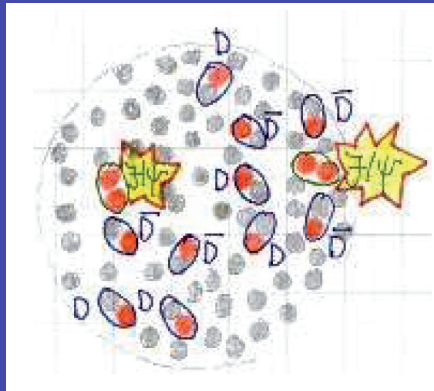


Heavy quark bound states in a quark-gluon plasma: dissociation and recombination



Imprints of the Quark-Gluon Plasma
EMMI workshop, MPI Heidelberg
April 16-17, 2015



European
Research
Council

Jean-Paul Blaizot, IPHT-Saclay

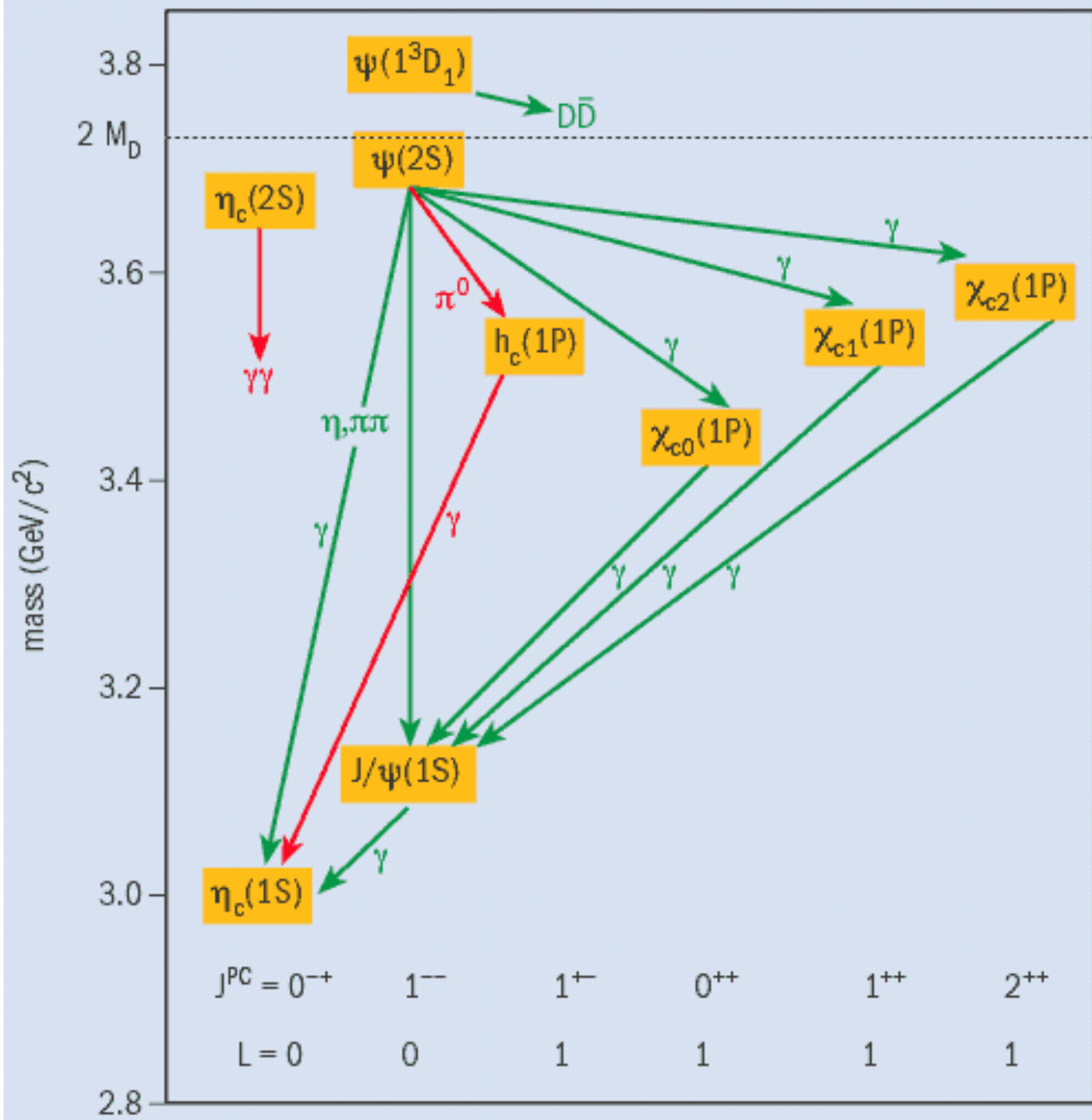


A very nice idea....

The charmonium is a « non relativistic » system

$$H = 2m_c + \frac{p_1^2}{2m_c} + \frac{p_2^2}{2m_c} + V(r)$$

$$V(r) = -\frac{\alpha}{r} + \sigma r$$



Screening of binding forces in a quark-gluon plasma

Disappearance of the string tension

$$\sigma(T > T_c) \rightarrow 0$$

Screened potential

$$V(r) = -\frac{\alpha}{r} e^{-r/r_D(T)}$$

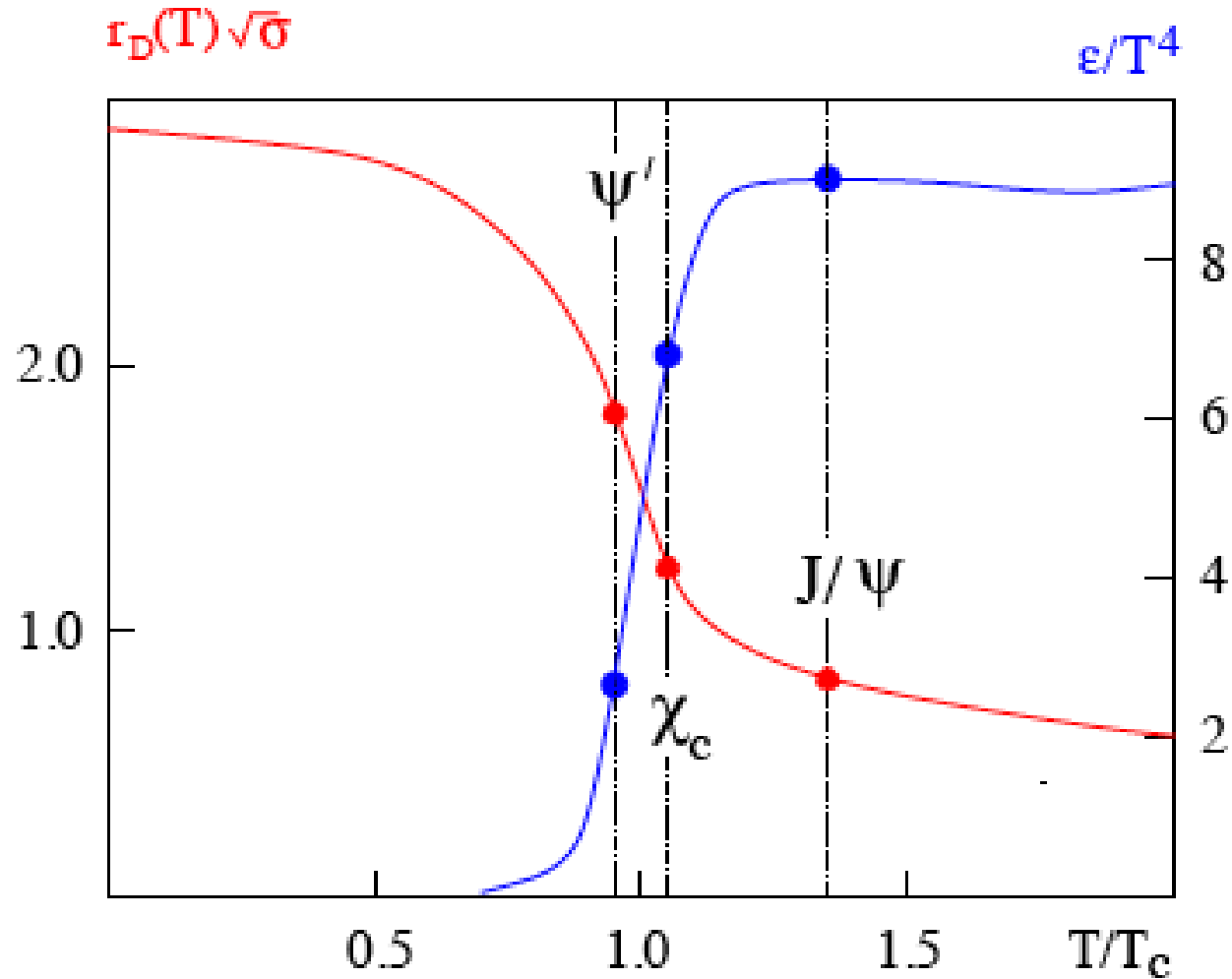
Bound state exists for

$$r_D(T) > r_D^{\min}$$

that is, for

$$T < T_D$$

Melting temperature depends on size of bound state

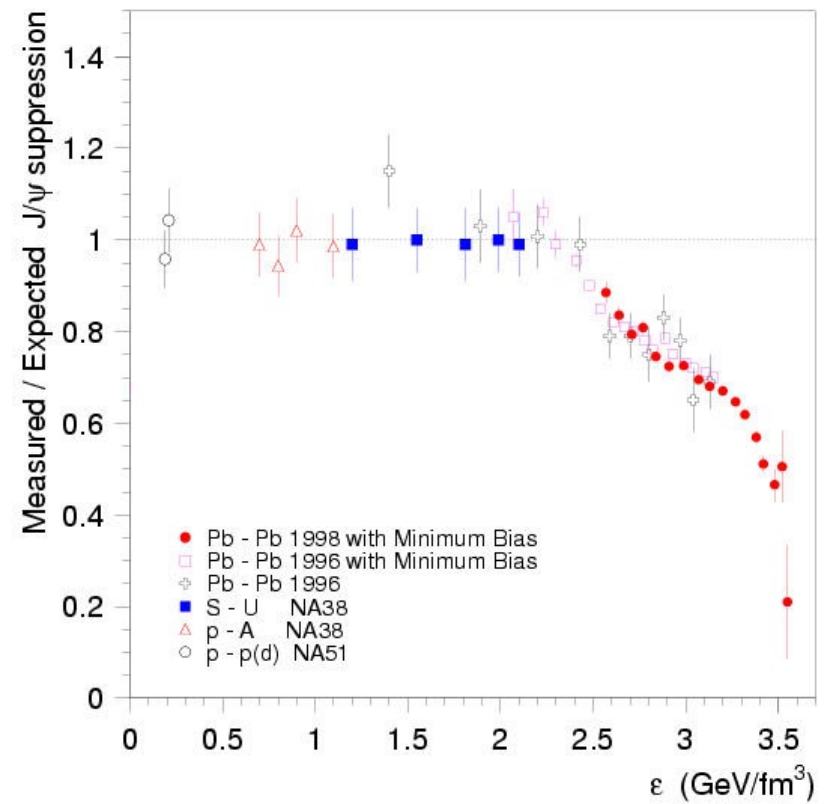
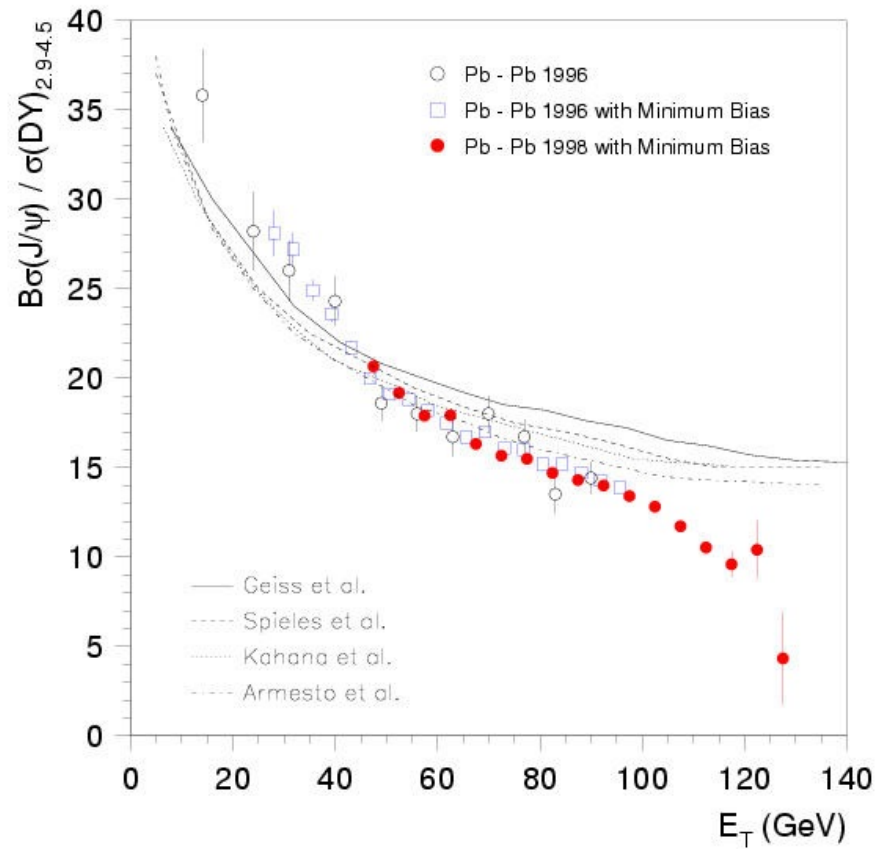


(from H. Satz, hep-ph/0602245)

