MAX-PLANCK-GESELLSCHAFT

Laser-driven relativistic recollisions and vacuum polarization effects

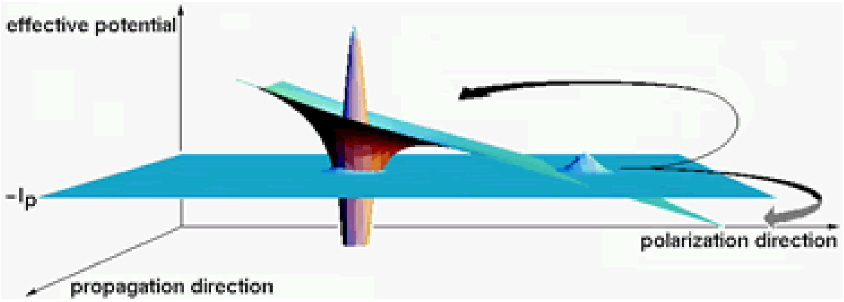
K. Z. Hatsagortsyan, M. Klaiber, C. Müller, A. Di Piazza, and C. H. Keitel

Content

- Relativistic recollisions
  - HHG in relativistic regime
    - in IR laser assisted with a strong APT
    - with counter-propagating super-strong APTs
  - Laser-driven collider
    - Coherent recollisions/high luminosity
    - Muon pair production
- Vacuum polarization effects in laser fields
    - Nonperturbative effects
    - Harmonics via laser and proton beam collision
    - High-order harmonics

Motivation

Laser-driven recollisions in relativistic regime: 100 keV – MeV energy



- Coherent hard x-rays/γ-rays
- Initiation of nuclear reactions
- Time-resolved nuclear spectroscopy?

HHG via laser pulse reflection from overdense plasma : $n_c \sim 4\xi^2$

HHG via three-step process : $n_c \approx 3mc^2\xi^2/4\hbar\omega$

Parameter of the relativistic regime for free electrons: $\xi = \frac{v_E}{c} = \frac{eE}{mc\omega} \geq 1$

Highly charged ions/underdense plasma

1) Single atom response: relativistic drift

Recollison is suppressed by the relativistic drift in laser propagation direction: **vxB**

2) Macroscopic response: phase-mismatch

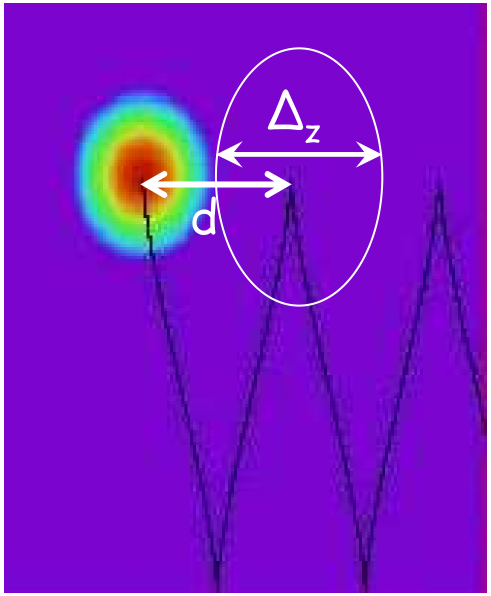
When relativistic effects are important?

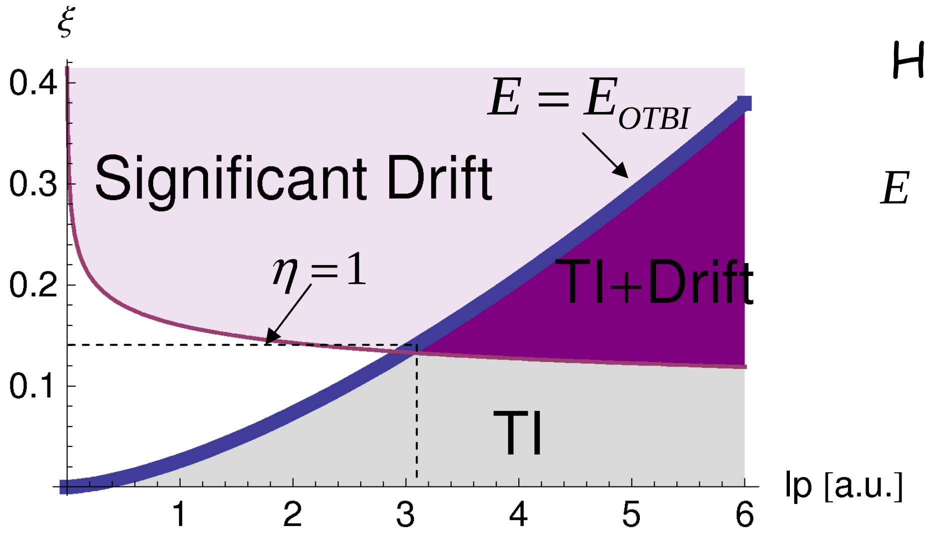
Drift distance due to **vxB** : $d \approx \frac{c}{\omega}\frac{\xi^2}{2}$

Wave packet spreading: $\Delta_z \approx \frac{\Delta v_z}{\omega} \approx \frac{1}{\omega}\frac{2^{3/4}\sqrt{eE\hbar}}{m^{3/4} I_p^{1/4}}$

$\Downarrow$

$$\boxed{\eta = \left(\frac{d}{\Delta_z}\right)^2 = \sqrt{\frac{2 I_p}{m c^2}} \frac{\xi^3}{16} \frac{m c^2}{\hbar \omega} \geq 1}$$





HHG

$E < E_{OTBI} \quad \Rightarrow \quad \xi < \frac{1}{2^{5/2}} \frac{mc^2}{\hbar\omega}\left(\frac{I_p}{mc^2}\right)^{3/2}$

$\xi \approx 1 \qquad I \approx 10^{18}\, W/cm^2$ (IR)

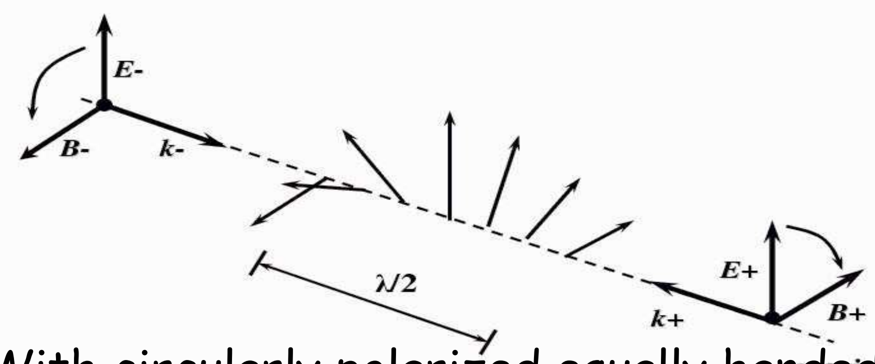
$\xi > 0.15 \qquad I > 5\times10^{16}\, W/cm^2$

$U_p \approx 3 keV$

Proposals to circumvent the drift

Ions counterpropagating the laser beam

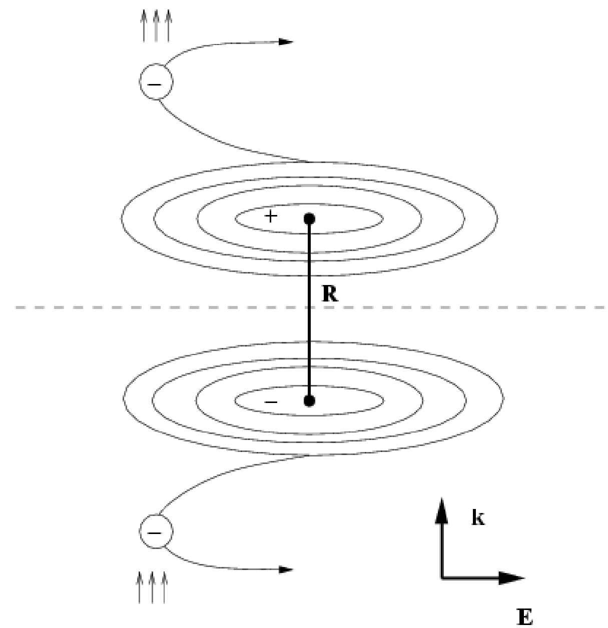
C.C. Chirilă, et al. PRL 93, 243603 (2004)
G. Mocken, C.H. Keitel, JPB 37, L275 (2004)



With circularly polarized equally handed counterpropagating laser waves

N. Milosevic, et al. PRL 92, 013002 (2004)

Employing antisymmetric molecular orbitals



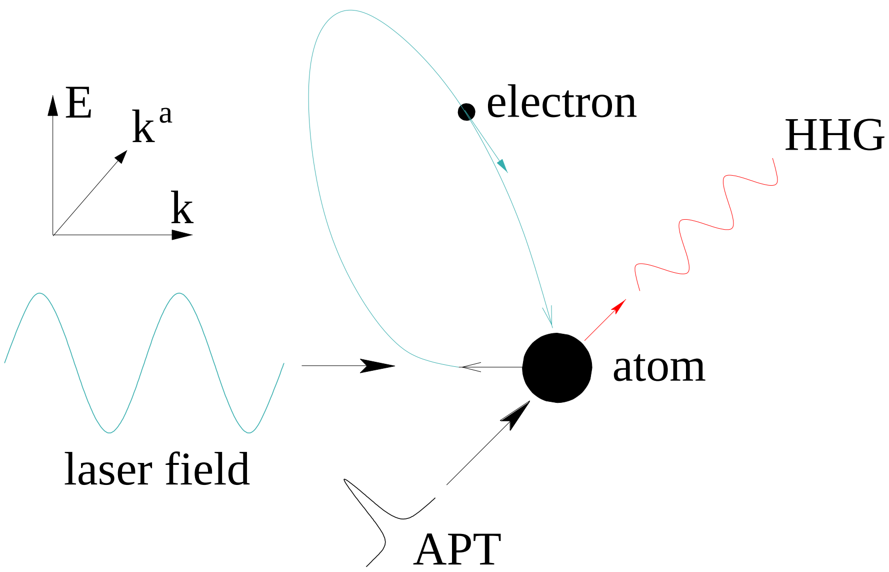
R. Fischer, et al.
PRL 97, 143901 (2006)

What we propose

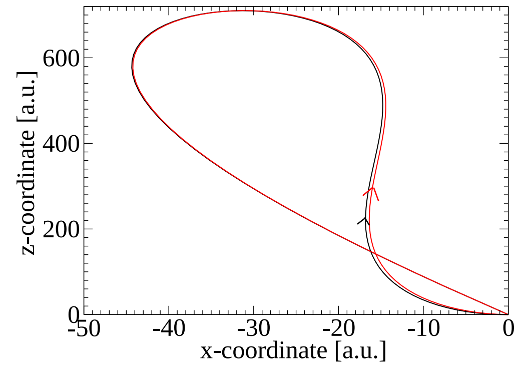
to use the emerging tool of attosecond pulses for the problem of relativistic drift

