Exotic states from the lattice

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Introduction

Several exotic states:

1. $X(3872)$ near DD^* threshold, $J^{PC}=1^{++}$,the isospin remains unsetlled because of its sizable decay to $J/\psi\rho$;
   $X(3872)\to J/\psi\rho,\ J/\psi\omega,\ D\bar{D}^*$ etc.;
2. $Z_c^{\pm}(3900)$, $C,I,J,P-?$. It was discovered in $J/\psi\pi^{\pm}$ inv. mass by BESII and confirmed by Belle and CLEOc, possible quark structure is $\bar{c}c\bar{d}u$;
3. $X(4140)$ was found in J/ϕ inv. mass by CDF and by CMS and D0.
4. $C=+1$: $X(3940)$, $Z(4050)^{\pm}$ and $Z(4250)^{\pm}$ can have $J^{PC}=1^{++}$;

1. $D^{(*)}\bar{D}$ molecules, composed of a charmed meson $D^{(*)}$ and antimeson $\bar{D}$,
2. tetraquark states consisting of diquark-antidiquark pairs, bound by QCD forces,
3. $\bar{c}cg$ hybrid states consisting of charm-anticharm quark pair and additional gluons, and
4. a compact $\bar{c}c$ core, bound inside a light meson, hadro-charmonium.

Lattice QCD

Gauge field is a path depending phase factor.
K.G.Wilson, Phys.Rev., vol. 010, p. 2445, 1974.
Lagrangian of the theory $\rightarrow$ it's discretized version $\rightarrow$ simulate numerically the corresponding functional integral in 4-dimensional Euclidean space-time.
Gauge field ansamble of gluonic configurations: Monte-Carlo methods with weight $e^{-S_{QCD}}$, corresponding to Boltzmann distribution.
S_{QCD} is decretized version of the action on the lattice.

The errors of the simplest discretizations: $O(a^2)$. Improved actions allow to deacrease these errrors.

Idea: exchange of the continuous space-time by discretized one.

Lattice spacing a playes the role of the energy cutoff.

Broken relativistic invariance in discrete space $\rightarrow$, but it will be restored in the continuum $(a \rightarrow 0)$.
Gauge invariance is not broken.

$$\mathcal{G}_E(x_1,...,x_n) = \langle 0|\Phi(x_1)..\Phi(x_n)|0\rangle_E = \frac{\int D\Phi(\prod_j^n \Phi(x_j))\exp\{-S^E_{QCD}\}}{D\Phi\exp\{-S^E_{QCD}\}} =$$

$$= \frac{1}{\mathcal{Z}}\int D\Phi(\prod_j^n \Phi(x_j))\exp\{-S^E_{QCD}\}.$$

$$\mathcal{Z} = \int DA_\mu D\psi D\bar{\psi}\exp\{-S_{QCD}\},$$

$$S_{QCD} = \int d^4x\mathcal{L} = \int d^4x\left(\frac{1}{4}F^a_{\mu\nu}(x)F^{a\mu\nu}(x) + \sum_q \bar{\psi}^i_q(x)(\gamma_\mu D_\mu(x) + m_q)_{ij}\psi^j_q(x)\right) =$$

$$= \int d^4x\left(\frac{1}{4}F^a_{\mu\nu}(x)F^{a\mu\nu}(x) + \sum_q \bar{\psi}^i_q(x)[M_q]_{ij}\psi^j_q(x)\right).$$

$$S_{QCD} = S_{gl} + S_{quark} = \int d^4x\exp\left(\frac{1}{4}F^a_{\mu\nu}(x)F^{a\mu\nu}(x)\right) - \sum_q \ln(Det[M_q]_{ij}),$$

$$\mathcal{Z} = \int DA_\mu DetM\exp\left(\frac{1}{4}F^a_{\mu\nu}(x)F^{a\mu\nu}(x)\right).$$

Simulations and approximations

detM has the central role in the full QCD, describes sea quark and antiquarks interaction in QCD.
Approximation: $det\, M = const$ corresponds to the quenched approxiation, $m_q \to \infty$, decreasing the time of the calculations.
Observables on the lattice:

$$\langle \mathcal{O}[U_\mu(x)] \rangle = \frac{1}{\mathcal{Z}} \int DU \mathcal{O}[U_\mu(x)] \exp\{-S[U_\mu(x)]\},$$

usual integration $\to$ integration over the Haar measure.
Finite ensemble of the gauge field configurations $(U_1(x), .., U_{M_c}(x))$
Perform averaging

$$\langle \mathcal{O}[U_\mu(x)] \rangle = \lim_{M_c \to \infty} \frac{1}{M_c} \sum_{i=1}^{M_c} \mathcal{O}[(U_\mu(x))_i].$$

