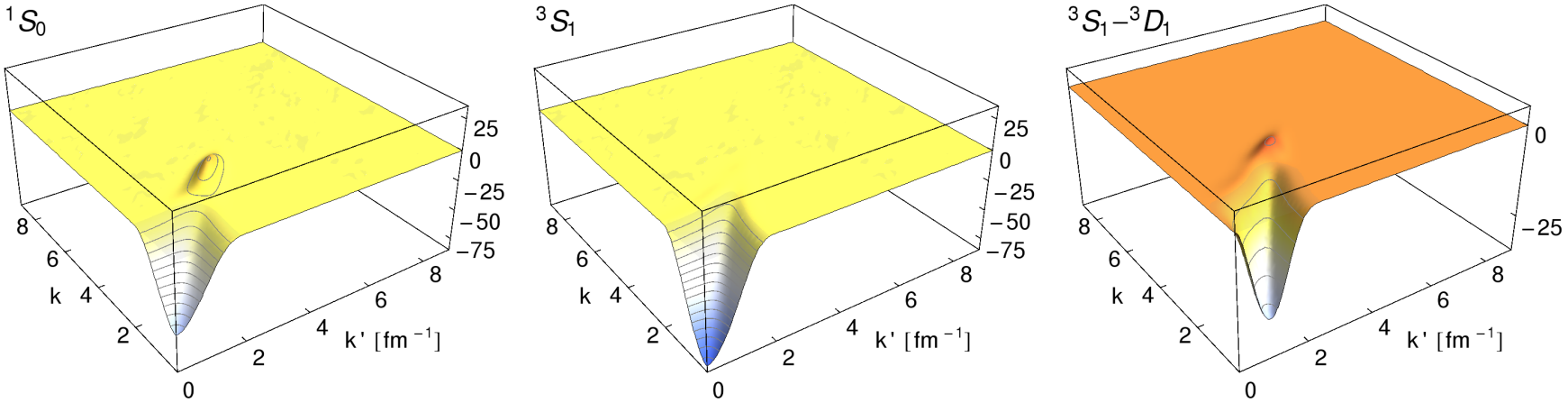
Similarity Renormalization Group

Evolution of the potential

AV8'



N^{3}LO

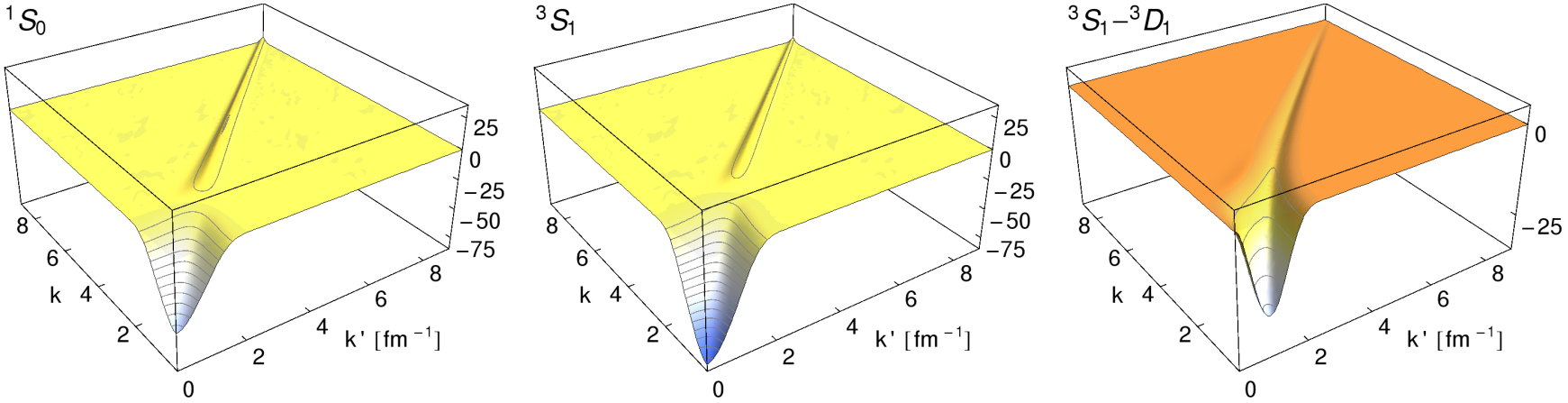


$V(k,k') - \alpha = 0.20\text{fm}^4$

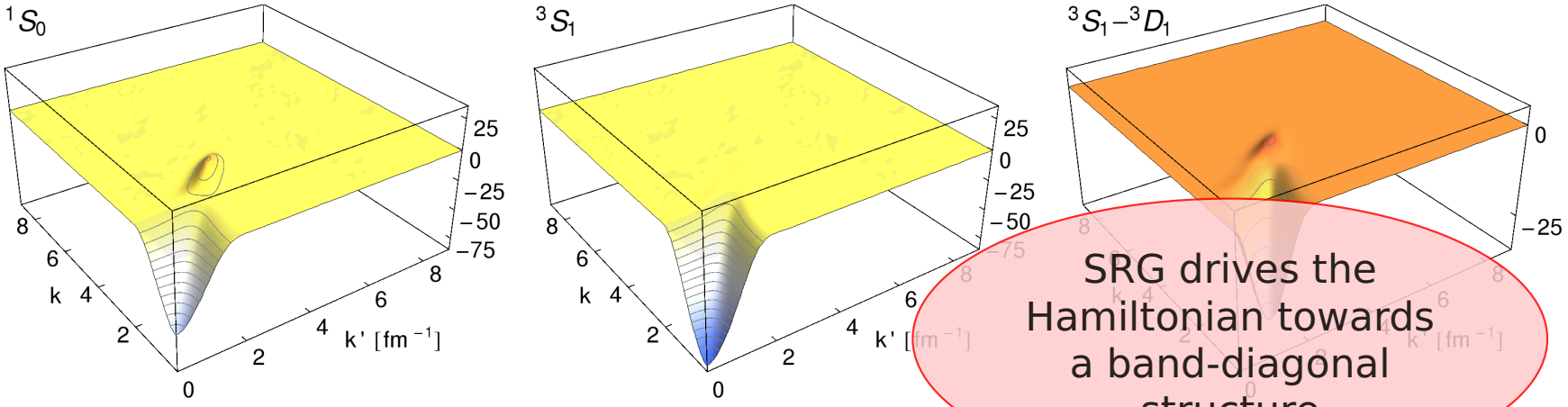
Similarity Renormalization Group

Evolution of the potential

AV8'



N^{3}LO



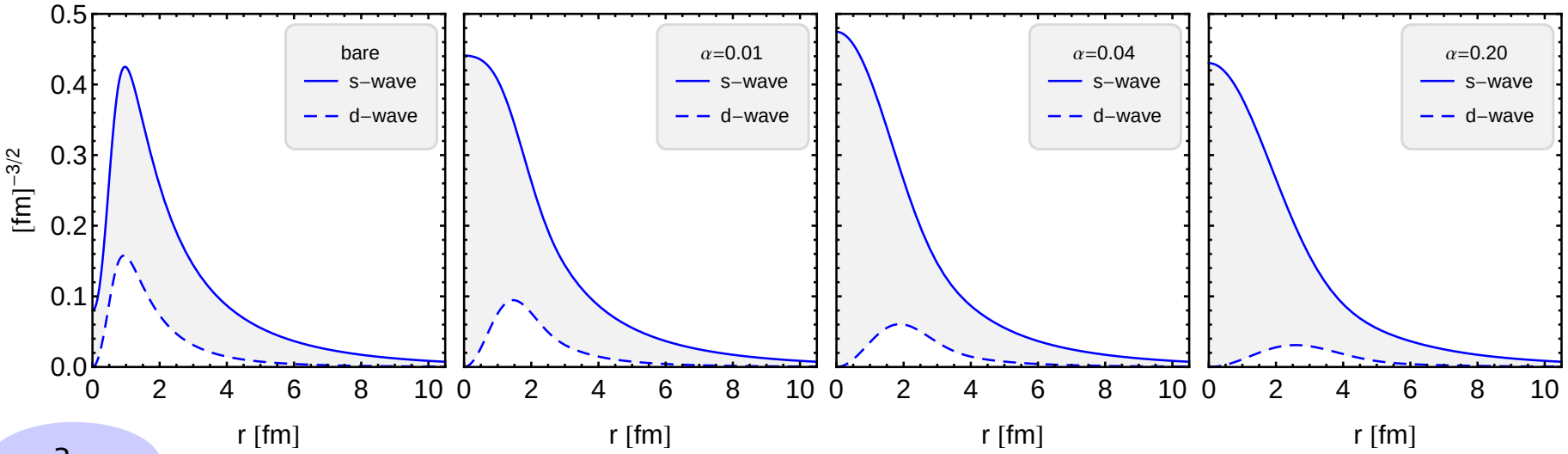
SRG drives the Hamiltonian towards a band-diagonal structure

$V(k,k') - \alpha = 0.20\text{fm}^4$

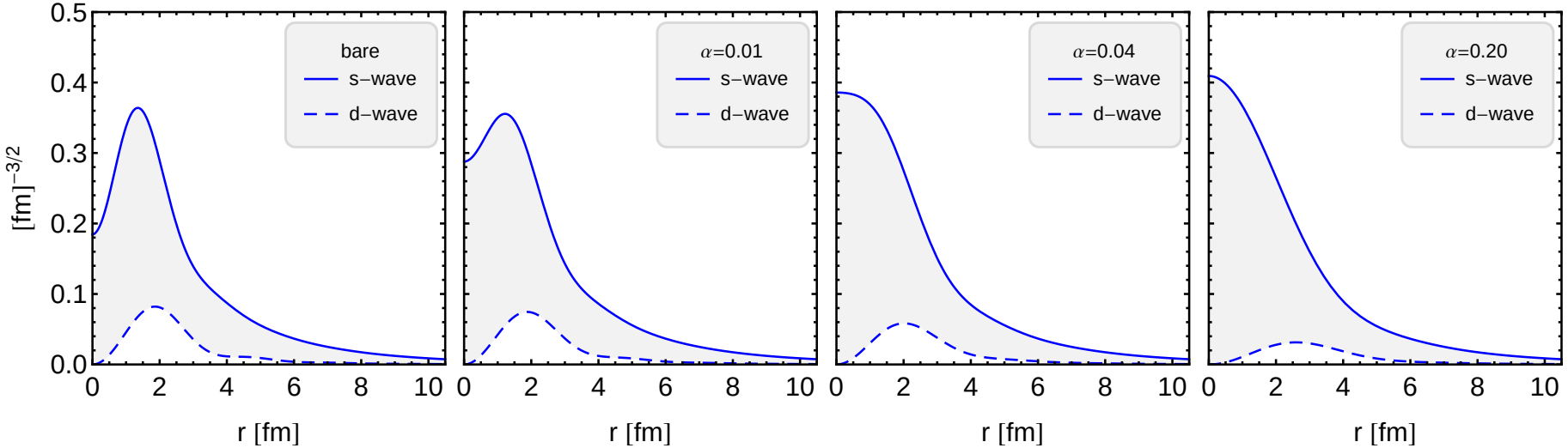
Similarity Renormalization Group

Deuteron – Coordinate Space Wave Functions

AV8'



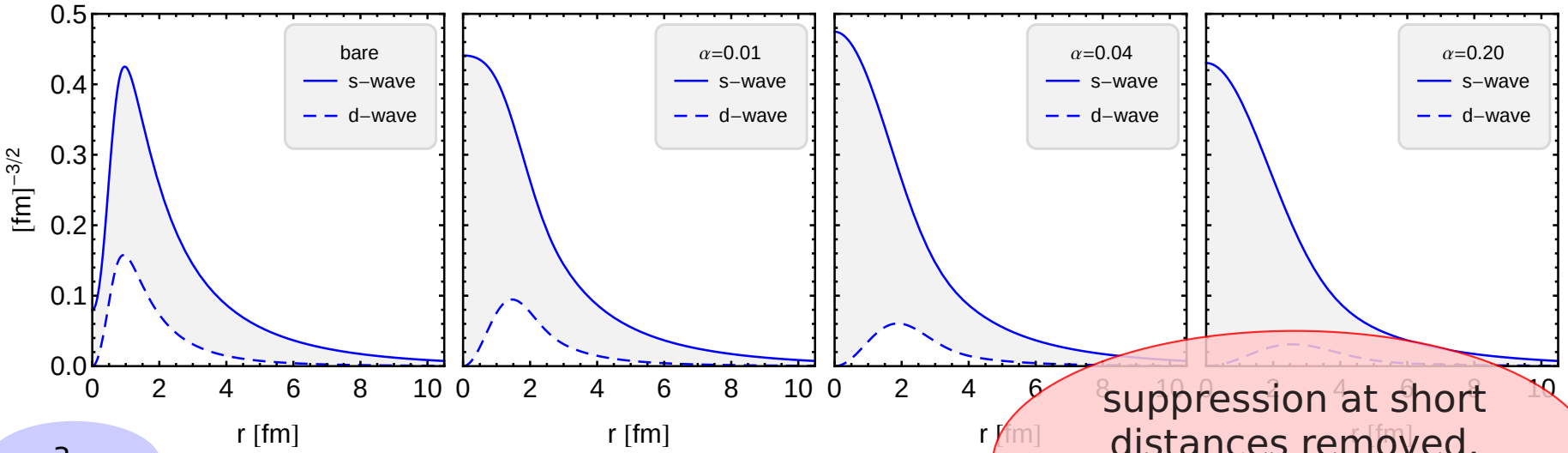
N^{3}LO



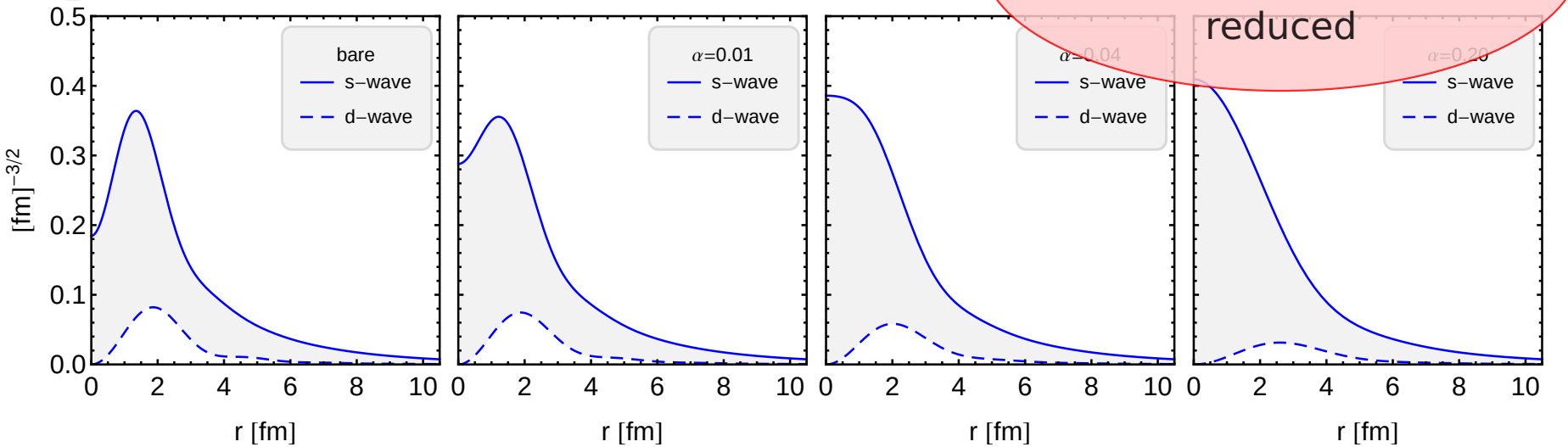
Similarity Renormalization Group

Deuteron – Coordinate Space Wave Functions

AV8'



