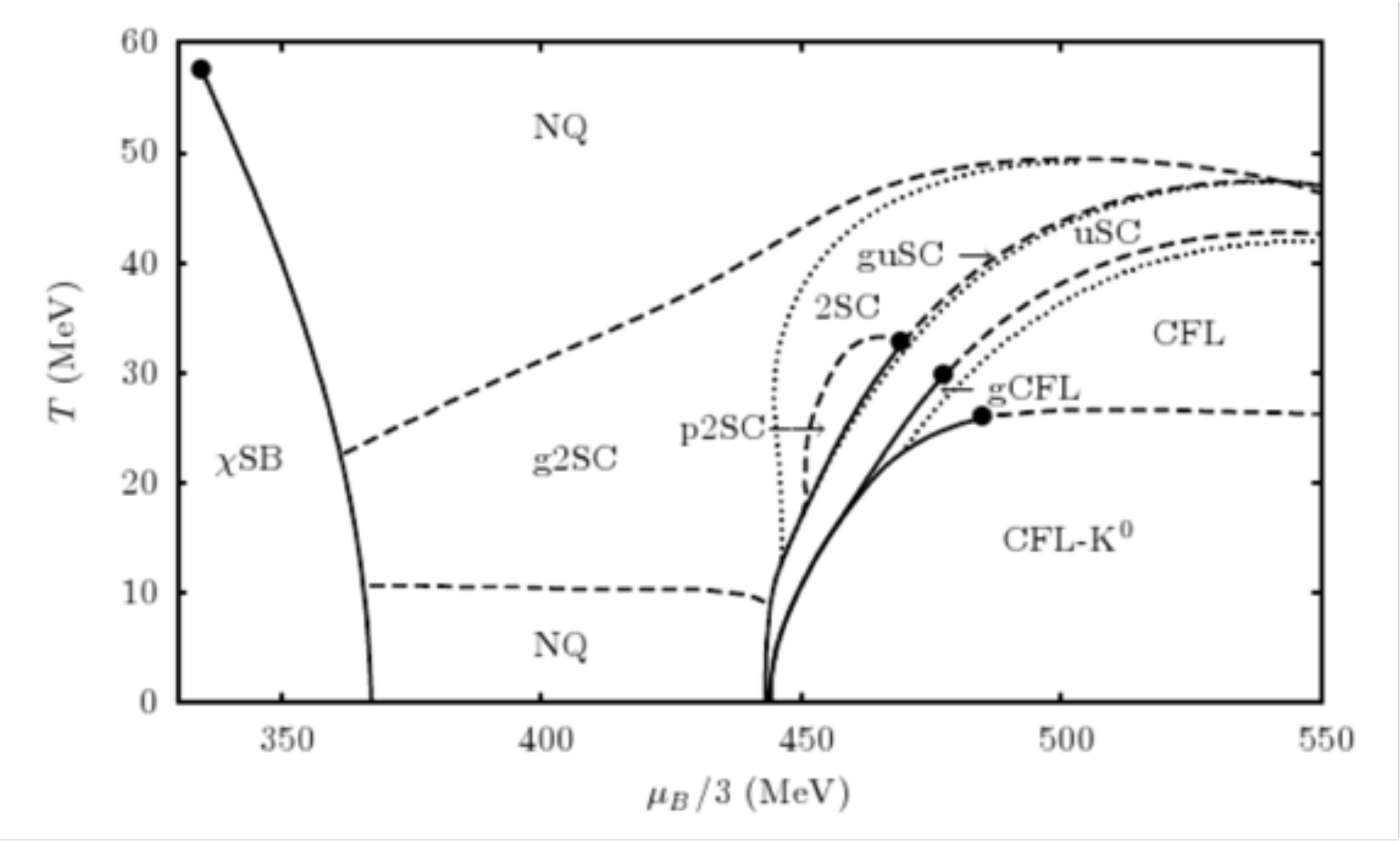
Fluctuations and the Phase structure of QCD

- The Phase(s) of QCD
- Remarks on the Phase diagram
- Fluctuations and correlations (Theory vs. Exp)

Lots of Phases ….

