Three-Flavor Chiral Effective Model with Four Baryonic Multiplets within the Mirror Assignment

Lisa Olbrich[1]

in collaboration with Miklós Zétényi[2], Francesco Giacosa[1,3] and Dirk H. Rischke[1]

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FAIRness, February 16th 2016

Outline

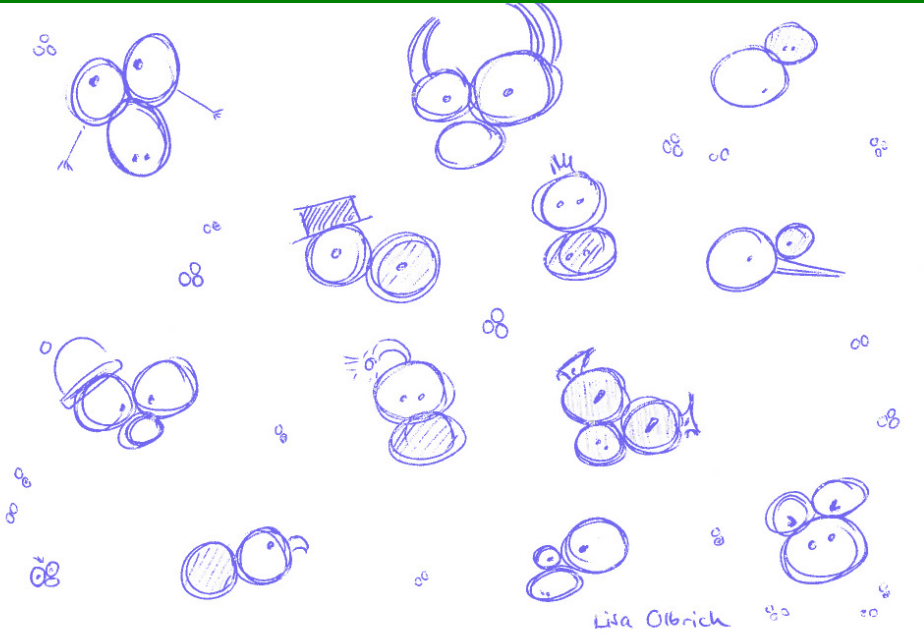
1. Introduction

2. The Model and Its Implications

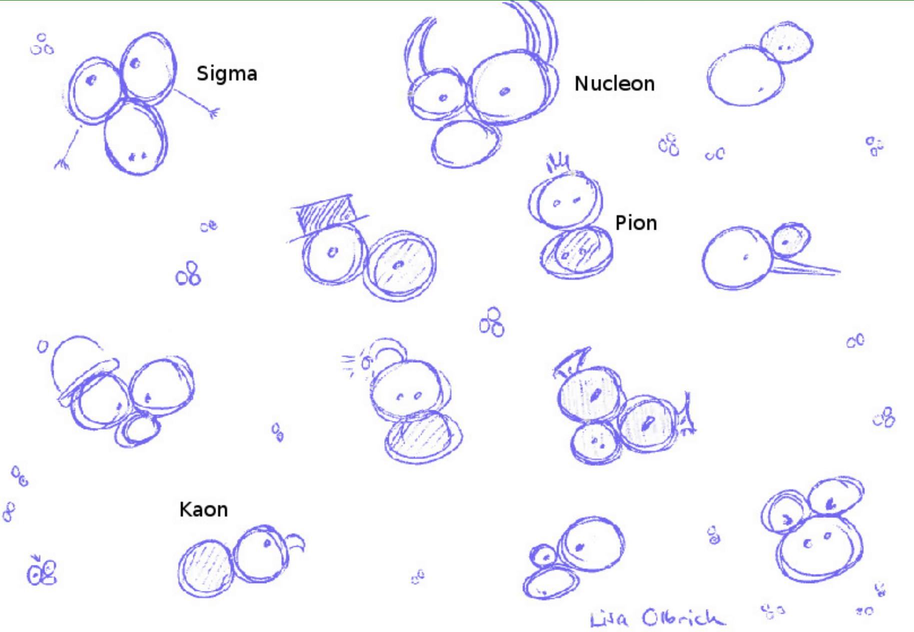
3. Fit and Results

4. Conclusions and Outlook

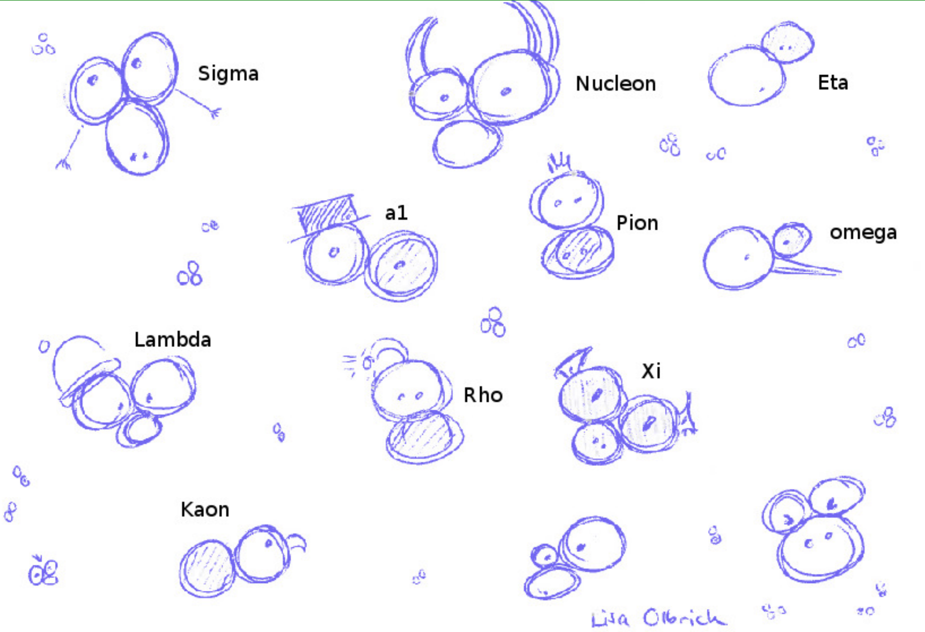
The zoo of hadrons

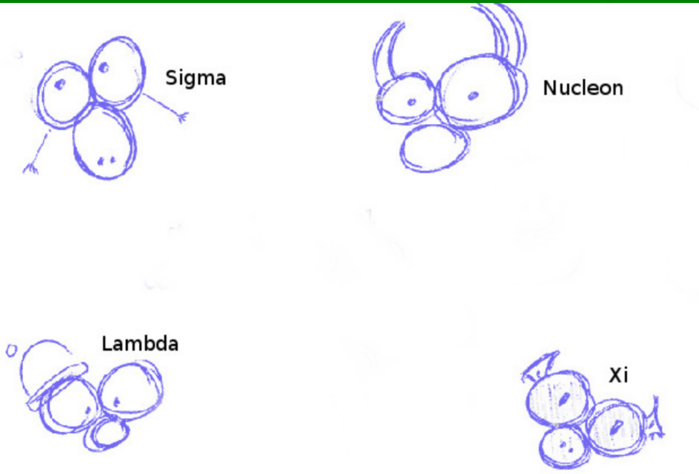


The zoo of hadrons

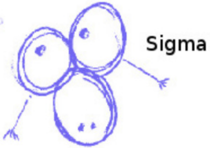


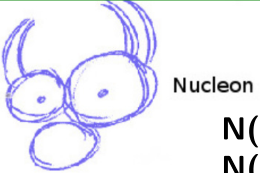
The zoo of hadrons



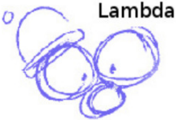


Octet baryons with three flavors





N(939)
N(1440)
N(1535)
N(1650)



Octet baryons with three flavors

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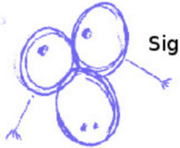

Sigma

$\Sigma(1193)$
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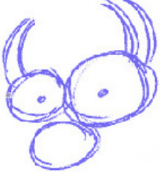

Nucleon

N(939)
N(1440)
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$\Lambda(1116)$
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Octet baryons with three flavors

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Our aim is to

describe the three-flavor octet baryons

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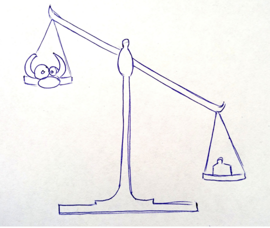
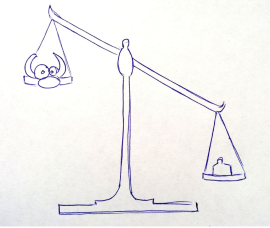
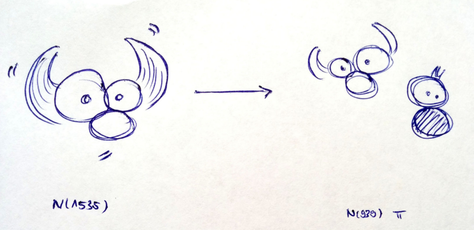


masses and

Our aim is to

describe the three-flavor octet baryons

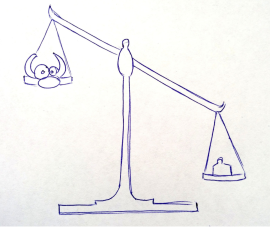


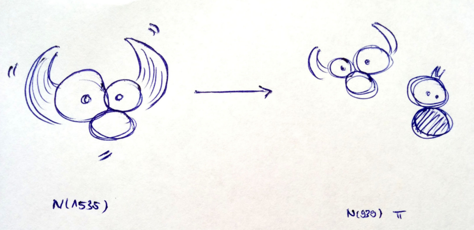


masses and interactions / decay widths.