N^{3}LO

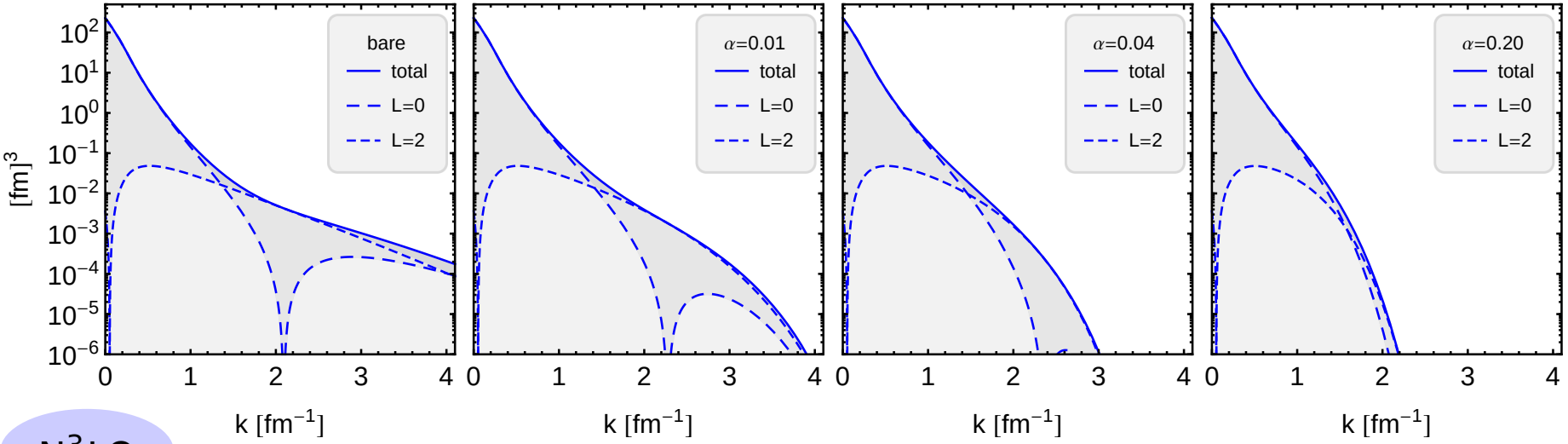


suppression at short distances removed, *d*-wave admixture reduced

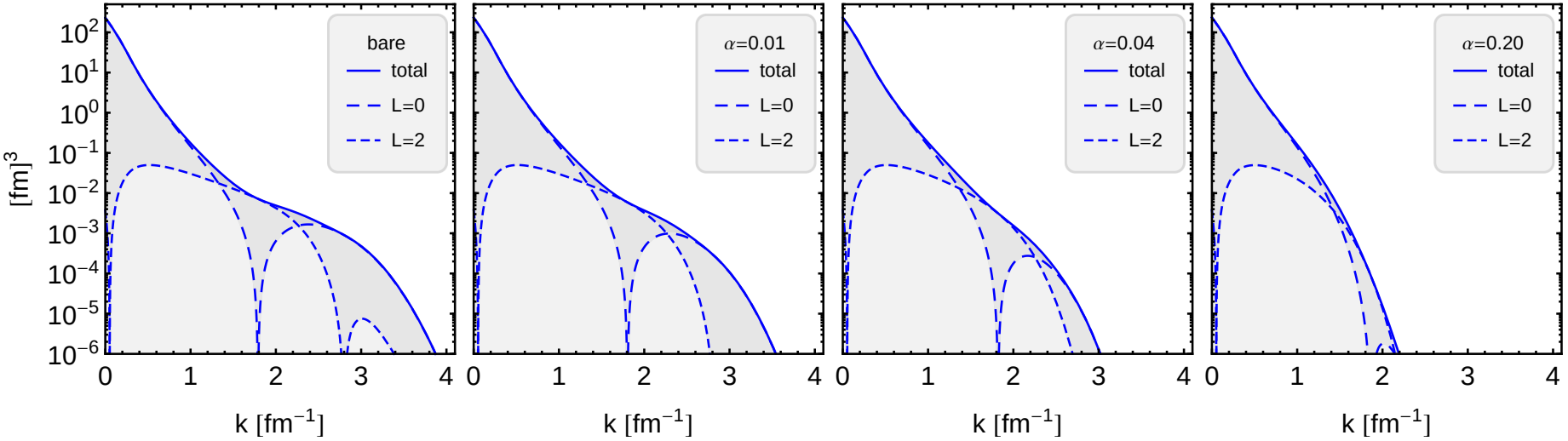
Similarity Renormalization Group

Deuteron – Momentum Space Densities

AV8'



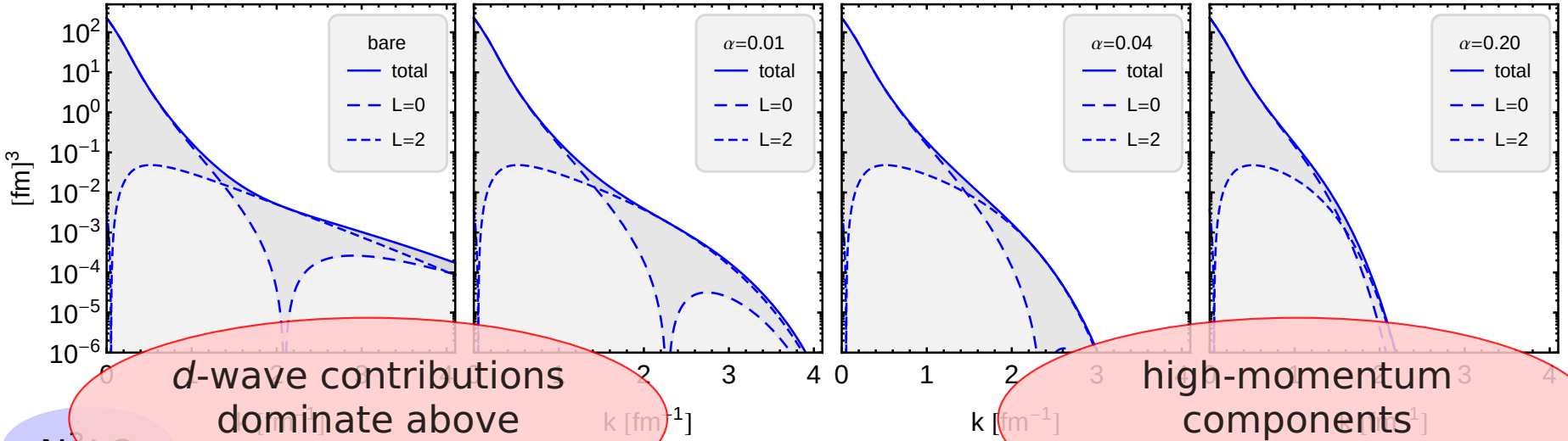
N^{3}LO



Similarity Renormalization Group

Deuteron – Momentum Space Densities

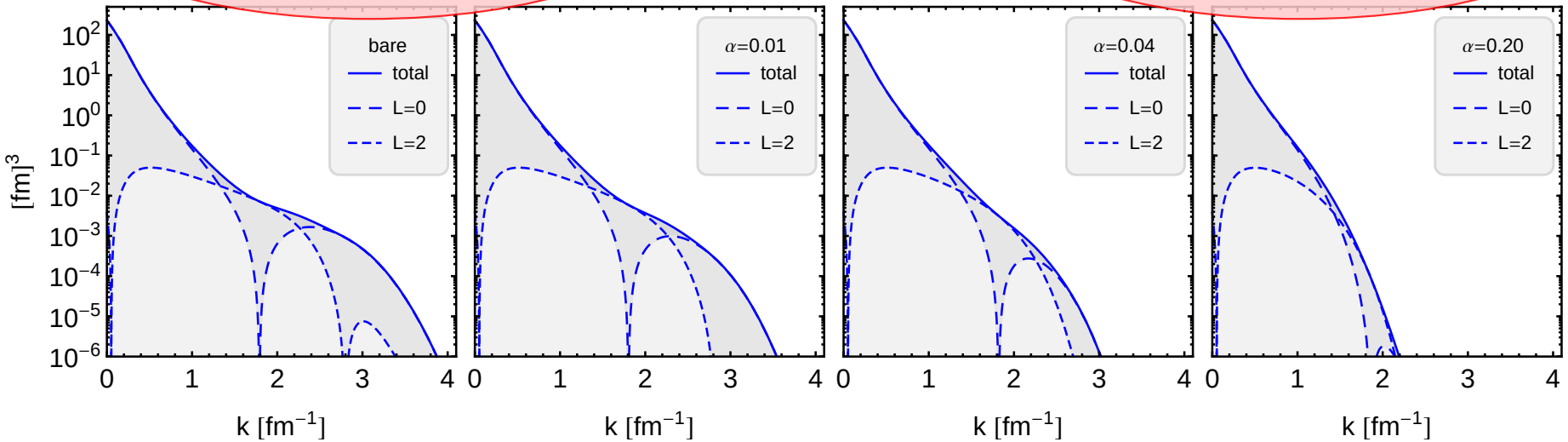
AV8'



d-wave contributions dominate above Fermi momentum

high-momentum components eliminated

N^{3}LO



Two-Body Densities calculated with NCSM and SRG

Outline of Calculation

- simultaneous SRG evolution for transformed Hamiltonian and transformation matrix **on the two-body level**

$$\frac{d\hat{H}_\alpha}{d\alpha} = \left[\hat{\eta}_\alpha, \hat{H}_\alpha\right], \qquad \frac{d\hat{U}_\alpha}{d\alpha} = -\hat{U}_\alpha \hat{\eta}_\alpha, \qquad \hat{\eta}_\alpha = (2\mu)^2 \left[\hat{T}_{\text{int}}, \hat{H}_\alpha\right]$$

- Solve many-body problem with SRG transformed Hamiltonian in the No-Core Shell Model (ANTOINE)

$$\hat{H}_\alpha \left|\Psi_\alpha\right\rangle = E_\alpha \left|\Psi_\alpha\right\rangle$$

- Calculate two-body densities with “bare” and “effective” density operators

$$\rho_{\text{bare}} = \langle\Psi_\alpha| \hat{\rho} |\Psi_\alpha\rangle, \qquad \rho_{\text{effective}} = \langle\Psi_\alpha| \hat{U}_\alpha^\dagger \hat{\rho} \hat{U}_\alpha |\Psi_\alpha\rangle$$

- Investigate **convergence** of NCSM calculations and **α-dependence of two-body densities**

^{4}He Two-body densities

Why ^{4}He ?

- two-body densities available for bare AV8' interaction
- bare N^{3}LO can be converged in NCSM

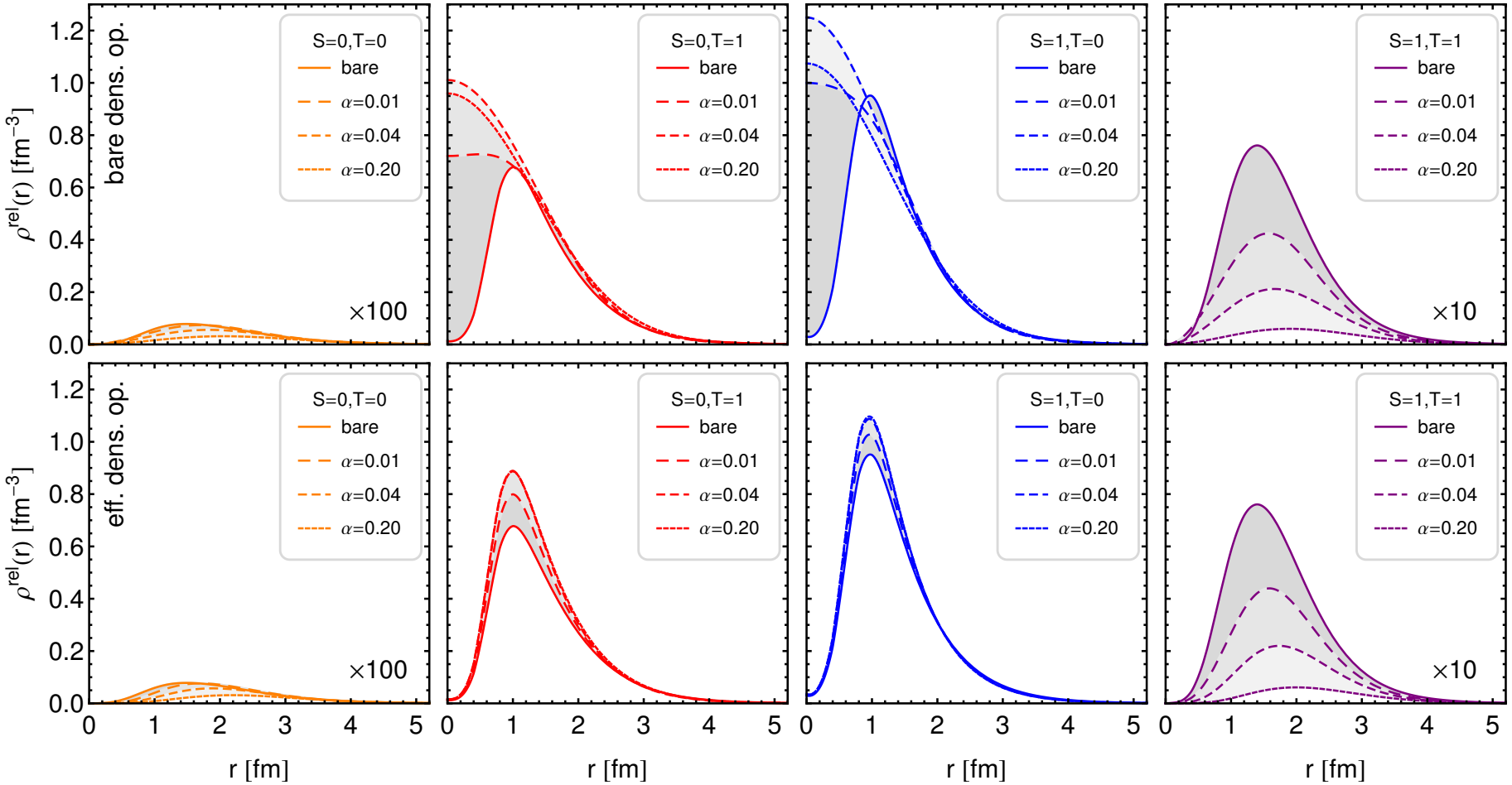
Objectives

- Compare AV8' and N^{3}LO results
- Check for NCSM convergence
- Check flow dependence $\alpha = 0, 0.01, 0.04, 0.20\text{fm}^4$ ($\Lambda = \infty, 3.16, 2.24, 1.50\text{fm}^{-1}$)
- Can we see many-body effects ?

