Claudio Ciofi degli Atti

MANY-BODY CALCULATIONS
of
SHORT-RANGE CORRELATIONS

EMMI WORKSHOP

on

Cold Dense Nuclear Matter

October 13 - 16 2015, GSI, Germany

OUTLINE

1. What are SRCs and why we care about them?

2. Ab initio solutions of the non relativistic many-body problem and theoretical predictions of SRC's.

3. The novel picture of nuclei at short range and high momenta: universality of SRCs in configuration space and factorization of the nuclear wave function at high momenta.

4. Factorization and the convolution model of nucleon momentum distributions and spectral functions, the two main quantities which are necessary to study SRCs.

5. A brief comparison between theory and experiments.

6. Few words about "Nuclear Contacts".

1 *WHAT ARE SRCs AND WHY WE CARE ABOUT THEM?*

- Many properties of nuclei measured at low Q^2 and generated by the average and collective motions of point-like nucleons can be successfully described in terms of the nuclear Mean Field (Shell Model).
- Nowadays it is possible to investigate nuclei at high Q^2, probing distances of the order of the nucleon radius ($\simeq 1 fm$), and the following longstanding questions arise:

1. can we get information on in-medium short-range nucleon dynamics which cannot be obtained by free NN scattering? What is the role at short inter-nucleon distances of nucleon, meson, quark and gluon d.o.f.?
2. Is the two-nucleon short-range behavior strongly affected by the surrounding nucleons?

Answering these questions implies the study of SRCs i.e. "SRCs" is a short-hand notation for "in-medium short-range nucleon dynamics".

2 *AB INITIO SOLUTIONS OF THE NUCLEAR MANY-BODY PROBLEM AND THEORETICAL PREDICTIONS OF SRCs*

THE STANDARD MODEL OF NUCLEI

QCD $\Longrightarrow$ Nuclei- non perturbative regime $\Longrightarrow$ too difficult
Many-body systems $\Longrightarrow$ single out the leading effective d.o.f.
Effective d.o.f. in Nuclei $\Longrightarrow$ nucleons and gauge bosons.
Reduction of a field theoretical description to an instantaneous potential description (Schroedinger equation) $\Longrightarrow$ two-body, three-body,........,A-body potentials are generated.

Primakoff,Holstein 1944

$$\text{(m-body potential)} \simeq \left(\frac{v_N}{c}\right)^{\text{(m-2)}} \times \text{(two-body potential)}$$

$$\left[-\frac{\hbar^2}{2\,m_N}\sum_i \hat{\nabla}_i^2 + \sum_{i<j}\hat{v}_2(i,j) + \sum_{i<j<k}\hat{v}_3(i,j,k)\right]\Psi_n = E_n\Psi_n$$

$$\Psi_n \equiv \Psi_n(1\ldots A) \qquad i \equiv \mathbf{x}_i \equiv \{\sigma_i, \tau_i, \mathbf{r}_i\} \qquad \sum_{i=1}^{A}\boldsymbol{r}_i = 0$$

Theoretical framework: Solve *ab initio* the standard model with realistic interactions $\Longrightarrow$ compare with experimental data (energy, form factors, transition matrix elements, etc); if agreement $\Longrightarrow$ OK; if not $\Longrightarrow$ look for new d.o.f.

Modern bare two-nucleon interactions ($\simeq$ *2000 phase shifts*)

$$\hat{v}_2(x_i, x_j) = \sum_{n=1}^{18} v^{(n)}(r_{ij})\hat{\mathcal{O}}_{ij}^{(n)} \qquad r_{ij} \equiv |\mathbf{r}_i - \mathbf{r}_j|$$

$$\mathcal{O}_{ij}^{(1)} = 1, \qquad \mathcal{O}_{ij}^{(2)} = \sigma_i \cdot \sigma_j, \quad \mathcal{O}_{ij}^{(3)} = \tau_i \cdot \tau_j$$

$$\mathcal{O}_{ij}^{(4)} = (\sigma_i \cdot \sigma_j)(\tau_i \cdot \tau_j), \quad \mathcal{O}_{ij}^{(5)} = \hat{S}_{ij}, \quad \mathcal{O}_{ij}^{(6)} = \hat{S}_{ij}\tau_i \cdot \tau_j,$$

$$\hat{S}_{ij} = 3(\hat{r}_{ij} \cdot \sigma_i)(\hat{r}_{ij} \cdot \sigma_j) - \sigma_i \cdot \sigma_j$$

- short-range repulsion (common to many systems)
- intermediate- to long-range tensor character (unique to nuclei)

THE MEAN FIELD APPROXIMATION

$$\sum_{i<j} \hat{v}_2(i,j) + \sum_{i<j<k} \hat{v}_3(i,j,k) \Longrightarrow \sum_i V_i(i).$$

$$\Downarrow$$

$$\left[-\frac{\hbar^2}{2\,m_N}\sum_i \hat{\nabla}_i^2 + \sum_i V(r_i)\right]\Phi_o(1,\ldots,A) = \epsilon_o\,\Phi_o(1,\ldots,A)$$

Mean-field (shell model) wave function

$$\Phi_0(1,2,\ldots,A) = \hat{\mathcal{A}}\prod_i^A \phi_{\alpha_i}(x_i) \equiv \Phi_{0p0h} \quad \alpha_i > \alpha_F, \phi_{\alpha_i} = 0$$

Exact correlated wave function

$$\Psi_0(1,2,\ldots,A) = C_{0p0h}\,\phi_{0p0h} + C_{1p1h}\,\Phi_{1p1h} + C_{2p2h}\,\Phi_{2p2h} + \ldots$$

$$SRC \longrightarrow \sum_{n=1}^{\infty} C_{np\,nh}\Phi_{np\,nh}$$

VARIOUS *ab initio* THEORETICAL METHODS

- Direct solution for few-body systems
- Expansion in complete set of basis functions
- Introduction of correlations into the mean field wave function by proper correlation operators
- Variational Monte Carlo (VMC) with Correlated basis functions (ARGONNE)
- Correlated basis functions and cluster expansion:

$$\Psi_o = \hat{\mathbf{F}} \, \Phi_o$$

$$\hat{\mathbf{F}} = \hat{\mathcal{S}} \prod_{i<j} \hat{f}_{ij} = \hat{\mathcal{S}} \prod_{i<j} \left[\sum_n f^{(n)}(r_{ij}) \, \hat{\mathcal{O}}_{ij}^{(n)} \right]$$

NO FREE PARAMETERS

THE RELEVANT QUANTITY: DENSITY MATRICES

Diagonal one-body density matrix (***1BDM***)(***matter distribution***):

$$\rho_{(1)}(\boldsymbol{r}_1) = \int |\Psi_0(\boldsymbol{r}_1, \boldsymbol{r}_2 \ldots, \boldsymbol{r}_A)|^2 \prod_{i=2}^{A} d\boldsymbol{r}_i$$

Non diagonal (***1BDM***) (***One-body density fluctuations***):

$$\rho_{(1)}(\boldsymbol{r}_1, \boldsymbol{r}_1') = \int \Psi_0^*(\boldsymbol{r}_1, \boldsymbol{r}_2 \ldots, \boldsymbol{r}_A)\, \boldsymbol{r}_i)\Psi_0(\boldsymbol{r}_1', \boldsymbol{r}_2 \ldots, \boldsymbol{r}_A) \prod_{i=2}^{A} d\boldsymbol{r}_i$$

Non diagonal 2-body density matrix (***2BDM***)(two body density fluctuations):

$$\rho_{(2)}(\boldsymbol{r}_1, \boldsymbol{r}_2; \boldsymbol{r}_1', \boldsymbol{r}_2') = \int \Psi_0^*(\boldsymbol{r}_1, \boldsymbol{r}_2 \ldots, \boldsymbol{r}_A)\, \Psi_0(\boldsymbol{r}_1', \boldsymbol{r}_2' \ldots, \boldsymbol{r}_A) \prod_{i=3}^{A} d\boldsymbol{r}_i$$

Diagonal 2BDM:

$$\rho_{(2)}(\boldsymbol{r}_1, \boldsymbol{r}_2) = \int |\Psi_0(\boldsymbol{r}_1, \boldsymbol{r}_2 \ldots, \boldsymbol{r}_A)|^2 \prod_{i=3}^{A} d\boldsymbol{r}_i$$

The relative (rel) and center-of-mass (CM) density matrices

$$\boldsymbol{r} = \boldsymbol{r}_1 - \boldsymbol{r}_2 \qquad \boldsymbol{R} = (\boldsymbol{r}_1 + \boldsymbol{r}_2)/2$$

$$\rho_{(2)}(\boldsymbol{r}, \boldsymbol{R}) = \int |\Psi_0(\boldsymbol{R} + \frac{\boldsymbol{r}}{2}, \boldsymbol{R} - \frac{\boldsymbol{r}}{2}, \boldsymbol{r}_3 \ldots, \boldsymbol{r}_A)|^2 \prod_{i=3}^{A} d\boldsymbol{r}_i$$

$$\rho_{CM}(\boldsymbol{R}) = \int \rho_{(2)}(\boldsymbol{r}, \boldsymbol{R}) d\boldsymbol{r}$$

$$\rho_{rel}(\boldsymbol{r}) = \int \rho_{(2)}(\boldsymbol{r}, \boldsymbol{R}) d\boldsymbol{R}$$

The relative 2BDM has been calculated by different groups within different many-body approaches and realistic *b*are NN interactions.

3 THE NOVEL PICTURE OF NUCLEI AT SHORT RANGE AND HIGH MOMENTA: UNIVERSALITY OF SRCs IN CONFIGURATION SPACE AND FACTORIZATION OF THE NUCLEAR WAVE FUNCTION AT HIGH MOMENTA.

The RELATIVE 2BDM and the CORRELATION HOLE in FEW-NUCLEON SYSTEMS

Schiavilla *et al*, Nucl. Phys. A267 (1987) 267

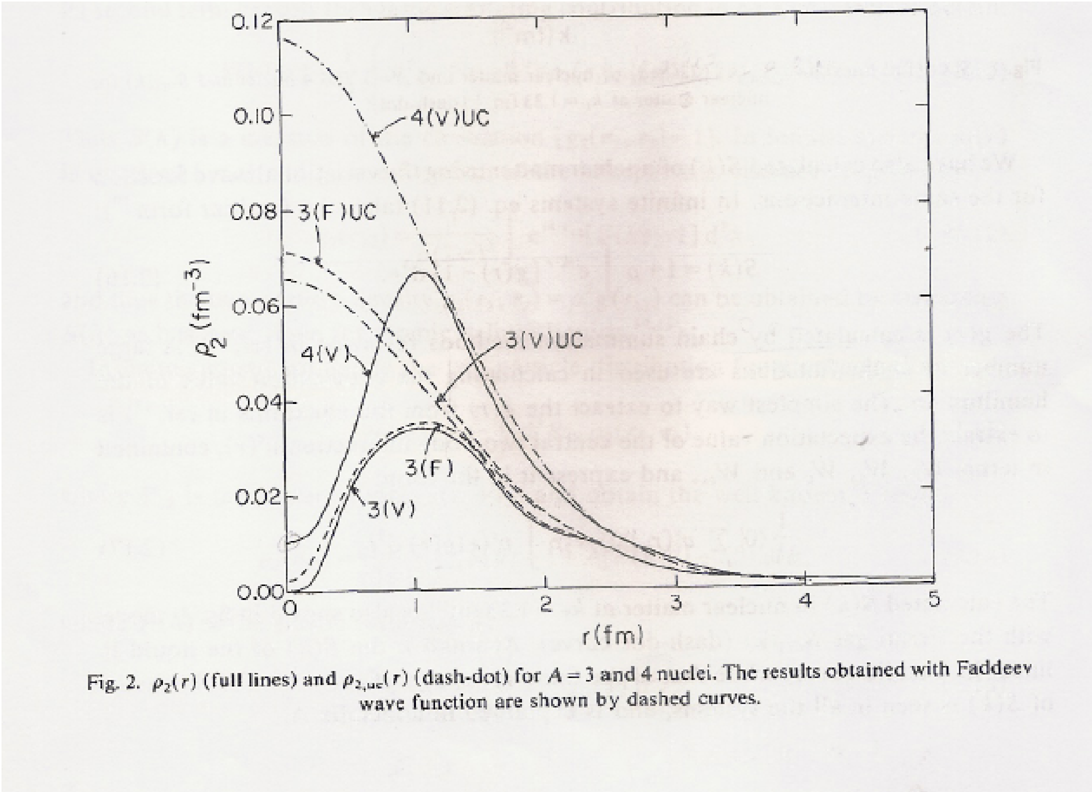
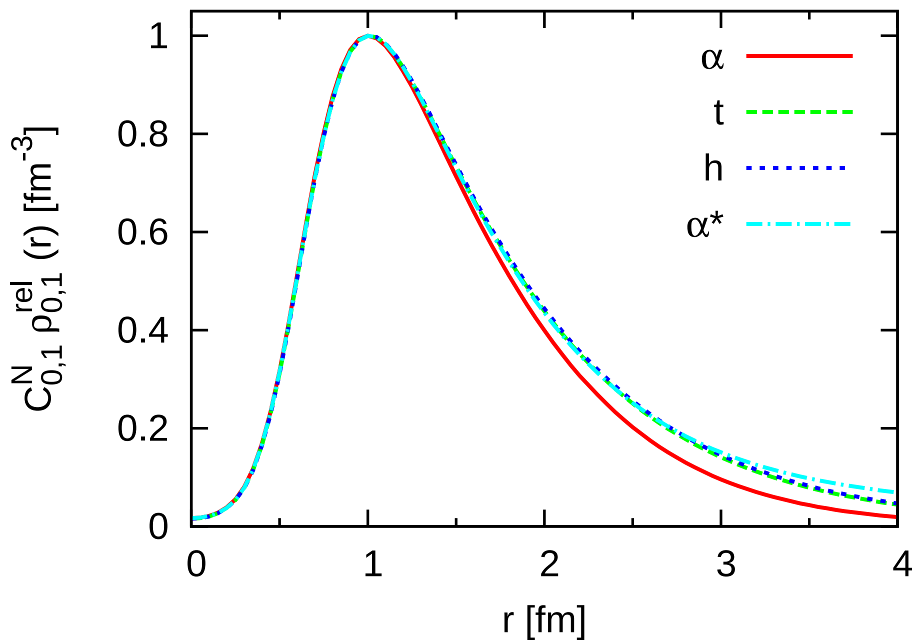


Figure 1: The two-body relative distribution in ^3He and ^4He (After Ref. [?])