Our aim is to

describe the three-flavor octet baryons





masses and interactions / decay widths.

Determined by the strong interaction.

Glass of wine

Glass of wine

Can we enjoy the wine also here?

Will the wine be frozen?

Exact description using Classical Mechanics:

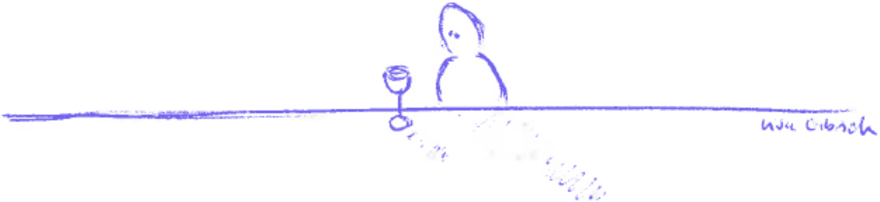
$$\Gamma(\vec{r}_1(t), \vec{r}_2(t), \dots, \vec{p}_1(t), \vec{p}_2(t), \dots)$$

Effective description using Thermodynamics:

$$\Gamma(T, \dots)$$

What have we learned?

Effective Models can simplify our everyday lives.



Quantum Chromodynamics

$$\mathcal{L}_{\mathsf{QCD}} = \bar{q}(x)\big(i\gamma^{\mu}D_{\mu} - m\big)q(x) - \frac{1}{2}\,\mathrm{Tr}(\mathcal{G}_{\mu\nu}\mathcal{G}^{\mu\nu})$$

Quantum Chromodynamics

$$\mathcal{L}_{\mathsf{QCD}} = \bar{q}(x)\big(i\gamma^{\mu} D_{\mu} - m\big)q(x) - \frac{1}{2}\,\mathrm{Tr}(\mathcal{G}_{\mu\nu}\mathcal{G}^{\mu\nu})$$

- Only a few parameters,
- but not analytically solvable.

Coupling ‘constant’ of strong interaction gets arbitrary large at low energies. I.e., at small energies, perturbative expansion of interaction terms in powers of the coupling constant not possible.

Extended Linear Sigma Model (meson part)

$$
\begin{aligned}
\mathcal{L}_{\text{meson}} =& \operatorname{Tr}\left[(D^\mu\Phi)^\dagger D_\mu\Phi\right] - m_0^2 \operatorname{Tr}\left(\Phi^\dagger\Phi\right) - \lambda_1 \left[\operatorname{Tr}\left(\Phi^\dagger\Phi\right)\right]^2 - \lambda_2 \operatorname{Tr}\left[\left(\Phi^\dagger\Phi\right)^2\right] \\
&- \frac{1}{4}\operatorname{Tr}\left(L_{\mu\nu}L^{\mu\nu} + R_{\mu\nu}R^{\mu\nu}\right) + \operatorname{Tr}\left[\left(\frac{m_1^2}{2} + \Delta\right)\left(L_\mu L^\mu + R_\mu R^\mu\right)\right] \\
&+ \operatorname{Tr}\left[H\left(\Phi + \Phi^\dagger\right)\right] + c_1\left(\det\Phi - \det\Phi^\dagger\right)^2 \\
&+ i\frac{g_2}{2}\left[\operatorname{Tr}\left(L_{\mu\nu}\left[L^\mu, L^\nu\right]\right) + \operatorname{Tr}\left(R_{\mu\nu}\left[R^\mu, R^\nu\right]\right)\right] \\
&+ \frac{h_1}{2}\operatorname{Tr}\left(\Phi^\dagger\Phi\right)\operatorname{Tr}\left(L_\mu L^\mu + R_\mu R^\mu\right) + h_2 \operatorname{Tr}\left[(L_\mu\Phi)^\dagger(L^\mu\Phi) + (\Phi R_\mu)^\dagger(\Phi R^\mu)\right] \\
&+ 2h_3 \operatorname{Tr}\left(\Phi R^\mu \Phi^\dagger L^\mu\right) + g_3\left[\operatorname{Tr}\left(L_\mu L_\nu L^\mu L^\nu\right\} + \operatorname{Tr}\left\{R_\mu R_\nu R^\mu R^\nu\right)\right] \\
&+ g_4\left[\operatorname{Tr}\left(L_\mu L^\mu L_\nu L^\nu\right) + \operatorname{Tr}\left(R_\mu R^\mu R_\nu R^\nu\right)\right] + g_5 \operatorname{Tr}\left(L_\mu L^\mu\right)\operatorname{Tr}\left(R_\nu R^\nu\right) \\
&+ g_6\left[\operatorname{Tr}\left(L_\mu L^\mu\right)\operatorname{Tr}\left(L_\nu L^\nu\right) + \operatorname{Tr}\left(R_\mu R^\mu\right)\operatorname{Tr}\left(R_\nu R^\nu\right)\right]
\end{aligned}
$$

D. Parganlija, P. Kovacs, G. Wolf, F. Giacosa and D. H. Rischke, Phys. Rev. D **87** (2013) 014011
S. Janowski, F. Giacosa and D. H. Rischke, Phys. Rev. D **90** (2014) 11, 114005

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& - \frac{1}{4}\mathrm{Tr}\left(L_{\mu\nu}L^{\mu\nu} + R_{\mu\nu}R^{\mu\nu}\right) + \mathrm{Tr}\left[\left(\frac{m_1^2}{2} + \Delta\right)\left(L_{\mu}L^{\mu} + R_{\mu}R^{\mu}\right)\right] \\
& + \mathrm{Tr}\left[H\left(\Phi + \Phi^{\dagger}\right)\right] + c_1\left(\det\Phi - \det\Phi^{\dagger}\right)^2 \\
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& + g_6\left[\mathrm{Tr}\left(L_{\mu}L^{\mu}\right)\mathrm{Tr}\left(L_{\nu}L^{\nu}\right) + \mathrm{Tr}\left(R_{\mu}R^{\mu}\right)\mathrm{Tr}\left(R_{\nu}R^{\nu}\right)\right]
\end{aligned}
$$

- A lot more parameters,
- but solvable using perturbation theory.
- Good results already at tree level.

D. Parganlija, P. Kovacs, G. Wolf, F. Giacosa and D. H. Rischke, Phys. Rev. D **87** (2013) 014011
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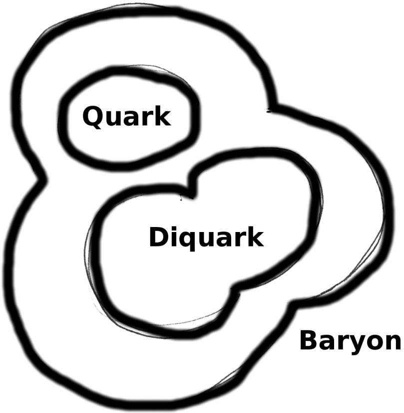
Baryonic fields as quark-diquark states



$J^P = \frac{1}{2}^+$ baryon octet in the $N_f = 3$ case:

$$\underbrace{\begin{pmatrix} [d,s] \\ -[u,s] \\ [u,d] \end{pmatrix}}_{\text{diquark}} \underbrace{(u,d,s)}_{\text{quark}} \hat{=} \begin{pmatrix} uds & uus & uud \\ dds & uds & udd \\ dss & uss & uds \end{pmatrix}$$

$$\sim \begin{pmatrix} \frac{\Lambda}{\sqrt{6}} + \frac{\Sigma^0}{\sqrt{2}} & \Sigma^+ & p \\ \Sigma^- & \frac{\Lambda}{\sqrt{6}} - \frac{\Sigma^0}{\sqrt{2}} & n \\ \Xi^- & \Xi^0 & -\frac{2\Lambda}{\sqrt{6}} \end{pmatrix}$$

Chiral Transformation – Mirror Assignment

- Two matrices N_1 and N_2, which behave under chiral transformations as

$$N_{1R} \to \; U_R N_{1R} U_R^\dagger \,, \qquad N_{1L} \to \; U_L N_{1L} U_R^\dagger ,$$
$$N_{2R} \to \; U_R N_{2R} U_L^\dagger \,, \qquad N_{2L} \to \; U_L N_{2L} U_L^\dagger \,.$$

- And two matrices M_1 and M_2 whose chiral transformation from the left is 'mirror-like':

$$M_{1R} \to \; U_L M_{1R} U_R^\dagger \,, \qquad M_{1L} \to \; U_R M_{1L} U_R^\dagger$$
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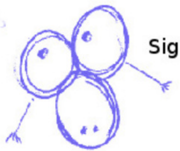

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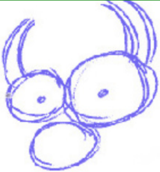

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N(1440)
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N(1650)

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$\Lambda(1116)$
$\Lambda(1600)$
$\Lambda(1670)$
$\Lambda(1800)$

Xi

$\Xi(1338)$
$\Xi(1690)$
$\Xi(?)$
$\Xi(?)$

Experimentally known correspondence?!

