

# Heavy quark bound states in a quark-gluon plasma

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Council

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The quark-gluon plasma is a unique many-body system:

- color particles
- special interactions
- long range forces
- confinement/deconfinement

Hard probes (jets, quarkonia, etc)

- they are special
- they share common features
- they play the role of 'test' particles
- they can help understand transport properties of the QGP

# Outline

A very nice idea

A considerable experimental effort

A difficult many-body problem !

...but some recent progress



*A very nice idea*

The quarkonium is a « non relativistic » system

$$H = 2m_c + \frac{p_1^2}{2m_c} + \frac{p_2^2}{2m_c} + V(r)$$

$$V(r) = -\frac{\alpha}{r} + \sigma r$$

Heavy quarks are "heavy"

$$m_c \approx 1.5\text{GeV} \quad m_b \approx 5\text{GeV} \quad m_Q \gg \Lambda_{\text{QCD}}$$

Hierarchy of scales

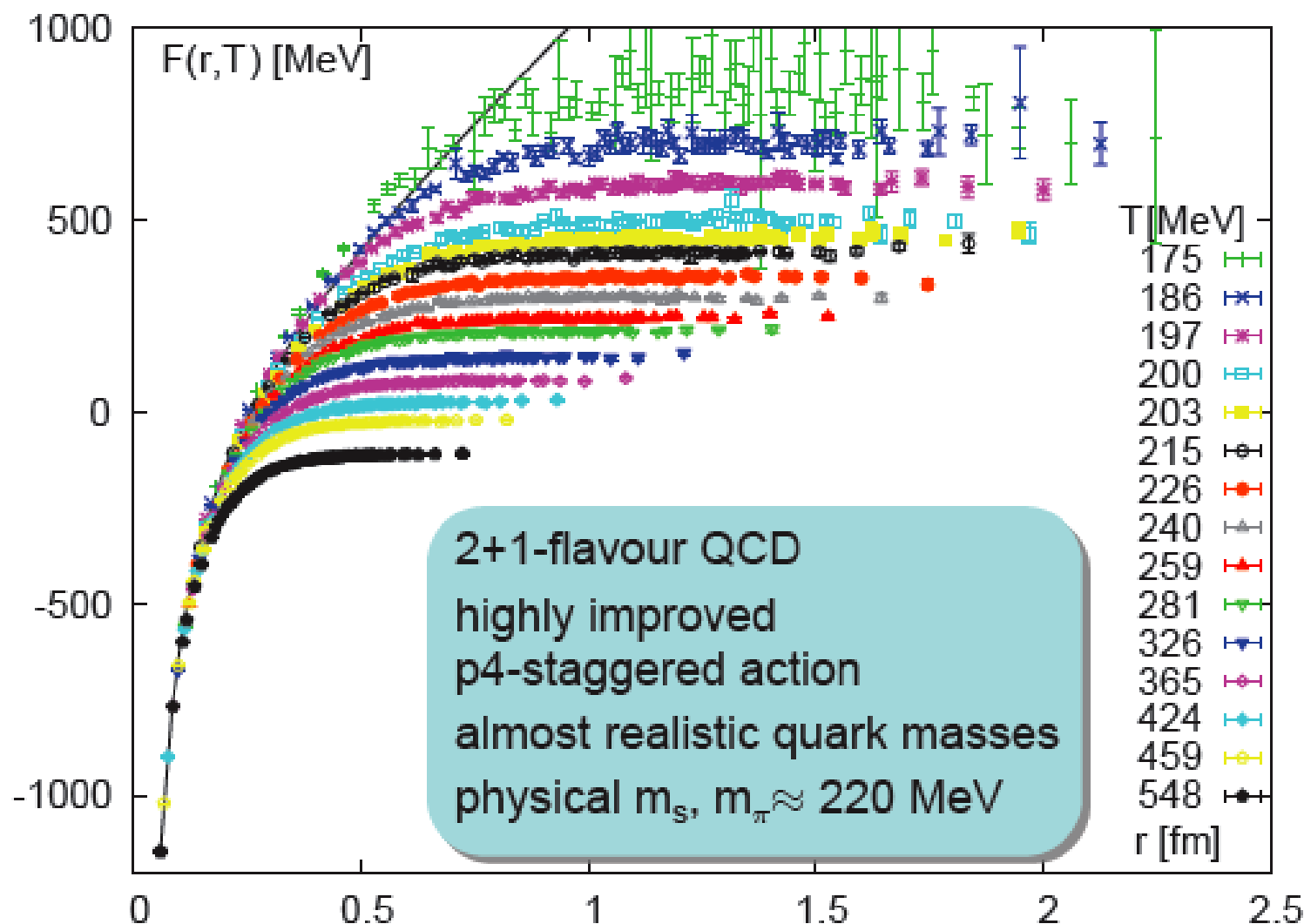
$$m \ll mv \ll mv^2 \quad v \ll 1$$

$$v_c^2 \sim 0.3 \quad v_b^2 \sim 0.1 \quad mv \sim 1/r$$

well suited for application of effective field theory

[Brambilla, Ghiglieri, Petreczky, Vairo, Escobedo, Soto]

# Heavy quarks free energy from lattice calculations



(O. Kaczmarek et al., PLB543(2002)41,  
S. Gupta et al., Phys.Rev.D77(2008)034503)

# Screening of binding forces in a quark-gluon plasma

Screened potential

$$V(r) = -\frac{\alpha}{r} e^{-r/r_D(T)}$$

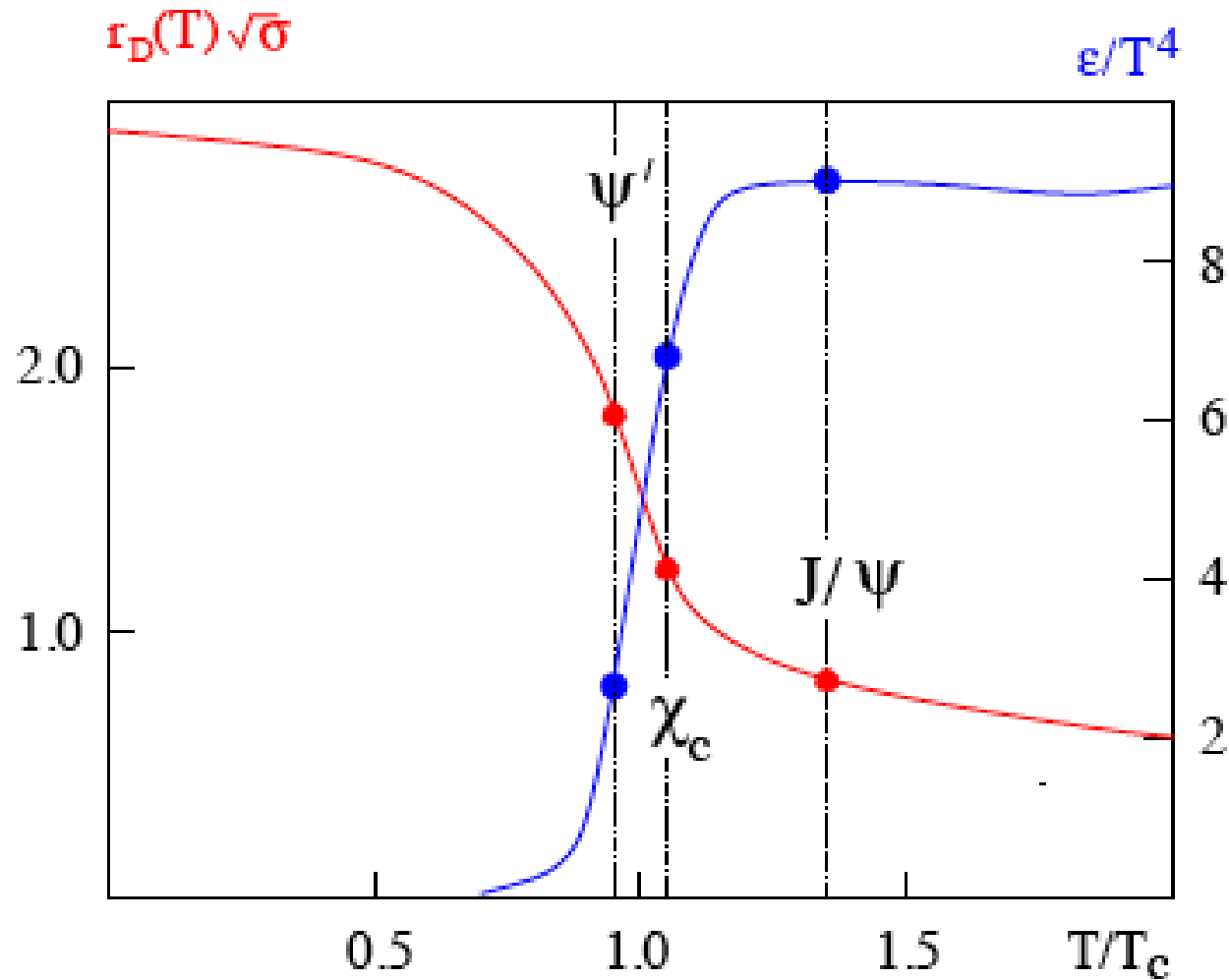
Bound state exists for

$$r_D(T) > r_D^{\min}$$

that is, for

$$T < T_D$$

Melting temperature depends on size of bound state

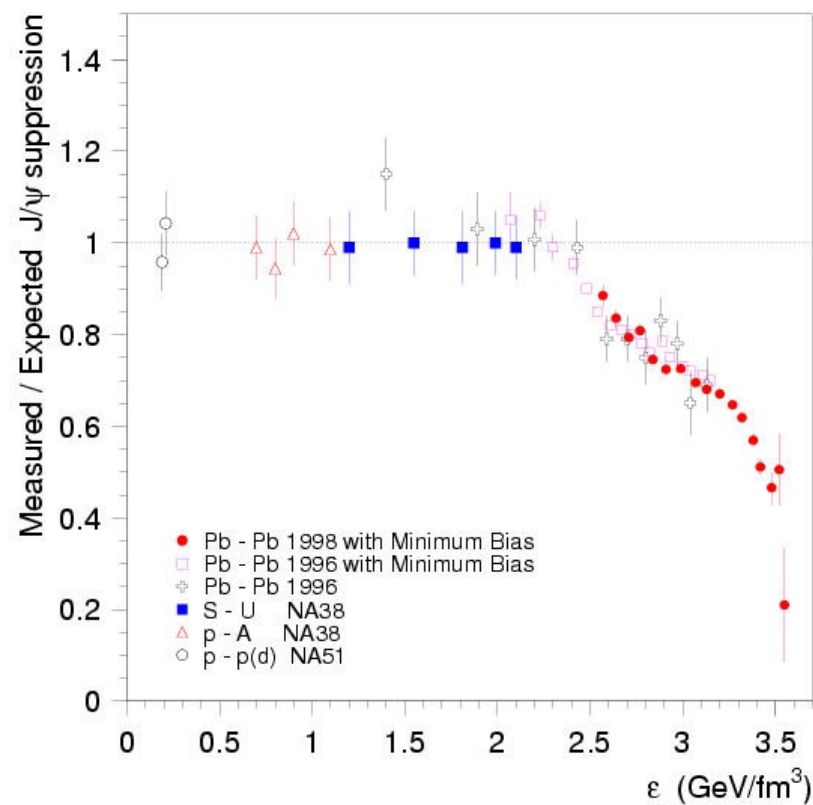
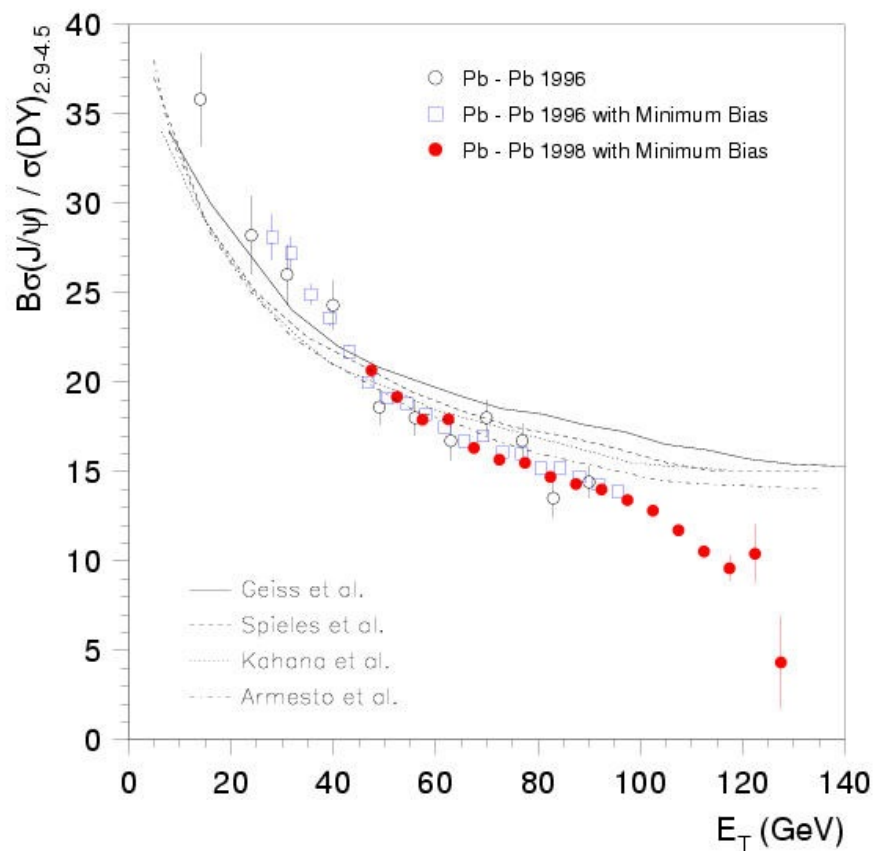


