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Cosmological Inflation

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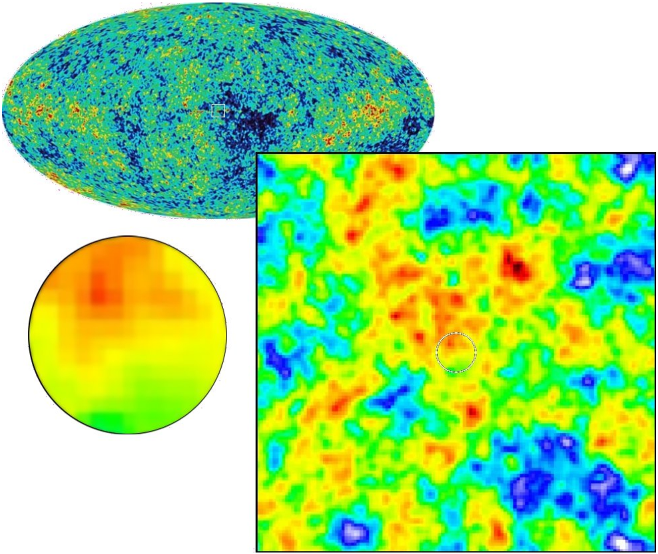
Causality and the need for inflation

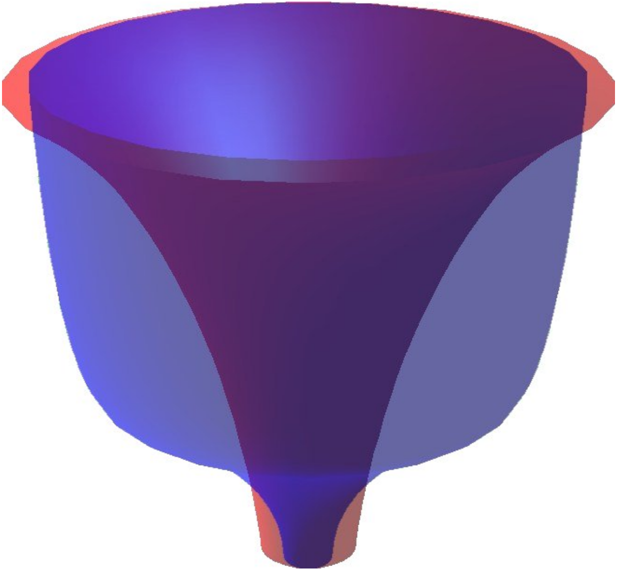


Cosmological standard model

- General relativity
- Symmetry assumptions: spatial homogeneity and isotropy
- Only degree of freedom is the scale factor $a(t)$, controlled by Friedman's equations
- Hubble function $H(a) := \dot{a}/a$ sets inverse time scale (Hubble time)
- Comoving Hubble radius, causality:

$$\frac{c}{aH} < \lambda = \frac{2\pi}{k}$$

Solution for causality



Solution for causality

- Comoving Hubble radius needs to *shrink* for causal contact
- By Friedman's equations, this implies

$$P < \frac{\rho c^2}{3}$$

- Postulate scalar field φ (inflaton) with Lagrangian

$$\mathcal{L} = \partial_\mu \varphi \partial^\mu \varphi - V(\varphi)$$

- Leads to

$$\frac{P}{\rho} = \frac{\dot{\varphi}^2/2 - V}{\dot{\varphi}^2/2 + V}$$

- Negative pressure possible if $\dot{\varphi}^2/2 < V$

Slow-roll inflation

- Inflation requires "slow roll": $\dot{\varphi}^2 \ll V$ for long enough
- Potential slope needs to be small,

