Strong-field effects for the production of lepton pairs and photons in collisions of relativistic nuclei

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The discussed problems were considered in many books and reviews, for example:

Heitler "The quantum theory of radiation," 1954;

Greiner, Müller, Rafelsky "QED of strong fields," 1985

Baur, Hencken, Trautmann. Phys. Rep. 453, 1 (2007)

Baltz et all Phys. Rep. 458, 1 (2008)

This report is based mainly on papers
(Dresden, Heidelberg, Leipzig and Novosibirsk Universities):

Ginzburg, Jenschura, Karshenboim, Krauss, Serbo, Soff.
Phys. Rev. C 58, 3565 (1998);

Ivanov, Schiller, Serbo. *Phys. Lett. B 454, 155 (1999)*;

Lee, Milstein, Serbo. *Phys. Rev. A* **65**, *022102 (2002)*;

Ginzburg, Jentschura, Serbo, *Phys. Lett. B* **658**, *125 (2008)*;

Ginzburg, Jentschura, Serbo, *Eur. Phys. J. C* **54**, *267 (2008).*

Jentschura, Hencken, Serbo (in preparation)

PART I

LEPTON PAIR PRODUCTION

in collisions of relativistic nuclei

1. Introduction and main results for e^+e^- pairs

1.1. Basic numbers

For the RHIC and LHC colliders, the charge numbers of nuclei $Z_1 = Z_2 \equiv Z$ and their Lorentz factors $\gamma_1 = \gamma_2 \equiv \gamma$ are given as follows...

Colliders and the Born cross sections for lepton pair production

Collider	Z	γ	$\sigma_{\text{Born}}^{e^+e^-}$ [kb]	$\sigma_{\text{Born}}^{\mu^+\mu^-}$ [b]
RHIC, Au-Au	79	108	36.0	0.23
LHC, Pb-Pb	82	3000	227	2.6

The cross section of **one pair production in the Born approximation** (described by Feynman diagram of Fig. 1)

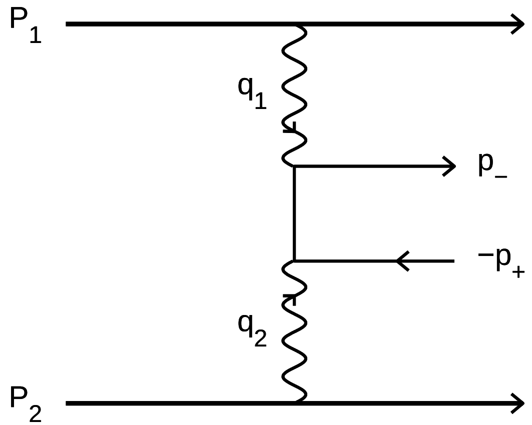

Fig. 1

with **two photon production** was obtained many years ago by *Landau, Lifshitz (1934)* and *Racah (1937)*:

$$\sigma_{\rm Born} = \frac{28}{27\pi}\,\sigma_0 \left[L^3 - 2.198\, L^2 + 3.821\, L - 1.632 \right] ,$$

where

$$\sigma_0 = \frac{(Z_1\alpha Z_2\alpha)^2}{m^2}, \quad \alpha = \frac{1}{137}, \quad L = \ln(\gamma_1\gamma_2) \gtrsim 10,$$

$\mathbf{m}$ is the electron mass.

1.2. Importance

Since **the Born cross section is huge** (see Table 1), pair production can be a serious background for many experiments:
Hencken, Sadovsky, Kharlov, ALICE Note ALICE-INT-2002-11.

It is also important for the problem of beam lifetime and luminosity of colliders (see the reviews *Bertulani, Baur. Phys. Rep.* **163**, *299 (1988); Baur et al. Phys. Rep.* **364**, *359 (2002) and Klein, NIM A59 (2001) 51*).

It means that the various corrections to the Born cross section are of great importance.

Similar there are ideas to use multiple pair production as a possible trigger for ultra-peripheral collisions at ALICE. A good knowledge of the multiple pair production cross section is needed for this.

Since **the parameter $Z\alpha$ may be not small**
($Z\alpha \approx 0.6$ for Au-Au and Pb-Pb collisions),
the whole series in $Z\alpha$ has to be summed to obtain the cross section with sufficient accuracy.

Fortunately, there is **important small parameter**

$$\frac{1}{L} < 0.11\,, \qquad L = \ln(\gamma^2)\,,$$

therefore, it is sufficient to calculate the corrections in **the leading logarithmic approximation (LLA) only**.

In the literature, there were a lot of controversial and incorrect statements in papers devoted to this subject. Some corresponding references and critical remarks can be found in

Ivanov, Schiller, Serbo. Phys. Lett. B 454 (199) 155;

Lee, Milstein. Phys. Rev. A 61 (2000) 032103;

Phys. Rev. A 64 (2001) 032106 (2001);

Lee, Milstein, Serbo. Phys. Rev. A **65***, 022102 (2002);*

Aste, Baur, Hencken, Trautmann, Scharf. EPJ C23 (2002) 545

1.3. Results for e^+e^- pair production

The exact cross section for **one pair production** σ_1 can be written in the form

$$\sigma_1 = \sigma_{\text{Born}} + \sigma_{\text{Coul}} + \sigma_{\text{unit}}\,,$$

where **two different types of corrections** have been distinguished.

The typical electric field of nucleus is **very strong**

$$\mathcal{E} \sim \frac{Ze}{\rho^2}\gamma = \gamma\, Z\alpha\, \mathcal{E}_{\text{Schwinger}} \ \text{ at } \ \rho = \frac{\hbar}{m_e c}$$

since $\gamma\, Z\alpha = 60$ for RHIC and 1800 for LHC.