Dependence on Flow Parameter

AV8' Interaction

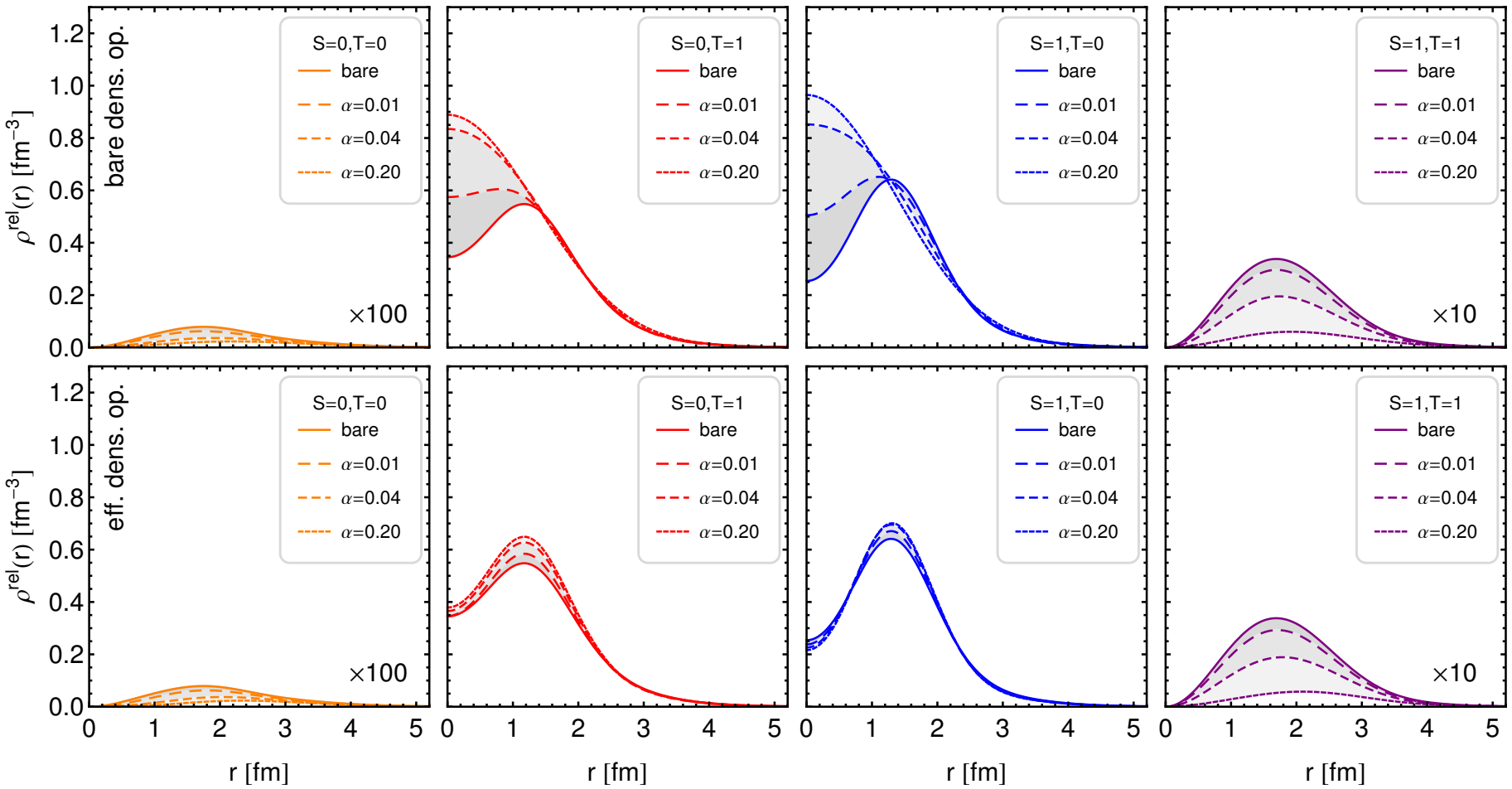
^{4}He Two-body Density in Coordinate Space



Dependence on Flow Parameter

N^{3}LO Interaction

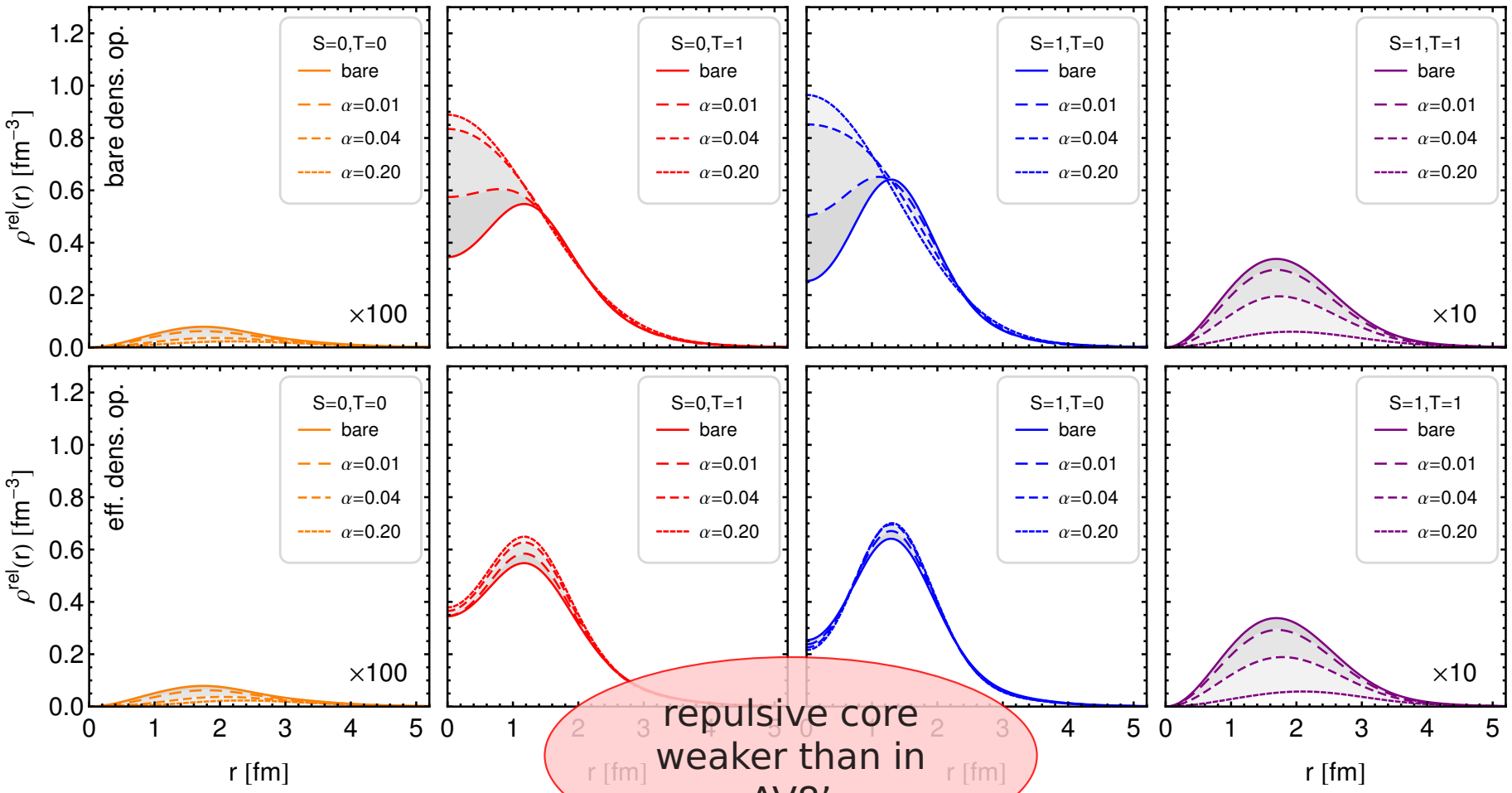
^{4}He Two-body Density in Coordinate Space



Dependence on Flow Parameter

N^{3}LO Interaction

^{4}He Two-body Density in Coordinate Space

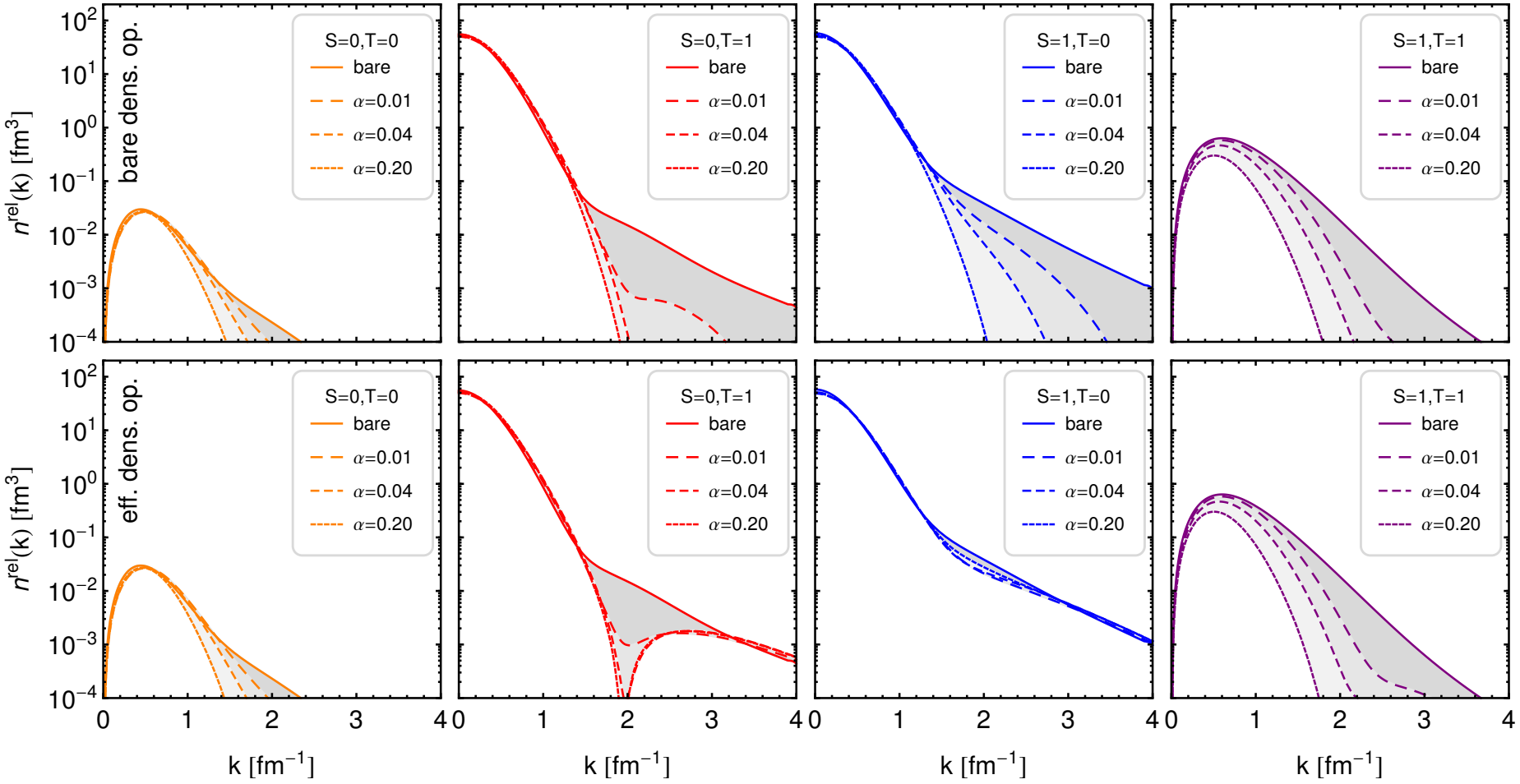


repulsive core weaker than in AV8'

Dependence on Flow Parameter

AV8' Interaction

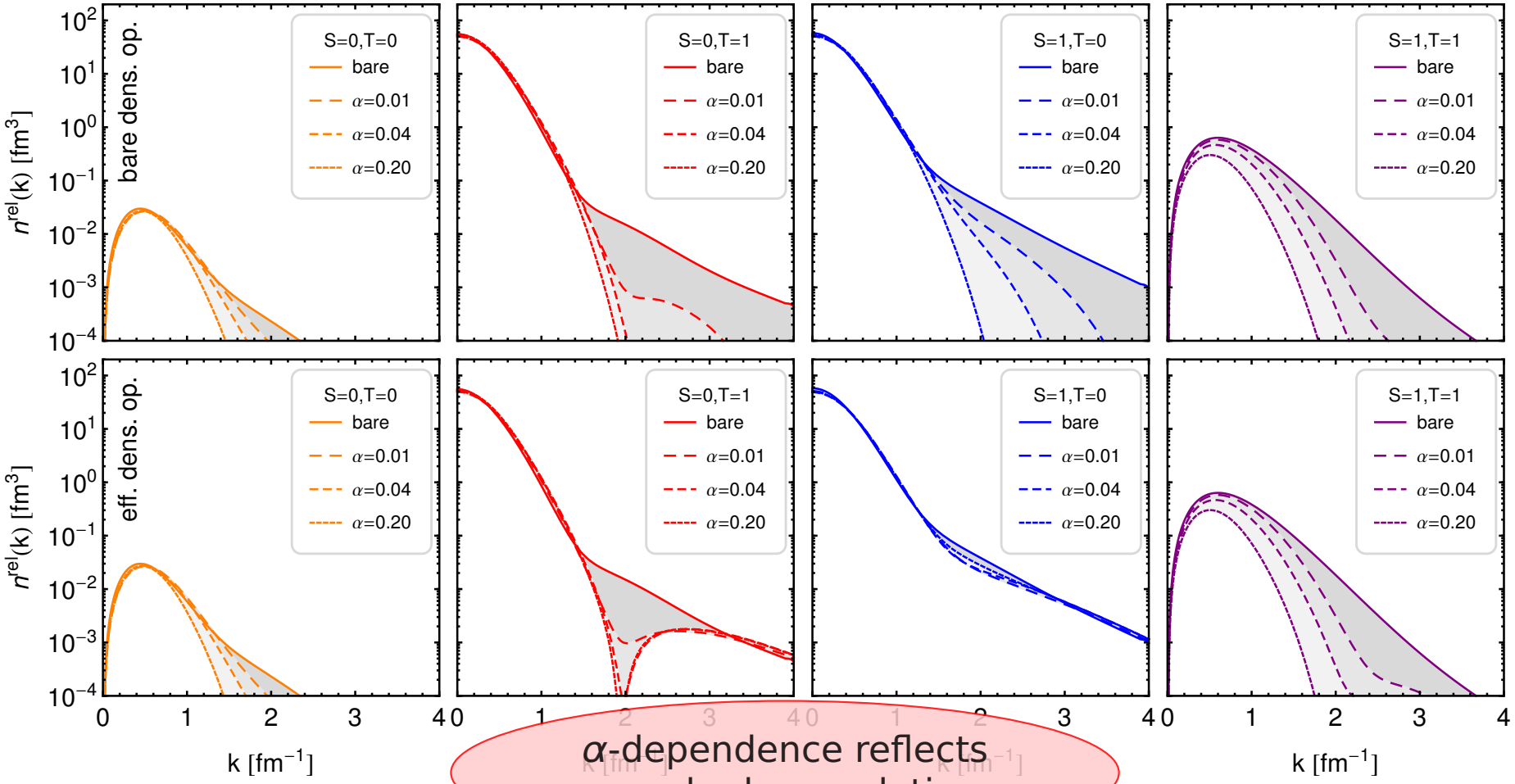
^{4}He Two-body Density in Momentum Space