- Laser pulse + strong attosecond pulse train:
  generate electron with large initial momentum
- Counterpropagating super-strong attosecond pulses:
  revert the relativistic drift

Relativistic HHG with IR laser & APT



Control of the ionization energy and phase by APT



Electron can recollide if it is tunneled with an initial momentum in the laser propagation direction: $p_{z0} \approx -mc\xi^2/4$

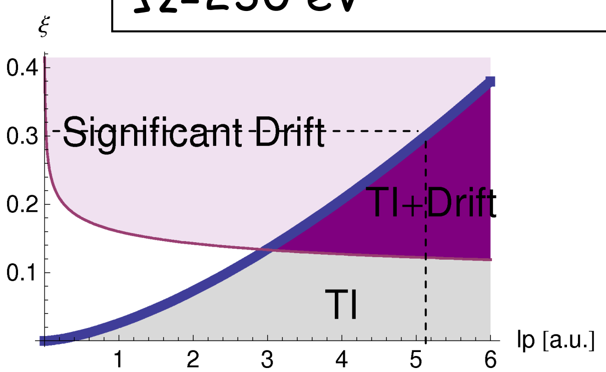
$\xi \approx 0.3$; Ip=5.29 a.u. (Ar^{7+})

Ω=230 eV

via tunneling: $$W \sim \exp\left\{-\frac{2\left(2mI_p + P_{z0}^2\right)^{3/2}}{3me\hbar\left|E(\eta_0)\right|}\right\}$$

with APT: $$\varepsilon_0 = \hbar\Omega - I_p$$

$$\varepsilon_0 \approx p_d^2/2m \approx mc^2\xi^4/32$$

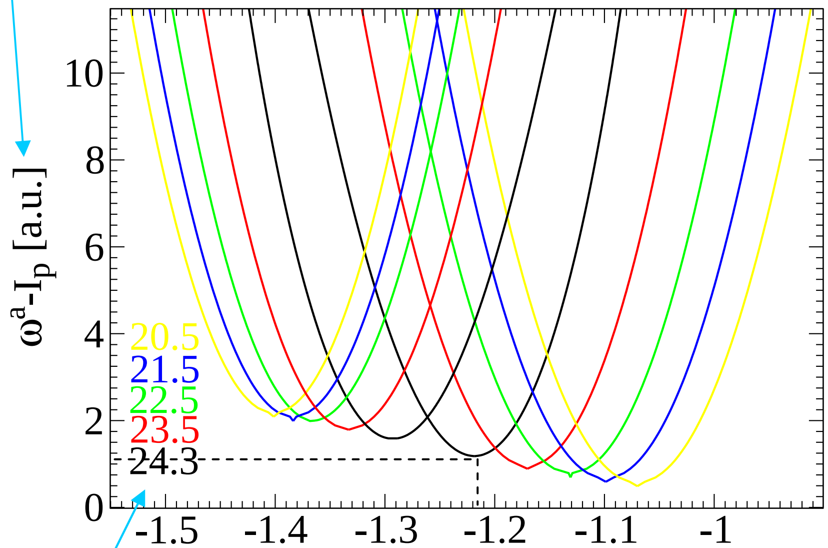


Cutoff energy = 3.17 Up ≈ 39 keV

Relativistic HHG with IR laser & APT

Favorable APT parameters for the drift compensation

Electron initial energy



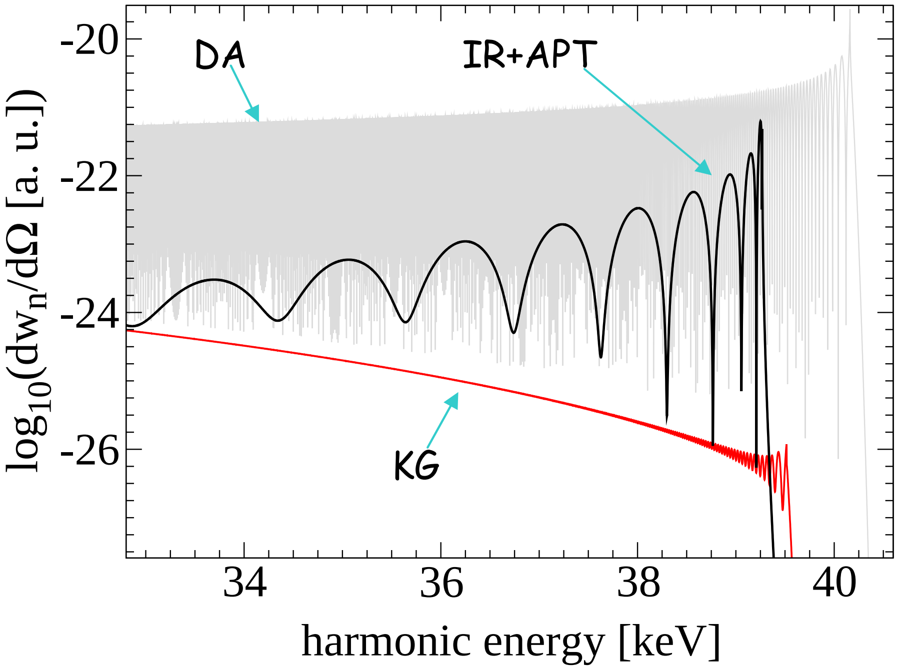
Recollision energy

Ionization phase

For instance, the electron will have a rescattering energy of 24.3 keV if it is ionized at -1.2 rad with a XUV photon of 5.8 a.u.

The drift compensation can occur only at a specific phase of ionization for a certain XUV frequency. This is controlled by the phase delay between the APT and IR fields for a certain harmonic emission.

Relativistic HHG with IR laser & APT



Cutoff energy ≈ 39 keV

100 photons per pulse
at 39 keV (in 1 keV window)

APT pulse energy: 300 pJ

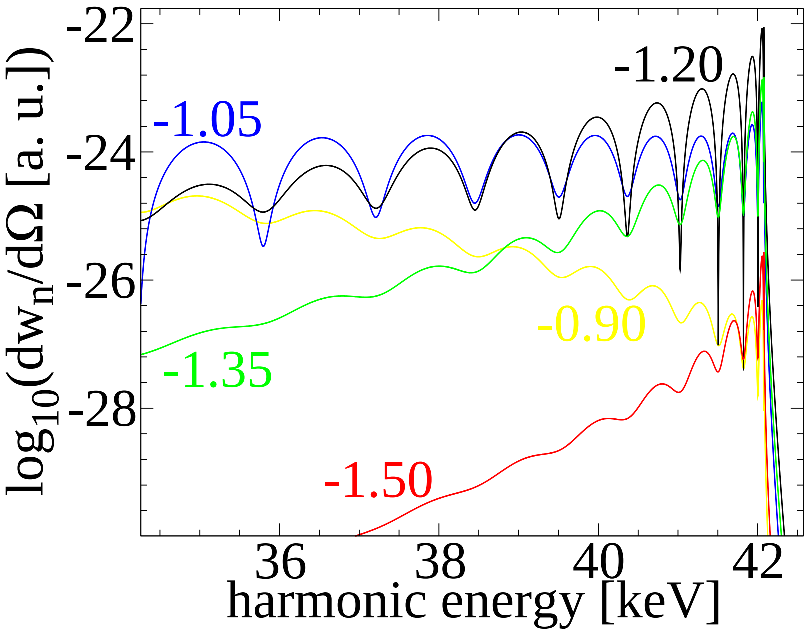
ion density: 10^{16} cm^{-3}

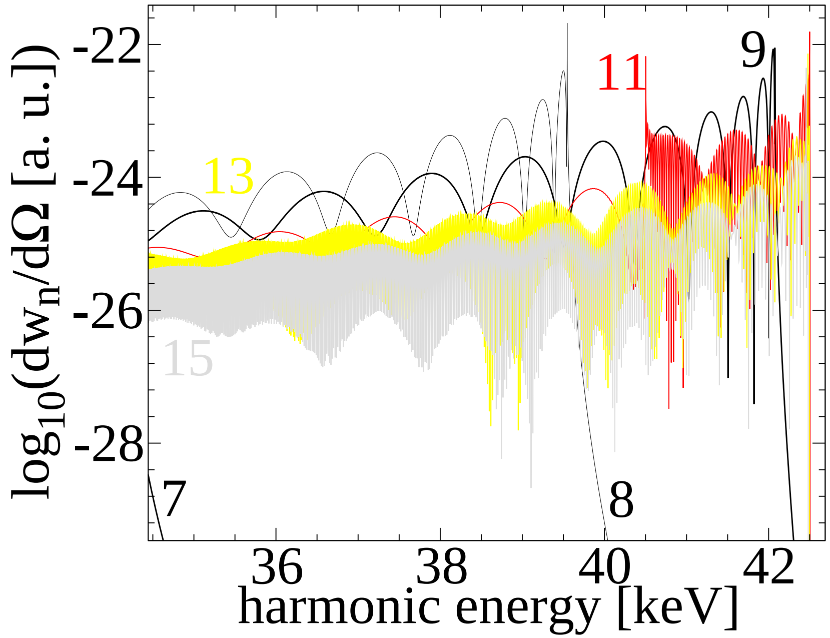
- Relativistic SFA: ξ=0.3
- APT field is perturbation: $eE_a a_0 \ll Ip$; E_a=0.02 a.u.; Ip=5.29 a.u. (Ar^{7+})
- APT has no influence on the excursion: $\omega\tau \ll 1$; $\tau \approx 100$ as
- DA for APT: $c/\Omega \gg a_0$; $\hbar\Omega \ll mc^2$; $\xi_a = eE_a/mc\Omega \ll 1$; Ω=8.5 a.u.≈230 eV

M. Klaiber et al., Opt. Lett. 33, 411 (2008)

Relativistic HHG with IR laser & APT

Stability against variation of APT parameters





Variable phase delay

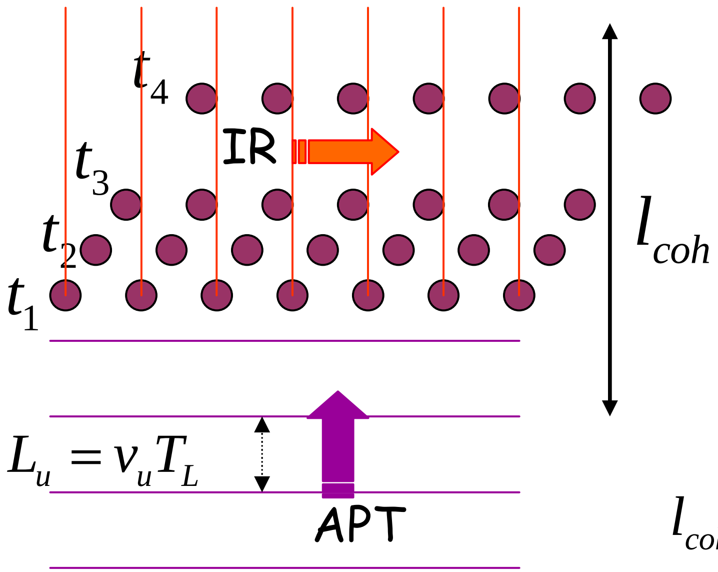
Variable Ω

Phase delay change of 0.15 rad has minor influence on HHG: $\omega\tau$=0.2 rad

Ω<8.5 a.u. HHG is strongly reduced.
Ω>8.5 a.u. HHG decreases slowly.