The Coulomb corrections σ_{Coul} correspond to **multi-photon exchange** of the produced $e^{\pm}$ with nuclei:

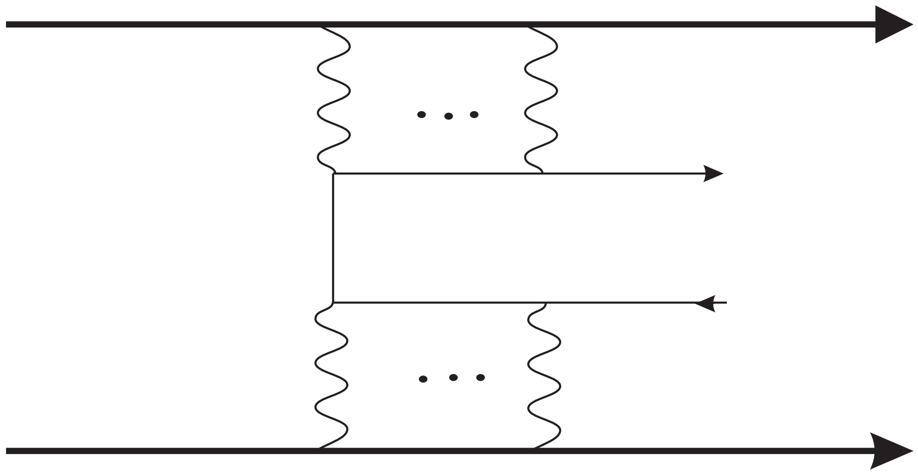

Fig. 2

They were calculated by *D.Yu. Ivanov, A. Schiller, V.G. Serbo. Phys. Lett. B 454 (1999) 155*.

The unitarity corrections σ_{unit} correspond to the exchange of the virtual light-by-light blocks between nuclei

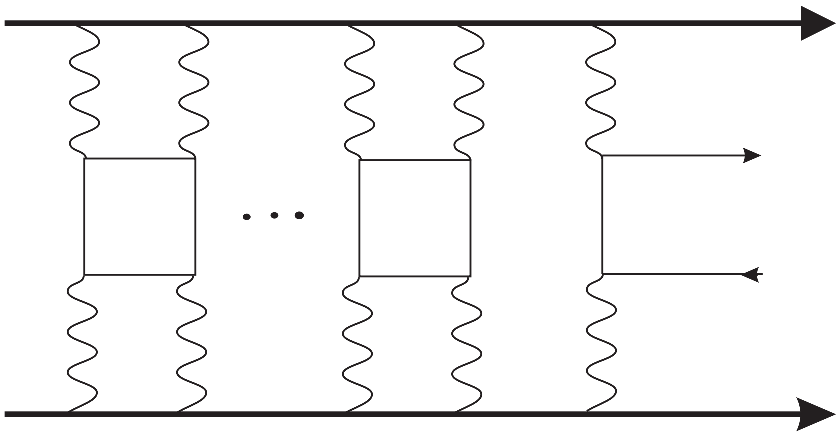

Fig. 3

They were calculated by *R.N. Lee, A.I. Milstein, V.G. Serbo. Phys. Rev. A 65 (2002) 022102*
and updated by
U.D. Jentschura, K. Hencken, V.G. Serbo (in preparation)

It was found that **the Coulomb corrections are large** while **the unitarity corrections are small**:

Coulomb and unitarity corrections to the e^+e^- pair production

Collider	$\frac{\sigma_{\text{Coul}}}{\sigma_{\text{Born}}}$	$\frac{\sigma_{\text{unit}}}{\sigma_{\text{Born}}}$
RHIC, Au-Au	-25%	-5.0%
LHC, Pb-Pb	-14%	-4.0%

Multiple production of e^+e^- pairs

$$Z + Z \to Z + Z + n(e^+e^-)$$

If $Z\alpha$ is small, the corresponding cross section grows as L^n:

$$\sigma_n = C_n \, \frac{(Z\alpha)^{4n}}{m^2} \, L^n \, , \;\; n \geq 2 \, ,$$

$$C_2 = 2.21 \, , \; C_3 = 0.443 \, , \; C_4 = 0.119 \, .$$

R.N. Lee, A.I. Milstein, V.G. Serbo. Phys. Rev. A 65 (2002) 022102
U.D. Jentschura, K. Hencken, V.G. Serbo (in preparation)
In particular,

$$\sigma_2 = 0.114 \text{ barn for Ca-Ca at LHC.}$$

For large values of $Z\alpha$ there are only numerical calculations of σ_n for a particular values of γ

A.Alscher, K.Henken, D. Trautman, G.Baur. Phys. Rev. C 59 (1999) 811

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NOW FEW WORDS ABOUT THEORY

2. Coulomb correction to σ_1 for e^+e^- pair production

Selection of the leading diagrams. Let $\mathcal{M}$ be the sum of amplitudes $M^n_{n'}$ of Fig. 4. The case $n \geq 2$ and $n' \geq 2$ is difficult for calculation, **but** the corresponding contribution is **small**!

