Basic Aspects of Short Range Correlations

Based on work with
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Motivation and Content

- Off-shell properties and spectral functions in transport theory
- Application: nucleon spectral functions, comparison with many-body theory
- Connection with EFT NN potentials????

Single-particle propagators

- One particle Green‘s function:

for $t_1 > t_{1'}$:

$$g^>(1,1') \quad \equiv \quad -i\langle\Psi(1)\Psi^\dagger(1')\rangle = g(1,1')$$

for $t_1 < t_{1'}$:

$$g^<(1,1') \quad \equiv \quad i\langle\Psi^\dagger(1')\Psi(1)\rangle = g(1,1')$$

- Interpretation:
  - - i $g^{<(}$1,1) = particle density
  - +i $g^{>}$(1,1) = hole density

Transport equation

- Kadanoff-Baym (or BUU) equation

$$\left(\frac{\partial}{\partial t} + \frac{\vec{p}}{m}\cdot\vec{\nabla}_x - \vec{\nabla}_x V \cdot \vec{\nabla}_p + \text{KB terms}\right) g^{<}$$
$$= \underbrace{-i\Sigma^{>}g^{<}}_{\text{Loss term}} + \underbrace{i\Sigma^{<}g^{>}}_{\text{Gain Term}}$$

- LHS: drift term
- RHS: collision term = - loss + gain terms

First: equilibrium nuclear matter

Propagators
for equilibrium matter

$$g^{<}(\omega, p) = i a(\omega, p) f(\omega, p),$$
$$g^{>}(\omega, p) = -i a(\omega, p)\big(1 - f(\omega, p)\big),$$

$$a(\omega, p) = \frac{\Gamma(\omega, p)}{\left(\omega - \frac{p^2}{2m} - \Sigma^{\mathrm{mf}} - \operatorname{Re}\Sigma^{\mathrm{ret}}(\omega, p)\right)^2 + \frac{1}{4}\Gamma^2(\omega, p)},$$

f = energy-momentum distribution

Equilibrium nuclear matter

- *In equilibrium* collision term in KB equation vanishes:

$$
\begin{aligned}
\Sigma^{>}(\omega,\vec{p}) &= -i\Gamma(\omega,\vec{p})(1-f(\omega,\vec{p}) \\
\Sigma^{<}(\omega,\vec{p}) &= i\Gamma(\omega,\vec{p})f(\omega)
\end{aligned}
$$

➔ $$-\Sigma^{>}g^{<}+\Sigma^{<}g^{>}=0$$

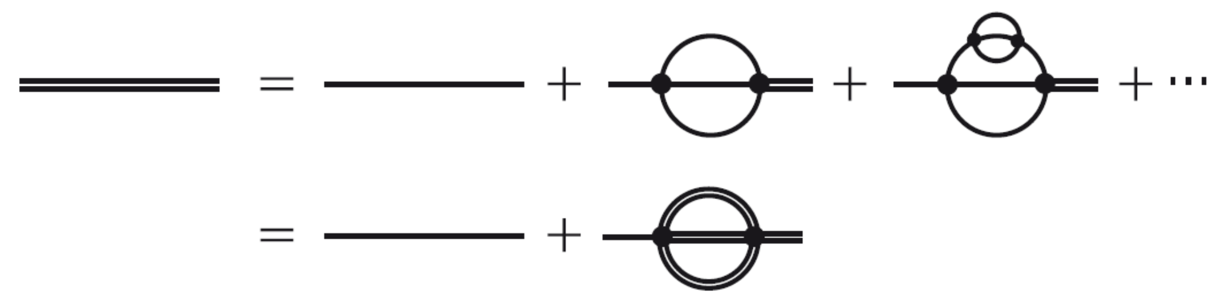
Equilibrium nuclear matter

- Phase space distribution in nuclear matter for $T = 0$:

  $f(\omega) = \Theta(\omega_F - \omega)$
- $\omega < \omega_F$ $$\Sigma^{>}(\omega, p) \;=\; 0, \qquad \Gamma(\omega, p) = -i\Sigma^{<}(\omega, p)$$
- $\omega > \omega_F$ $$\Sigma^{<}(\omega, p) \;=\; 0, \qquad \Gamma(\omega, p) = i\Sigma^{>}(\omega, p)$$

$$\Rightarrow \Gamma(\omega_F, p) = 0$$

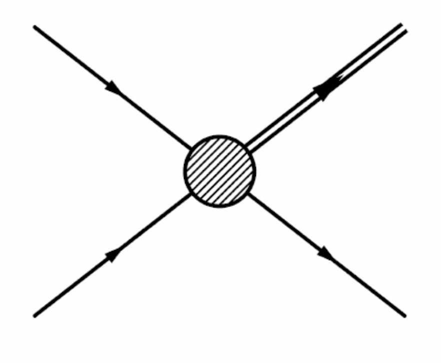
Selfconsistent particle propagator for zero-range interactions



Dyson equation for propagator resummed

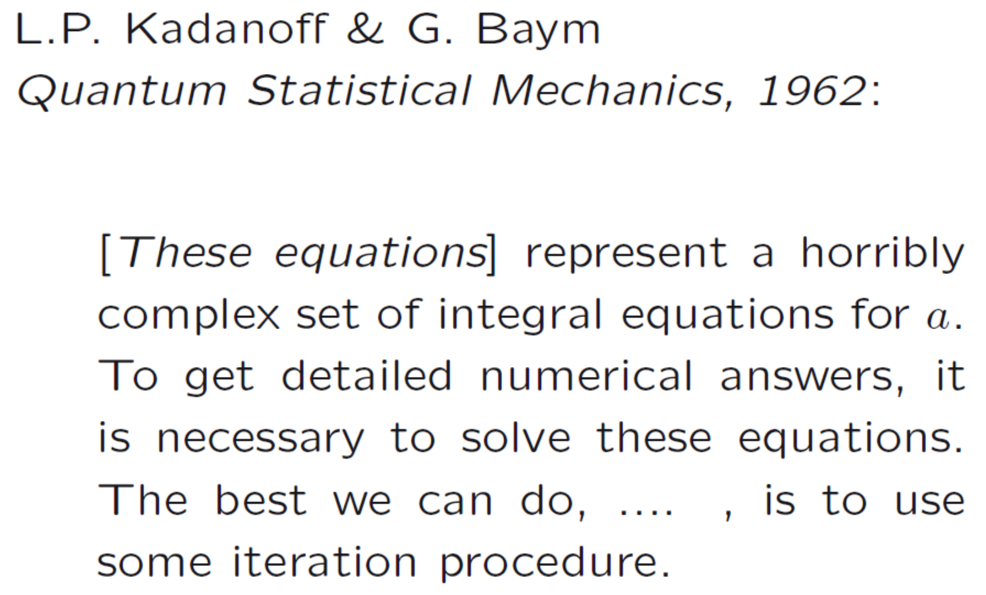
Gain term explicitly for NN collisions

$$
\begin{aligned}
\Sigma^{<}(\omega,p) \;=\; & g\int \frac{d^3p_2 d\omega_2}{(2\pi)^4}\frac{d^3p_3 d\omega_3}{(2\pi)^4}\frac{d^3p_4 d\omega_4}{(2\pi)^4} \\
& (2\pi)^4\delta^4(p+p_2-p_3-p_4) \\
& |\mathcal{M}|^2 \\
& \times g^{>}(\omega_2,p_2)g^{<}(\omega_3,p_3)g^{<}(\omega_4,p_4)
\end{aligned}
$$



Collision rate of hole in Born approximation: 1p2h
Propagators *g* are dressed, depend on Σ
Solved by iterative procedure

50 years Kadanoff & Baym



L.P. Kadanoff & G. Baym

Quantum Statistical Mechanics, 1962:

> [*These equations*] represent a horribly complex set of integral equations for a. To get detailed numerical answers, it is necessary to solve these equations. The best we can do, ... , is to use some iteration procedure.