$$\epsilon_V := \frac{M_{\mathrm{Pl}}^2}{2V^2}(V')^2 \ll 1 \,,$$

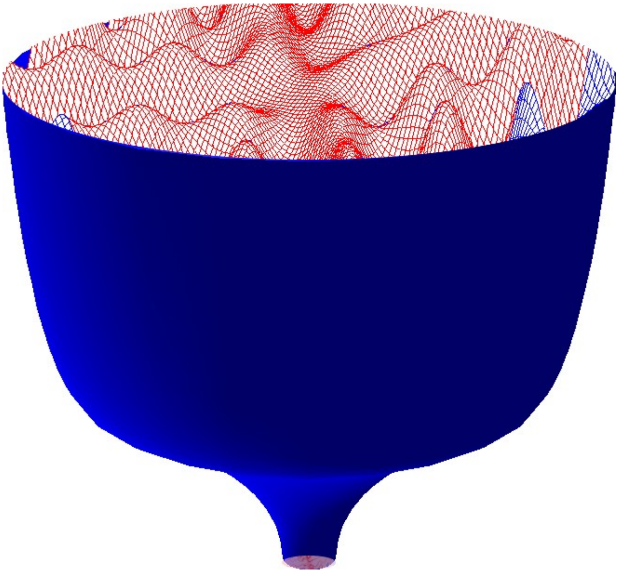
- for sufficiently long time,

$$\eta_V := \frac{M_{\mathrm{Pl}}^2}{V} V'' \ll 1$$

- By Friedman's equations, leads to

$$H^2 \approx \frac{V}{3M_{\mathrm{Pl}}}$$

Structures from Inflation



Structures from Inflation

- Vacuum fluctuations $\delta\varphi$ cause curvature fluctuations,

$$\mathcal{R} = -H\frac{\delta\varphi}{\varphi}$$

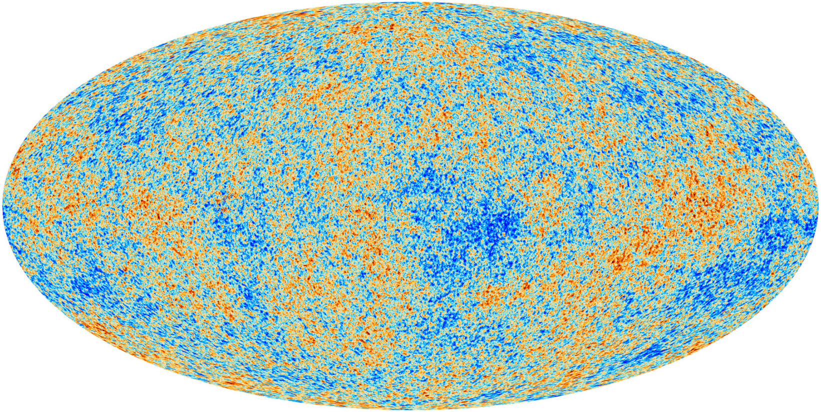
- At the same time, tensor fluctuations $h_k^{+,\times}$ are produced
- Power spectra

$$\mathcal{P}_\mathcal{R} = A_s\left(\frac{k}{k_*}\right)^{n_s-1+\mathrm{d}\ln n_s/\mathrm{d}\ln k\,\ln k+\ldots}$$

$$\mathcal{P}_t = A_t\left(\frac{k}{k_*}\right)^{n_t+\mathrm{d}\ln n_t/\mathrm{d}\ln k\,\ln k+\ldots}$$

for scalar and for tensor modes can be predicted

Observables



Observables

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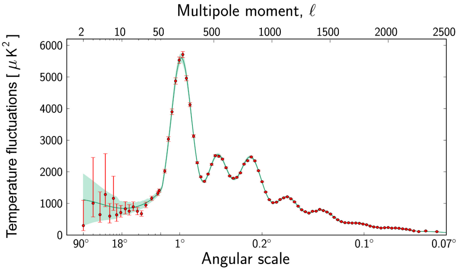
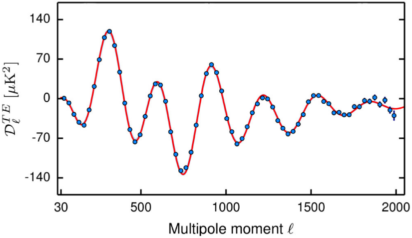
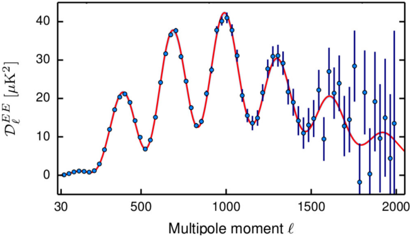






Observables, slow-roll parameters

- Observables:

$$A_s, A_t, n_s, n_t, \frac{\mathrm{d}\ln n_s}{\mathrm{d}\ln k}, r := \frac{\mathcal{P}_t}{\mathcal{P}_\mathcal{R}}$$

