# Statistical uncertainties on the NN interaction and light nuclei

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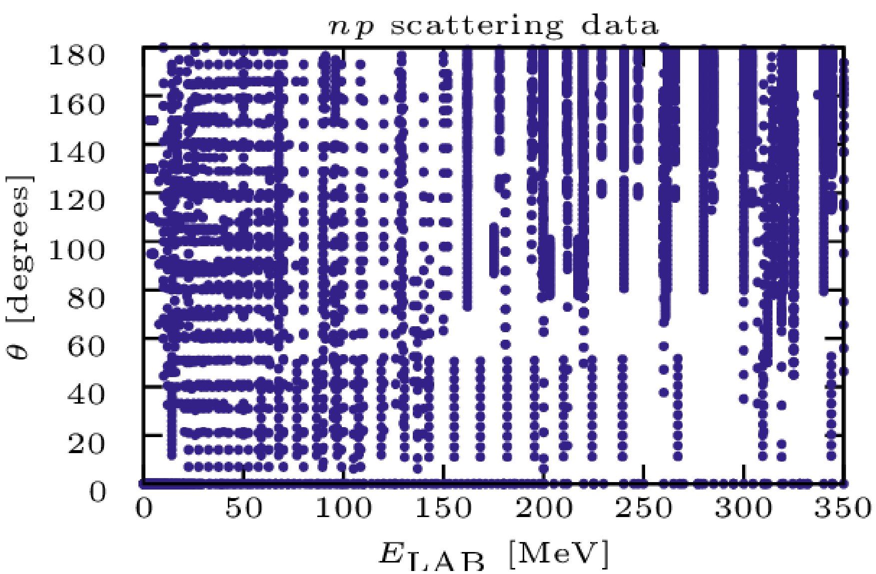
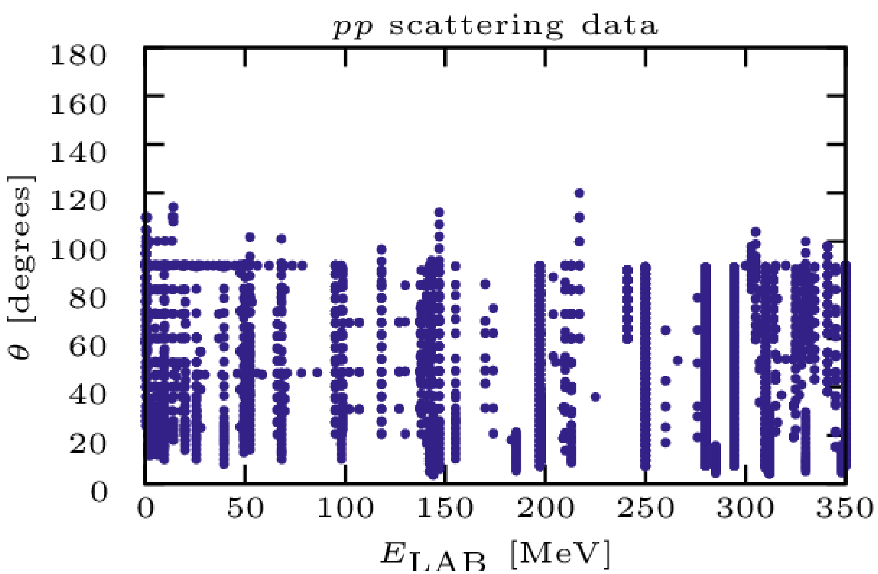
# Motivation

- More than 7800 elastic scattering data since the 1950's
- Several potentials and PWA
  - Hamada Johnston, Yale, Paris, Bonn, Nijmegen, Argonne V18, …
- $\chi^2$/d.o.f. ≈ 1 possible by 1993

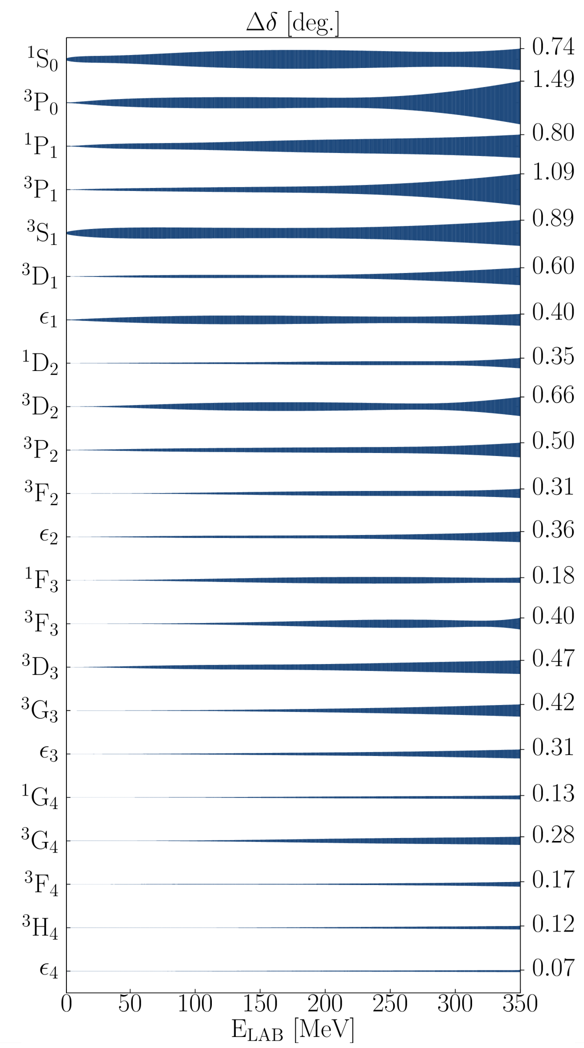


# Motivation

## The NN interaction has no unique representation

- Different phenomenological potentials
  - Fitted to experimental scattering data
  - High accuracy $\chi^2$/d.o.f. ≈ 1
  - Dispersion in Phaseshifts
  - OPE for the long range interaction
  - ~ 40 parameters for the short and intermediate range
- Repulsive core in most of them
  - Complicates nuclear structure
- Statistical uncertainties are recent



# Motivation
## Chiral Potentials

- pp PWA by the Nijmegen group

  [Rentmeester et al, Phys. Rev. Lett. 82 (1999), 4992]

  - Improvement in the $\chi^2$ value compared to OPE only
- Accurate N3LO chiral potential up to 290 MeV

  [Entem & Machleidt, Phys. Rev. C68 (2003), 041001]
- Optimized NNLO (sim, sat)

  [Carlson et al, arXiv:1506.02466 ]
- TPE including $\Delta$ resonances up to 300 MeV

  [Piarulli et al, Phys. Rev. C91 (2015) 024003]

Reproduction of statistical uncertainties is still in progress

# Motivation

## Sources of uncertainty

- Numerical (Implementation)
  - Inexact solution method
  - Inherent to any numerical calculation
- Systematic (Model dependence)
  - Any model makes assumptions
  - Different representations for the NN interaction
- Statistical (Fitting bias)
  - Statistical fluctuations in any measurement
  - Uncertainty in data ➡ Uncertainty in parameters

Assuming independence among them

$$(\Delta F)^2 = (\Delta F^{\,num})^2 + (\Delta F^{\,sys})^2 + (\Delta F^{\,stat})^2$$