Each configuration have been generated with the probability distribution P_i

$$P_i \propto \begin{cases} \exp\{-S[U_\mu(x)]\}, & \text{quenched QCD;} \\ DetM \cdot \exp\{-S[U_\mu(x)]\}, & \text{full QCD.} \end{cases}$$

$\mathcal{O}$ - any combination of operators. $(M^{-1})^{y,j,b}_{x,i,a}$ - amplitude of propagation from x spin and color i and a to y, j, b.

Standard approach for hadron spectrum calculation

- **The variational method** The energy levels E_n are extracted from the decay of two-point Green functions in Euclidean time,

$$C_{ij}(t) = \langle O_i(t) O_j^\dagger(t) \rangle = \sum_{n \geq 1} v_i^n v_j^{n*} e^{-E_n t}, \quad v_i^n = \langle 0 | \hat{O}_i | n \rangle$$

Interpolating operator $\hat{O}_i^\dagger$ creates states of an isospin I, charm number, a given momentum and $J^{P(C)}$ quantum numbers.
Need to solve the symmetrized eigenvalue problem

$$C^{-1}(t_0) C(t) u^n(t, t_0) = \lambda^n(t, t_0) u^n(t, t_0)$$

$$\lambda^n(t, t_0) \propto e^{-(t-t_0)E_n}[1 + \mathcal{O}(e^{-(t-t_0)\triangle E_n})]$$

B. Blossier, R.Sommer et al. [ALPHA Collab.], arXiv: 0902.1265 [hep-lat]
The effective energy levels or $\vec{p} = 0$ masses are obtained from the eigenvalues

$$m_{n,eff}^{t_0}(t + a/2) = a^{-1} \frac{\lambda^n(t, t_0)}{\lambda^n(t + a, t_0)}$$

States well below open charm threshold

$$m = \sqrt{E^2 - P^2}$$

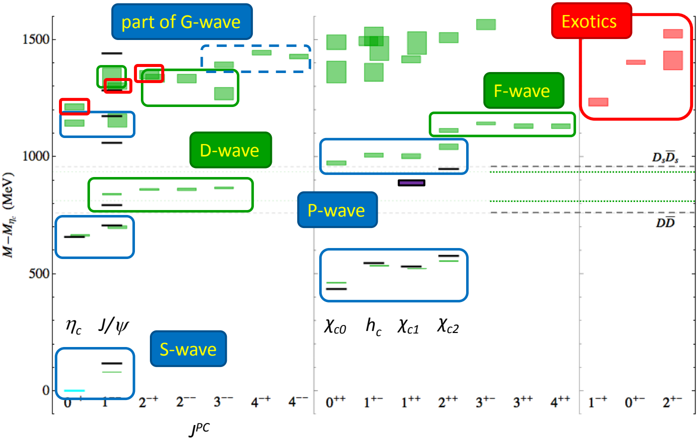
If $P = 0$ then $m = E$.
Perform extrapolations: $L \to \infty$, $a \to 0$, $m_q \to m_{phys}$
(lattice collab.: HPQCD, FNAL/MILC, χQCD).

"Single-meson" treatment of excited states is not good

1. using only quark-antiquark interpolating fields $O \sim \bar{q}q$ for mesons;
2. assuming that all energy levels correspond to $one - particle$ states;
3. that the mass of the state equals the measured energy level $m = E$.

These assumptions are too strong for the resonances, which are not asymptotic states. This approach also ignores the effect of the threshold and near-threshold states.

$\bar{c}c$ spectrum by the HSC, G.Moir et.al. arXiv: 1301.7670



$N_f = 2+1, m_u = m_d \neq m_s$, lattices $16^3 \times 128$, $24^3 \times 128$ with $m_\pi \approx 400$ MeV, $L \simeq 1.9,\ 2.9$ fm, $a_s \sim 0.12$ fm, $a_s/a_t \sim 3.5$, $\eta_c = 2980\ MeV$.