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Baryon Fields for $N_f = 3$ - Parity Eigenstates

Baryonic fields with definite parity are given by

$$B_N = \frac{N_1 - N_2}{\sqrt{2}}, \qquad B_{N\star} = \frac{N_1 + N_2}{\sqrt{2}},$$

$$B_M = \frac{M_1 - M_2}{\sqrt{2}}, \qquad B_{M\star} = \frac{M_1 + M_2}{\sqrt{2}} ,$$

where now B_N and B_M have positive parity and B_{N*} and B_{M*} have negative parity.

Assignment to physical particles/resonances

In the limit of zero mixing the fields can be assigned to particles as follows

B_N to $\{N(939),\ \Lambda(1116), \Sigma(1193), \Xi(1338)\}$,
B_M to $\{N(1440), \Lambda(1600), \Sigma(1660), \Xi(1690)\}$,
B_{N*} to $\{N(1535), \Lambda(1670), \Sigma(1620), \Xi(?)\}$
B_{M*} to $\{N(1650), \Lambda(1800), \Sigma(1750), \Xi(?)\}$.

The detailed study of the mixing is performed for the two-flavor case.

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B_{N*} to $\{$**N(1535)**$, \Lambda(1670), \Sigma(1620), \Xi(?)\}$
B_{M*} to $\{$**N(1650)**$, \Lambda(1800), \Sigma(1750), \Xi(?)\}$.

The detailed study of the mixing is performed for the two-flavor case.

The Lagrangian $(N_f = 3)$

$$
\begin{aligned}
\mathcal{L}_{N_f=3} = & \operatorname{Tr}\left\{\bar{N}_{1L} i\gamma_\mu D^\mu_{2L} N_{1L} + \bar{N}_{1R} i\gamma_\mu D^\mu_{1R} N_{1R} + \bar{N}_{2L} i\gamma_\mu D^\mu_{1L} N_{2L} + \bar{N}_{2R} i\gamma_\mu D^\mu_{2R} N_{2R}\right\} \\
& + \operatorname{Tr}\left\{\bar{M}_{1L} i\gamma_\mu D^\mu_{4R} M_{1L} + \bar{M}_{1R} i\gamma_\mu D^\mu_{3L} M_{1R} + \bar{M}_{2L} i\gamma_\mu D^\mu_{3R} M_{2L} + \bar{M}_{2R} i\gamma_\mu D^\mu_{4L} M_{2R}\right\} \\
& - g_N \operatorname{Tr}\left\{\bar{N}_{1L} \Phi N_{1R} + \bar{N}_{1R} \Phi^\dagger N_{1L} + \bar{N}_{2L} \Phi N_{2R} + \bar{N}_{2R} \Phi^\dagger N_{2L}\right\} \\
& - g_M \operatorname{Tr}\left\{\bar{M}_{1L} \Phi^\dagger M_{1R} + \bar{M}_{1R} \Phi M_{1L} + \bar{M}_{2L} \Phi^\dagger M_{2R} + \bar{M}_{2R} \Phi M_{2L}\right\} \\
& - m_{0,1} \operatorname{Tr}\left\{\bar{N}_{1L} M_{1R} + \bar{M}_{1R} N_{1L} + \bar{N}_{2R} M_{2L} + \bar{M}_{2L} N_{2R}\right\} \\
& - m_{0,2} \operatorname{Tr}\left\{\bar{N}_{1R} M_{1L} + \bar{M}_{1L} N_{1R} + \bar{N}_{2L} M_{2R} + \bar{M}_{2R} N_{2L}\right\} \\
& - \kappa_1 \operatorname{Tr}\left\{\bar{N}_{1R} \Phi^\dagger N_{2L} \Phi + \bar{N}_{2L} \Phi N_{1R} \Phi^\dagger\right\} - \kappa_1' \operatorname{Tr}\left\{\bar{N}_{1L} \Phi N_{2R} \Phi + \bar{N}_{2R} \Phi^\dagger N_{1L} \Phi^\dagger\right\} \\
& - \kappa_2 \operatorname{Tr}\left\{\bar{M}_{1R} \Phi M_{2L} \Phi + \bar{M}_{2L} \Phi^\dagger M_{1R} \Phi^\dagger\right\} - \kappa_2' \operatorname{Tr}\left\{\bar{M}_{1L} \Phi^\dagger M_{2R} \Phi + \bar{M}_{2R} \Phi M_{1L} \Phi^\dagger\right\} \\
& - \epsilon_1 \left(\operatorname{Tr}\left\{\bar{N}_{1L} \Phi\right\} \operatorname{Tr}\left\{N_{2R} \Phi\right\} + \operatorname{Tr}\left\{\bar{N}_{2R} \Phi^\dagger\right\} \operatorname{Tr}\left\{N_{1L} \Phi^\dagger\right\}\right) \\
& - \epsilon_2 \left(\operatorname{Tr}\left\{\bar{M}_{1R} \Phi\right\} \operatorname{Tr}\left\{M_{2L} \Phi\right\} + \operatorname{Tr}\left\{\bar{M}_{2L} \Phi^\dagger\right\} \operatorname{Tr}\left\{M_{1R} \Phi^\dagger\right\}\right) \\
& - \epsilon_3 \operatorname{Tr}\left\{\Phi^\dagger \Phi\right\} \operatorname{Tr}\left\{\bar{N}_{1L} M_{1R} + \bar{M}_{1R} N_{1L} + \bar{N}_{2R} M_{2L} + \bar{M}_{2L} N_{2R}\right\} \\
& - \epsilon_4 \operatorname{Tr}\left\{\Phi^\dagger \Phi\right\} \operatorname{Tr}\left\{\bar{N}_{1R} M_{1L} + \bar{M}_{1L} N_{1R} + \bar{N}_{2L} M_{2R} + \bar{M}_{2R} N_{2L}\right\}
\end{aligned}
$$