The 2BDM $\rho_{(2)}$ in few-nucleon systems in (ST)=(10) and (01) states

Suzuki, Horiuchi, Nucl. Phys. A818, 188 (2009)

Feldmaier, Horiuchi, Neff, Suzuki, Phys. Rev. C84,054013(2011)



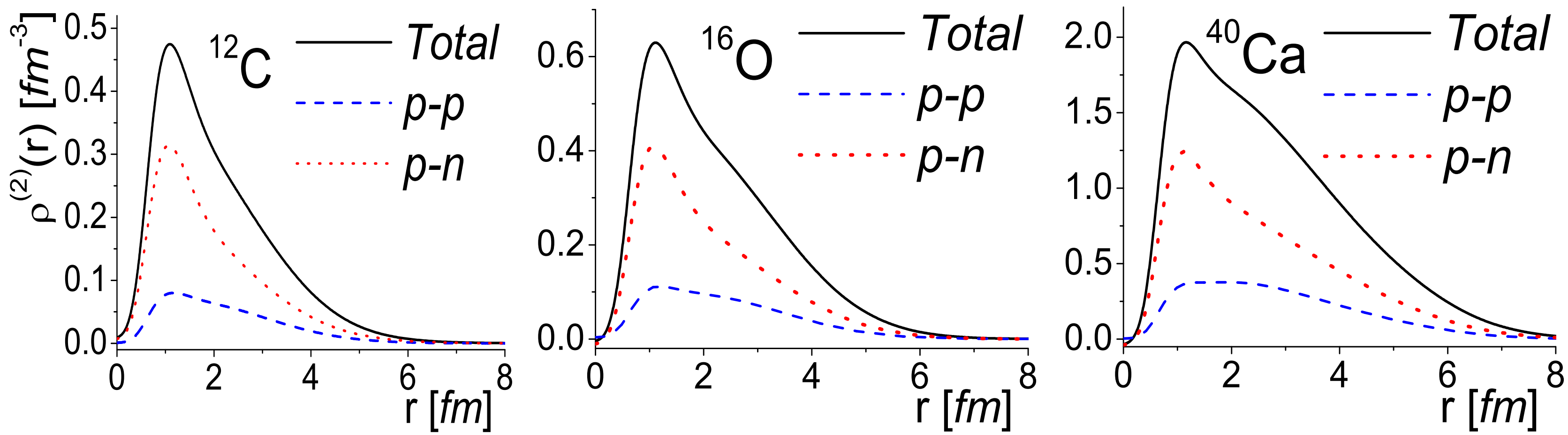
At $r < 1.5 fm$ the 2BDM exhibits A-independence and is similar to the deuteron one

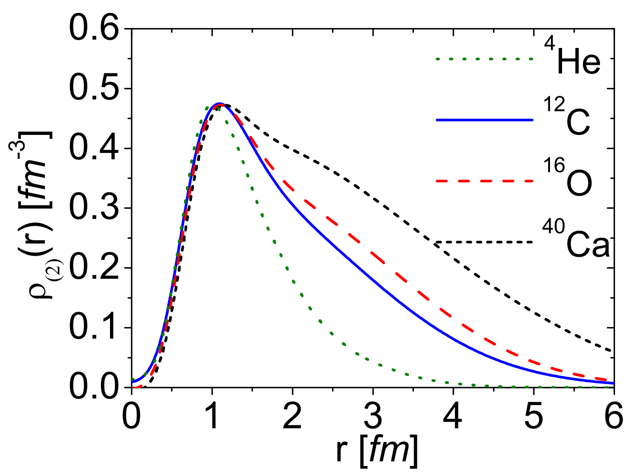
$\Downarrow$

UNIVERSALITY of SRC

The 2BDM $\rho_{(2)}(r)$ in COMPLEX NUCLEI

Alvioli, CdA, Morita, ArXiv: 0709:3989 (2007)





At $r < 1.5 fm$ the 2BDM exhibits A-independence and is similar to the deuteron one in complex nuclei as well

$\Downarrow$

UNIVERSALITY of SRC

The Correlated 2BDM versus the Mean-Field 2BDM

Pieper, Wiringa, Pandharipande, Phys. Rev. C46 1741 (2000)

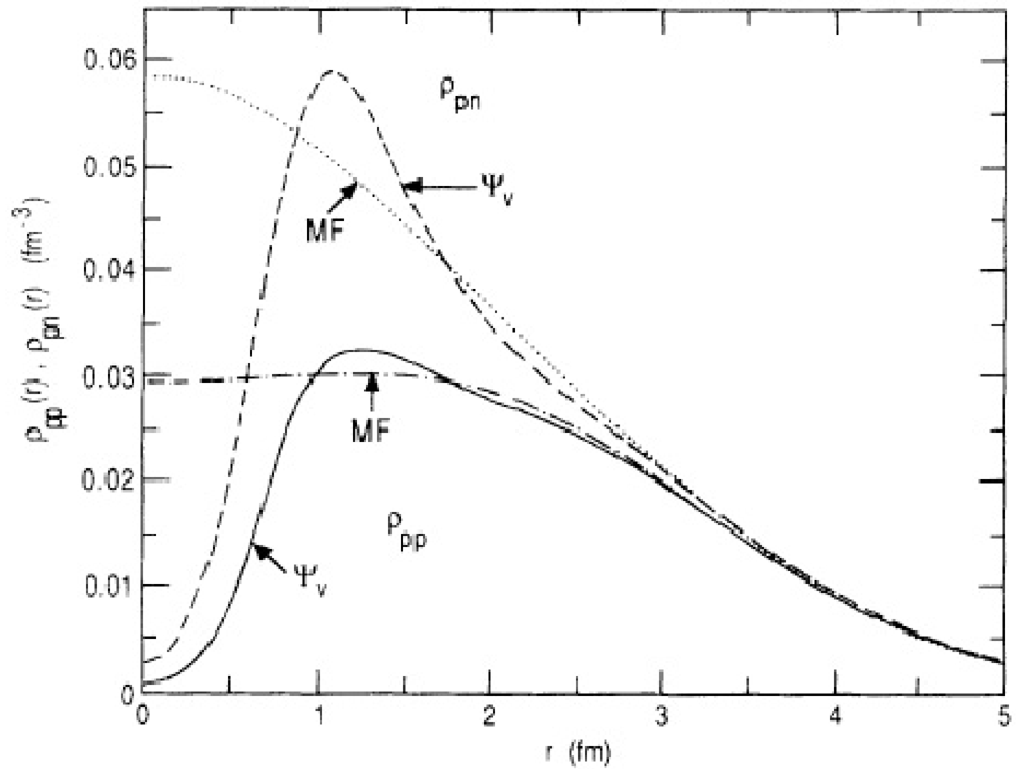


Figure 2: The two body density distribution within realistic and mean-field approaches for ^{16}O

SRCs in configuration space: summary

- SRC are characterized by the ***correlation hole,*** generated by the cooperation of the ***short-range repulsive interaction*** and the ***intermediate-range tensor attraction*** . The basic features of the correlation hole are independent of the mass A $\Longrightarrow$ universality of SRC.

- SRC in configuration space can be defined as follows: ***"They represent the deviation of realistic many-body $\rho_{(2)}(r)$ from the mean-field $\rho_{(2)}(r)$ at $r \leq 1.5 - 2 fm^{-1}$."***

- How can we investigate the existence and the properties of the correlation hole ? To this end we have to shift to momentum space. What do we expect? We expect: (i) an increase of nucleon high momentum components, and (ii) peculiar momentum configurations in the nuclear wave function..

(i) *increase of the high momentum content of the wave function*

Mean-field (shell model) wave function

$$\Phi_0(1,2,\ldots,A) = \hat{\mathcal{A}} \prod_i^A \phi_{\alpha_i}(x_i) \equiv \Phi_{0p0h} \quad \alpha_i > \alpha_F, \phi_{\alpha_i} = 0$$

Correlated wave function

$$\Psi_0(1,2,\ldots,A) = C_{0p0h}\,\phi_{0p0h} + C_{1p1h}\,\Phi_{1p1h} + C_{2p2h}\,\Phi_{2p2h} + \ldots$$

$$SRC \Longrightarrow \sum_{n=1}^{\infty} C_{npnh}\Phi_{npnh}$$

Thus :

SRC populate states (n particle-n hole) with momentum much higher than the Fermi momentum $k_F \simeq 1.4 fm^{-1}$!!!

(ii) *SRC generate peculiar wave function configurations*

Momentum conservation

$$\sum_{1}^{A} \vec{k}_i = 0$$

Consider a nucleon with high momentum $\vec{k}_1$

In a mean-field configuration

$$\vec{k}_1 \simeq -\sum_{2}^{A} \vec{k}_i \quad \vec{k}_i \simeq \frac{\vec{k}_1}{A}$$

In a two-nucleon correlation configuration

$$\vec{k}_1 \simeq -\vec{k}_2 \quad \vec{K}_{A-2} = \sum_{3}^{A} \vec{k}_i \simeq 0 \quad \vec{k}_{rel} \simeq \vec{k}_1 \quad \vec{K}_{CM} = -\vec{K}_{A-2} \simeq 0$$

SRC :HIGH relative and LOW CM momenta of a pair.

Frankfurt, Strikman, Phys. Rep. 1988

THE SPIN-ISOSPIN STRUCTURE OF THE NUCLEAR WAVE FUNCTION.

Pauli Principle: L+S+T-odd

Shell Model (IPM):

$A \leq 4$: $L-$even,(10),(01)
$A > 4$: $L-$even,(10),(01); $L=$odd,(00),(11)

SRCs:

they create states (00) and (11) (L-odd) also in $A \leq 4$ nuclei and change the percentage of (01) in favor of (11) state

The number of NN pairs in various spin-isospin (ST) states Phys. Rev. C87(2013)034603

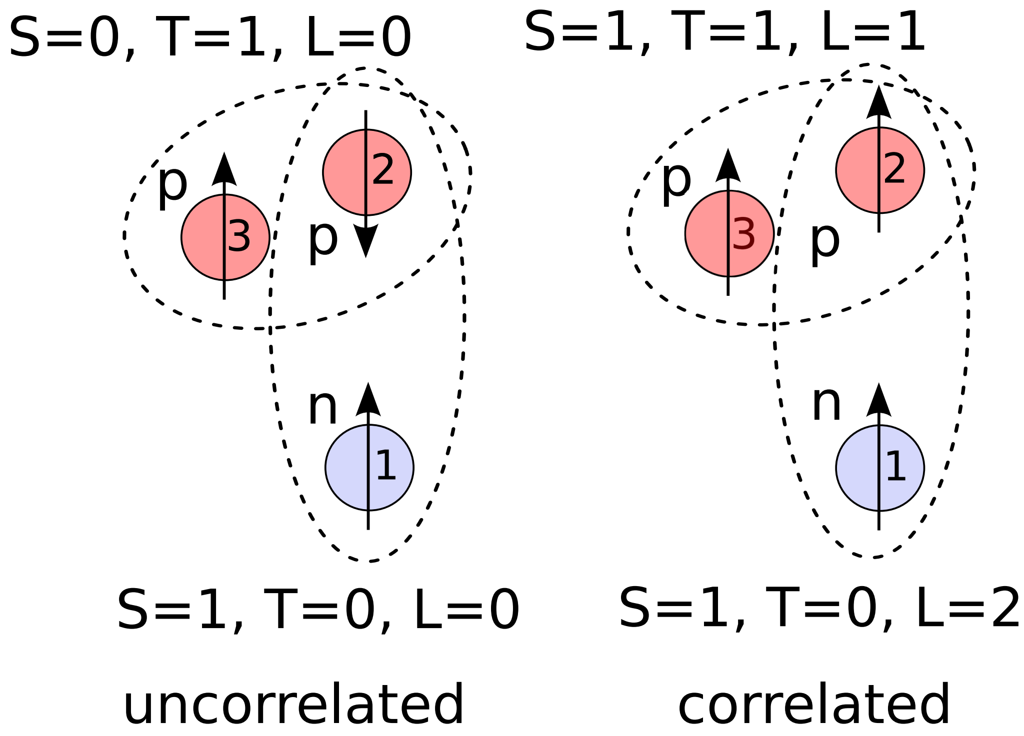
Nucleus		(ST)			
		(10)	(01)	(00)	(11)
^{2}H		1	-	-	-
^{3}He	IPM	1.50	1.50	-	-
	SRC (Present work)	1.488	1.360	0.013	0.139
	SRC (Forest et al, 1996)	1.50	1.350	0.01	0.14
	SRC (Feldmeier et al, 2011)	1.489	1.361	0.011	0.139
^{4}He	IPM	3	3	-	-
	SRC (Present work)	2.99	2.57	0.01	0.43
	SRC (Forestet al,1996)	3.02	2.5	0.01	0.47
	SRC (Feldmeier et al, 2011)	2.992	2.572	0.08	0.428
^{16}O	IPM	30	30	6	54
	SRC (Present work)	29.8	27.5	6.075	56.7
	SRC (Forest et al, 1996)	30.05	28.4	6.05	55.5
^{40}Ca	IPM	165	165	45	405
	SRC (Present work)	165.18	159.39	45.10	410.34