Dependence on Flow Parameter

AV8' Interaction

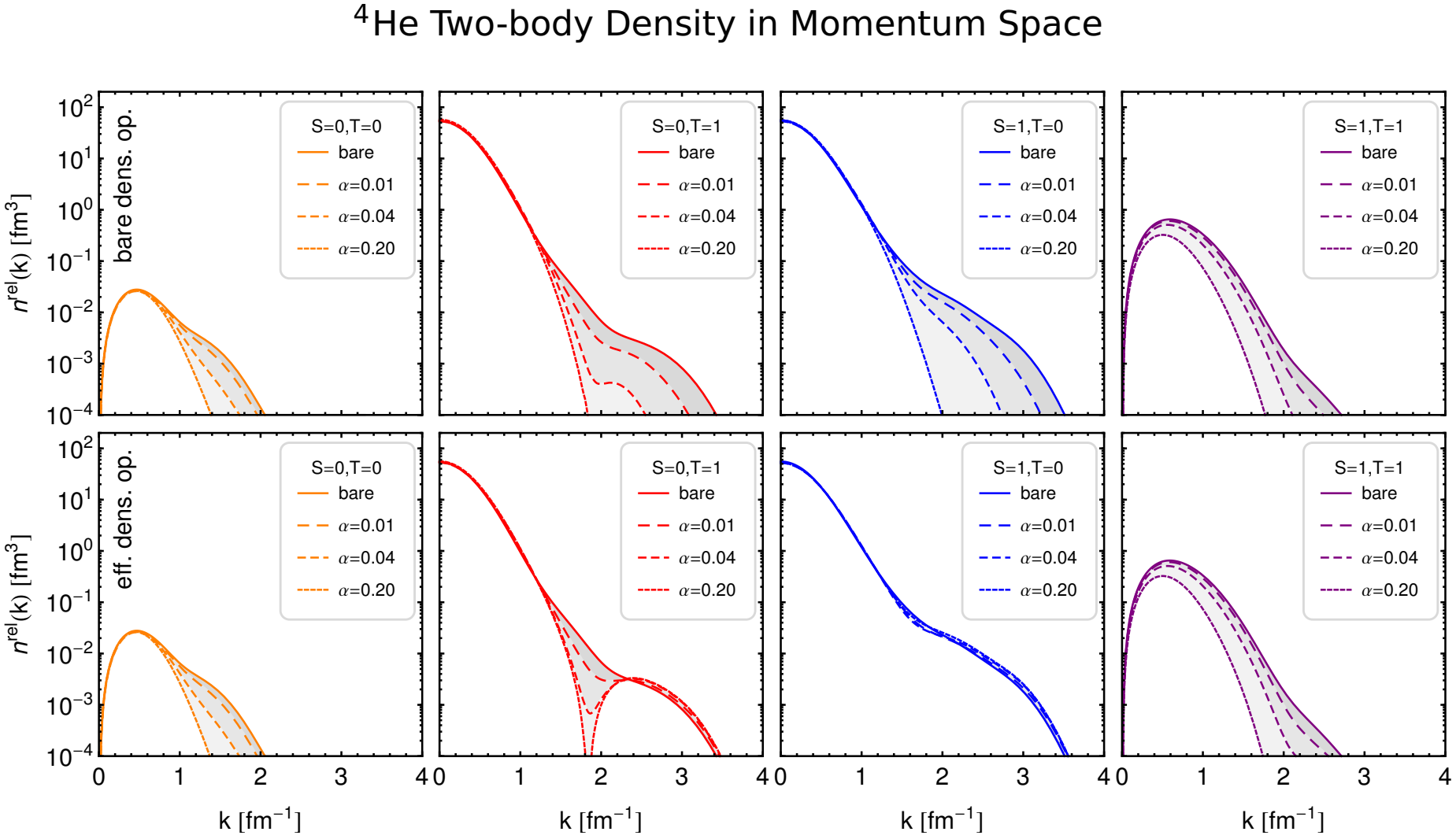
^{4}He Two-body Density in Momentum Space



α-dependence reflects many-body correlations

Flow dependence

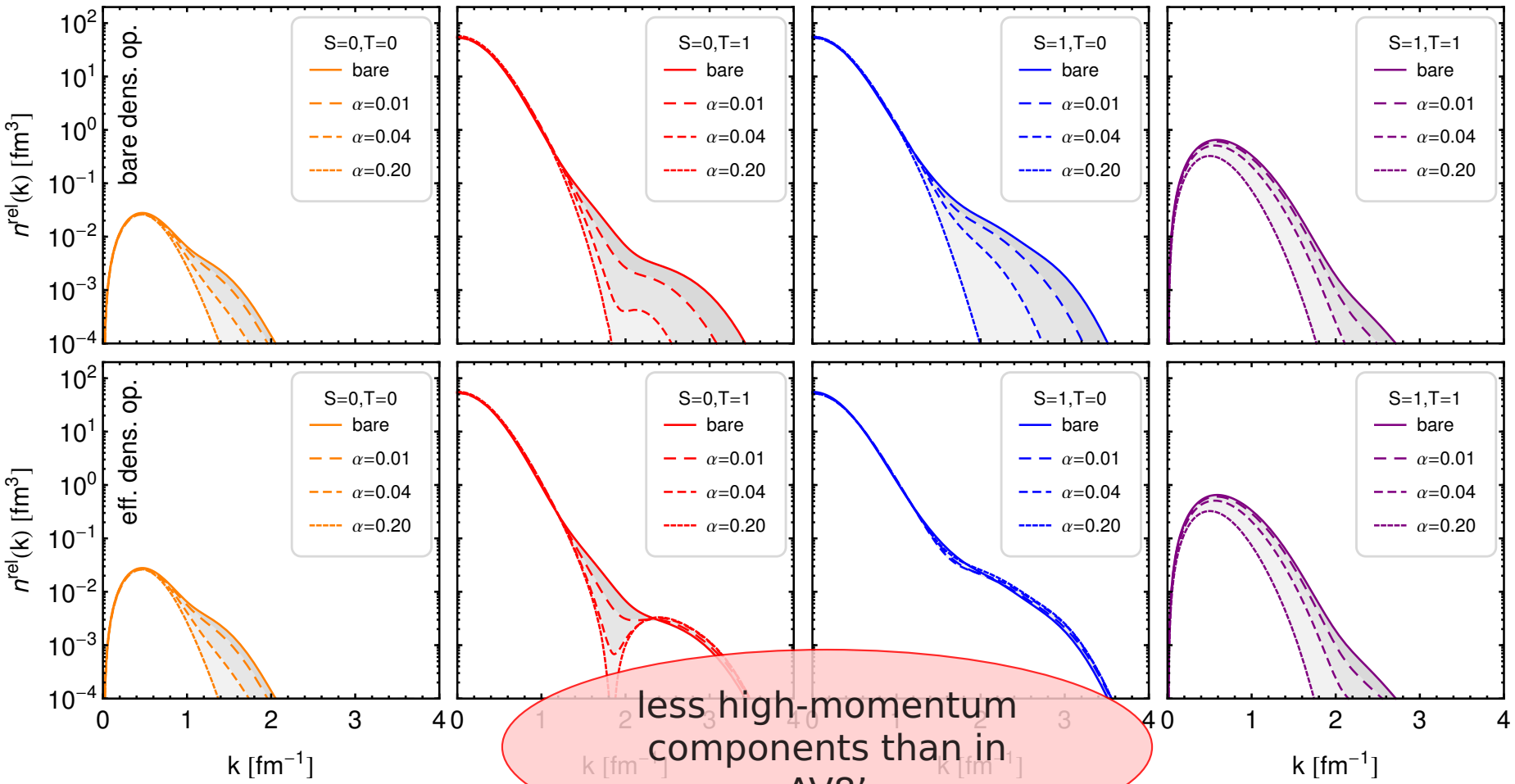
N^{3}LO Interaction



Flow dependence

N^{3}LO Interaction

^{4}He Two-body Density in Momentum Space



less high-momentum components than in AV8'

^{4}He many-body correlations

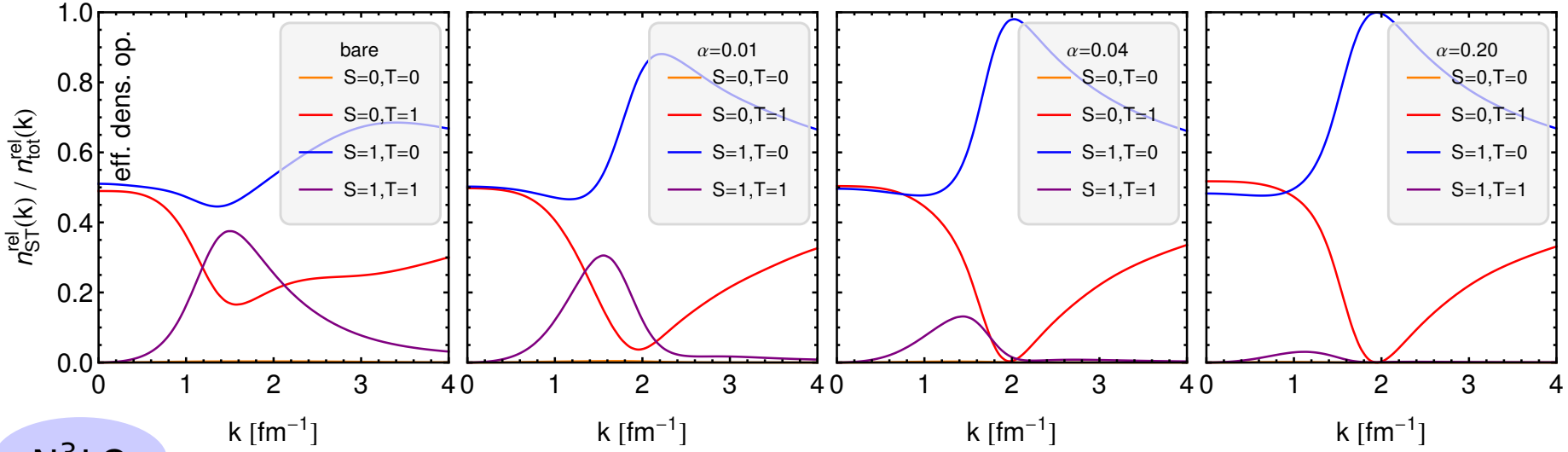
number of pairs in (S,T) channels

Interaction	(0,0)	(0,1)	(1,0)	(1,1)
AV8'	0.008	2.572	2.992	0.428
AV8' $\alpha = 0.01\,\mathrm{fm}^4$	0.008	2.708	2.992	0.292
AV8' $\alpha = 0.04\,\mathrm{fm}^4$	0.007	2.821	2.993	0.179
AV8' $\alpha = 0.20\,\mathrm{fm}^4$	0.005	2.925	2.995	0.075
N^{3}LO	0.009	2.710	2.991	0.290
N^{3}LO $\alpha = 0.01\,\mathrm{fm}^4$	0.007	2.745	2.992	0.255
N^{3}LO $\alpha = 0.04\,\mathrm{fm}^4$	0.006	2.817	2.994	0.183
N^{3}LO $\alpha = 0.20\,\mathrm{fm}^4$	0.004	2.921	2.995	0.079

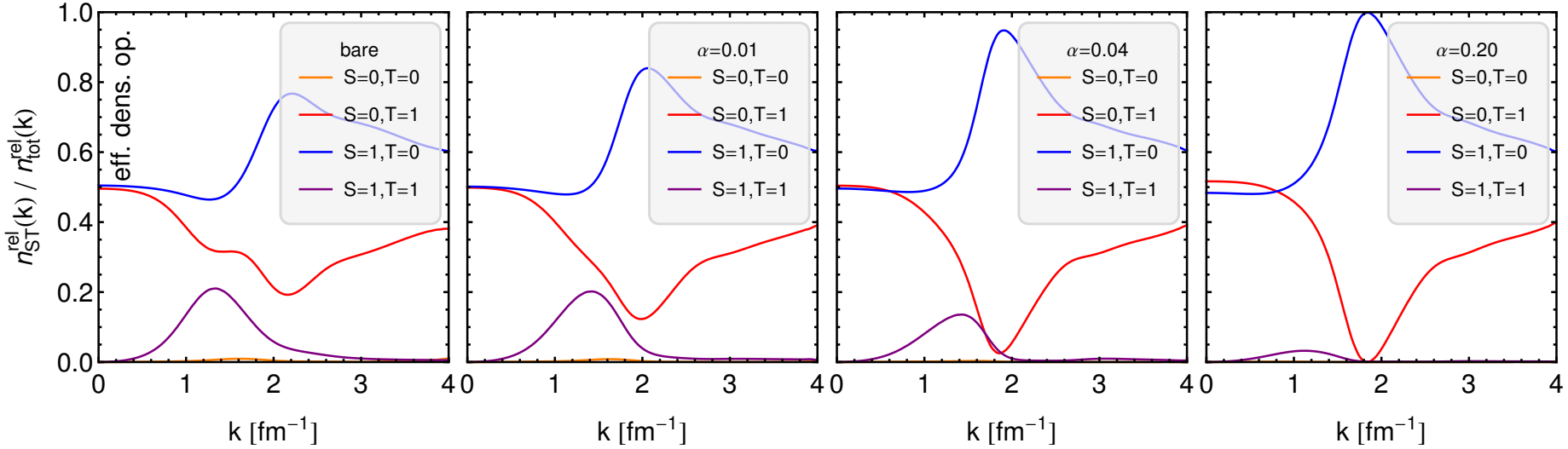
- occupation of $(S,T) = (0,1)$ and $(S,T) = (1.0)$ channels reflect many-body correlations
- AV8' induces stronger many-body correlations than N^{3}LO
- with increasing flow parameter many-body correlations become weaker

Relative contributions of ST channels

AV8'