Relativistic HHG with IR laser & APT

Phase-matching problem



Strong laser pulse of ≈10^{17}W/cm^2, multiply ionized plasma Ar^{7+} :

The refraction of the laser wave only changes the geometry of the regions of HHG. The HHG process in the considered setup is triggered by APT. The coherence length of HHG is:

$$l_{coh} = \frac{\pi}{\omega_h \left|1/v_h - 1/v_u\right|} \approx 2\ cm$$

$$\left|1 - 1/v_u\right| \approx 10^{-9}$$

$$\left|1 - 1/v_h\right| \approx 3 \times 10^{-14}$$

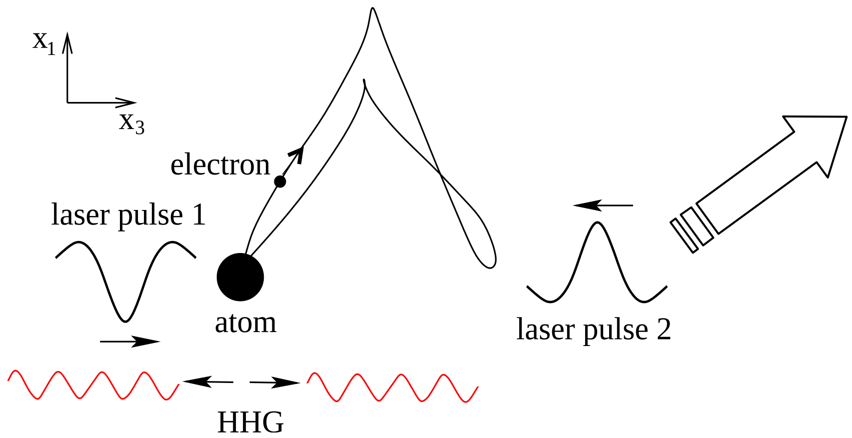
$$\rho = 10^{16}\ cm^{-3}$$

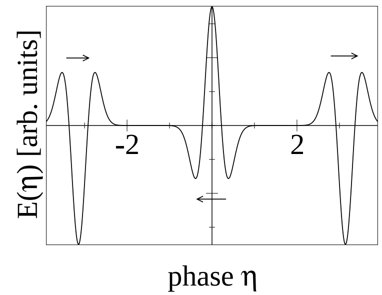
$$\omega_h = 39\ keV$$

The increase of duration of APT pulse due to dispersion is negligible.

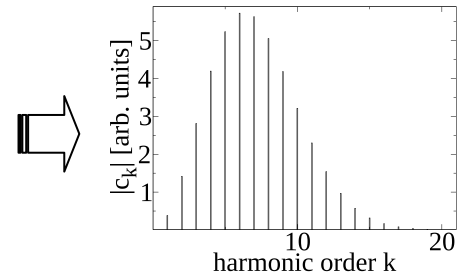
Relativistic HHG with counterpropagating APTs

Super-strong attosecond pulse trains





Pulse shape



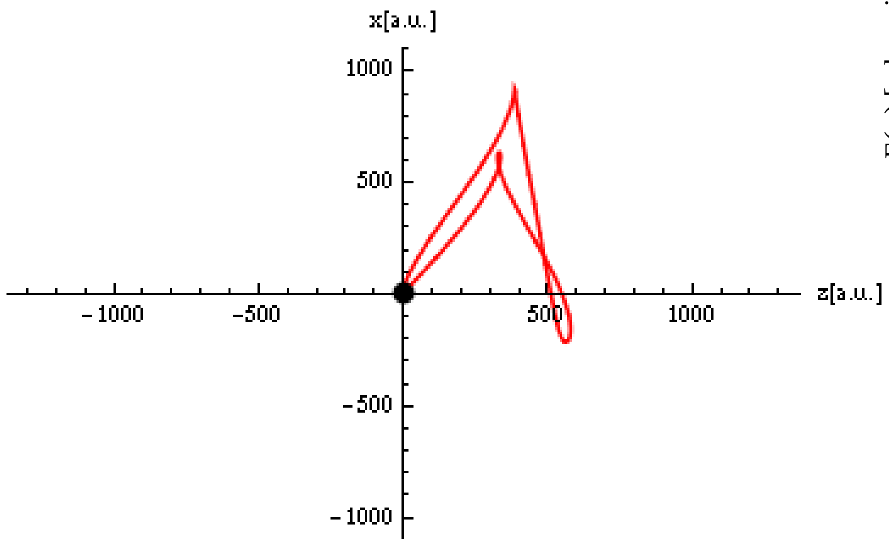
Pulse spectrum

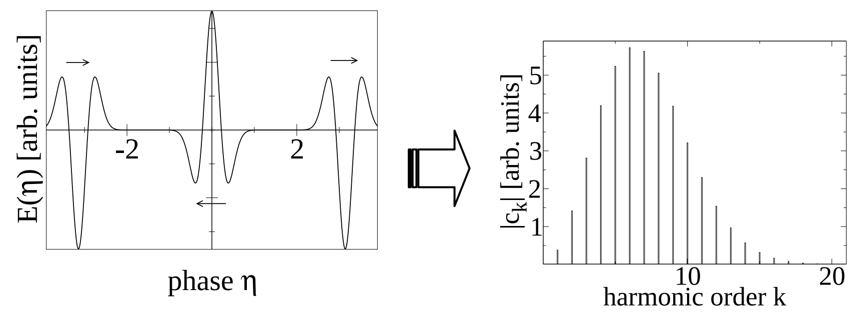
In this setup recollisions of the ionized electrons can be achieved in the highly relativistic regime by a reversal of the commonly deteriorating drift.

M. Klaiber et al., JOSAB 25, 92 (2008)

Relativistic HHG with counterpropagating APTs

Super-strong attosecond pulse trains





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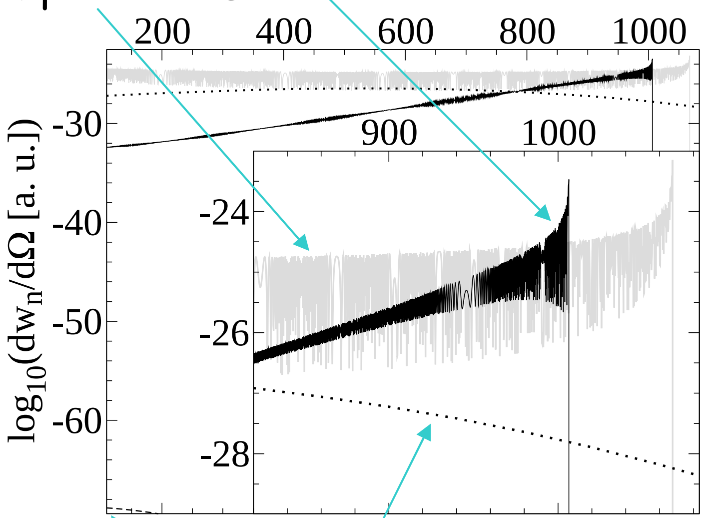
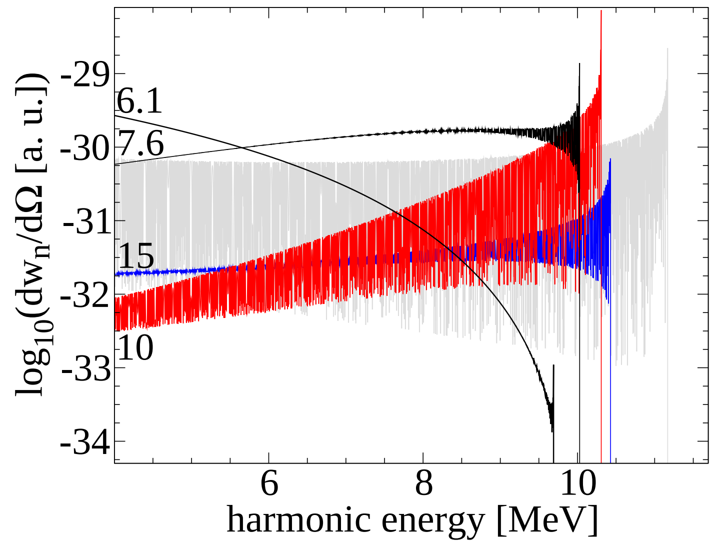
Relativistic HHG with counterpropagating APTs

Coherent γ-rays

1 MeV cutoff



10 MeV cutoff



Peak Intensity 8×10^{21} W/cm^2;
Ip=325 a.u. (Ar^{16+})

Peak Intensity 6×10^{20} W/cm^2;
Pulse energy 60 mJ
Ip=63 a.u. (Mg^{10+})

Advantages:
- drift is totally compensated
- no strict requirments on the pulse shape

Relativistic HHG with counterpropagating APTs

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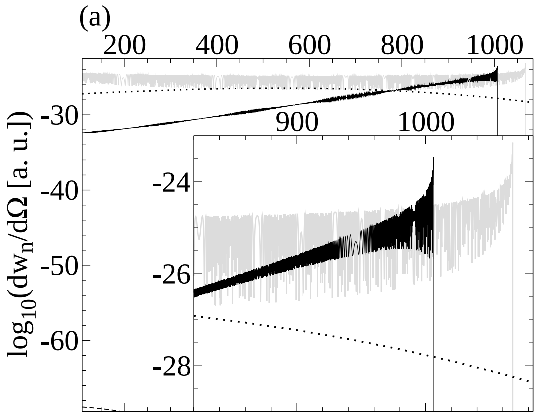
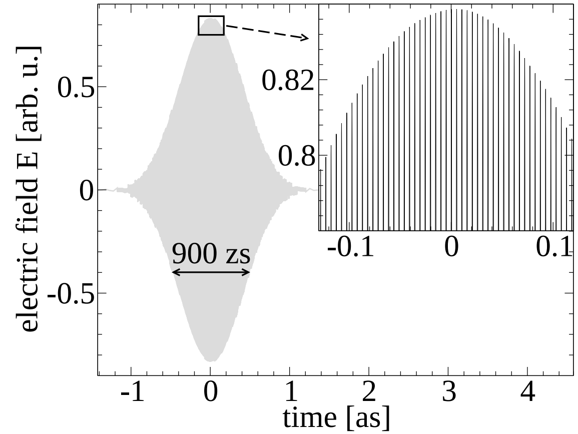
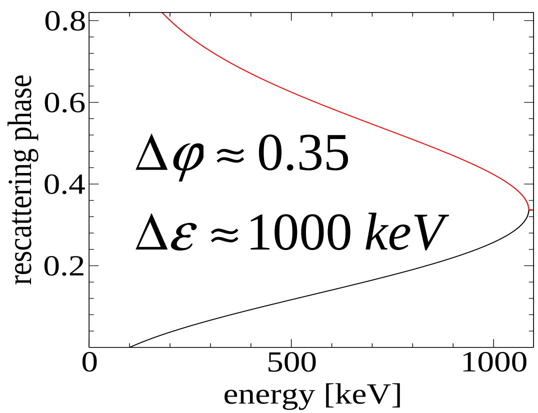
Zeptosecond γ-ray pulses