Rigorous treatment of near-threshold states X(3872)

$$X(3872) \to J/\psi\omega, J/\psi\rho$$

Consider also discrete scattering levels DD^* and $J/\psi V$, where $V = \omega$ for $I = 0$ and $V = \omega$ for $I = 1$.
The eigenstates are also the s-wave scaterring states $D(\vec{p})D^*(-\vec{p})$ and $J/\psi(\vec{p})V(-\vec{p})$ with discrete momenta $\vec{p}$.
All states carrying the same quantum numbers, including the single-particle and multi-particle states, in principle contribute to the eigenstates of Hamiltonian.

In the absence of interaction $p = p^{n,i} = 2\pi|n|/L$ and the scattering levels appear at

$$E^{n,i} = E_1(p^{n,i}) + E_2(p^{n,i})$$

In the presence of interaction the scattering levels $E = E_1(p) + E_2(p)$ are shifted with respect to $E^{n,i}$ since momentum outside the interaction region is defferent from $p^{n,i}$.

This energy shift provides rigorous information on the DD^* interaction.

Bound states and resonances lead to levels in addition to the scattering levels.

$X(3872)$ from DD^* scattering on the lattice

The interpolating fields O_i have to couple well to $\bar{c}c$ as well as the scattering states to study the system with $J^{PC}=1^{++}$, $\vec{p}=0$ and $I=0$ or $I=1$.

$$O^{\bar{c}c}=\bar{c}\hat{M}_i c(0)\text{, where}$$

$\hat{M}=\gamma_i,\ \gamma_t\gamma_i,\ \vec{\nabla}_i,\ \epsilon_{ijk}\gamma_j\gamma_5\vec{\nabla}_i,\ \overleftarrow{\nabla}_i\gamma_i\vec{\nabla}_i,\ \overleftarrow{\nabla}_i\gamma_t\gamma_i\vec{\nabla}_i,\ \overleftarrow{\Delta}_i\gamma_i\vec{\Delta}_i,\ \overleftarrow{\Delta}_i\gamma_t\gamma_i\vec{\Delta}_i$ (only for I=0)

$$O_1^{DD^*}=[\bar{c}\gamma_5 u(0)\bar{u}\gamma_i c(0)-\bar{c}\gamma_i u(0)\bar{u}\gamma_5 c(0)]+f_I\{u\to d\}$$

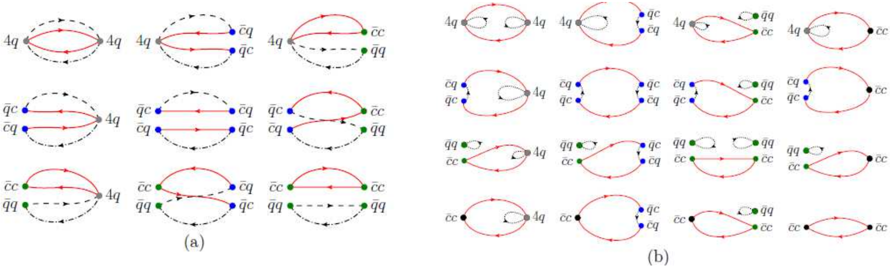
$$O_2^{DD^*}=[\bar{c}\gamma_5\gamma_t u(0)\bar{u}\gamma_i\gamma_t c(0)-\bar{c}\gamma_i\gamma_t u(0)\bar{u}\gamma_5\gamma_t c(0)]+f_I\{u\to d\}$$

$$O_3^{DD^*}=\sum_{e_k=\pm e_{x,y,z}}[\bar{c}\gamma_5 u(e_k)\bar{u}\gamma_i c(-e_k)-\bar{c}\gamma_i u(e_k)\bar{u}\gamma_5 c(-e_k)]+f_I\{u\to d\}$$

$$O_1^{J/\psi V}=\epsilon_{ijk}\bar{c}\gamma_j c(0)[\bar{u}\gamma_k u(0)+f_I\bar{d}\gamma_k d(0)]$$

$$O_1^{J/\psi V}=\epsilon_{ijk}\bar{c}\gamma_j\gamma_t c(0)[\bar{u}\gamma_k\gamma_t u(0)+f_I\bar{d}\gamma_k\gamma_t d(0)]$$

where $f_I=1$ and $V=\omega$ for $I=0$, while $f_I=-1$ and $V=\rho$ for $I=1$.