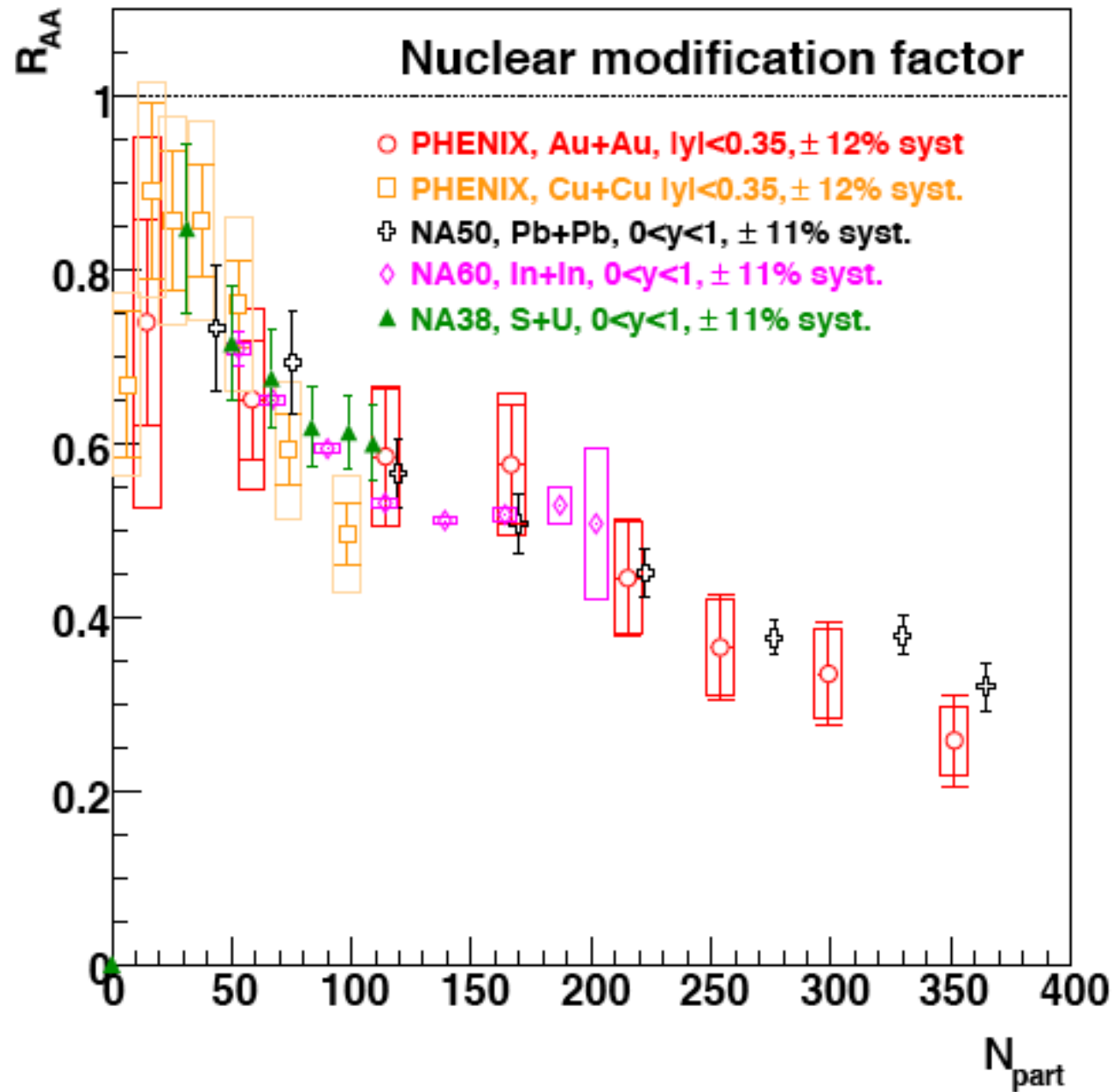
A considerable experimental effort

Summary of early measurements (NA38, NA50)

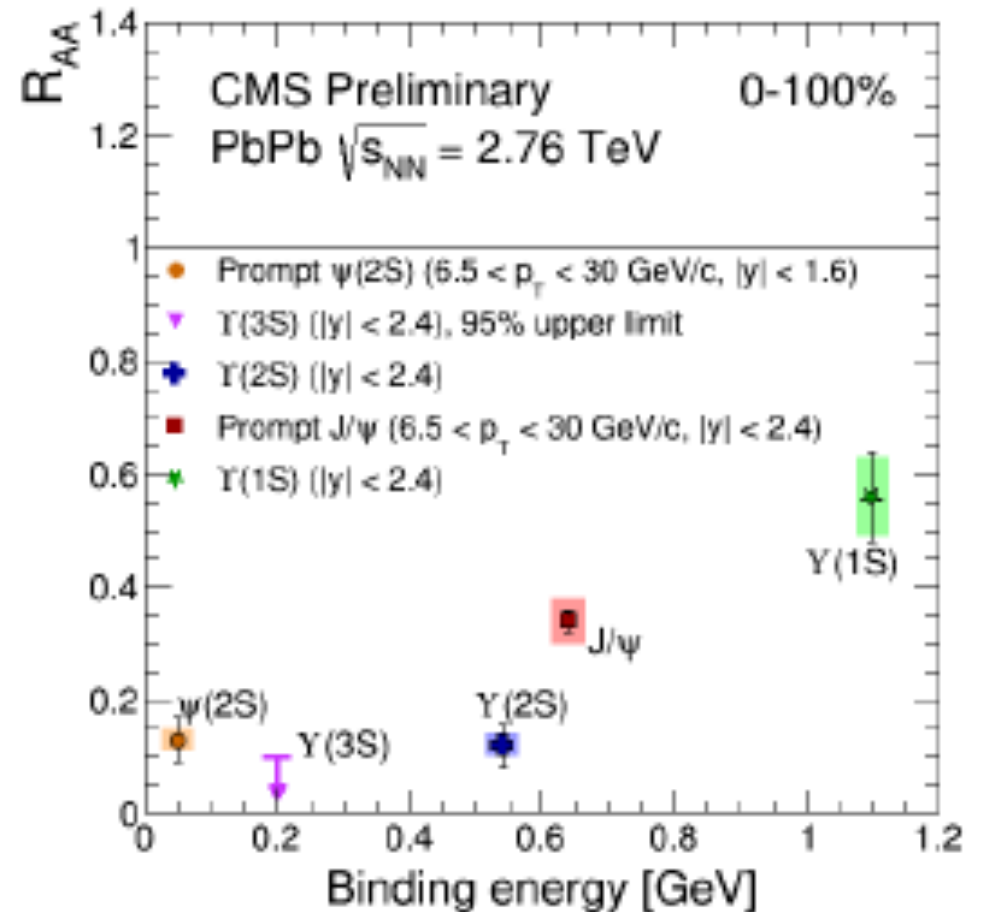
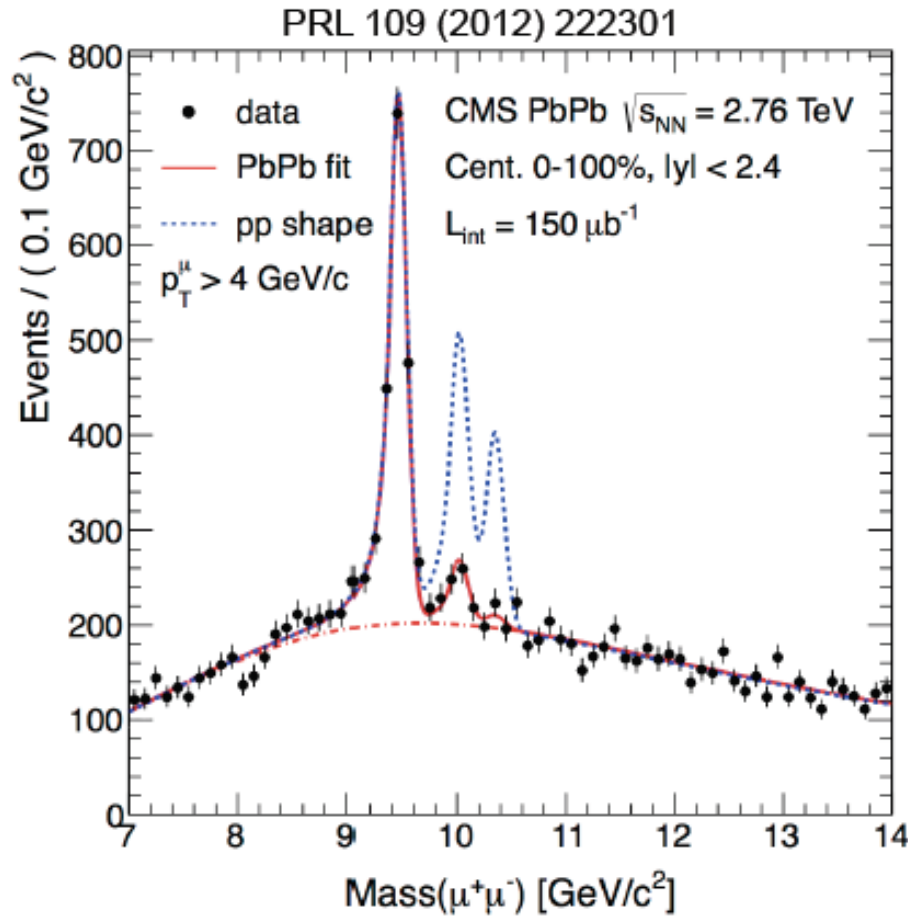
(CERN, 2000)



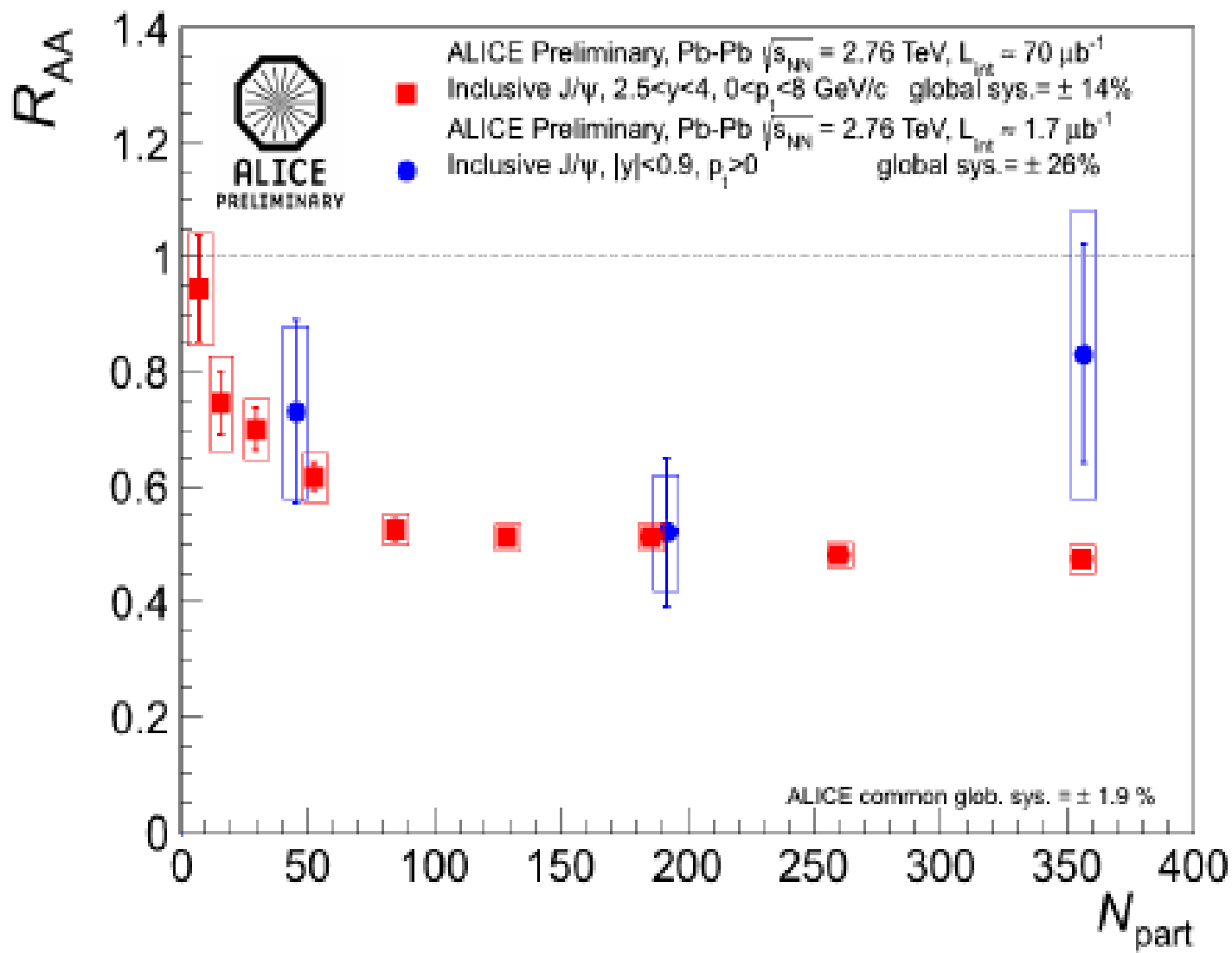
What about RHIC?



Υ suppression



excited states are more 'fragile'...
findings in line with expectations....



A very nice idea....

a considerable experimental effort

but a very difficult many-body problem !

a large variety of theoretical approaches

- potential models
- spectral functions
- Euclidean correlators (lattice), maximum entropy techniques
- coupled channels
- path integrals
- open quantum systems
- effective field theory, non relativistic heavy quark effective theory
- strong coupling techniques
- etc

Which problem do we need to solve ?

