BB and $B\bar{B}$ four-quark systems from lattice QCD

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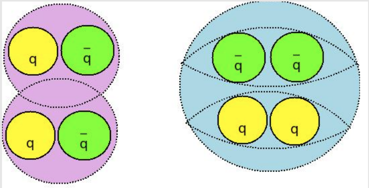
Study of tetraquark states

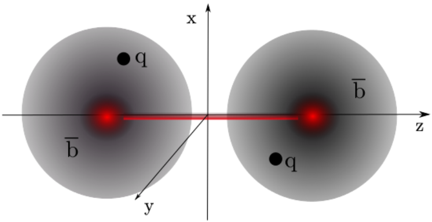
Motivation

- A number of mesons found in particle detectors (LHCb, Belle) are not well understood.
- E.g. charged charmonium and bottomonium states (${Z_c}^{\pm}$ and ${Z_b}^{\pm}$)
- They include bottomonium $b\bar{b}$ or charmonium $c\bar{c}$, but are also charged: must be 4-quark states



Approach

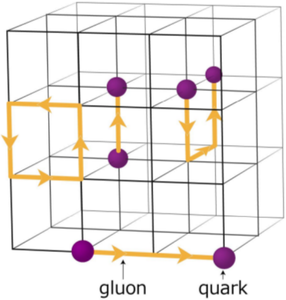
- Computation of 4-quark state very difficult
- If 2 quarks are heavy and 2 quarks are light: Treat degrees of freedom independently in two steps (Born-Oppenheimer approximation)
  1. Lattice computation of the potential of two static quarks in the presence of two light quarks



  2. Solve Schrödinger's equation to check whether potentials are sufficiently attractive to form a bound state.

Lattice QCD

- QCD: Theory to describe quarks and gluons and the forces between them
- No analytical solution for low energy observables
- No pertubative approach on the potential
  $\rightsquigarrow$ numerical solution (Lattice QCD)



[http://www.jicfus.jp/en/promotion/pr/mj/guido-cossu/, April 28, 2015]

Hadron spectroscopy I

- A hadron is completely described by its isospin (flavor content), total angular momentum J, parity P and charge conjugation C.
- Application of a suitable operator $\mathcal{O}(t)$ on the vacuum generates field excitations which are similar to the hadron of interest.
- Here: Use two static quarks and two quarks of finite mass
  - Static quarks: no spin, no contribution to total angular momentum and isospin
  - BB: $\bar{Q}\bar{Q}qq$ and $B\bar{B}$: $Q\bar{Q}q\bar{q}$ with $Q = b$ and $q \in \{u, d, s, c\}$
  - e.g. $B\bar{B}$ four-quark operator

$$\mathcal{O}_{B\bar{B}}(t) = \Gamma_{AB}\tilde{\Gamma}_{CD} \underbrace{\bar{Q}^a_C(\vec{x},t) u^a_A(\vec{x},t)}_{B \text{ meson at } \vec{x}} \underbrace{\bar{d}^b_B(\vec{y},t) Q^b_D(\vec{y},t)}_{\bar{B} \text{ meson at } \vec{y}}$$

- Obtain the correlation function in time for each separation r of the static quarks via

$$C(t,r) = \langle\Omega| \mathcal{O}^\dagger(t)\mathcal{O}(0) |\Omega\rangle$$

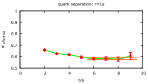
Requires $\mathcal{O}(\text{months})$ of computing time on high performance computers.

Hadron spectroscopy II

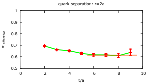
- For large t one finds the potential $V(r)$ of the hadronic state $\mathcal{O}(t)$:

$$\lim_{t\to\infty} \langle\Omega|\,\mathcal{O}^\dagger(t)\mathcal{O}(0)\,|\Omega\rangle \propto \exp(-V(r)t)$$

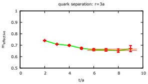
- Get a value of the potential for each quark separation r and obtain the complete potential:



r=1a

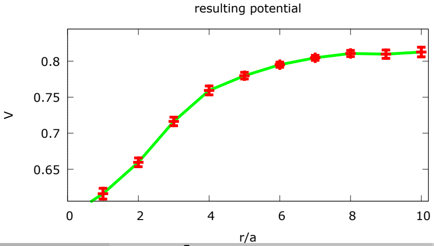


r=2a



r=3a

and so on...