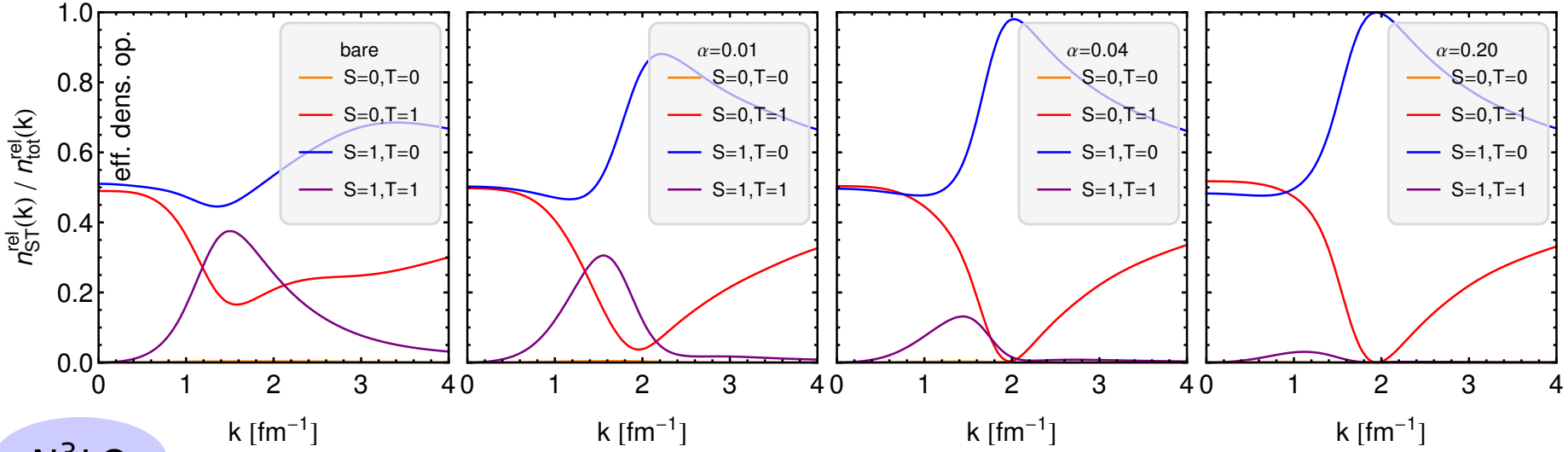


N^{3}LO

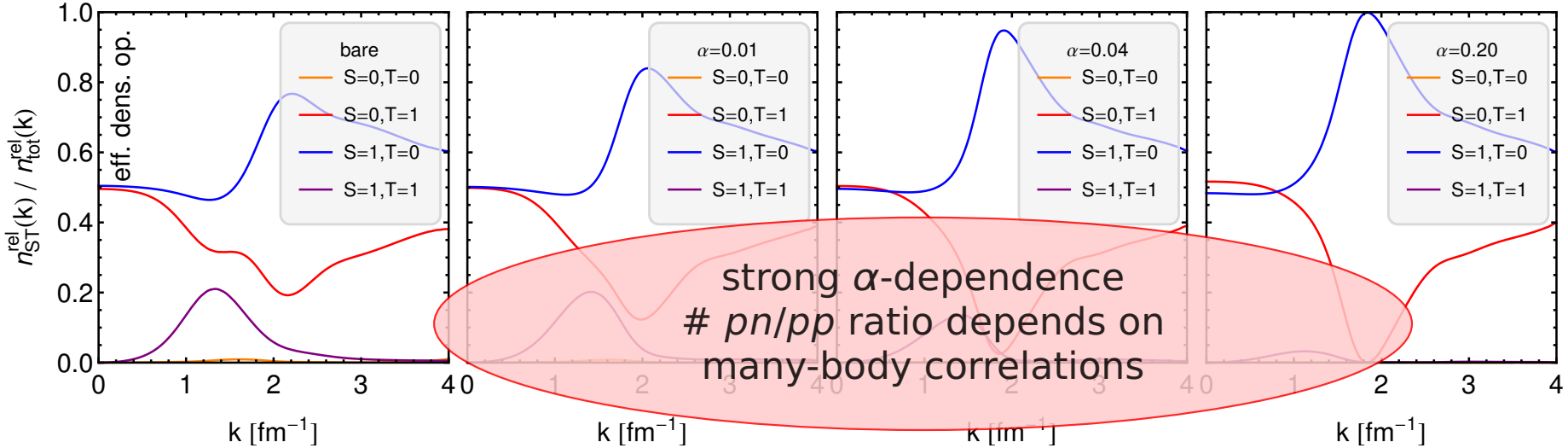


Relative contributions of ST channels

AV8'

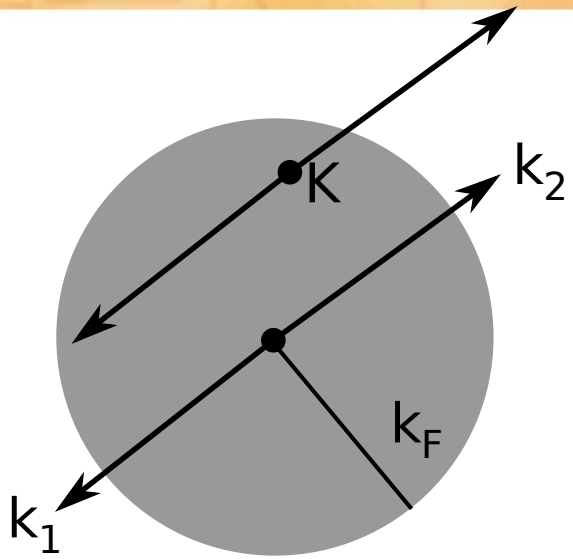


N^{3}LO



strong α-dependence
pn/*pp* ratio depends on
many-body correlations

Pair momentum $K = 0$



- for vanishing pair momentum and $k > k_F$ only high-momentum nucleons are sampled
- vanishing pair momentum implies vanishing pair angular momentum
- look only at pairs with $K = 0$:

$$n^{(2)}_{SM_S,TM_T}(\boldsymbol{K}=0,\boldsymbol{k}) = \langle\Psi|\sum_{i<j}^{A}\hat{P}^{SM_S}_{ij}\hat{P}^{TM_T}_{ij}\,\delta^3(\hat{\boldsymbol{k}}_i+\hat{\boldsymbol{k}}_j)\,\delta^3\big(\frac{1}{2}(\hat{\boldsymbol{k}}_i-\hat{\boldsymbol{k}}_j)-\boldsymbol{k}\big)\,|\Psi\rangle$$

Translational invariant two-body densities

- To calculate the translational invariant two-body density the single-particle coordinates are transformed to Jacobi coordinates

$$(\boldsymbol{x}_1, \ldots, \boldsymbol{x}_{A-1}, \boldsymbol{x}_A) \rightarrow (\boldsymbol{\xi}_0, \boldsymbol{\xi}_1, \ldots, \boldsymbol{\xi}_{A-3}, \boldsymbol{\eta}, \boldsymbol{\vartheta})$$

with

$$\boldsymbol{\eta} = \sqrt{\frac{2(A-2)}{A}} \left[\frac{1}{A-2}(\boldsymbol{x}_1 + \ldots + \boldsymbol{x}_{A-2}) - \frac{1}{2}(\boldsymbol{x}_{A-1} + \boldsymbol{x}_A) \right], \qquad \boldsymbol{\vartheta} = \sqrt{\frac{1}{2}}(\boldsymbol{x}_{A-1} - \boldsymbol{x}_A)$$

- The two-body density in the lab system $\rho^{(2)}$ is then given by a linear transformation of the translational invariant two-body density $\tilde{\rho}^{(2)}$

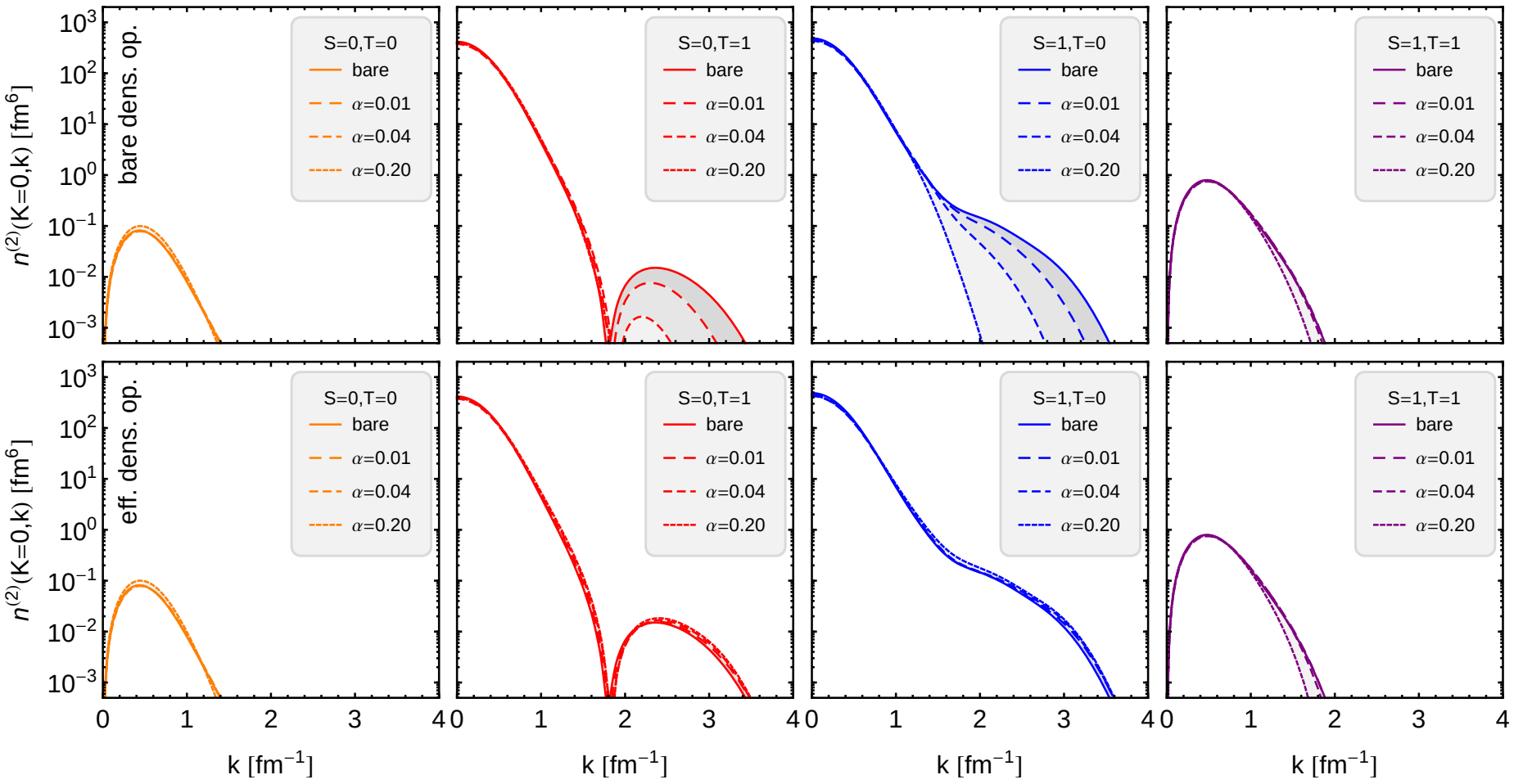
$$\sum_{M_1} \rho^{(2)}_{n_1' l_1 m_1, N_1' L_1 M_1; n_1 l_1 m_1, N_1 L_1 M_1} = \sum_{NN'L} M^{(0)}_{N_1' L_1, N_1 L_1; N'L, NL} \sum_{M} \tilde{\rho}^{(2)}_{n_1' l_1 m_1, N'LM; n_1 l_1 m_1, NLM}$$

- invert transformation to obtain translational invariant density from NCSM wave function

Flow dependence – $K = 0$

N^{3}LO Interaction

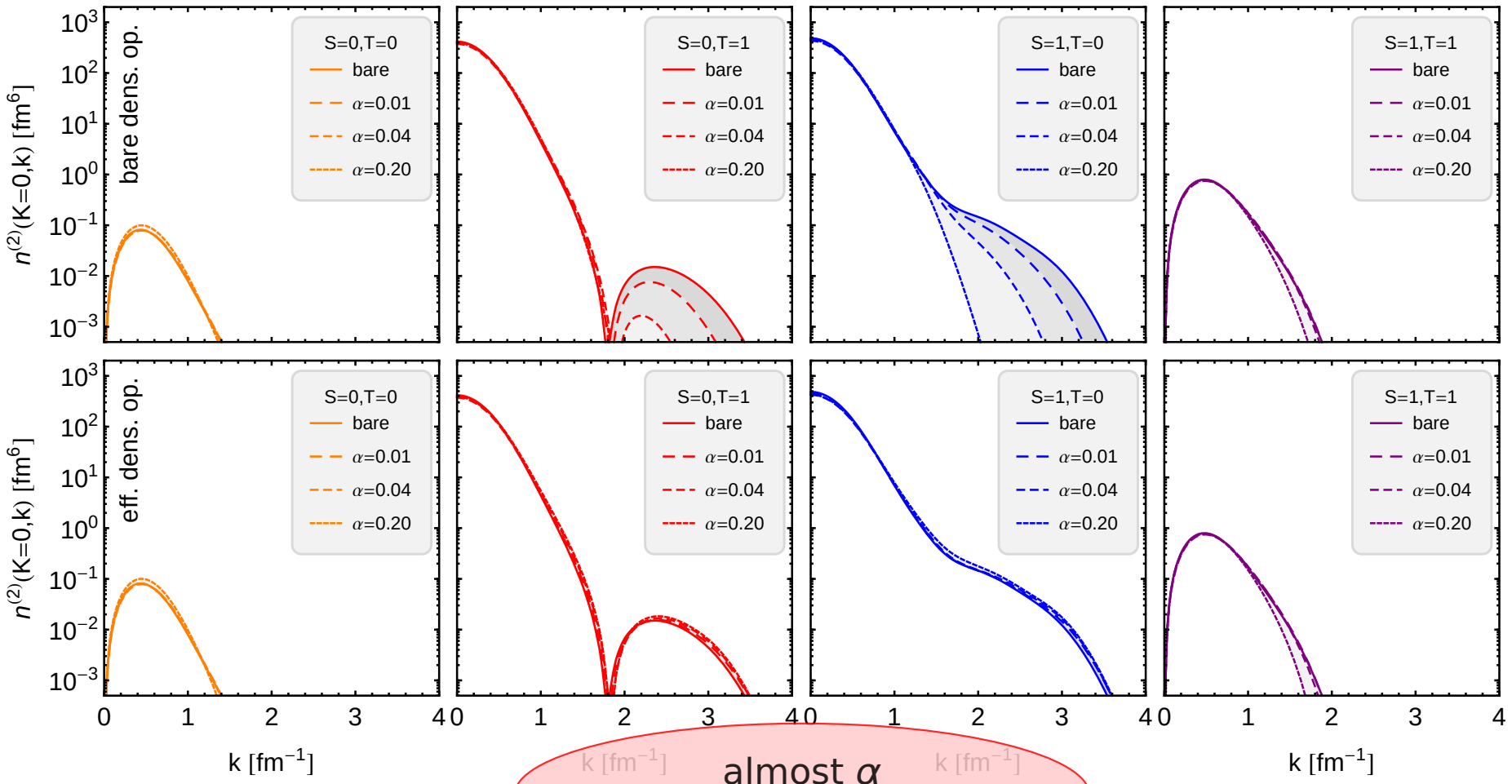
^{4}He Two-body Density in Momentum Space