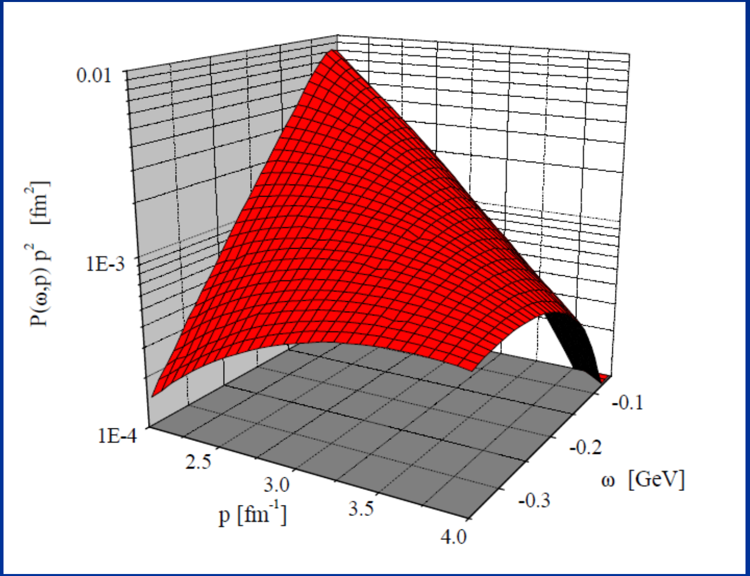
Simplified collision term: src

- *First:* Assume zero range interaction
  → *M* = const in momentum space

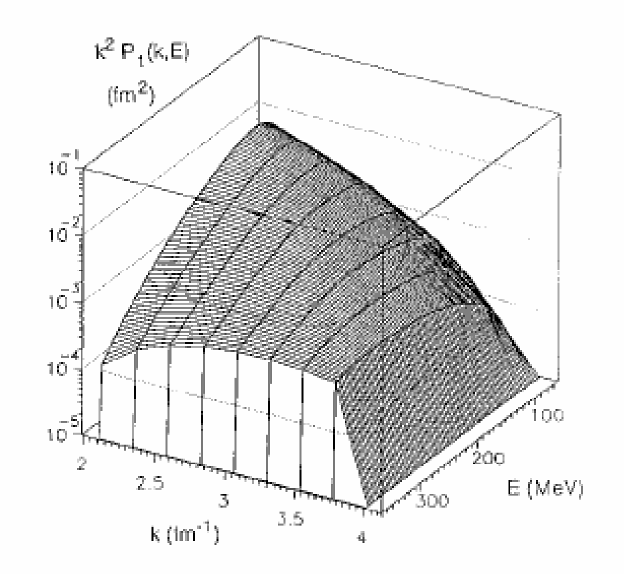
  no spin or isospin dependence
- Start with free Fermi-gas, then iterate selfenergies with this interaction
- *Later:* improve model for *M*