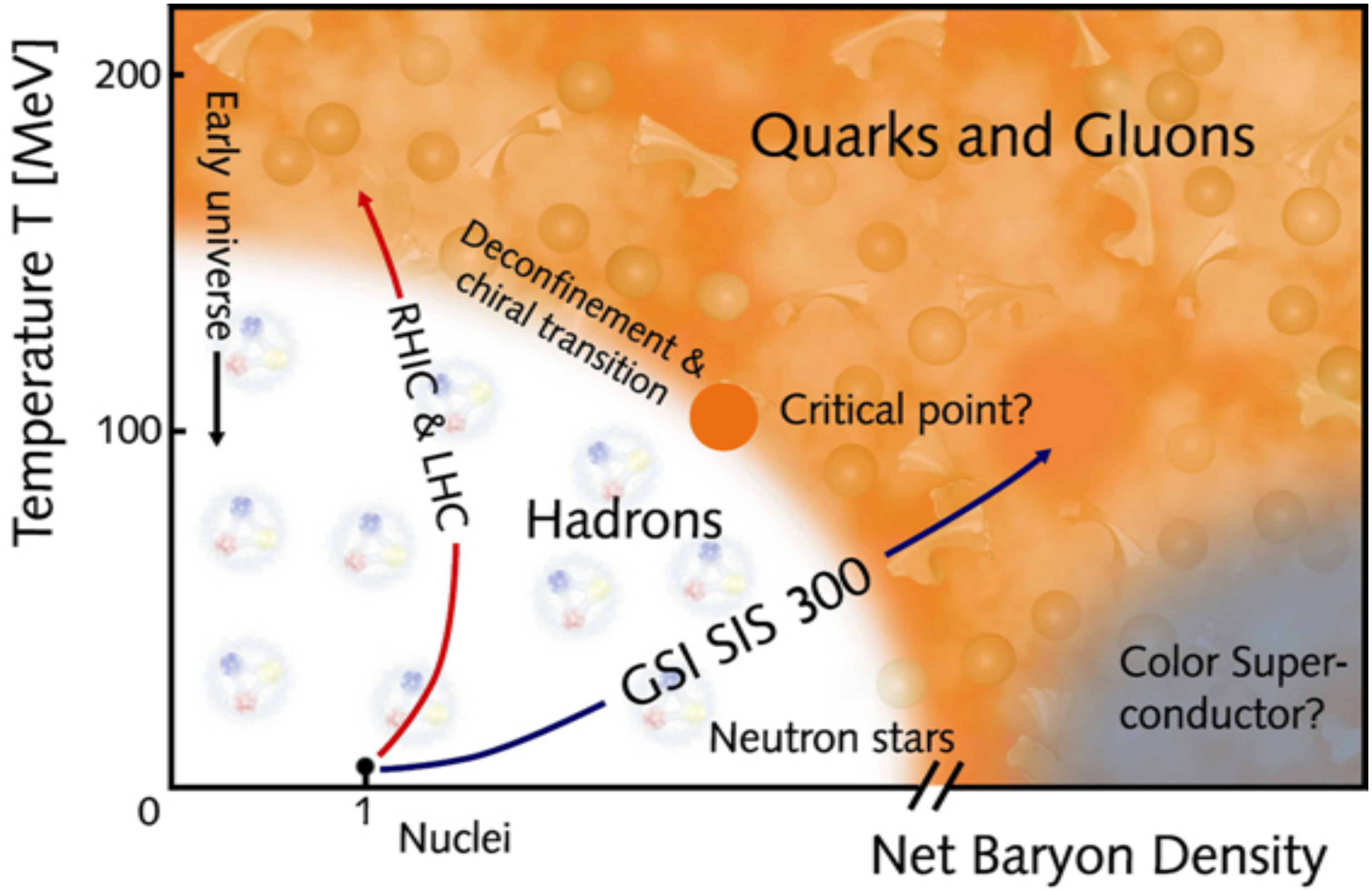


H. Warringa hep-ph/0606063

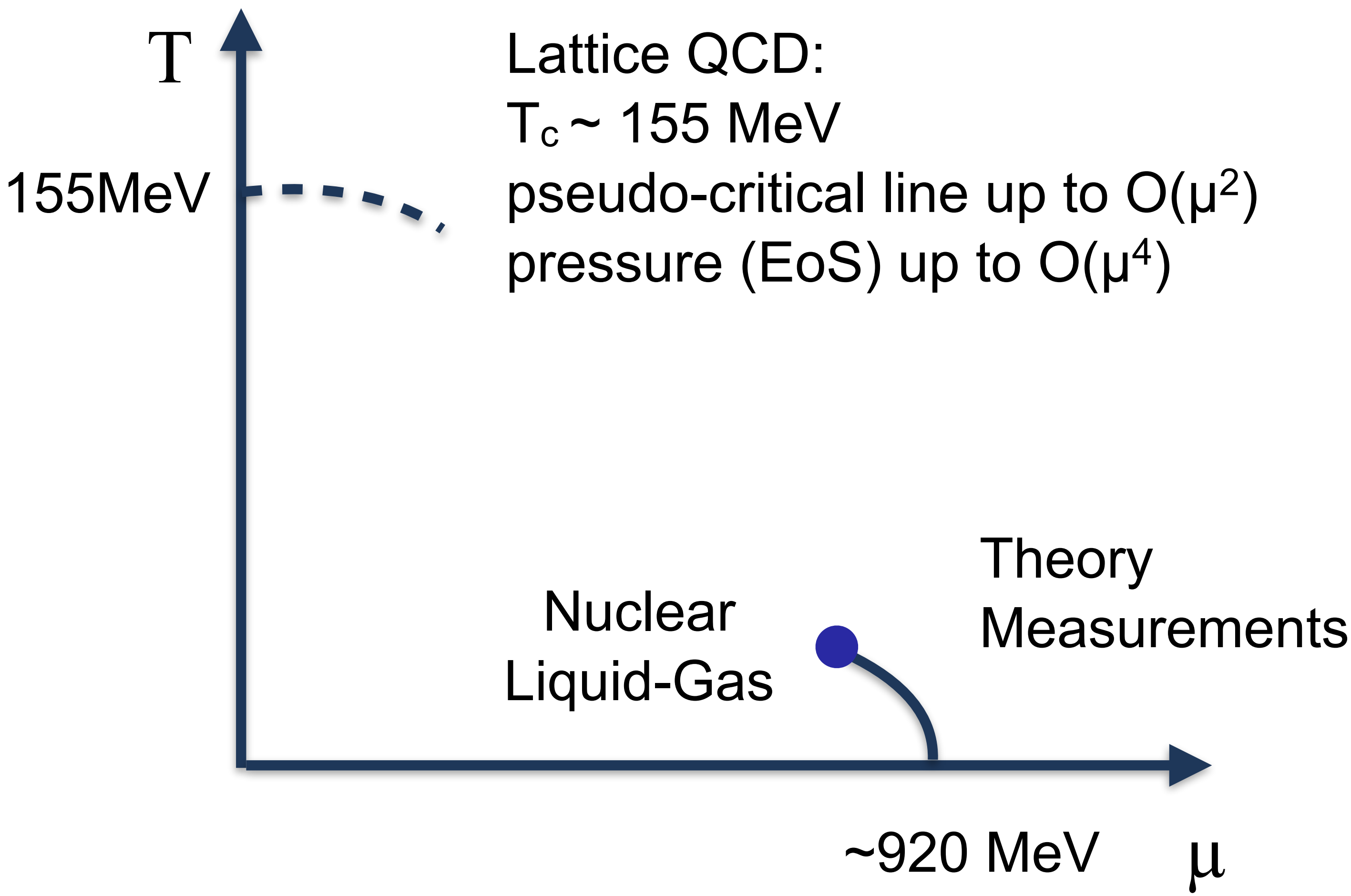
At least theoretically……

The Face of the QGP