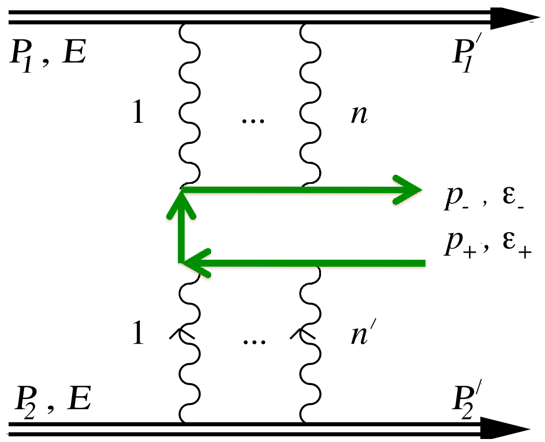

Fig. 4

It can be presented in the form

$$\mathcal{M} = \sum_{nn'\geq 1} M^n_{n'} = \mathcal{M}_{\text{Born}} + \mathcal{M}_1 + \tilde{\mathcal{M}}_1 + \mathcal{M}_2\,,$$

$$\mathcal{M}_1 = \sum_{n'\geq 2} M^1_{n'}\,, \quad \tilde{\mathcal{M}}_1 = \sum_{n\geq 2} M^n_1\,, \quad \mathcal{M}_2 = \sum_{nn'\geq 2} M^n_{n'}\,.$$

$\mathcal{M}_{\text{Born}}$ contains **the one–photon** exchange **both** with the first and the second nucleus;

$\mathcal{M}_1$ ($\tilde{\mathcal{M}}_1$) contains **the one–photon** exchange **only** with the first (second) nucleus;

$\mathcal{M}_2$ has **no one–photon** exchange.

According to this classification we write the total cross section as

$$\sigma = \sigma_{\text{Born}} + \sigma_1 + \tilde{\sigma}_1 + \sigma_2$$

where

$$\begin{aligned} d\sigma_{\text{Born}} &\propto |M_{\text{Born}}|^2 \,, \\ d\sigma_1 &\propto 2\,\text{Re}\,(\mathcal{M}_{\text{Born}}\mathcal{M}_1^*) + |\mathcal{M}_1|^2 \,, \\ d\tilde{\sigma}_1 &\propto 2\,\text{Re}\,(\mathcal{M}_{\text{Born}}\tilde{\mathcal{M}}_1^*) + |\tilde{\mathcal{M}}_1|^2 \,, \end{aligned}$$

and

$$d\sigma_2 \propto 2\,\text{Re}\,\left(\mathcal{M}_{\text{Born}}\mathcal{M}_2^* + \mathcal{M}_1\tilde{\mathcal{M}}_1^* + \mathcal{M}_1\mathcal{M}_2^* + \tilde{\mathcal{M}}_1\mathcal{M}_2^*\right)$$
$$+|\mathcal{M}_2|^2 \,.$$

Due to C-parity conservation the ratio $\sigma_i/\sigma_{\text{Born}}$ is a function of $(Z\alpha)^2$ only but not of $Z\alpha$ itself.

The integration over the transferred momentum squared q_1^2 and q_2^2 results in **two** large Weizsäcker–Williams logarithms $\sim L^2$ for σ_{Born}; in **one** WW logarithm $\sim L$ for σ_1 and $\tilde{\sigma}_1$; σ_2 contains **no** WW logarithm. Therefore,

$$\frac{\sigma_1}{\sigma_{\text{Born}}} \sim \frac{\tilde{\sigma}_1}{\sigma_{\text{Born}}} \sim \frac{(Z\alpha)^2}{L}$$

and

$$\frac{\sigma_2}{\sigma_{\text{Born}}} \sim \frac{(Z\alpha)^2}{L^2} < 0.4\,\%\,.$$

As a result, with an accuracy of the order of 1% **one can neglect** σ_2 in the total cross section and to calculate

$$\sigma_{\text{Coul}} = \sigma_1 + \tilde{\sigma}_1$$

in LLA only using EPA (see *Ivanov, Schiller, Serbo. Phys. Lett. B 454 (1999) 155*):

$$\sigma_{\text{Coul}} = -\frac{28}{9\pi}\left[f(Z_1\alpha) + f(Z_2\alpha)\right]\sigma_0\, L^2\,,$$

$$f(x) = x^2 \sum_{n=1}^{\infty} \frac{1}{n(n^2+x^2)}\,.$$

It was also shown by ISS that **the Coulomb corrections disappear** for **large transverse momenta of the produced leptons**, at $p_{\pm\perp} \gg m$.

3. Unitarity corrections and σ_n

Due to $Z_1 Z_2 \alpha \gg 1$ for $\gamma \gg 1$ it is possible to treat the nuclei as **sources of the external field** and calculate the probability of n-pair production $P_n(\rho)$ in collision of two nuclei at a given impact parameter ρ.

The cross section is then found as:

$$\sigma_n = \int P_n(\rho)\, d^2\rho \, .$$

What we know about

$$P_n(\rho)\,?$$

It was realized many years ago that in the Born approximation

$$P_1(\rho) \sim (Z\alpha)^4 L \quad \text{at } \rho \sim 1/m$$

and, therefore, this probability **can be greater than 1** *Baur. Phys. Rev. A 42 (1990) 5736*.

It means that one **should take into account the unitarity corrections** and that the cross section for multiple pair production should be large enough.

The unitarity corrections come from **the unitarity requirement for the S-matrix**.

It was argued in papers

Baur. Phys. Rev. **D 41**, *3535 (1990); Roades-Brown and Wenes. Phys. Rev.* **A 44**, *330 (1991); Best, Greiner, and Soff. Phys. Rev* **A 46**, *261 (1992); Henken, Trautman, and Baur. Phys. Rev.* **A 51**, *998 (1995)*

that the factorization of the multiple pair production probability is valid with a good accuracy given by **the Poisson distribution**:

$$P_n(\rho) = \frac{[\bar{n}(\rho)]^n}{n!} \mathrm{e}^{-\bar{n}(\rho)} \,,$$

where $\bar{n}(\rho)$ is **the average number of pairs**.