Nucleon spectral functions

M = 207 MeV fm^3, only free parameter fitted to Benhar‘s results

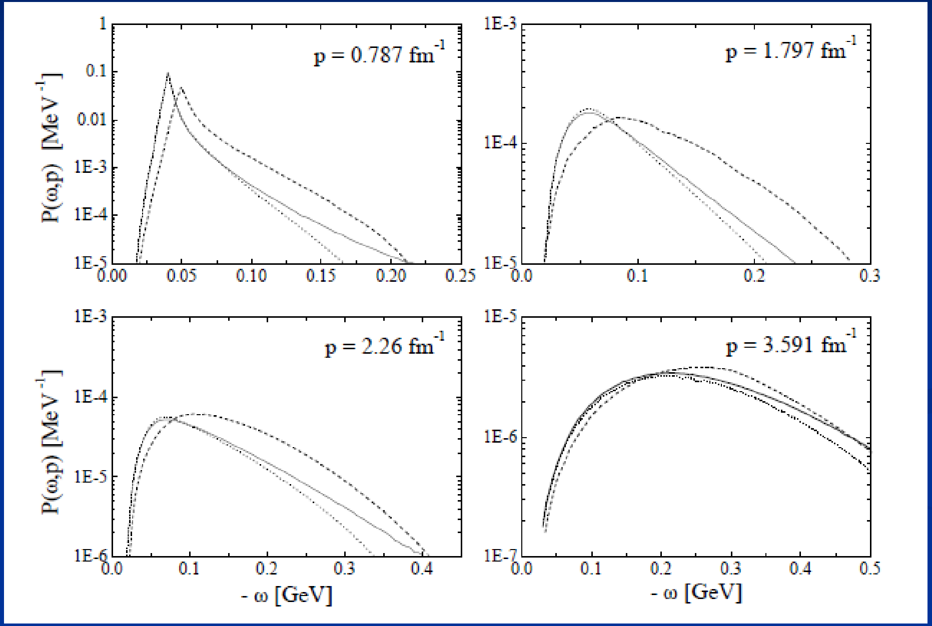


Lehr et al, 2000
Strength *M* obtained by fit to Benhar results



Benhar et al, 1992

Spectral functions



Solid: Transport
Dashed:
Benhar et al

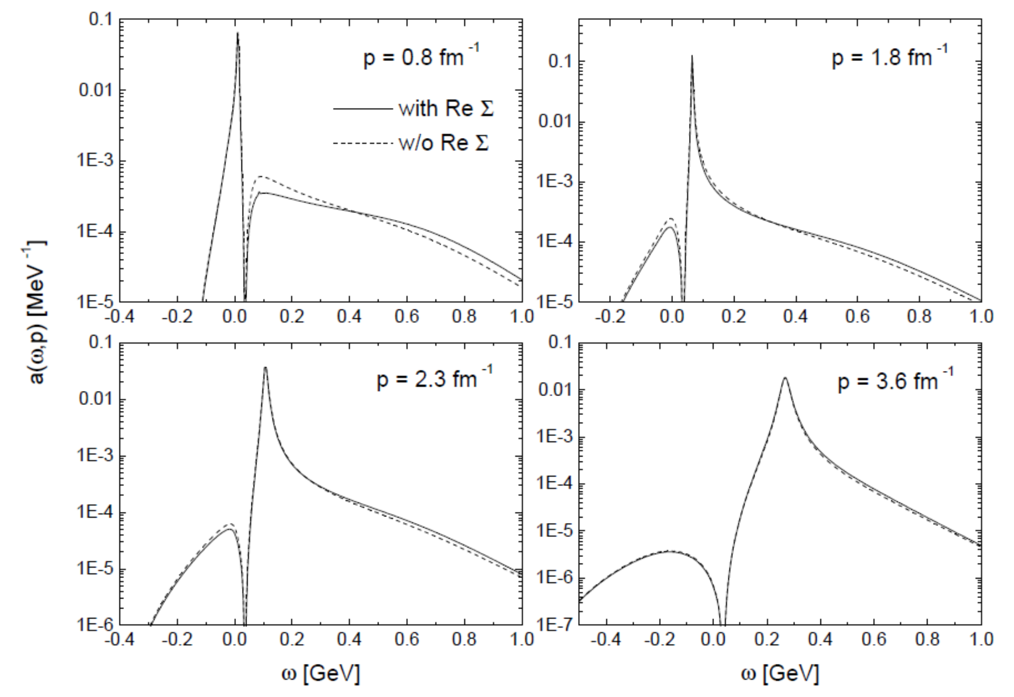
Strength *M*
fitted to Benhar‘s
Results (dashed)