1 MeV cutoff



4 keV window





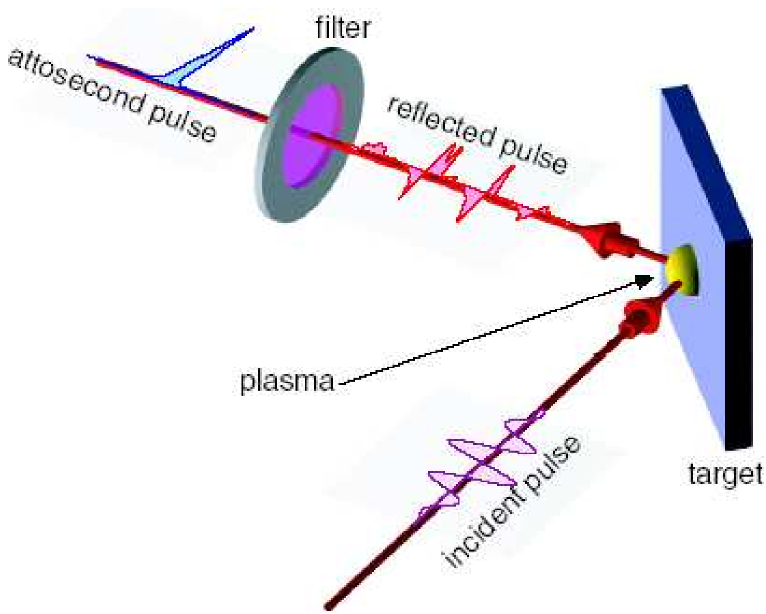
Zeptosecond pulse is feasible due to:
- main contribution in HHG from one trajectory
- harmonic chirp is not significant

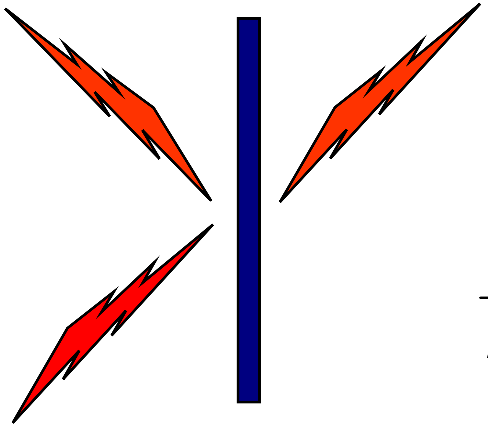
N_γ=100 photons/pulse

Ways for strong APT realization

Laser beam reflection from overdense plasma

Laser-plasma interaction in sliding mirror regime





$$\frac{n}{n_{cr}}\frac{\pi L}{\lambda} \approx a_0$$

High-energy APT generation without spectral filtering

Conversion efficiency ~ a few %

G.D. Tsakiris, et al., NJP **8**, 19 (2006)

A.S. Pirozhkov, et al., PP **13** , 013107 (2006)

Conclusion

- In IR laser field with an assistance of a weak APT HHG of several hundred keV can be produced
- In strong counter-propagating APTs HHG up to MeV energies and short pulses of γ-rays can be achieved.

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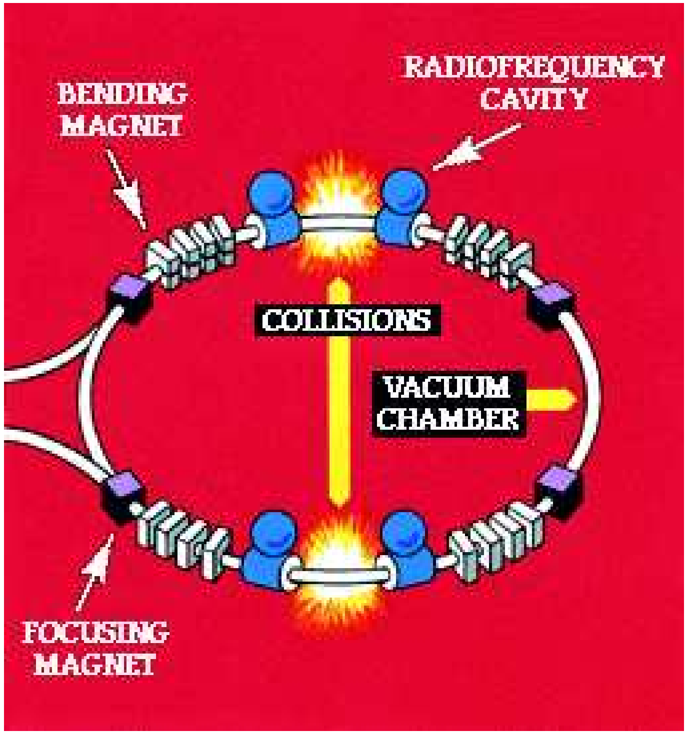
Laser-driven collider

Conventional collider



ε~100 -1000 GeV

L~10^{32} - 10^{34} cm^{-2} s^{-1}

Can particle reactions be initiated with laser fields?

r =1 fm

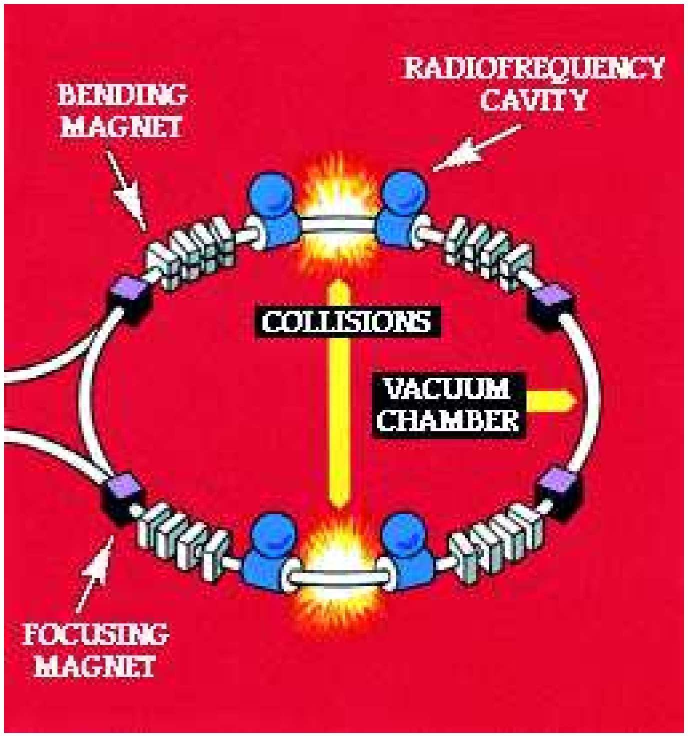
$\dot{N}= \sigma L \approx r_0^2 L \sim 1\ s^{-1}$

$\varepsilon \sim ch/r \sim 1$ GeV

$\boxed{L \sim 10^{26}\text{-}10^{27}\ cm^{-2}\ s^{-1}}$

Laser-driven collider

Conventional collider



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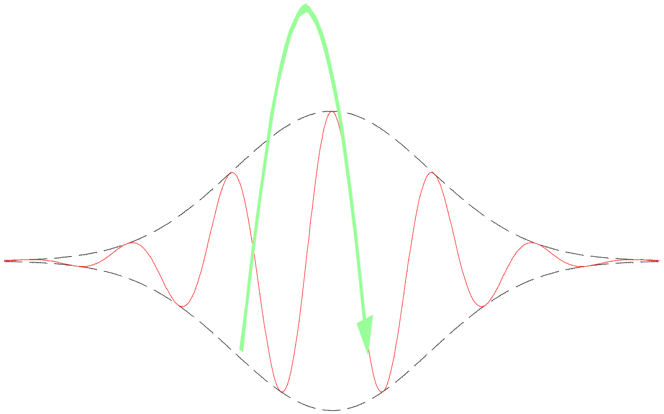
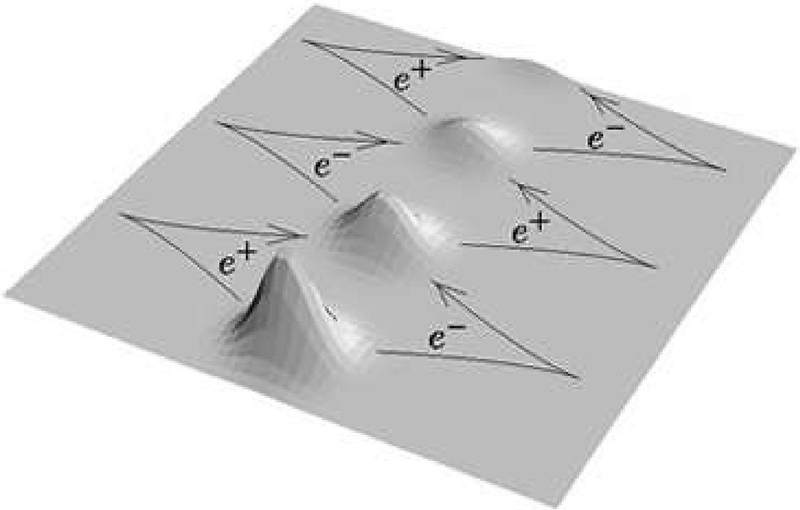
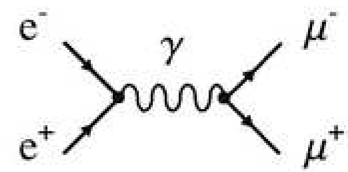
Combine acceleration, focusing and collision in a single stage in a laser field

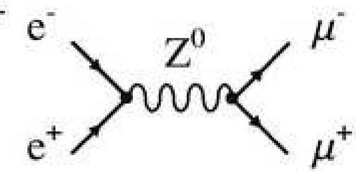


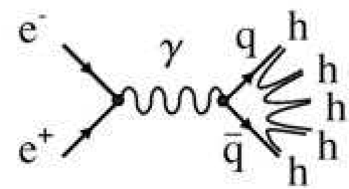
Laser-driven collider

Positronium in a laser field









Identical charge-to-mass ratio:

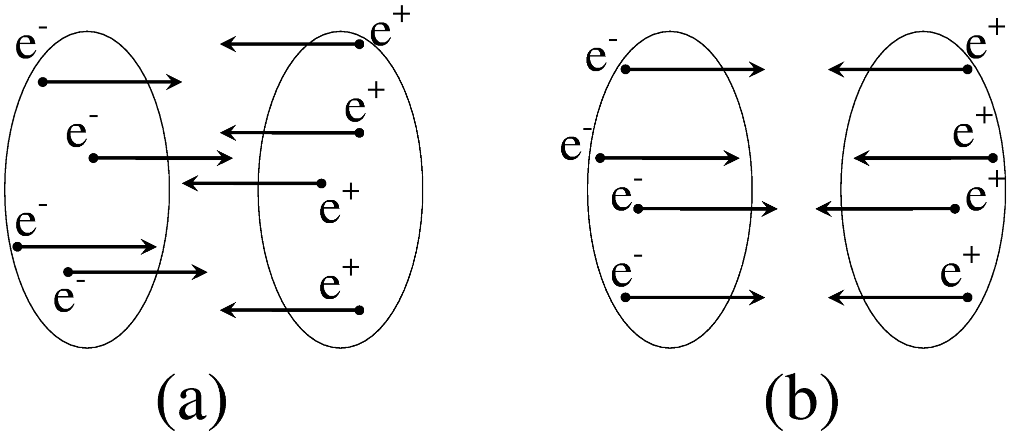
$$F = \frac{e}{c}\left(\vec{v} \times \vec{B}\right) \quad \vec{v} \propto e\vec{E}$$

Identical relativistic drift
Periodic electron-positron recollisions

B. Henrich et al. PRL **93**, 013601 (2004)

Laser-driven collider

Coherent recollisions



incoherent

coherent

Conventional colliders:

mean impact parameter ~ beam size a_b

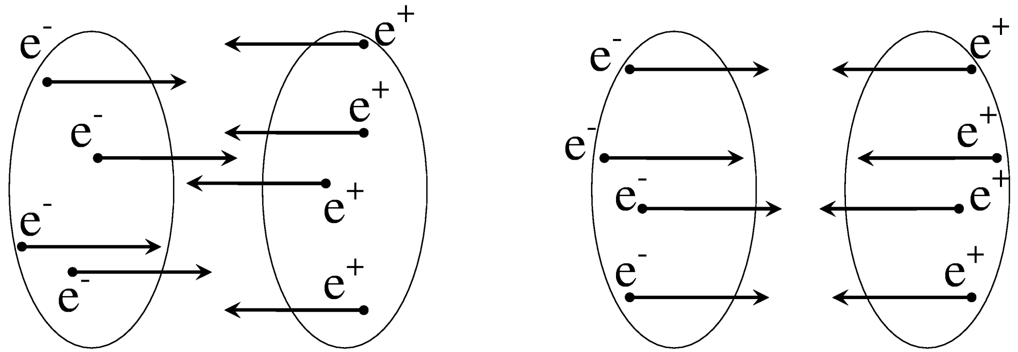
Laser-driven Ps:

mean impact parameter ~ electron wave packet size a_w

K. Hatsagortsyan et al. EPL **76**, 29 (2006)

Laser-driven collider

Coherent recollisions



(a) incoherent

(b) coherent

$$\frac{dN}{dt} = \sigma L$$

$$L = \left[\frac{N_e (N_e - 1)}{a_b^2} + \frac{N_e}{a_w^2} \right] f$$

Luminosity enhancement due to coherent component

Conventional colliders:

mean impact parameter ~ beam size a_b

Laser-driven Ps:

mean impact parameter ~ electron wave packet size a_w

K. Hatsagortsyan et al. EPL **76**, 29 (2006)

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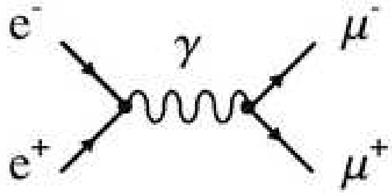
Laser-driven collider

Muon production

$$e^+e^- \to \mu^+\mu^-$$



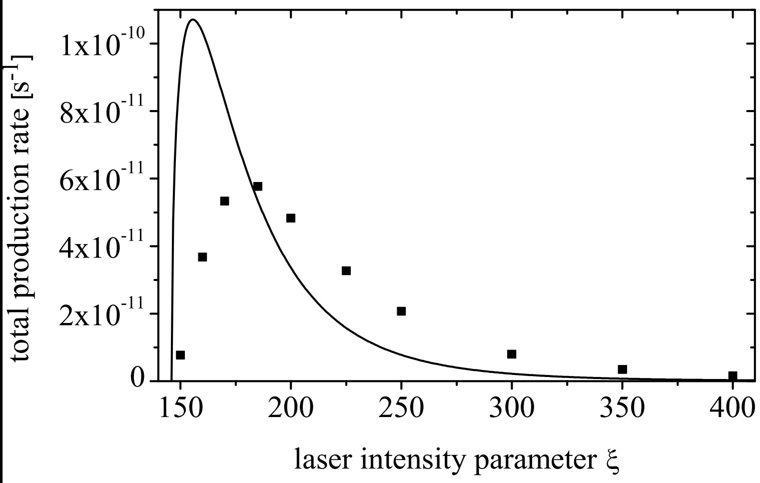
Threshold: $2mc^2\xi \ge 2Mc^2$ I>4x10²² W/cm²

Rigorous QED result can be estimated by simpleman model via field free cross-section σ and electron wave packet spreading a_0:

$$\boxed{R \sim \frac{c\sigma}{a_0^3}}$$

Field-free cross-section

$$\sigma \approx \frac{4\pi}{3}\frac{r_0^2}{\gamma^2} \qquad \gamma \sim \xi$$



Wave packet spreading:

$$a_0 \sim v_a t_r$$

$$t_r \sim 2\pi\gamma/\omega \sim 2\pi\xi/\omega$$

C. Müller et al. PLB 659, 209 (2008)

MAX-PLANCK-GESELLSCHAFT

Laser-driven collider

Muon production

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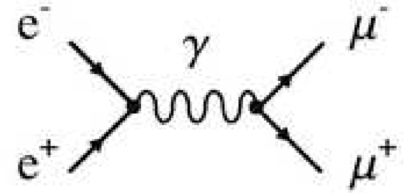

Threshold: $2mc^2\xi \geq 2Mc^2$ $I > 4\times10^{22}\ W/cm^2$

Rigorous QED result can be estimated by simpleman model via field free cross-section σ and electron wave packet spreading a_0:

$$\boxed{R \sim \frac{c\sigma}{a_0^3}}$$

Field-free cross-section

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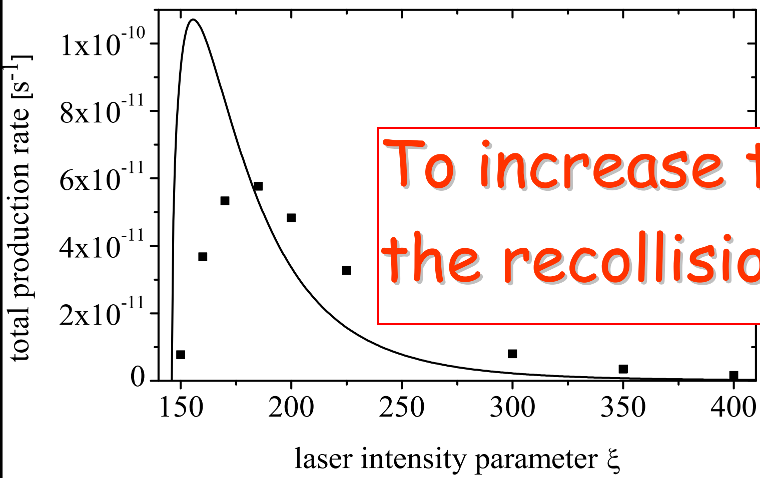


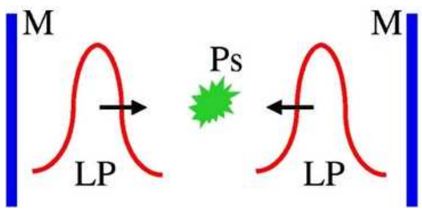
Wave packet spreading:

To increase the reaction rate in LDC, the recollision time should be reduced.

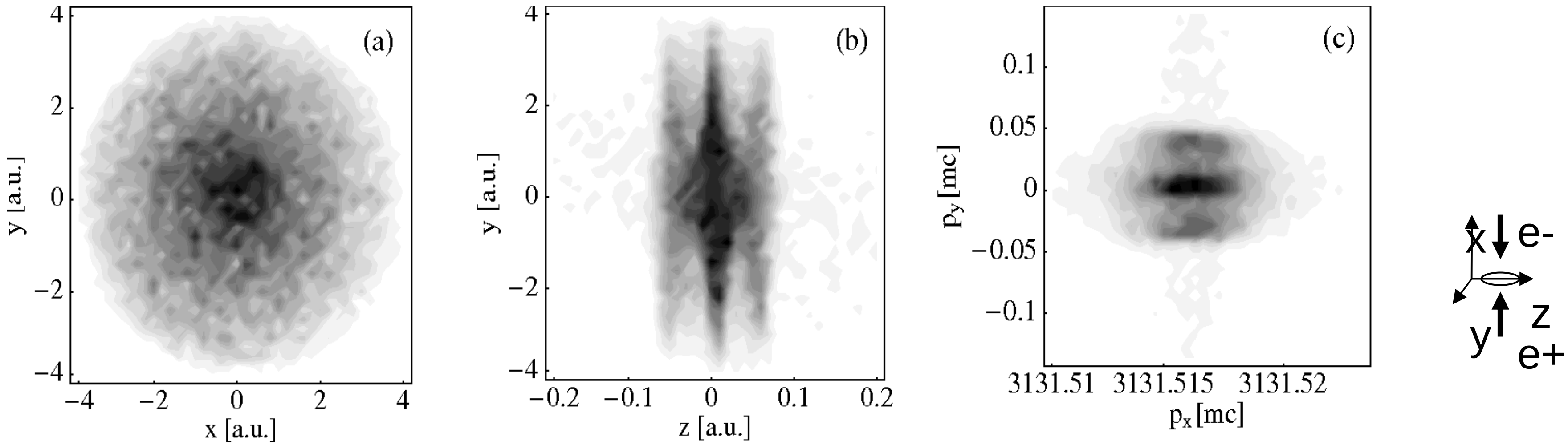
C. Müller et al. PLB 659, 209 (2008)

Laser-driven collider

Counter-propagating laser pulses

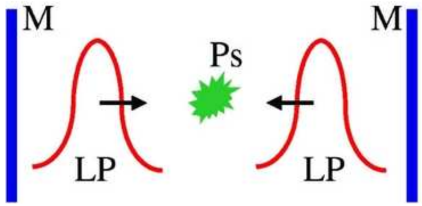


Short recolision time ~ T/2
Wave packet spreading is not large: $a_o<4a_B$
Scattering energy: $\varepsilon = mc^2\xi$