- **NN interaction doesn't practically affect the state (10) but appreciably reduces the state (01) giving rise to a "visible" content of the (11) state; this is due to a three-body mechanism originating from the tensor force.**

THE THREE-BODY MECHANISM

H. Feldemeier, W. Horiuchi, T. Neff, Y. Suzuki

Phys. Rev. C84, 054003 (2011)



IPM: only L=0 (10), (01) states are possible
Correlated particles: tensor interaction in the p-n pair in L=2 can induce a spin flip in the p-p pair with creation of a state L=1, (11) of the pair.
Three-body effect.

ONE-BODY MOMENTUM DISTRIBUTIONS AND SRCs

$$\rho(\boldsymbol{r}_1, \boldsymbol{r}'_1) = \int \Psi_0^*(\boldsymbol{r}_1, \boldsymbol{r}_2 \ldots, \boldsymbol{r}_A)\, \Psi_0(\boldsymbol{r}'_1, \boldsymbol{r}_2 \ldots, \boldsymbol{r}_A) \prod_{i=2}^{A} d\boldsymbol{r}_i$$

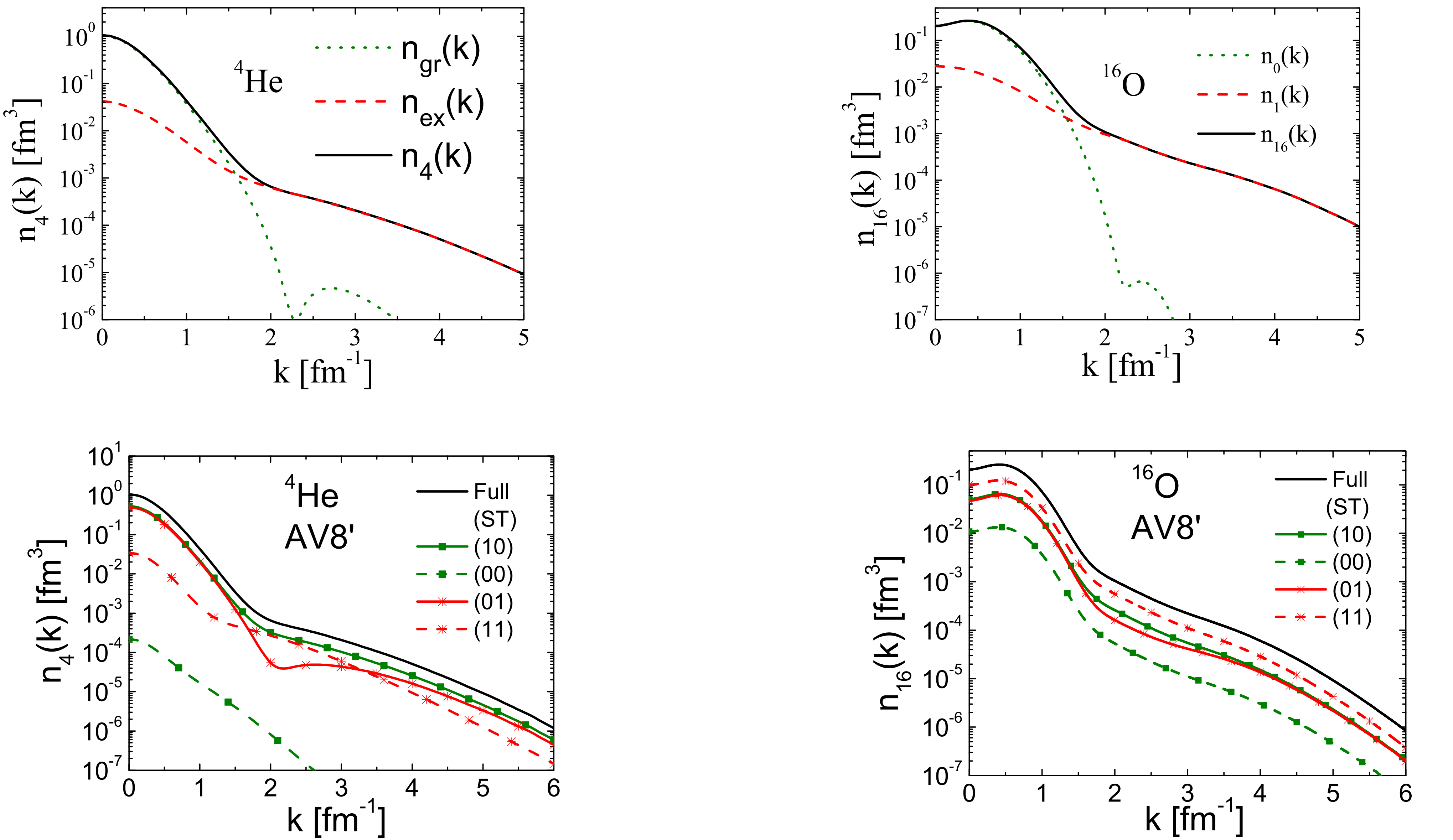
$$n(\boldsymbol{k}) = \int e^{-i\,\boldsymbol{k}\cdot(\boldsymbol{r}_1 - \boldsymbol{r}'_1)} \rho(\boldsymbol{r}_1, \boldsymbol{r}'_1) d\boldsymbol{r}_1 d\boldsymbol{r}'_1$$

$$n_A(\boldsymbol{k}_1) = \sum_{ST} n_A^{(ST)}(\boldsymbol{k}_1) =$$

$$= \int d\boldsymbol{r_1}\, d\boldsymbol{r}'_1 e^{i\,\boldsymbol{k}_1\cdot(\boldsymbol{r}_1 - \boldsymbol{r}'_1)} \sum_{ST} \int d\boldsymbol{r_2} \rho_{ST}^{N_1 N_2}(\boldsymbol{r}_1, \boldsymbol{r}'_1; \boldsymbol{r}_2)$$

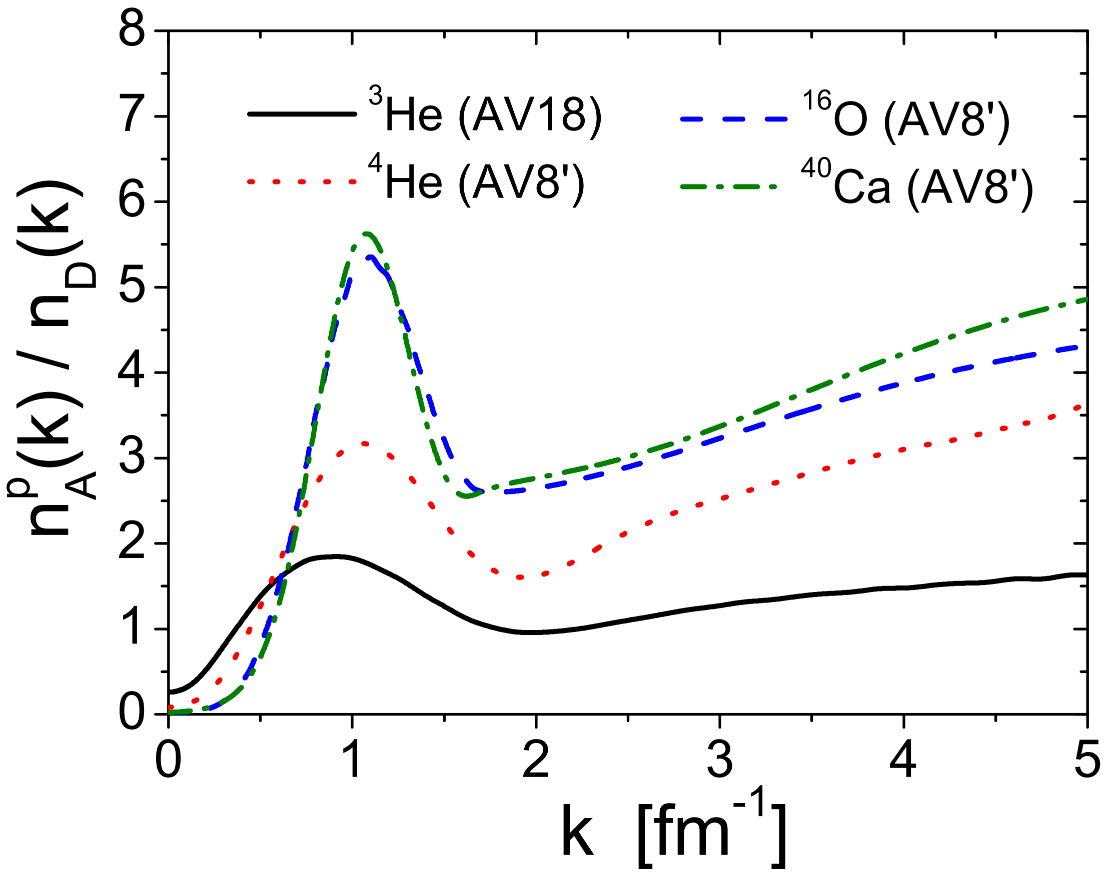
Alvioli, CdA, Kaptari, Mezzetti, Morita, Phys. Rev. C87 (2013) 709 (arXiv:1211.0134v1[nucl-th])

4th order linked cluster expansion AV8' NN interaction (Alvioli's talk)



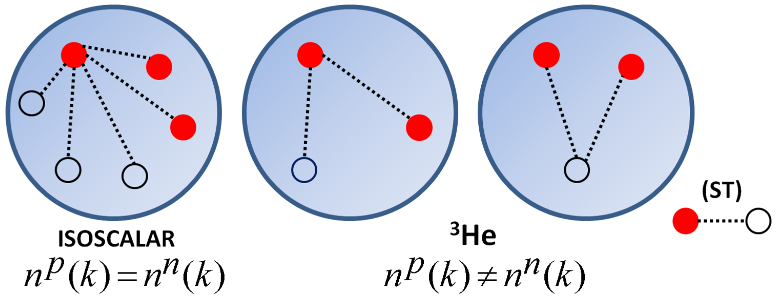
^{4}He
$n_{gr}(k)$
$n_{ex}(k)$
$n_4(k)$
$n_4(k)$ [fm^3]
k [fm^{-1}]
^{16}O
$n_0(k)$
$n_1(k)$
$n_{16}(k)$
$n_{16}(k)$ [fm^3]
k [fm^{-1}]
^{4}He
AV8'
Full
(ST)
(10)
(00)
(01)
(11)
$n_4(k)$ [fm^3]
k [fm^{-1}]
^{16}O
AV8'
Full
(ST)
(10)
(00)
(01)
(11)
$n_{16}(k)$ [fm^3]
k [fm^{-1}]

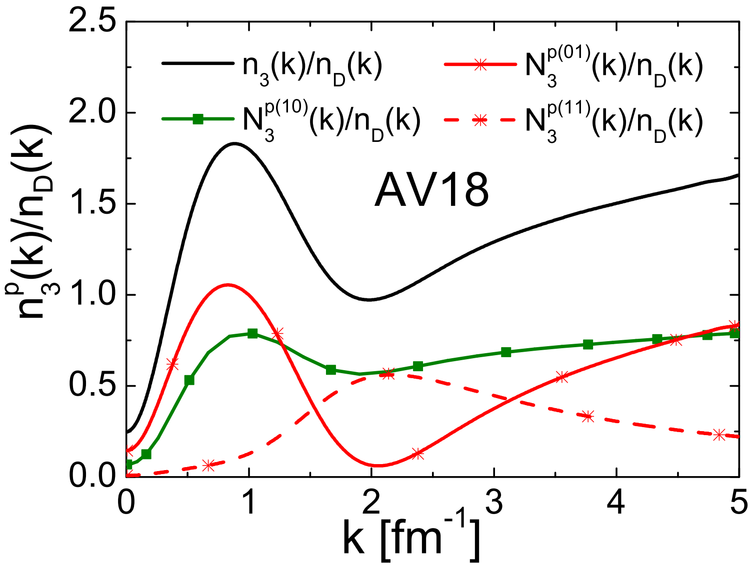
The ratio $n_A(k)/n_D(k)$ according to many-body calculations

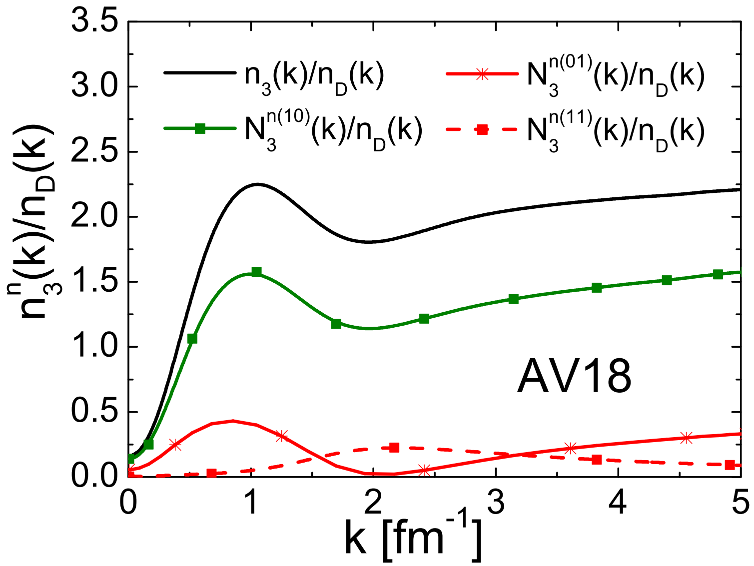


The increase of the ratio with k originates from the spin-isospin dependence of the momentum distributions and from the CM motion of the pair in the nucleus.