Shape-diffs due to
real part of selfenergy

Reasonable agreement shows that

SF shape is determined by phase-space

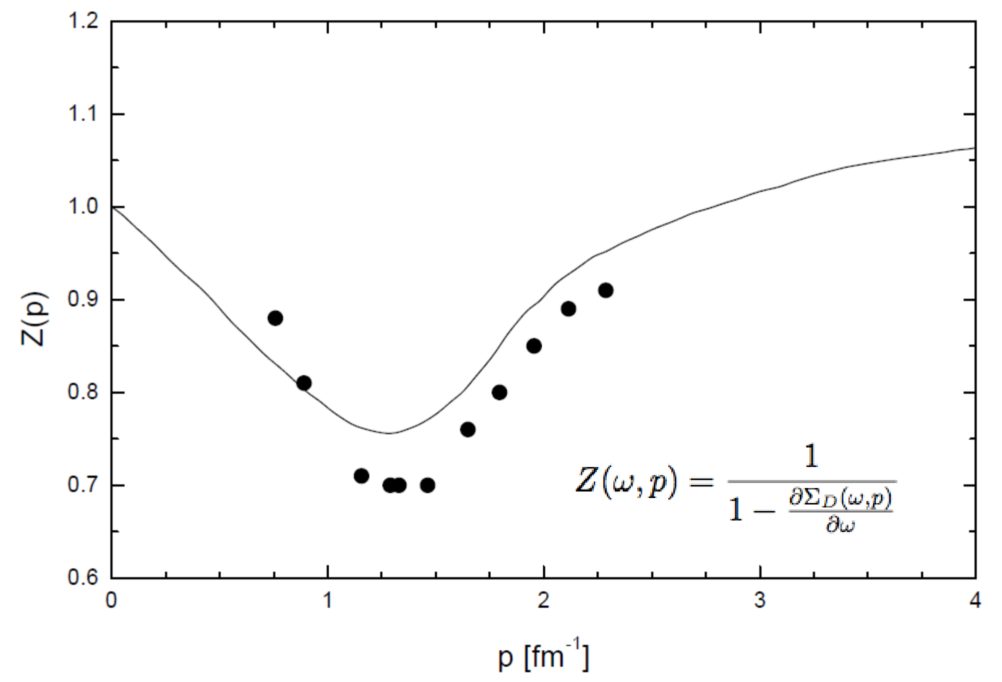
Nucleon spectral functions



Hole and particle spectral functions, only small influence of real parts

Spectral functions shape determined by phase-space alone

Nucleon spectroscopic factor

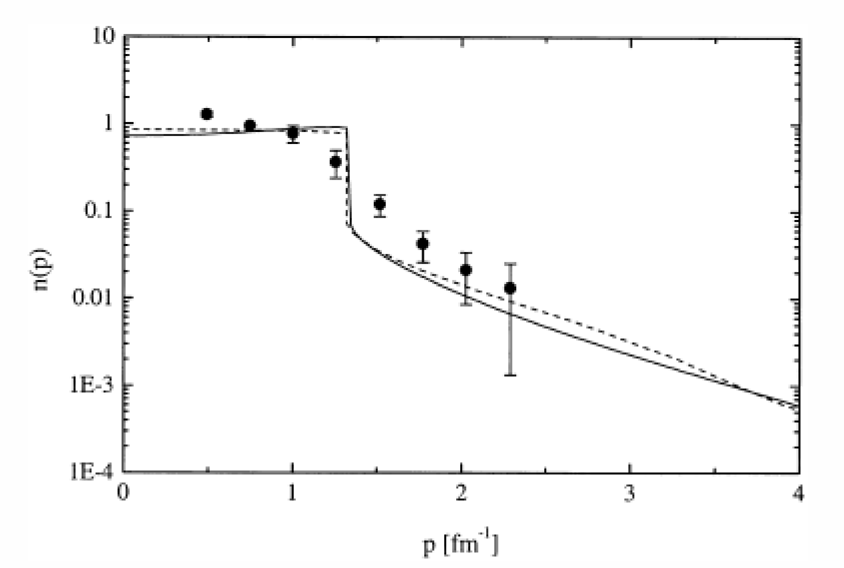


$$Z(\omega,p) = \frac{1}{1 - \frac{\partial \Sigma_D(\omega,p)}{\partial \omega}}$$