BB systems

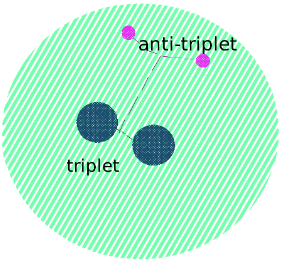
The *BB* system - Expectations

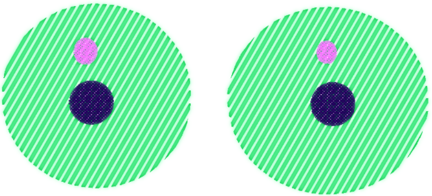
small separations of the static antiquarks:

- interaction due to 1-gluon exchange
- bound state: static $\bar{Q}\bar{Q}$ pair in a color triplet (attractive) $\longrightarrow$ antidiquark

large separations of the static antiquarks:

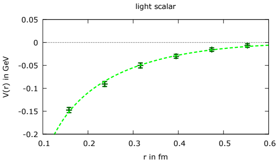
- screening of the antiquark-antiquark interaction due to light quarks (stronger, the more massive the light quarks)
- basically 2 static-light mesons

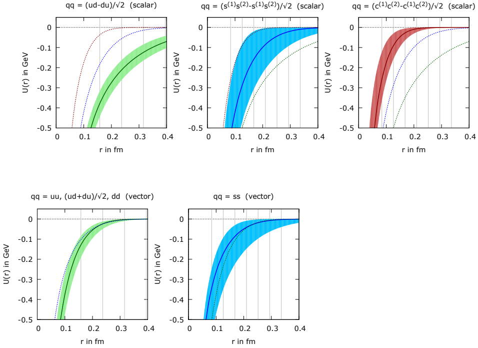




Fit an ansatz to the potentials:

$$V(r) = -\underbrace{\frac{\alpha}{r}}_{\text{Coulomb}-\text{like}} \underbrace{e^{-\left(\frac{r}{d}\right)^2}}_{\text{colour screening}}$$





Observation: Potentials show more promise for binding the less massive the light quarks are! Strongest attraction for scalar isosinglet $qq = \frac{ud-du}{\sqrt{2}}$.

Solve Schrödinger's equation

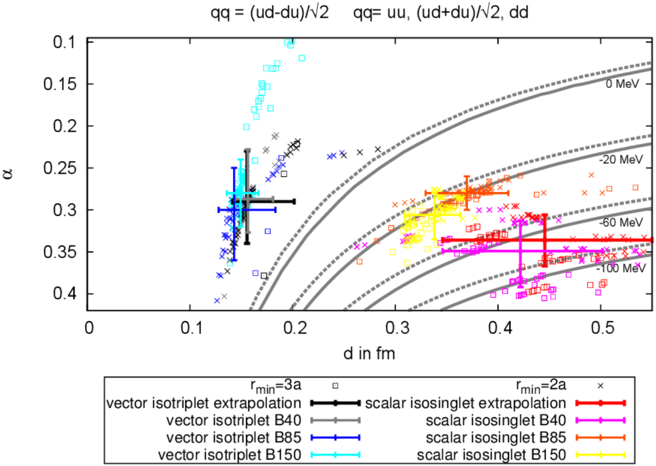
- notice: Born-Oppenheimer approximation is valid for $m_q \ll m_Q$
- solve Schrödinger's equation:

$$\boxed{\left(-\frac{1}{2\mu}\frac{\mathrm{d}^2}{\mathrm{d}r^2} + V(r)\right) R(r) = E_B R(r) \quad , \qquad \psi(r) = R(r)/r}$$

- lowest eigenvalue $E_B < 0$ binding (4-quark bound state), $E_B > 0$ no binding (2 mesons)

Most promising channel for a bound state: scalar $ud\bar{b}\bar{b}$

Extrapolation to the physical quark mass: Binding increases



Summary *BB*

- *BB* systems are experimentally hard to observe, but theoretically easier to investigate
- Here: *BB* with light quarks $qq = ud$ with quantum numbers $I(J^P) = 0(1^+)$ (attractive channel)
- We find $E_B = -90^{+43}_{-36} \longrightarrow$ binding with more than 2σ confidence level
- $qq = ss, cc$: no binding

$B\bar{B}$ systems

The $B\bar{B}$ system - Expectations

- For the experimentally most interesting case of isospin $|I_z| = 1$, $B\bar{B}$ has the same quantum numbers as $Q\bar{Q}$ and π.
- So there are several states to be taken into account:
  - four-quark state
  - 2 mesons
  - $Q\bar{Q} + \pi$

Consequently, one wonders...