(from H. Satz, hep-ph/0602245)

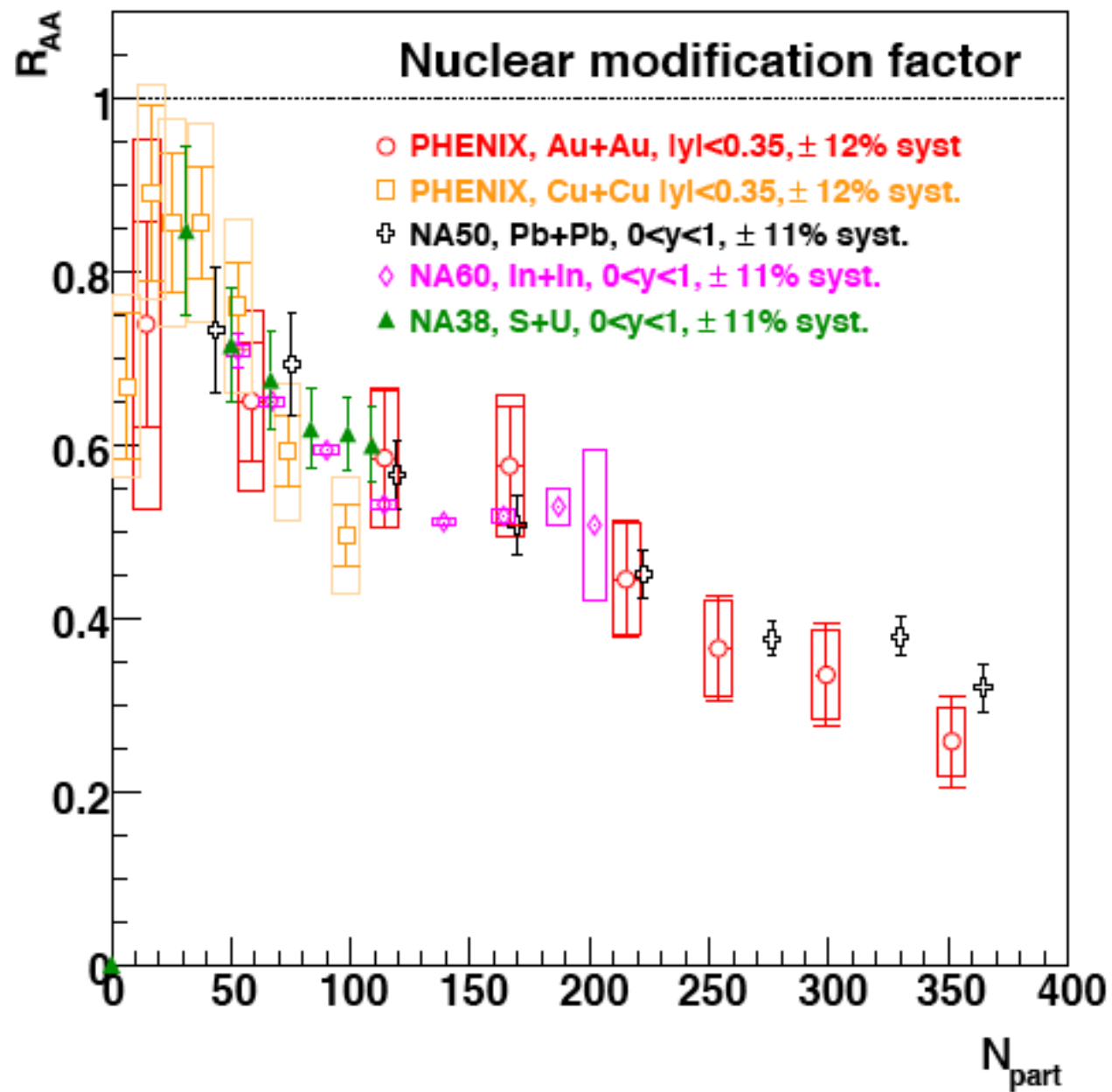
*A considerable experimental effort*

# Summary of early measurements (NA38, NA50)

(CERN, 2000)

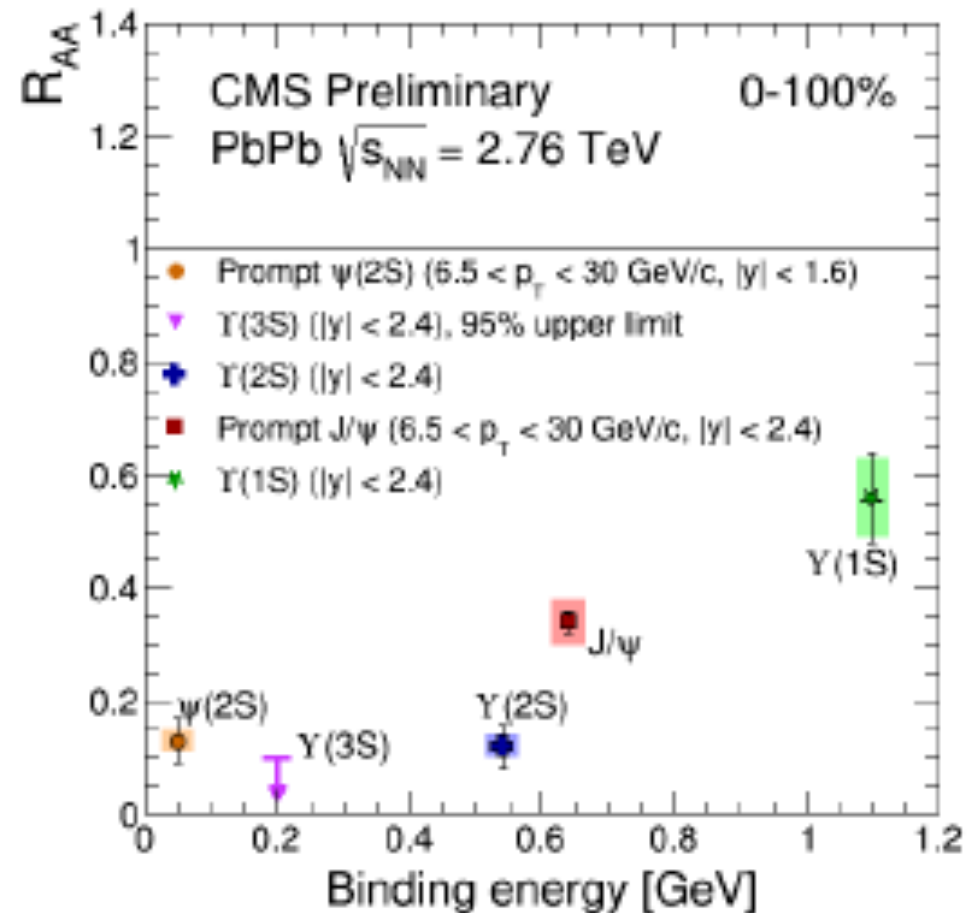
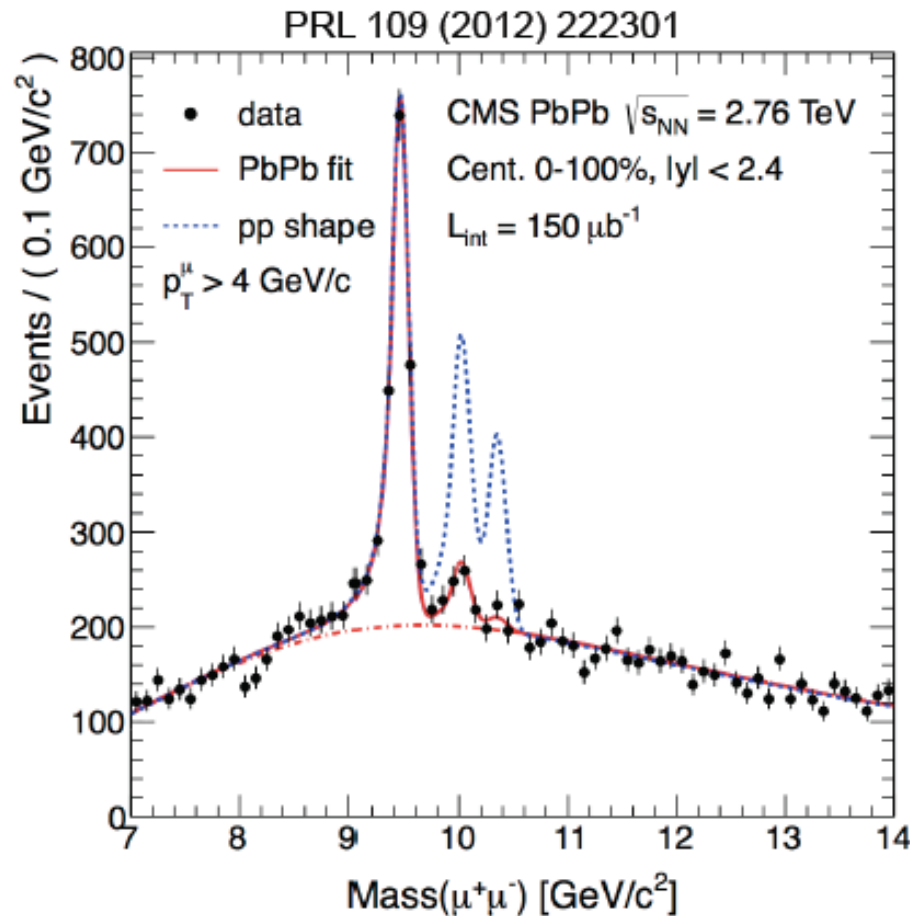


What about RHIC ?