Recently, it was proved in paper

Bartoš, Gevorkyan, Kuraev, Nikolaev. Phys. Lett. B 538 (2002) 45

by a direct summation of the Feynman diagram in LLA.

The unitarity requirement is fulfilled by the Poisson distribution, whose sum over n gives one.

The probability for producing one pair, given in perturbation theory by $\bar{n}_e$, should be modified to read $\bar{n}_e \exp(-\bar{n}_e)$.

For the one-pair production it corresponds to replacement:

$$\sigma_{e^+e^-} = \int \bar{n}_e(\rho)\, \mathrm{d}^2\rho \quad \rightarrow \quad \sigma_{e^+e^-} + \sigma_{e^+e^-}^{\mathrm{unit}} = \int \bar{n}_e(\rho)\, \mathrm{e}^{-\bar{n}_e(\rho)}\, \mathrm{d}^2\rho\,, \quad (1)$$

where

$$\sigma_{e^+e^-}^{\mathrm{unit}} = -\int \bar{n}_e(\rho)\, \left[1 - \mathrm{e}^{-\bar{n}_e(\rho)}\right] \mathrm{d}^2\rho$$

is the **unitarity** correction.

The main contribution to $\sigma_{e^+e^-}$ comes from $\rho \gg 1/m$,
But, the main contribution to $\sigma_{e^+e^-}^{\mathrm{unit}}$ comes from $\rho \sim 1/m$.

The function $\bar{n}_e(\rho)$ is a very important quantity for the evaluation of unitarity corrections.

It was found for $\gamma \gg 1$ in closed form
(taken into account $(Z\alpha)^n$ terms exactly) by
Baltz, McLerran. Phys. Rev. C 58 (1998) 1679;
Segev, Wells. Phys. Rev. A 57 (1998) 1849;
Baltz, Gelis, McLerran, Peshier. Nucl. Phys. A 695 (2001) 395 .

The problem of its **proper regularization** was solved by
Lee, Milstein. Phys. Rev. A 64 (2001) 032106.

But! The obtained close form for $\bar{n}_e(\rho)$ is, in fact, **nine-fold integral** and its calculation is **very laborious**.

More simpler approximate expression for $\bar{n}_e(\rho)$ is very welcome. The functional form of this function reads

$$\bar{n}_e(\rho, \gamma, Z) = (Z\alpha)^4 F(x, Z) \, [L - G(x, Z)] \, , \quad L = \ln(\gamma^2) \, , \; x = m\,\rho \, .$$

The simple analytical expressions for functions $F(x, Z)$ and $G(x, Z)$ is obtained by Lee, Milstein, Serbo (2002) only at large values of the impact parameters, $\rho \gg 1/m$.

On the other hand, for calculation of the unitarity corrections we need $F(x, Z)$ and $G(x, Z)$ in the range $\rho \sim 1/m$.

In the recent paper by *Lee, Milstein nucl-th/0610008* it was given the detailed consideration of the function $F(x)$ including its **tables and the compact integral form — five-fold Integral**.

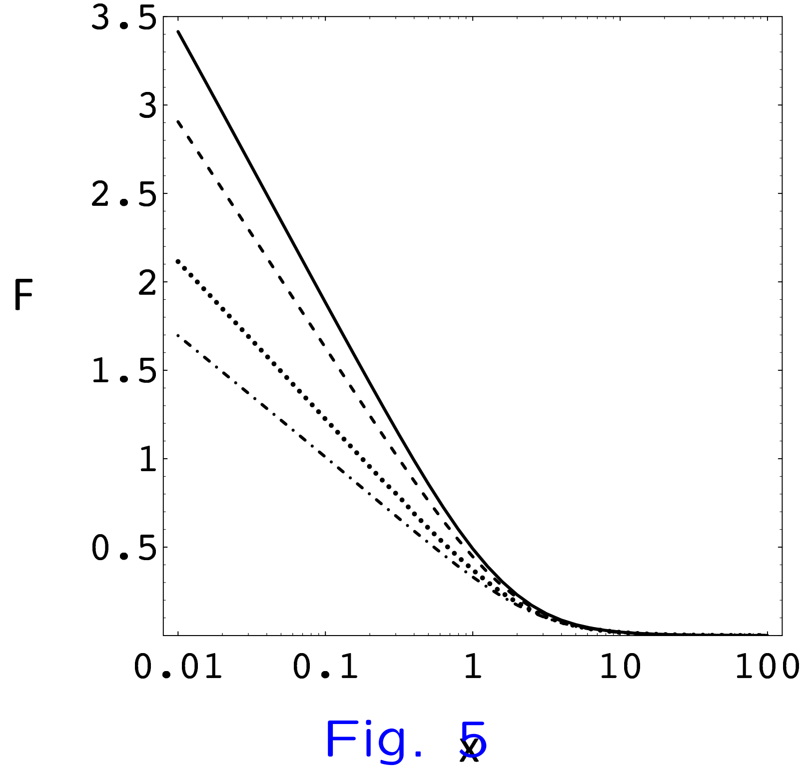

Fig. 5

As an example, in Fig. 5 it is shown the function $F(x)$ from *Lee, Milstein* paper, for $Z_1 = 92$ (dash-dotted line) , $Z_1 = 79$ (dotted line), $Z_1 = 47$ (dashed line), and the Born approximation (solid line).

Using some numerical calculations for the function $\bar{n}_e(\rho, \gamma, Z)$, **we find a simple approximation**

$$G(x, Z) \approx 1.5 \ln(x + 1.4) + 1.9\,.$$

As a result, the approximate expression

$$\bar{n}_e(\rho, \gamma, Z) = (Z\alpha)^4\, F(x, Z)\ [L - 1.5 \ln(x + 1.4) - 1.9]\ ,$$

$$L = \ln\,(\gamma^2)\,, \quad x = m\,\rho$$

with the function $F(x, Z)$ from paper of Lee, Milstein (2006) can be used for calculation of unitarity corrections with an accuracy of the order of 5 %.