- Is the potential contaminated by $Q\bar{Q} + \pi$?
- How to find a four-quark signal that is precise enough to be distinguishable from $Q\bar{Q} + \pi$?
- How to exclude "inadvertent" and unnecessary expensive computation of the $Q\bar{Q} + \pi$ instead of four-quark state?

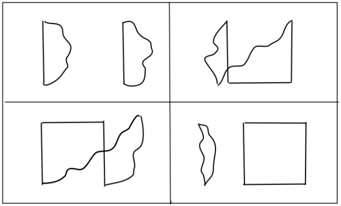
Solution:

Build a 2×2-matrix $C(t)$

- $C_{00}(t) = \langle \mathcal{O}^\dagger_{B\bar{B}}(t) \mathcal{O}_{B\bar{B}}(0) \rangle$
- $C_{01}(t) = \langle \mathcal{O}^\dagger_{B\bar{B}}(t) \mathcal{O}_{Q\bar{Q}+\pi}(0) \rangle$
- $C_{10}(t) = \langle \mathcal{O}^\dagger_{Q\bar{Q}+\pi}(t) \mathcal{O}_{B\bar{B}}(0) \rangle$
- $C_{11}(t) = \langle \mathcal{O}^\dagger_{Q\bar{Q}+\pi}(t) \mathcal{O}_{Q\bar{Q}+\pi}(0) \rangle$

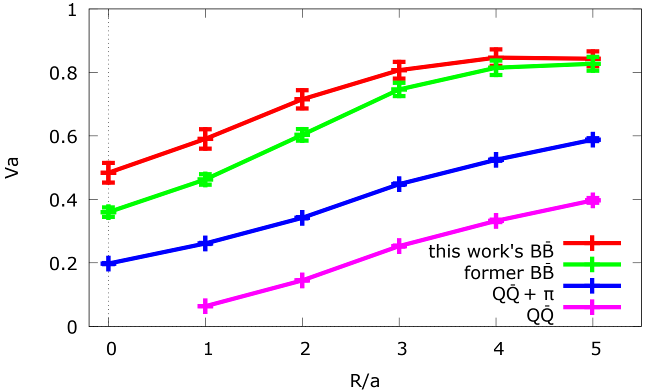
and extract the first excited state with the Generalized Eigenvalue Problem (GEP).

The matrix $C(t)$

$C(t) =$ 

- Derive different elements $C_{ij}(t)$ of the matrix analytically.
- Implement them.
- Identify the symmetries of $C_{ij}(t)$ (time-reversal, parity, charge conjugation, hermiticity and cubic rotations).
- Average according to the symmetries to increase statistics.

Preliminary results I



Potentials obtained

- the $Q\bar{Q}$ potential for comparison
- $Q\bar{Q} + \pi$: ground state
- $B\bar{B}$: includes contributions of $Q\bar{Q} + \pi$
- first excited state of the 2x2 matrix: free of contributions of $Q\bar{Q} + \pi$

Preliminary results II

Binding energy

A very preliminary analysis yields for quantum numbers $I(J^P) = 1(1^+)$:

$$E_B = (-170 \pm 100)\ \mathrm{MeV}$$

Summary $B\bar{B}$ system

- $B\bar{B}$ is experimentally more easy to investigate than BB.
- $B\bar{B}$ with light quarks $qq = u\bar{d}$ with quantum numbers $I(J^P) = 1(1^+)$ is a tetraquark candidate.
- Work in progress

Summary BB and $B\bar{B}$ systems

- BB systems with light quarks are able to form a bound state.
- $B\bar{B}$ systems are experimentally more easy to access than BB systems, but theoretically more challenging.
- Candidate for a binding $B\bar{B}$ state is currently investigated.