Flow dependence – $K = 0$

N^{3}LO Interaction

^{4}He Two-body Density in Momentum Space

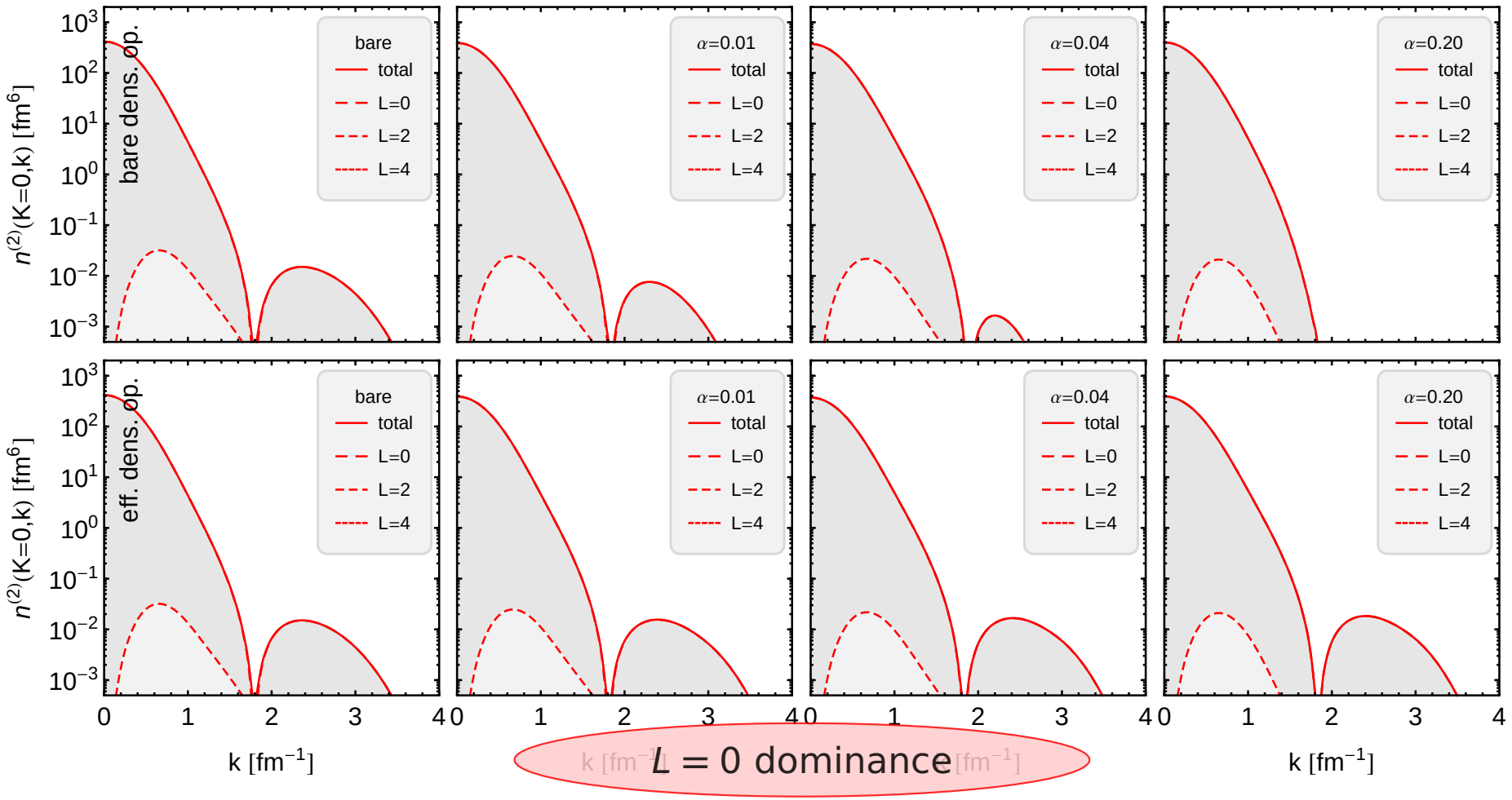


almost α independent

Contributions from different angular momenta – $K = 0$

N^{3}LO Interaction

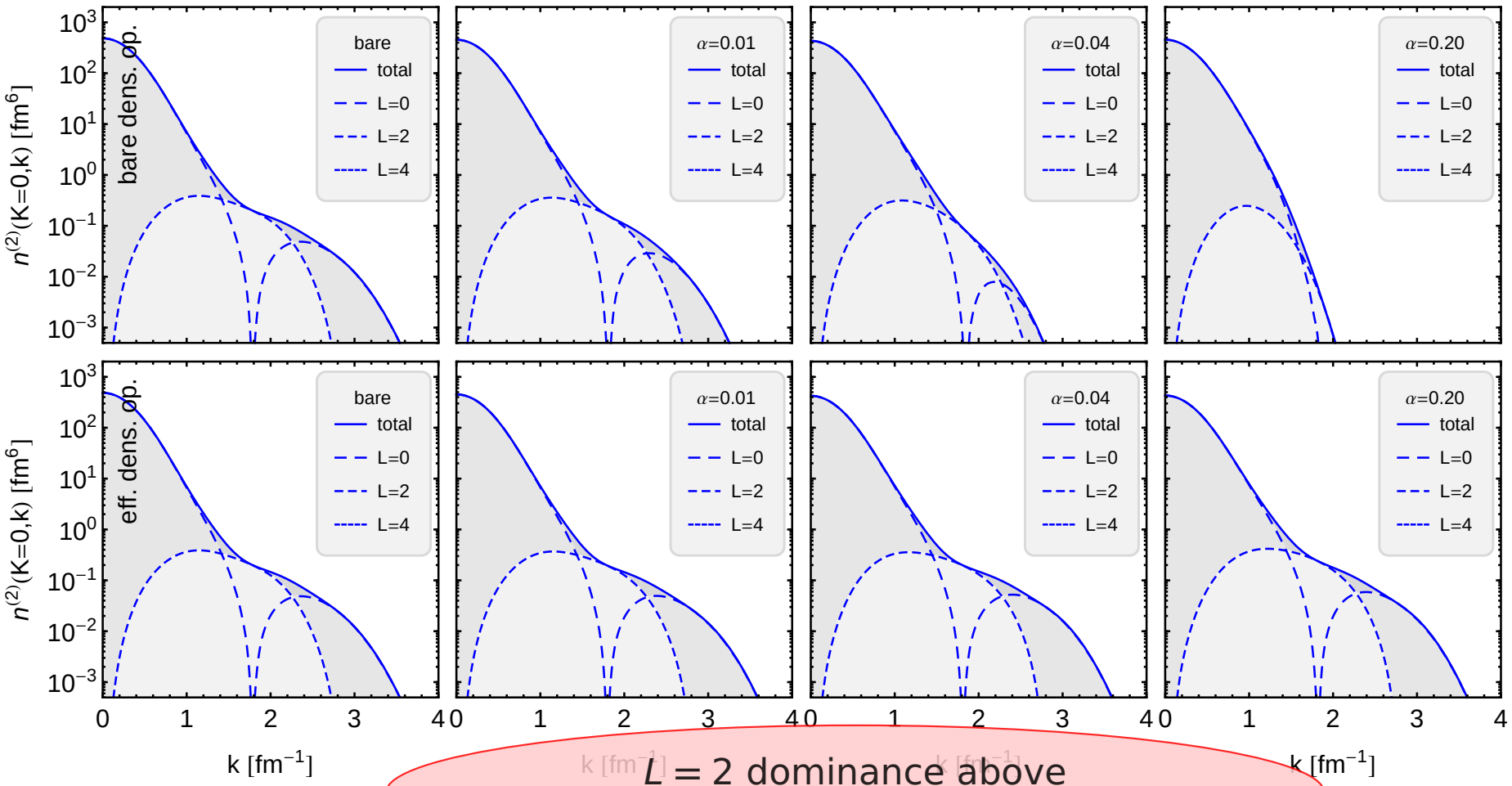
^{4}He Two-body Density in Momentum Space $S = 0, T = 1$