- full dynamics, including plasma expansion
- dynamics of bound state formation (stationary states are not enough)
- dynamics of dissociation and recombination within the same framework

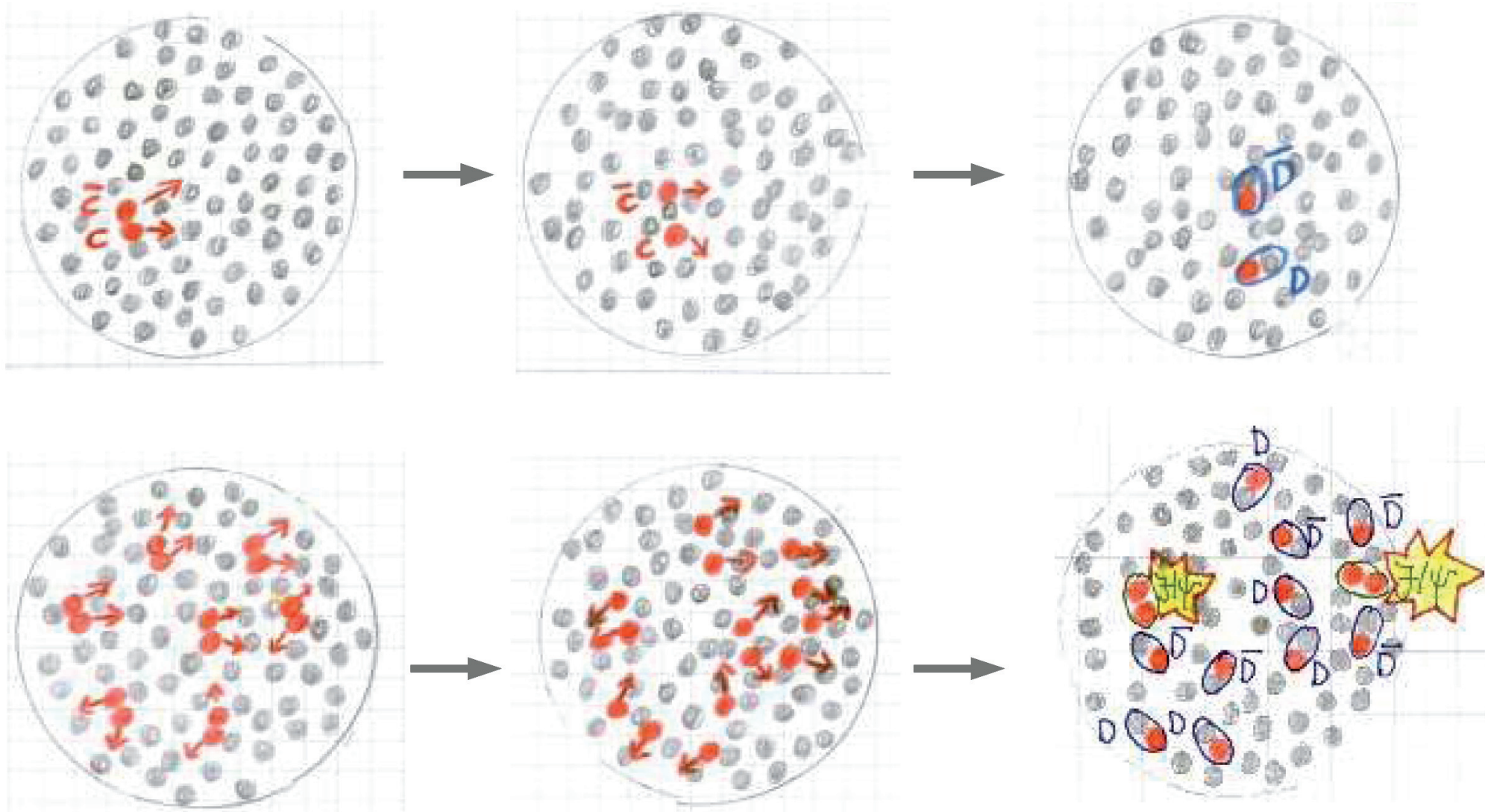
WORK IN PROGRESS !

Results presented are based on

A. Beraudo, JPB, C. Ratti, NPA 806 (2008) 312 [arXiv: 0712.4394]

A. Beraudo, JPB, P. Faccioli and G. Garberoglio [arXiv: 1005.1245]

JPB, D. de Boni, P. Faccioli and G. Garberoglio [arXiv: 1503.03857]



On the Charm Production in Ultra-relativistic Heavy Ion Collisions *

T. Matsui

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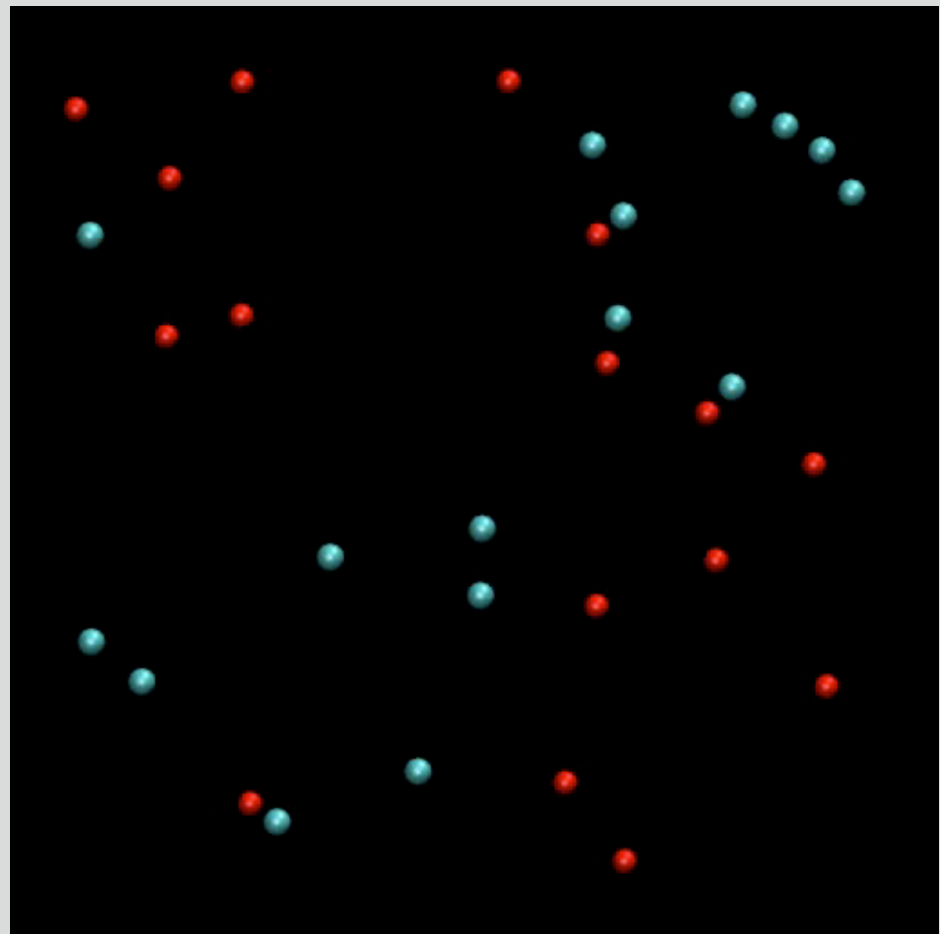
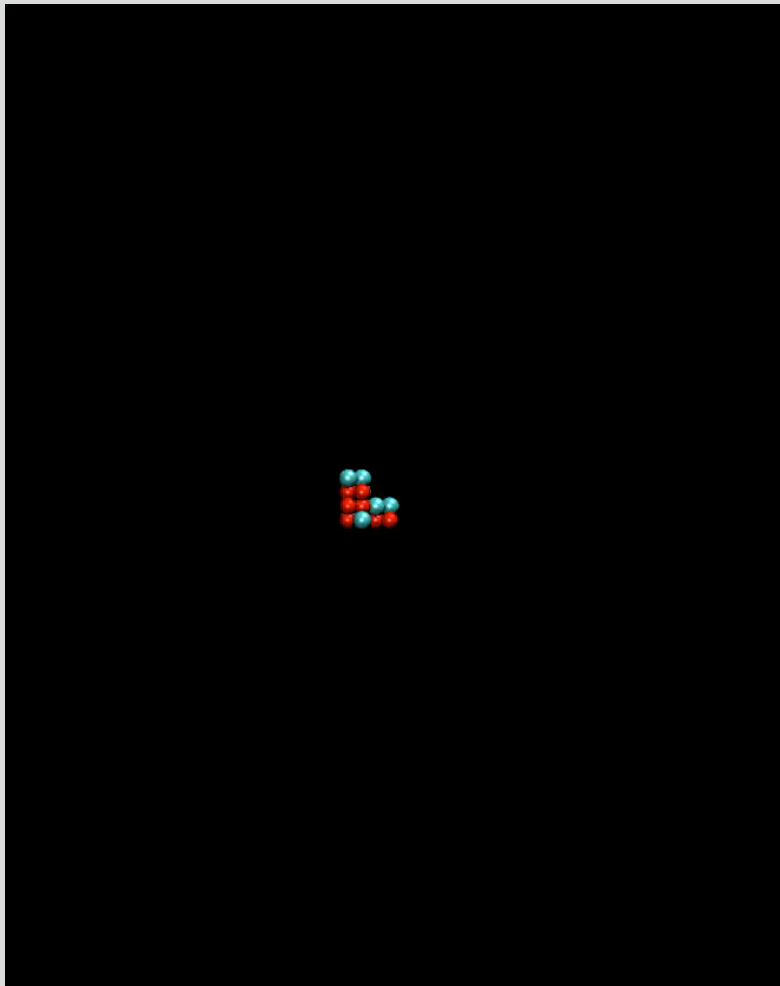
There are several reasons why it is important to measure the charm production cross section in ultrarelativistic heavy ion collisions.

1) Charm carries an information of the very early stage of the collision process: Since the charm quark is so massive ($m_c = 1.5 - 1.8$ GeV), it is likely that its creation takes place only at the very beginning of the whole collision process and the charm quark abundance will be essentially frozen in the later stage of the matter evolution. Hence it can be used to probe the early stage of matter formation and to test dynamical models of particle production.

2) If there is a strong enhancement of charm production in heavy ion collisions, in comparison with non-charm particle production, it would spoil some interesting signals of the plasma formation: J/ψ suppression by the plasma screening effect¹ will be compensated by the enhanced recombination of $c\bar{c}$ into the J/ψ during the hadronization stage; semileptonic decay of charmed mesons produces a large background for the dilepton signals from the plasma².

In this short report, I will first make a crude and rather conservative estimate of the expected charm abundance in nucleus-nucleus collisions based on the measured charm production cross section in pp interactions, and then discuss a possible coherent soft process which would lead to a further enhancement of the charm production in the case of heavy ion collisions. This talk is based on the work which is presently in progress in collaboration with Larry McLerran and Ben Svetitsky.

A rough conservative estimate:



Dynamics

(Abelian approximation)

$$H = H_Q + H_{med} + H_{int}$$

Heavy quark

$$H_Q = M \int d^3\mathbf{r} \psi^\dagger(\mathbf{r})\psi(\mathbf{r}) + \int d^3\mathbf{r} \psi^\dagger(\mathbf{r}) \left(-\frac{\nabla^2}{2M} \right) \psi(\mathbf{r})$$

linearly coupled to gauge field

$$H_{int} = g \int d^3\mathbf{r} \psi^\dagger(\mathbf{r})\psi(\mathbf{r})A_0(\mathbf{r})$$

The hot plasma

$$H_{med} = \int d^3r \xi^\dagger(\mathbf{r})h_0 \xi(\mathbf{r}) + \frac{1}{2} \int d^3r d^3r' \hat{\rho}(\mathbf{r}) \frac{g^2}{4\pi|\mathbf{r} - \mathbf{r}'|} \hat{\rho}(\mathbf{r}')$$

Path integral and influence functional

$$P(Q_f, t_f | Q_i, t_i) = \int_{\mathcal{C}} DQ e^{iS_0[Q]} e^{i\Phi[Q]}$$

$$e^{i\Phi[Q]} = \int DA_0 e^{-i \int_{\mathcal{C}} d^4x g \rho(x) A_0(x)} e^{iS_2[A_0]}$$

$$\rho(x) = \sum_{j=1}^N (\delta(\mathbf{x} - \mathbf{q}_j(t)) - \delta(\mathbf{x} - \bar{\mathbf{q}}_j(t)))$$

'Integrate out' the light particles and keep the quadratic part of the resulting action (HTL approximation)

$$S_2[A_0] = -\frac{1}{2} \int_{\mathcal{C}} dx (A_0(x) \nabla^2 A_0(x)) - i \text{Tr} \ln [i\gamma^\mu \partial_\mu - m - e\gamma^0 A_0(x)]$$

The diagram shows a wavy line on the left, followed by an equals sign, then a wavy line, a plus sign, a wavy line with a fermion loop (a circle with an arrow) attached to it, another plus sign, and finally an ellipsis.

$$\Phi[Q] = \frac{g^2}{2} \iint_{\mathcal{C}} d^4x d^4y \rho(x) \Delta_c(x-y) \rho(y)$$

$$\Delta(x-y) \equiv i \langle T_{\mathcal{C}} [A_0(x) A_0(y)] \rangle$$

Physical content of the influence functional

$$\Phi[\mathbf{Q}] = \frac{g^2}{2} \iint_{\mathcal{C}} d^4x d^4y \rho(x) \Delta_c(x-y) \rho(y)$$

$$\Delta(x-y) \equiv i \langle T_{\mathcal{C}} [A_0(x) A_0(y)] \rangle$$

$$V(x) \sim \Delta_{11}(\omega = 0, x) \quad \text{Heavy quark potential (complex)}$$