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& + \mathrm{Tr}\left\{\bar{M}_{1L} i\gamma_\mu D_{4R}^{\mu} M_{1L} + \bar{M}_{1R} i\gamma_\mu D_{3L}^{\mu} M_{1R} + \bar{M}_{2L} i\gamma_\mu D_{3R}^{\mu} M_{2L} + \bar{M}_{2R} i\gamma_\mu D_{4L}^{\mu} M_{2R}\right\} \\
& - g_N\, \mathrm{Tr}\left\{\bar{N}_{1L} \Phi N_{1R} + \bar{N}_{1R} \Phi^\dagger N_{1L} + \bar{N}_{2L} \Phi N_{2R} + \bar{N}_{2R} \Phi^\dagger N_{2L}\right\} \\
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& - \kappa_1\, \mathrm{Tr}\left\{\bar{N}_{1R} \Phi^\dagger N_{2L} \Phi + \bar{N}_{2L} \Phi N_{1R} \Phi^\dagger\right\} - \kappa_1'\, \mathrm{Tr}\left\{\bar{N}_{1L} \Phi N_{2R} \Phi + \bar{N}_{2R} \Phi^\dagger N_{1L} \Phi^\dagger\right\} \\
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& - \epsilon_1 \left(\mathrm{Tr}\left\{\bar{N}_{1L} \Phi\right\} \mathrm{Tr}\left\{N_{2R} \Phi\right\} + \mathrm{Tr}\left\{\bar{N}_{2R} \Phi^\dagger\right\} \mathrm{Tr}\left\{N_{1L} \Phi^\dagger\right\}\right) \\
& - \epsilon_2 \left(\mathrm{Tr}\left\{\bar{M}_{1R} \Phi\right\} \mathrm{Tr}\left\{M_{2L} \Phi\right\} + \mathrm{Tr}\left\{\bar{M}_{2L} \Phi^\dagger\right\} \mathrm{Tr}\left\{M_{1R} \Phi^\dagger\right\}\right) \\
& - \epsilon_3\, \mathrm{Tr}\{\Phi^\dagger \Phi\}\, \mathrm{Tr}\{\bar{N}_{1L} M_{1R} + \bar{M}_{1R} N_{1L} + \bar{N}_{2R} M_{2L} + \bar{M}_{2L} N_{2R}\} \\
& - \epsilon_4\, \mathrm{Tr}\{\Phi^\dagger \Phi\}\, \mathrm{Tr}\{\bar{N}_{1R} M_{1L} + \bar{M}_{1L} N_{1R} + \bar{N}_{2L} M_{2R} + \bar{M}_{2R} N_{2L}\}
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$$
\begin{aligned}
\mathcal{L}_{N_f=3} = {} & \mathrm{Tr}\left\{\bar{N}_{1L} i\gamma_\mu D^\mu_{2L} N_{1L} + \bar{N}_{1R} i\gamma_\mu D^\mu_{1R} N_{1R} + \bar{N}_{2L} i\gamma_\mu D^\mu_{1L} N_{2L} + \bar{N}_{2R} i\gamma_\mu D^\mu_{2R} N_{2R}\right\} \\
& + \mathrm{Tr}\left\{\bar{M}_{1L} i\gamma_\mu D^\mu_{4R} M_{1L} + \bar{M}_{1R} i\gamma_\mu D^\mu_{3L} M_{1R} + \bar{M}_{2L} i\gamma_\mu D^\mu_{3R} M_{2L} + \bar{M}_{2R} i\gamma_\mu D^\mu_{4L} M_{2R}\right\} \\
& - g_N \, \mathrm{Tr}\left\{\bar{N}_{1L} \Phi N_{1R} + \bar{N}_{1R} \Phi^\dagger N_{1L} + \bar{N}_{2L} \Phi N_{2R} + \bar{N}_{2R} \Phi^\dagger N_{2L}\right\} \\
& - g_M \, \mathrm{Tr}\left\{\bar{M}_{1L} \Phi^\dagger M_{1R} + \bar{M}_{1R} \Phi M_{1L} + \bar{M}_{2L} \Phi^\dagger M_{2R} + \bar{M}_{2R} \Phi M_{2L}\right\} \\
& - m_{0,1} \, \mathrm{Tr}\left\{\bar{N}_{1L} M_{1R} + \bar{M}_{1R} N_{1L} + \bar{N}_{2R} M_{2L} + \bar{M}_{2L} N_{2R}\right\} \\
& - m_{0,2} \, \mathrm{Tr}\left\{\bar{N}_{1R} M_{1L} + \bar{M}_{1L} N_{1R} + \bar{N}_{2L} M_{2R} + \bar{M}_{2R} N_{2L}\right\} \\
& - \kappa_1 \, \mathrm{Tr}\left\{\bar{N}_{1R} \Phi^\dagger N_{2L} \Phi + \bar{N}_{2L} \Phi N_{1R} \Phi^\dagger\right\} - \kappa_1' \, \mathrm{Tr}\left\{\bar{N}_{1L} \Phi N_{2R} \Phi + \bar{N}_{2R} \Phi^\dagger N_{1L} \Phi^\dagger\right\} \\
& - \kappa_2 \, \mathrm{Tr}\left\{\bar{M}_{1R} \Phi M_{2L} \Phi + \bar{M}_{2L} \Phi^\dagger M_{1R} \Phi^\dagger\right\} - \kappa_2' \, \mathrm{Tr}\left\{\bar{M}_{1L} \Phi^\dagger M_{2R} \Phi + \bar{M}_{2R} \Phi M_{1L} \Phi^\dagger\right\} \\
& - \epsilon_1 \left(\mathrm{Tr}\left\{\bar{N}_{1L} \Phi\right\} \mathrm{Tr}\left\{N_{2R} \Phi\right\} + \mathrm{Tr}\left\{\bar{N}_{2R} \Phi^\dagger\right\} \mathrm{Tr}\left\{N_{1L} \Phi^\dagger\right\}\right) \\
& - \epsilon_2 \left(\mathrm{Tr}\left\{\bar{M}_{1R} \Phi\right\} \mathrm{Tr}\left\{M_{2L} \Phi\right\} + \mathrm{Tr}\left\{\bar{M}_{2L} \Phi^\dagger\right\} \mathrm{Tr}\left\{M_{1R} \Phi^\dagger\right\}\right) \\
& - \epsilon_3 \, \mathrm{Tr}\left\{\Phi^\dagger \Phi\right\} \mathrm{Tr}\left\{\bar{N}_{1L} M_{1R} + \bar{M}_{1R} N_{1L} + \bar{N}_{2R} M_{2L} + \bar{M}_{2L} N_{2R}\right\} \\
& - \epsilon_4 \, \mathrm{Tr}\left\{\Phi^\dagger \Phi\right\} \mathrm{Tr}\left\{\bar{N}_{1R} M_{1L} + \bar{M}_{1L} N_{1R} + \bar{N}_{2L} M_{2R} + \bar{M}_{2R} N_{2L}\right\}
\end{aligned}
$$

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The Lagrangian ($N_f = 3$)

$$
\begin{aligned}
\mathcal{L}_{N_f=3} = {} & \mathrm{Tr}\left\{\bar{N}_{1L} i\gamma_\mu D^\mu_{2L} N_{1L} + \bar{N}_{1R} i\gamma_\mu D^\mu_{1R} N_{1R} + \bar{N}_{2L} i\gamma_\mu D^\mu_{1L} N_{2L} + \bar{N}_{2R} i\gamma_\mu D^\mu_{2R} N_{2R}\right\} \\
& + \mathrm{Tr}\left\{\bar{M}_{1L} i\gamma_\mu D^\mu_{4R} M_{1L} + \bar{M}_{1R} i\gamma_\mu D^\mu_{3L} M_{1R} + \bar{M}_{2L} i\gamma_\mu D^\mu_{3R} M_{2L} + \bar{M}_{2R} i\gamma_\mu D^\mu_{4L} M_{2R}\right\} \\
& - g_N \,\mathrm{Tr}\left\{\bar{N}_{1L}\Phi N_{1R} + \bar{N}_{1R}\Phi^\dagger N_{1L} + \bar{N}_{2L}\Phi N_{2R} + \bar{N}_{2R}\Phi^\dagger N_{2L}\right\} \\
& - g_M \,\mathrm{Tr}\left\{\bar{M}_{1L}\Phi^\dagger M_{1R} + \bar{M}_{1R}\Phi M_{1L} + \bar{M}_{2L}\Phi^\dagger M_{2R} + \bar{M}_{2R}\Phi M_{2L}\right\} \\
& - m_{0,1}\,\mathrm{Tr}\left\{\bar{N}_{1L} M_{1R} + \bar{M}_{1R} N_{1L} + \bar{N}_{2R} M_{2L} + \bar{M}_{2L} N_{2R}\right\} \\
& - m_{0,2}\,\mathrm{Tr}\left\{\bar{N}_{1R} M_{1L} + \bar{M}_{1L} N_{1R} + \bar{N}_{2L} M_{2R} + \bar{M}_{2R} N_{2L}\right\} \\
& - \kappa_1 \,\mathrm{Tr}\left\{\bar{N}_{1R}\Phi^\dagger N_{2L}\Phi + \bar{N}_{2L}\Phi N_{1R}\Phi^\dagger\right\} - \kappa_1' \,\mathrm{Tr}\left\{\bar{N}_{1L}\Phi N_{2R}\Phi + \bar{N}_{2R}\Phi^\dagger N_{1L}\Phi^\dagger\right\} \\
& - \kappa_2 \,\mathrm{Tr}\left\{\bar{M}_{1R}\Phi M_{2L}\Phi + \bar{M}_{2L}\Phi^\dagger M_{1R}\Phi^\dagger\right\} - \kappa_2' \,\mathrm{Tr}\left\{\bar{M}_{1L}\Phi^\dagger M_{2R}\Phi + \bar{M}_{2R}\Phi M_{1L}\Phi^\dagger\right\} \\
& - \epsilon_1 \left(\mathrm{Tr}\left\{\bar{N}_{1L}\Phi\right\}\mathrm{Tr}\left\{N_{2R}\Phi\right\} + \mathrm{Tr}\left\{\bar{N}_{2R}\Phi^\dagger\right\}\mathrm{Tr}\left\{N_{1L}\Phi^\dagger\right\}\right) \\
& - \epsilon_2 \left(\mathrm{Tr}\left\{\bar{M}_{1R}\Phi\right\}\mathrm{Tr}\left\{M_{2L}\Phi\right\} + \mathrm{Tr}\left\{\bar{M}_{2L}\Phi^\dagger\right\}\mathrm{Tr}\left\{M_{1R}\Phi^\dagger\right\}\right) \\
& - \epsilon_3 \,\mathrm{Tr}\left\{\Phi^\dagger\Phi\right\}\mathrm{Tr}\left\{\bar{N}_{1L} M_{1R} + \bar{M}_{1R} N_{1L} + \bar{N}_{2R} M_{2L} + \bar{M}_{2L} N_{2R}\right\} \\
& - \epsilon_4 \,\mathrm{Tr}\left\{\Phi^\dagger\Phi\right\}\mathrm{Tr}\left\{\bar{N}_{1R} M_{1L} + \bar{M}_{1L} N_{1R} + \bar{N}_{2L} M_{2R} + \bar{M}_{2R} N_{2L}\right\}
\end{aligned}
$$