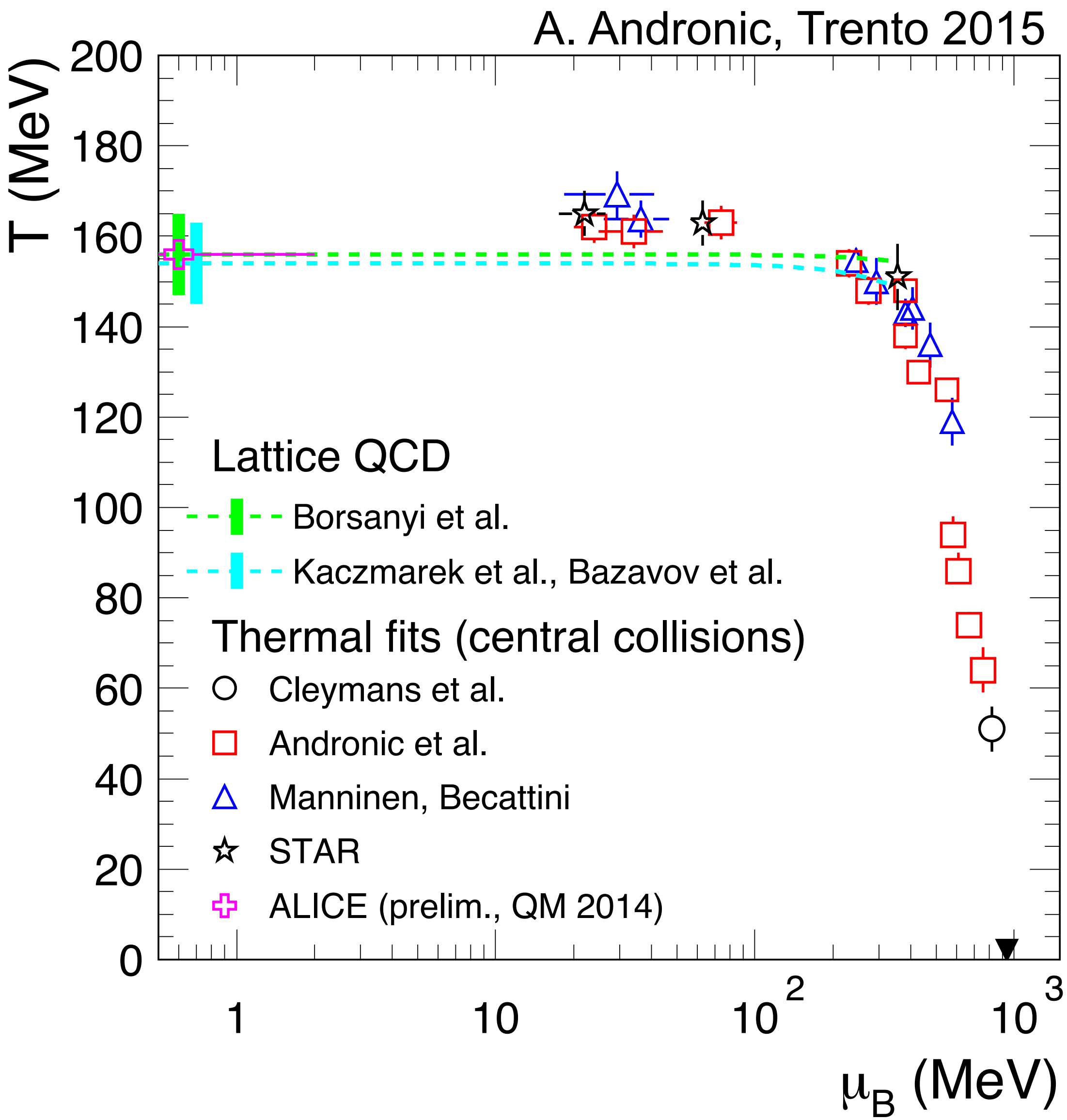
The QCD Phase diagram



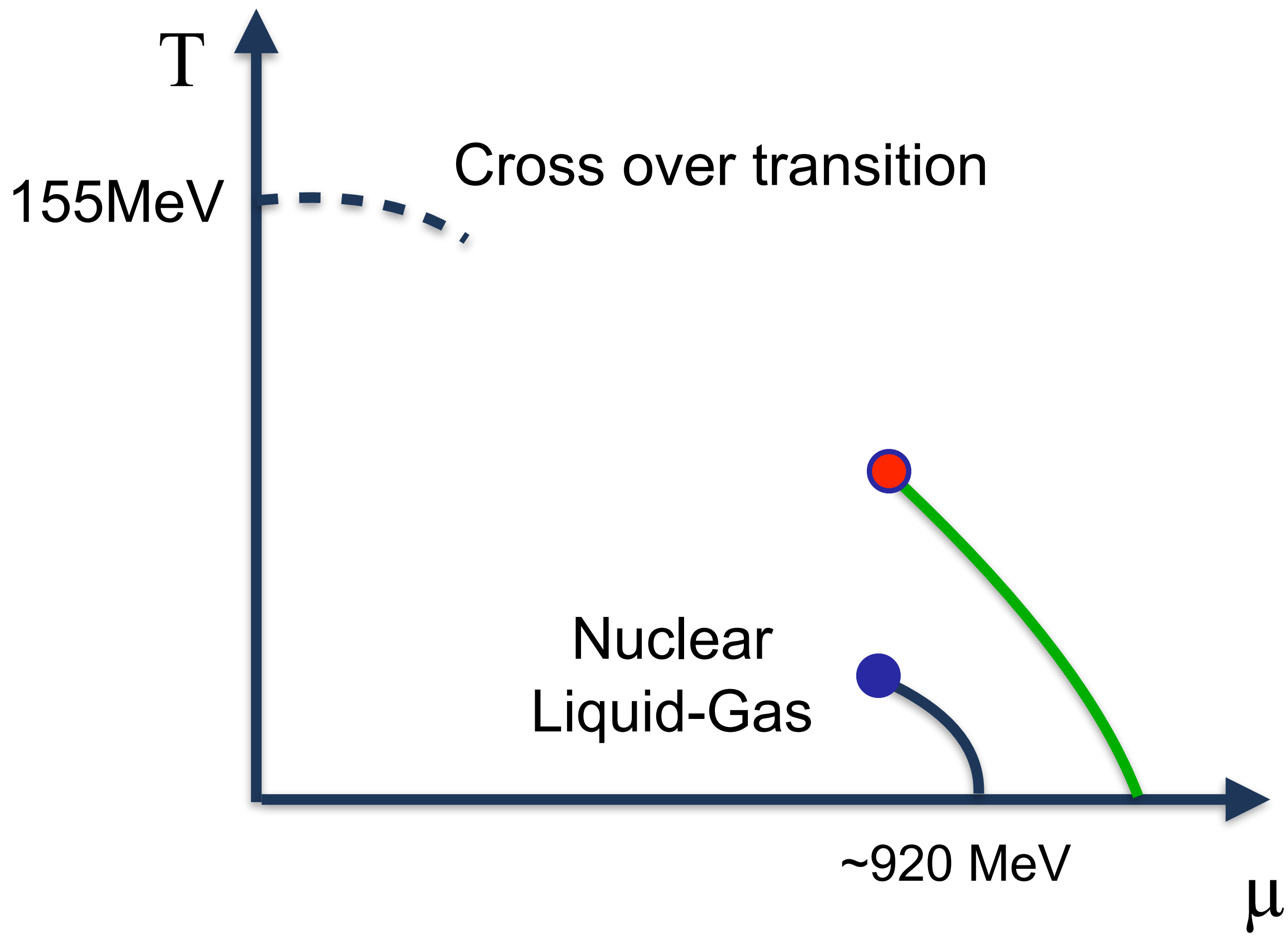
What we know about the Phase Diagram