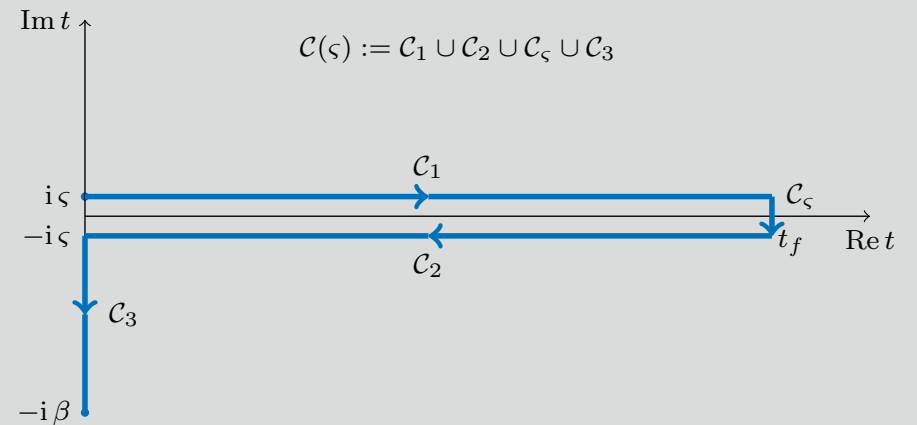
$$D(x) \sim \Delta_{12}(\omega = 0, x) \sim \text{Im}V(x) \quad \text{dissipation}$$

$$\frac{g^2}{2MT} \left. \frac{\partial^2 D}{\partial x_i \partial x_j} \right|_{x=0} = \delta_{ij} \gamma \quad \gamma \quad \text{friction coefficient}$$

Low frequency expansion

$$r_i = \frac{1}{2}(q_{i,1} + q_{i,2})$$

$$y_i = q_{i,1} - q_{i,2}$$



$$P(R_f, t_f | R_i, t_i) = \int_{R_i}^{R_f} DR \int_0^1 DY e^{\int_{t_i}^{t_f} dt \mathcal{L}(R, Y)}$$

$$\mathcal{L}(R, Y) = -i Y \left(M \ddot{R} + \frac{\beta}{2} \mathcal{H}(R) \dot{R} - \mathbf{F}(R) \right) - \frac{1}{2} Y \mathcal{H}(R) Y$$

$$\mathbf{F}(R) \sim \nabla \text{Re} V(R) \quad \mathcal{H}_{ij} \sim \left. \frac{\partial^2 D}{\partial x_i \partial x_j} \right|_{x=0}$$

Equivalent Langevin equation

$$M \ddot{R} = -\frac{\beta}{2} \mathcal{H}(R) \dot{R} + \mathbf{F}(R) + \Psi(R, t)$$

$$\mathcal{H}_{ij} \sim \left. \frac{\partial^2 D}{\partial x_i \partial x_j} \right|_{x=0} \quad \mathbf{F}(R) \sim \nabla \text{Re}V(R)$$

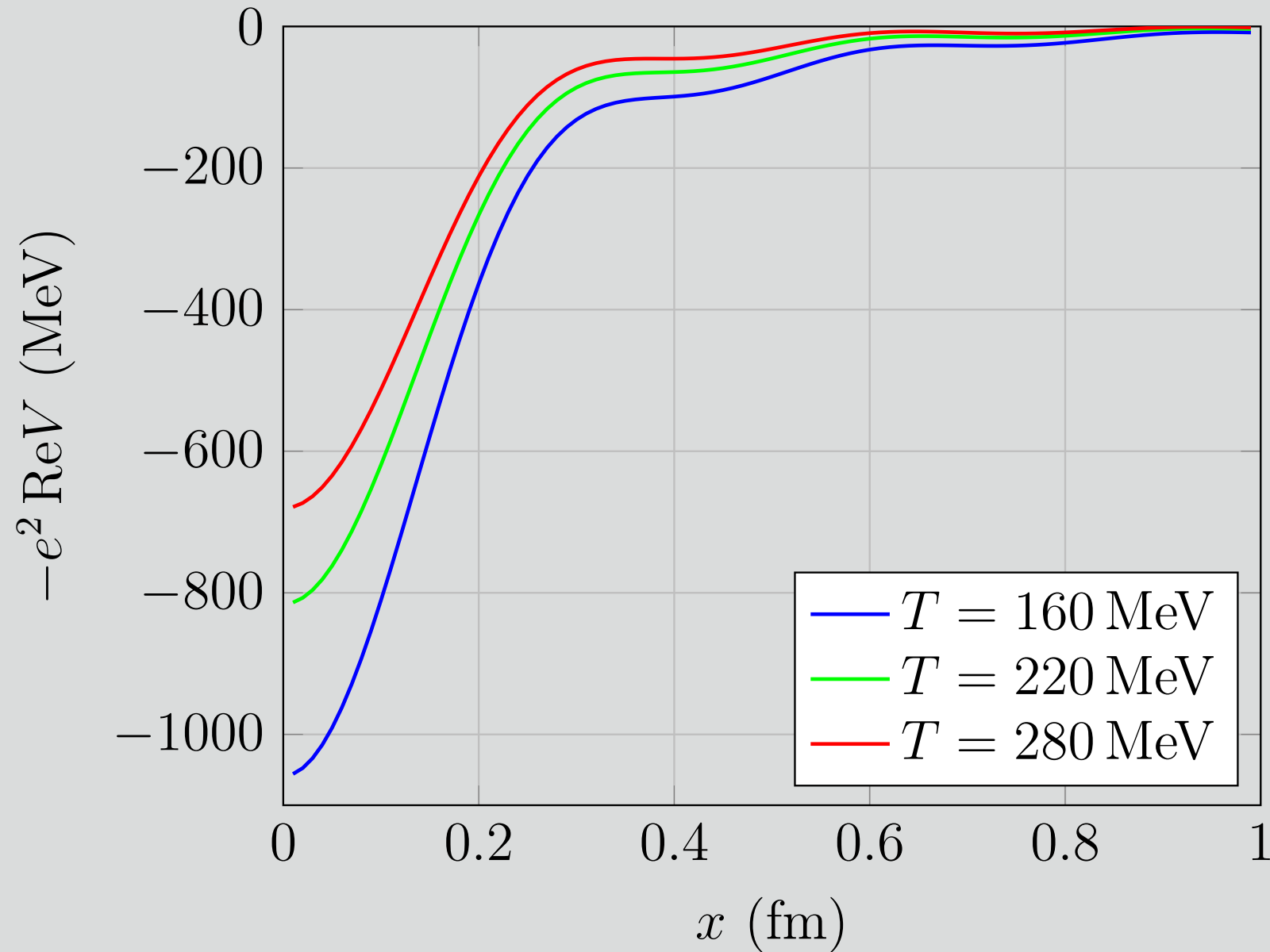
$$\langle \Psi(R, t) \rangle = 0$$

$$\langle \Psi_k(R, t) \Psi_m(R, t') \rangle = \mathcal{H}_{km}(R) \delta(t - t')$$