# Anatomy of phenomenological potentials
## fitted to the Granada database

### Short and Intermediate range

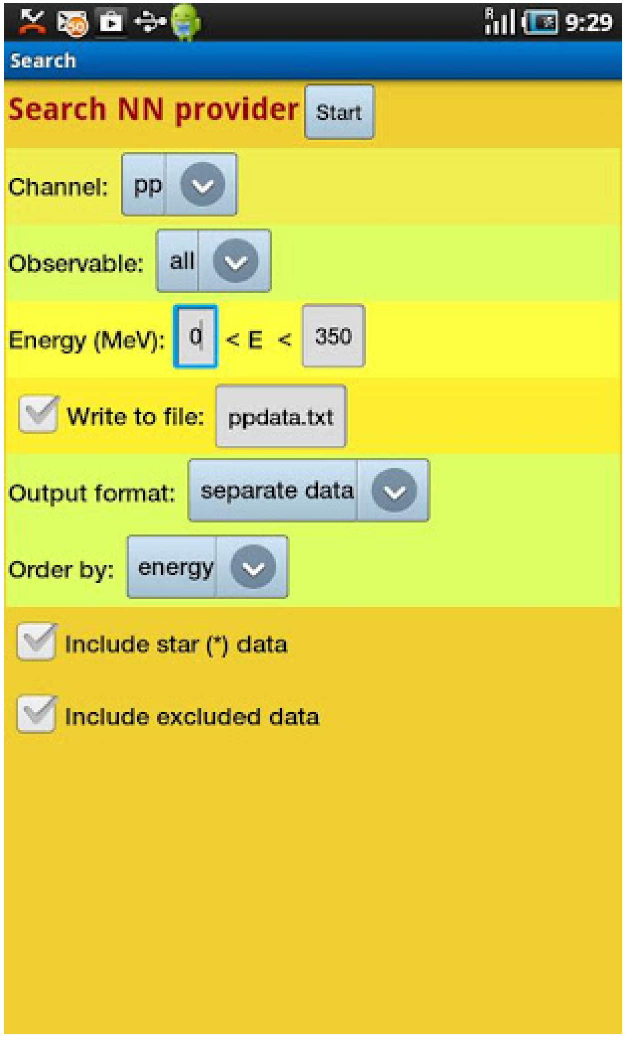
- Delta Shells
  - Coarse grained
  - Simplified calculations
  - High momentum components
- Sum of Gaussian functions
  - Smooth and soft
  - Nuclear structure calculations
  - Not as fast

### Long range

- Electromagnetic contributions
  - Small but crucial
- One pion exchange
  - Proper analytic behavior
- Optional
  - Two pion exchange
  - $\Delta$ degree of freedom
    - Born approximation

**Six different phenomenological potentials**

# Granada database




- NN scattering data from 1950 to 2013
  - http://nn-online.org/
  - http://gwdac.phys.gwu.edu/
  - NN Provider for Android
    - Google play store

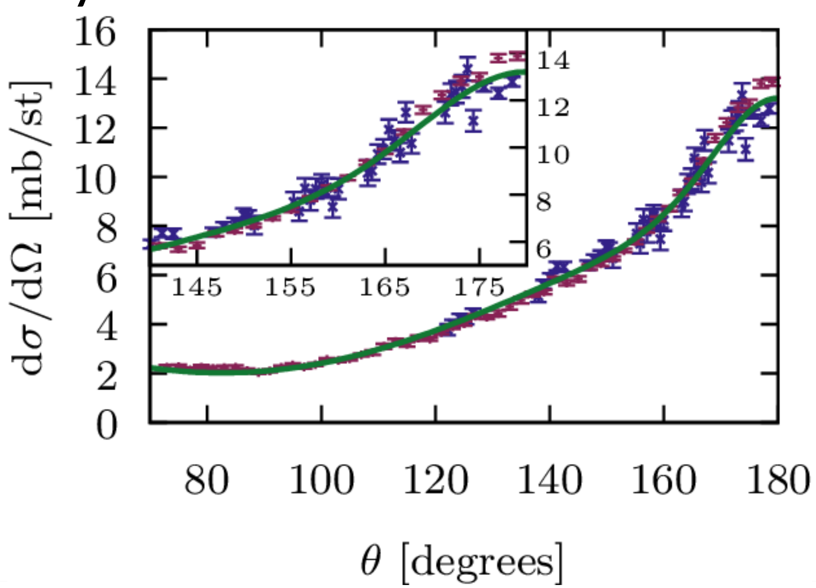
[Amaro, RNP, Ruiz-Arriola]

  - http://www.ugr.es/~amaro/nndatabase/
- 2868 pp and 4991 np data

# Fitting NN scattering observables

## Selection of data

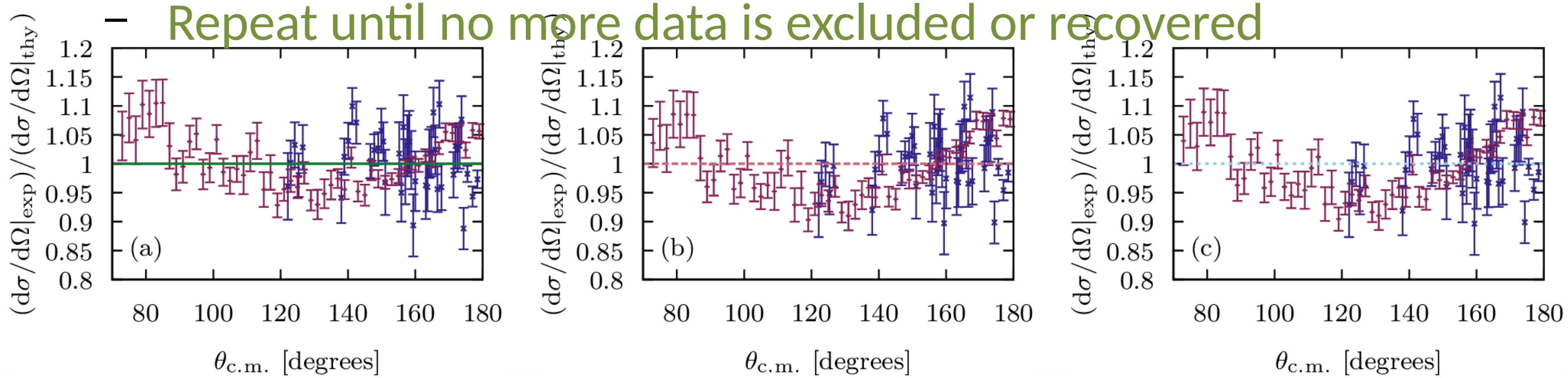
- Direct fits to all data NEVER give $\chi^2$/d.o.f. ≈ 1
  - Restrictive model ? ➡ Improve model
  - Mutually incompatible data ➡ Reject incompatible data
- np *dσ/dΩ* at 162 MeV
- Statistical and systematic errors may be over or underestimated
- 3*σ* criterion
  - Fit all data ($\chi^2$/d.o.f. > 1)
  - Remove sets with improbably high or low $\chi^2$
  - Refit parameters



# Fitting NN scattering observables

## Recovering data

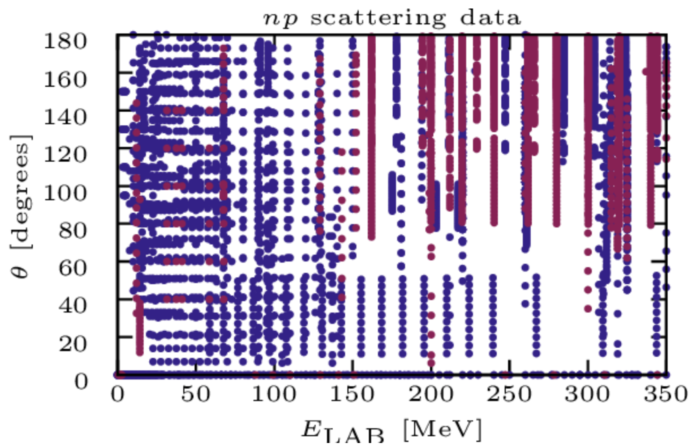
- Mutually incompatible data
  - Which experiment is correct?
  - Is any of the two correct?
  - Maximization of experimental consensus
- Exclude data sets inconsistent with the rest of the database
  - Fit to all data ($\chi^2$/d.o.f. > 1)
  - Remove data sets with improbably high or low $\chi^2$ (3$\sigma$ criterion)
  - Refit parameters
  - Re-apply 3$\sigma$ criterion to all data
  - Repeat until no more data is excluded or recovered