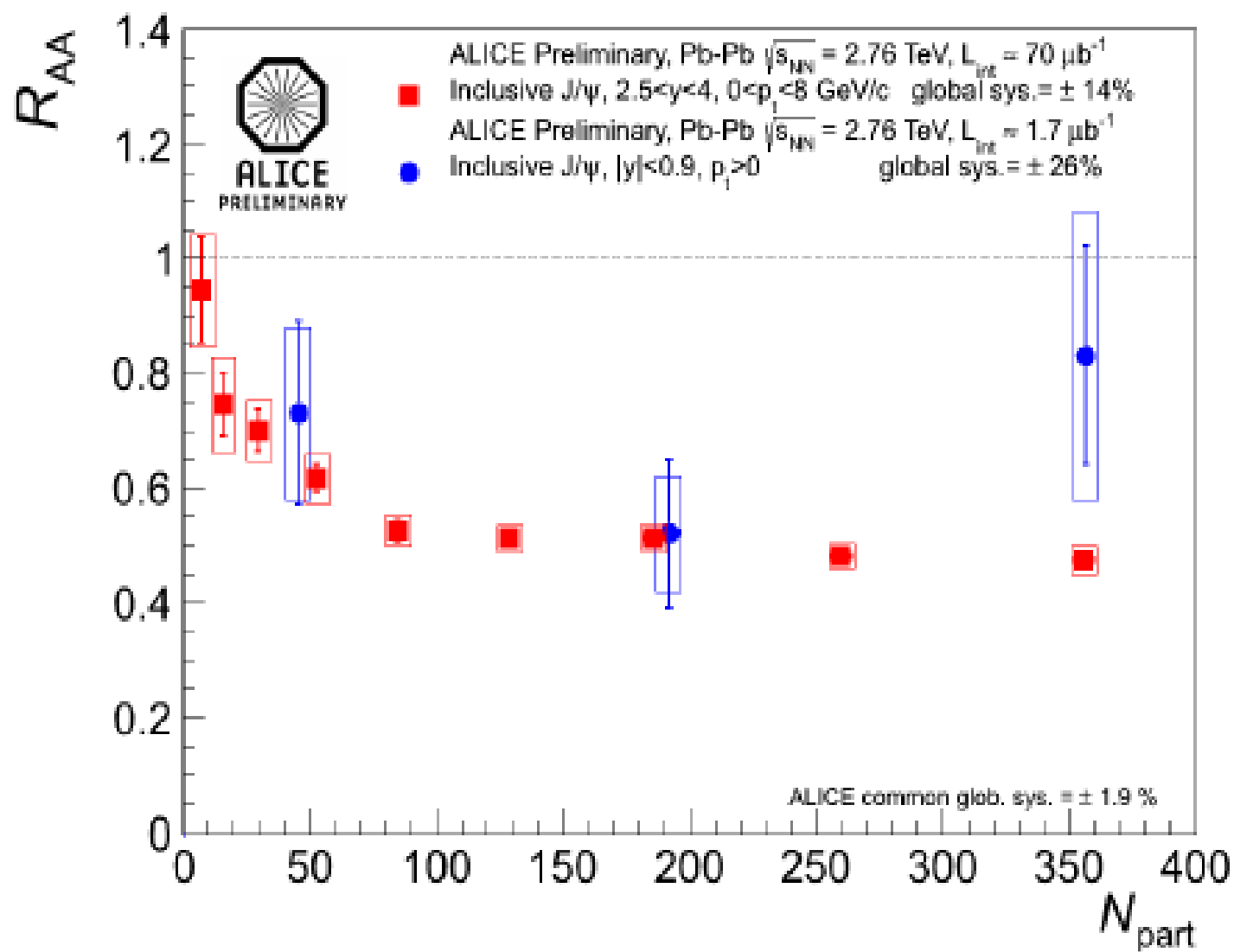




# $\Upsilon$ suppression



excited states are more 'fragile'....  
findings in line with expectations....



*A difficult many-body problem*

# a large variety of theoretical approaches

- potential models
- spectral functions
- Euclidean correlators (lattice), maximum entropy techniques
- coupled channels
- path integrals
- open quantum systems
- effective field theory, non relativistic heavy quark effective theory
- strong coupling techniques
- etc

## Which problem do we need to solve ?

- full dynamics, including plasma expansion
- dynamics of bound state formation (stationary states are not enough)
- dynamics of dissociation and recombination

**TIME SCALES MATTER!**

## Our goals:

- address full dynamics, keeping consistency between different processes
- provide unifying perspective and clarify the relations between different approaches
- deepen connection between energy loss, momentum broadening and quarkonium dynamics

Note: we may not have to worry about the dynamics, all is "thermal"

"principle" of statistical hadronization

PBM and JS [PLB490 (2000)196]

Heavy quarks hadronize  
statistically  
at the 'phase boundary'

# some recent progress

Results presented are based on

A. Beraudo, JPB, C. Ratti, NPA 806 (2008) 312 [arXiv: 0712.4394]

A. Beraudo, JPB, P. Faccioli and G. Garberoglio [arXiv: 1005.1245]

JPB, D. de Boni, P. Faccioli and G. Garberoglio, [arXiv:1503.03857]

WORK IN PROGRESS !

Similar effort by Y. Akamatsu, A. Rothkopf and others  
(‘open quantum systems’)

# Dynamics

(Abelian approximation)

$$H = H_Q + H_{med} + H_{int}$$

Heavy quark

$$H_Q = M \int d^3\mathbf{r} \psi^\dagger(\mathbf{r})\psi(\mathbf{r}) + \int d^3\mathbf{r} \psi^\dagger(\mathbf{r}) \left( -\frac{\nabla^2}{2M} \right) \psi(\mathbf{r})$$

linearly coupled to gauge field

$$H_{int} = g \int d^3\mathbf{r} \psi^\dagger(\mathbf{r})\psi(\mathbf{r})A_0(\mathbf{r})$$

The hot plasma

$$H_{med} = \int d^3r \xi^\dagger(\mathbf{r})h_0 \xi(\mathbf{r}) + \frac{1}{2} \int d^3r d^3r' \hat{\rho}(\mathbf{r}) \frac{g^2}{4\pi|\mathbf{r} - \mathbf{r}'|} \hat{\rho}(\mathbf{r}')$$



# Path integral and influence functional

$$P(Q_f, t_f | Q_i, t_i) = \int_C DQ e^{iS_0[Q]} e^{i\Phi[Q]}$$

$$e^{i\Phi[Q]} = \int DA_0 e^{-i \int_C d^4x g \rho(x) A_0(x)} e^{iS_2[A_0]}$$

$$\rho(x) = \sum_{j=1}^N (\delta(\mathbf{x} - \mathbf{q}_j(t)) - \delta(\mathbf{x} - \bar{\mathbf{q}}_j(t)))$$

'Integrate out' the light particles and keep the quadratic part of the resulting action (HTL approximation)

$$S_2[A_0] = -\frac{1}{2} \int_C dx (A_0(x) \nabla^2 A_0(x)) - i \text{Tr} \ln [i\gamma^\mu \partial_\mu - m - e\gamma^0 A_0(x)]$$



$$\text{wavy line with dot} = \text{wavy line} + \text{wavy line with loop} + \dots$$

$$\Phi[Q] = \frac{g^2}{2} \iint_C d^4x d^4y \rho(x) \Delta_c(x-y) \rho(y)$$

$$\Delta(x-y) \equiv i \langle T_C [A_0(x) A_0(y)] \rangle$$

# Physical content of the influence functional

$$\Phi[\mathbf{Q}] = \frac{g^2}{2} \iint_C d^4x d^4y \rho(x) \Delta_c(x-y) \rho(y)$$

$$\Delta(x-y) \equiv i \langle T_C [A_0(x) A_0(y)] \rangle$$

$$\Delta_{11}(\omega = 0, x) \sim V(x) \quad \text{Heavy quark potential (complex)}$$

$$V(r) = \alpha_s \left[ m_D + \frac{e^{-m_D r}}{r} \right] - i \alpha_s T \phi(m_D r)$$

(first observed by M. Laine et al hep-ph/0611300)

$$\Delta_{12}(\omega = 0, x) \sim \text{Im} V(x) = W(x) \quad \text{collisions}$$

$$\mathcal{H}_{ij} \sim \frac{\partial^2 W}{\partial x_i \partial x_j} \quad \text{friction}$$

# Low frequency expansion yields Langevin equation

$$M \ddot{R} = -\frac{\beta}{2} \mathcal{H}(R) \dot{R} + \mathbf{F}(R) + \Psi(R, t)$$

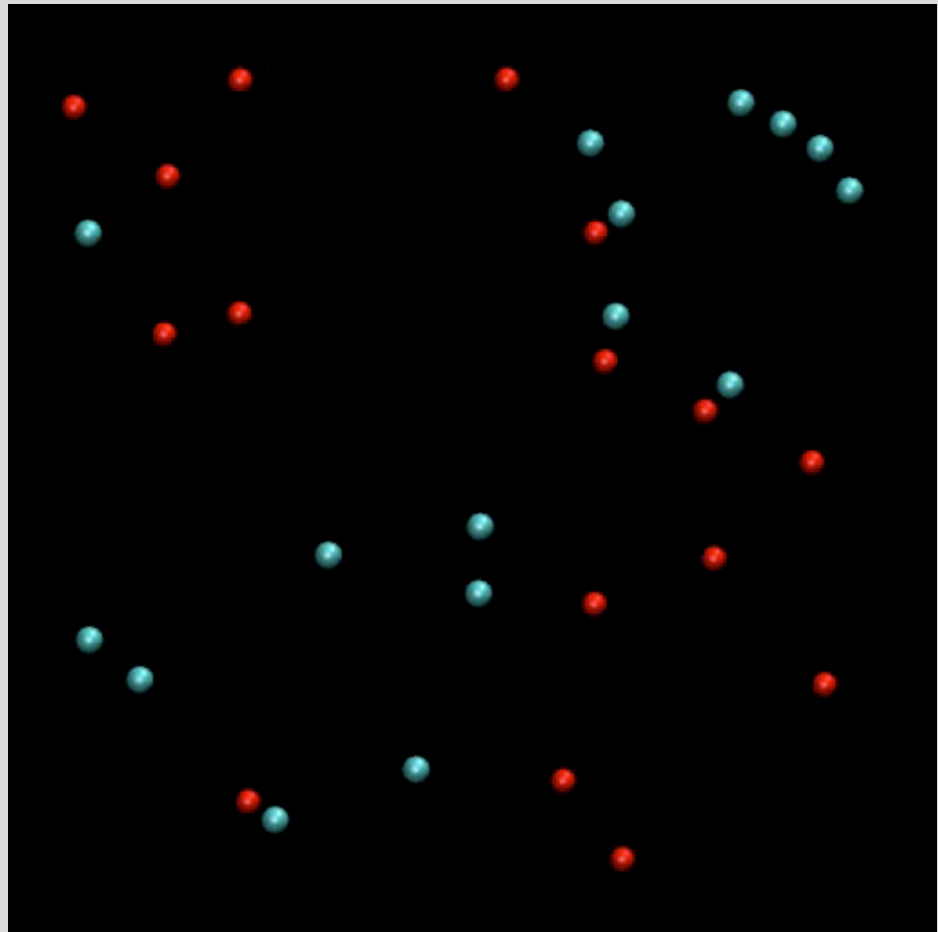
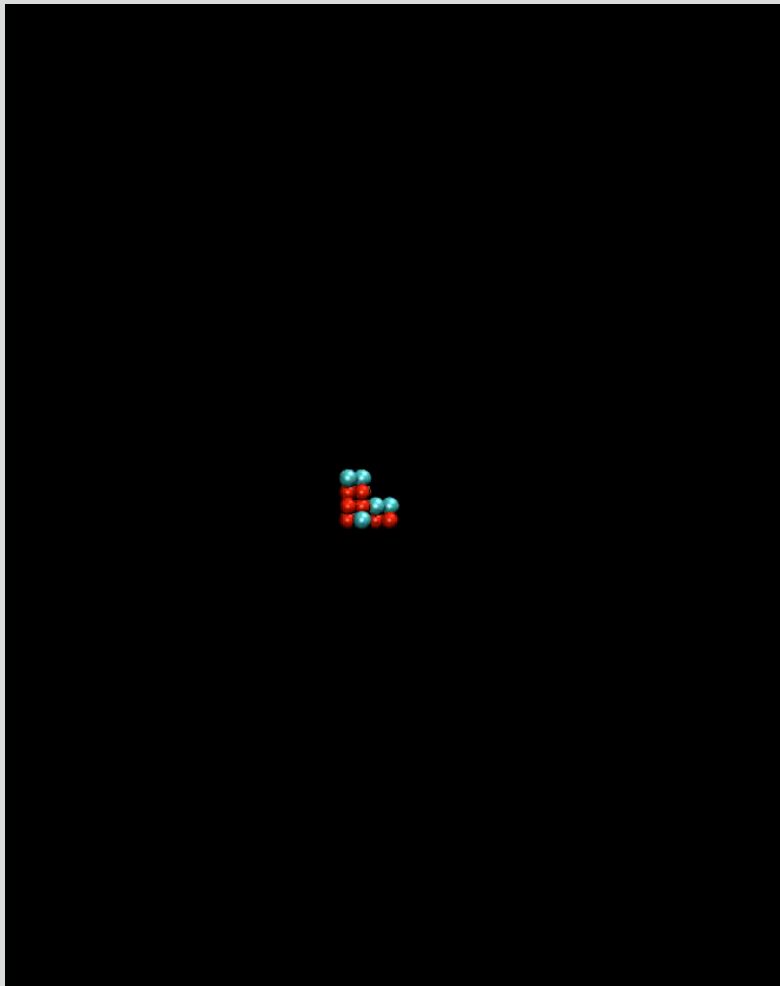
$$\mathcal{H}_{ij} \sim \frac{\partial^2 W}{\partial x_i \partial x_j} \quad \mathbf{F}(R) \sim \nabla \text{Re} V(R)$$