What we know: Chemical freeze out

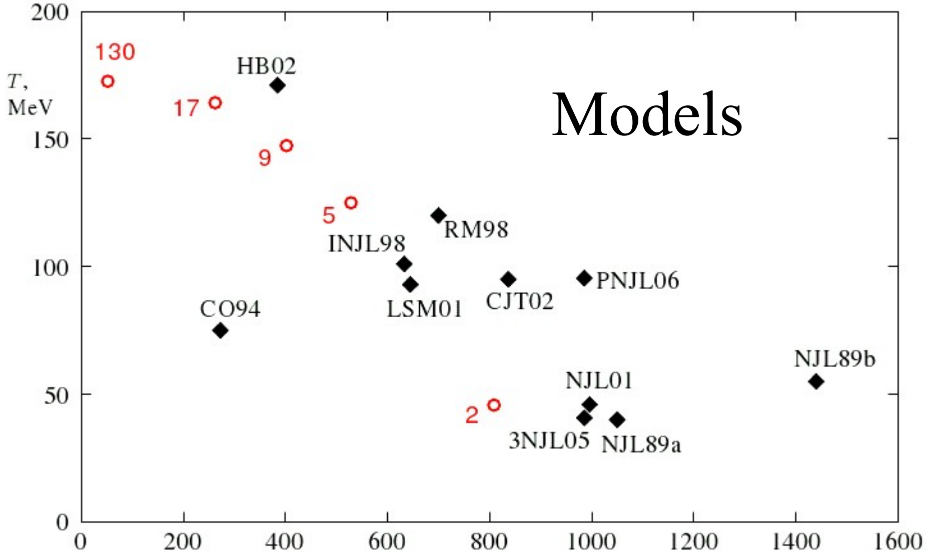


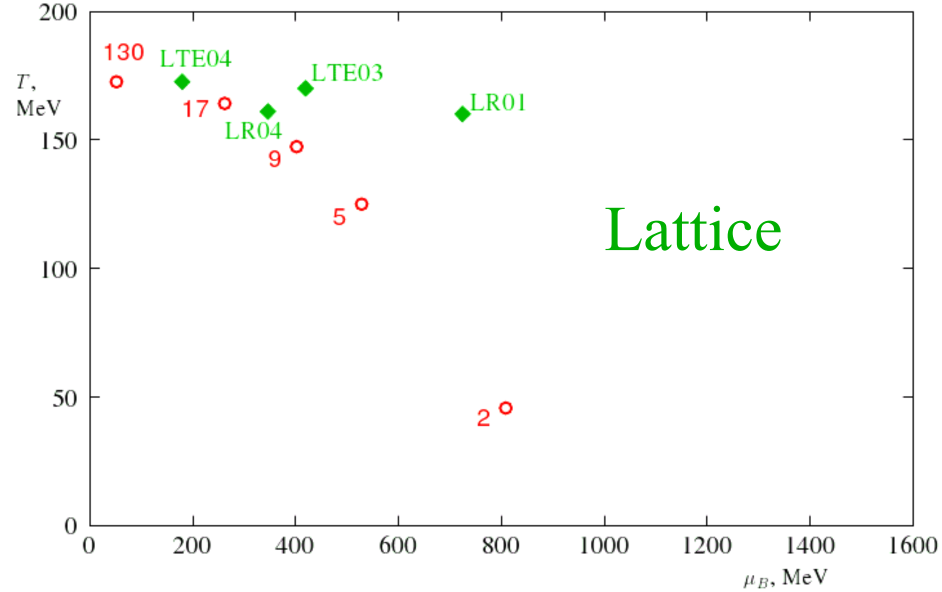
What we “hope” for



Is there a critical point?