Non trivial noise

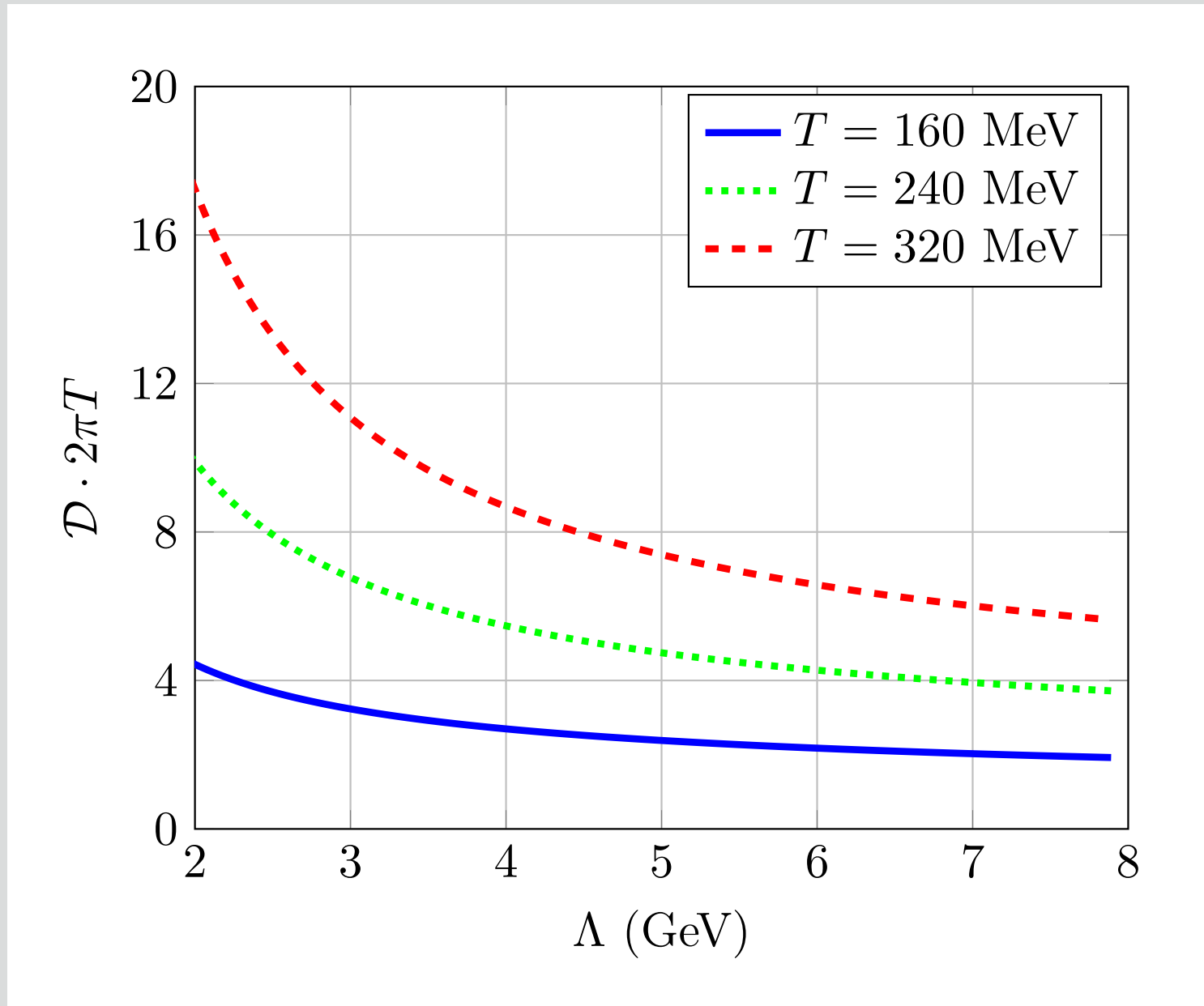
Selected results

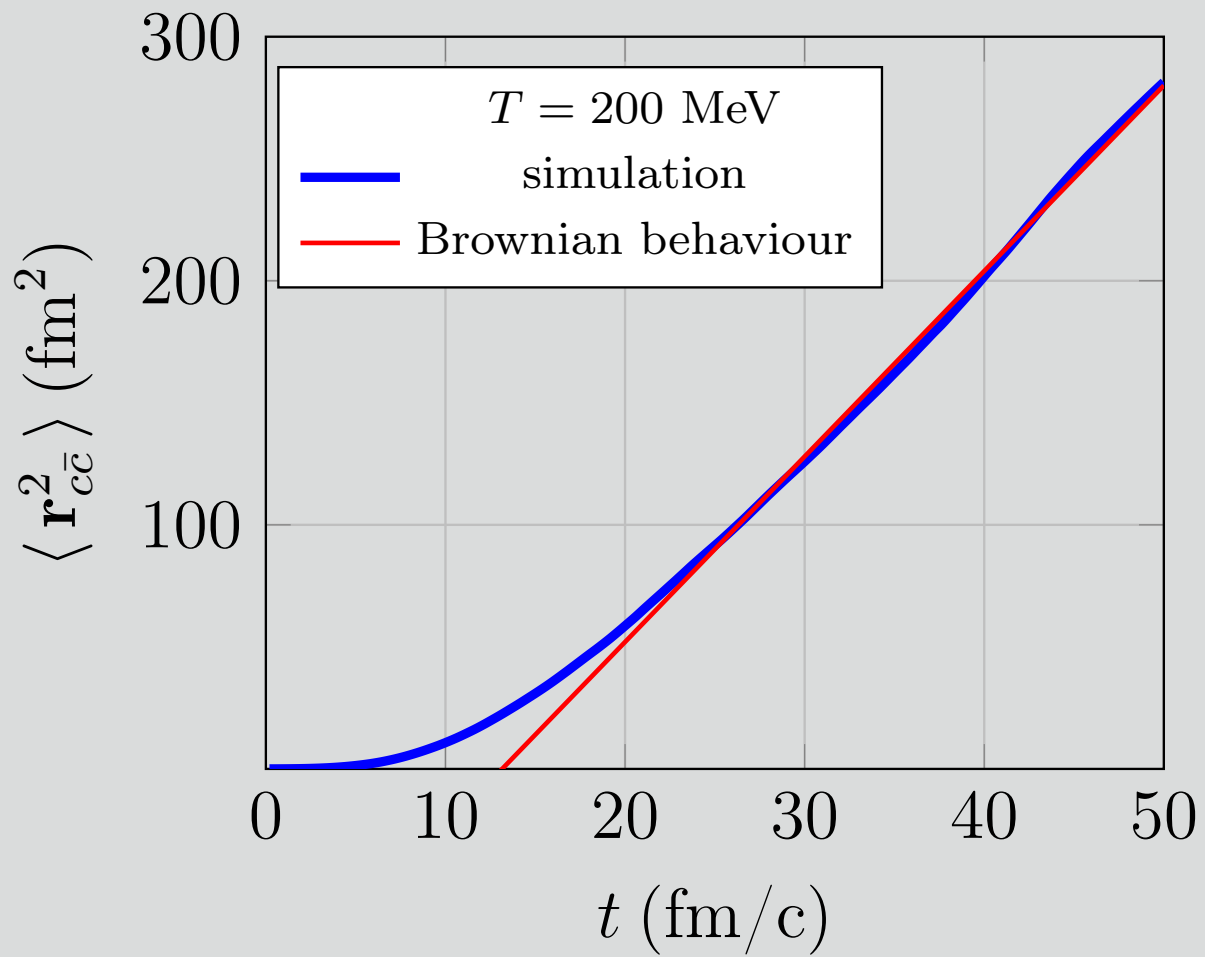
Potential (real part) - charmonium



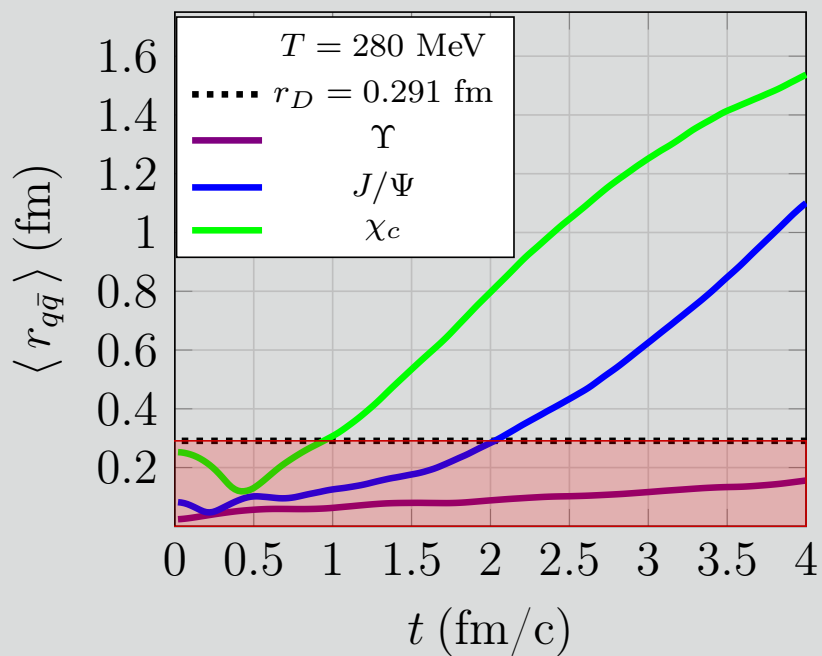
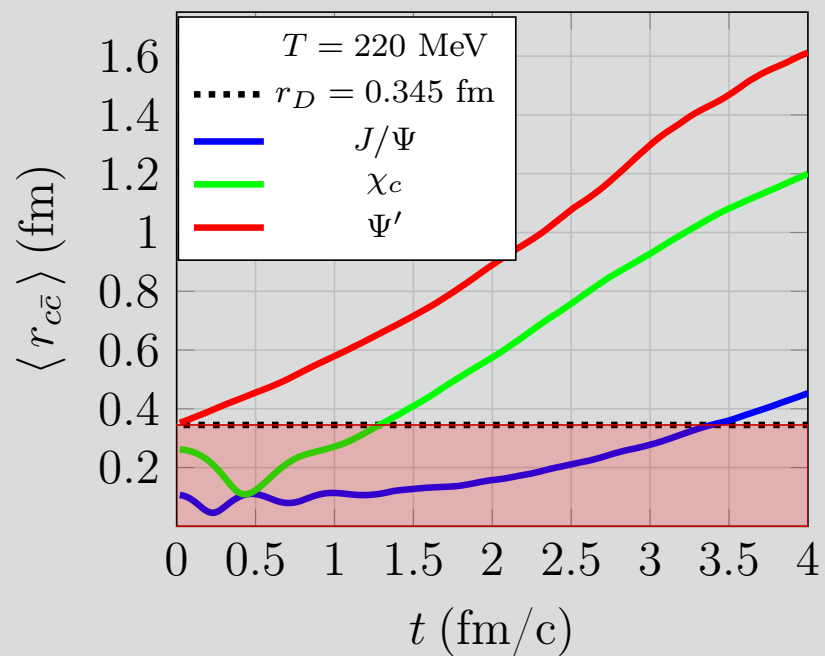
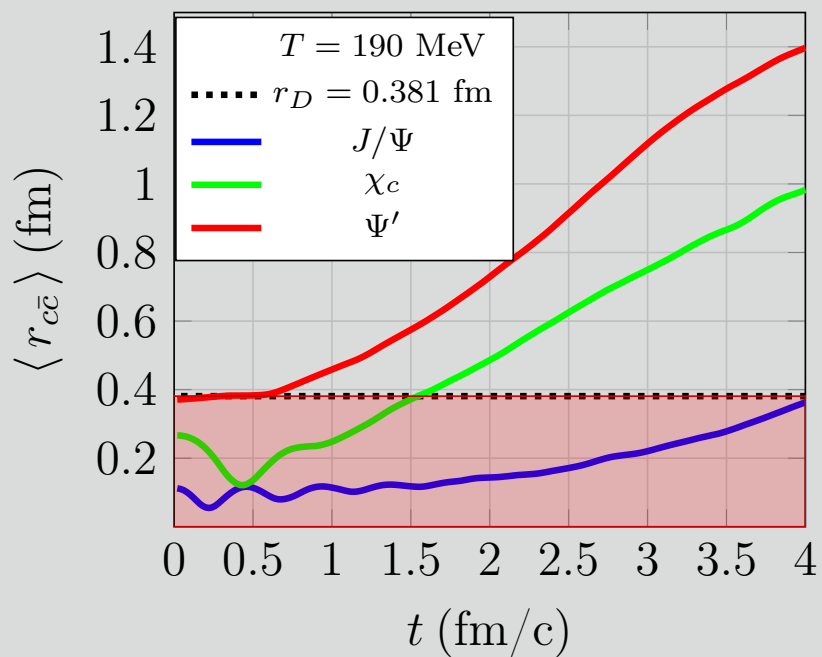
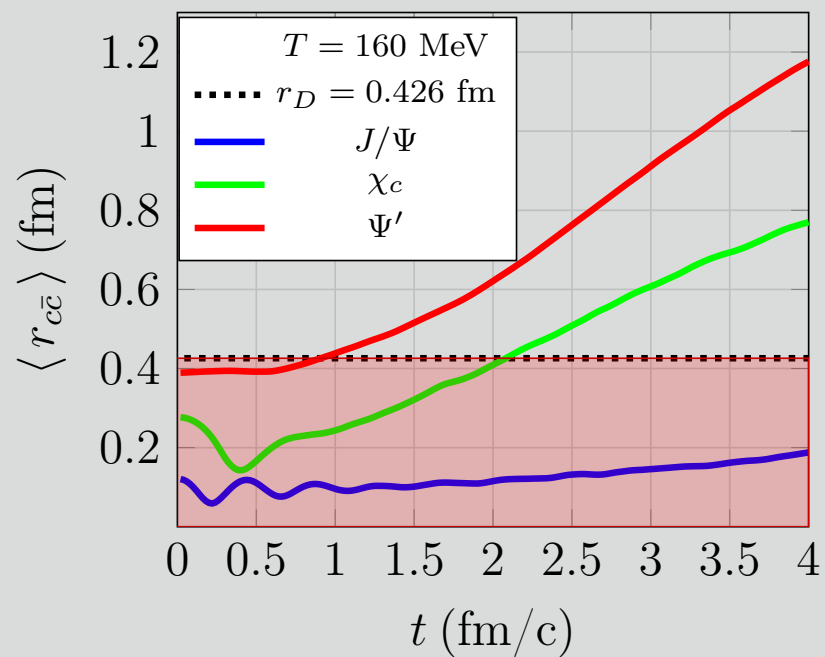
Diffusion constant

$$\mathcal{D} = \frac{T}{M\gamma}$$

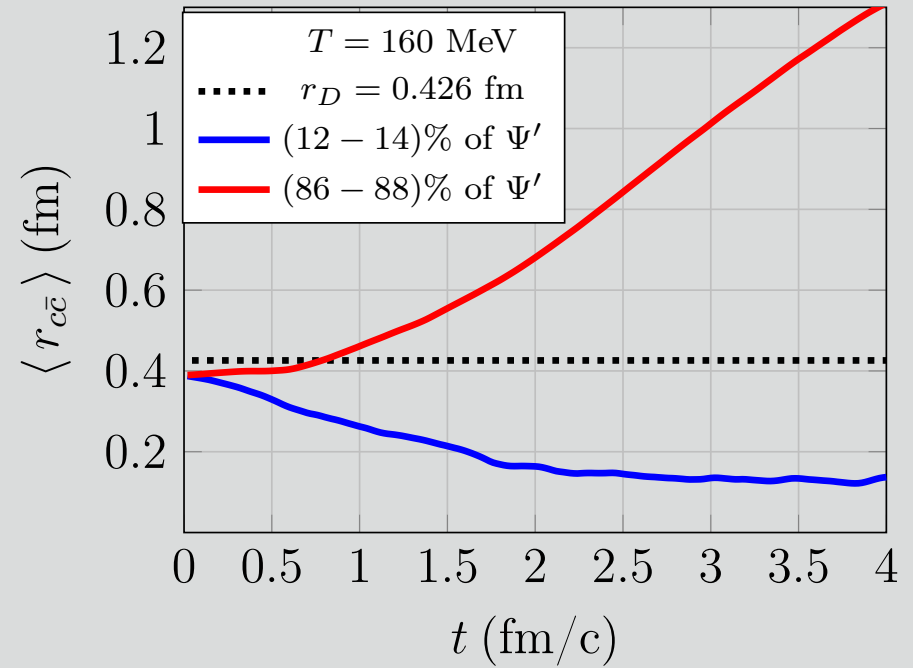
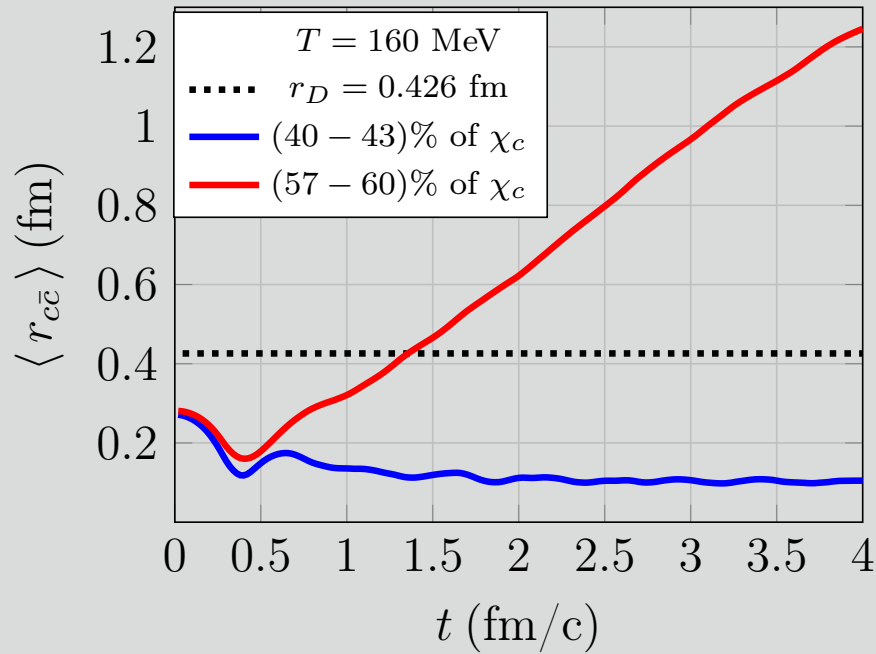