Neutron rich nuclei: the p and n momentum distributions in ^{3}He







A proton is correlated with one p-n and one p-p pair; a neutron with two n-p pair → Tensor dominance in neutron (proton) distributions in ^3He (^3H) and in neutron-rich nuclei.

TWO-BODY MOMENTUM DISTRIBUTIONS

$$\boldsymbol{k}_{rel} \equiv \boldsymbol{k} = \frac{1}{2}(\boldsymbol{k}_1 - \boldsymbol{k}_2) \qquad \boldsymbol{K}_{CM} \equiv \boldsymbol{K} = \boldsymbol{k}_1 + \boldsymbol{k}_2$$

1. $n(\boldsymbol{k}_1, \boldsymbol{k}_2) = n(\boldsymbol{k}_{rel}, \boldsymbol{K}_{CM}) = n(k_{rel}, K_{CM}, \theta) =$

$$= \frac{1}{(2\pi)^6} \int d\boldsymbol{r} d\boldsymbol{r}' d\boldsymbol{R} d\boldsymbol{R}' \, e^{-i\,\boldsymbol{K}\cdot(\boldsymbol{R}-\boldsymbol{R}')} \, e^{-i\,\boldsymbol{k}\cdot(\boldsymbol{r}-\boldsymbol{r}')} \rho^{(2)}(\boldsymbol{r}, \boldsymbol{r}'; \boldsymbol{R}, \boldsymbol{R}')$$

2. $n(k_{rel}, K_{CM} = 0)$

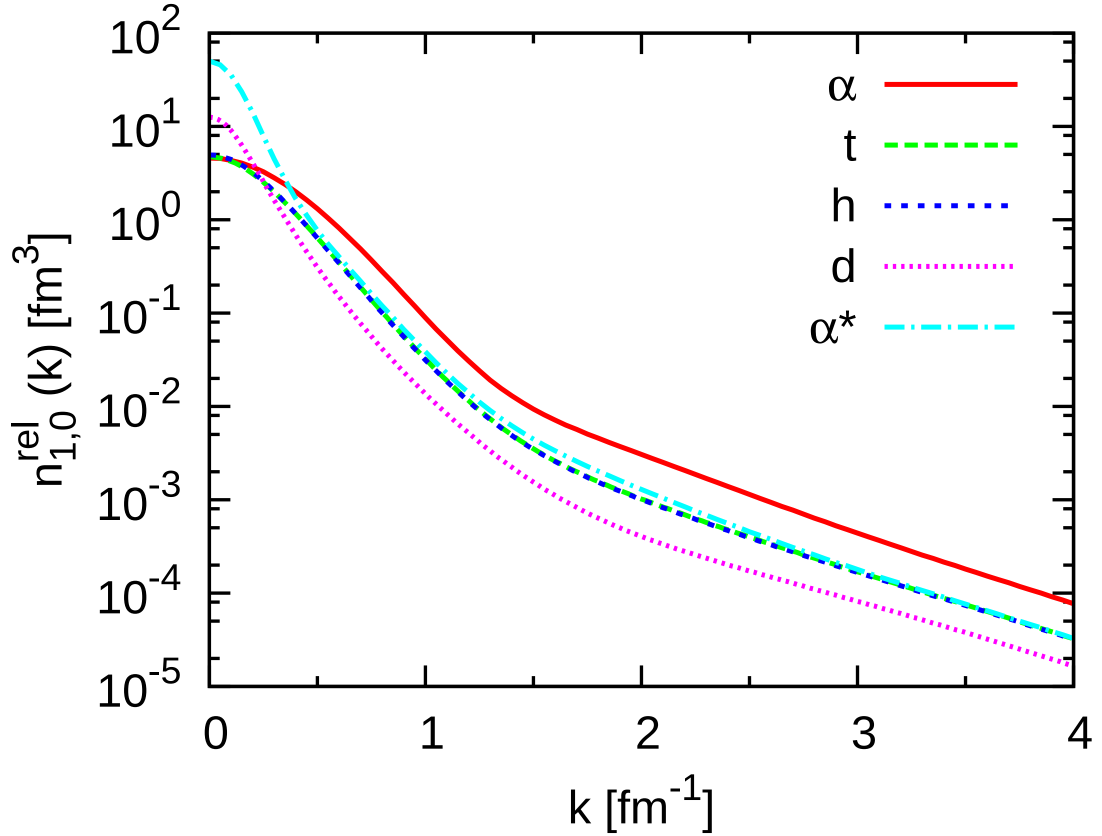
$$K_{CM} = 0 \quad \Longrightarrow \quad \boldsymbol{k}_2 = -\boldsymbol{k}_1,$$

back-to-back nucleons, like in the deuteron

3. $n_{rel}(k) = \frac{1}{(2\pi)^3} \int n(\boldsymbol{k}, \boldsymbol{K}) \, d\boldsymbol{K}$ **4.** $n_{CM}(K) = \frac{1}{(2\pi)^3} \int n(\boldsymbol{k}, \boldsymbol{K}) \, d\boldsymbol{k}$

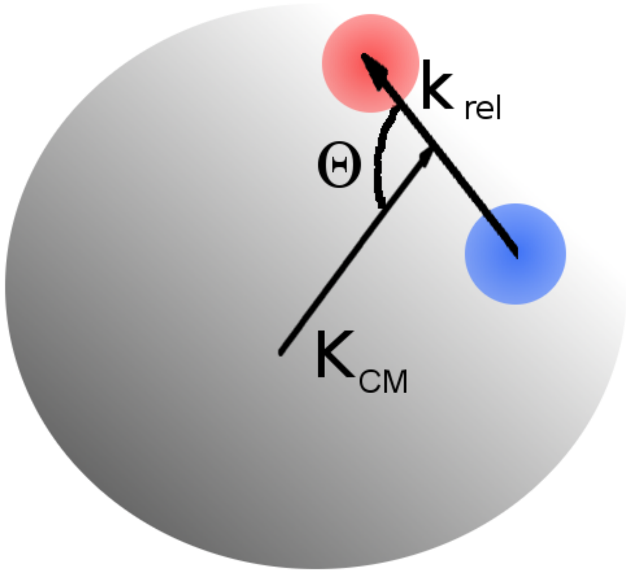
SPIN-ISOSPIN DEPENDENCE of $n_{rel}(k_{rel})$ in FEW-NUCLEON SYSTEMS

H. Feldmaier, W. Horiuchi, T. Neff, Y. Suzuki, Phys. Rev. (2011)



UNIVERSALITY: $n^A_{rel}(k_{rel}) \simeq C_A n_D(k)$ (in (10) state!!)

THE 3D PICTURE OF $n(\boldsymbol{k}_{rel}, \boldsymbol{K}_{CM}) = n(k_{rel}, K_{CM}, \Theta)$

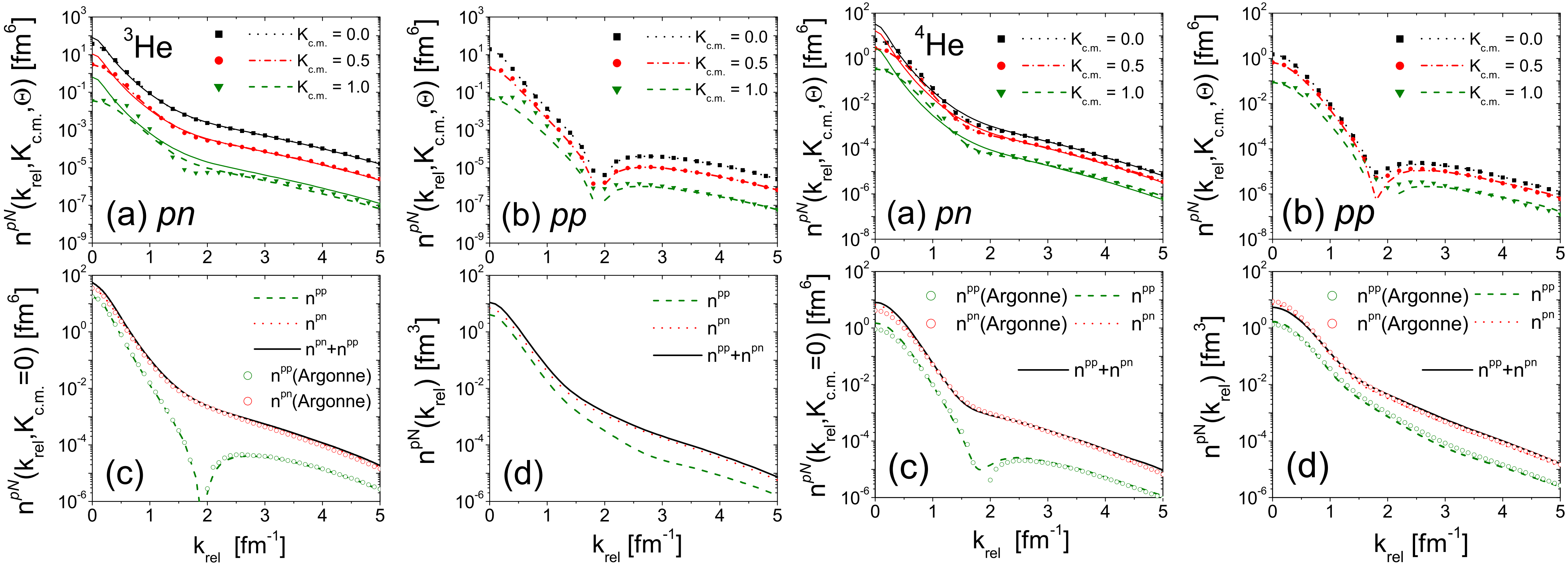


! VERY IMPORTANT !

- If $n(k_{rel}, K_{CM}, \Theta)$ is θ independent, it means that $n(\boldsymbol{k}_{rel}, \boldsymbol{K}_{CM}) = n(k_{rel})\, n(K_{CM})$ i.e. the relative and CM motions factorize.

FEW-NUCLEON SYSTEMS

$n(k_{rel}, K_{CM}, \theta)$ symbols-$\Theta = 90^o$, dashes-$\Theta = 180^o$, full-^2H.



Alvioli, CdA, Kaptari, Mezzetti, Morita, Scopetta, Phys. Rev. C85(2012)021001

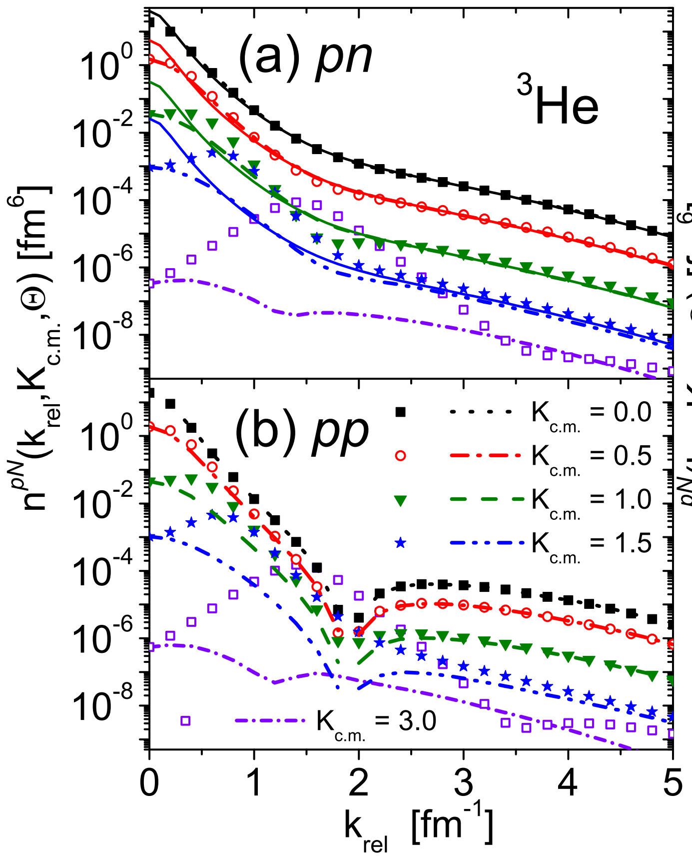
Alvioli, CdA, Morita PRELIMINARY!