Dots: Benhar
Curve: Transport,

at on-shell point

Momentum distribution



‚Data‘ from Benhar-Sick analysis for nuclear mattter

High momentum tails generated from Fermi-gas and constant interaction

Scaling of high-momentum tails

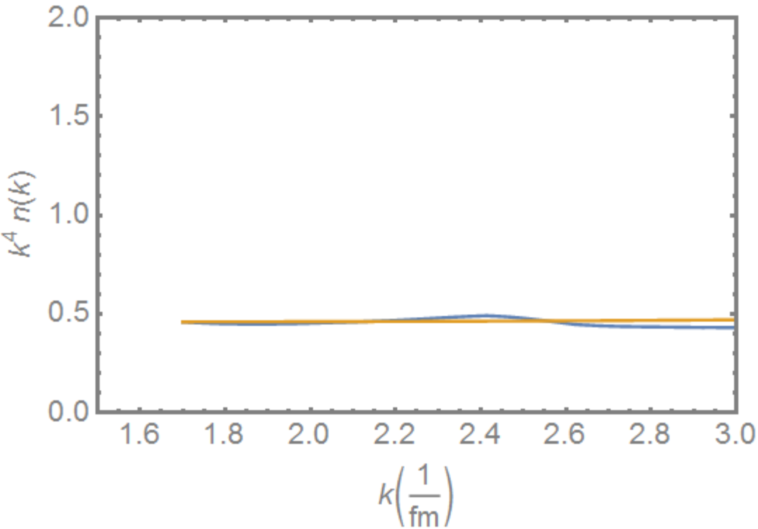
- O. Hen et al (arXiv:1407.8175) notice scaling of high-momentum tails with $1/k^4$ for nuclei and atomic gases

- Question: scaling accidental or deeper reason?

- Hen et al assign the scaling to tensor interactions and np pairs

Scaling of high-momentum tails

- n(k) scales as $1/k^4$



Scaling appears here as a consequence of phase-space alone

Scaling of high-momentum tails

- Scaling as general property of *any* fermion system with zero-range interactions, no details of interaction essential, in particular no assumptions on pp vs nn vs pn interactions

- This explains why the same scaling is observed in nuclear and atomic systems

Improve collision strength M

- So far: collision strength was fitted, no isospin dependence

- Now: obtain collision strength as short-range part of nuclear energy-density functional
(Konrad et al., Nucl. Phys. A 756 (2005))

Potentials from LM Theory

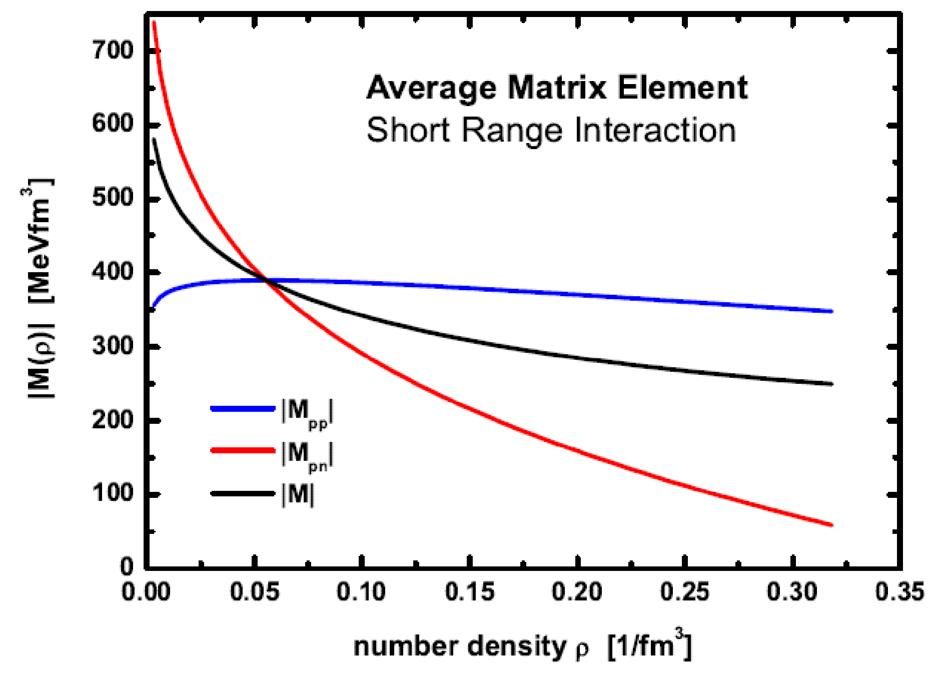
- Improve modeling of matrixelement *M:*

  Use as guideline Landau-Migdal Theory:

$$V^{qq'} = f^{qq'} + g'^{qq'}\vec{\sigma}_1\vec{\sigma}_2$$

- Calculate spin singlet contrib *f* and spin triplet contrib *g* from short range part of Skyrme (or any other) energy-density functional. No tensor interaction!