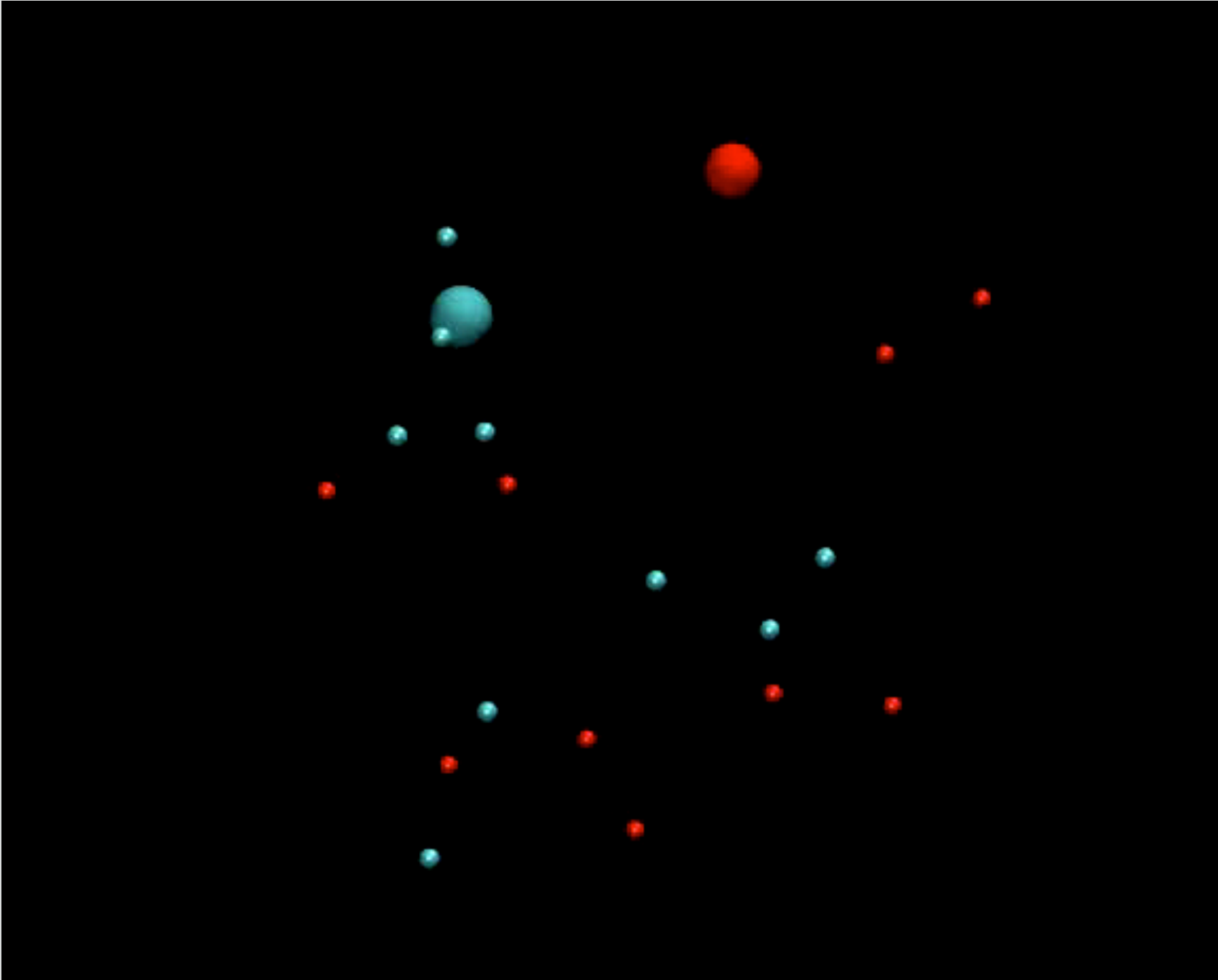
$$\langle \Psi(R, t) \rangle = 0$$

$$\langle \Psi_k(R, t) \Psi_m(R, t') \rangle = \mathcal{H}_{km}(R) \delta(t - t')$$

Non trivial noise (depends on the  
configuration  $R$  of the heavy quarks)

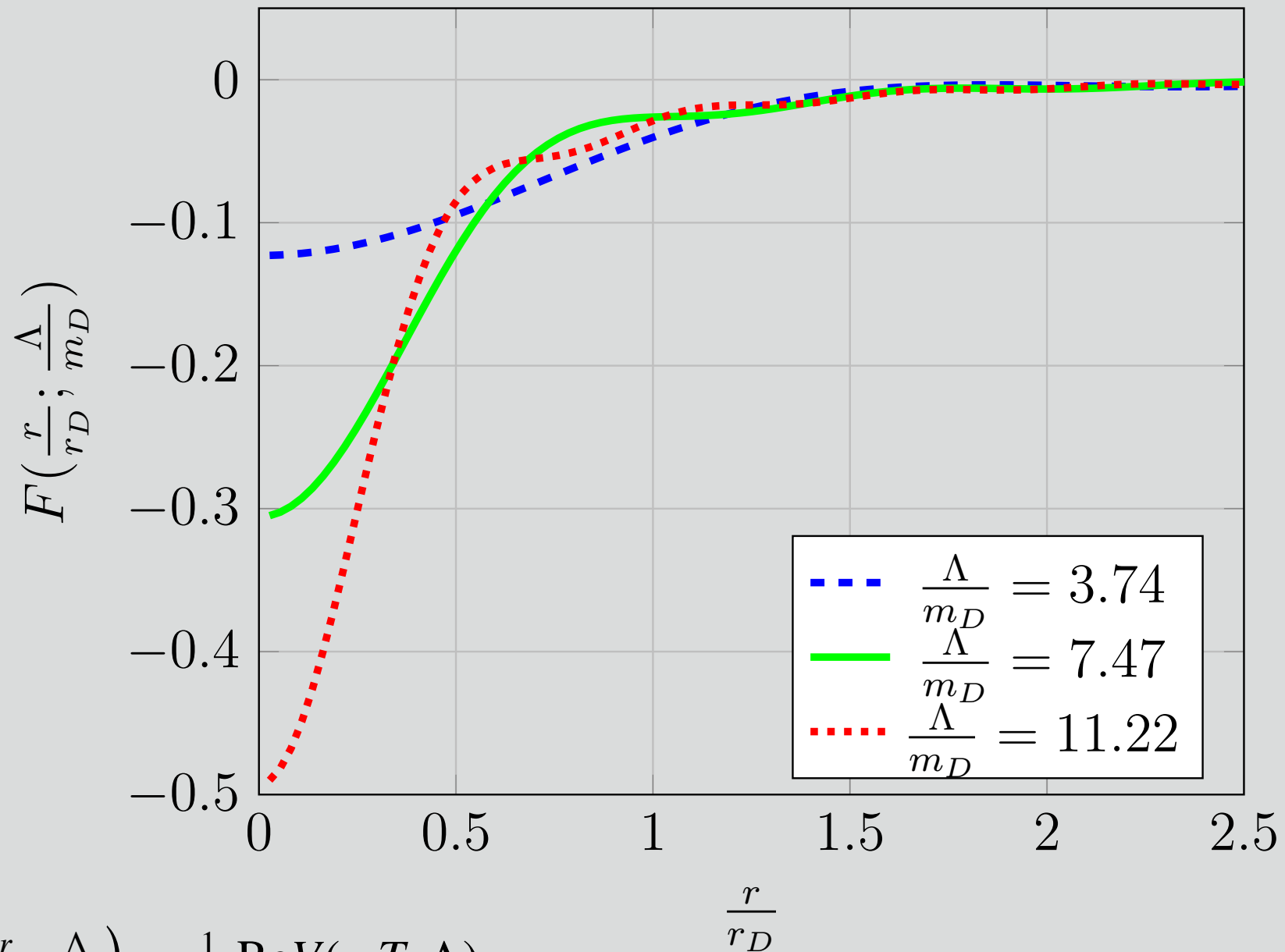
*Selected results*





Fixing a few "details"

# Regularized Coulomb potential

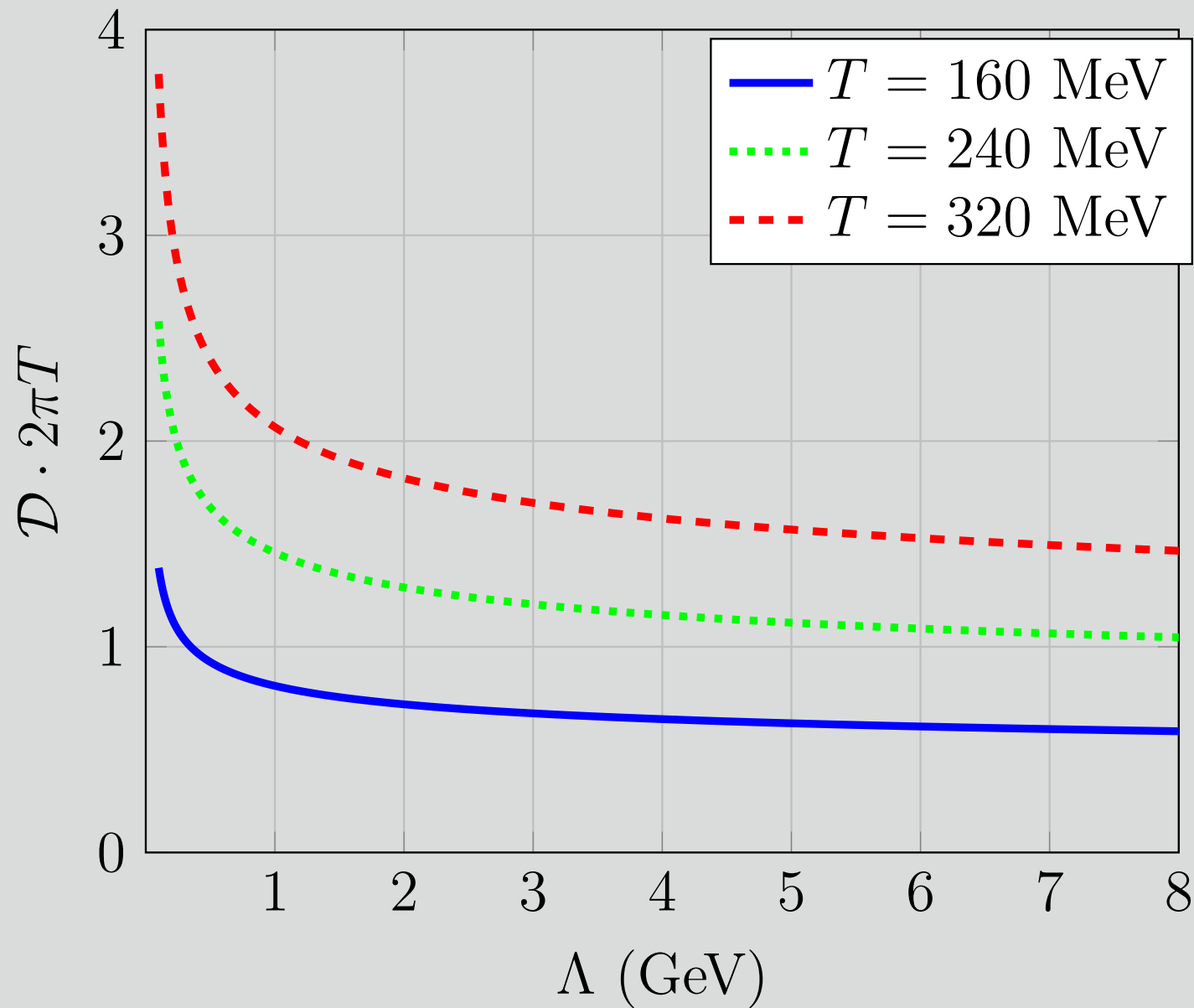


$$F\left(\frac{r}{r_D}, \frac{\Lambda}{m_D}\right) = \frac{1}{m_D} \text{Re}V(r, T, \Lambda)$$