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The Lagrangian $(N_f = 3)$

$$\begin{aligned}
\mathcal{L}_{N_f=3} = &\operatorname{Tr}\left\{\bar{N}_{1L} i\gamma_\mu D^\mu_{2L} N_{1L} + \bar{N}_{1R} i\gamma_\mu D^\mu_{1R} N_{1R} + \bar{N}_{2L} i\gamma_\mu D^\mu_{1L} N_{2L} + \bar{N}_{2R} i\gamma_\mu D^\mu_{2R} N_{2R}\right\} \\
&+ \operatorname{Tr}\left\{\bar{M}_{1L} i\gamma_\mu D^\mu_{4R} M_{1L} + \bar{M}_{1R} i\gamma_\mu D^\mu_{3L} M_{1R} + \bar{M}_{2L} i\gamma_\mu D^\mu_{3R} M_{2L} + \bar{M}_{2R} i\gamma_\mu D^\mu_{4L} M_{2R}\right\} \\
&- g_N \operatorname{Tr}\left\{\bar{N}_{1L}\Phi N_{1R} + \bar{N}_{1R}\Phi^\dagger N_{1L} + \bar{N}_{2L}\Phi N_{2R} + \bar{N}_{2R}\Phi^\dagger N_{2L}\right\} \\
&- g_M \operatorname{Tr}\left\{\bar{M}_{1L}\Phi^\dagger M_{1R} + \bar{M}_{1R}\Phi M_{1L} + \bar{M}_{2L}\Phi^\dagger M_{2R} + \bar{M}_{2R}\Phi M_{2L}\right\} \\
&- m_{0,1} \operatorname{Tr}\left\{\bar{N}_{1L} M_{1R} + \bar{M}_{1R} N_{1L} + \bar{N}_{2R} M_{2L} + \bar{M}_{2L} N_{2R}\right\} \\
&- m_{0,2} \operatorname{Tr}\left\{\bar{N}_{1R} M_{1L} + \bar{M}_{1L} N_{1R} + \bar{N}_{2L} M_{2R} + \bar{M}_{2R} N_{2L}\right\} \\
&- \kappa_1 \operatorname{Tr}\left\{\bar{N}_{1R}\Phi^\dagger N_{2L}\Phi + \bar{N}_{2L}\Phi N_{1R}\Phi^\dagger\right\} - \kappa_1' \operatorname{Tr}\left\{\bar{N}_{1L}\Phi N_{2R}\Phi + \bar{N}_{2R}\Phi^\dagger N_{1L}\Phi^\dagger\right\} \\
&- \kappa_2 \operatorname{Tr}\left\{\bar{M}_{1R}\Phi M_{2L}\Phi + \bar{M}_{2L}\Phi^\dagger M_{1R}\Phi^\dagger\right\} - \kappa_2' \operatorname{Tr}\left\{\bar{M}_{1L}\Phi^\dagger M_{2R}\Phi + \bar{M}_{2R}\Phi M_{1L}\Phi^\dagger\right\} \\
&- \epsilon_1 \left(\operatorname{Tr}\left\{\bar{N}_{1L}\Phi\right\}\operatorname{Tr}\left\{N_{2R}\Phi\right\} + \operatorname{Tr}\left\{\bar{N}_{2R}\Phi^\dagger\right\}\operatorname{Tr}\left\{N_{1L}\Phi^\dagger\right\}\right) \\
&- \epsilon_2 \left(\operatorname{Tr}\left\{\bar{M}_{1R}\Phi\right\}\operatorname{Tr}\left\{M_{2L}\Phi\right\} + \operatorname{Tr}\left\{\bar{M}_{2L}\Phi^\dagger\right\}\operatorname{Tr}\left\{M_{1R}\Phi^\dagger\right\}\right) \\
&- \epsilon_3 \operatorname{Tr}\left\{\Phi^\dagger\Phi\right\}\operatorname{Tr}\left\{\bar{N}_{1L} M_{1R} + \bar{M}_{1R} N_{1L} + \bar{N}_{2R} M_{2L} + \bar{M}_{2L} N_{2R}\right\} \\
&- \epsilon_4 \operatorname{Tr}\left\{\Phi^\dagger\Phi\right\}\operatorname{Tr}\left\{\bar{N}_{1R} M_{1L} + \bar{M}_{1L} N_{1R} + \bar{N}_{2L} M_{2R} + \bar{M}_{2R} N_{2L}\right\}
\end{aligned}$$

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The Lagrangian ($N_f = 3$)

$$
\begin{aligned}
\mathcal{L}_{N_f=3} = & \operatorname{Tr}\left\{\bar{N}_{1L} i\gamma_\mu D^\mu_{2L} N_{1L} + \bar{N}_{1R} i\gamma_\mu D^\mu_{1R} N_{1R} + \bar{N}_{2L} i\gamma_\mu D^\mu_{1L} N_{2L} + \bar{N}_{2R} i\gamma_\mu D^\mu_{2R} N_{2R}\right\} \\
& + \operatorname{Tr}\left\{\bar{M}_{1L} i\gamma_\mu D^\mu_{4R} M_{1L} + \bar{M}_{1R} i\gamma_\mu D^\mu_{3L} M_{1R} + \bar{M}_{2L} i\gamma_\mu D^\mu_{3R} M_{2L} + \bar{M}_{2R} i\gamma_\mu D^\mu_{4L} M_{2R}\right\} \\
& - g_N \operatorname{Tr}\left\{\bar{N}_{1L} \Phi N_{1R} + \bar{N}_{1R} \Phi^\dagger N_{1L} + \bar{N}_{2L} \Phi N_{2R} + \bar{N}_{2R} \Phi^\dagger N_{2L}\right\} \\
& - g_M \operatorname{Tr}\left\{\bar{M}_{1L} \Phi^\dagger M_{1R} + \bar{M}_{1R} \Phi M_{1L} + \bar{M}_{2L} \Phi^\dagger M_{2R} + \bar{M}_{2R} \Phi M_{2L}\right\} \\
& - m_{0,1} \operatorname{Tr}\left\{\bar{N}_{1L} M_{1R} + \bar{M}_{1R} N_{1L} + \bar{N}_{2R} M_{2L} + \bar{M}_{2L} N_{2R}\right\} \\
& - m_{0,2} \operatorname{Tr}\left\{\bar{N}_{1R} M_{1L} + \bar{M}_{1L} N_{1R} + \bar{N}_{2L} M_{2R} + \bar{M}_{2R} N_{2L}\right\} \\
& - \kappa_1 \operatorname{Tr}\left\{\bar{N}_{1R} \Phi^\dagger N_{2L} \Phi + \bar{N}_{2L} \Phi N_{1R} \Phi^\dagger\right\} - \kappa_1' \operatorname{Tr}\left\{\bar{N}_{1L} \Phi N_{2R} \Phi + \bar{N}_{2R} \Phi^\dagger N_{1L} \Phi^\dagger\right\} \\
& - \kappa_2 \operatorname{Tr}\left\{\bar{M}_{1R} \Phi M_{2L} \Phi + \bar{M}_{2L} \Phi^\dagger M_{1R} \Phi^\dagger\right\} - \kappa_2' \operatorname{Tr}\left\{\bar{M}_{1L} \Phi^\dagger M_{2R} \Phi + \bar{M}_{2R} \Phi M_{1L} \Phi^\dagger\right\} \\
& - \epsilon_1 \left(\operatorname{Tr}\left\{\bar{N}_{1L} \Phi\right\} \operatorname{Tr}\left\{N_{2R} \Phi\right\} + \operatorname{Tr}\left\{\bar{N}_{2R} \Phi^\dagger\right\} \operatorname{Tr}\left\{N_{1L} \Phi^\dagger\right\}\right) \\
& - \epsilon_2 \left(\operatorname{Tr}\left\{\bar{M}_{1R} \Phi\right\} \operatorname{Tr}\left\{M_{2L} \Phi\right\} + \operatorname{Tr}\left\{\bar{M}_{2L} \Phi^\dagger\right\} \operatorname{Tr}\left\{M_{1R} \Phi^\dagger\right\}\right) \\
& - \epsilon_3 \operatorname{Tr}\left\{\Phi^\dagger \Phi\right\} \operatorname{Tr}\left\{\bar{N}_{1L} M_{1R} + \bar{M}_{1R} N_{1L} + \bar{N}_{2R} M_{2L} + \bar{M}_{2L} N_{2R}\right\} \\
& - \epsilon_4 \operatorname{Tr}\left\{\Phi^\dagger \Phi\right\} \operatorname{Tr}\left\{\bar{N}_{1R} M_{1L} + \bar{M}_{1L} N_{1R} + \bar{N}_{2L} M_{2R} + \bar{M}_{2R} N_{2L}\right\}
\end{aligned}
$$