The Wick contractions considered in the calculations. Left: connected contraction diagrams. Right: diagrams, in which the light/strange quarks do not propagate from source to sink. The correlation function in the $\bar{c}c(\bar{u}u + \bar{d}d)$ and $\bar{c}c\bar{s}s$ cases (depending on isospin) are linear combinations of all these diagrams; the correlation function between operators with quark content $\bar{c}c\bar{u}d$ are constructed from the left part.

$X(3872)$ from DD^* scattering on the lattice

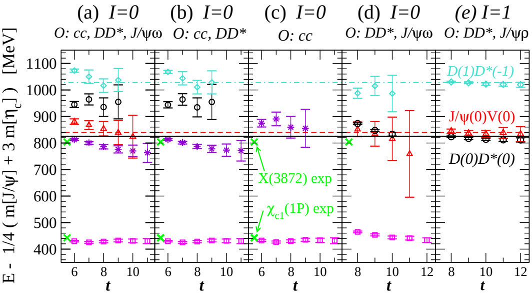
Solve

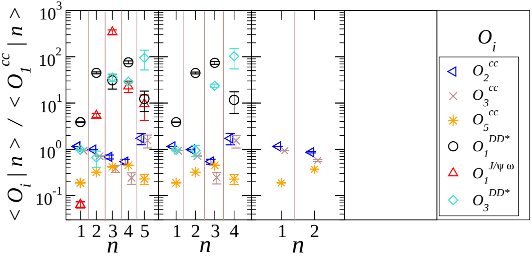
$$C^{-1}(t_0)C(t)u^n(t,t_0) = \lambda^n(t,t_0)u^n(t,t_0)$$

Then find the ratios of overlaps for state n

$$\frac{\langle O_i|n\rangle}{\langle O_j|n\rangle} = \frac{\sum_k C_{ik}(t)u_k^n(t)}{\sum_{k'} C_{ik'}(t)u_{k'}^n(t)}$$

Simulations: $N_f = 2$, $m_u = m_d$, $m_{val} = m_{dyn}$, $m_\pi = 266(4)$ MeV,
$a = 0.1239(13)$ fm,
the lattice volume $V = 16^3 \times 32$
The main drawback: small spatial size $L \simeq 2$ fm because $X(3872)$ is larger in size.


(a) I=0
O: cc, DD*, J/ψω
(b) I=0
O: cc, DD*
(c) I=0
O: cc
(d) I=0
O: DD*, J/ψω
(e) I=1
O: DD*, J/ψρ
E - 1/4 (m[J/ψ] + 3 m[η_c]) [MeV]
D(1)D*(-1)
J/ψ(0)V(0)
D(0)D*(0)
X(3872) exp
χ_c1(1P) exp
t


< O_i | n > / < O_1^cc | n >
n
O_i
O_2^cc
O_3^cc
O_5^cc
O_1^DD*
O_1^J/ψ ω
O_3^DD*

$1^{++}, I = 1$

for $I = 1$

The first possibility: $X(3872)$ has charged partners. Experimental facts speak against this.

The second possibility: its isospin is broken in the decay or its linear combination of $I = 0$ and $I = 1$. It is impossible to see this because $m_u = m_d$ in the lattice simulations.

Only the scattering levels appear $D(0)D^*(0)$, $J/\psi(0)\rho(0)$ and $D(1)D^*(-1)$, Their energies are equal to non-interacting energies.

Also no candidates for $Z(4050)^+$ and $Z(4250)^+$ in the channel 1^{++}.

$1^{++}, I = 0$

for $I = 0$

The first possibility: $a_0(DD^*) < 0$

then the stars → weakly bound state $X(3872)$

the circles → scattering state $D(0)D^*(0)$.