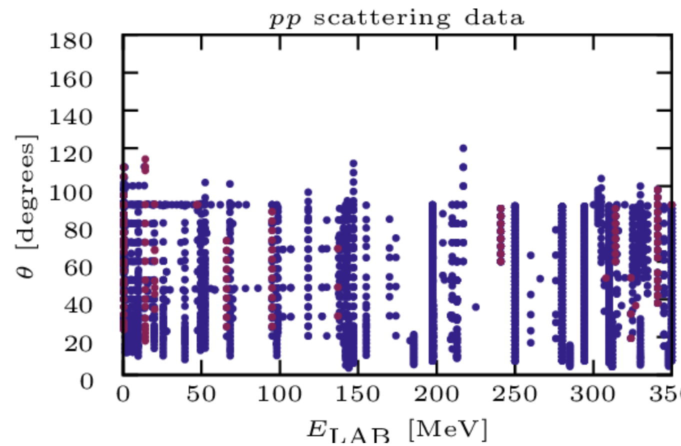


# Fitting NN scattering observables

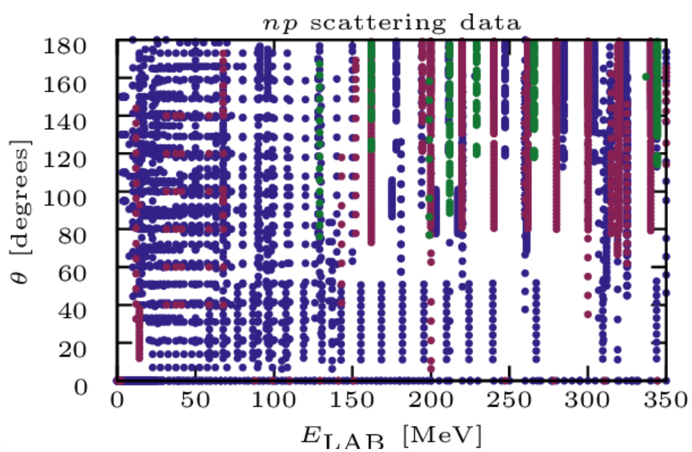
## Recovering data

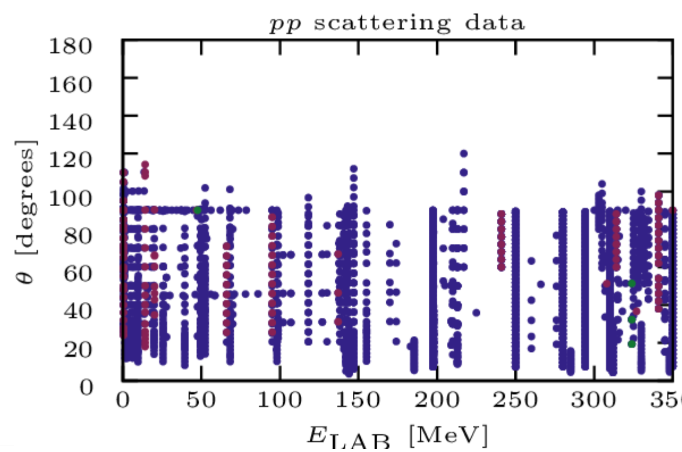
Usual Nijmegen 3σ criterion (1677 rejected data)



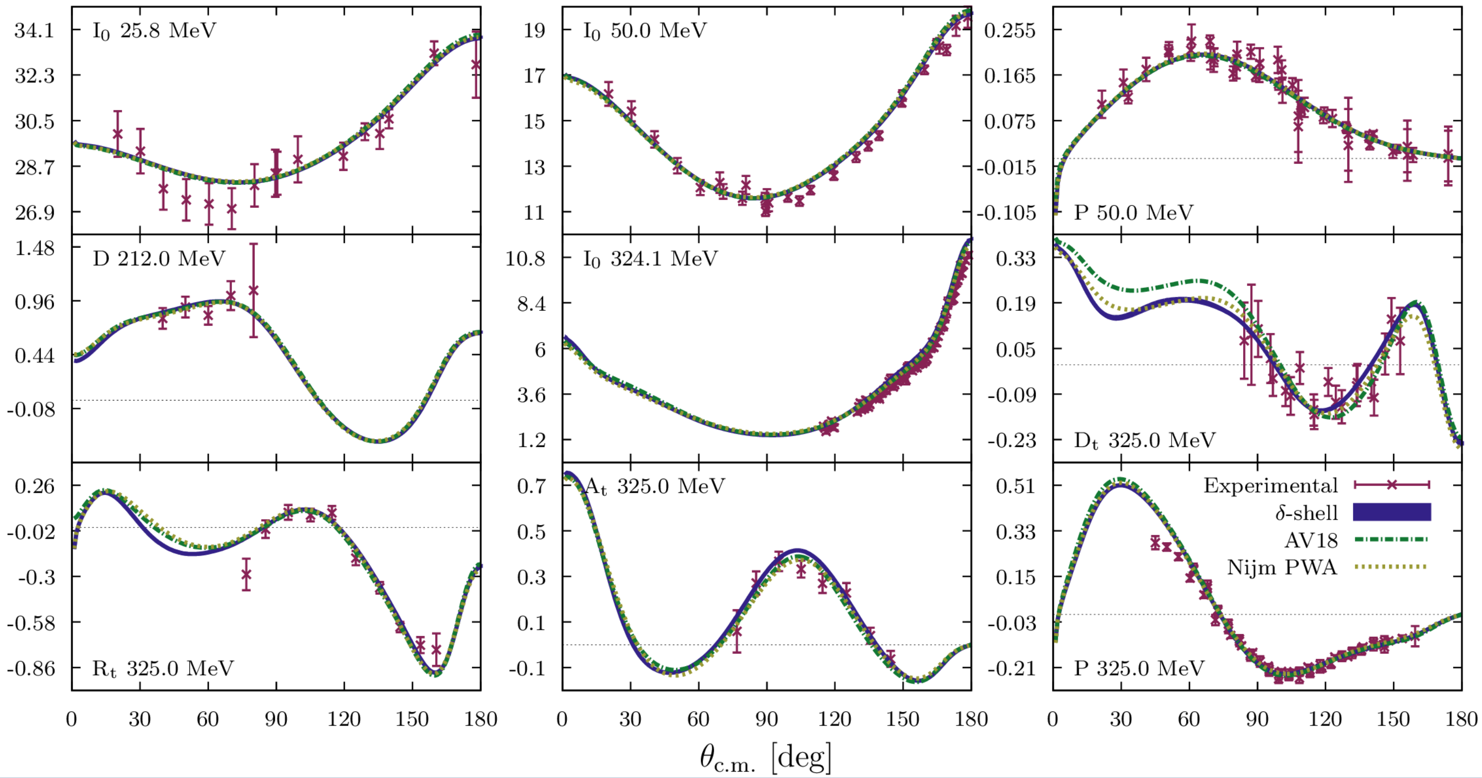

300 recovered data with Granada procedure (consistent database)



# Fitting NN scattering observables

- Comparing with Potentials and Experimental data




$\chi^2$/d.o.f. = 1.06 with $N = 2747|_{pp} + 3691|_{np}$

[RNP, Amaro & Ruiz-Arriola. Phys.Rev.C88 (2013) 024002]

# Chiral Two Pion Exchange

- Can χTPE interaction describe the same data
  - OPE, TPE(NLO) and TPE(NNLO)
  - Different cut radious rc = 3.0, 2.4, 1.8 fm
- Fitting the consistent database
  - No further data is excluded or added