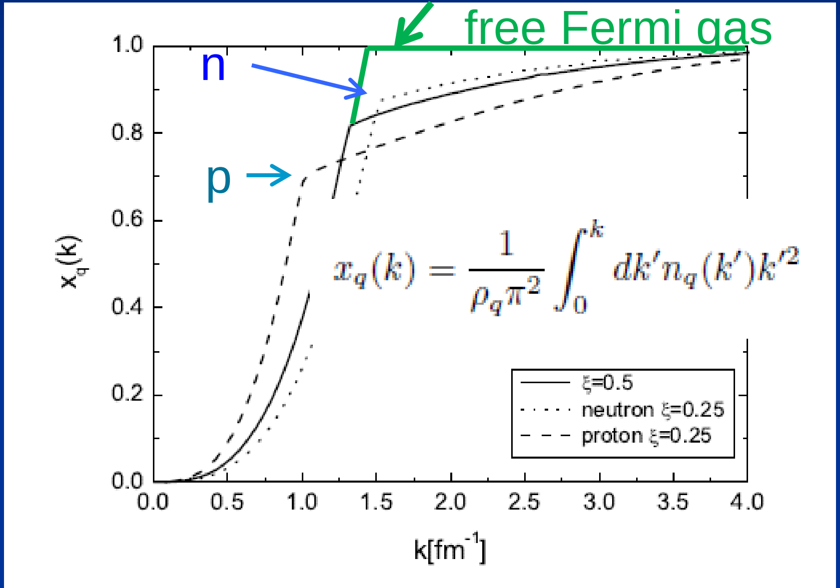
Isospin-Dependence of *M*



pn large only for small densities,

determined by durface-symmetry energies in EDF

Proton Momenta in Asymmetric Matter



In asymmetric matter protons carry a larger part of their momentum in the high-momentum src tails

Again: consequence of phase-space alone (different Fermi surfaces for protons and neutrons), pn interactions not essential

Now to non-equilibrium

- Solve full, time-dependent BUU equation with KB term for off-shell transport

- KB term ensures that medium-broadened hadrons assume their free spectral functions when they leave the nucleus

- Method encoded in GiBUU

Institut für Theoretische Physik, JLU Giessen

GiBUU

The Giessen Boltzmann-Uehling-Uhlenbeck Project

- **GiBUU : Theory and Event Generator**
  based on an approx. solution of Kadanoff-Baym equations

- Physics content (and code available): **Phys. Rept. 512 (2012) 1**
  further details and code on: gibuu.hepforge.org

- **GiBUU** describes (within the same unified theory and code)
  - heavy ion reactions, particle production and flow
  - pion and proton induced reactions
  - low and high energy photon and electron induced reactions
  - neutrino induced reactions

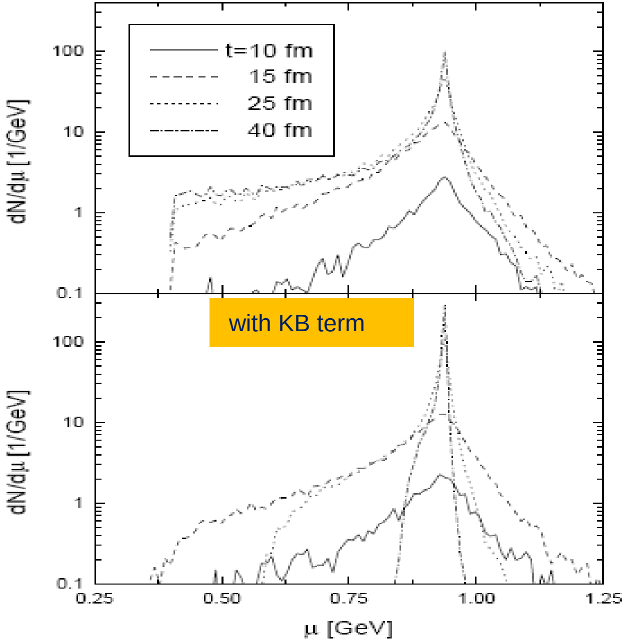
……..using the same physics input! And the same code!