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The Lagrangian ($N_f = 3$)

$$
\begin{aligned}
\mathcal{L}_{N_f=3} = & \,\mathrm{Tr}\left\{\bar{N}_{1L} i\gamma_\mu D^\mu_{2L} N_{1L} + \bar{N}_{1R} i\gamma_\mu D^\mu_{1R} N_{1R} + \bar{N}_{2L} i\gamma_\mu D^\mu_{1L} N_{2L} + \bar{N}_{2R} i\gamma_\mu D^\mu_{2R} N_{2R}\right\} \\
& + \mathrm{Tr}\left\{\bar{M}_{1L} i\gamma_\mu D^\mu_{4R} M_{1L} + \bar{M}_{1R} i\gamma_\mu D^\mu_{3L} M_{1R} + \bar{M}_{2L} i\gamma_\mu D^\mu_{3R} M_{2L} + \bar{M}_{2R} i\gamma_\mu D^\mu_{4L} M_{2R}\right\} \\
& - g_N \,\mathrm{Tr}\left\{\bar{N}_{1L} \Phi N_{1R} + \bar{N}_{1R} \Phi^\dagger N_{1L} + \bar{N}_{2L} \Phi N_{2R} + \bar{N}_{2R} \Phi^\dagger N_{2L}\right\} \\
& - g_M \,\mathrm{Tr}\left\{\bar{M}_{1L} \Phi^\dagger M_{1R} + \bar{M}_{1R} \Phi M_{1L} + \bar{M}_{2L} \Phi^\dagger M_{2R} + \bar{M}_{2R} \Phi M_{2L}\right\} \\
& - m_{0,1} \,\mathrm{Tr}\left\{\bar{N}_{1L} M_{1R} + \bar{M}_{1R} N_{1L} + \bar{N}_{2R} M_{2L} + \bar{M}_{2L} N_{2R}\right\} \\
& - m_{0,2} \,\mathrm{Tr}\left\{\bar{N}_{1R} M_{1L} + \bar{M}_{1L} N_{1R} + \bar{N}_{2L} M_{2R} + \bar{M}_{2R} N_{2L}\right\} \\
& - \kappa_1 \,\mathrm{Tr}\left\{\bar{N}_{1R} \Phi^\dagger N_{2L} \Phi + \bar{N}_{2L} \Phi N_{1R} \Phi^\dagger\right\} - \kappa_1' \,\mathrm{Tr}\left\{\bar{N}_{1L} \Phi N_{2R} \Phi + \bar{N}_{2R} \Phi^\dagger N_{1L} \Phi^\dagger\right\} \\
& - \kappa_2 \,\mathrm{Tr}\left\{\bar{M}_{1R} \Phi M_{2L} \Phi + \bar{M}_{2L} \Phi^\dagger M_{1R} \Phi^\dagger\right\} - \kappa_2' \,\mathrm{Tr}\left\{\bar{M}_{1L} \Phi^\dagger M_{2R} \Phi + \bar{M}_{2R} \Phi M_{1L} \Phi^\dagger\right\} \\
& - \epsilon_1 \left(\mathrm{Tr}\left\{\bar{N}_{1L} \Phi\right\} \mathrm{Tr}\left\{N_{2R} \Phi\right\} + \mathrm{Tr}\left\{\bar{N}_{2R} \Phi^\dagger\right\} \mathrm{Tr}\left\{N_{1L} \Phi^\dagger\right\}\right) \\
& - \epsilon_2 \left(\mathrm{Tr}\left\{\bar{M}_{1R} \Phi\right\} \mathrm{Tr}\left\{M_{2L} \Phi\right\} + \mathrm{Tr}\left\{\bar{M}_{2L} \Phi^\dagger\right\} \mathrm{Tr}\left\{M_{1R} \Phi^\dagger\right\}\right) \\
& - \epsilon_3 \,\mathrm{Tr}\left\{\Phi^\dagger \Phi\right\} \mathrm{Tr}\left\{\bar{N}_{1L} M_{1R} + \bar{M}_{1R} N_{1L} + \bar{N}_{2R} M_{2L} + \bar{M}_{2L} N_{2R}\right\} \\
& - \epsilon_4 \,\mathrm{Tr}\left\{\Phi^\dagger \Phi\right\} \mathrm{Tr}\left\{\bar{N}_{1R} M_{1L} + \bar{M}_{1L} N_{1R} + \bar{N}_{2L} M_{2R} + \bar{M}_{2R} N_{2L}\right\}
\end{aligned}
$$

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The Lagrangian ($N_f = 2$)

$$
\begin{aligned}
\mathcal{L} = \quad & \bar{\Psi}_{NR} i\gamma_\mu D^\mu_{NR} \Psi_{NR} + \bar{\Psi}_{NL} i\gamma_\mu D^\mu_{NL} \Psi_{NL} + \bar{\Psi}_{N*R} i\gamma_\mu D^\mu_{NR} \Psi_{N*R} + \bar{\Psi}_{N*L} i\gamma_\mu D^\mu_{NL} \Psi_{N*L} \\
& + \bar{\Psi}_{MR} i\gamma_\mu D^\mu_{ML} \Psi_{MR} + \bar{\Psi}_{ML} i\gamma_\mu D^\mu_{MR} \Psi_{ML} + \bar{\Psi}_{M*R} i\gamma_\mu D^\mu_{ML} \Psi_{M*R} + \bar{\Psi}_{M*L} i\gamma_\mu D^\mu_{MR} \Psi_{M*L} \\
& + c_{A_N} \left(\bar{\Psi}_{NR} i\gamma_\mu R^\mu \Psi_{N*R} + \bar{\Psi}_{N*R} i\gamma_\mu R^\mu \Psi_{NR} - \bar{\Psi}_{NL} i\gamma_\mu L^\mu \Psi_{N*L} - \bar{\Psi}_{N*L} i\gamma_\mu L^\mu \Psi_{NL} \right) \\
& + c_{A_M} \left(\bar{\Psi}_{MR} i\gamma_\mu L^\mu \Psi_{M*R} + \bar{\Psi}_{M*R} i\gamma_\mu L^\mu \Psi_{MR} - \bar{\Psi}_{ML} i\gamma_\mu R^\mu \Psi_{M*L} - \bar{\Psi}_{M*L} i\gamma_\mu R^\mu \Psi_{ML} \right) \\
& - g_N \left(\bar{\Psi}_{NL} \Phi \Psi_{NR} + \bar{\Psi}_{NR} \Phi^\dagger \Psi_{NL} + \bar{\Psi}_{N*L} \Phi \Psi_{N*R} + \bar{\Psi}_{N*L} \Phi^\dagger \Psi_{N*R} \right) \\
& - g_M \left(\bar{\Psi}_{ML} \Phi^\dagger \Psi_{MR} + \bar{\Psi}_{MR} \Phi \Psi_{ML} + \bar{\Psi}_{M*L} \Phi^\dagger \Psi_{M*R} + \bar{\Psi}_{M*L} \Phi \Psi_{M*R} \right) \\
& - \frac{m_{0,1} + m_{0,2}}{2} \left(\bar{\Psi}_{NL} \Psi_{MR} + \bar{\Psi}_{NR} \Psi_{ML} + \bar{\Psi}_{N*L} \Psi_{M*R} + \bar{\Psi}_{N*R} \Psi_{M*L} \right. \\
& \qquad \left. + \bar{\Psi}_{ML} \Psi_{NR} + \bar{\Psi}_{MR} \Psi_{NL} + \bar{\Psi}_{M*L} \Psi_{N*R} + \bar{\Psi}_{M*R} \Psi_{N*L} \right) \\
& - \frac{m_{0,1} - m_{0,2}}{2} \left(\bar{\Psi}_{NL} \Psi_{M*R} - \bar{\Psi}_{NR} \Psi_{M*L} - \bar{\Psi}_{ML} \Psi_{N*R} + \bar{\Psi}_{MR} \Psi_{N*L} \right. \\
& \qquad \left. - \bar{\Psi}_{N*L} \Psi_{MR} + \bar{\Psi}_{N*R} \Psi_{ML} + \bar{\Psi}_{M*L} \Psi_{NR} - \bar{\Psi}_{M*R} \Psi_{NL} \right) \\
& - \frac{\kappa_1' + \kappa_1}{2} \frac{\varphi_S}{\sqrt{2}} \left(-\bar{\Psi}_{NL} \Phi \Psi_{NR} - \bar{\Psi}_{NR} \Phi^\dagger \Psi_{NL} + \bar{\Psi}_{N*L} \Phi \Psi_{N*R} + \bar{\Psi}_{N*R} \Phi^\dagger \Psi_{N*L} \right) \\
& - \frac{\kappa_1' - \kappa_1}{2} \frac{\varphi_S}{\sqrt{2}} \left(\bar{\Psi}_{NL} \Phi \Psi_{N*R} - \bar{\Psi}_{NR} \Phi^\dagger \Psi_{N*L} - \bar{\Psi}_{N*L} \Phi \Psi_{NR} + \bar{\Psi}_{N*R} \Phi^\dagger \Psi_{NL} \right) \\
& - \frac{\kappa_2' + \kappa_2}{2} \frac{\varphi_S}{\sqrt{2}} \left(-\bar{\Psi}_{ML} \Phi^\dagger \Psi_{MR} - \bar{\Psi}_{MR} \Phi \Psi_{ML} + \bar{\Psi}_{M*L} \Phi^\dagger \Psi_{M*R} + \bar{\Psi}_{M*R} \Phi \Psi_{M*L} \right) \\
& - \frac{\kappa_2' - \kappa_2}{2} \frac{\varphi_S}{\sqrt{2}} \left(\bar{\Psi}_{ML} \Phi^\dagger \Psi_{M*R} - \bar{\Psi}_{MR} \Phi \Psi_{M*L} - \bar{\Psi}_{M*L} \Phi^\dagger \Psi_{MR} + \bar{\Psi}_{M*R} \Phi \Psi_{ML} \right)
\end{aligned}
$$