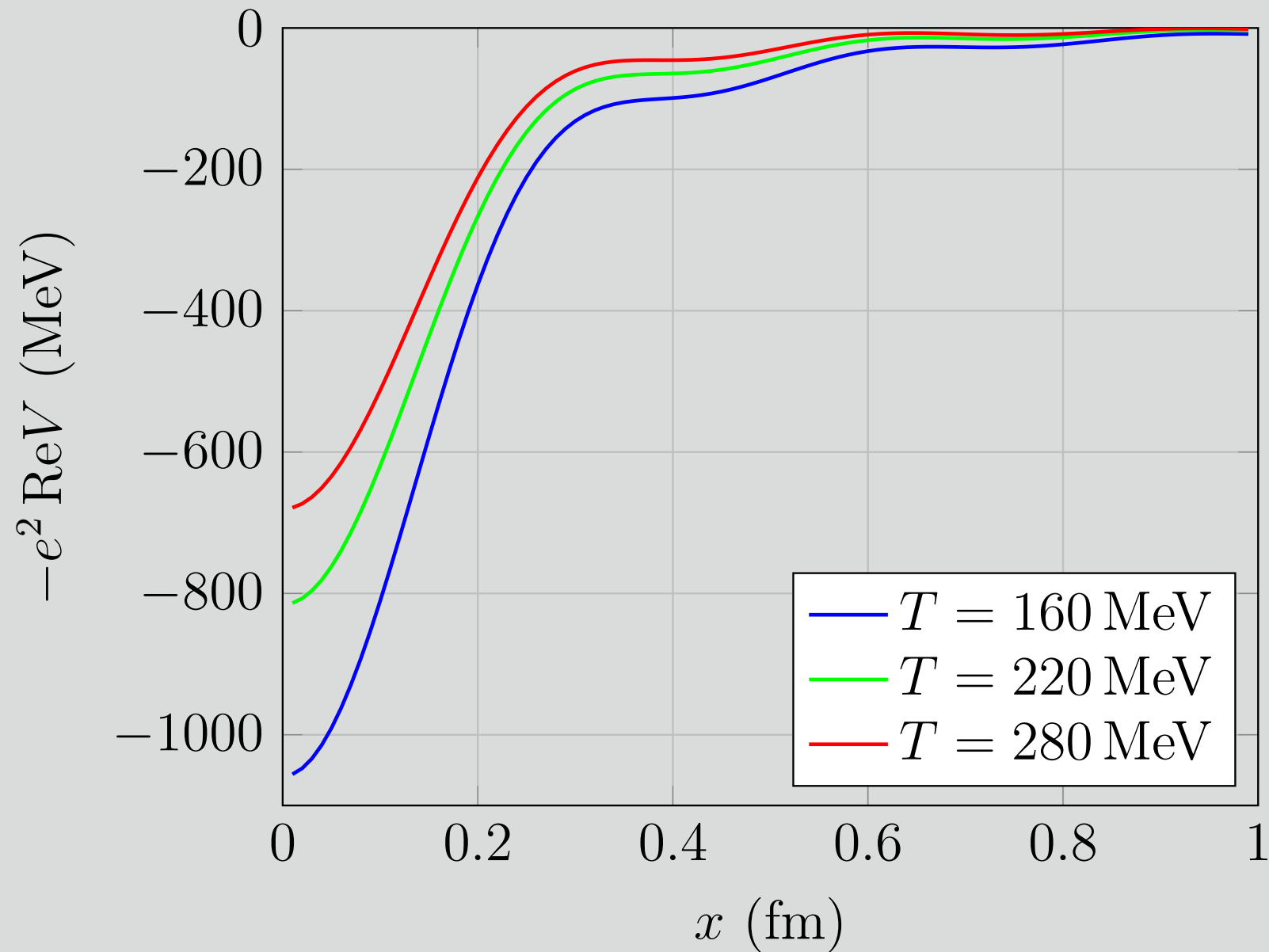


# Diffusion constant

$$\mathcal{D} = \frac{T}{M_\gamma}$$

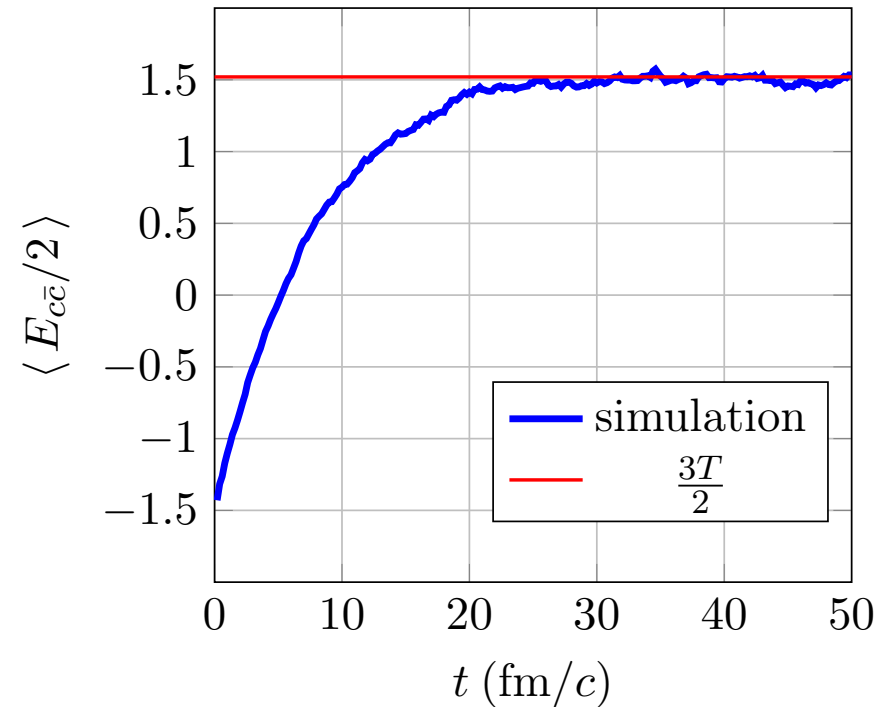
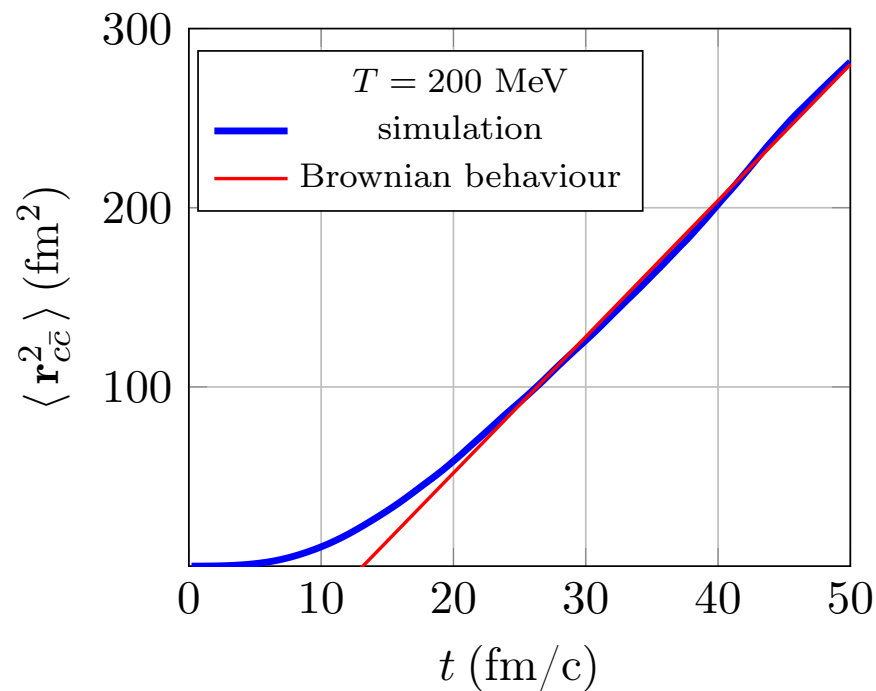