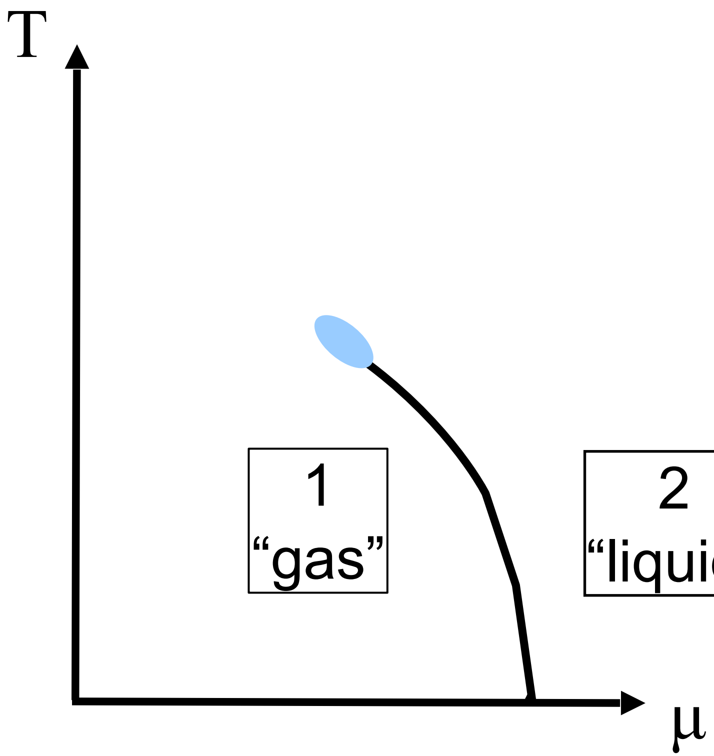
Is there a critical point?



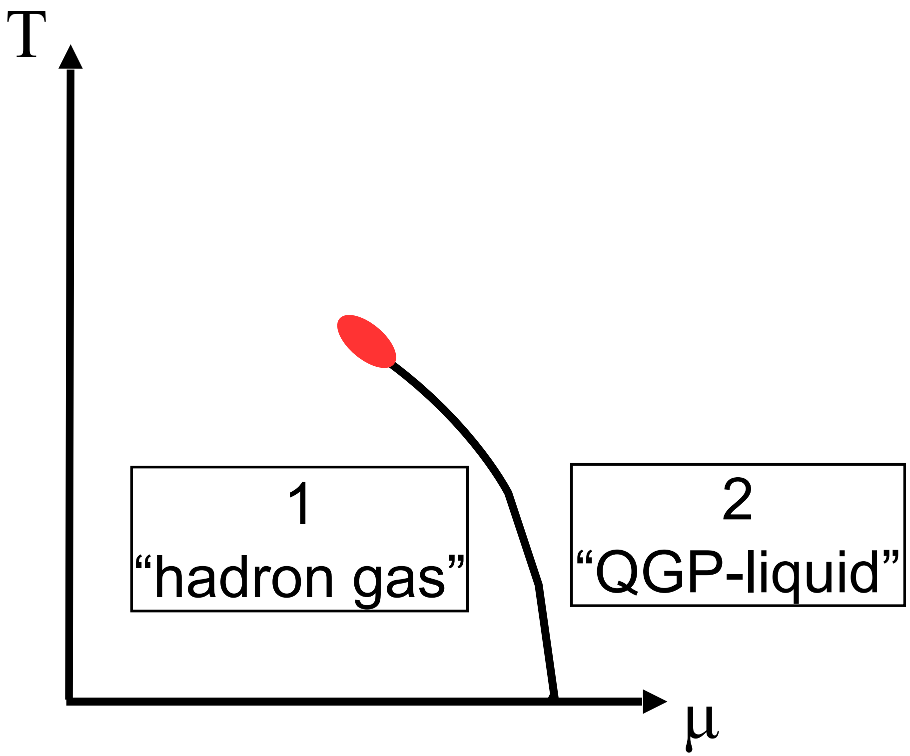


Lots of them!