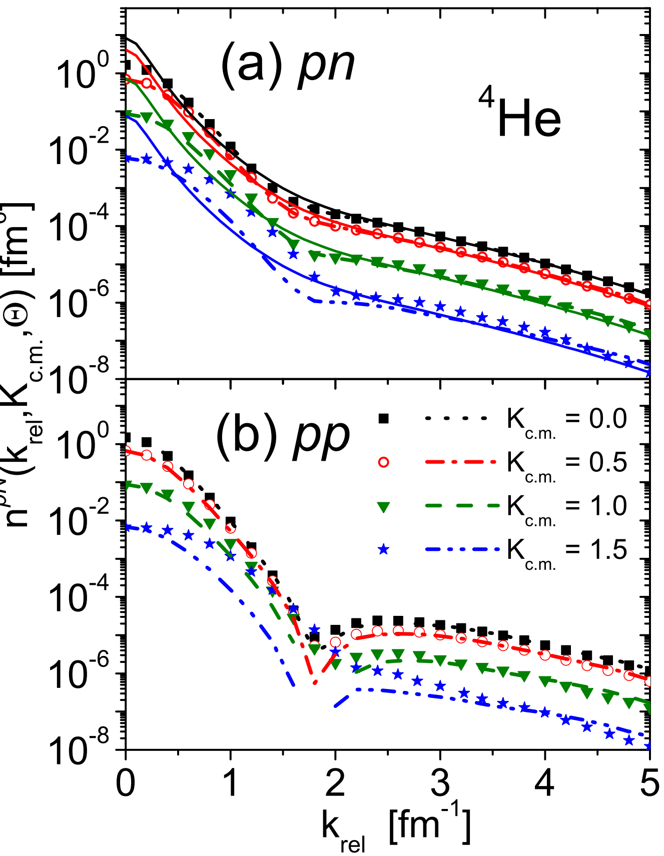
Full lines in pn represent $C_A^{pn} n_D(k_{rel}) n_{c.m.}(K_{c.m.})$

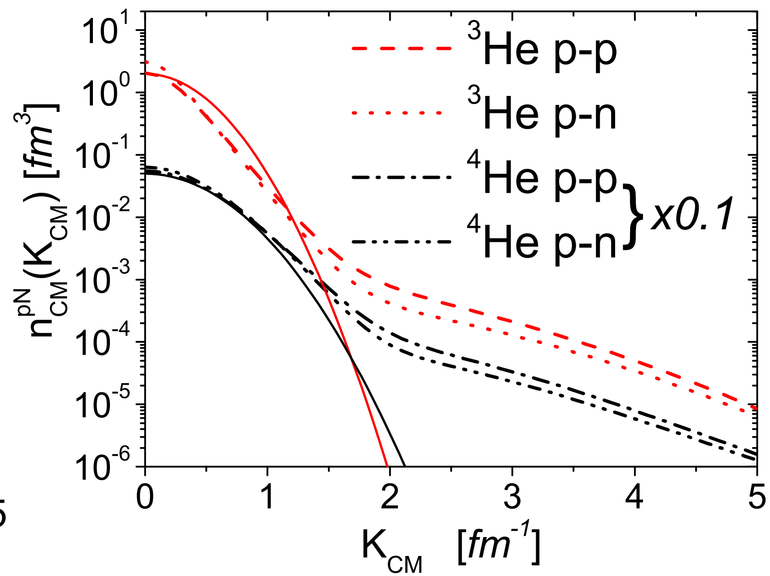
$\Downarrow$

FACTORIZATION at LARGE k_{rel}- SMALL $K_{c.m.}$!!!

$n^{pn}(k_{rel}) >> n^{pp}(k_{rel})$ Acceptable agreement with VMC





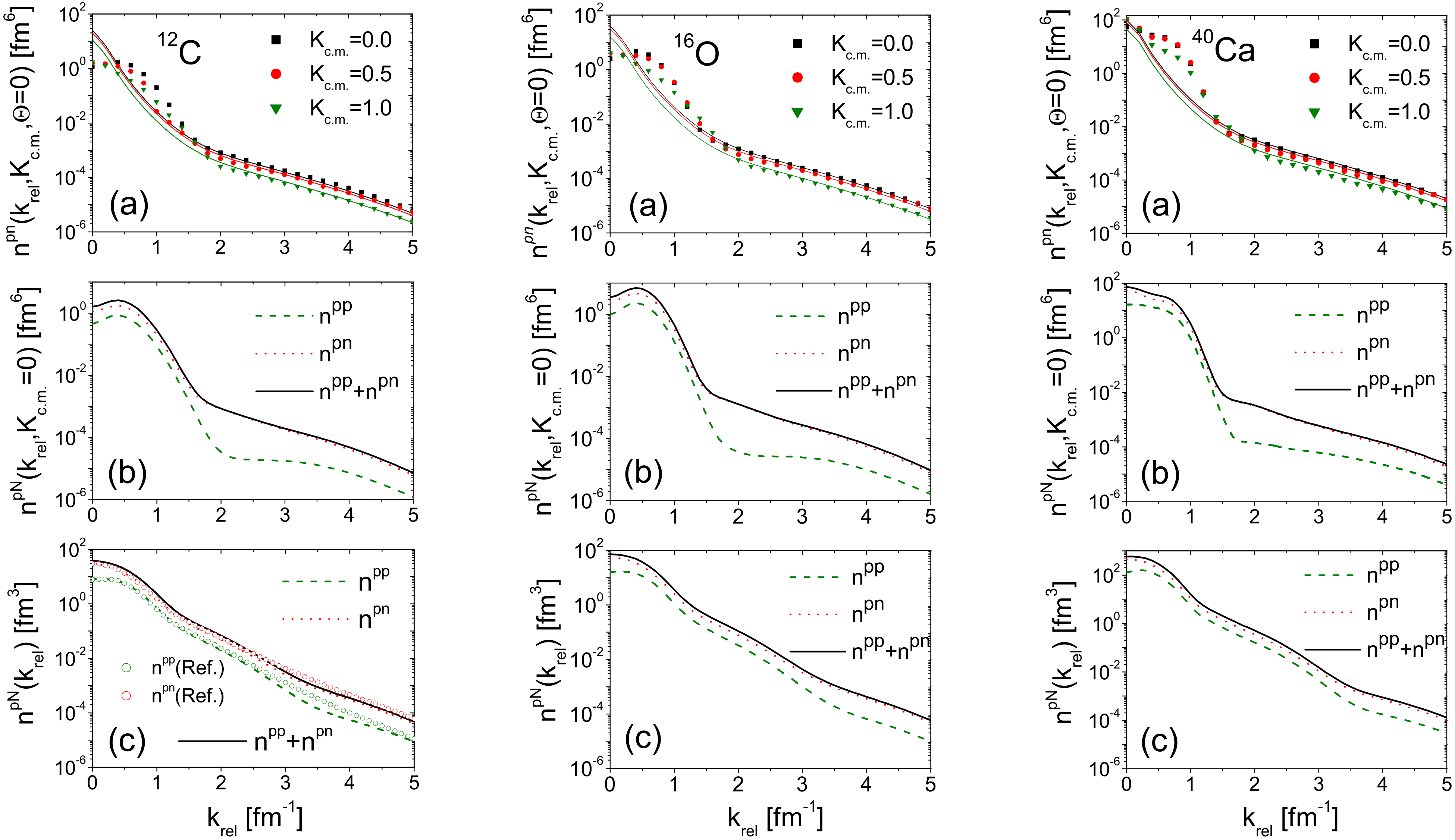


at large values of k_{rel} and small values of K_{CM} we have :

$$n^{pn}(\boldsymbol{k}_{rel}, \boldsymbol{K}_{CM}) \Longrightarrow n^{pn}(k_{rel}, K_{CM}) \simeq C_A^{pn} n^D(k_{rel}) n_{CM}(K_{CM})$$

Factorization will be shown to be a general property of many-body wave functions at high momenta

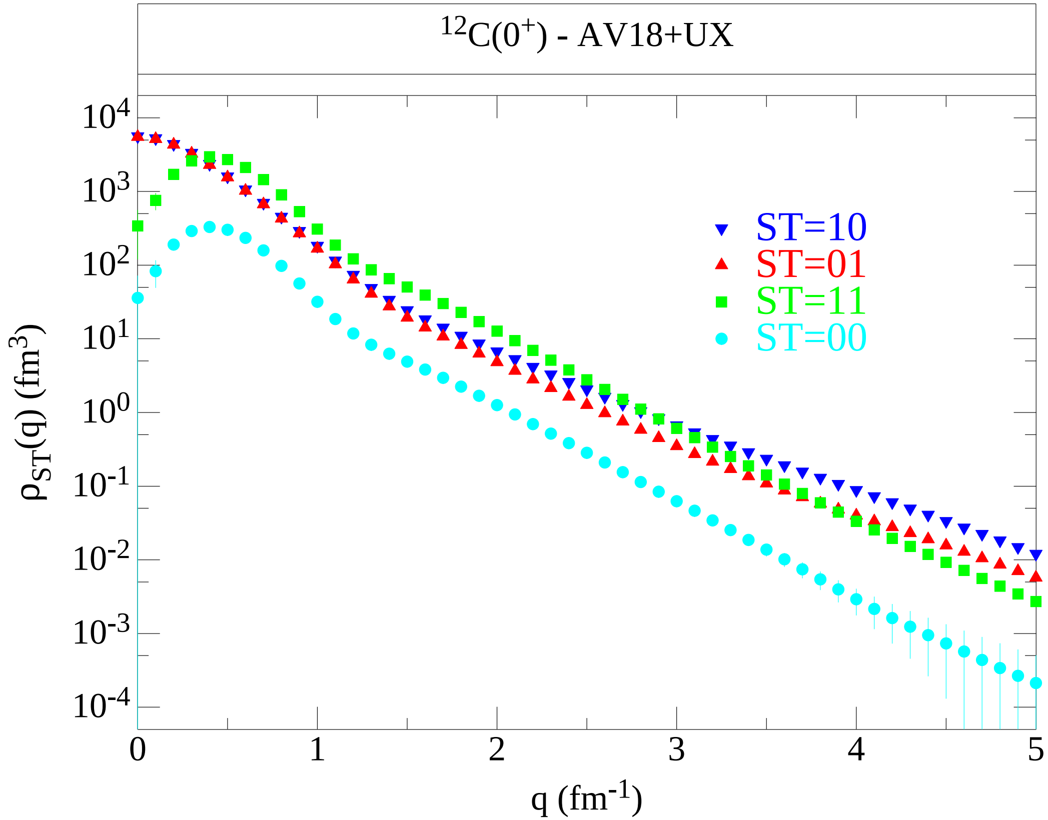
COMPLEX NUCLEI



Alvioli,CdA, Morita, PRELIMINARY!