Contributions from different angular momenta – $K = 0$

N^{3}LO Interaction

^{4}He Two-body Density in Momentum Space $S = 1, T = 0$

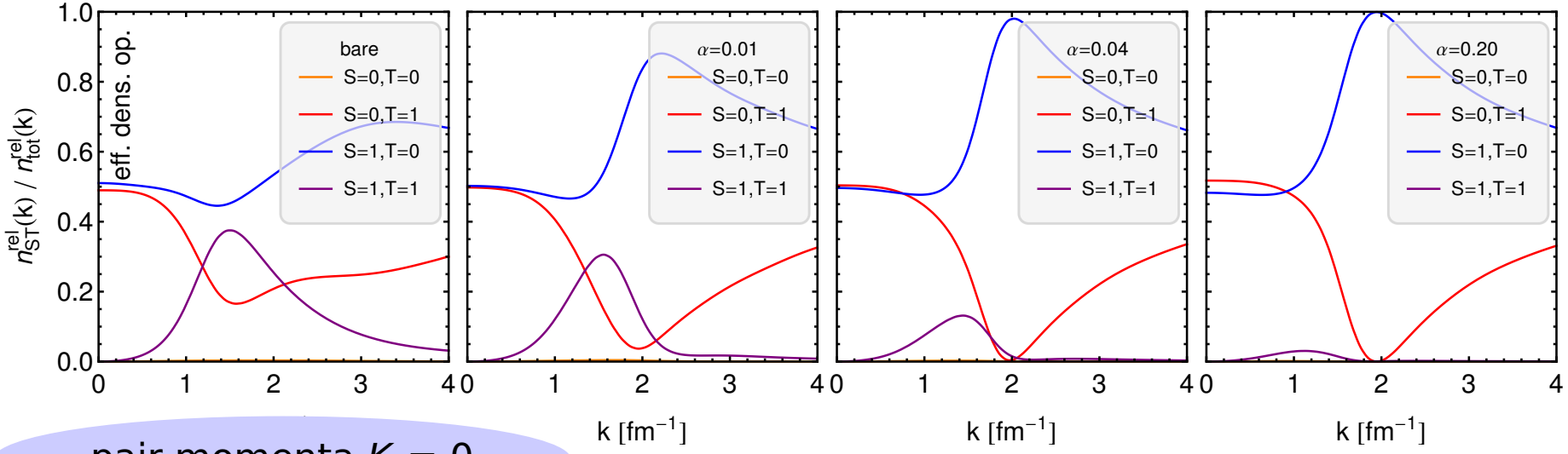


$L = 2$ dominance above Fermi momentum

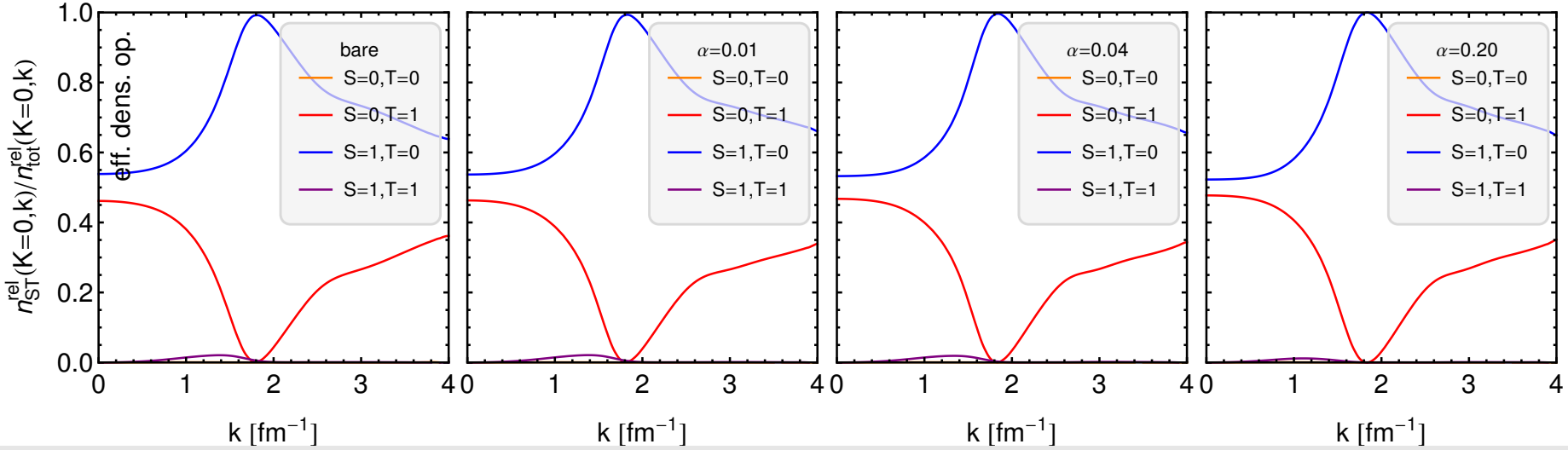
Relative contributions of ST channels

N^{3}LO interaction

all pair momenta



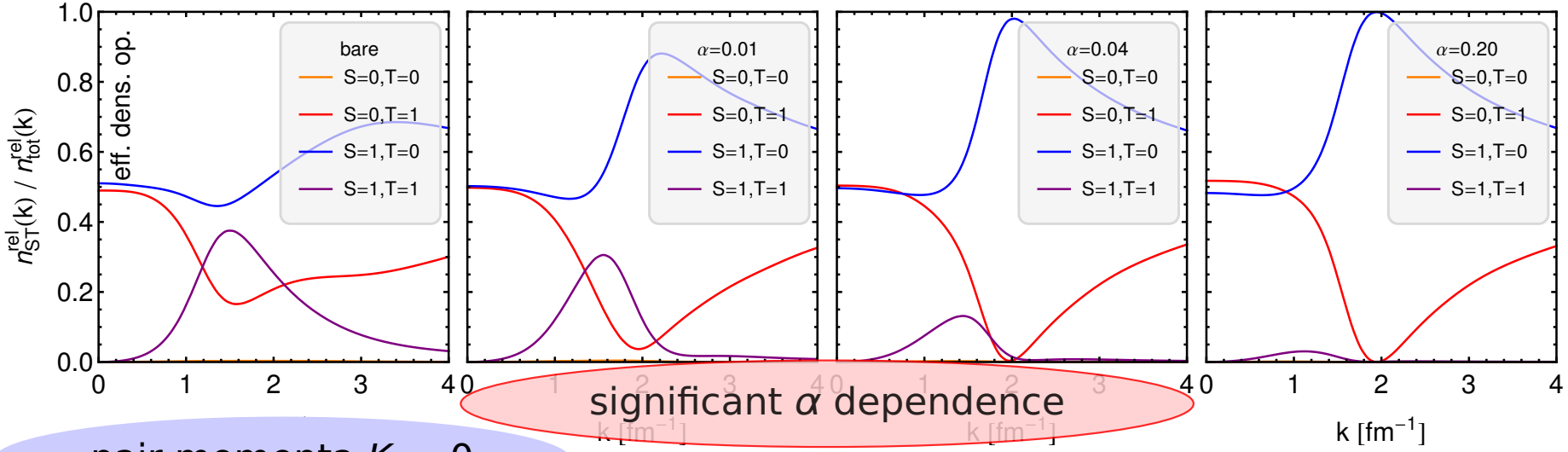
pair momenta $K = 0$



Relative contributions of ST channels

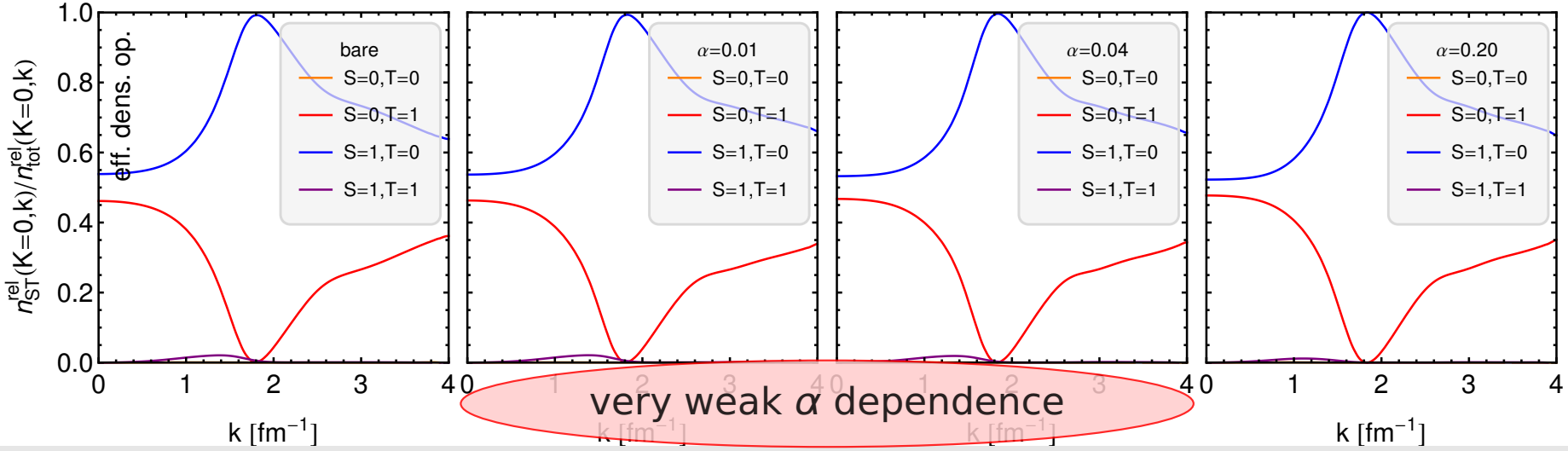
N^{3}LO interaction

all pair momenta



significant α dependence

pair momenta $K = 0$



very weak α dependence

^{4}He,^{6}He,^{9}Be,^{12}C Results

Heavier nuclei

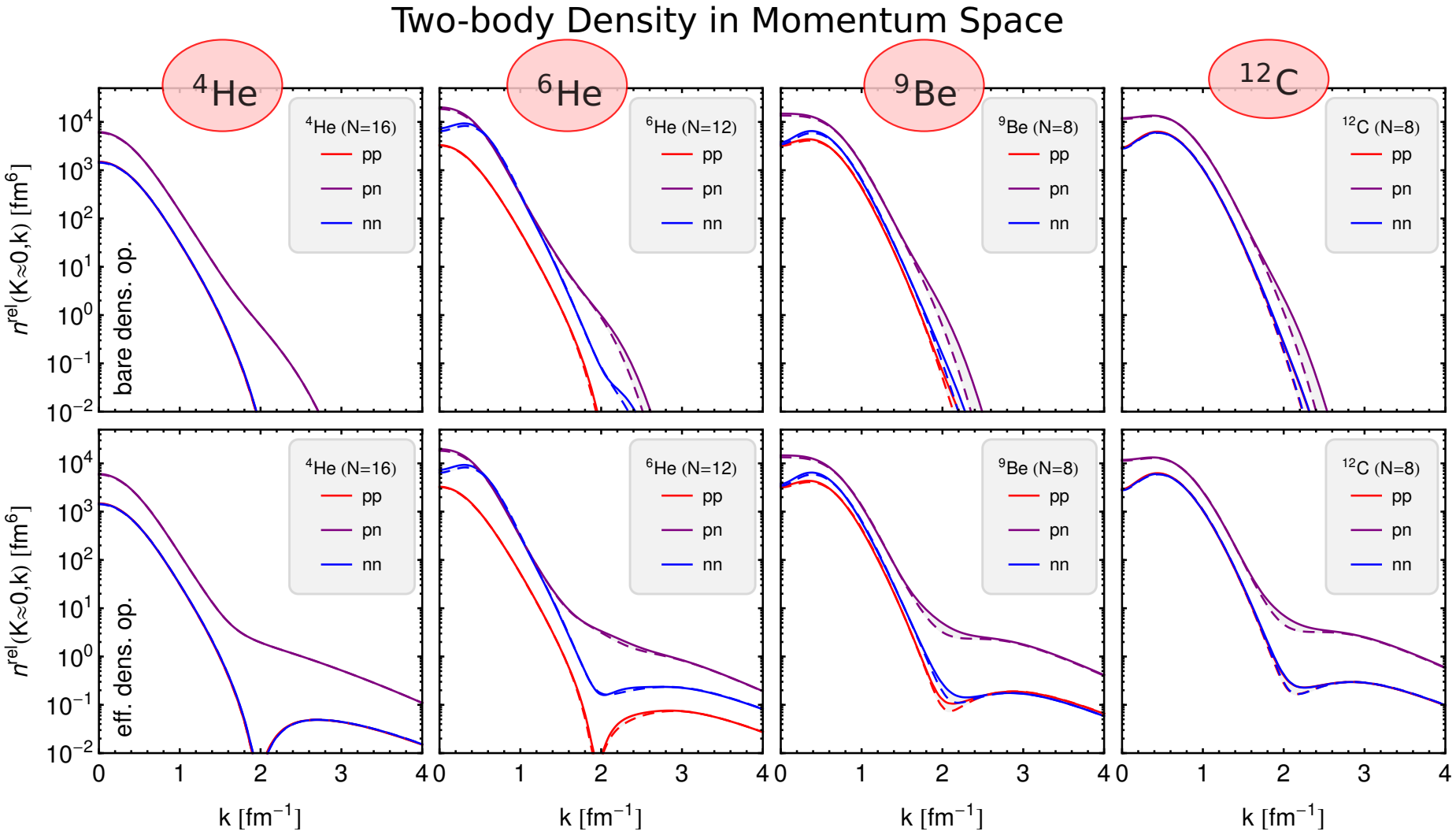
Calculation

- bare AV18 and N^{3}LO can not be converged for heavier nuclei
- NCSM converges only for larger flow parameters

Objectives

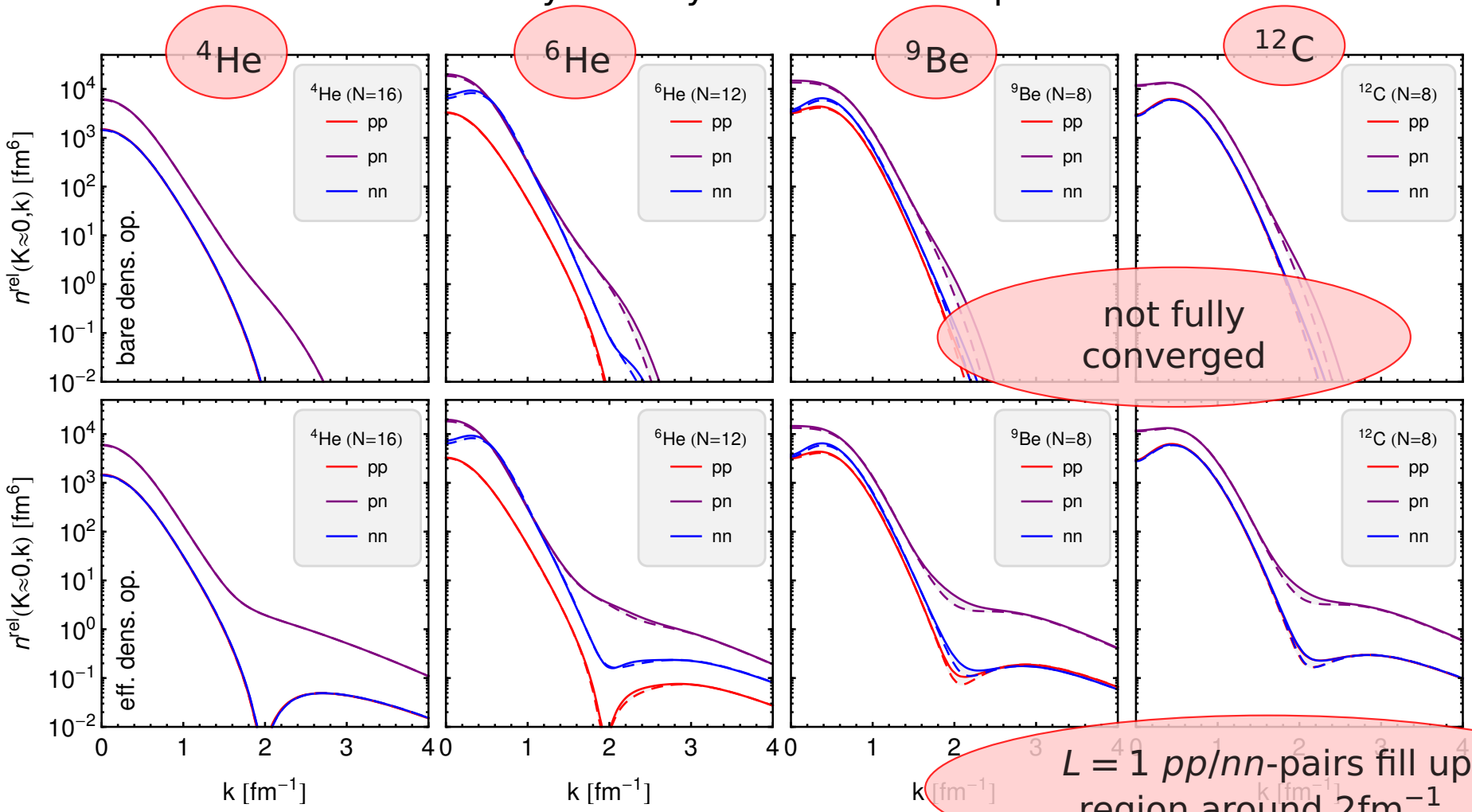
- Compare AV18 and N^{3}LO results
- Check for NCSM convergence
- Check flow dependence $\alpha = 0.04, 0.08, 0.20\text{fm}^4$ ($\Lambda = 2.24, 1.88, 1.50\text{fm}^{-1}$)
- What is different from ^{4}He ?

AV18, $K \approx 0$, α=0.04 fm^4



AV18, $K \approx 0$, α=0.04 fm^4

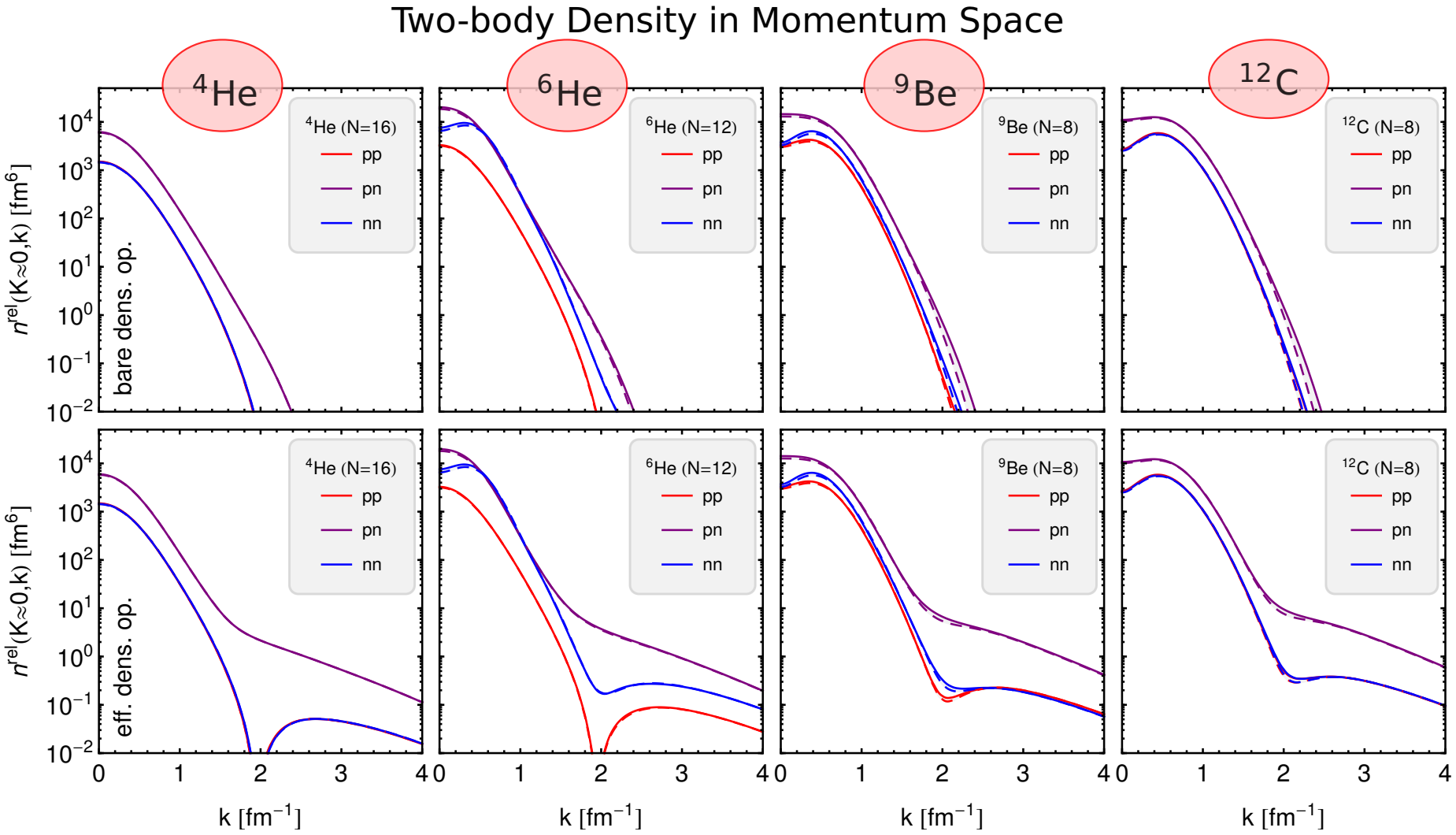
Two-body Density in Momentum Space



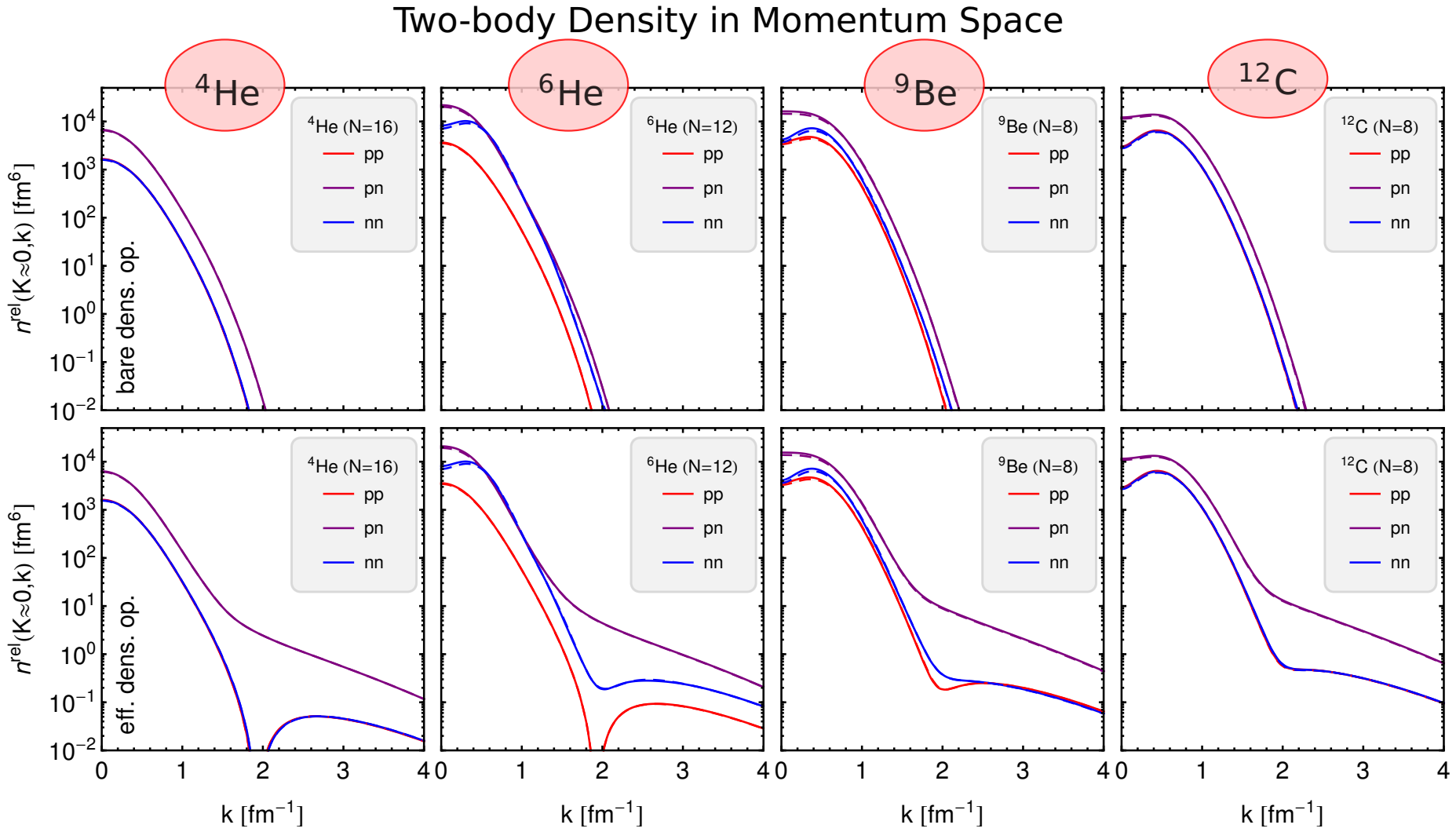
not fully converged

$L = 1$ *pp*/*nn*-pairs fill up region around 2fm^{-1}.