4. Motivation and main results for $\mu^+\mu^-$ pair

Motivation: **muon pair production may be easier for an experimental observation**.
Technique: **the calculation scheme for the $\mu^+\mu^-$ pair production is quite different from that for the e^+e^- pair production**.

This process was recently considered in detail by
Hencken, Kuraev, Serbo. Phys. Rev. C 75 (2007) 034903;
Jetschura, Hencken, Serbo (in preparation).

It was found out that:

1. **The Coulomb** corrections are **small**, while **unitarity** corrections are **large**;

2. **The exclusive** cross section **differs** considerable from its Born value, but it is difficult for the experimental observation;

3. **The inclusive** cross section **coincides** with the Born cross section.

5. Exclusive $\mu^+\mu^-$ pair production

5.1. Born cross section for one $\mu^+\mu^-$ pair production

Let us consider the production of **one $\mu^+\mu^-$ pair**

$$Z_1 + Z_2 \to Z_1 + Z_2 + \mu^+\mu^- \,,$$

using EPA, but **taking into account** nucleus electromagnetic **form factors**.

The Born differential cross section $d\sigma_{\rm B}$ for the considered process is related to the cross section $\sigma_{\gamma\gamma}$ for **the real $\gamma\gamma \to \mu^+\mu^-$ process** by the equation

$$d\sigma_{\rm B} = dn_1 dn_2\, d\sigma_{\gamma\gamma}\,,$$

where dn_i is the number of equivalent photons.

As a result,

$$\sigma_{\rm B} = \frac{(Z_1\alpha Z_2\alpha)^2}{\pi\mu^2}\, J(\gamma\Lambda/\mu)\,,$$

where the function $J(\gamma\Lambda/\mu)$ is plotted in Fig. 6. An accuracy of this calculation is of the order of 5 %.

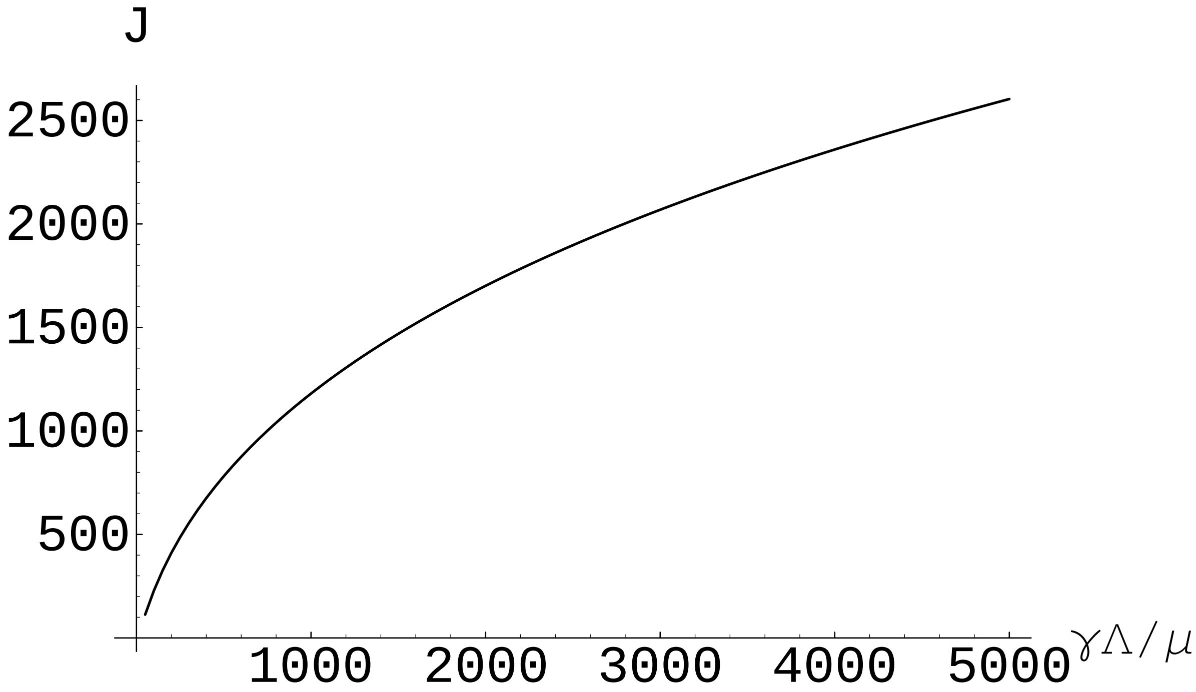
J
2500
2000
1500
1000
500
1000
2000
3000
4000
5000
$\gamma\Lambda/\mu$

Fig. 6

5.2. Probability $P_{\text{B}}(\rho)$

Let us now consider **the probability of muon pair production** $P_{\text{B}}(\rho)$ in the Born approximation. In the LLA:

$$P_{\text{B}}(\rho) = \int dn_1 dn_2\, \delta(\rho_1 - \rho_2 - \rho)\, \sigma_{\gamma\gamma} =$$

$$= \frac{28}{9\pi^2} \frac{(Z_1\alpha Z_2\alpha)^2}{(\mu\rho)^2} \Phi(\rho)\,.$$

There are **two scales** in dependence of function $\Phi(\rho)$ on ρ:

$$\Phi(\rho) = \left(4\ln\frac{\gamma}{\mu\rho} + \ln\frac{\rho}{R}\right)\ln\frac{\rho}{R} \quad \text{at} \quad R \ll \rho \le \gamma/\mu\,,$$

$$\Phi(\rho) = \left(\ln\frac{\gamma^2}{\mu^2\rho R}\right)^2 \quad \text{at} \quad \gamma/\mu \le \rho \ll \gamma^2/(\mu^2 R)\,.$$

We compare Eqs. for $\Phi(\rho)$ with the numerical calculations based on **the exact matrix element**. **There is a good agreement** for the Pb-Pb collisions: **the discrepancy is less then 10 % at** $\mu\rho > 10$ **and it is less then 15 % at** $\mu\rho > 2\mu R = 7.55$.