The Lagrangian ($N_f = 2$)

$$\begin{aligned}
\mathcal{L} = \quad & \bar{\Psi}_{NR} i\gamma_\mu D^\mu_{NR} \Psi_{NR} + \bar{\Psi}_{NL} i\gamma_\mu D^\mu_{NL} \Psi_{NL} + \bar{\Psi}_{N*R} i\gamma_\mu D^\mu_{NR} \Psi_{N*R} + \bar{\Psi}_{N*L} i\gamma_\mu D^\mu_{NL} \Psi_{N*L} \\
& + \bar{\Psi}_{MR} i\gamma_\mu D^\mu_{ML} \Psi_{MR} + \bar{\Psi}_{ML} i\gamma_\mu D^\mu_{MR} \Psi_{ML} + \bar{\Psi}_{M*R} i\gamma_\mu D^\mu_{ML} \Psi_{M*R} + \bar{\Psi}_{M*L} i\gamma_\mu D^\mu_{MR} \Psi_{M*L} \\
& + c_{A_N} \left(\bar{\Psi}_{NR} i\gamma_\mu R^\mu \Psi_{N*R} + \bar{\Psi}_{N*R} i\gamma_\mu R^\mu \Psi_{NR} - \bar{\Psi}_{NL} i\gamma_\mu L^\mu \Psi_{N*L} - \bar{\Psi}_{N*L} i\gamma_\mu L^\mu \Psi_{NL} \right) \\
& + c_{A_M} \left(\bar{\Psi}_{MR} i\gamma_\mu L^\mu \Psi_{M*R} + \bar{\Psi}_{M*R} i\gamma_\mu L^\mu \Psi_{MR} - \bar{\Psi}_{ML} i\gamma_\mu R^\mu \Psi_{M*L} - \bar{\Psi}_{M*L} i\gamma_\mu R^\mu \Psi_{ML} \right) \\
& - g_N \left(\bar{\Psi}_{NL} \Phi \Psi_{NR} + \bar{\Psi}_{NR} \Phi^\dagger \Psi_{NL} + \bar{\Psi}_{N*L} \Phi \Psi_{N*R} + \bar{\Psi}_{N*L} \Phi^\dagger \Psi_{N*R} \right) \\
& - g_M \left(\bar{\Psi}_{ML} \Phi^\dagger \Psi_{MR} + \bar{\Psi}_{MR} \Phi \Psi_{ML} + \bar{\Psi}_{M*L} \Phi^\dagger \Psi_{M*R} + \bar{\Psi}_{M*L} \Phi \Psi_{M*R} \right) \\
& - \frac{m_{0,1} + m_{0,2}}{2} \left(\bar{\Psi}_{NL} \Psi_{MR} + \bar{\Psi}_{NR} \Psi_{ML} + \bar{\Psi}_{N*L} \Psi_{M*R} + \bar{\Psi}_{N*R} \Psi_{M*L} \right. \\
& \qquad \left. + \bar{\Psi}_{ML} \Psi_{NR} + \bar{\Psi}_{MR} \Psi_{NL} + \bar{\Psi}_{M*L} \Psi_{N*R} + \bar{\Psi}_{M*R} \Psi_{N*L} \right) \\
& - \frac{m_{0,1} - m_{0,2}}{2} \left(\bar{\Psi}_{NL} \Psi_{M*R} - \bar{\Psi}_{NR} \Psi_{M*L} - \bar{\Psi}_{ML} \Psi_{N*R} + \bar{\Psi}_{MR} \Psi_{N*L} \right. \\
& \qquad \left. - \bar{\Psi}_{N*L} \Psi_{MR} + \bar{\Psi}_{N*R} \Psi_{ML} + \bar{\Psi}_{M*L} \Psi_{NR} - \bar{\Psi}_{M*R} \Psi_{NL} \right) \\
& - \frac{\kappa_1' + \kappa_1}{2} \frac{\varphi_S}{\sqrt{2}} \left(-\bar{\Psi}_{NL} \Phi \Psi_{NR} - \bar{\Psi}_{NR} \Phi^\dagger \Psi_{NL} + \bar{\Psi}_{N*L} \Phi \Psi_{N*R} + \bar{\Psi}_{N*R} \Phi^\dagger \Psi_{N*L} \right) \\
& - \frac{\kappa_1' - \kappa_1}{2} \frac{\varphi_S}{\sqrt{2}} \left(\bar{\Psi}_{NL} \Phi \Psi_{N*R} - \bar{\Psi}_{NR} \Phi^\dagger \Psi_{N*L} - \bar{\Psi}_{N*L} \Phi \Psi_{NR} + \bar{\Psi}_{N*R} \Phi^\dagger \Psi_{NL} \right) \\
& - \frac{\kappa_2' + \kappa_2}{2} \frac{\varphi_S}{\sqrt{2}} \left(-\bar{\Psi}_{ML} \Phi^\dagger \Psi_{MR} - \bar{\Psi}_{MR} \Phi \Psi_{ML} + \bar{\Psi}_{M*L} \Phi^\dagger \Psi_{M*R} + \bar{\Psi}_{M*R} \Phi \Psi_{M*L} \right) \\
& - \frac{\kappa_2' - \kappa_2}{2} \frac{\varphi_S}{\sqrt{2}} \left(\bar{\Psi}_{ML} \Phi^\dagger \Psi_{M*R} - \bar{\Psi}_{MR} \Phi \Psi_{M*L} - \bar{\Psi}_{M*L} \Phi^\dagger \Psi_{MR} + \bar{\Psi}_{M*R} \Phi \Psi_{ML} \right)
\end{aligned}$$

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Fit Result of the Free Parameters

Using a standard χ^2 procedure we find that three acceptable and almost equally deep minima exist.