Remarks on Phase diagram

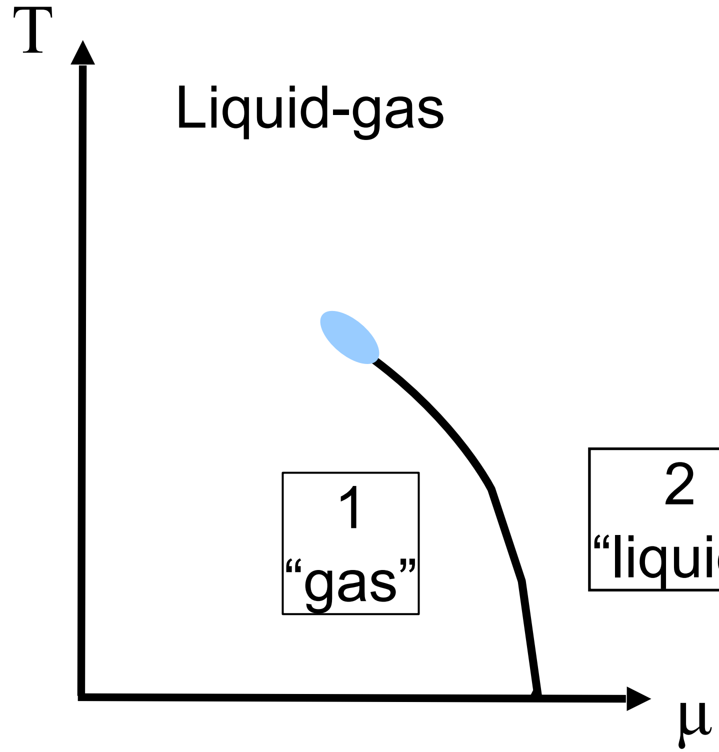


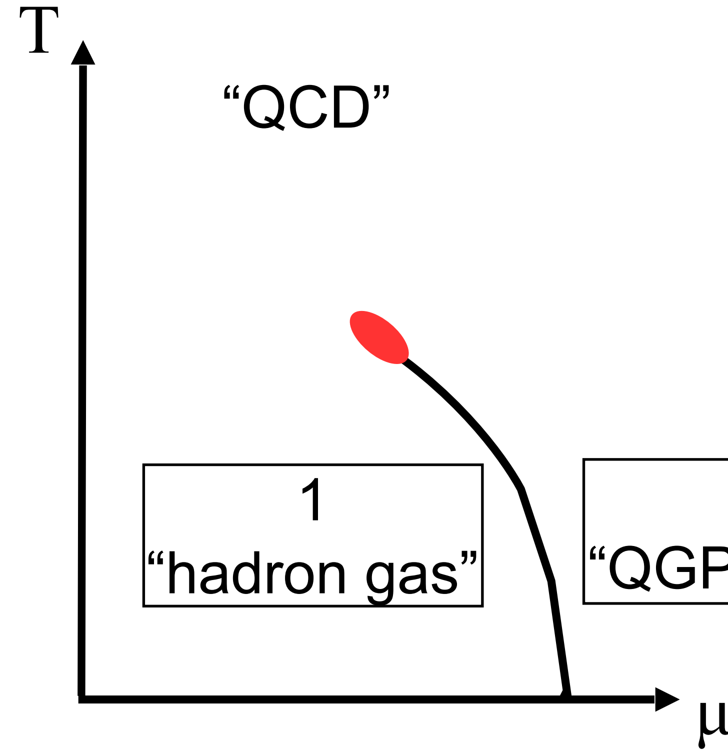
Liquid-Gas
Water, nuclear matter, ...

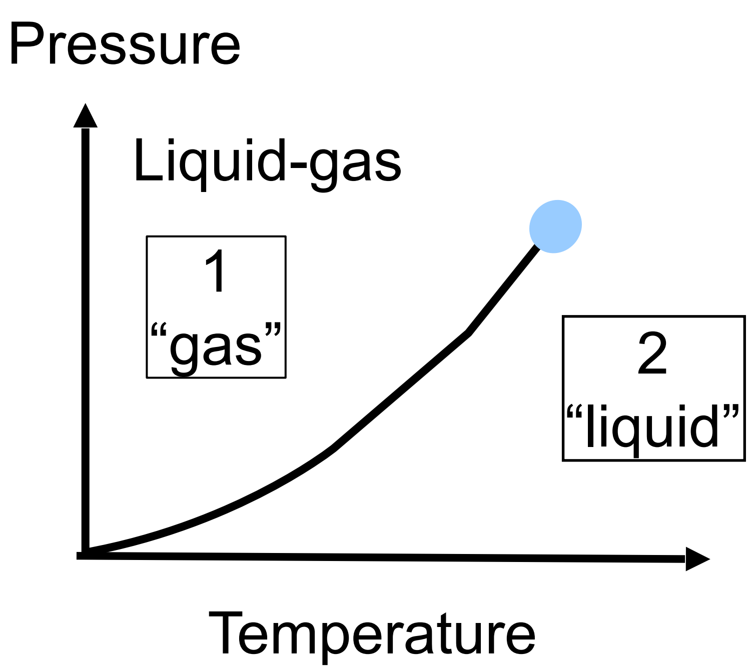


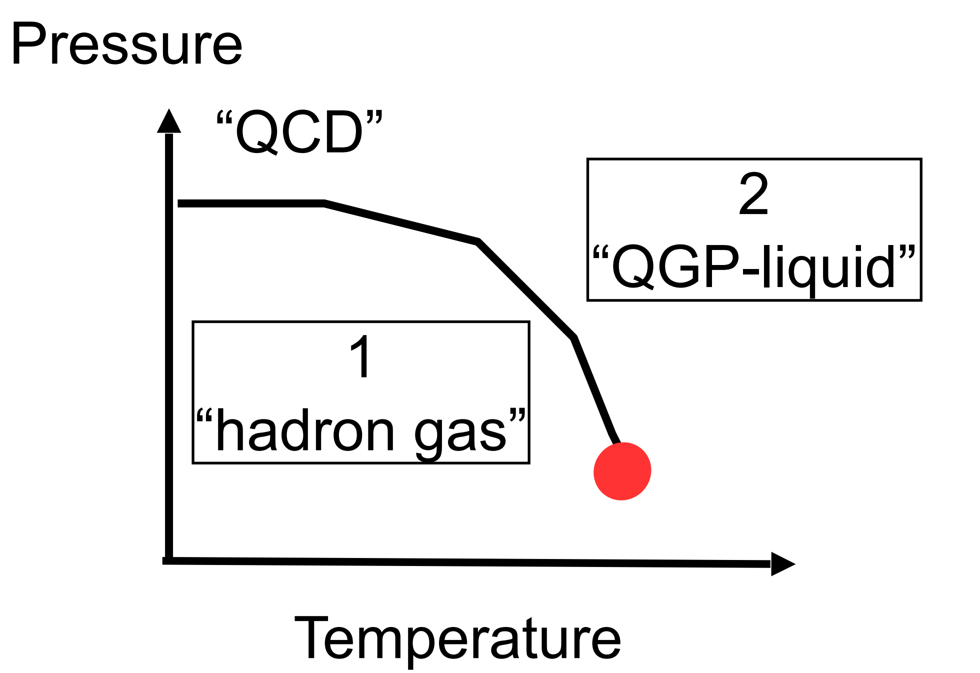
“QCD”