5.3. Coulomb and unitarity corrections

The Coulomb correction corresponds to Feynman diagram of Fig. 7 with **the multi-photon exchange**.

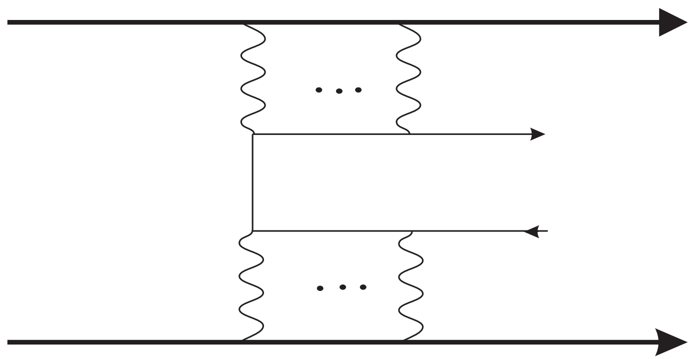

Fig. 7

Due to **restriction of transverse momenta** of additional exchange photons on the level of $1/R$ (**nuclear form factor!**), **the effective parameter** of the perturbation series is not $(Z\alpha)^2$, but $(Z\alpha)^2/((R\mu)^2 L)$.

Therefore, **the real suppression parameter** is of the order of

$$\eta_2 = \frac{(Z\alpha)^2}{(R\mu)^2 L}, \quad L = \ln\left(\gamma^2\right),$$

which corresponds to **the Coulomb correction less then $1\,\%$**.

The unitarity correction σ_{unit} to **one muon pair production** is described by exchange of blocks, corresponding light-by-light scattering via the **virtual electron loop**, between nuclei.

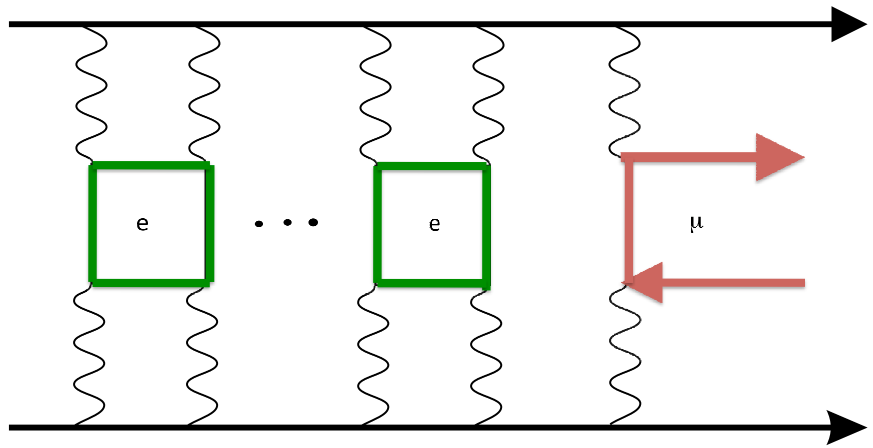

Fig. 8

As usual,

$$\sigma_{\text{B}} = \int_{2R}^{\infty} P_{\text{B}}(\rho)\, d^2\rho \to \sigma_{\text{B}} + \sigma_{\text{unit}} = \int_{2R}^{\infty} P_{\text{B}}(\rho)\, \mathrm{e}^{-\bar{n}_e(\rho)}\, d^2\rho$$

and

$$\sigma_{\text{unit}} = -\int_{2R}^{\infty} \left[1 - \mathrm{e}^{-\bar{n}_e(\rho)}\right] P_{\text{B}}(\rho)\, d^2\rho$$

is **the unitarity correction for the exclusive production of one muon pair**. **In LLA we find**

$$\delta_{\text{unit}} = \frac{\sigma_{\text{unit}}}{\sigma_{\text{B}}} = -49\ \% \text{ for the Pb-Pb collisions at LHC.}$$

It is seen that unitarity corrections are **large**, in other words, **the exclusive production of one muon pair differs considerable from its Born value**.

6. Inclusive production of one $\mu^+\mu^-$ pair

The experimental study of **the exclusive** muon pair production seems to be a very difficult task.

Indeed, this process requires that **the muon pair should be registered without any electron-positron pair production** including $e^\pm$ emitted at **very small angles**.

Otherwise, the corresponding inclusive cross section will be close to the Born cross section.

To prove this we consider **the production of one $\mu^+\mu^-$ pair** and n **electron-positron pairs** in collision of two ultra-relativistic nuclei

$$Z_1 + Z_2 \to Z_1 + Z_2 + \mu^+\mu^- + n\,(e^+e^-)$$

taking into account the unitarity corrections which corresponds to the exchange of the blocks of light-by-light scattering via the virtual electron loop.