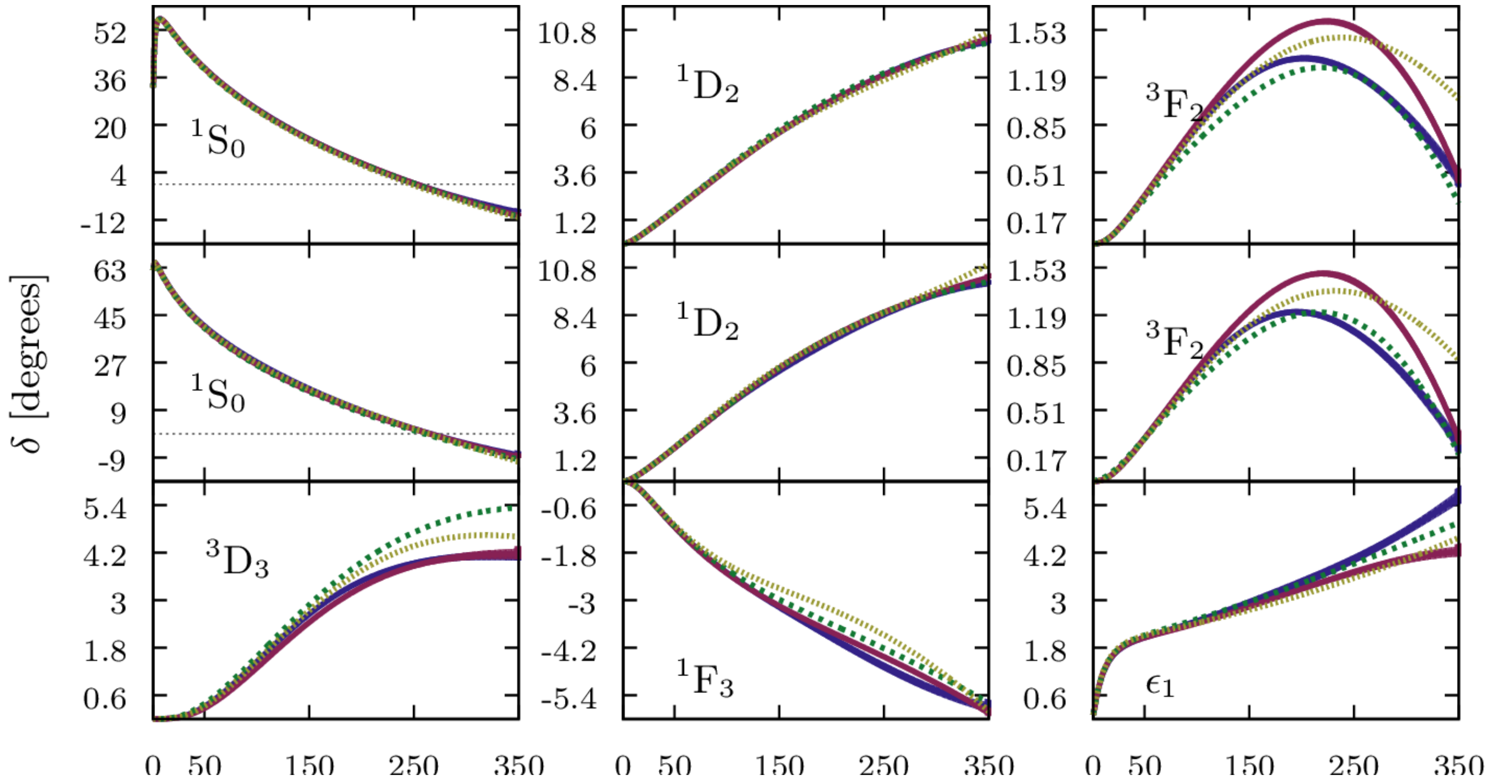
| $r_c$ [fm] | *1.8*<br>*Np - $\chi^2$/d.o.f.* | 2.4<br>*Np - $\chi^2$/d.o.f.* | 3.0<br>*Np - $\chi^2$/d.o.f.* |
|---|---|---|---|
| OPE | 31 – 1.37 | 39 – 1.09 | 46 – 1.06 |
| TPE (NLO) | 31 – 1.26 | 38 – 1.08 | 46 – 1.06 |
| TPE (NNLO) | 30+3 – 1.10 | 38+3 – 1.08 | 46+3 – 1.06 |

[RNP, Amaro & Ruiz Arriola. Phys.Rev.C89 (2014) 024004]

# Chiral Two Pion Exchange

## Phase-shifts

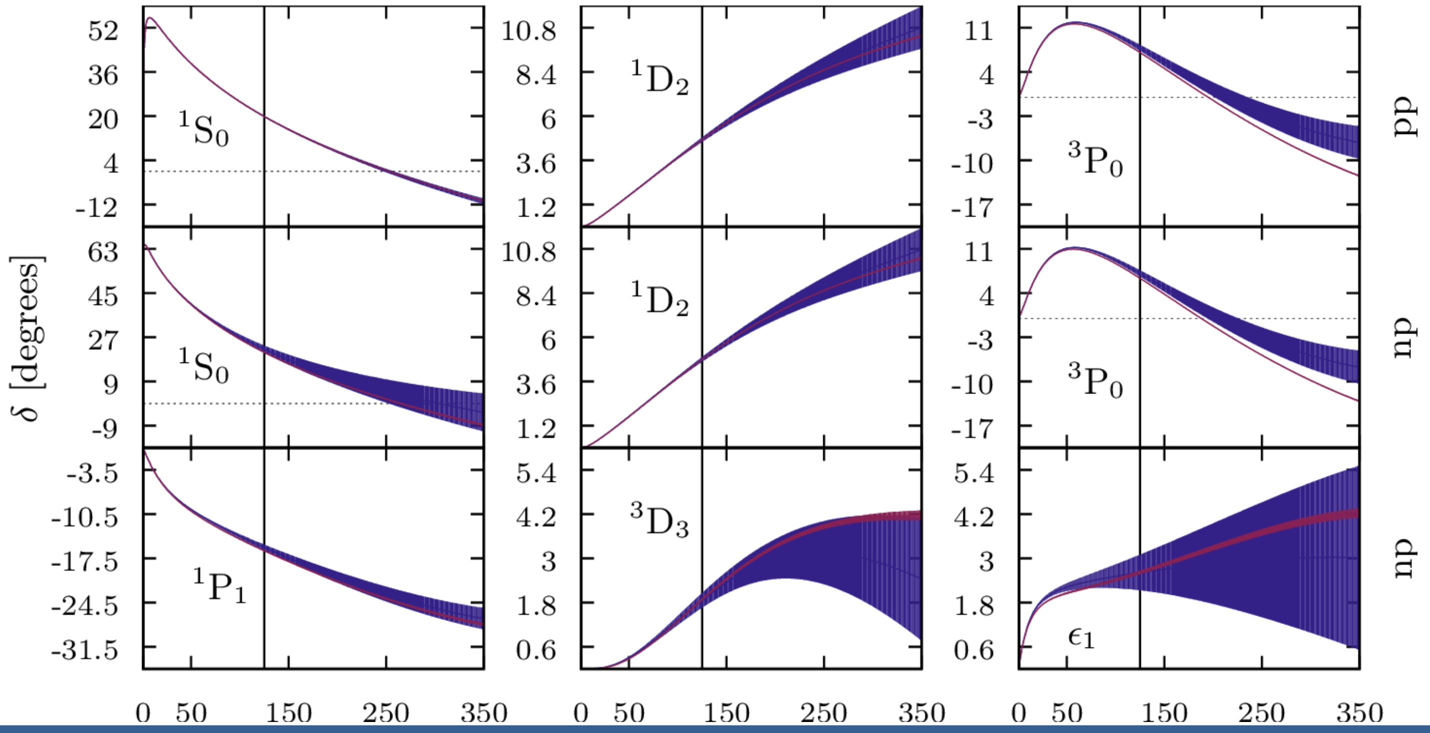
- Comparison of OPE and χTPE




**Discrepancies in phase-shifts account for systematic uncertainties**

# Chiral Two Pion Exchange
## Phase-shifts

- Lowering the Energy fitting range from 350 to 125 MeV
  - 20+3 parameters with $\chi^2$/d.o.f. = 1.02

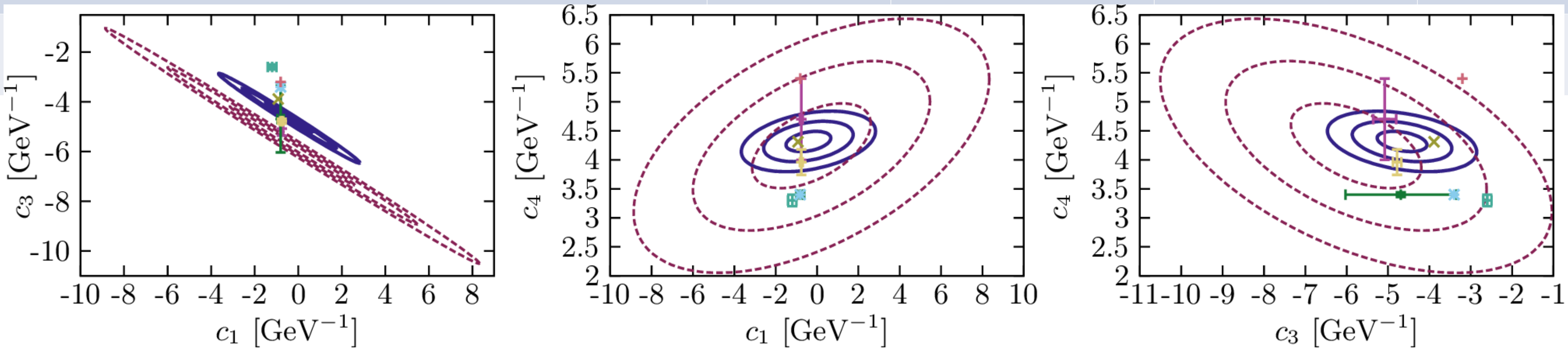



**Significant increase of statistical uncertainties**

[RNP, Amaro & Ruiz-Arriola Phys.Rev.C91 (2015) 054002]