Steinheimer et al, Phys.Rev. C89 (2014) 034901

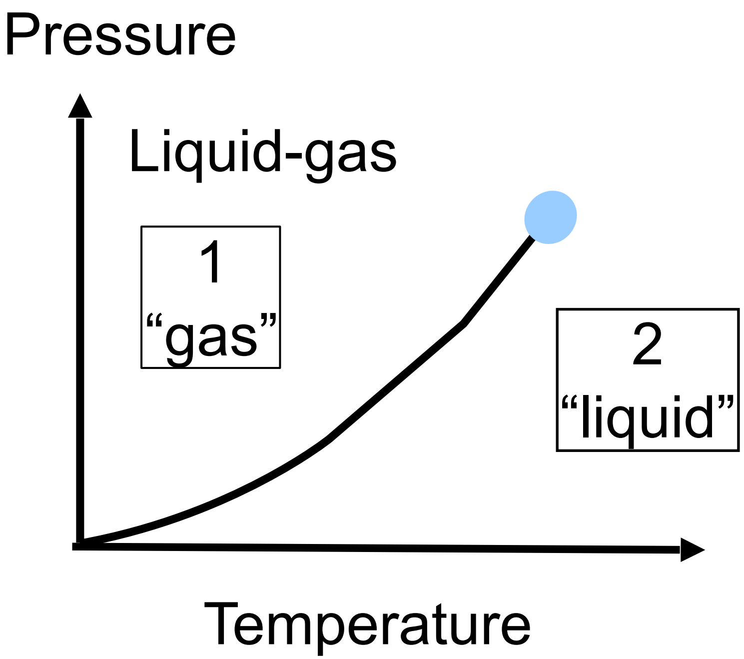

T
Liquid-gas
1
"gas"
2
"liquid"
μ

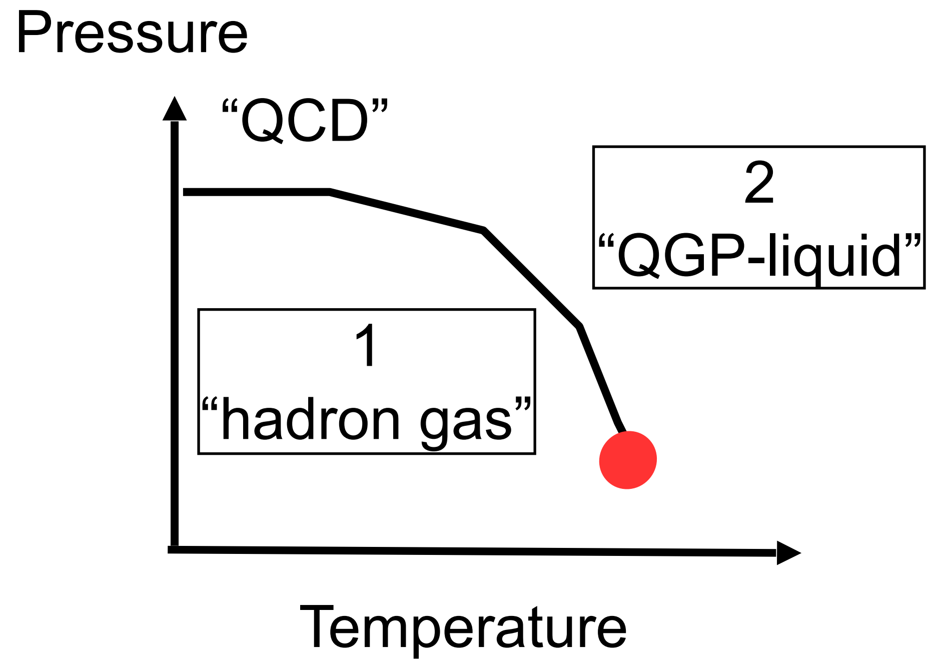

T
"QCD"
1
"hadron gas"
2
"QGP-liquid"
μ


Pressure
Liquid-gas
1
"gas"
2
"liquid"
Temperature


Pressure
"QCD"
2
"QGP-liquid"
1
"hadron gas"
Temperature

Liquid Gas vs QCD PT





Clausius-Clapeyron: $\frac{dP}{dT} = \frac{S_1/B_1 - S_2/B_2}{1/\rho_1 - 1/\rho_2}$ $\quad \rho_2 > \rho_1 \rightarrow (1/\rho_1 - 1/\rho_2) > 0$

$$\frac{dP}{dT} > 0 \rightarrow \; S_1/B_1 > S_2/B_2$$

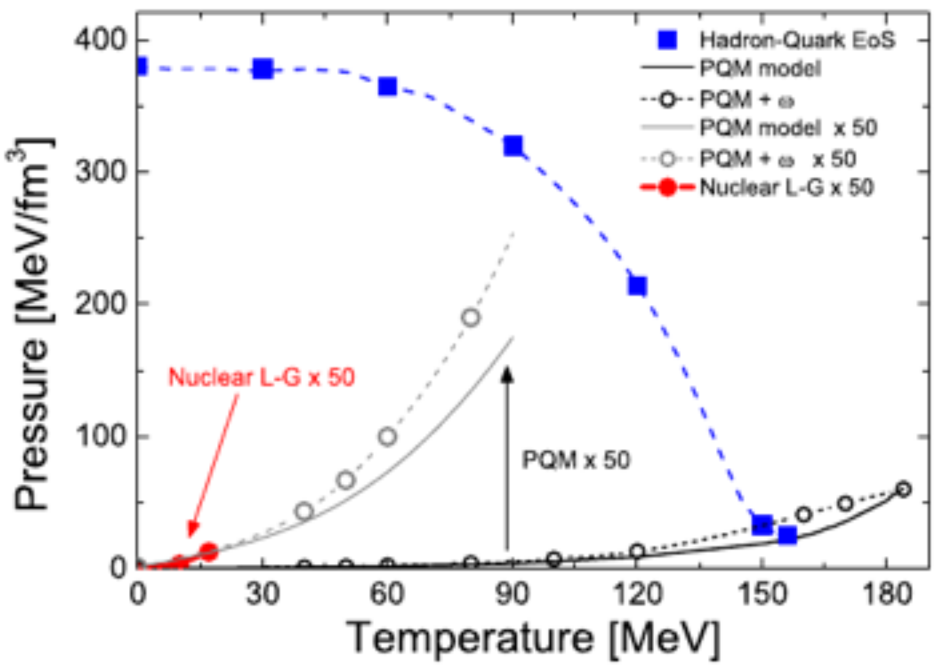
$$\left(\frac{S}{B}\right)_{gas} > \left(\frac{S}{B}\right)_{liquid}$$