# Potential (real part) - charmonium

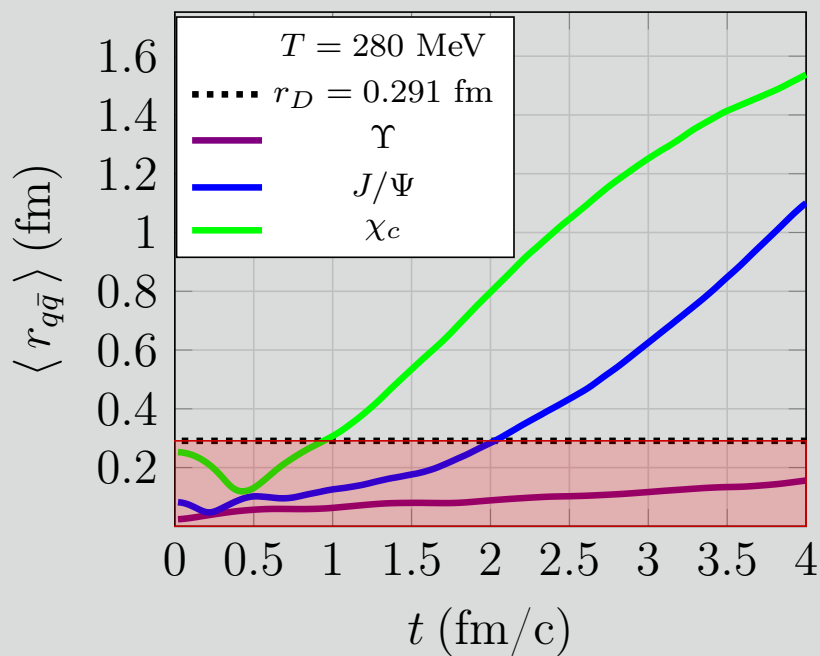
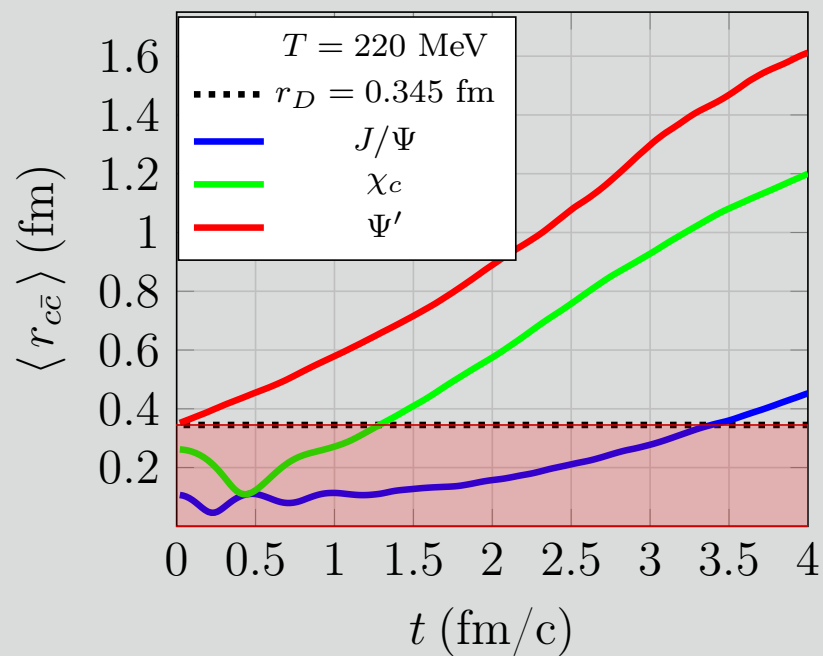
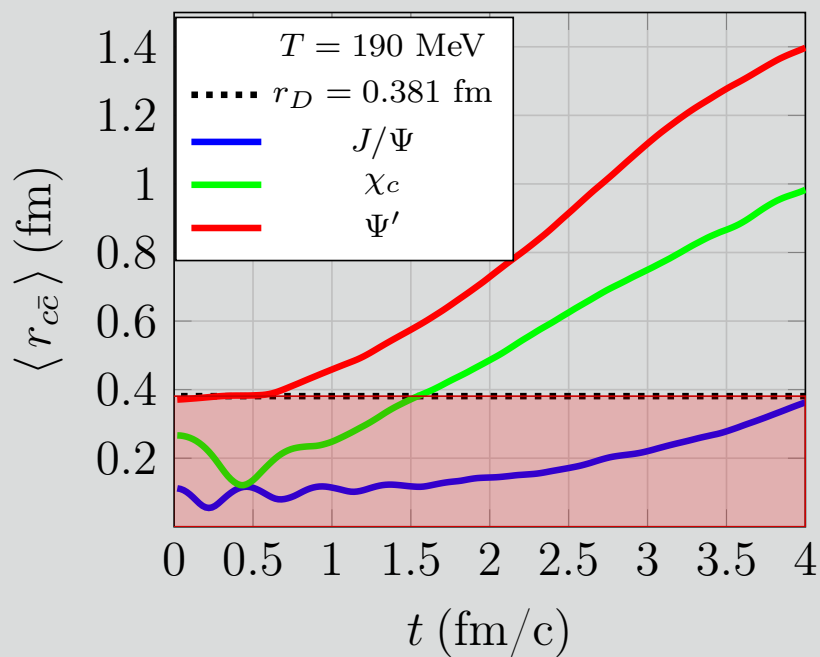
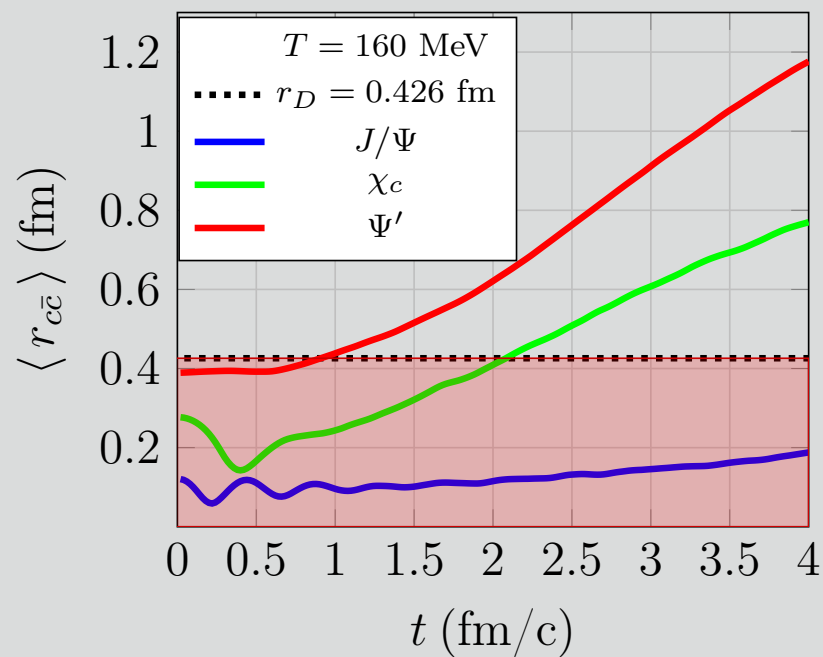


One quark-antiquark pair in the plasma

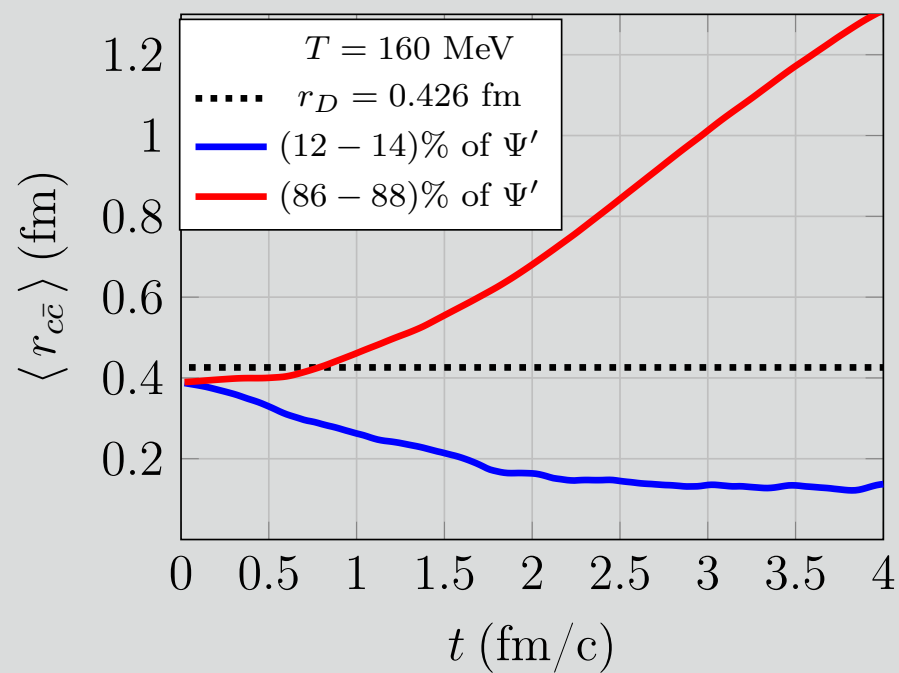
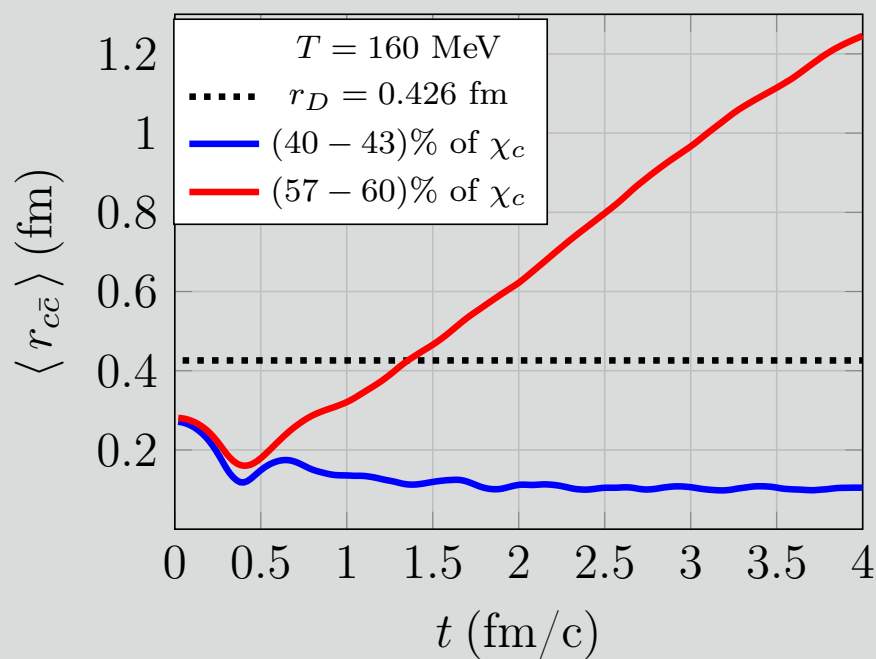
If  $T$  high enough, heavy quarks thermalize