The corresponding cross section

$$d\sigma_{1+n}$$

can be calculated by a simple generalization of the Eqs. obtained in paper of

Bartoš, Gevorkyan, Kuraev, Nikolaev. Phys. Lett. B 538 (2002) 45

for the process **without muon pair production**:

$$Z_1 + Z_2 \to Z_1 + Z_2 + n\,(e^+e^-)\,.$$

Our results is the following:

$$\frac{d\sigma_{1+n}}{d^2\rho} = P_{\mathsf{B}}(\rho)\,\frac{[\bar{n}_e(\rho)]^n}{n!}\,\mathrm{e}^{-\bar{n}_e(\rho)}\,,$$

where $\bar{n}_e(\rho)$ is the average number of the e^+e^- pairs.

It is clearly seen from this expression that **after summing up over all possible electron pairs** **we obtain the Born cross section**

$$\sum_{n=0}^{\infty} \sigma_{1+n} = \sigma_{\mathsf{B}}\,.$$

Therefore, there is a very definite prediction:

the inclusive muon pair production coincides with the Born limit.

This direct consequence of calculations taking into account strong field effects may be more easier for an experimental test that the prediction for cross sections of several e^+e^- pair production.

One more prediction

Let us discuss the relation of the obtained cross sections for the muon pair production with the **the differential cross section of the e^+e^- pair production in the region of large transverse momenta of** $e^\pm$, for example at $p_{\pm\perp} \gtrsim 100$ MeV.

It is clear that for the e^+e^- pair production in this region, the situation is **similar** to the considered case for $\mu^+\mu^-$ pair production.

Therefore, we expect that

the inclusive production of a single e^+e^- pair with large transverse momenta of $e^\pm$

(together with several **unobserved** e^+e^- pairs in the region of small transverse momenta of $e^\pm$ of the order of m_e)

has small Coulomb and unitarity corrections and, therefore,

coincides with the Born limit.

PART II

NUCLEAR BREMSSTAHLUNG

Based mainly on papers

I.F. Ginzburg, U.D. Jentschura, V.G. Serbo, *Phys. Lett. B* **658**, *125 (2008)*;
I.F. Ginzburg, U.D. Jentschura, V.G. Serbo, *Eur. Phys. J. C* **54**, *267 (2008)*;
U.D. Jentschura, K. Hencken, V.G. Serbo (in preparation)

7. Ordinary nuclear bremsstrahlung

The ordinary nuclear bremsstrahlung without excitation of the final nuclei is given by Feynman diagrams of Fig. 1

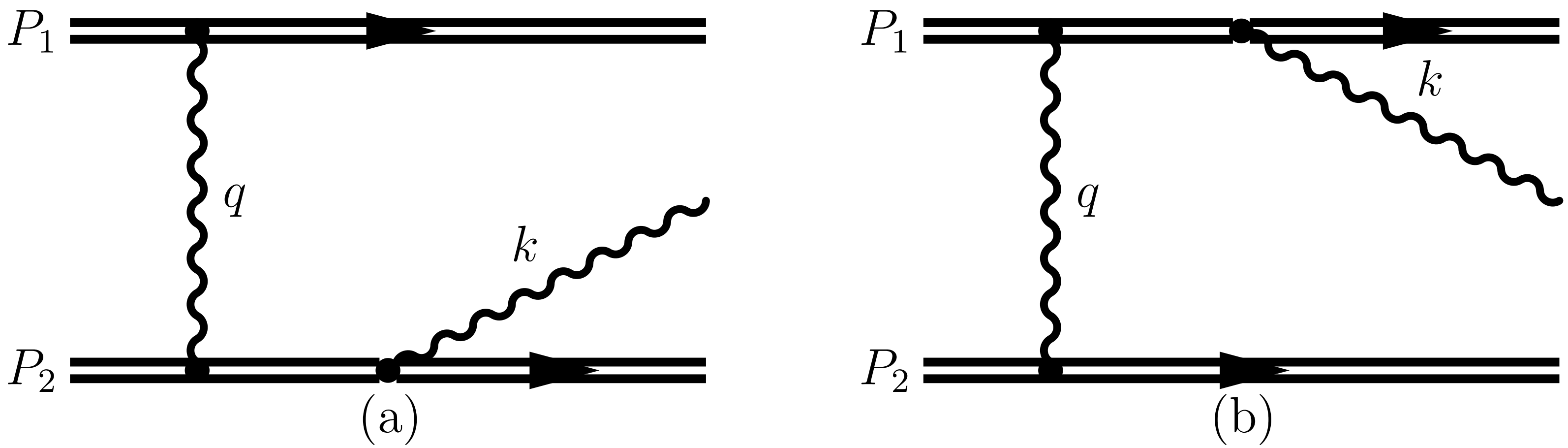

Fig. 9

and was known in detail many years ago
Bertulany, Baur Phys. Rep. **163**, *299 (1988)*

It can be described as **the Compton scattering** of the equivalent photon off opposite nucleus:

$$\mathrm{d}\sigma_{\mathrm{br}} = \mathrm{d}\sigma^{a}_{\mathrm{br}} + \mathrm{d}\sigma^{b}_{\mathrm{br}}\,,$$

and

$$\mathrm{d}\sigma^{a}_{\mathrm{br}} = \mathrm{d}n_1\;\mathrm{d}\sigma_{\mathrm{C}}(\omega, E_\gamma, E_2, Z_2)\,.$$

Here, $\mathrm{d}n_1$ is the number of equivalent photons emitted by nucleus 1 and $\mathrm{d}\sigma_{\mathrm{C}}(\omega, E_\gamma, E_2, Z_2)$ is the differential cross section for the Compton scattering off nucleus Z_2.

