

Hybrid Quarkonium with Non-Relativistic Effective Field Theories

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based on: Phys. Rev. **D92** (2015) 11, 114019 [arXiv:1510.04299]

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17.02.2016

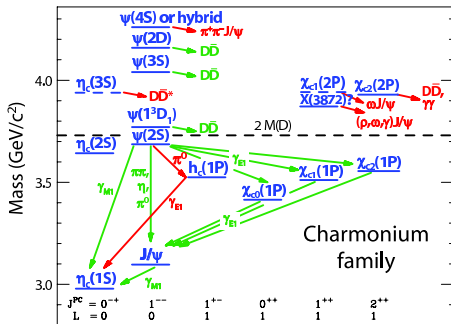


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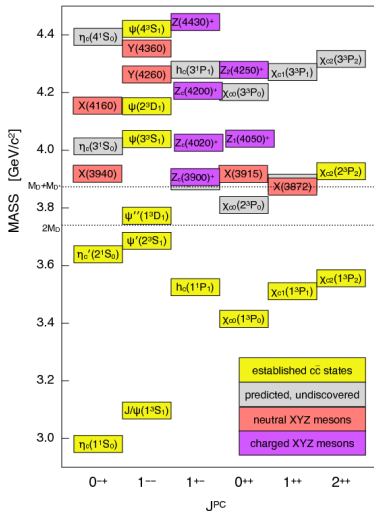
Introduction: Quarkonium and XYZ exotics

Eichten et al 2008



- quarkonium: $Q\bar{Q}$ bound state
- hybrids: gluonic excitation
- additional light quarks: tetraquark, meson molecule, diquarkonium, etc

Olsen 2015



Hybrids: theoretical approaches

Constituent gluon picture

Horn and Mandula 1978

- treat hybrids as a three-body system $Q\bar{Q}g$
- add J^{PC} quantum numbers of gluon and quarkonium

Fluxtube model

Isgur and Paton 1983

- gluons assumed to form string between heavy quarks
- hybrids correspond to vibrational excitations of string

Born-Oppenheimer (BO) approximation

Griffiths, Michael and Rakow 1983

- determine energy spectrum of static quarks
- solve Schrödinger equation with static energy as potential

Effective Field Theory (EFT) treatment

- exploit scale hierarchy $M \gg p_{\text{rel}} \gg E_{\text{kin}}$ in systematic expansion
- integrate out M (NRQCD) and p_{rel} (pNRCD)

Revision: QM perturbation theory

Split H into free and perturbation part, determine free spectrum:

$$H = H_0 + H_1, \quad H_0|n\rangle_0 = E_n^{(0)}|n\rangle_0, \quad H|n\rangle = E_n|n\rangle$$

Naive convergence criterion: ${}_0\langle n|H_1|n\rangle_0 \ll E_n^{(0)}$, but at higher orders:

$$E_n = E_n^{(0)} + {}_0\langle n|H_1|n\rangle_0 + \sum_{k \neq n} \frac{|{}_0\langle k|H_1|n\rangle_0|^2}{E_n^{(0)} - E_k^{(0)}} + \dots$$

→ NLO convergence criterion: $|{}_0\langle k|H_1|n\rangle_0| \ll E_n^{(0)} - E_k^{(0)}$.

Degenerate PT: new basis of states $|h\rangle_0 \in \text{span} \left\{ |n\rangle_0 \left| E_n^{(0)} \approx E_{n'}^{(0)} \right. \right\}$

such that $H_0 + H_1$ diagonal: ${}_0\langle h'|H_0 + H_1|h\rangle_0 = \delta_{h'h} E_h^{(1)}$

$$E_h = E_h^{(1)} + \sum_{k \notin \{h\}} \frac{|{}_0\langle k|H_1|h\rangle_0|^2}{E_h^{(1)} - E_k^{(0)}} + \dots$$

QM perturbation theory for $1/M$ expansion

$1/M$ expanded Hamiltonian through EFT (NRQCD)

Caswell and Lepage 1986

Bodwin, Braaten and Lepage 1995

$$H_0 = \int d^3x \text{Tr} [\mathbf{E}^2 + \mathbf{B}^2]$$

$$H_1 = \int d^3x \psi^\dagger \left(-\frac{D^2}{2M_Q} - c_F \frac{g\mathbf{B} \cdot \boldsymbol{\sigma}}{2M_Q} \right) \psi + \int d^3x \chi^\dagger \left(\frac{D^2}{2M_{\bar{Q}}} + c_F \frac{g\mathbf{B} \cdot \boldsymbol{\sigma}}{2M_{\bar{Q}}} \right) \chi$$