	minimum 1		minimum 2		minimum 3	
$m_{0,1}$ [GeV]	0.1393	$\pm$ 0.0026	0.14	$\pm$ 0.11	-1.078	$\pm$ 0.017
$m_{0,2}$ [GeV]	-0.2069	$\pm$ 0.0027	-0.18	$\pm$ 0.12	0.894	$\pm$ 0.019
c_N	-2.071	$\pm$ 0.023	-2.83	$\pm$ 0.39	-33.6	$\pm$ 2.2
c_M	12.4	$\pm$ 1.3	11.7	$\pm$ 1.8	-19.1	$\pm$ 3.1
c_{A_N}	-1.00	$\pm$ 0.23	0.03	$\pm$ 0.40	-2.68	$\pm$ 0.80
c_{A_M}	-51.0	$\pm$ 2.8	80	$\pm$ 41	-71.7	$\pm$ 6.5
g_N	15.485	$\pm$ 0.012	15.24	$\pm$ 0.36	10.58	$\pm$ 0.24
g_M	17.96	$\pm$ 0.17	18.26	$\pm$ 0.52	13.07	$\pm$ 0.33
κ_1 [GeV^{-1}]	37.80	$\pm$ 0.26	59.9	$\pm$ 8.5	32.4	$\pm$ 4.2
κ_1' [GeV^{-1}]	57.12	$\pm$ 0.29	29.8	$\pm$ 6.6	55.2	$\pm$ 4.0
κ_2 [GeV^{-1}]	-20.7	$\pm$ 2.5	32	$\pm$ 13	-20	$\pm$ 13
κ_2' [GeV^{-1}]	41.5	$\pm$ 3.2	-8	$\pm$ 13	48.9	$\pm$ 4.5
χ^2	10.3		10.7		10.3	

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Comparison of predictions of the model to experimental and lattice results – masses

	minimum 1		minimum 2		minimum 3	
m_N [GeV]	0.9389	$\pm$ 0.0010	0.9389	$\pm$ 0.0010	0.9389	$\pm$ 0.0010
$m_{N(1440)}$ [GeV]	1.430	$\pm$ 0.071	1.432	$\pm$ 0.073	1.429	$\pm$ 0.074
$m_{N(1535)}$ [GeV]	1.561	$\pm$ 0.065	1.585	$\pm$ 0.069	1.559	$\pm$ 0.069
$m_{N(1650)}$ [GeV]	1.658	$\pm$ 0.076	1.619	$\pm$ 0.071	1.663	$\pm$ 0.081

	experiment [PDG]	
m_N [GeV]	0.9389	$\pm$ 0.001
$m_{N(1440)}$ [GeV]	1.43	$\pm$ 0.07
$m_{N(1535)}$ [GeV]	1.53	$\pm$ 0.08
$m_{N(1650)}$ [GeV]	1.65	$\pm$ 0.08

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Comparison of predictions of the model to experimental and lattice results – decay widths

	minimum 1		minimum 2		minimum 3	
$\Gamma_{N(1440)\to N\pi}$ [GeV]	0.195	$\pm$ 0.087	0.195	$\pm$ 0.088	0.196	$\pm$ 0.087
$\Gamma_{N(1535)\to N\pi}$ [GeV]	0.072	$\pm$ 0.019	0.073	$\pm$ 0.019	0.072	$\pm$ 0.019
$\Gamma_{N(1535)\to N\eta}$ [GeV]	0.0055	$\pm$ 0.0025	0.0062	$\pm$ 0.0024	0.0055	$\pm$ 0.0027
$\Gamma_{N(1650)\to N\pi}$ [GeV]	0.112	$\pm$ 0.033	0.114	$\pm$ 0.033	0.112	$\pm$ 0.033
$\Gamma_{N(1650)\to N\eta}$ [GeV]	0.0117	$\pm$ 0.0038	0.0109	$\pm$ 0.0038	0.0119	$\pm$ 0.0038

	experiment [PDG]	
$\Gamma_{N(1440)\to N\pi}$ [GeV]	0.195	$\pm$ 0.087
$\Gamma_{N(1535)\to N\pi}$ [GeV]	0.068	$\pm$ 0.019
$\Gamma_{N(1535)\to N\eta}$ [GeV]	0.063	$\pm$ 0.018
$\Gamma_{N(1650)\to N\pi}$ [GeV]	0.105	$\pm$ 0.037
$\Gamma_{N(1650)\to N\eta}$ [GeV]	0.015	$\pm$ 0.008

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compare S. Gallas, F. Giacosa and D. H. Rischke, Phys. Rev. D **82** (2010) 014004 [arXiv:0907.5084 [hep-ph]].

Comparison of predictions of the model to experimental and lattice results – axial coupling constants

	minimum 1		minimum 2		minimum 3	
g_A^N	1.2670	$\pm$ 0.0025	1.2670	$\pm$ 0.0025	1.2670	$\pm$ 0.0025
$g_A^{N(1440)}$	1.20	$\pm$ 0.20	1.19	$\pm$ 0.20	1.21	$\pm$ 0.21
$g_A^{N(1535)}$	0.20	$\pm$ 0.30	0.21	$\pm$ 0.30	0.20	$\pm$ 0.31
$g_A^{N(1650)}$	0.55	$\pm$ 0.20	0.55	$\pm$ 0.20	0.55	$\pm$ 0.20

	experiment/lattice	
g_A^N	1.267	$\pm$ 0.003
$g_A^{N(1440)}$	1.2	$\pm$ 0.2
$g_A^{N(1535)}$	0.2	$\pm$ 0.3
$g_A^{N(1650)}$	0.55	$\pm$ 0.2

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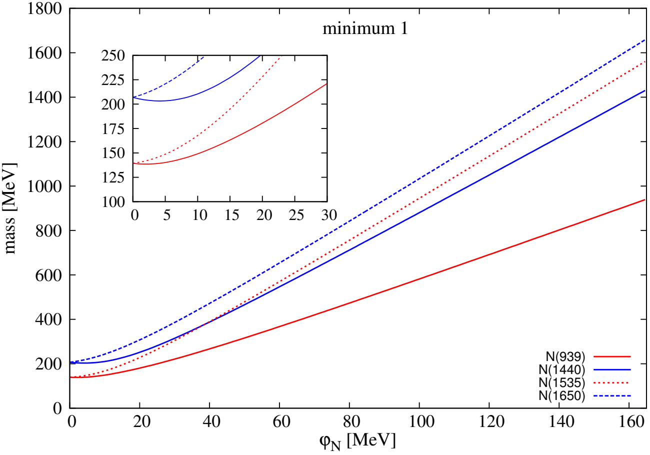
Chiral Partner of the Nucleon



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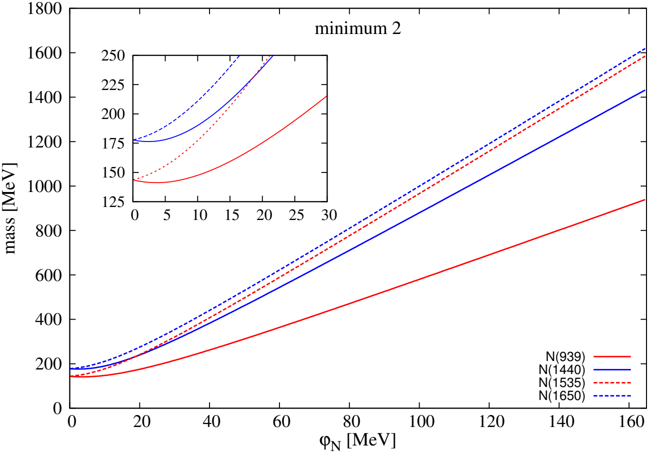

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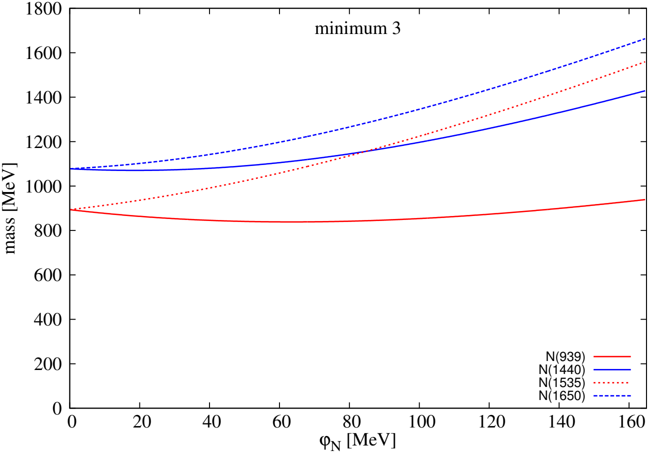

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Chiral Partner of the Nucleon

Chiral partners are (for all three minima)

$N(939)$ and $N(1535)$,

and

$N(1440)$ and $N(1650)$.

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Outline

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Conclusions

- Generalization of eLSM to the three-flavor case, thus including baryons with strangeness.
- Using a quark-diquark model and requiring chirally invariant mass terms naturally leads to the consideration of four baryonic multiplets.
- Reduction to $N_f = 2$ and fit.
- Three existing minima yield good results except for the $N(1535) \to N\eta$ decay width.
- The pairs $N(939)$, $N(1535)$ and $N(1440)$, $N(1650)$ form chiral partners.

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The decay width of $N(1535) \to N\eta$

- Our theoretical values are too small compared to the experimental value.
- This result is stable under parameter variations.
- Further studies are needed to understand the resonance $N(1535)$.
- Some authors say that $N(1535)$ may contain a sizable amount of $s\bar{s}$.
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