# "Sequential" suppression

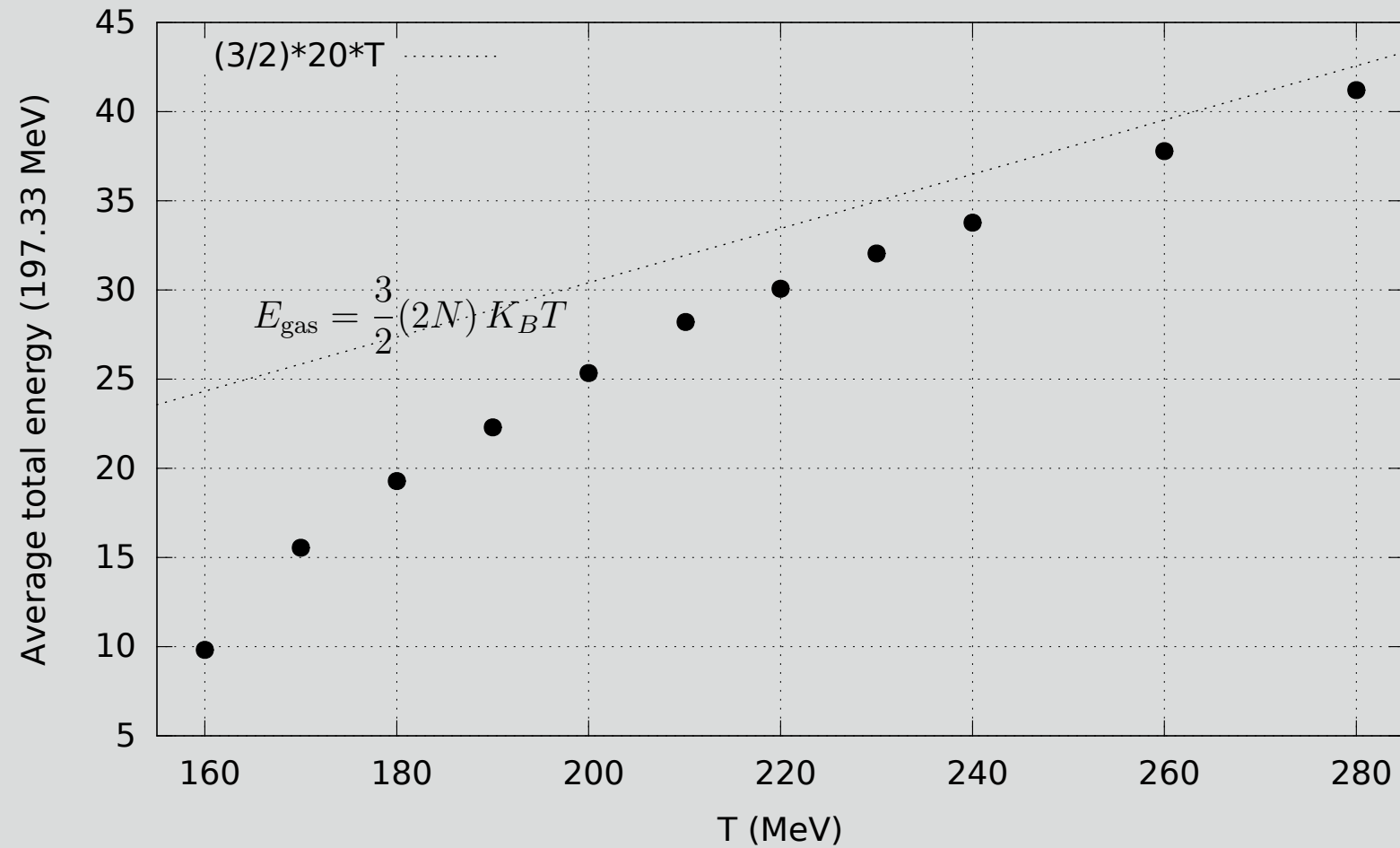


# Effective feed down from excited states!



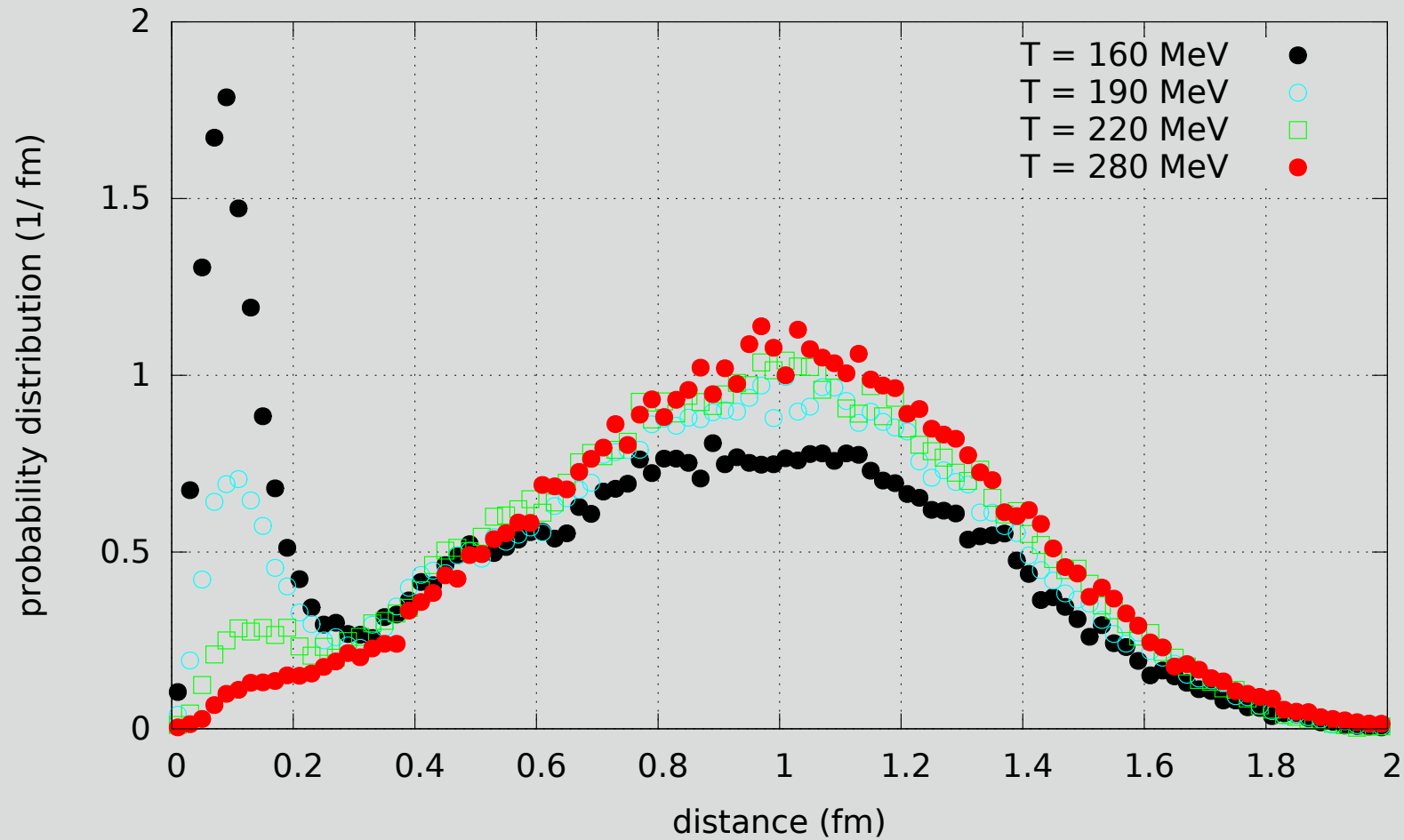
Many pairs in the plasma

# 10 pairs in plasma



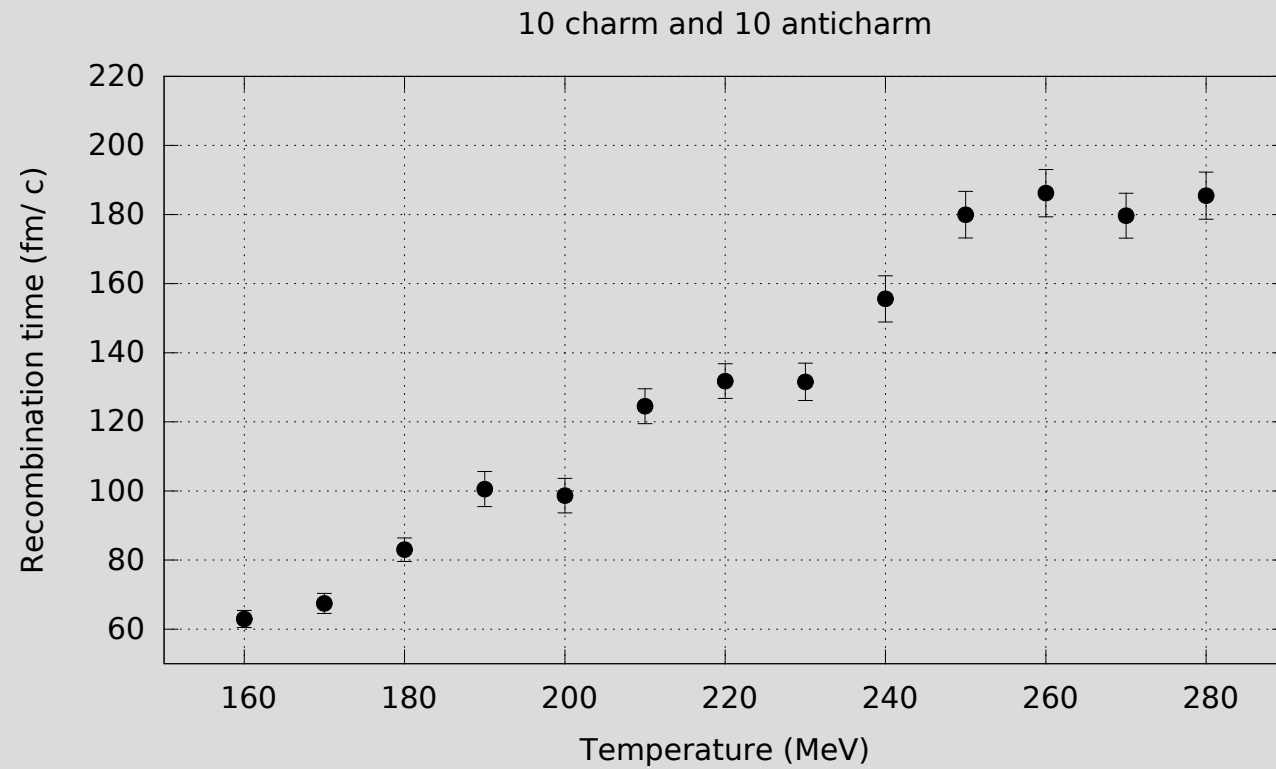


# Probability distribution of distance to nearest neighbor

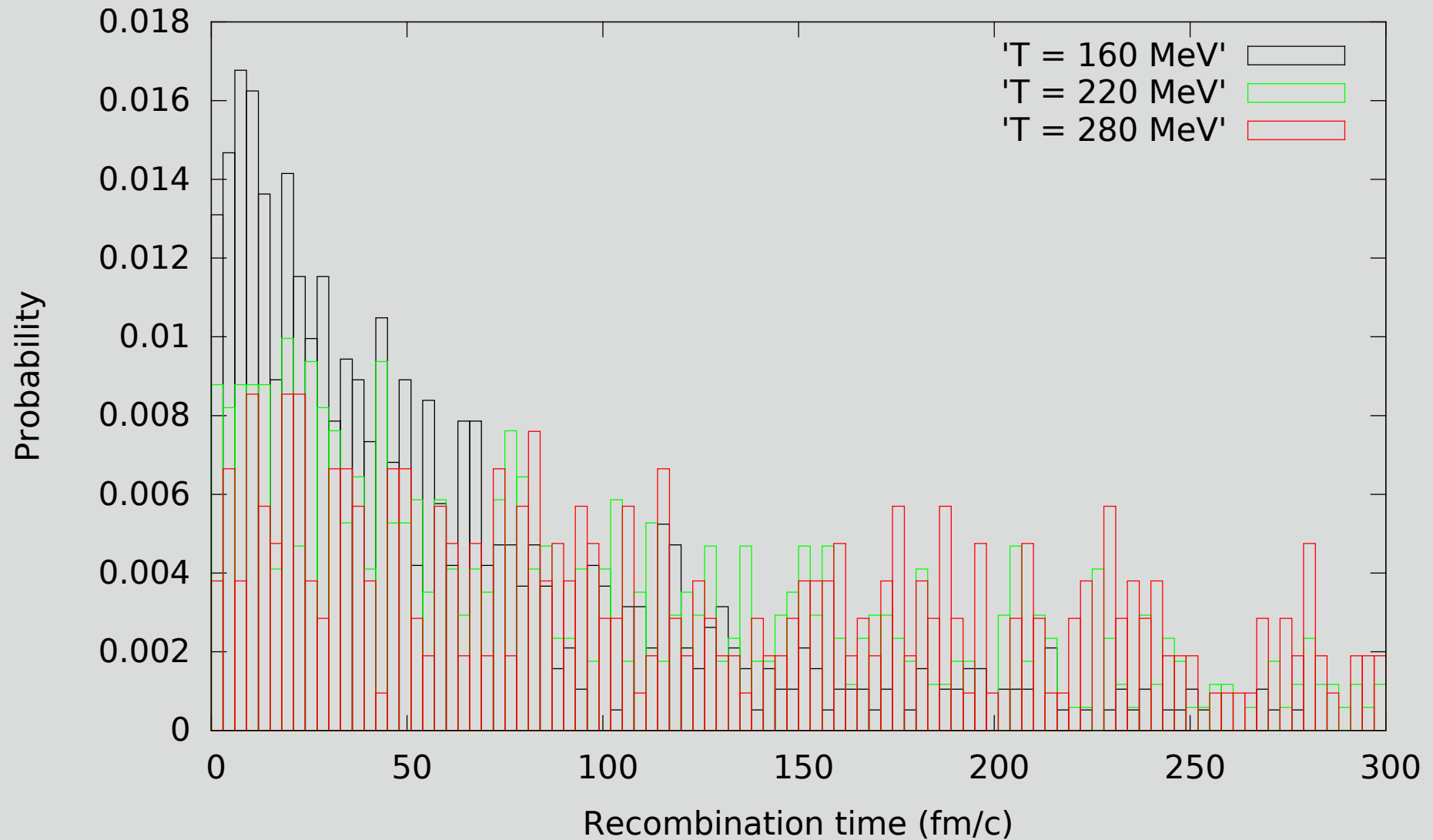


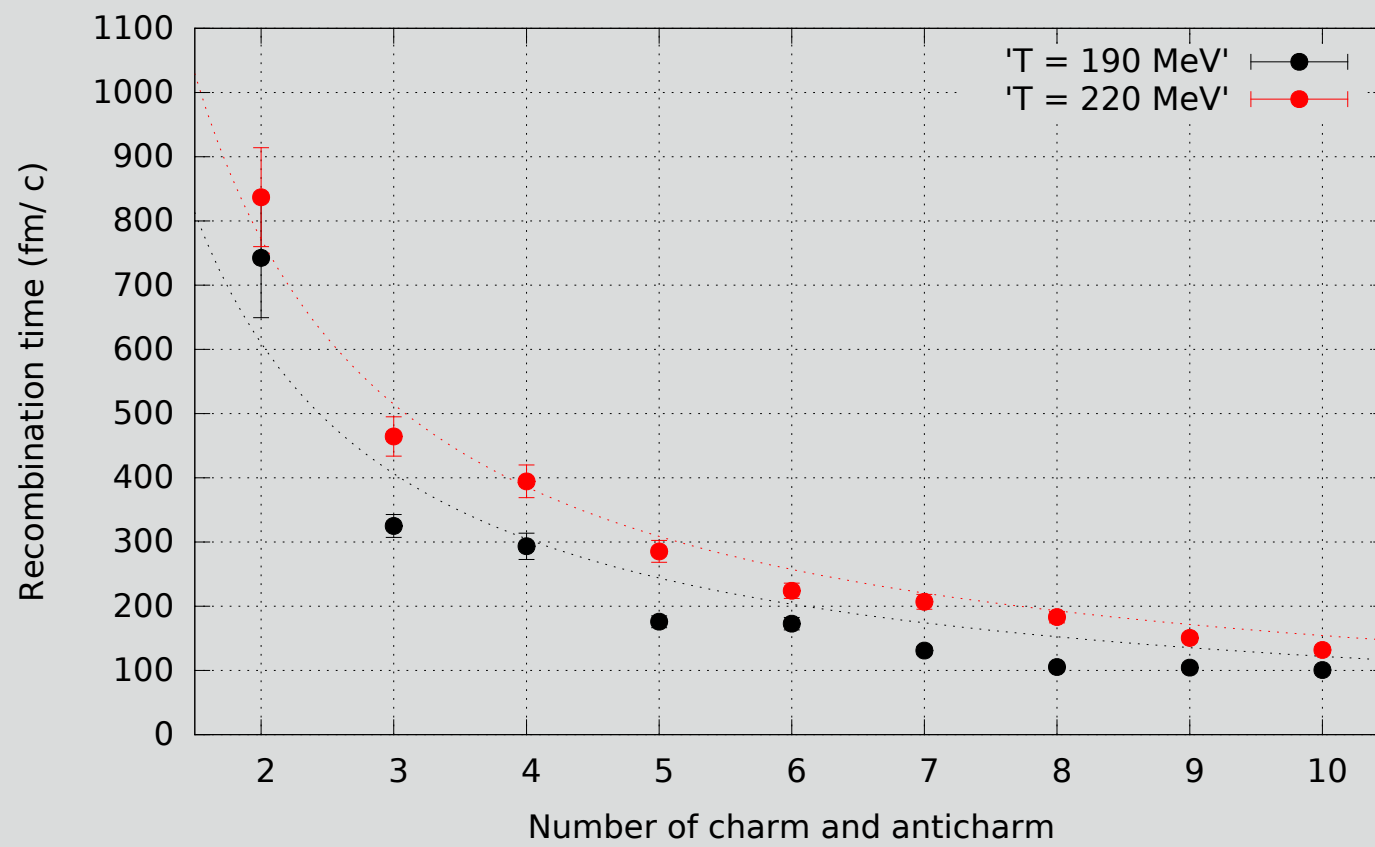
$$P_{q\bar{q}}^{\text{ideal}}(r) = \frac{3}{a} \left( \frac{r}{a} \right)^2 \left( 1 - \left( \frac{r}{a} \right)^3 \frac{1}{N} \right)^{N-1} \stackrel{N \gg 1}{\simeq} \frac{3}{a} \left( \frac{r}{a} \right)^2 e^{-(r/a)^3}$$

# Recombination time

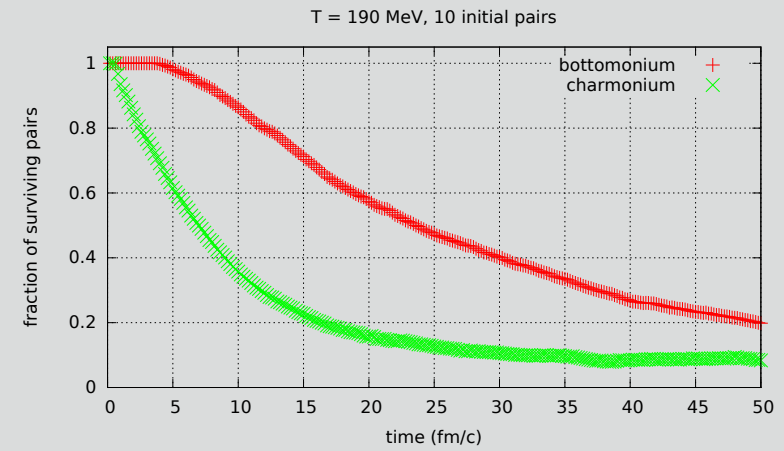
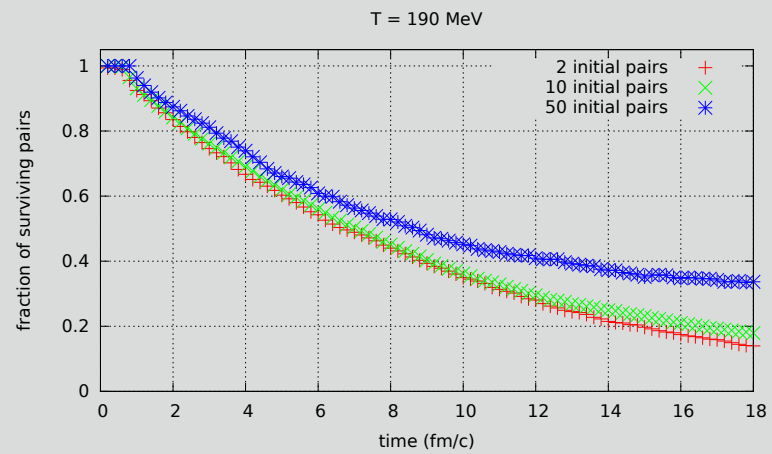
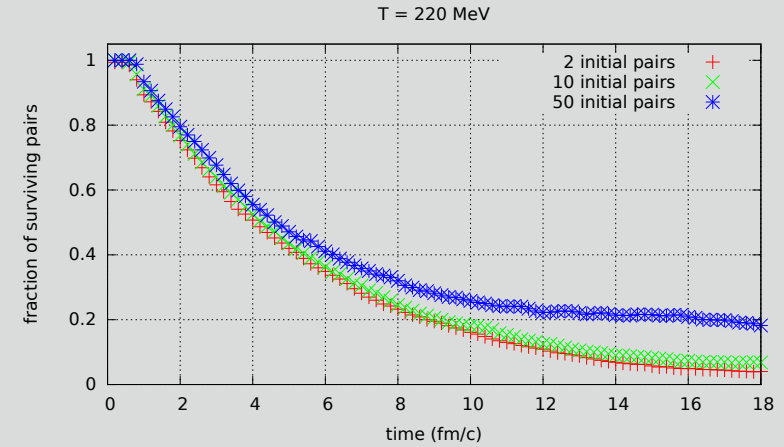
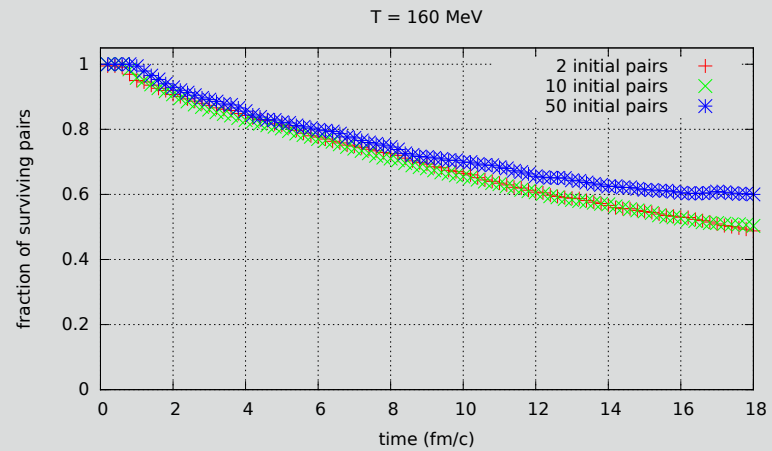