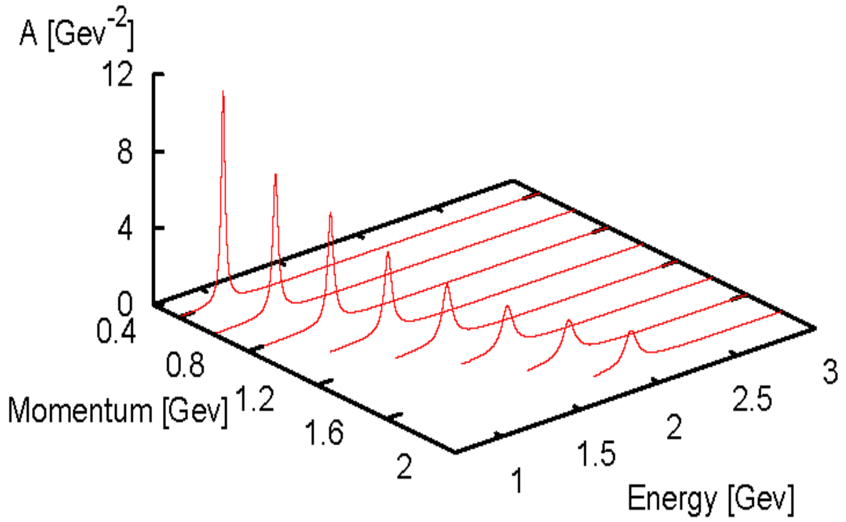
Off-shell time-development

Hole Spectral Function

Spectral Function $\Gamma = \rho\ \sigma\ v$

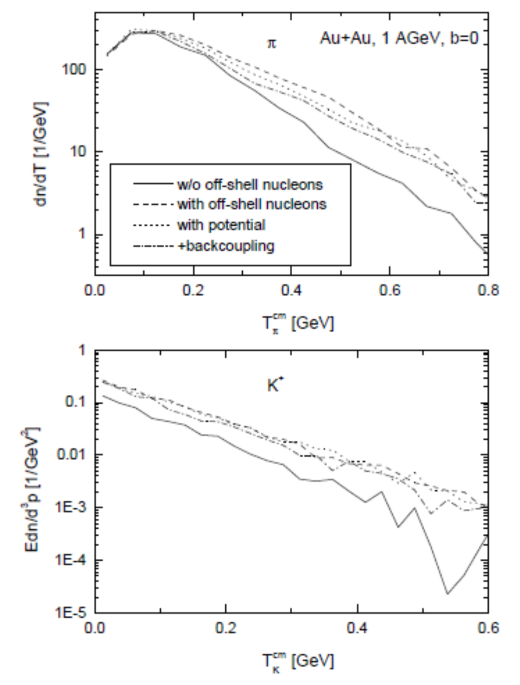
Effenberger





Pb + Pb, 1.5 A GeV

Off-shell effects on particle production



Particle production yields are affected by nucleon spectral functions:

- Particles below theshold (pions) get hardened
- Particles above threshold (kaons) get enhanced for all momenta

Effective field theory (EFT)
NN Potentials

- EFT potentials contain src (contact terms):
  - 24 contact terms (parameters) in N^{3}LO, contribute to low partial waves ($l \leq 2$), in addition regulators

- Propose to fit these these parameters under constraint of empirical spectral functions

SRC in EFT

- EFT contact terms in LO, NLO, N^{3}LO

$$\frac{1}{m_\omega^2+Q^2}\approx\frac{1}{m_\omega^2}\left(1-\frac{Q^2}{m_\omega^2}+\frac{Q^4}{m_\omega^4}-+\cdots\right),$$

Skyrme-like terms

In LO: $$V_{\mathrm{ct}}^{(0)}(\vec{p}\,',\vec{p})=C_S+C_T\,\vec{\sigma}_1\cdot\vec{\sigma}_2\,,$$

Same structure as for LM force

NN EFT and Migdal force

- LO contact Lagrangian and Migdal force identical.
- LO contact terms can be chosen such that they produce realistic spectral functions.

Summary

- SRC in transport theory generate selfconsistently nucleon spectral functions
- High-momentum tails scale with $1/k^4$, independent of any specifics of the interaction. ‚Deeper reason‘: Fermi-Gas and Pauli-principle
- Transport theory can describe off-shell transport
- Contact interactions in EFT NN potentials should be determined with constraints from spectral functions

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