# Determination of Chiral LEC's

| | Source | $c_1$ GeV$^{-1}$ | $c_3$ GeV$^{-1}$ | $c_4$ GeV$^{-1}$ |
|---|---|---|---|---|
| RNP, Amaro and Ruiz-Arriola 350 | NN | -0.42±1.08 | -4.66±0.60 | 4.32±0.17 |
| RNP, Amaro and Ruiz-Arriola 125 | NN | -0.27±2.87 | -5.77±1.58 | 4.24±0.73 |
| Nijmegen | pp | -0.76±0.07 | -5.08±0.28 | 4.70±0.70 |
| Entem & Machleidt | NN | -0.81 | -3.40 | 3.40 |
| Ekström et. al. | NN | -0.92 | -3.89 | 4.31 |
| Buetikker & Meissner | πN | -0.81±0.15 | -4.69±1.34 | 3.40±0.04 |



# Fitting NN scattering observables

- Different potentials fitted to the *same* database

| Potential | $T_{LAB}$ | $N_{Data}$ | $N_{parameters}$ | $\chi^2$/d.o.f. |
|---|---|---|---|---|
| DS - OPE | 350 | 6713 | 46 | 1.05 |
| DS - χTPE | 350 | 6712 | 33 | 1.08 |
| DS - ΔBorn | 350 | 6719 | 31 | 1.06 |
| Gauss - OPE | 350 | 6712 | 42 | 1.07 |
| Gauss - χTPE | 350 | 6712 | 31 | 1.09 |
| Gauss - ΔBorn | 350 | 6712 | 30 | 1.14 |

[RNP, Amaro & Ruiz Arriola. ArXiv:1410.8097v3]

Predictions are different
Source of *systematic* uncertainties

# Fitting NN scattering observables

## Testing the normality of residuals

- Experiments by counting events → Poissonian statistics

- Large number of events → Normal statistics
- Crucial assumption

$$R_i = \frac{O_i^{\exp} - O_i^{theor}\left(p_1, p_2, \ldots p_P\right)}{\Delta O_i^{\exp}}$$

  follows the standard normal distribution

- $\chi^2$/d.o.f. = 1 ± (2/d.o.f.)$^{1/2}$

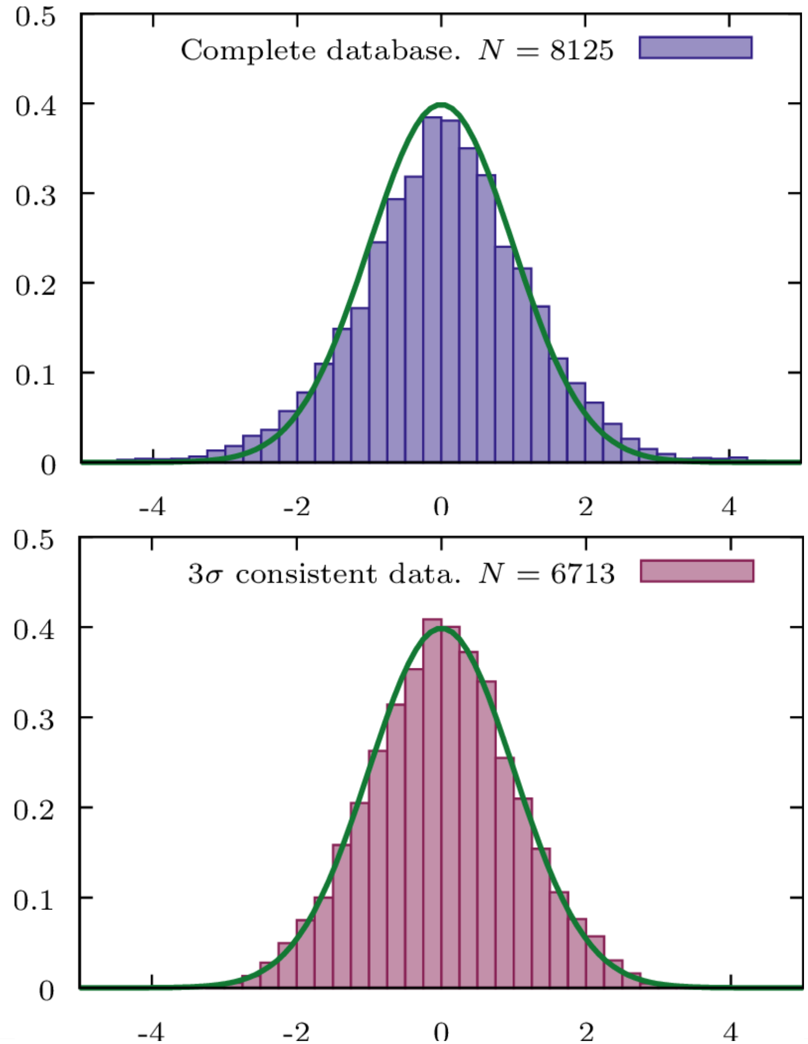
- Can be different from *N*(0,1), but it has to be known

Can only be checked *a posteriori*

# Fitting NN scattering observables

## Testing the normality of residuals

- Empirical distribution $P_{emp}$
- Normal distribution N(0,1)
- Finite size fluctuations
- Discrepancies between $P_{emp}$ and $N(0,1)$
- How large is too large?
- Normality tests
  - Quantifying discrepancies
  - Test statistic $T$
  - Critical values



# Fitting NN scattering observables

## Tail Sensitive test

- Quantitative test with a graphical representation

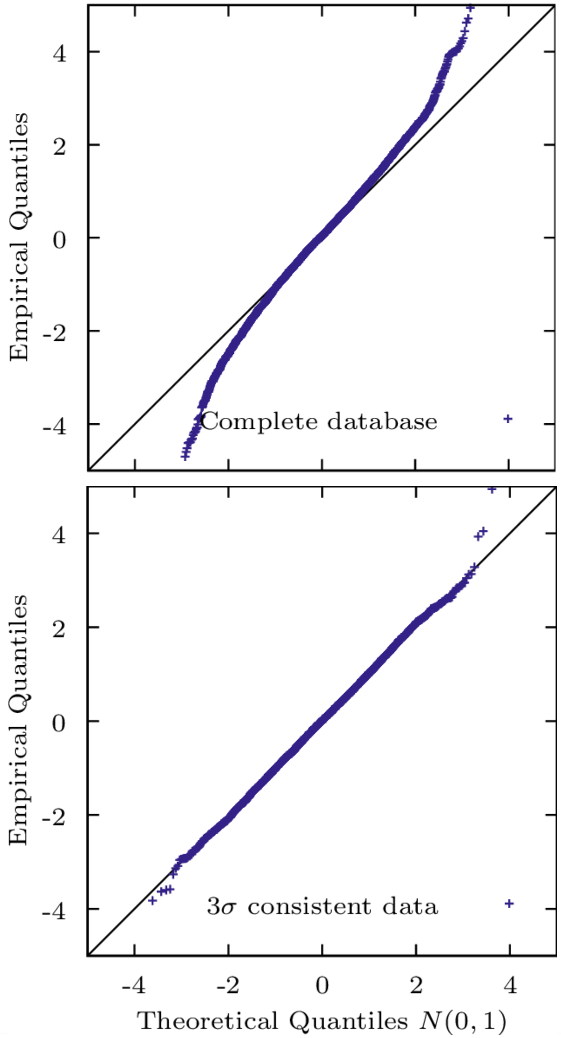
- Quantile-Quantile plot
  - Theoretical quantiles