Laser-driven collider

Counter-propagating laser pulses



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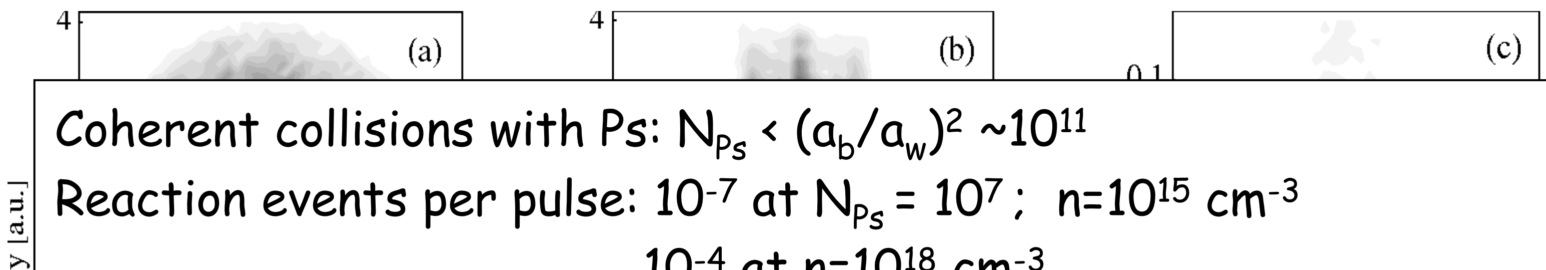


Coherent collisions with Ps: $N_{Ps} < (a_b/a_w)^2 \sim 10^{11}$

Reaction events per pulse: 10^{-7} at $N_{Ps} = 10^7$; $n=10^{15}$ cm^{-3}

10^{-4} at $n=10^{18}$ cm^{-3}

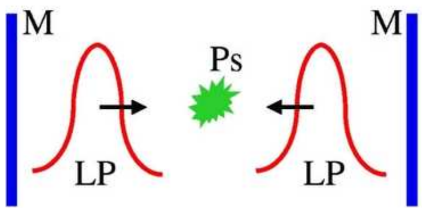
Cassidy et al. Nature 449, 195 (2005)

Eff. Luminosity: $L=10^{23}$-10^{26} f cm^{-2}s^{-1}

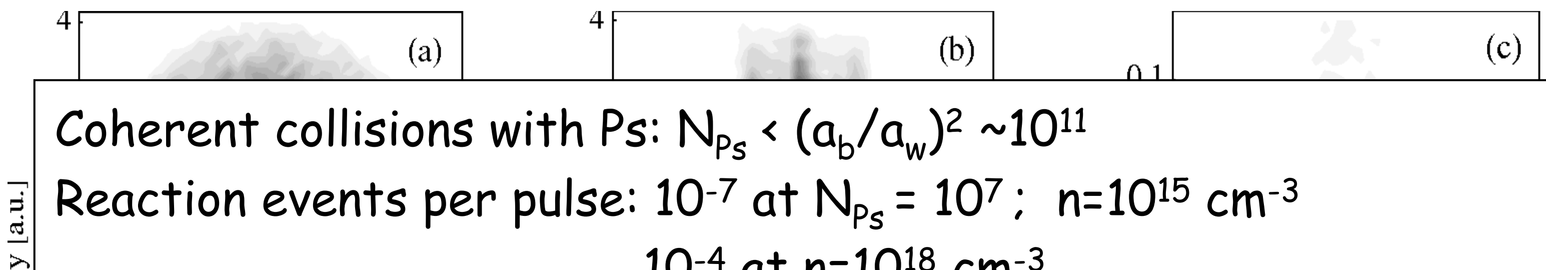
One reaction event per sec at f=1 kHz

Laser-driven collider

Counter-propagating laser pulses



Short recolision time ~ T/2
Wave packet spreading is not large: $a_o < 4a_B$
Scattering energy: $\varepsilon = mc^2 \xi$



Coherent collisions with Ps: $N_{Ps} < (a_b/a_w)^2 \sim 10^{11}$
Reaction events per pulse: 10^{-7} at $N_{Ps} = 10^7$; $n=10^{15}$ cm^{-3}
10^{-4} at $n=10^{18}$ cm^{-3}
Cassidy et al. Nature 449, 195 (2005)
Eff. Luminosity: $L=10^{23}\text{-}10^{26}$ f cm^{-2}s^{-1}
One reaction event per sec at f=1 kHz

Incoherent collisions with e+e- plasma:
Reaction events per pulse: 10^{-8} at $n = 10^{16}$ cm^{-3} Surko et al. PP 11, 2333 (2004)
10^{-4} at $n= 10^{18}$ cm^{-3}

Eff. Luminosity: $L=10^{22}\text{-}10^{26}$ f cm^{-2}s^{-1}

Conclusion

- Laser-driven collider (LDC) can be realised using e+e- coherent recollisions from Ps atom in a laser field.
- LDC based on a gas of Ps atoms in the field of two crossed laser beams will enable a high scattering luminosity by using coherent head-on-head collisions.
- In a dense electron-positron plasma LDC can operate with incoherent collisions.

Content

- Relativistic recollisions
  - HHG in relativistic regime
    - in IR laser assisted with a strong APT
    - with counter-propagating super-strong APTs
  - Laser-driven collider
    - Coherent recollisions/high luminosity
    - Muon pair production
- Vacuum polarization effects in laser fields
    - Nonperturbative effects
    - Harmonics via laser and proton beam collision
    - High-order harmonics

Vacuum polarization in laser fields

Can nonperturbative vacuum polarization effects be observed with strong lasers?

Classical parameter of strong fields:

$$\xi = \frac{e\sqrt{A_\mu A^\mu}}{mc^2} = \frac{eE\lambda}{2\pi mc^2} = \frac{eE\lambda\!\!\!^{-}_c}{\hbar\omega}; \qquad \lambda\!\!\!^{-}_c = \frac{\hbar}{mc}$$

ξ=1, I≈10^{18} W/cm^2

ξ<<1, no multiphoton effects
ξ>>1, adiabatic limit

Quantum parameter of strong fields:

$$\chi = \frac{e\sqrt{(F_{\mu\nu}p^\nu)^2}}{(mc^2)(mc)}\lambda_c = \left.\frac{eE\lambda_c}{mc^2}\right|_{r.f.} = \left.\frac{E}{E_{cr}}\right|_{r.f.} \ or \ = \left.\frac{\Omega}{m}\frac{E}{E_{cr}}\right|_{r.f.}$$

ξ<<1 or χ<<1
perturbative regime

$\Downarrow$

χ=1, I≈10^{29} W/cm^2

Spontaneous
electron-positron
pair production

χ<1 vacuum is stable

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⇩

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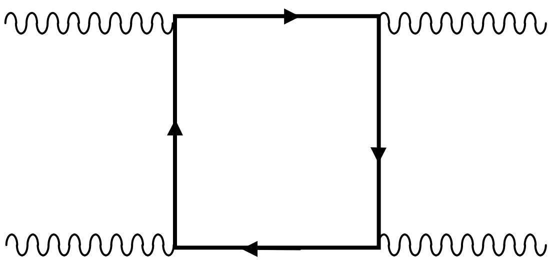

Vacuum dispersive nonlinearities

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$\xi=1$, $I \approx 10^{18}$ W/cm^2

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$\xi \ll 1$ or $\chi \ll 1$
perturbative regime

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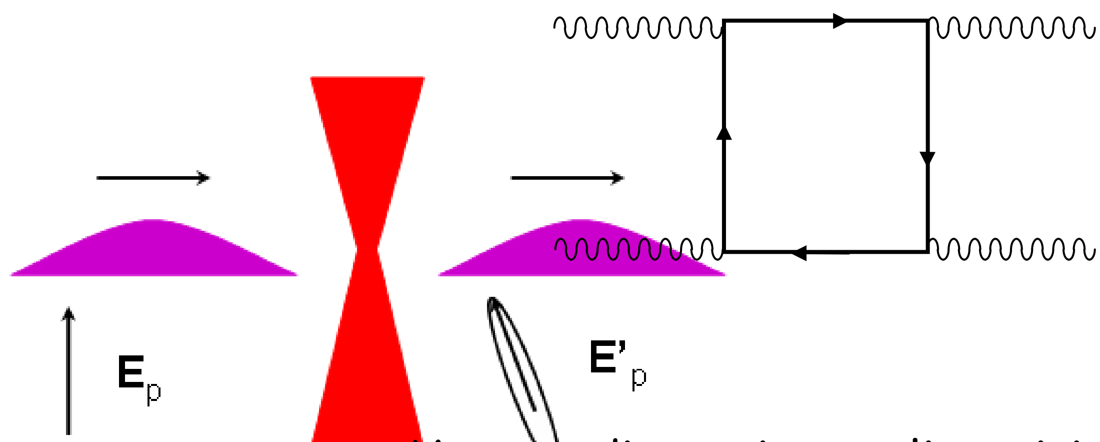
Spontaneous electron-positron pair production

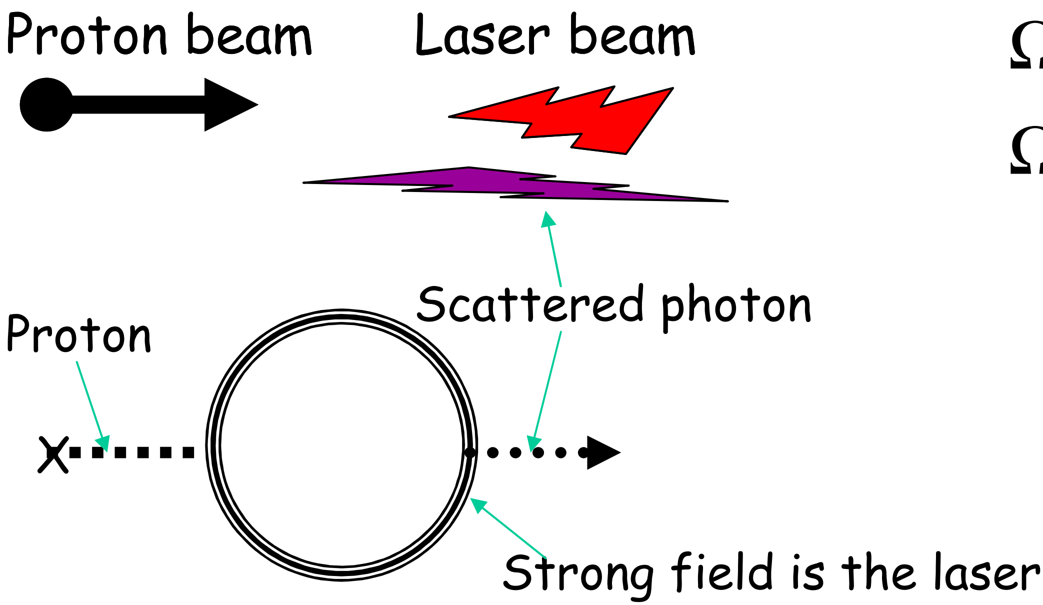
$\chi<1$ vacuum is stable



Vacuum dispersive nonlinearities

Vacuum polarization in laser fields

Photon fusion during laser and proton beam collision



$$\Omega \approx 2\gamma\omega_L; \quad E \approx 2\gamma E_L$$

$$\Omega \sim m; \quad E \sim E_{cr}$$

Di Piazza et al. PRL 100, 010403 (2008)