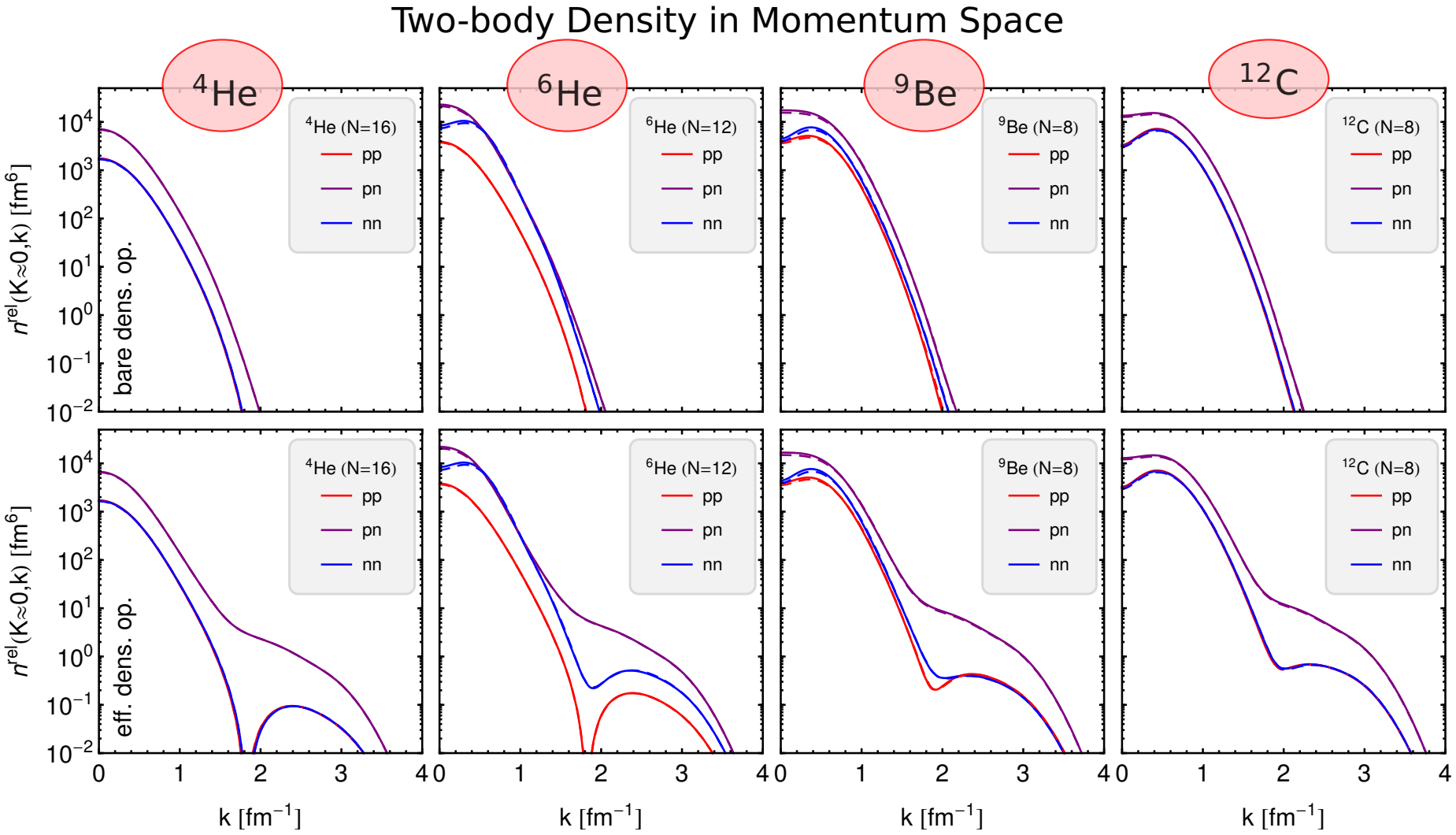
AV18, $K \approx 0$, α=0.08 fm^4



AV18, $K \approx 0$, α=0.20 fm^4

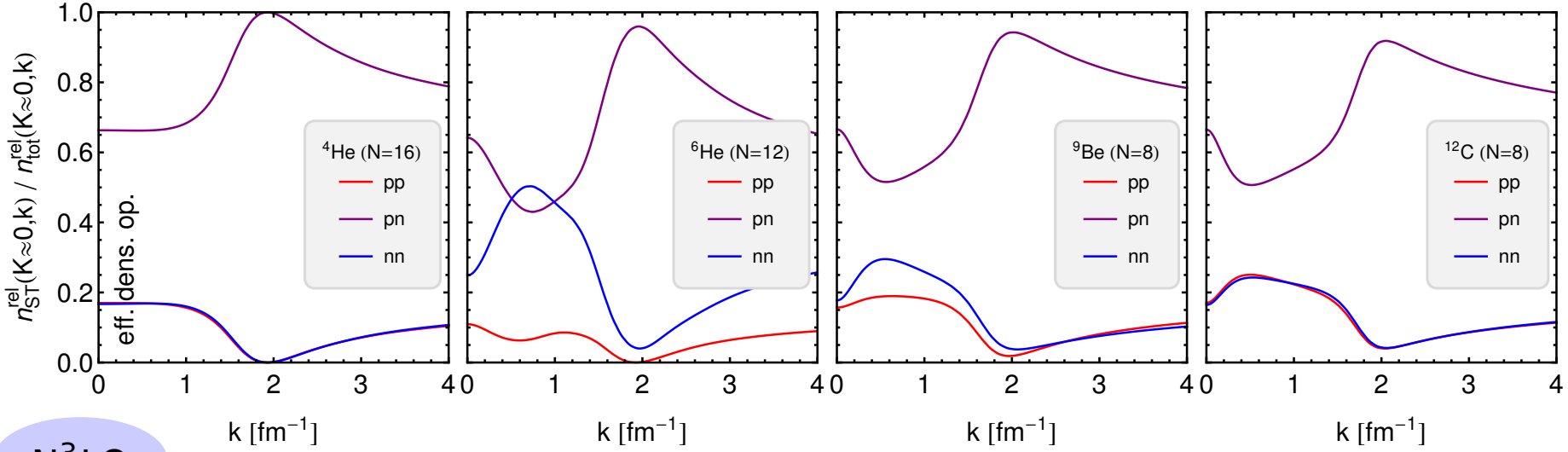


N^{3}LO, $K \approx 0$, α=0.20 fm^4

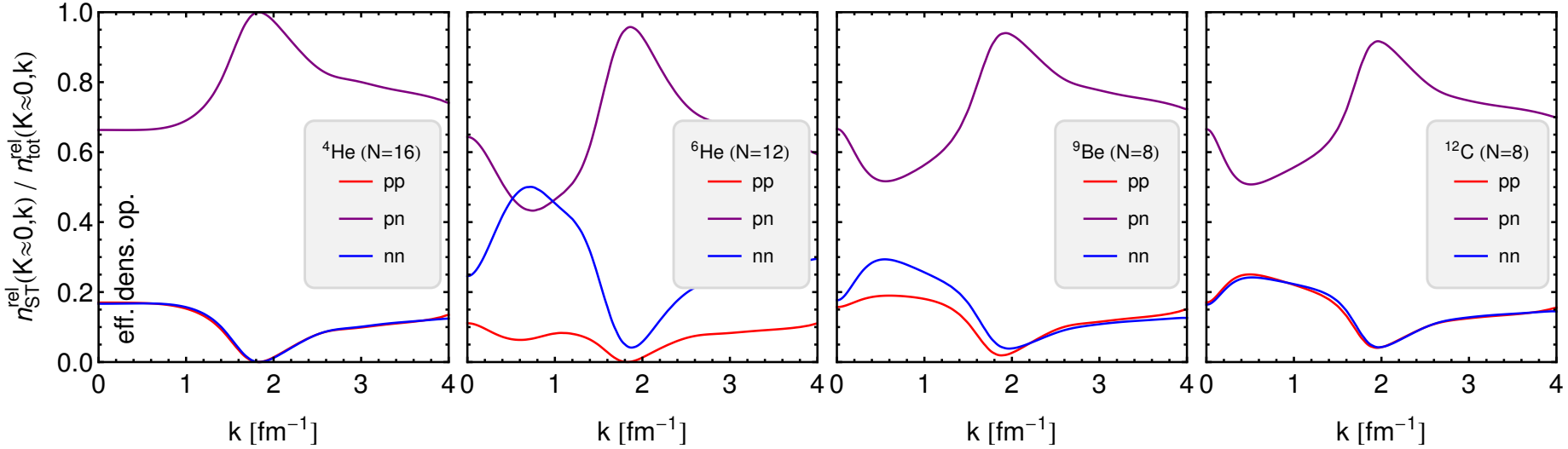


pp, pn, nn relative contributions, $K \approx 0$

AV18

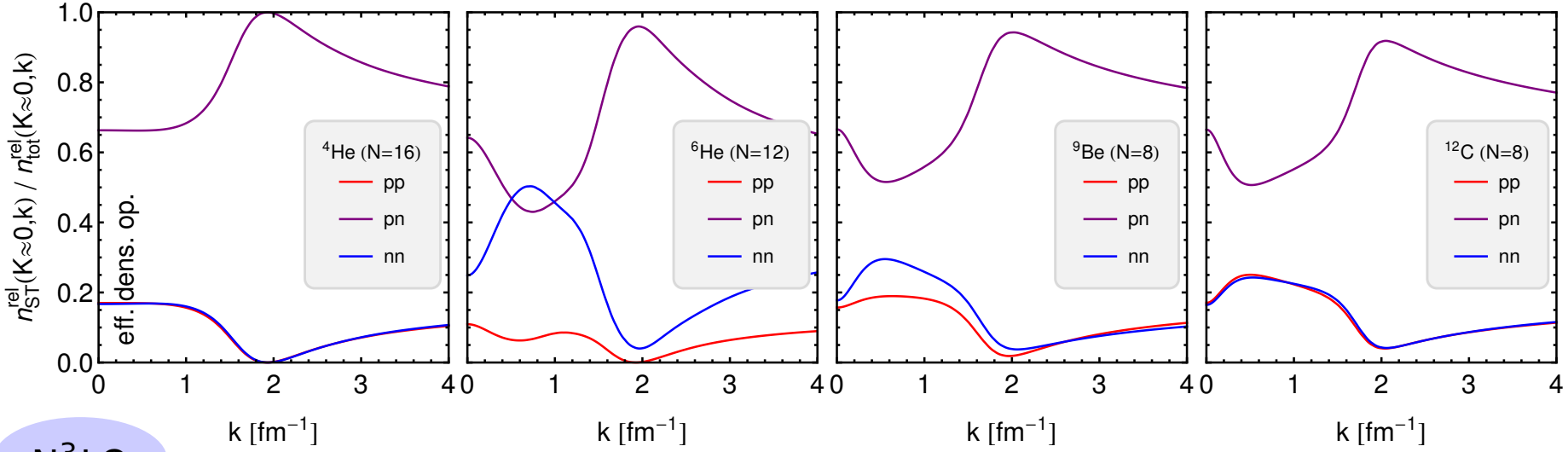


N^{3}LO

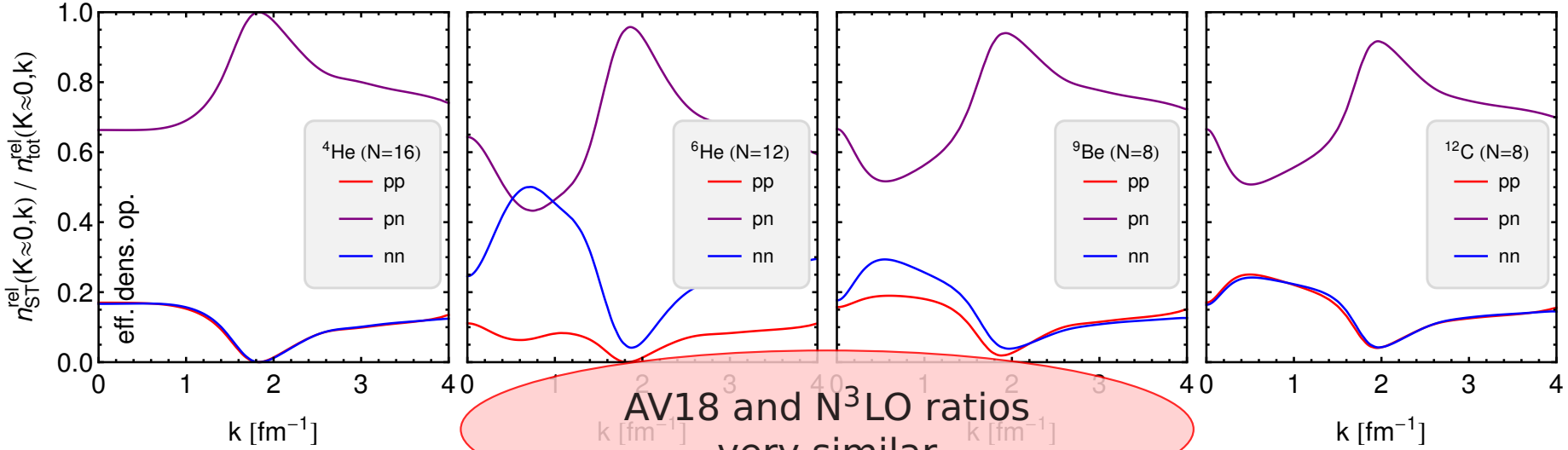


pp, pn, nn relative contributions, $K \approx 0$

AV18



N^{3}LO



AV18 and N^{3}LO ratios very similar

Summary

Similarity Renormalization Group

- SRG evolved Hamiltonian $\hat{H}_\alpha$ and unitary transformation matrix $\hat{U}_\alpha$
- “bare” and “effective” density operators

^{4}He two-body densities

- stronger short-range correlations for AV8’ than N^{3}LO
- short-range and high-momentum components described by effective operators
- high-momentum components above the Fermi momentum dominated by $S = 1, T = 0, L = 2$ pairs (tensor correlations)
- flow-dependence in $T = 1$ channels reflects three-body correlations induced by the tensor force
- $K = 0$ momentum distributions (almost) renormalization flow independent
  - good choice for studying two-body correlations experimentally -

^{6}He,^{9}Be,^{12}C two-body densities

- Similar behavior in all nuclei – *pn* pairs dominate momentum distribution
- $T = 1$ pairs with $L = 1$ fill up the *pp*/*nn* momentum distributions above the Fermi momentum
- AV18 and N^{3}LO provide similar results up to $k \approx 3\text{fm}^{-1}$