$$\frac{i}{N+1} = \int_{-\infty}^{x_i^{th}} N(t)\,dt$$

  - Empirical Quantiles

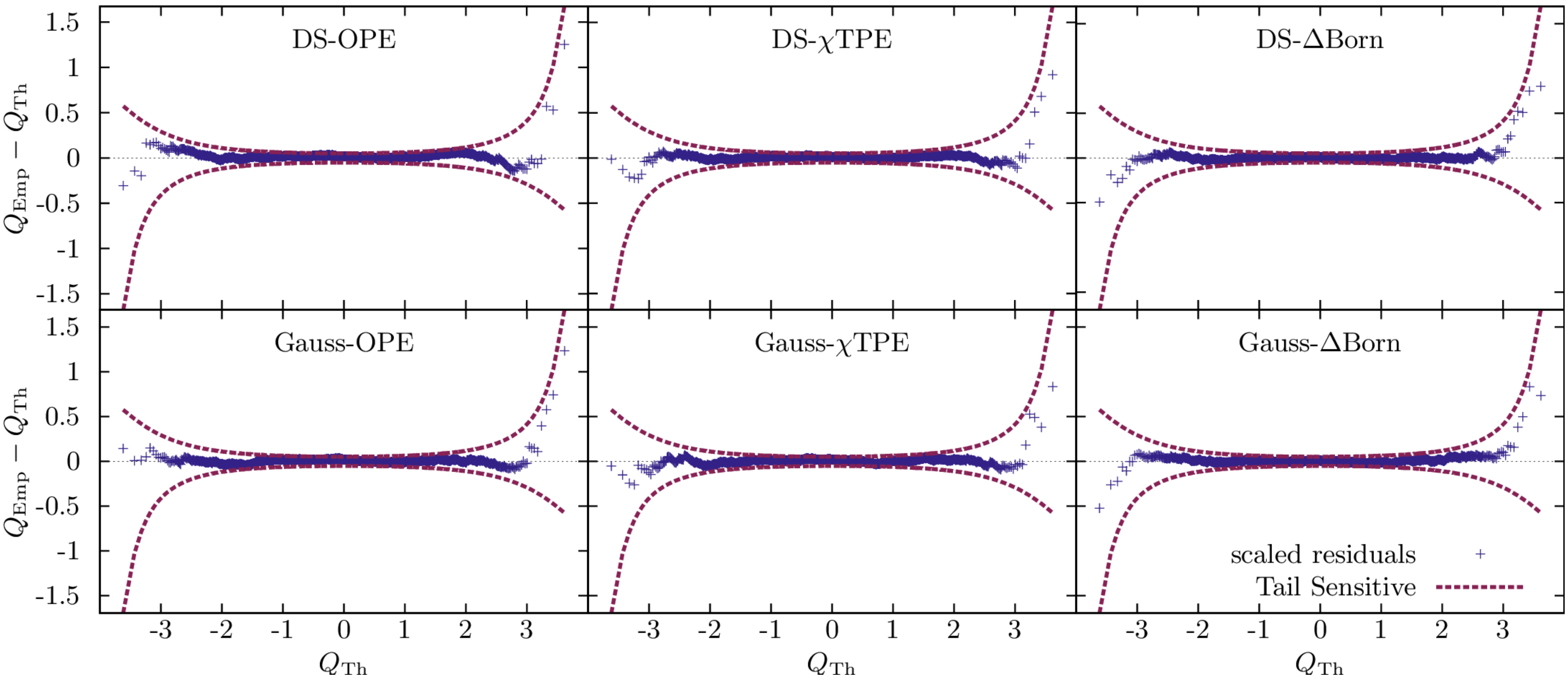
$$x_1^{emp} < x_2^{emp} < \ldots < x_N^{emp}$$

  - Mapping ($x_i^{th}$, $x_i^{emp}$)
  - $\mathrm{Lim}_{N\to\infty}$ ($x_i^{emp}$ – $x_i^{th}$) = 0
  - Confidence bands



# Fitting NN scattering observables

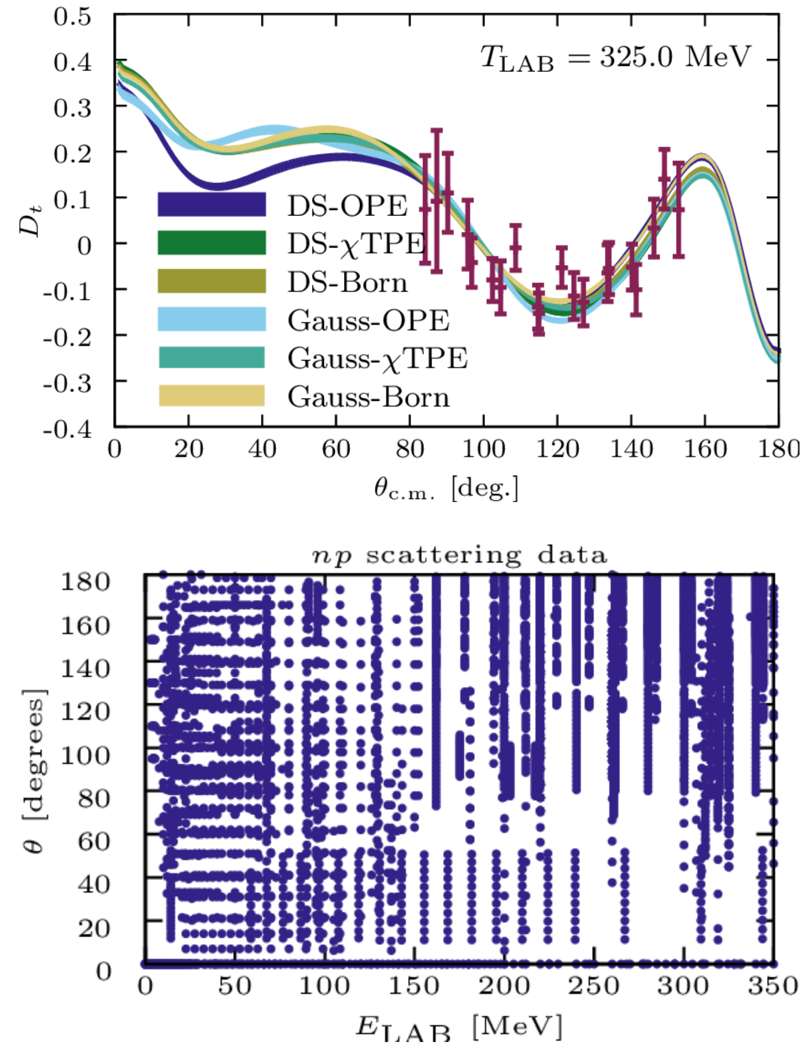
## Testing the normality of residuals



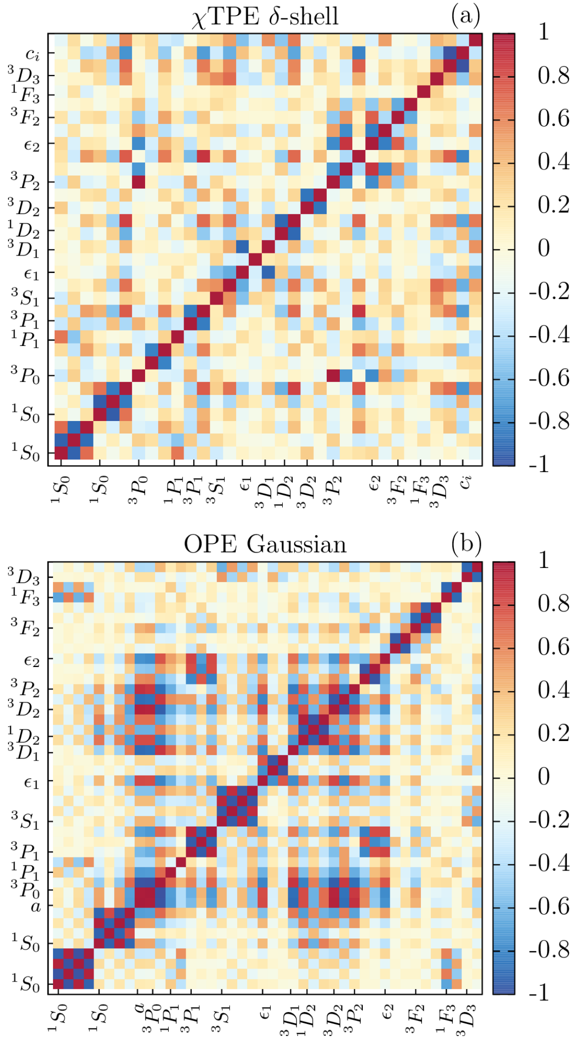

Six statistically equivalent representations of the NN interaction
Their discrepancies won't come from the data