The second possibility: $a_0(DD^*) > 0$

the opposite case to the first one, attraction between D and D^* is ruled out because the analysis of lattice data leads to

$$a_0^{DD^*} = -1.7 \pm 0.4\ fm \quad \rightarrow \quad m(X(3872)) - (m_D + m_{D^*}) = -11 \pm 7\ MeV$$

arXiv:1307.5172 [hep-lat] , S.Prelovsek, L. Leskovec

$X(3872)$ from DD^* scattering on the lattice

9 new interpolators have been added into consideration

$$m(X(3872)) - (m_D + m_{D^*}) = -8 \pm (15)\ MeV$$

The part of them corresponds to the tetraquark structure $[\bar{c}\bar{q}]_g[cq]_g$

$$m(X(3872)) - (m_D + m_{D^*}) = -9 \pm (8)\ MeV$$

arXiv:1503.03257 [hep-lat] , S.Prelovsek, L. Leskovec

$$m(X(3872)) - (m_D + m_{D^*}) = -13 \pm (6)\ MeV$$

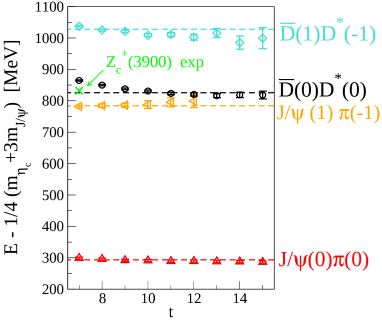
simulations with $N_f = 2 + 1 + 1$ dynamical quarks
arXiv:1411.1389 [hep-lat] , S.Lee et al (Fermilab Lattice and MILC)

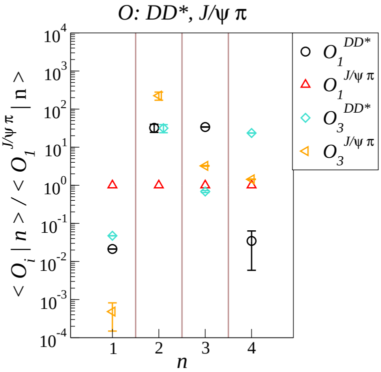
Search for $Z_c^+(3900)$ exotic state in 1^+, $I = 1$

BESIII, Belle: $Z_c^{\pm}(3900)$, CLEO: $Z_c^0(3900)$,
$m(Z_c^+) = 3899.0 \pm 3.6 \pm 4.9$ MeV, suggest quarks structure $\bar{c}c\bar{d}u$.

$$Z_c^+ \to J/\psi\pi^+,$$

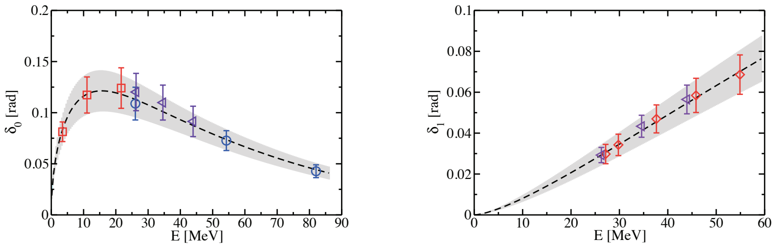
J and P quantum numbers are experimentally unknown
The search on the lattice by simulating the channel with $J^P = 1^+$ and $I = 1$ was performed with $N_f = 2$ of dynamical quarks with $m_u = m_d$.
Parameters of simulations: $V = 16^3 \times 32$, $a = 0.1239(13)$ fm, $L = 1.98$ fm, $m_\pi = 266(4)$ MeV.
No candidate was found. Possible reasons: $Z_c^+(3900)$ has another quantum numbers or the set of interpolation operators are not diverse enough.





arXiv:1411.1389 [hep-lat] , S.Lee et al (Fermilab Lattice and MILC)
arXiv:1310.4354 [hep-lat] , S. Prelovsek

$X(4140), J^P = 1^+$



The phase shift for s-wave and p-wave scattering of $J\psi\phi$. No resonant structure was found, ignoring of $\bar{s}s$ annihilation contribution.
For the one near-threshold bound state the phase shift have to start at $\delta(p=0)=\pi$ and fall down to $\delta(p\to\infty)=0$. *S.Ozaki, arXiv:1211.5512*
No results at $I=0$ channel with flavor $\bar{c}c\bar{s}s$ and $\bar{c}c$.

Conclusions

1. The single meson treatment of states are not suited for excited states and resonances which are not asymptotic states of lattice correlators
2. $X(3872)$ is found below the DD^* slightly below the DD^* threshold
3. No candidate for $Z_c^+(3900)$ in $1^+ = 1^+$ and $I = 1$ channel
4. No $X(4140)$ resonant structure in $J^P = 1^+$, quantum numbers are unknown
5. No evidence in favour of $Z_c(4050)^+$ and $Z_c(4250)^+$ in 1^{++} channel