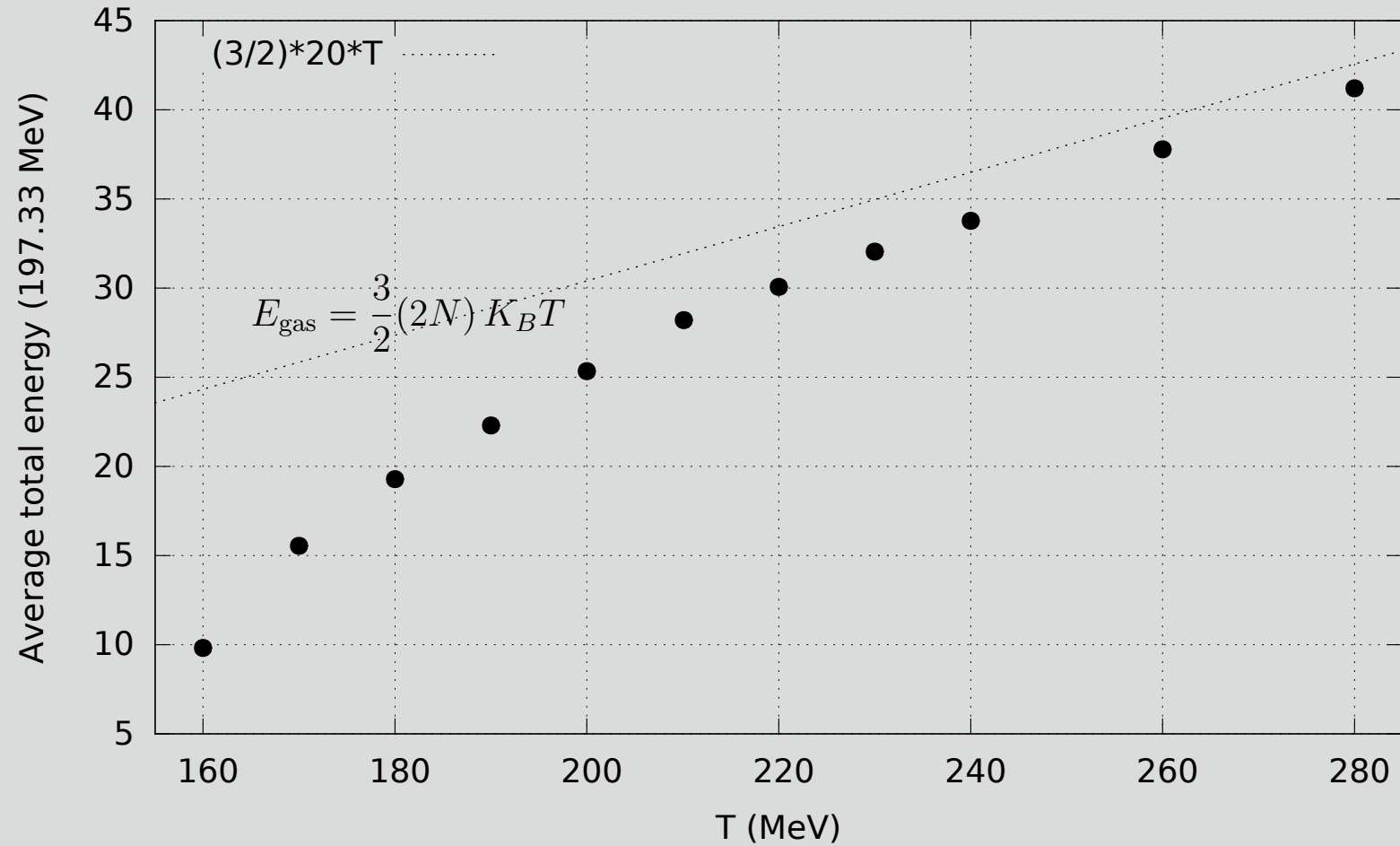
Sequential suppression



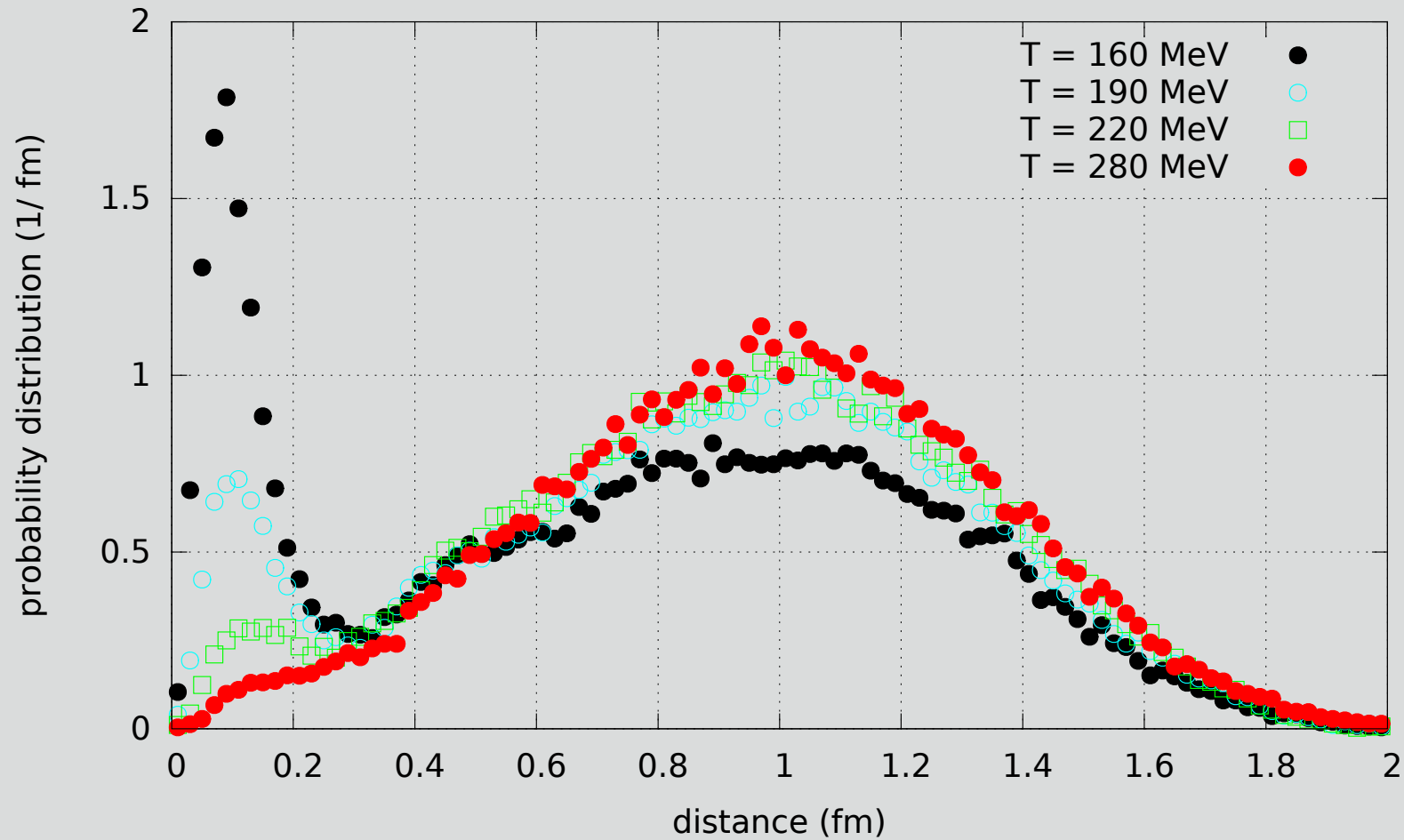
Effective feed down from excited states!