THE SPIN-ISOSPIN STRUCTURE OF ^{12}C FROM ARGONNE

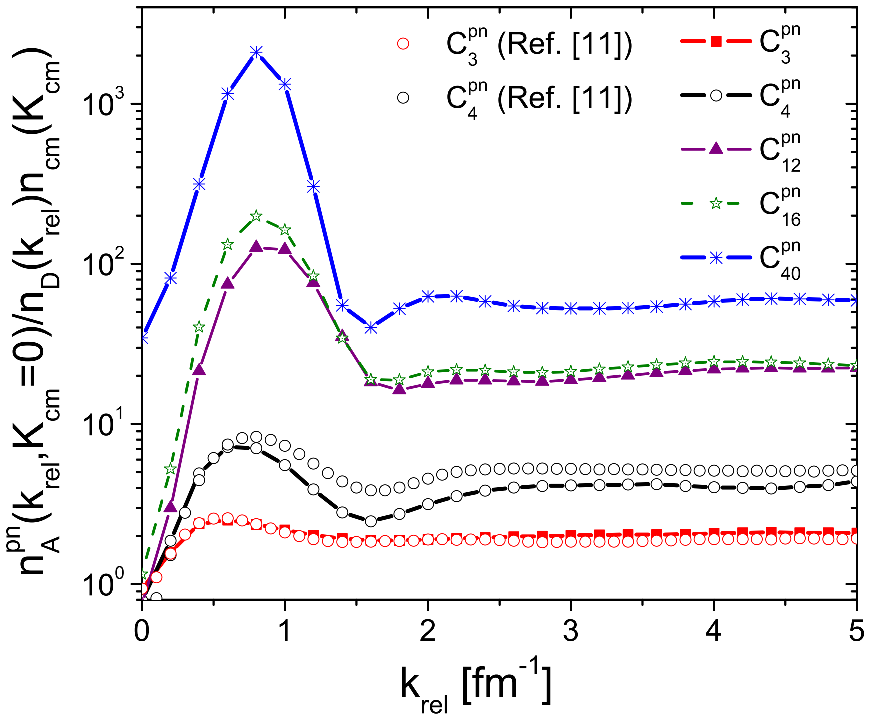
Wiringa, Schiavilla, Pieper, Carlson, Phys. Rev. C89(2014)



As in the case of the one-nucleon momentum distribution the state $(ST) = (11)$ (ODD L) plays a relevant role in the region $0.5 < q \equiv k_{rel} < 3.5\, fm^{-1}$

The value of C_A^{pn} in $n^{pn}(k_{rel}, K_{CM}) \simeq C_A^{pn} n^D(k_{rel}) n_{CM}(K_{CM})$:

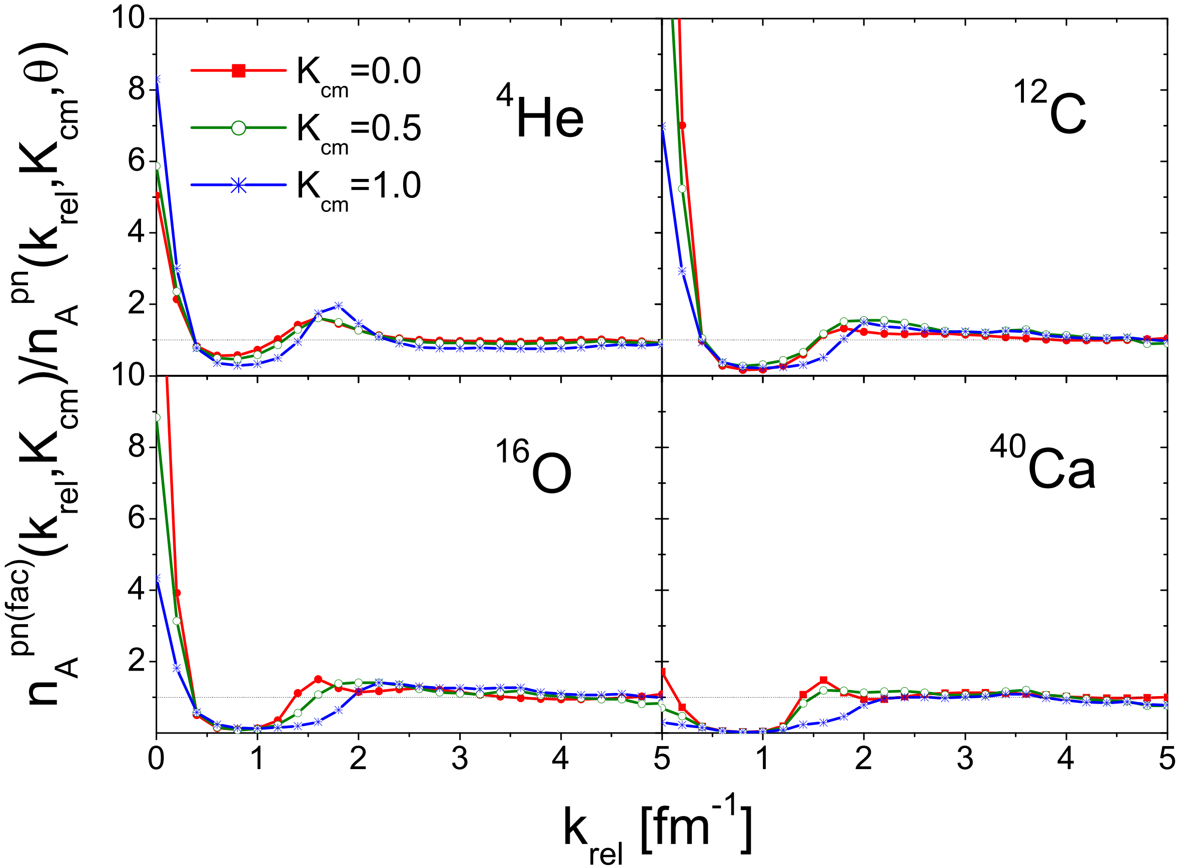
$$k_{rel} >> k_{rel}^{-} \simeq 2\, fm^{-1} \quad \Rightarrow \quad \frac{n_A^{pn}(k_{rel}, K_{c.m.}=0)}{n_{c.m.}^{pn}(K_{c.m.}=0) n_D(k_{rel})} \Rightarrow Const \equiv C_A^{pn}$$



Alvioli,CdA, Morita, PRELIMINARY!

Moreover, if factorization holds one should have:

$$k_{rel} >> k^{-}_{rel} \simeq 2\, fm^{-1} \qquad R^{pn}_{fact/exact} \equiv \frac{C^{pn}_{A} n_D(k_{rel}) n_{c.m.}(K_{c.m.})}{n^{pn}_{A}(k_{rel}, K_{cm}, \theta)} \Rightarrow 1$$



Alvioli,CdA, Morita, PRELIMINARY!

THE MICROSCOPIC MEANING OF C_A^{pn}

Number of back-to-back (BB) SRC pn pairs in the Deuteron:

$$N_D^{pn(BB)}(k_{rel}^- = 1.5) = 4\pi \int_{k_{rel}^-}^{\infty} n_D(k)\, k^2\, d\,k \simeq 0.04(AV18NN)$$

% Probability of BB SRC pn pairs in the Deuteron:

$$\frac{N_D^{pn(BB)}}{N_D^{pn}} \simeq 4$$

Number of BB SRC pn pairs in Nucleus A:

$$N_A^{pn(BB)}(k_{rel}^- = 1.5) = 4\pi \int_{k_{rel}^-}^{\infty} k_{rel}^2\, d\,k_{rel} \frac{n_A^{pn}(k_{\rm rel}, K_{c.m.} = 0)}{n_{c.m.}^{pn}(0)} \simeq C_A^{pn} N_D^{BB}$$

% Probability of BB SRC pn pairs in Nucleus A

$$\mathcal{P}_{BB/ALL}^{SRCpn} = \frac{C_A^{pn} N_D^{BB}}{N_{ALL}^{SRCpn}} \quad N_{ALL}^{SRCpn} = 4\pi \int_{k_{rel}^-=1.5}^{\infty} n_A^{pn}(k_{rel})\, k_{rel}^2\, d\,k_{rel}$$

THE NUMBER OF SHORT-RANGE BACK-to-BACK CORRELATED pn PAIRS vs THE TOTAL NUMBER OF CORRELATED pn PAIRS

Number of BB SRC pn pairs $N_A^{pn(BB)} = C_A^{pn} N_D^{pn}$

	^{2}H	^{3}He	^{4}He	^{12}C	^{16}O	^{40}Ca
	$N_2^{pn(BB)} = 0.04$	$N_3^{pn(BB)} = 0.08$	$N_4^{pn(BB)} = 0.16$	$N_{12}^{pn(BB)} = 0.6$	$N_{16}^{pn(BB)} = 0.8$	$N_{40}^{pn(BB)} = 2$

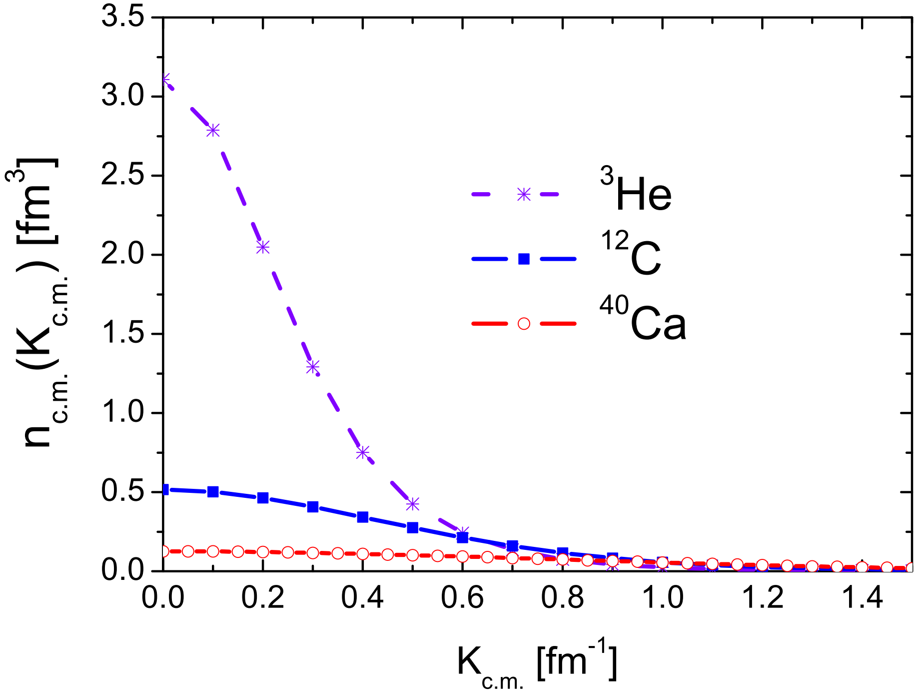
Probability of of (BB) SRC pn pairs with respect to ALL pn pairs

	^{2}H			^{3}He			^{4}He		
	N_{pn}	N_{pn}^{SRC}	$\mathcal{P}_{BB/ALL}^{SRC}(\%)$	N_{pn}	N_{pn}^{SRC}	$\mathcal{P}_{BB/ALL}^{SRC}(\%)$	N_{pn}	N_{pn}^{SRC}	$\mathcal{P}_{BB/ALL}^{SRC}(\%)$
pn($ALLSRC$)	1	0.04	100	2	0.093	86	4	0.243	66
pn($BBSRC$)		0.04			0.080			0.160	

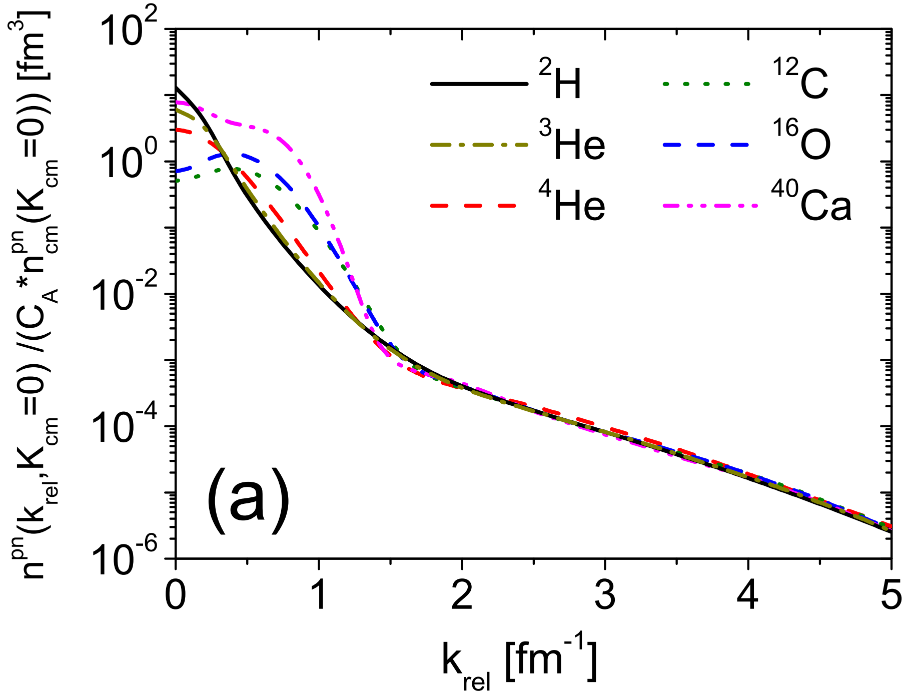
	^{12}C			^{16}O			^{40}Ca		
	N_{pn}	N_{pn}^{SRC}	$\mathcal{P}_{BB/ALL}^{SRC}(\%)$	N_{pn}	N_{pn}^{SRC}	$\mathcal{P}_{BB/ALL}^{SRC}(\%)$	N_{pn}	N_{pn}^{SRC}	$\mathcal{P}_{BB/ALL}^{SRC}(\%)$
pn($ALLSRC$)	36	2.28	27	64	4.6	17	400	24.08	8.5
pn($BBSRC$)		0.6			0.8			2	

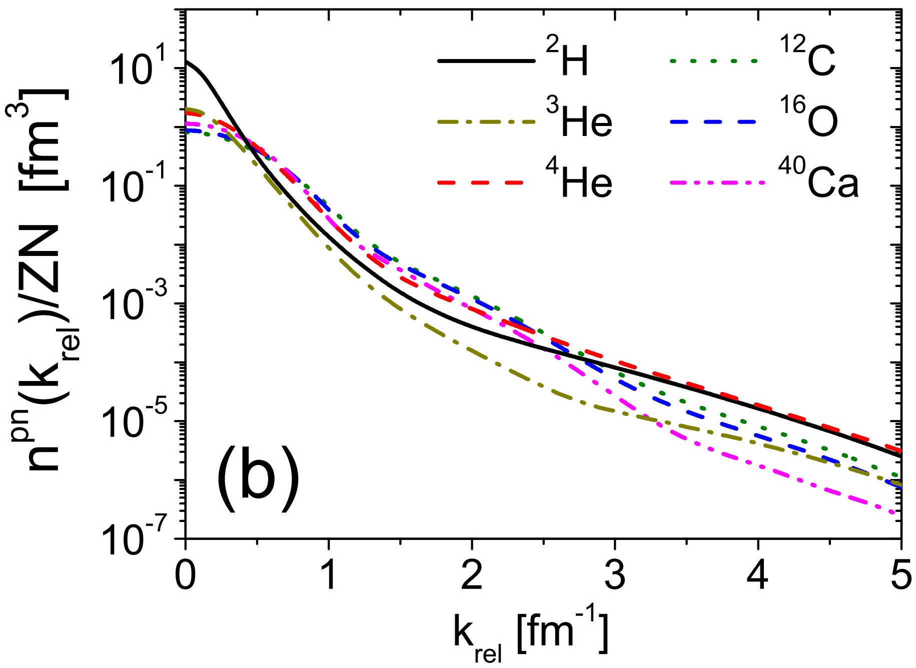
Alvioli,CdA, Morita, PRELIMINARY!