We can rewrite these Eqs. as

$$\mathrm{d}\sigma^{a}_{\mathrm{br}} = \mathrm{d}P_a(\rho)\,\mathrm{d}^2\rho\,,$$

where the differential probability $\mathrm{d}P_a(\rho)$ assumes the form

$$\mathrm{d}P_a(\rho) = \frac{Z_1^2\alpha}{\pi^2}\,\frac{\sigma_\mathrm{T}(Z_2)}{\rho^2}\,\left(1 - x_\gamma + \frac{3}{4}\,x_\gamma^2\right)\,\frac{\mathrm{d}E_\gamma}{E_\gamma}\,, \quad x_\gamma = \frac{E_\gamma}{E_2}$$

with the Thomson cross section (M is the mass of nucleus)

$$\sigma_\mathrm{T}(Z_2) = \frac{8\pi}{3}\,\frac{Z_2^4\alpha^2}{M^2}\,.$$

The unitarity correction

$$\delta^a_\mathrm{unit} = \frac{\mathrm{d}\sigma^a_\mathrm{unit}}{\mathrm{d}\sigma^a_\mathrm{br}}\,,$$

reads

$$\delta^a_\mathrm{unit} = -\frac{1}{L_\gamma}\int_{2R}^{\infty}\frac{\mathrm{d}\rho}{\rho}\,\left[1 - \mathrm{e}^{-\bar{n}_e(\rho)}\right]\,, \quad L_\gamma = \ln\left(\frac{2\gamma_1\,\gamma_2^2\,(1 - x_\gamma)}{R\,E_\gamma}\right).$$

An evaluation of this integral gives the following result

for the photon energy $E_\gamma = 1$ GeV,

$$\delta_{\text{unit}} = -\mathbf{19}\% \text{ for the RHIC}, \quad \delta_{\text{unit}} = -\mathbf{15}\% \text{ for the LHC}.$$

8. Large contribution of the virtual Delbrück scattering into nuclear bremsstrahlung

Recently we consider emisson of photons not via the virtual Compton subprocess, but via another one –

the virtual Delbrück scattering subprocess

— which gives an **essential** contribution to emission of photons at the nuclear collisions without excitation of the final nuclei (Fig. 10).

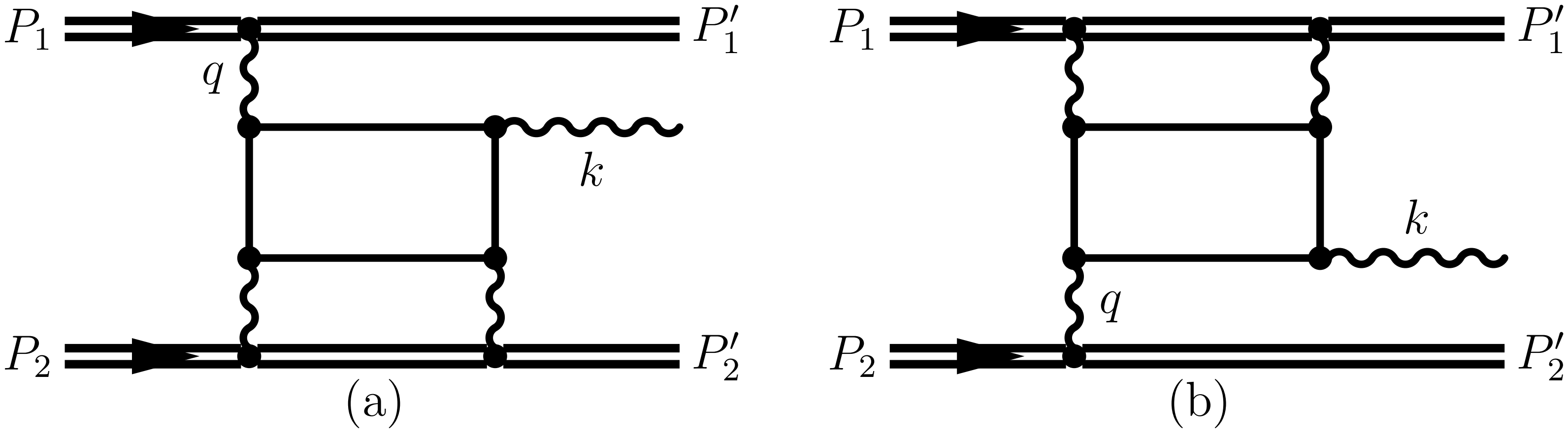

Fig. 10

First note: *Baur, Bertulany Z. f. Phys.* **A 330**, *77 (1988)*

At first sight, this is a process of **a very small cross section** since

$$\sigma \propto \alpha^7.$$

But at second sight, we should **add a very large factor**

$$Z^6 \sim 10^{11}$$

and take into account that the cross section **scale** is

$$1/m_e^2 .$$

And the last, but not the least, we found that this cross section has **an additional logarithmic enhancement** of the order of

$$L^2 \gtrsim 100 \,, \quad L = \ln\left(\gamma^2\right) .$$

Thus, the estimate is

$$\sigma \sim \frac{(Z\alpha)^6\,\alpha}{m_e^2}\, L^2 \,.$$

Our analytical result

$$\sigma = C \frac{(Z\alpha)^6 \, \alpha}{m_e^2} \, L^2$$

with

$$C \approx 0.4 \, .$$

This cross sections is **considerably larger** than that for ordinary nuclear bremsstrahlung in the considered photon energy range:

$$m_e \ll E_\gamma \ll m_e \, \gamma \, .$$

Thus, the discussed cross section for Au-Au collisions at the RHIC collider is

$$\sigma = \mathbf{14 \ barn}$$

and for Pb-Pb collisions at the LHC collider is

$$\sigma = \mathbf{50 \ barn}\,.$$

That is quite a serious number!

Note for comparison, that the last cross section is **6 times larger than for the total hadronic/nuclear cross section in Pb–Pb collisions**, which is roughly **8 barn**.