10 pairs in plasma

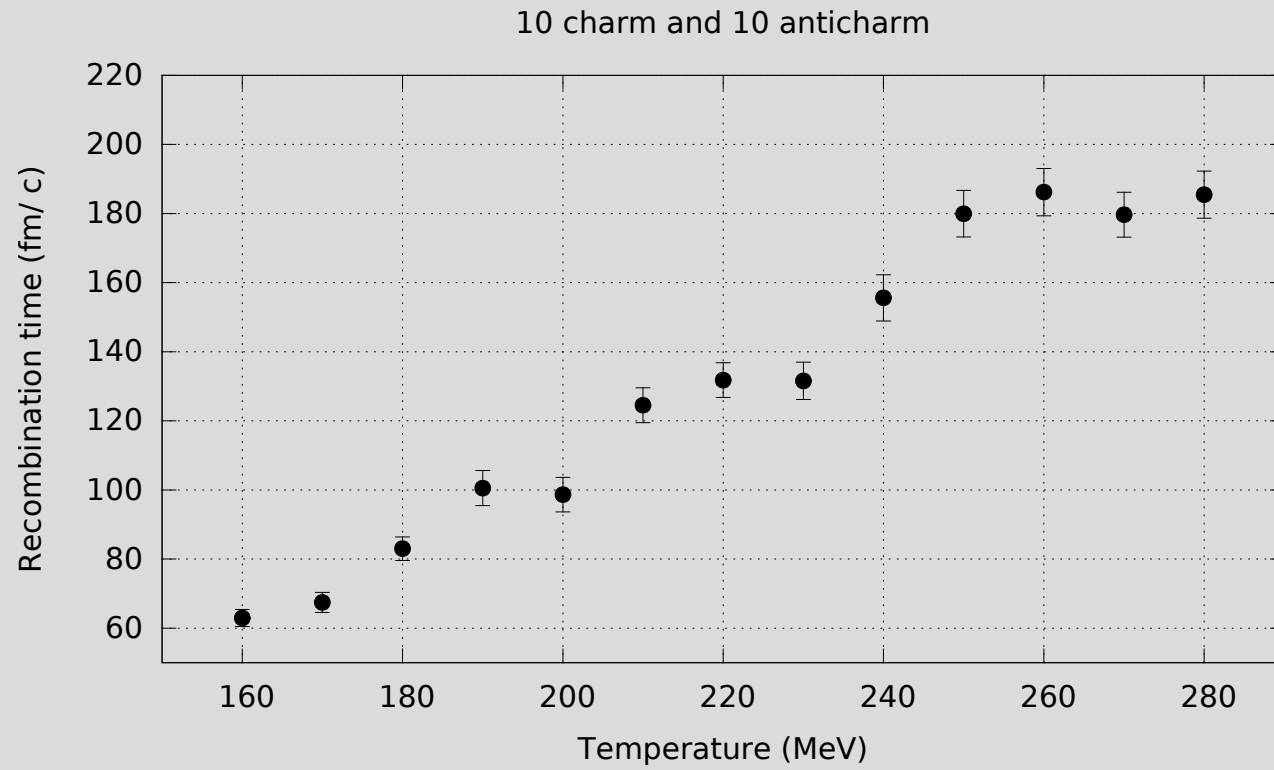


Probability distribution of distance to nearest neighbor

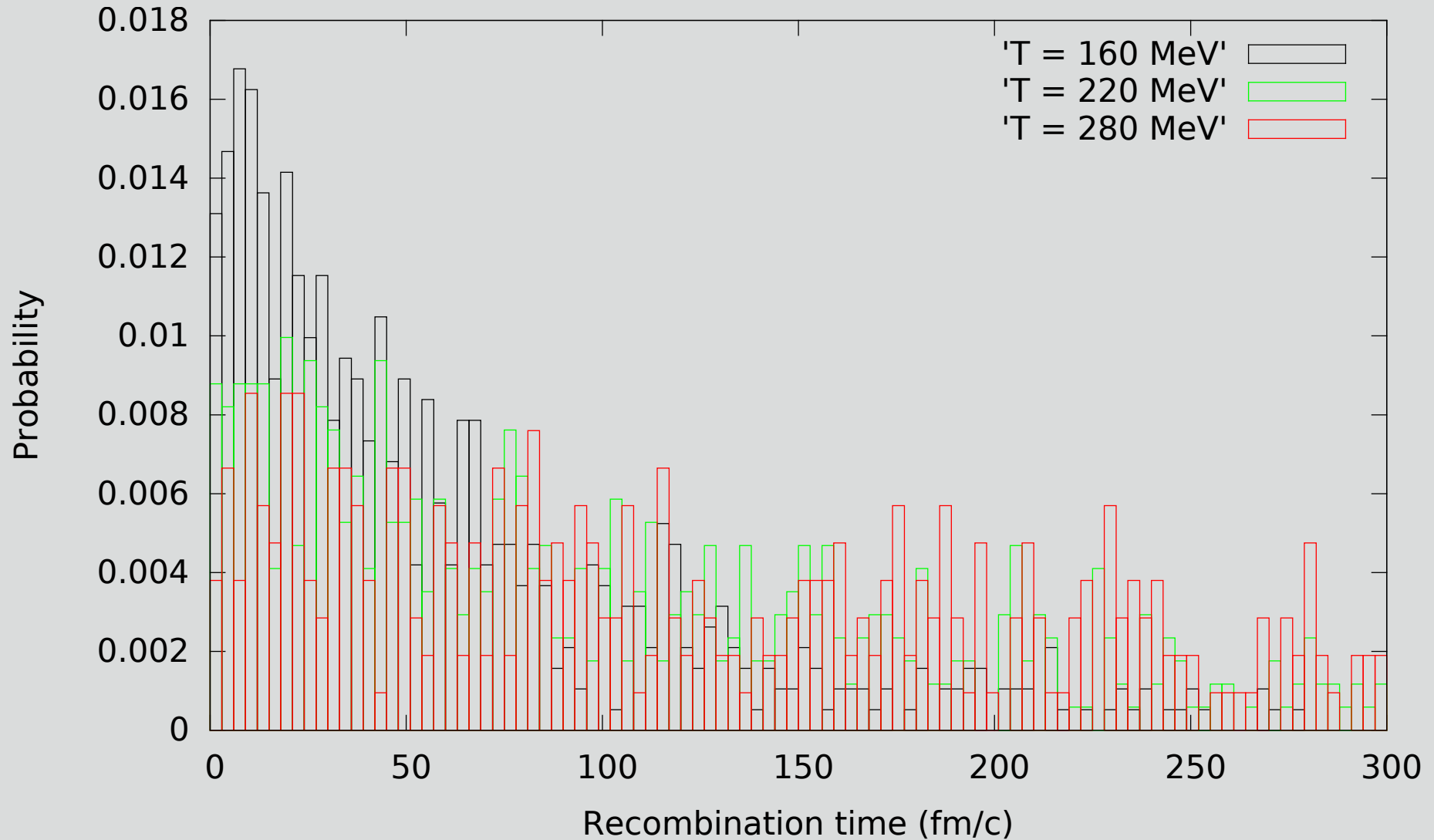


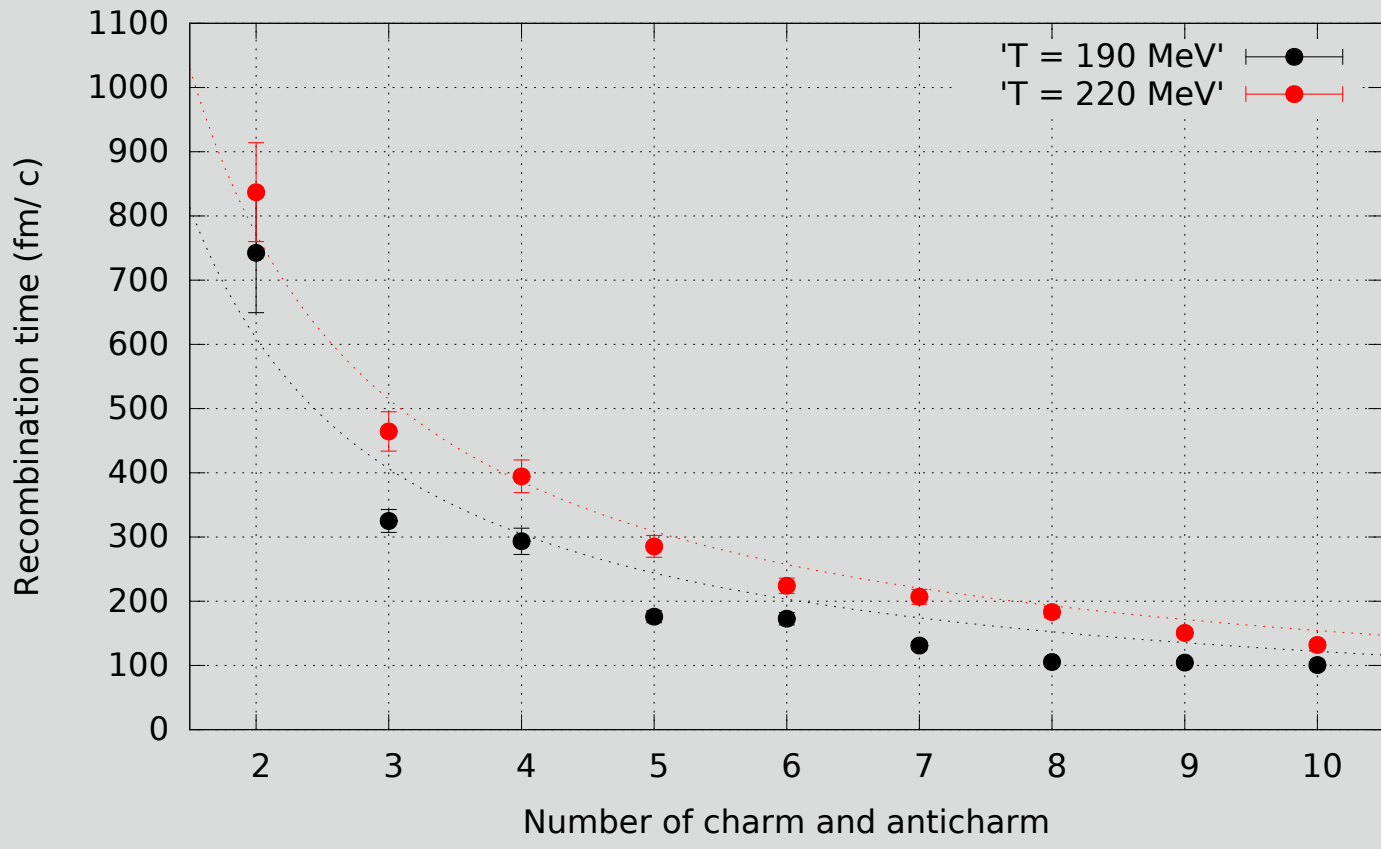
$$P_{q\bar{q}}^{\text{ideal}}(r) = \frac{3}{a} \left(\frac{r}{a}\right)^2 \left(1 - \left(\frac{r}{a}\right)^3 \frac{1}{N}\right)^{N-1} \stackrel{N \gg 1}{\approx} \frac{3}{a} \left(\frac{r}{a}\right)^2 e^{-(r/a)^3}$$

Recombination time

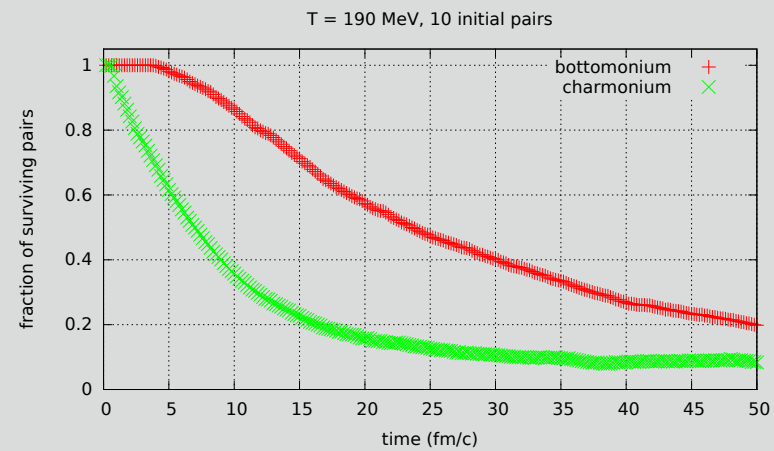
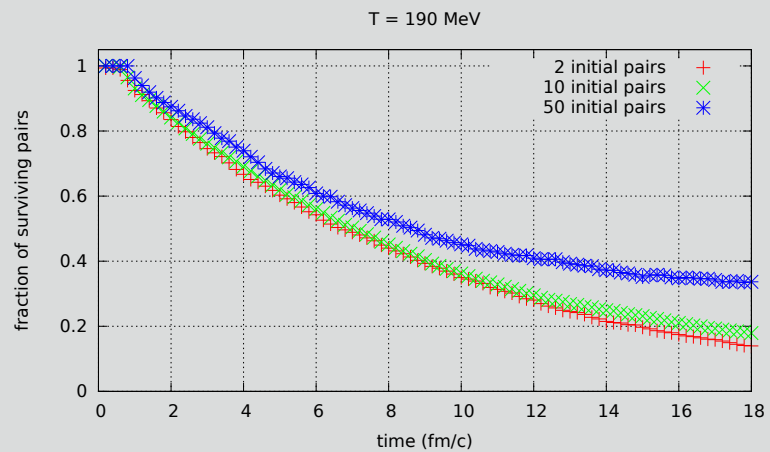
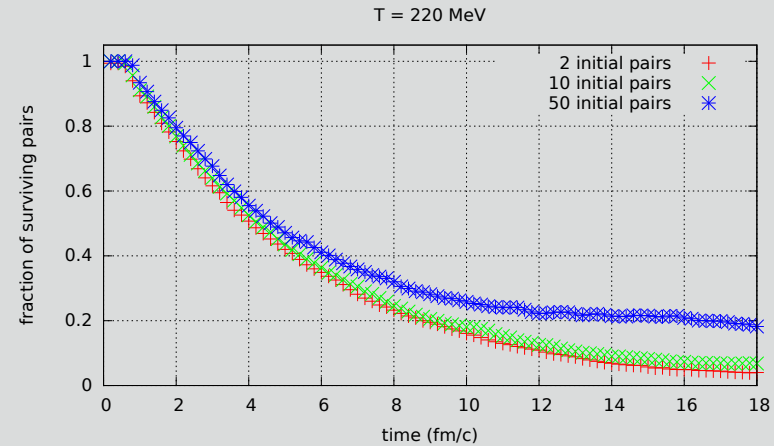
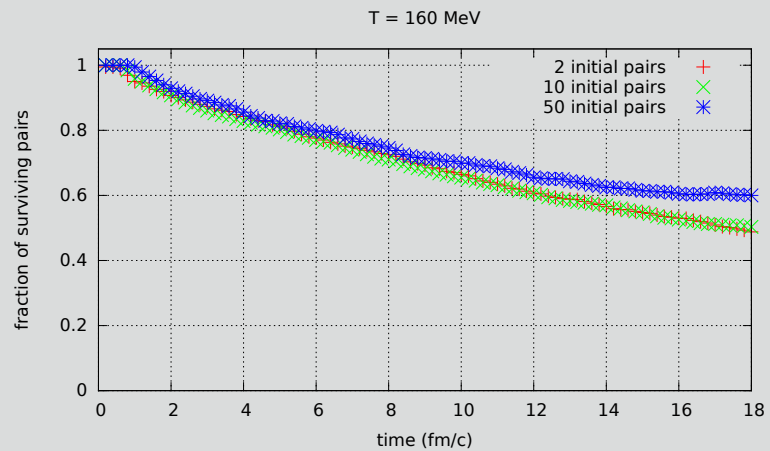


Distribution of recombination times

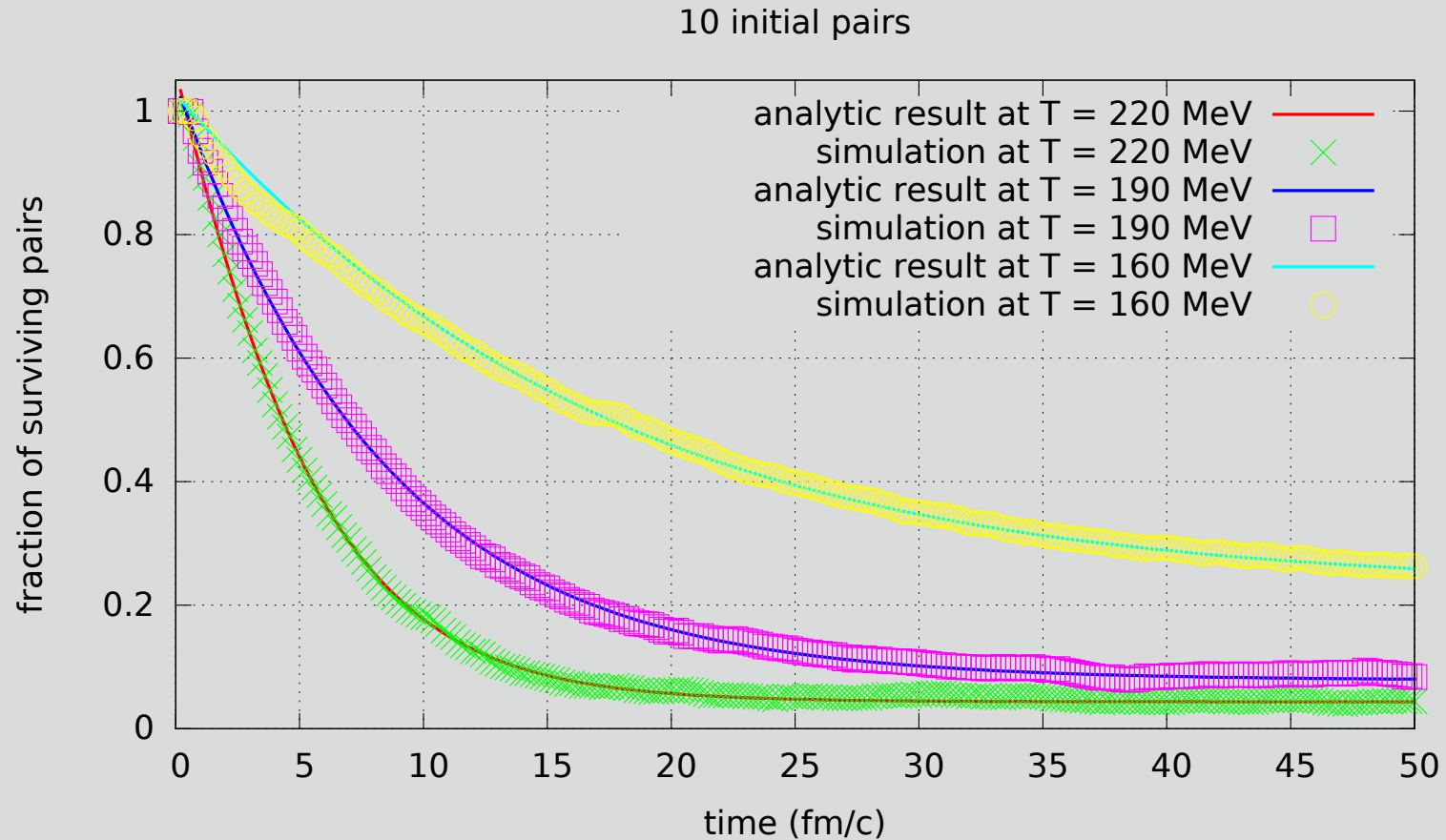




Dissociation/recombination



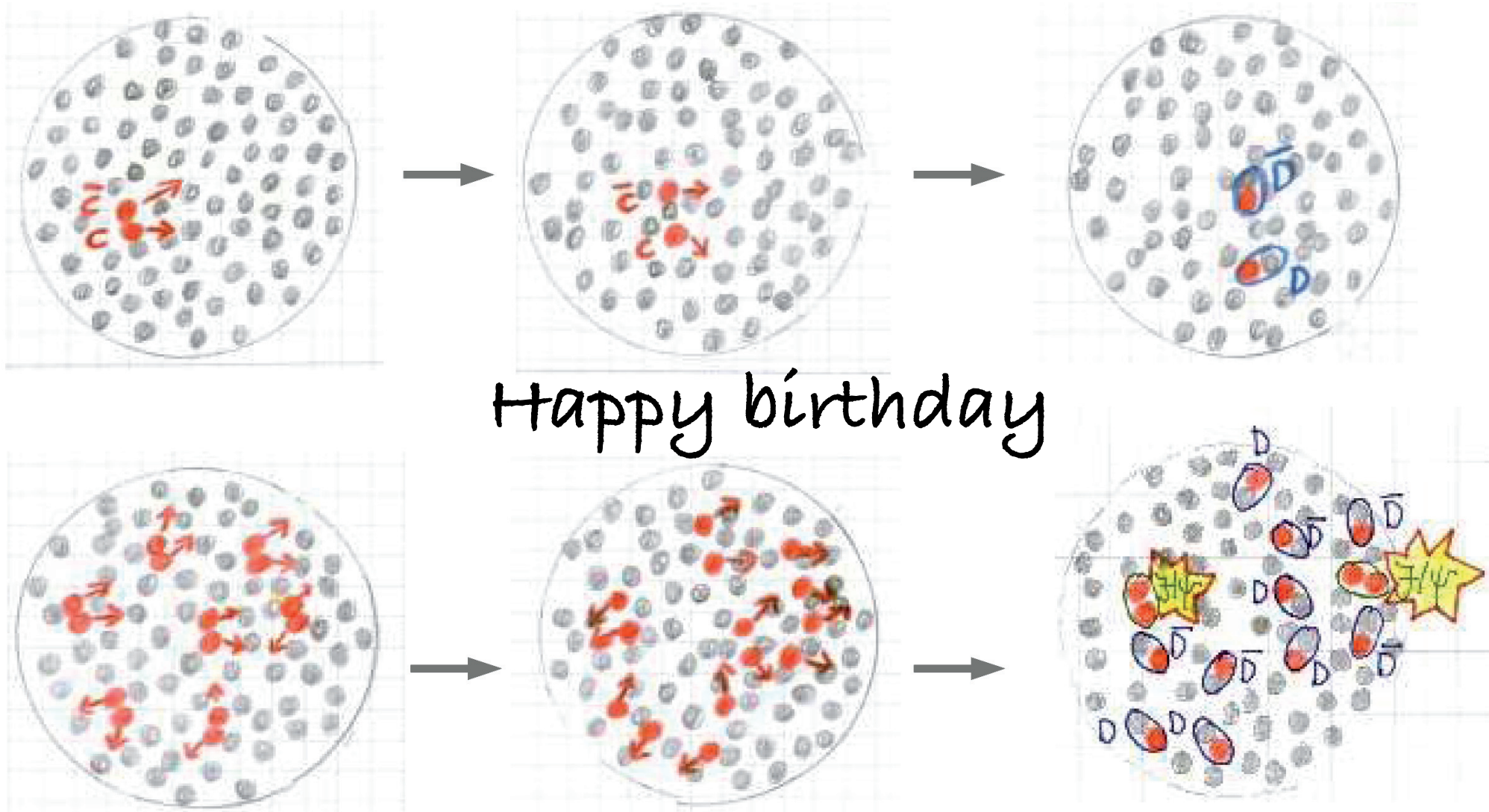
Evolution of population of bound states
is well described by a simple rate equation