(light quarks and higher $1/M$ terms are neglected, H_0 is static case)

Outline:

- find eigenstates and eigenvalues of H_0
- check if eigenvalues are degenerate
- calculate corrections with (non-)degenerate perturbation theory

Note: Different situation depending on the relative quark-antiquark distance r

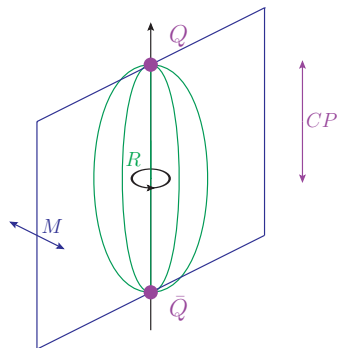
Symmetries of the static system

Static system has cylindric symmetry: $D_{\infty h}$

Elementary group transformations:

- Rotations R around $Q-\bar{Q}$ axis
- CP : Space inversion across center of $Q-\bar{Q}$ combined with charge conjugation
- Reflection M across plane with $Q-\bar{Q}$ axis

All other elements are combinations of these



Static States labeled with associated quantum numbers Λ_{η}^{σ} :

- Λ : rotational quantum number; labels Σ, Π, Δ correspond to $\Lambda = 0, 1, 2$
- η : eigenvalue of CP : $g \hat{=} +1$ (gerade), $u \hat{=} -1$ (ungerade)
- σ : sign of reflections M ; σ only relevant for Σ representations for $\Lambda \geq 1$: $\sigma = \pm$ corresponds to different projections of angular momentum

Eigenvalues of H_0

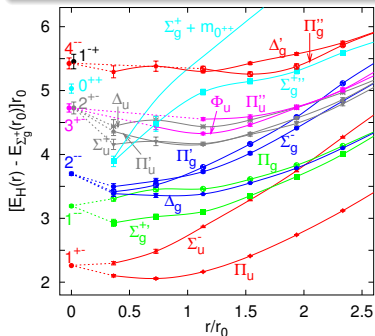
Eigenstates of H_0 are unknown, but lowest eigenvalues can be determined:

$$\langle X(T)|X(0)\rangle = \langle X|e^{-iH_0T} \sum_n |n\rangle_{00} \langle n|X\rangle = \sum_n |{}_0\langle n|X\rangle|^2 e^{-iE_n^{(0)}T}$$

If $|X_n\rangle$ has **same quantum numbers** as $|n\rangle_0$, then for those quantum numbers

$$E_n^{(0)} = \lim_{T \rightarrow \infty} \frac{i}{T} \ln \langle X_n(T)|X_n(0)\rangle$$

gives the **lowest eigenvalue!**



Bali and Pineda 2003

Lattice results:

- classification through quantum numbers (Λ_η^σ representations of $D_{\infty h}$ group)
- excited energies can be extracted with larger set of $|X_n\rangle$ states
- lowest states are Π_u and Σ_u^-
 - nearly degenerate at small r
 - well separated at large r

Study small r degeneracy with pNRQCD

Hybrids with pNRQCD

For $r \ll \Lambda_{\text{QCD}}^{-1}$ multipole expanded EFT: pNRQCD

Bali and Pineda 1997

Brambilla, Pineda, Soto, Vairo 1999

$$H^{(0,0)} = \int d^3R \text{Tr} [\mathbf{E}^2 + \mathbf{B}^2] + \int d^3r d^3R \text{Tr} [S^\dagger V_s S + O^\dagger V_o O]$$

$$H^{(0,1)} = \int d^3r d^3R \text{Tr} [V_A (S^\dagger \mathbf{r} \cdot g \mathbf{E} O + O^\dagger \mathbf{r} \cdot g \mathbf{E} S) + V_B O^\dagger \{\mathbf{r} \cdot g \mathbf{E}, O\}]$$

Convenient states: $|X_{\Pi_u}\rangle = \mathbf{r} \times \mathbf{B}^a O^{a\dagger} |\text{vac}\rangle$ $|X_{\Sigma_u^-}\rangle = \mathbf{r} \cdot \mathbf{B}^a O^{a\dagger} |\text{vac}\rangle$