# Distribution of recombination times

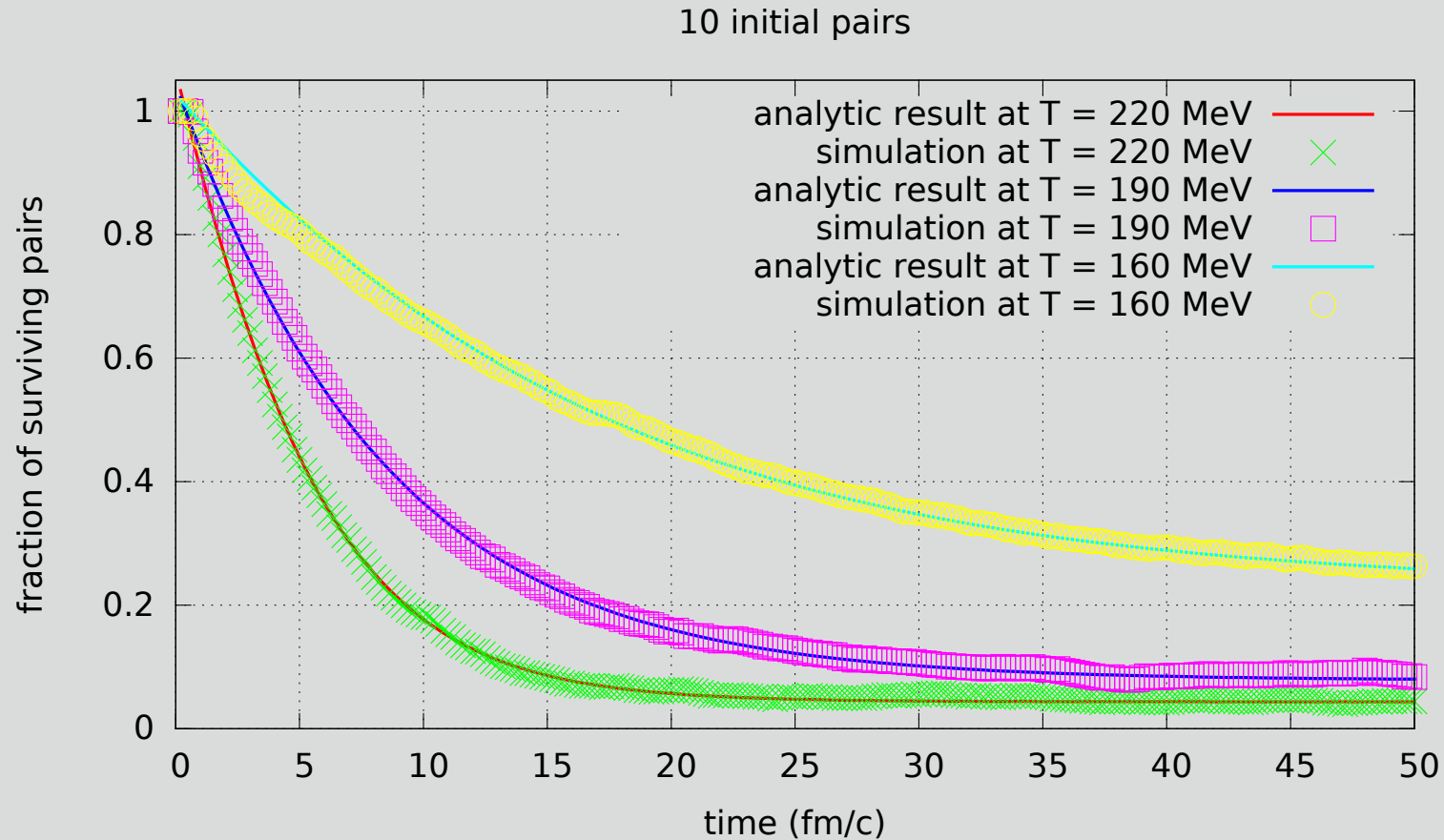




# Dissociation/recombination



Evolution of population of bound states  
is well described by a simple rate equation



$$\frac{dN(t)}{dt} = -\lambda_D N(t) + \lambda_R N_q(t) N_{\bar{q}}(t)$$

in agreement with previous works

# Summary

- first steps in an effort to describe the full dynamics of heavy quarks in a quark-gluon plasma, including bound state formation
- consistent treatment of various physical effects (screening, collisions, etc)
- systematic improvements are possible at many places (relax the abelian approximation, move from classical Langevin to quantum Schroedinger equation, etc)
- and, of course, comparison with data should be done.... after everything is under control