THE MOMENTUM DISTRIBUTION OF THE C.M. MOTION ON A LINEAR SCALE



In the deuteron $n_{c.m.}(K_{c.m.}) = \delta(\mathbf{K}_{c.m.})$ whereas for complex nuclei $n_{c.m.}(K_{c.m.})$ has a finite width increasing with A, therefore the number of pairs with $K_{c.m.} \neq 0$ increases with A and, correspondingly, the number of back-to-back pairs decreases.





- $n_A^{pn}(k_{rel}, K_{c.m.} = 0)/n_{c.m.}(0) \propto n_D(k_{rel})$ when $k_{rel} > 2\,fm^{-1}$.
- $n_A^{pn}(k_{rel}) \propto n_D(k_{rel})$ when $k_{rel} > 4\,fm^{-1}$.

4 *FACTORIZATION AND THE NUCLEON SPECTRAL FUNCTION EMBODYING SRCs*

Momentum conservation in a nucleus A:

$$\mathbf{k_1} + \mathbf{k_2} - \mathbf{K_{CM}} = \mathbf{0} \ \ (\mathbf{K_{CM}} = -\mathbf{K_{A-2}}),\ \mathbf{k_{rel}} = (\mathbf{k_1} - \mathbf{k_2})/\mathbf{2}, \quad \mathbf{k_2} = -\mathbf{k_1} + \mathbf{K_{CM}}$$

We demonstrated that in the region $\mathrm{k_{rel}} \geq \mathrm{k^{-}_{rel}}(\mathrm{KCM})$ factorization occurs:

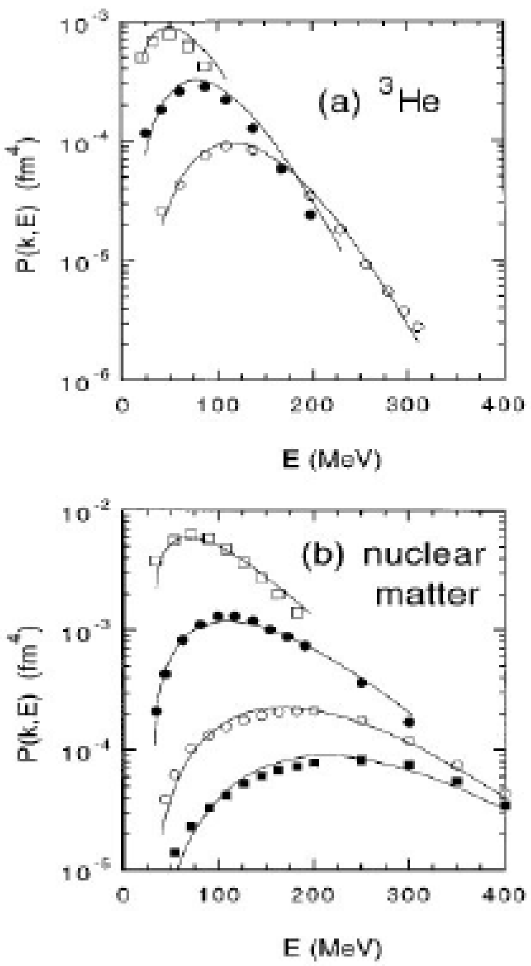
$$n_{pn}(k_{rel}, K_{CM}) \simeq n_D(k_{rel}) n_{CM}(K_{CM}) = n_D(|\boldsymbol{k}_1 - \frac{\boldsymbol{K}_{CM}}{2}|) n_{CM}(K_{CM})$$

which means

$$\begin{aligned} n^N(k_1) &\simeq \int n_D(|\boldsymbol{k}_1 - \frac{\boldsymbol{K}_{CM}}{2}|) n^N_{CM}(K_{CM})\, d\,\boldsymbol{K}_{CM} = \\ &= \int P^N(k_1, E^*_{A-1})\, d\,E^*_{A-1} \end{aligned}$$

where $P^N(k_1, E^*_{A-1})$ is the NUCLEON SPECTRAL FUNCTION

$$\begin{aligned} P^N(k_1, E^*_{A-1}) = & \int n_D(|\boldsymbol{k}_1 - \frac{\boldsymbol{K}^N_{CM}}{2}|) n^N_{CM}(K_{CM}) d\,\boldsymbol{K}_{CM} \times \\ & \times \delta\left(E^*_{A-1} - \frac{A-2}{2m_N(A-1)}\left[\boldsymbol{k}_1 - \frac{A-1}{A-2}\boldsymbol{K}_{CM}\right]^2\right) \end{aligned}$$



□ $k=1.5\,fm^{-1}$ ● $k=2.2\,fm^{-1}$
○ $k=3.0\,fm^{-1}$ ■ $k=3.5\,fm^{-1}$

Chiara Benedetta Mezzetti
Seattle, 05/11/2009

Points: numerical calculation of the spectral functions of ^{3}He (Ciofi degli Atti, Pace, Salmè, PRC 21 (1980)805) **and NM** (Benhar, Fabrocini, Fantoni, Nucl. Phys. A550(1992)201)

Curves: 2N correlation model

$$P_1^A(k,E) = \int d^3k_{cm}\, n_{rel}^A(|\vec{k} - \vec{k}_{cm}/2|)\, n_{cm}^A(|\vec{k}_{cm}|)$$

$$\delta\left[E - E_{thr}^{(2)} - \frac{(A-2)}{2M(A-1)}\cdot\left(\vec{k} - \frac{(A-1)\vec{k}_{cm}}{(A-2)}\right)^2\right]$$

Recently (Massimiliano's talk)

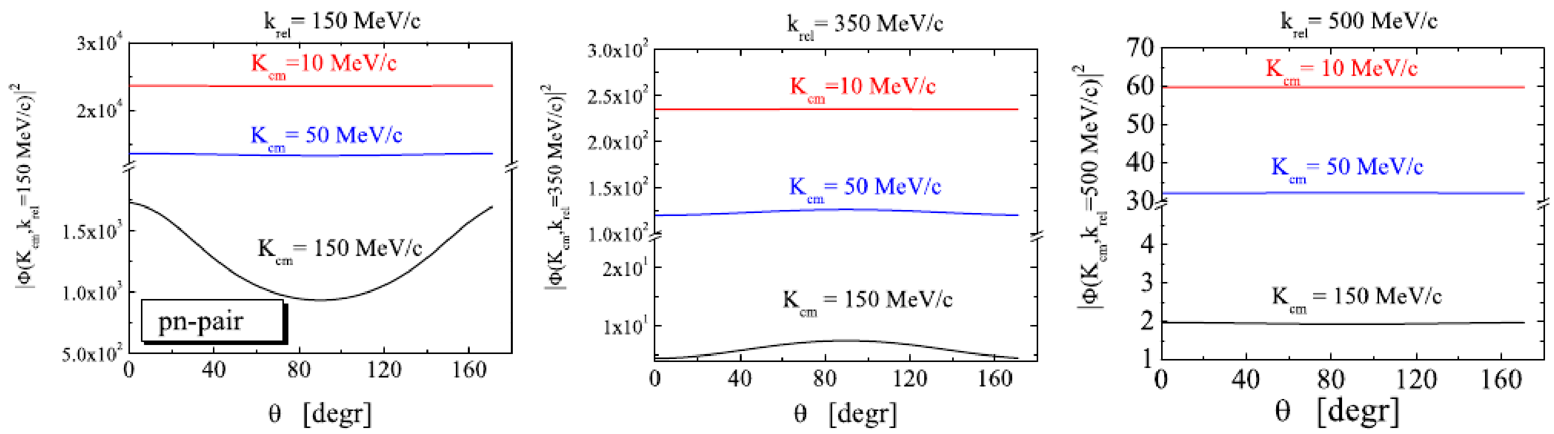
$$n_{cm}^A(k_{cm}) \qquad n_{rel}^A(k_{rel})$$

have been calculated by many-body approach
→ no free parameters!!

CdA, Frankfurt, Simula, Strikman, Phys. Rev. C41 (1990);
CdA, Simula Phys. Rev. C53(1996).

^{3}He WF factorization: the spectral function

$|\Phi_{\mathcal{S}}(\mathbf{K}_{cm}, \mathbf{k}_{rel})|^2$ is the 2-body quantity that the new generation of exclusive experiments would like to access. For ^{3}He, it can be calculated exactly.
Approaching the 2NC region, where $|\mathbf{K}_{cm}| << |\mathbf{k}_{rel}|$, the dependence upon the angle $\theta_{\widehat{\mathbf{K_{cm}k}_{rel}}}$ gets weaker and weaker:



This behavior is the one to be studied in forth-coming experiments, measuring $n(|\mathbf{K}_{cm}|)$ and $n(|\mathbf{k}_{rel}|)$

EFB21 Salamanca

August, 30^{th} 2010 Short range correlations and wave function factorization in light and finite nuclei – p.16/1!

CdA, Kaptari, Morita, Scopetta, Few-Body Systems 50(2011)243

ELSEVIER

Nuclear Physics A 604 (1996) 429–440

NUCLEAR
PHYSICS A

The high momentum and energy behaviour of the nucleon spectral function of nuclear matter within the Brueckner–Bethe–Goldstone approach

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Received 14 February 1996

Abstract

The nuclear single-particle spectral function is considered in the region of high momentum and high removal energy. For these kinematical conditions, far away from the quasi-particle peak, the spectral function is expected to be dominated by nucleon–nucleon correlations. It has been previously argued that the spectral function can be written as a convolution between the two-body relative momentum distribution and the corresponding centre-of-mass distribution of the correlated pairs which characterize the structure of the ground state in this energy–momentum region. It is shown that the convolution model can be microscopically derived from the Brueckner–Bethe–Goldstone (BBG) expansion. At the same time, this result also allows us to establish a direct link between the spectral function and the defect function of the BBG theory. From a numerical comparison with the microscopic spectral function the convolution model turns out to be highly accurate in the relevant momentum and energy range.