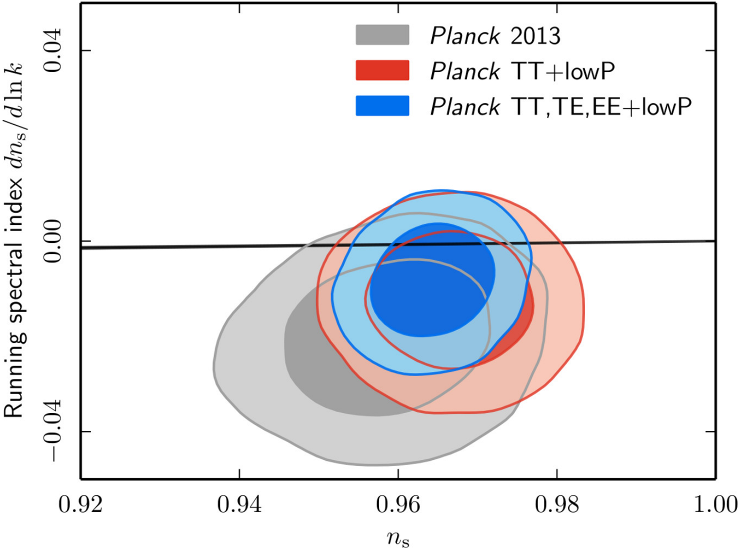
- They are related to the slow-roll parameters:

$$A_s \approx \frac{V}{24\pi^2 M_{\rm Pl}^4 \epsilon_V}\ , \qquad A_t \approx \frac{2V}{3\pi^2 M_{\rm Pl}^4}$$

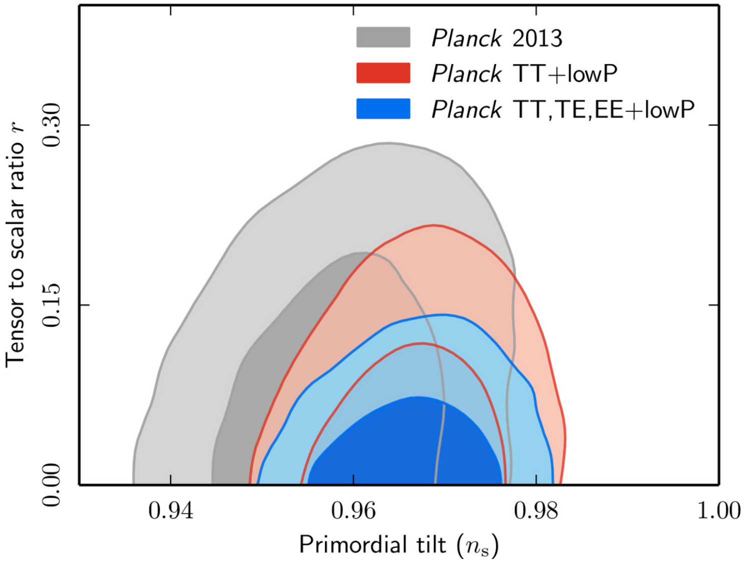
$$n_s \approx 1 - 6\epsilon_V + 2\eta_V\ , \qquad n_t \approx -2\epsilon_V$$

$$\frac{\mathrm{d}\ln n_s}{\mathrm{d}\ln k} \approx 16\epsilon_V\eta_V - 24\epsilon_V^2 - 2\xi_V^2\ , \qquad r \approx 16\epsilon_V \approx -8n_t$$