Vacuum polarization in laser fields

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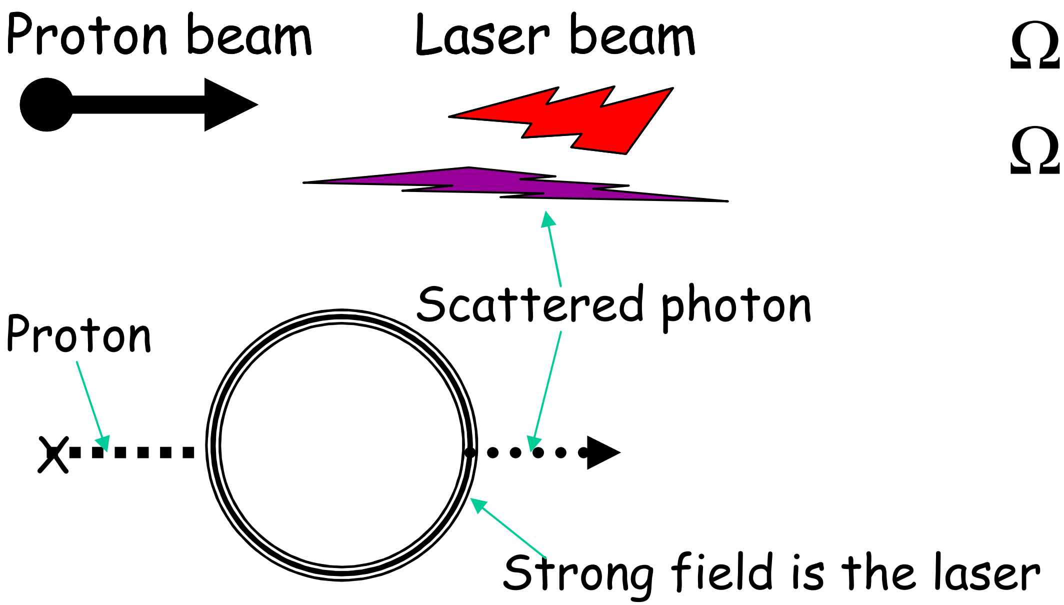

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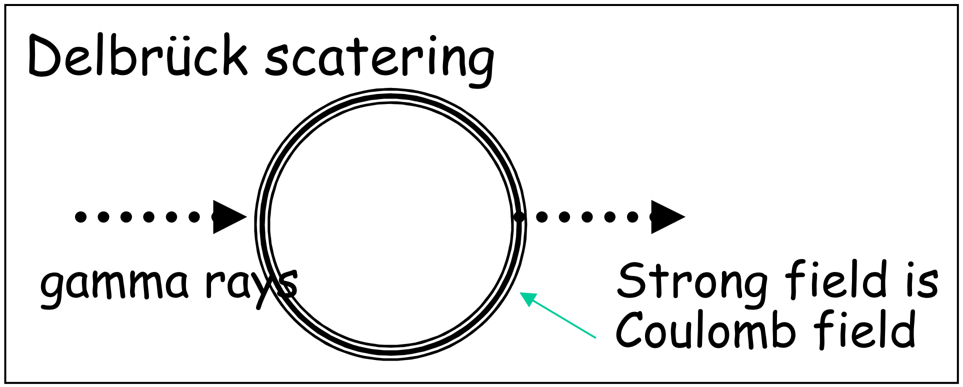

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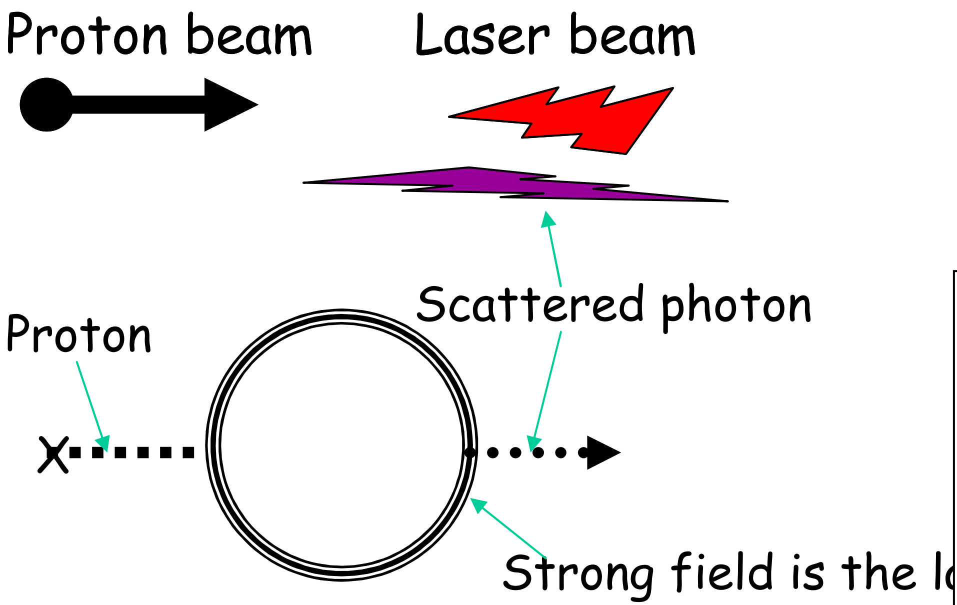


$$\Omega \approx 2\gamma\omega_L; \quad E \approx 2\gamma E_L$$

$$\Omega \sim m; \quad E \sim E_{cr}$$

Nonliner QED $\chi = \frac{2\Omega}{m}\frac{E}{E_{cr}} >> 1$

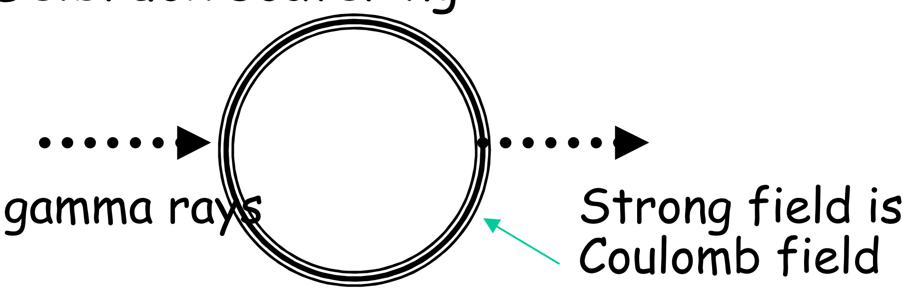
Perturbative: $c_n \sim \chi^{2n}$

Nonperturbative: $c_n \sim \chi^{2/3}$

Opening of multiphoton channels:

$$R_n \sim 1/n^5$$

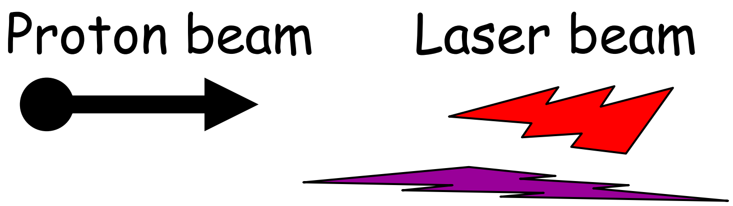
Delbrück scatering

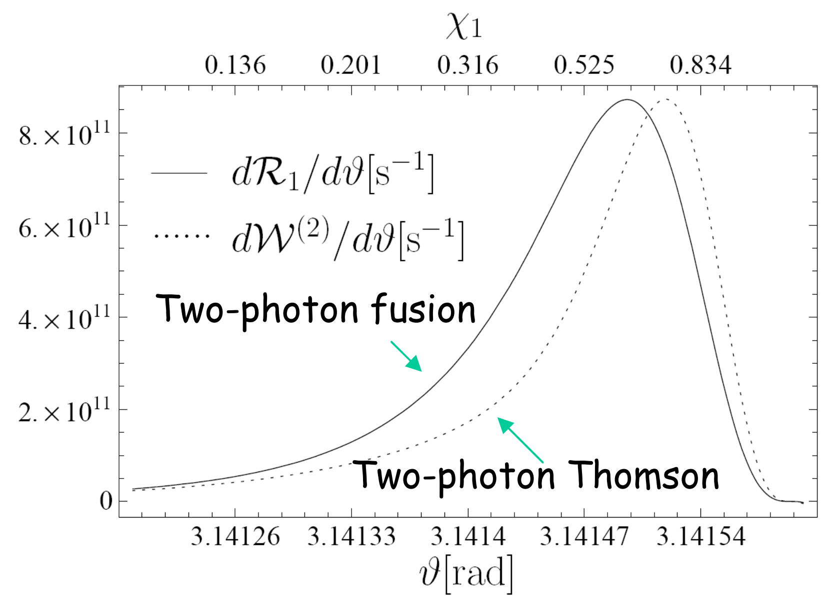


Di Piazza et al. PRL 100, 010403 (2008)

Vacuum polarization in laser fields

Photon fusion during laser and proton beam collision





LHC: Proton energy 7TeV; $N_p=10^{11}$

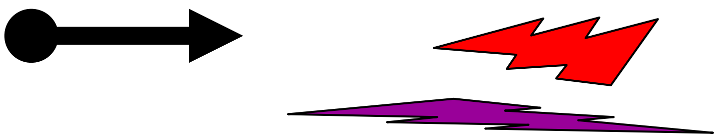
Laser: $I=3\times10^{22}$ W/cm^2, IR

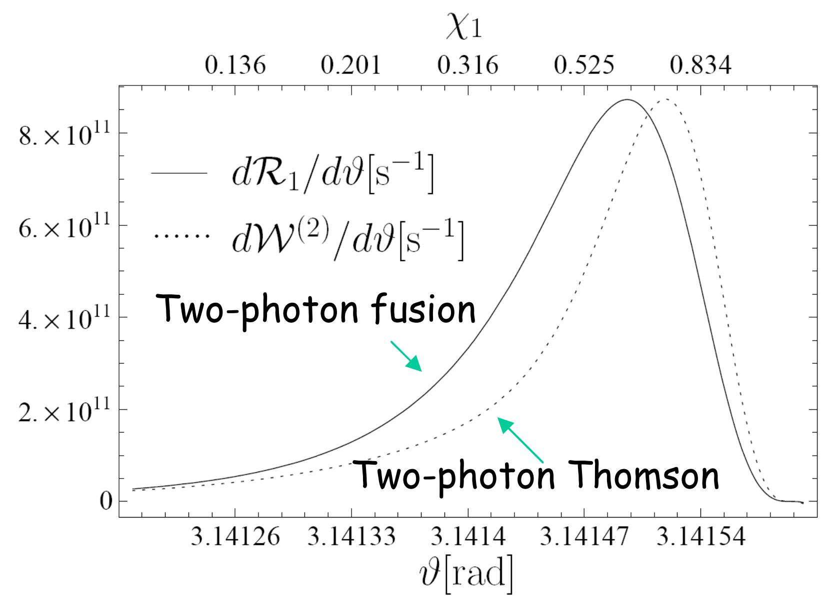
Second harmonic: 400 events/h
4th: 6 events/h