# NN Systematic Uncertainty

- Data is unevenly distributed on the ($T_{LAB}$,$\theta_{c.m.}$)
- Same description in probed regions
- Incompatible predictions in unexplored areas
- A uniform experimental exploration is necessary but unlikely

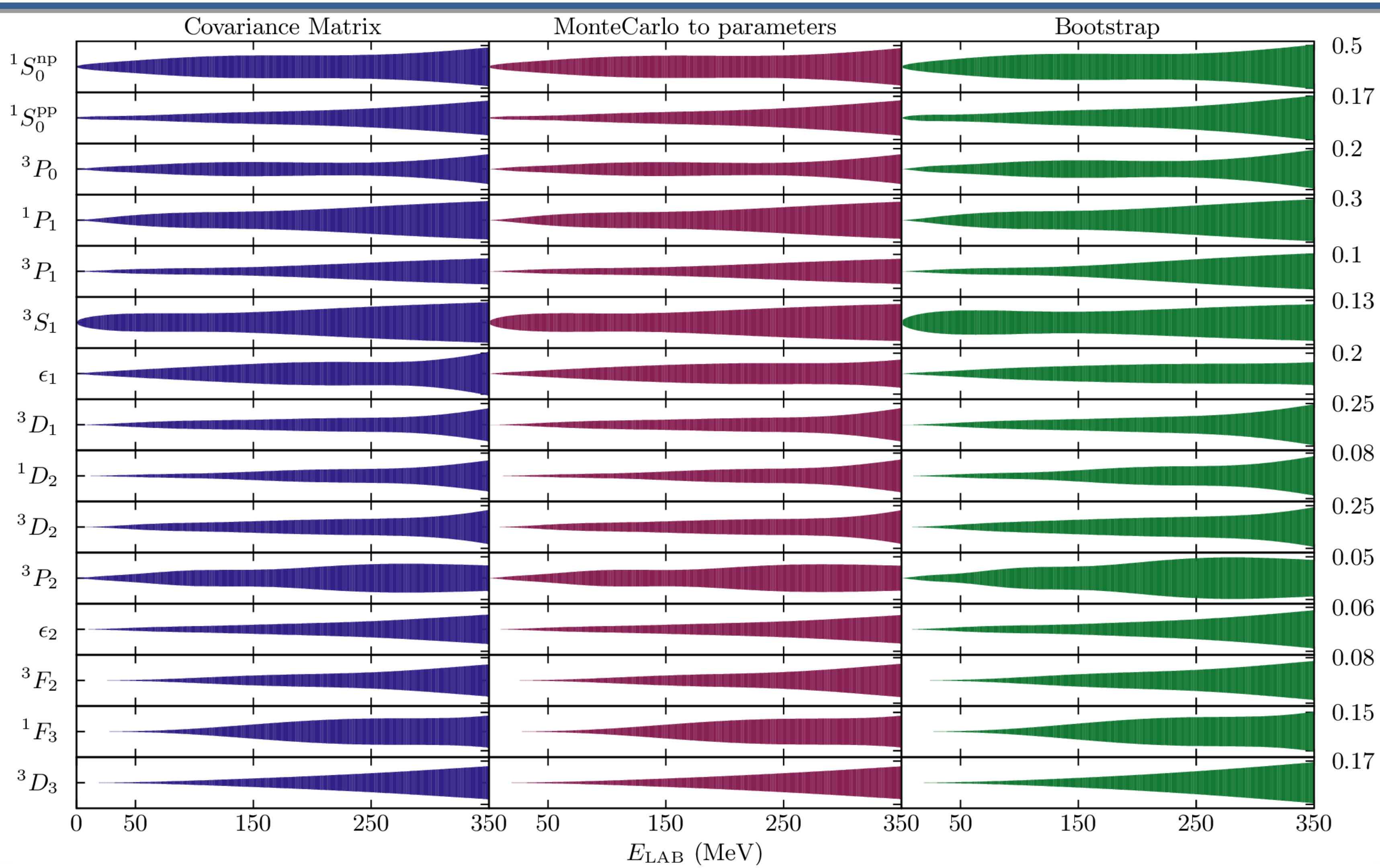


# Reproducing NN uncertainties from NN data




- Propagation with covariance matrix
  - Requires to calculate derivatives
- Monte-Carlo family of potentials
- Bootstrap the data
  - Simulate data ~ $N(O_i,\Delta o_i)$
  - Refit parameters
- Replicate parameters correlations
  - Simulate parameters
  - Faster, but real distribution may differ

# Reproducing NN uncertainties from NN data



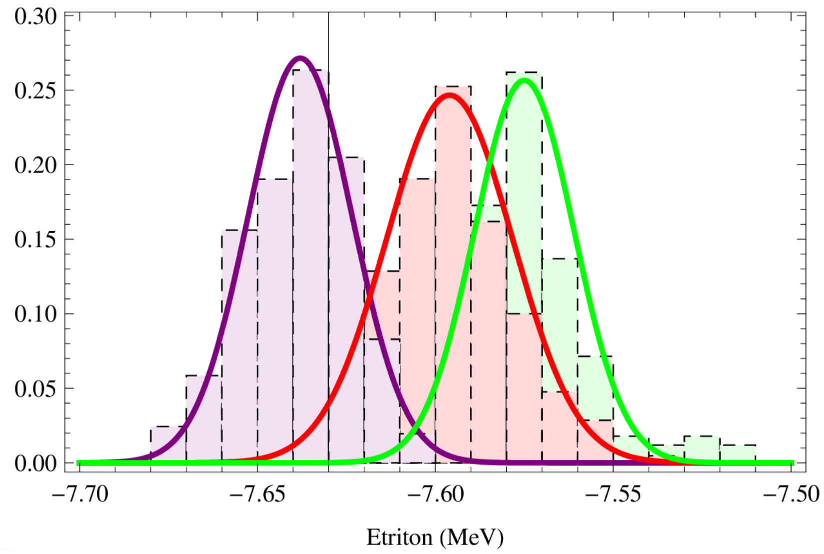
# Triton Binding Energy

## Hyperspherical Adiabatic Expansion Method

- Monte-Carlo simulation of *N* =250 potentials
- Error estimates in nuclear structure calculations
- $\Delta B_t^{stat}$ = 15(1) KeV, $\Delta B_t^{num}$ = 1 KeV

[RNP, Garrido, Amaro & Ruiz-Arriola. Phys.Rev.C99 (2014) 047001]

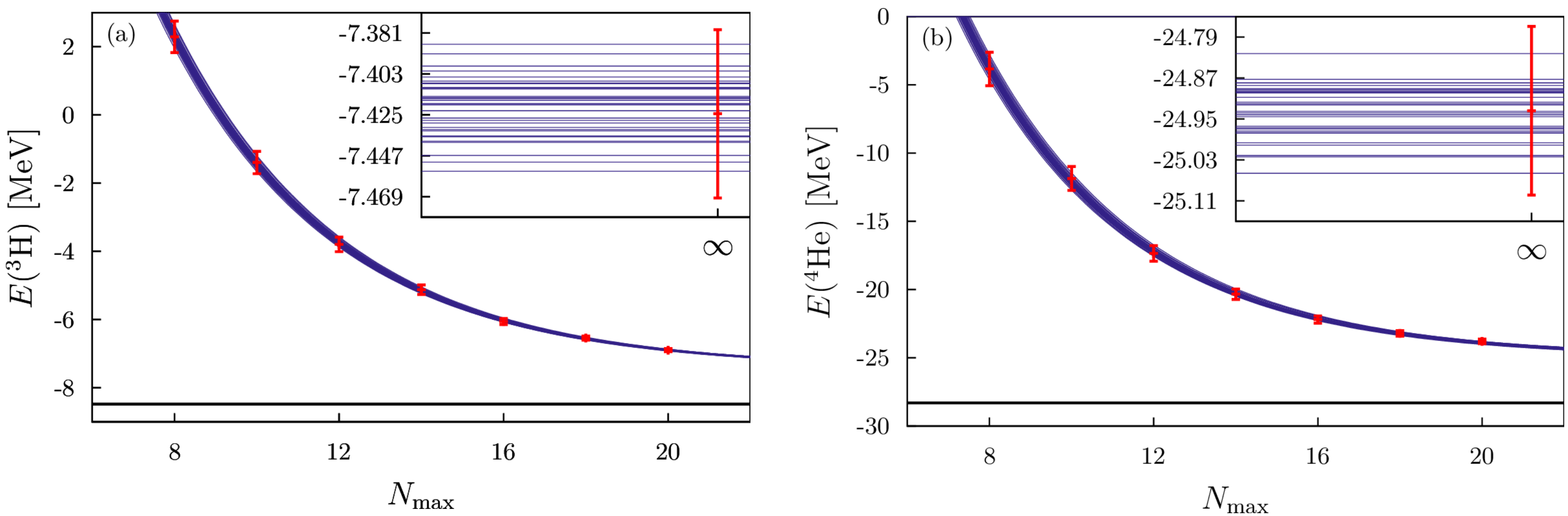
- *N* ~ 30 gives a fairly good estimate
- Reduction of target accuracy is possible
- $\Delta B_t^{sys}$ is even larger