3. The spectral function of nuclear matter within the BBG theory

In NM the spectral function corresponding to the nucleon self-energy $M(k,E) = V(k,E) + iW(k,E)$, is given by the well-known result [6]

$$P(k,E) = -\frac{1}{\pi}\mathrm{Im}\,\mathcal{G}(k,E) = \frac{1}{\pi}\frac{W(k,E)}{(-E-k^2/2m-V(k,E))^2+W(k,E)^2}, \qquad (9)$$

where $\mathcal{G}(k,E)$ is the single-particle Green function

$$\mathcal{G}(k,E) = \frac{1}{-E-k^2/2m-V(k,E)-iW(k,E)}. \qquad (10)$$

It has to be noticed that the real, $V(k,E)$, and imaginary parts, $W(k,E)$, of the self-energy are highly off-shell in the considered energy and momentum ranges. We are interested in the region where E is much greater than the Fermi energy E_F. For high k and E, one finds

$$E+\frac{k^2}{2m} \gg |V(k,E)|, |W(k,E)|, \qquad (11)$$

as can be seen from the results shown in Ref. [7], and the spectral function can thus

$$P(k,E) = \frac{\pi^2\rho^2}{16}\int\frac{d^3P}{(2\pi)^3}n_{\rm rel}(|\boldsymbol{k}-\tfrac{1}{2}\boldsymbol{P}|)n_{\rm cm}^{\rm FG}(P)$$
$$\times\delta\left(E-E_{\rm thr}^{(2)}-E^*-\frac{1}{2m}(\boldsymbol{P}-\boldsymbol{k})^2\right),$$

440 M. Baldo et al. / Nuclear Physics A 604 (1996) 429–440

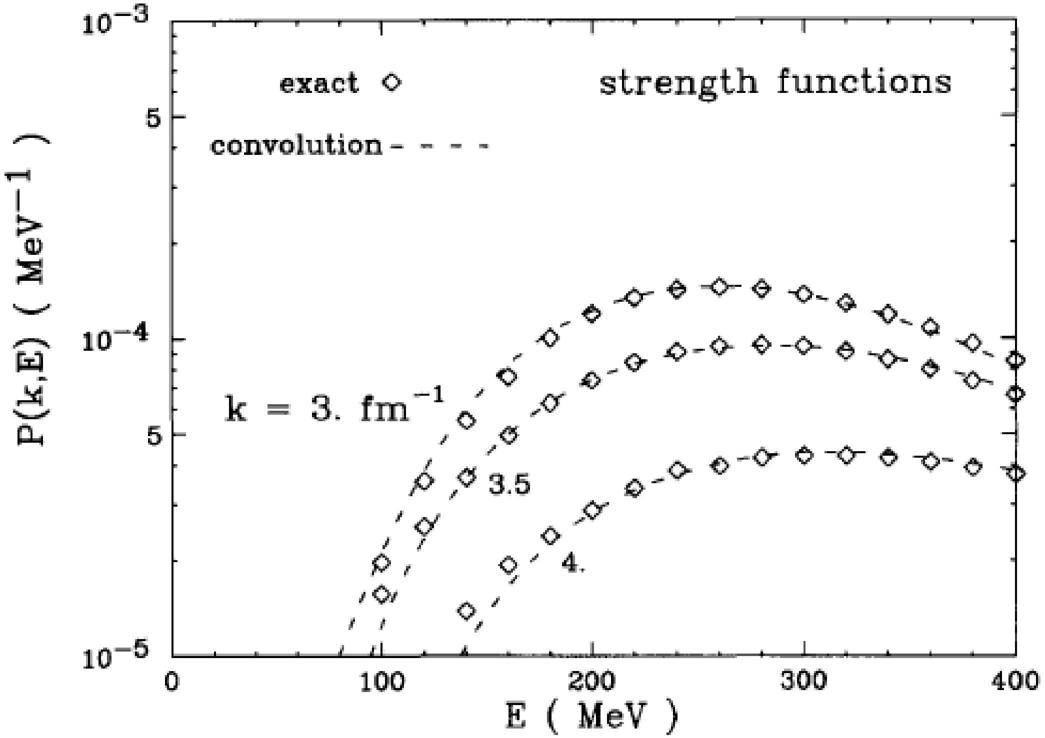


Fig. 4. Comparison between the SF obtained from the convolution model (dashed lines) and the one obtained from BBG theory (diamonds) for different values of the nucleon momentum *k*.

The convolution structure of $P(k,E)$ results from some general properties of of the many-body wave function at high momenta, in particular from the factorization property. The convolution model is the realistic starting point for the production of spectral function for finite nuclei.

See also:

Alvioli, CdA, Kaptari, Mezzetti, Morita Int. J. Mod. Phys. E22(2013)1330021

CdA Physics Report 590 (2015)

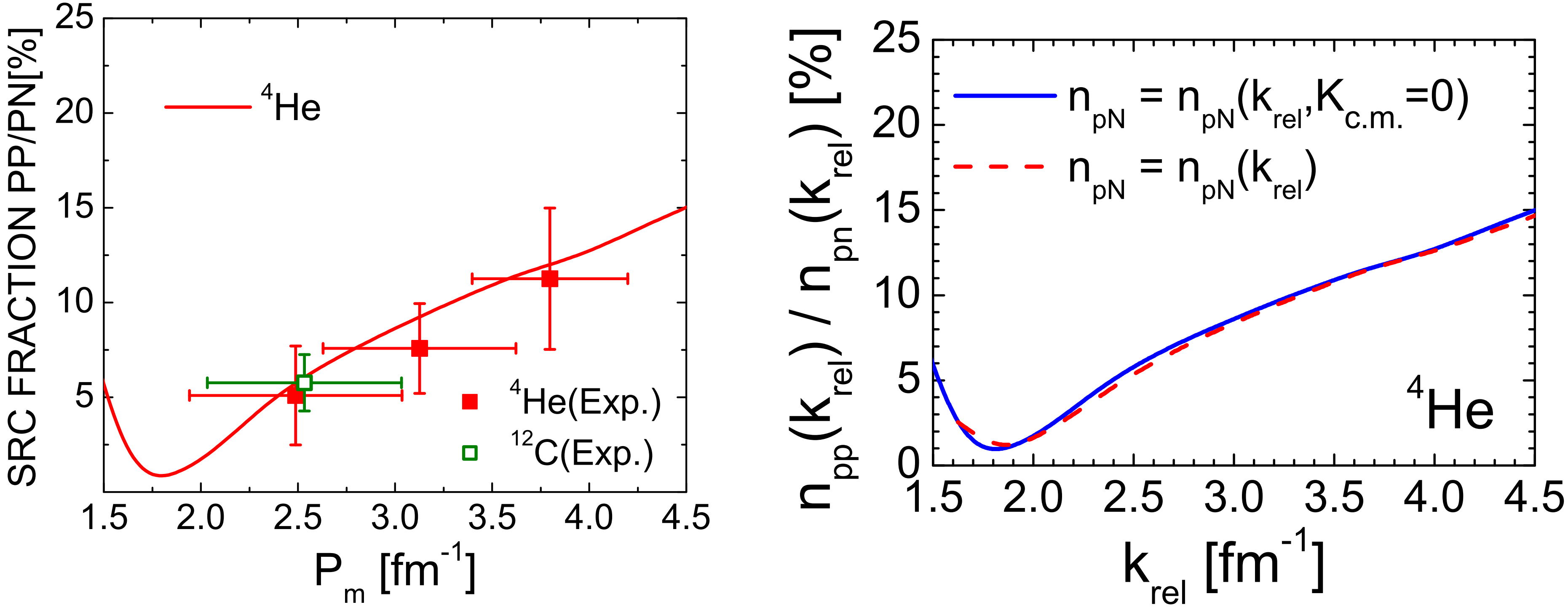
5 *A BRIEF COMPARISON BETWEEN THEORY AND EXPERIMENT*

Momentum distributions and spectral functions are not observable and the high removal energy and momentum produced by SRCs can only be extracted from various kinds of observable cross sections depending upon the full spectrum $\{\Psi_n\}$ of the Hamiltonian H

$$\sum_f | < \Psi_f | \hat{\mathcal{O}} | \Psi_0 > |^2$$

Ψ_0 - OK *ab initio*; $\Psi_f \to$ approximation (FSI); $\hat{\mathcal{O}} \to$ approximation (one- and two-body currents). Nonetheless the transition matrix elements appear to be under control at high Q^2 and $x_{Bj} > 1$.

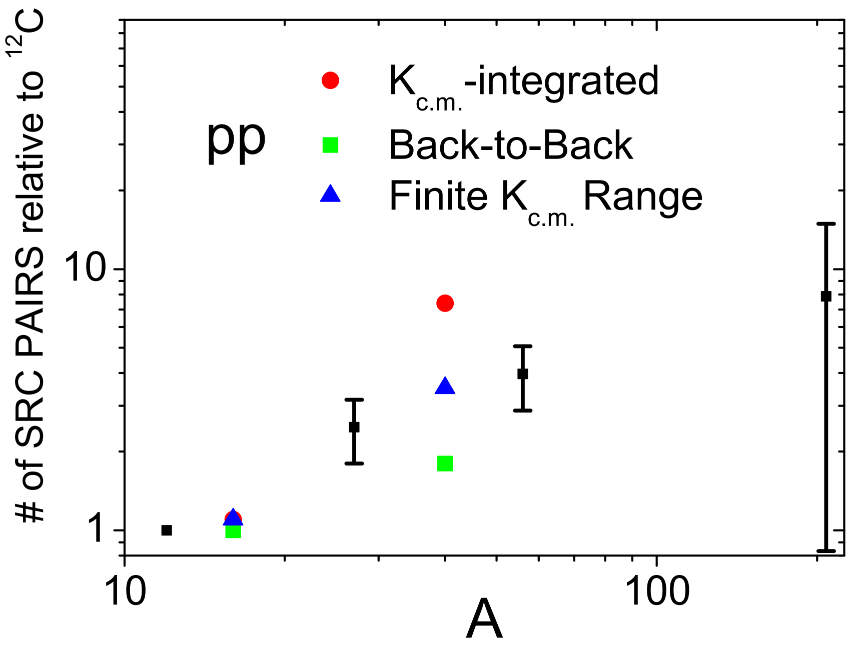
In what follows: a first order comparison between experimental data and theory based upon the calculated momentum distributions.

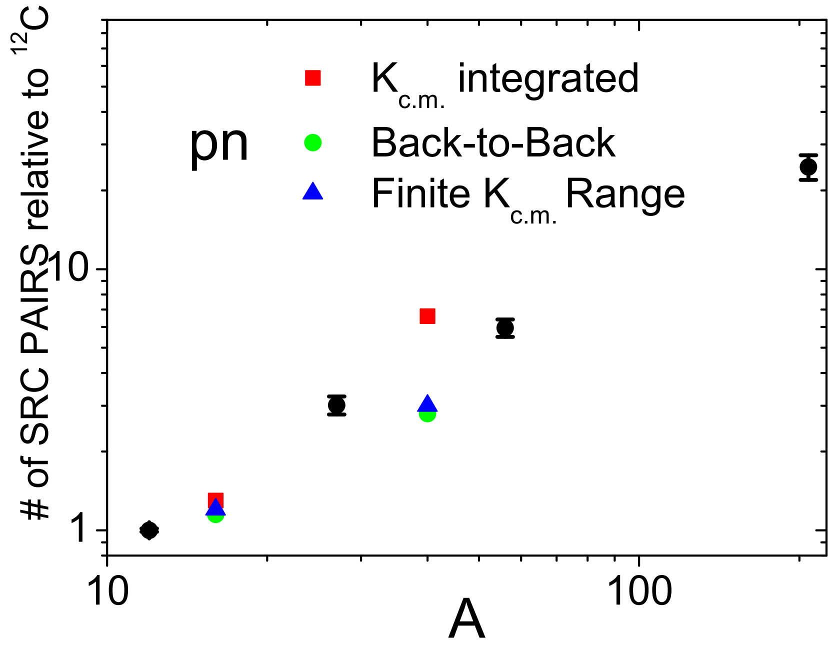


EXP: I. Korover *et al* Phys. Rev. Lett. 113 (2014);R. Shneor *et al* Phys. Rev. Lett. 99 (2007); R. Subedi *et al*, Science 320(2008);

THE: Alvioli,CdA, Morita, Preliminary

The right Figure represents further evidence of wave function factorization in the SRC region (cancelation of the c.m. momentum distribution in the numerator and the denominator)





EXP: R. Hen *et al* Science 346 (2014)

THE: Alvioli,CdA, Morita, Preliminary

For nuclei with $A > 40$ see: Colle, Hen, Cosyn, Korover *et al*, Phys. Rev. C92 (2015)

6 FEW WORDS ABOUT "NUCLEAR CONTACTS"

S.Tan-Ann. Phys. 323 (2008) described the short-range behavior of a two component Fermi gas in terms of a variable, called *"the contact"*, which measures the probability to find two unlike fermions at short range. The nucleus, even at short range, is different from a two component Fermi gas. However a *"nuclear contact"* can also be defined for atomic nuclei to describe the spin-isospin two-nucleon states at short range (Weiss, Bazak, Barnea, to appear; see also Or Hen's talk). It is important to stress that the main assumption to obtain the *"contacts"* , both in atomic and nuclear systems, is the assumption of WAVE FUNCTION FACTORIZATION

$$r_{ij} \longrightarrow 0 \qquad \Psi_0(\{\mathbf{x}\}_A) \longrightarrow \sum_{nm} \phi_n(\mathbf{x_{ij}}) \otimes \Phi_m(\{\mathbf{x}\}_{A-2})$$

As we have shown, factorization holds in *ab initio* nuclear wave functions for few-nucleon systems and nuclear matter and was used e.g. to obtain the convolution model of the spectral function in 1996. SRCs always implies wave function factorization and the quantity we have introduced

$$C_A^{pn}$$

can be called, if one likes so, the NUCLEAR CONTACT for BACK-to-BACK SHORT-RANGE CORRELATED Proton-Neutron PAIRS.

4. CONCLUSIONS

- **NN SRCs can be calculated *ab initio* with realistic bare NN interactions. Calculations by different groups seem to converge**

- **SRCs at $\mathbf{k_{rel} > 2\,fm^{-1}}$ and $\mathbf{K_{c.m.} < 1\,fm^{-1}}$ exhibit several universal (*independent of A*) features, in particular: (i) a factorization of the relative and center-of-mass motions; (ii) a deuteron-like behavior of the pn pairs; (iii) an appreciable spin-isospin dependence, e.g. tensor dominance around $k_{rel} \simeq 2\,fm^{-1}$**

- **The factorization of the nuclear wave function in the region of SRCs provide the basic justification of the convolution model of the nucleon momentum distributions and spectral functions**

- **SRCs and their experimental study may provide fundamental information on the nature of in-medium dynamics at short range in nuclei. Calculations with different types of NN interactions (soft NN bare interaction, chiral potentials, etc) are necessary and welcome.**