Degenerate energies: $E_{\Pi_u/\Sigma_u^-}^{(0)}(r) = V_o(r) + \Lambda_B + \mathcal{O}(r^2)$

with gluelump mass $\Lambda_B = \lim_{T \rightarrow \infty} \frac{i}{T} \ln \langle \text{vac} | \mathbf{B}^a(T) \cdot \mathbf{B}^a(0) | \text{vac} \rangle$

Degeneracy is broken through $\mathcal{O}(r^2)$ terms given through 4-gluon correlators

$$\langle \text{vac} | B_i^a(T) E_j^b(t) E_k^c(t') B_l^d(0) | \text{vac} \rangle$$

Beyond the static limit

Lift short distance degeneracy with next term in expansion

$$H^{(1,-2)} = \int d^3r d^3R \text{Tr} \left[-S^\dagger \frac{\nabla_r^2}{2\mu} S - O^\dagger \frac{\nabla_r^2}{2\mu} O \right]$$

In the pNRQCD power counting $H^{(1,-2)} \gg H^{(1,0)}$ and $H^{(1,-2)} \sim H^{(0,0)}$

In **degenerate PT**: find states that diagonalize $H^{(0,0)} + H^{(1,-2)}$

Write proper eigenstates of $H^{(0,0)}$ generically as $|n\rangle_0 = \hat{\mathbf{n}} \cdot \mathbf{G}^a O^{a\dagger} |\text{vac}\rangle$

For Σ_u^- and Π_u : projectors $\hat{\mathbf{n}}_\Sigma \parallel \mathbf{r}$, $\hat{\mathbf{n}}_\Pi \perp \mathbf{r}$, and \mathbf{G} some 1^{+-} gluon

Matrix elements given by:

$${}_0\langle n' | H^{(0,0)} + H^{(1,-2)} | n \rangle_0 = \delta_{n'n} E_n^{(0)}(r) + \hat{\mathbf{n}} \cdot \frac{\nabla_r^2}{2\mu} \hat{\mathbf{n}}$$

Differential operator, so need **wave function** to diagonalize!

Coupled radial Schrödinger equations

Projection vectors in matrix elements allow for two different solutions (coupled or uncoupled) for the Σ_u^- and Π_u radial wave functions:

1st solution

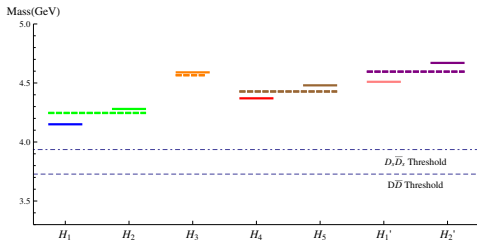
$$\left[-\frac{1}{2\mu r^2} \partial_r r^2 \partial_r + \frac{1}{2\mu r^2} \begin{pmatrix} l(l+1)+2 & 2\sqrt{l(l+1)} \\ 2\sqrt{l(l+1)} & l(l+1) \end{pmatrix} + \begin{pmatrix} E_\Sigma^{(0)} & 0 \\ 0 & E_\Pi^{(0)} \end{pmatrix} \right] \begin{pmatrix} \psi_\Sigma \\ \psi_\Pi \end{pmatrix} = \mathcal{E} \begin{pmatrix} \psi_\Sigma \\ \psi_\Pi \end{pmatrix}$$

2nd solution

$$\left[-\frac{1}{2\mu r^2} \partial_r r^2 \partial_r + \frac{l(l+1)}{2\mu r^2} + E_\Pi^{(0)} \right] \psi_\Pi = \mathcal{E} \psi_\Pi$$

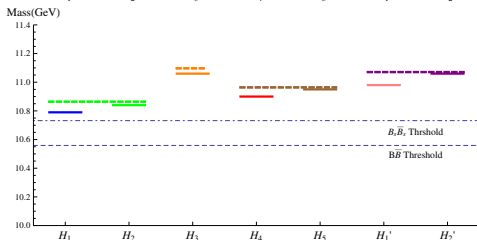
- energy eigenvalue \mathcal{E} gives hybrid mass: $m_H = m_Q + m_{\bar{Q}} + \mathcal{E}$
- $l(l+1)$ is the eigenvalue of angular momentum $\mathbf{L}^2 = (\mathbf{L}_{Q\bar{Q}} + \mathbf{L}_g)^2$
- the two solutions correspond to **opposite parity** states: $(-1)^l$ and $(-1)^{l+1}$
- corresponding eigenvalues under charge conjugation: $(-1)^{l+s}$ and $(-1)^{l+s+1}$
- Schrödinger equations can be solved numerically