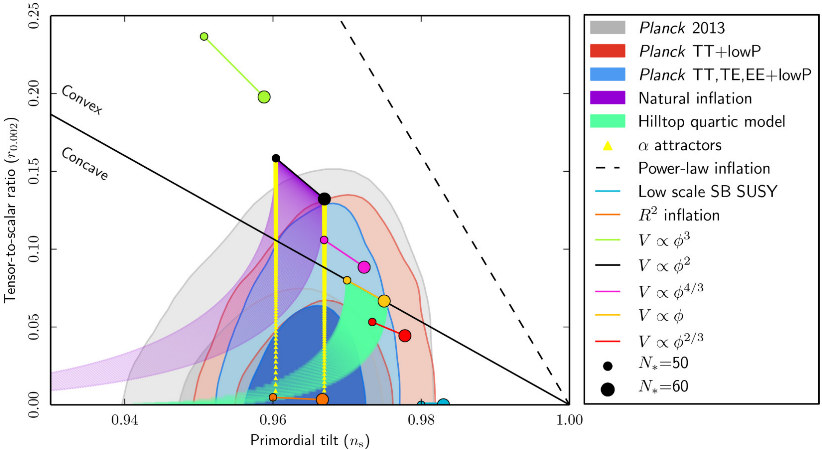
Results by Planck



Results by Planck



Results by Planck



Results by Planck

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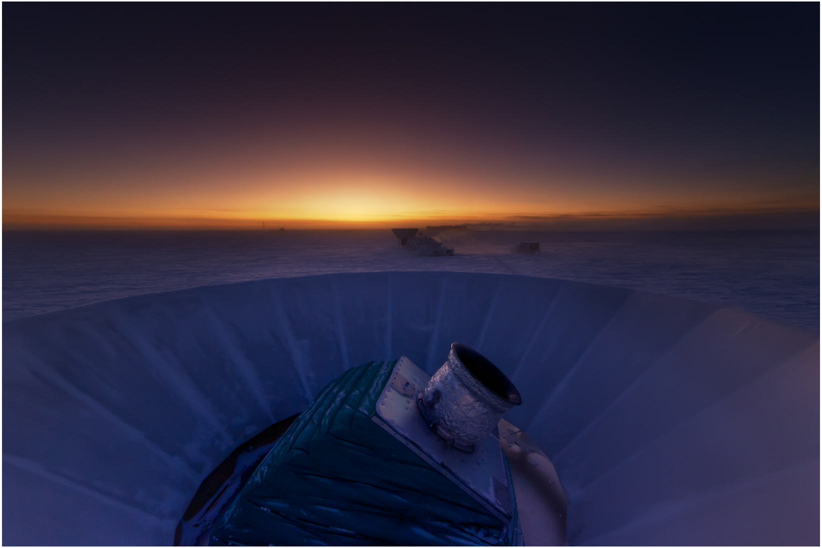
- Best constrains so far, combining temperature and polarisation information with gravitational lensing:

$$n_s = 0.9677 \pm 0.0060 \quad (68\ \%)$$
$$\frac{\mathrm{d}\ln n_s}{\mathrm{d}\ln k} = -0.0033 \pm 0.0074$$
$$r < 0.11$$

- Slow-roll parameters:

$$\epsilon_V < 0.011$$
$$\eta_V = -0.0092^{+0.0074}_{-0.0127}$$
$$\xi_V = 0.0044^{+0.0037}_{-0.0050}$$

Gravitational waves?



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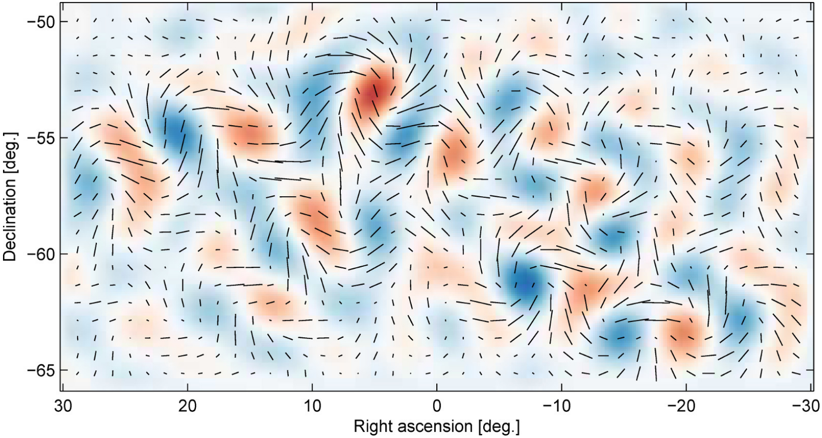
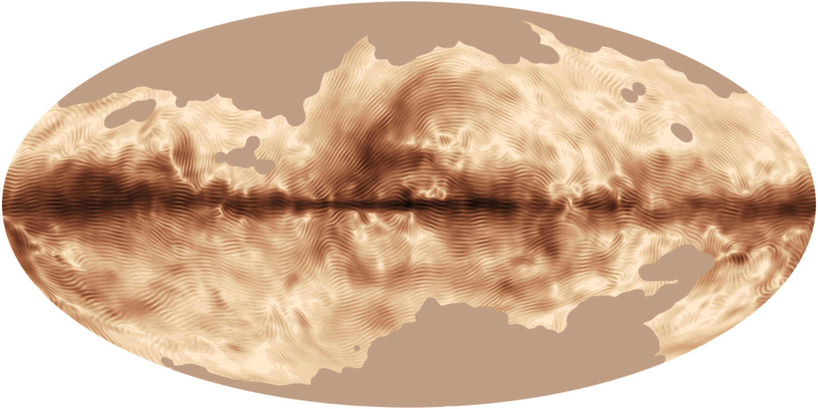


Gravitational waves?



Gravitational waves?