# $^{3}$H and $^{4}$He Binding Energy
## No Core Full Configuration Method

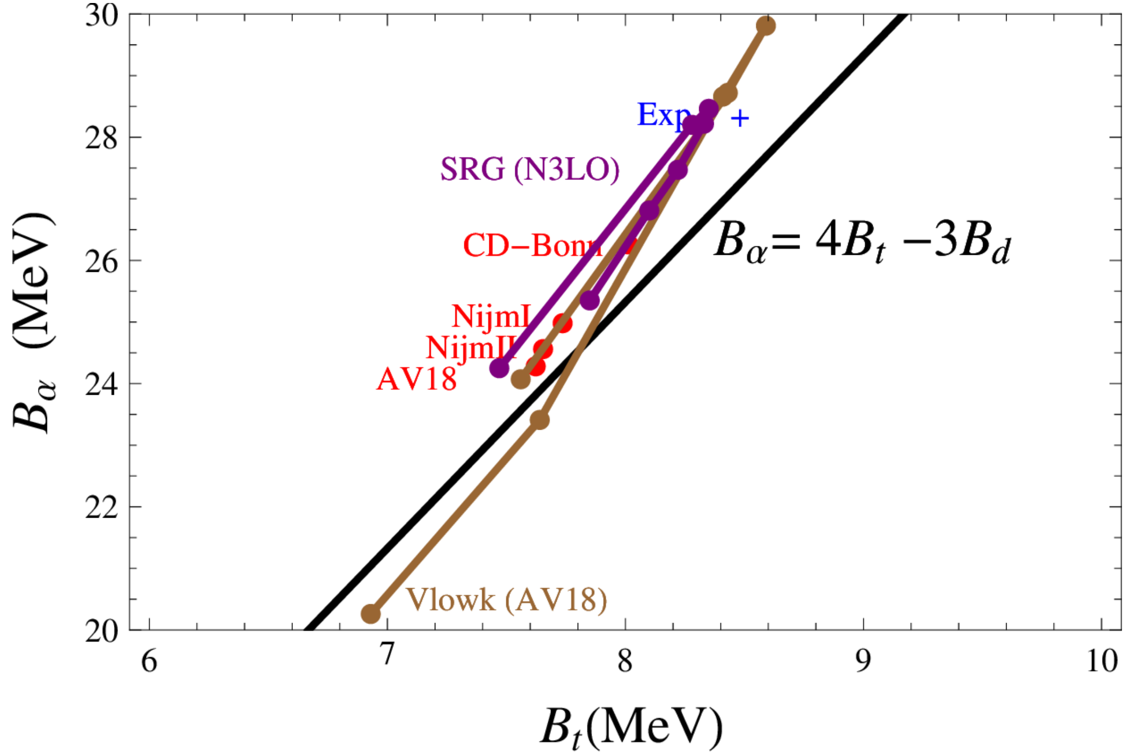
- Sum of Gaussians potential
- 33 Monte-Carlo potentials
- $\Delta(^{3}H)_t^{stat}$ = 15 KeV, $\Delta(^{4}He)_t^{stat}$ = 55 KeV

[RNP, Amaro, Ruiz-Arriola, Maris & Vary. Phys.Rev.C92 (2015) 064003]



# Tjon Line correlation

- Empirical correlation between binding energy calculations



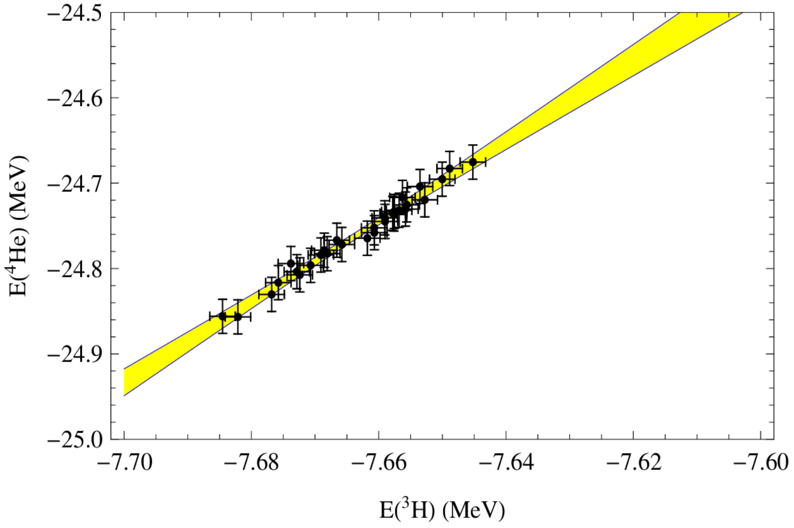

Similarity Renormalization Group: $B_\alpha = 4B_t + 3B_d$

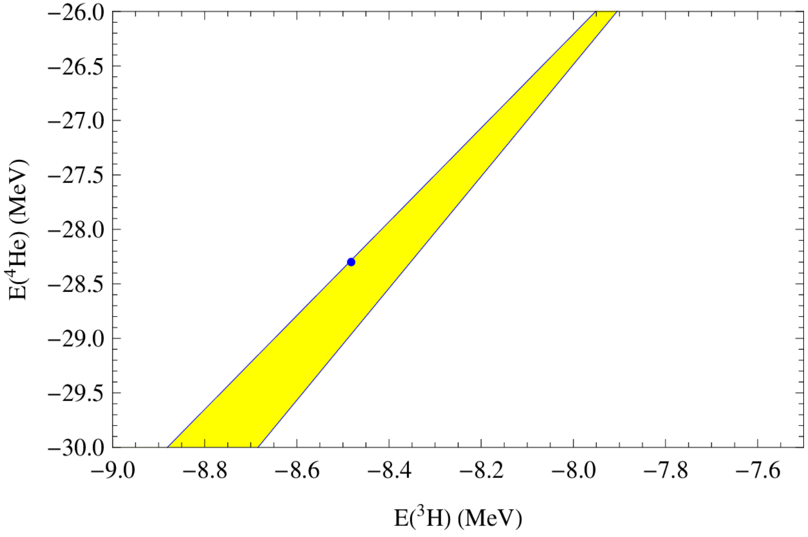
[Ruiz-Arriola, Szpiegel & Timoteo. Few Body Syst. 55 (2014) 971-975]

# Tjon Line Correlation

## Numerical accuracy.

- Preliminary Results
- $\Delta(^3H)_t^{num}$ = 1 KeV, $\Delta(^4He)_t^{num}$ = 20 KeV




4-Body forces are masked by the numerical noise in 3 and 4 body calculations

# Summary

- Determination of the NN interaction is not unique
- Fit to NN scattering data (on a desktop computer)
  - Good description of scattering observables (over 6400)
  - 6 different and statistically equivalent interactions
- Normality of residuals is crucial for a reliable error propagation
- Error propagation into bound states
  - Monte Carlo Simulation
  - 3H: $\Delta B^{stat}$ = 15 KeV
  - 4He: $\Delta B^{stat}$ = 55 KeV

Numerical accuracy in nuclear structure can be tailored to statistical and systematic uncertainties