Results I: Comparison to BO approximation



For Hybrid multiplets:

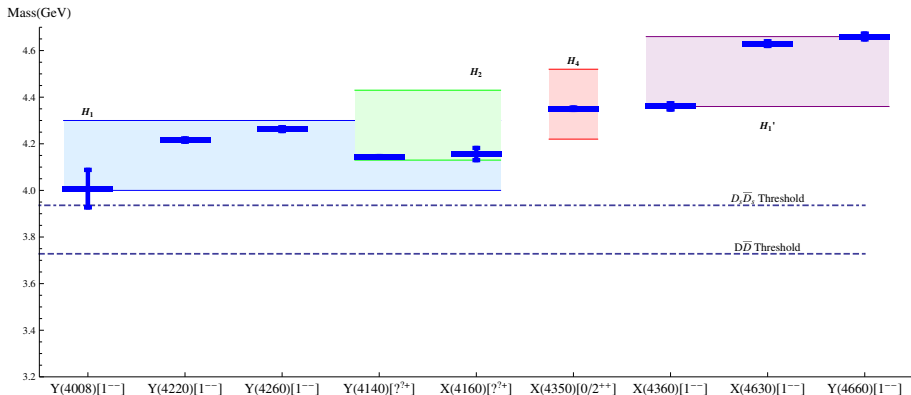
	l	$J^{PC} \{s = 0, s = 1\}$	$E_n^{(0)}$
H_1	1	$\{1^{--}, (0, 1, 2)^{-+}\}$	Σ_u^-, Π_u
H_2	1	$\{1^{++}, (0, 1, 2)^{+-}\}$	Π_u
H_3	0	$\{0^{++}, 1^{+-}\}$	Σ_u^-
H_4	2	$\{2^{++}, (1, 2, 3)^{+-}\}$	Σ_u^-, Π_u
H_5	2	$\{2^{--}, (1, 2, 3)^{-+}\}$	Π_u



- no distinction between opposite parity states in BO
- mixed states lie lower than pure
- discrepancy for H_2 , H_3 and H_5 due to different potential fits

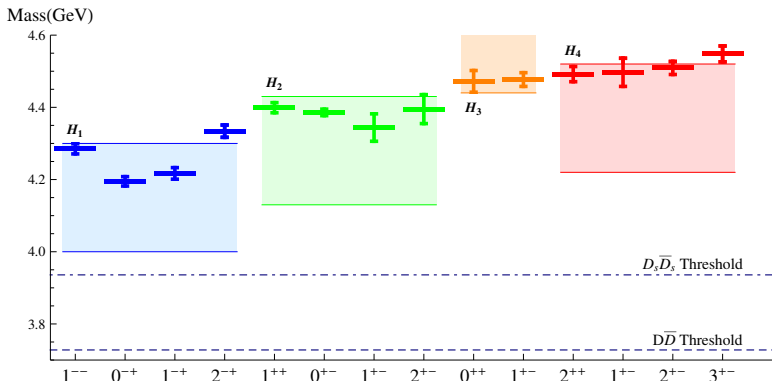
Braaten, Langmack and Smith 2014

Results II: Comparison to experiment



- comparison based on quantum numbers and mass
- error bands from uncertainty in gluon mass Λ_B
- several candidates, but most violate heavy quark spin symmetry in decays
- only compatible state is $Y(4220)$, but not yet well established

Results III: Comparison to lattice



Hadron Spectrum Collaboration 2012

- in EFT distinction between different spins only at order $1/M^2$
- good agreement for relative distance between spin-averaged multiplets
- some overall shift for absolute values

Conclusions and outlook

Conclusions

- study of hybrids in EFT framework (NRQCD and pNRQCD)
- study of the spectrum of the static Hamiltonian with non-static corrections
- short distance degeneracy of static energies
- mixing of different static states
- breaking of degeneracy between opposite parity states

Outlook

- development of full EFT treatment
- inclusion of spin dependent corrections
- inclusion of light quark contributions → study of tetraquarks or decays

Thank you for your attention!