$$\frac{dN(t)}{dt} = -\lambda_D N(t) + \lambda_R N_q(t) N_{\bar{q}}(t)$$

Summary

- a simple idea... but a difficult many-body problem
- very beautiful new data invites further theoretical efforts
- a consistent treatment of the full dynamics of heavy quarks in a quark-gluon plasma is within reach

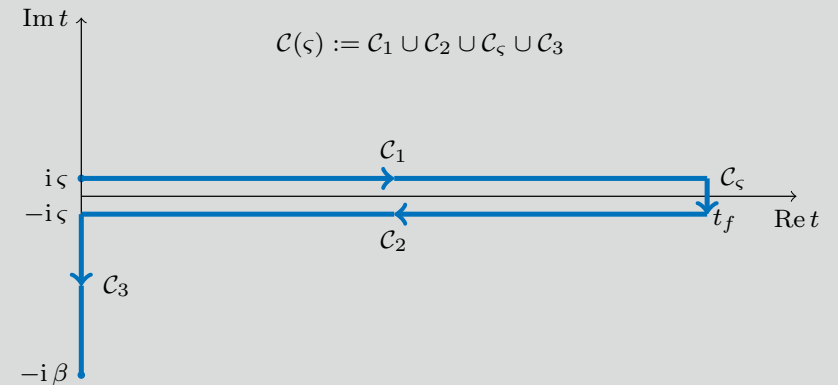


Happy birthday

Path integral formulation

$$(Q_f, t_f | Q_i, t_i) = \int_{x(t_i)=Q_i}^{x(t_f)=Q_f} [\mathcal{D}x(t)] \exp \left[i \int_{t_i}^{t_f} dt \left(\frac{1}{2} M \dot{x}^2 - V(x) \right) \right]$$

$$P(Q_f, t_f | Q_i, t_i) = |(Q_f, t_f | Q_i, t_i)|^2$$



$$P(Q_f, t_f | Q_i, t_i) = \int_C [\mathcal{D}x(t)] \exp \left[i \int_C dt_C \left(\frac{1}{2} M \dot{x}^2 - V(x) \right) \right]$$

$$V(x) = gA_0(x)$$

Some charmonium properties

| state | J/ψ | χ_c | ψ' | Υ | χ_b | Υ' | χ'_b | Υ'' |
|------------------|----------|----------|---------|------------|----------|-------------|-----------|--------------|
| mass [GeV] | 3.10 | 3.53 | 3.68 | 9.46 | 9.99 | 10.02 | 10.26 | 10.36 |
| ΔE [GeV] | 0.64 | 0.20 | 0.05 | 1.10 | 0.67 | 0.54 | 0.31 | 0.20 |
| ΔM [GeV] | 0.02 | -0.03 | 0.03 | 0.06 | -0.06 | -0.06 | -0.08 | -0.07 |
| r_0 [fm] | 0.50 | 0.72 | 0.90 | 0.28 | 0.44 | 0.56 | 0.68 | 0.78 |

ΔE is binding energy

(from H. Satz, hep-ph/0602245)

Infinite mass limit (single heavy quark)

$$G^>(t, \mathbf{r}) = \delta(\mathbf{r}) e^{-iMt} e^{iF(t)} \quad F(t) = \frac{g^2}{2} \int_0^t dt' \int_0^{t'} dt'' D(t' - t'', 0)$$

long time limit is determined by static response of plasma

$$F(t) \simeq \frac{g^2}{2} t D(\omega = 0, \mathbf{r} = 0) \equiv -t V_{opt}$$

'Optical potential'

$$\begin{aligned} V_{opt} &\equiv -\frac{g^2}{2} \int \frac{d\mathbf{q}}{(2\pi)^3} D(\omega = 0, \mathbf{q}) \\ &= \frac{g^2}{2} \int \frac{d\mathbf{q}}{(2\pi)^3} \left[\frac{1}{\mathbf{q}^2 + m_D^2} - \frac{1}{\mathbf{q}^2} - i \frac{\pi m_D^2 T}{|\mathbf{q}|(\mathbf{q}^2 + m_D^2)^2} \right] \\ &= -\frac{\alpha}{2} m_D - i \frac{\alpha T}{2}, \end{aligned}$$

Quark antiquark pair

Large time behaviour ($t m_D \gg 1$)

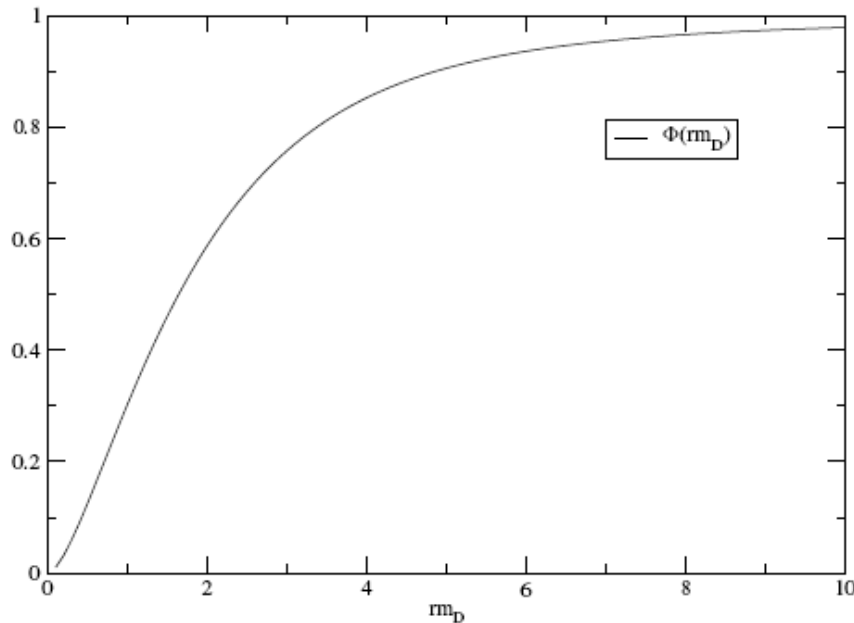
$$\bar{G}(t, r_1 - r_2) \underset{t \rightarrow \infty}{\sim} \exp[-iV_{\text{eff}}(r_1 - r_2)t]$$

V_{eff} has real and imaginary part (*)

$$\begin{aligned} V_{\text{eff}}(r_1 - r_2) &\equiv g^2 \int \frac{dq}{(2\pi)^3} (1 - e^{iq \cdot (r_1 - r_2)}) D_{00}(\omega = 0, q) \\ &= g^2 \int \frac{dq}{(2\pi)^3} (1 - e^{iq \cdot (r_1 - r_2)}) \left[\frac{1}{q^2 + m_D^2} - i \frac{\pi m_D^2 T}{|q|(q^2 + m_D^2)^2} \right] \\ &= -\frac{g^2}{4\pi} \left[m_D + \frac{e^{-m_D r}}{r} \right] - i \frac{g^2 T}{4\pi} \phi(m_D r) \end{aligned}$$

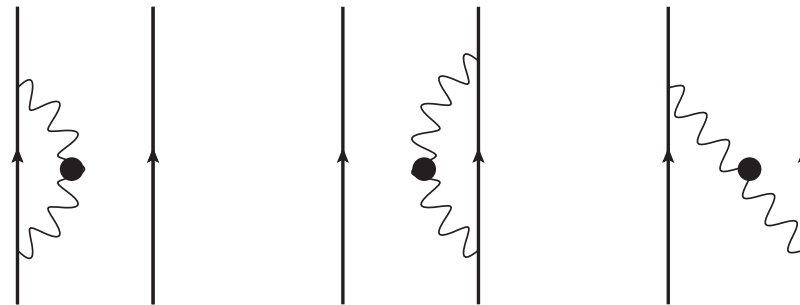
(*first observed by M. Laine et al hep-ph/0611300)

The imaginary part of the effective potential

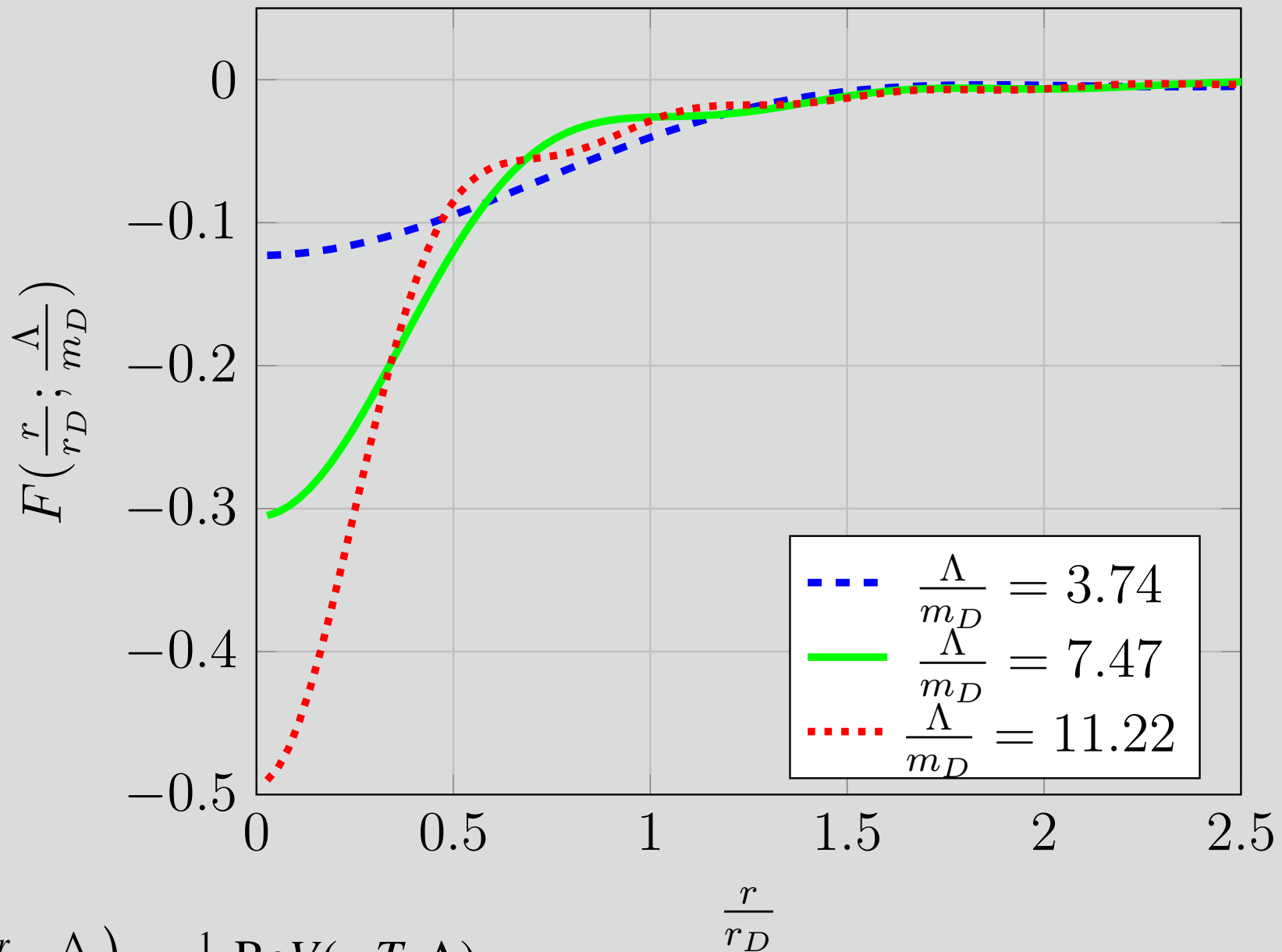


At large distance the imaginary part is twice the damping rate of the heavy quark

At short distance, interference produces cancellation: a small dipole does not "see" the electric field fluctuations.



Regularized Coulomb potential



$$F\left(\frac{r}{r_D}, \frac{\Lambda}{m_D}\right) = \frac{1}{m_D} \text{Re}V(r, T, \Lambda)$$