Vacuum polarization in laser fields

Photon fusion during laser and proton beam collision

Proton beam　　Laser beam





Tevatron: Proton energy 980 GeV; $N_p=10^{11}$

XUV Laser : $I=4\times10^{22}$ W/cm^2, ω=70 eV

Second harmonic: 500 events/h
4th: 7 events/h

LHC: Proton energy 7TeV; $N_p=10^{11}$

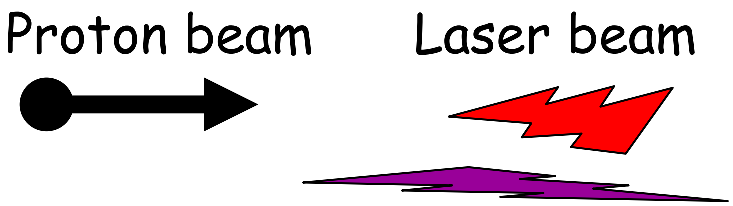
Laser: $I=3\times10^{22}$ W/cm^2, IR

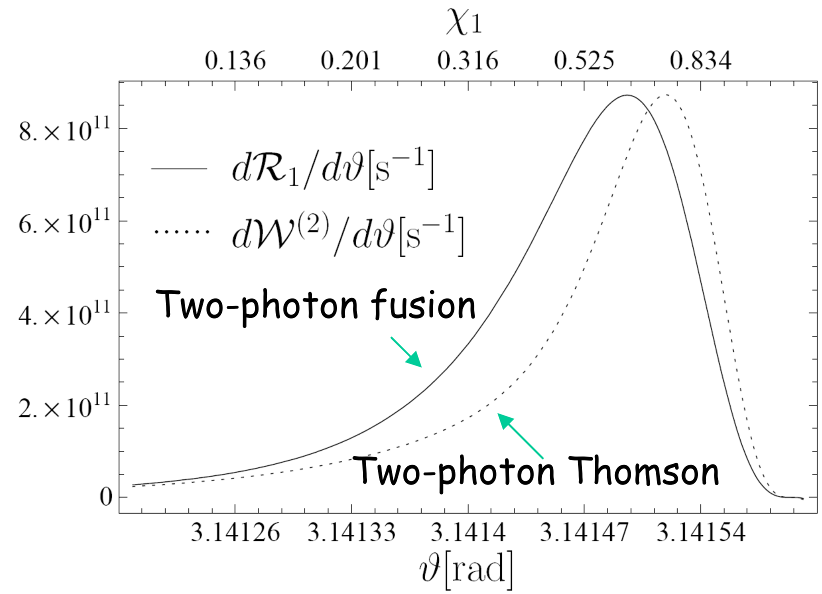
Second harmonic: 400 events/h
4th: 6 events/h

Vacuum polarization in laser fields

MAX-PLANCK-GESELLSCHAFT

Photon fusion during laser and proton beam collision





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XUV Laser : $I=4\text{x}10^{22}$ W/cm^2, $\omega=70$ eV

Second harmonic: 500 events/h
4th: 7 events/h

LHC: Proton energy 7TeV; $N_p=10^{11}$

=3x10^{22} W/cm^2, IR

harmonic: 400 events/h
vents/h

ELI: Proton energy 50 GeV; $N_p=10^{11}$

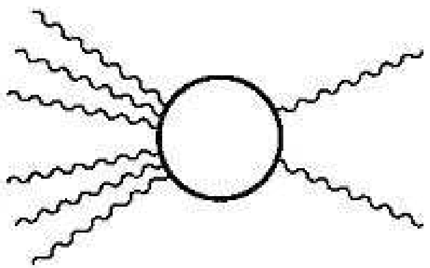
XUV Laser: $I=1.4\text{x}10^{24}$ W/cm^2, $\omega=200$ eV, 40 as
$I=2.5\text{x}10^{24}$ W/cm^2, IR 5 fs

Second harmonic: 5 photons/shot

Vacuum polarization in laser fields

High Harmonic Generation in counterpropagating laser beams

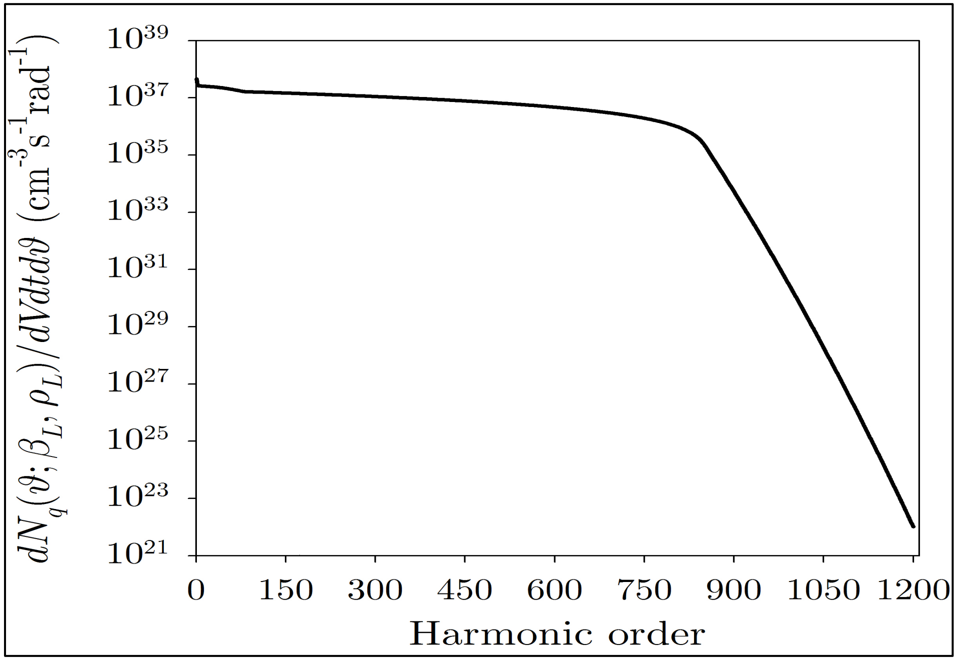
$E > E_c$



$$\boxed{\hbar\omega_{cutoff} \overset{\uparrow}{\neq} (\hbar e c E_L)^{1/2}}$$

$$ecE_L\Delta t \sim \hbar\omega_c$$

$$\Delta t \sim 1/\omega_c$$



$E/E_c = 10$, $E = 1.3 \times 10^{17}$ V/cm; $\omega = 12.5$ keV

Conclusion

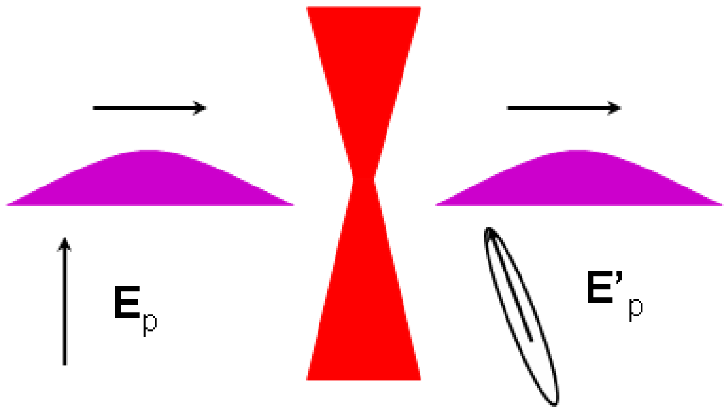
- The advancement of modern laser technique as in IR as well as in XUV spectral region combined with new ion accelerators open real perspectives for observation of nonperturbative vacuum polarization effects with laser fields.

Vacuum polarization in laser fields

Light-by-light diffraction

Euler-Heisenberg Lagrangian density:

$$L = \frac{1}{2}\left(E^2 - B^2\right) + \frac{2\alpha^2}{45m^4}\left[\left(E^2 - B^2\right)^2 + 7\left(\vec{E}\cdot\vec{B}\right)^2\right]$$

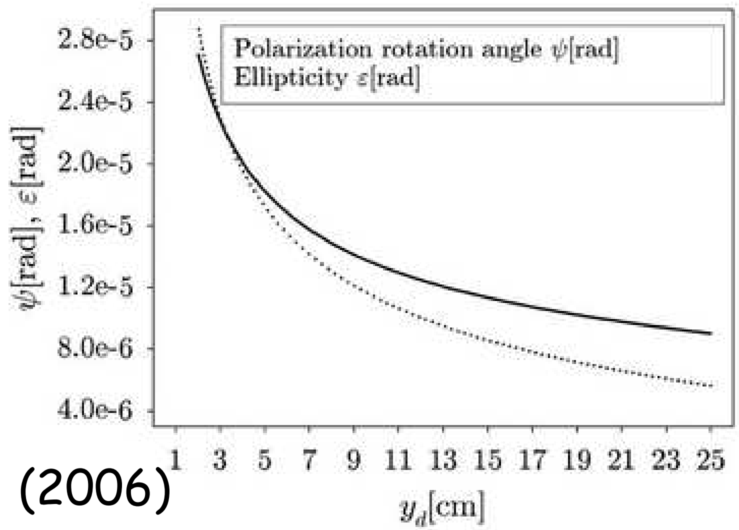


Polarization current:

$$\nabla^2 \vec{E} - \partial_t^2 \vec{E} = \vec{J}; \qquad \vec{J} \propto F^3$$

In far zone, D_0<<1 and D_p<<1, the diffraction is important

$$D_0 = \frac{w_0^2}{y_d \lambda_p}; \quad D_p = \frac{w_p^2}{y_d \lambda_p}$$



Di Piazza et al. PRL **97**, 083603 (2006)

Vacuum polarization in laser fields

Enhancement of vacuum polarization effects in plasma

When a strong laser pulse propagates through plasma near the threshold of the plasma transparency the vacuum polarization effects are enhanced.

$$n^2 = \varepsilon\mu \approx \varepsilon_p + \frac{2\alpha^2}{45\pi}\frac{E^2}{E_{cr}^2}\left(1-\varepsilon_p^2\right)$$

$$\varepsilon_p = 1 - \frac{\omega_p^2}{\omega^2}$$

In the proximity of this singular point $\omega \to \omega_p$, the plasma refractive index tends to zero, the field increases and the vacuum refractive index becomes more visible.

In plasma:

$$\varepsilon_p \to 0 \Rightarrow n_{pl} \approx \sqrt{\frac{2\alpha^2}{45\pi}\frac{E^2}{E_{cr}^2}}$$

In vacuum:

$$\varepsilon_p \to 1 \Rightarrow n_{vac} \approx 1 + \frac{\alpha^2}{45\pi}\frac{E^2}{E_{cr}^2}$$

$$n_{pl} >> n_{vac}$$

Di Piazza et al. PP **14**, 032102 (2007)