$$\frac{dP}{dT} < 0 \rightarrow \; S_1/B_1 < S_2/B_2$$

$$\left(\frac{S}{B}\right)_{hadron-gas} < \left(\frac{S}{B}\right)_{QGP-liquid}$$

See e.g. Hempel et al, arXiv:1302.2835

Liquid-gas vs QCD



Steinheimer et al,

Phys.Rev. C89 (2014) 034901

QCD: $p(T=T_c, \rho=0) \sim p(T=0, \rho \sim 2.5\, \rho_0)$

If T=0 phase transition happens above $2.5\, \rho_0$ $\rightarrow$ $\frac{dP}{dT} < 0$

Note: virtually ALL models predicting a QCD critical point have

$$\frac{dP}{dT} > 0$$

Lattice QCD: Slope of pressure along pseudo-critical line:

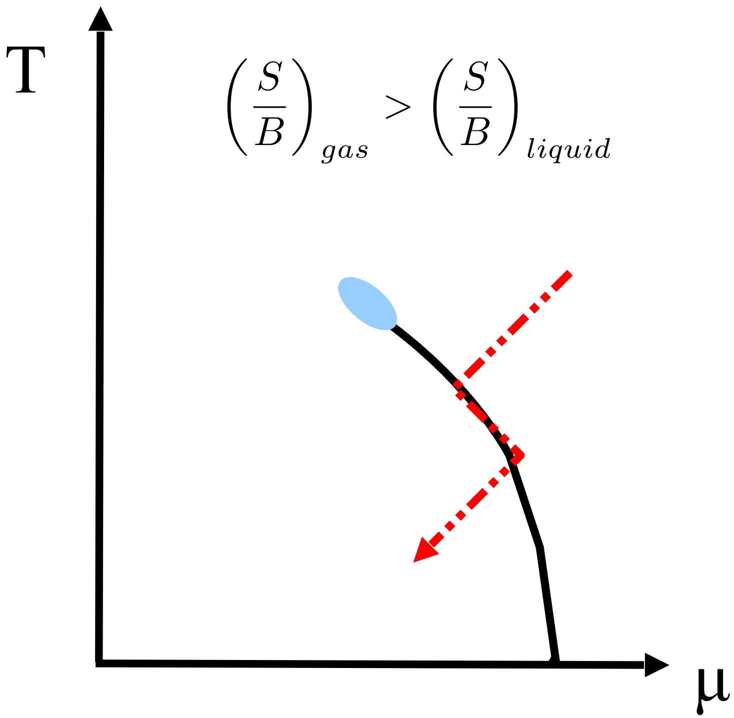
$$\frac{\partial}{\partial T} p_{\rm pc}(T, \mu = 0)|_{T=T_\times} = s(T_\times, \mu = 0) - \frac{T_\times^3}{2\kappa} \chi_2(T_\times) \ .$$

Sign depends on definition of pseudo-critical line 

Liquid-gas vs QCD

Liquid-Gas

$P(T=0)_{\text{co-exist}} = 0$

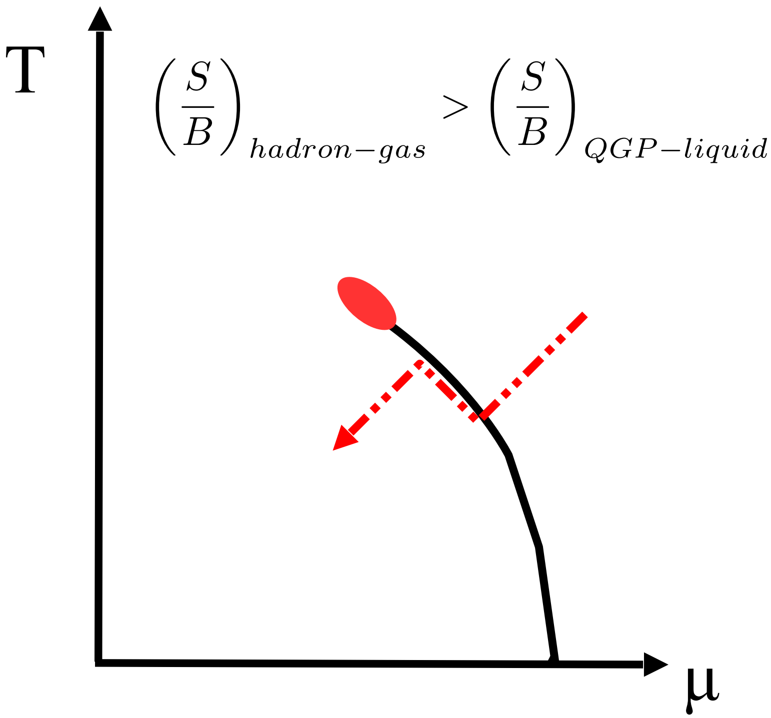


Droplets are stable in vacuum

$$\frac{dP}{dT} > 0$$

"QCD"

$P(T=0)_{\text{co-exist}} >> 0$

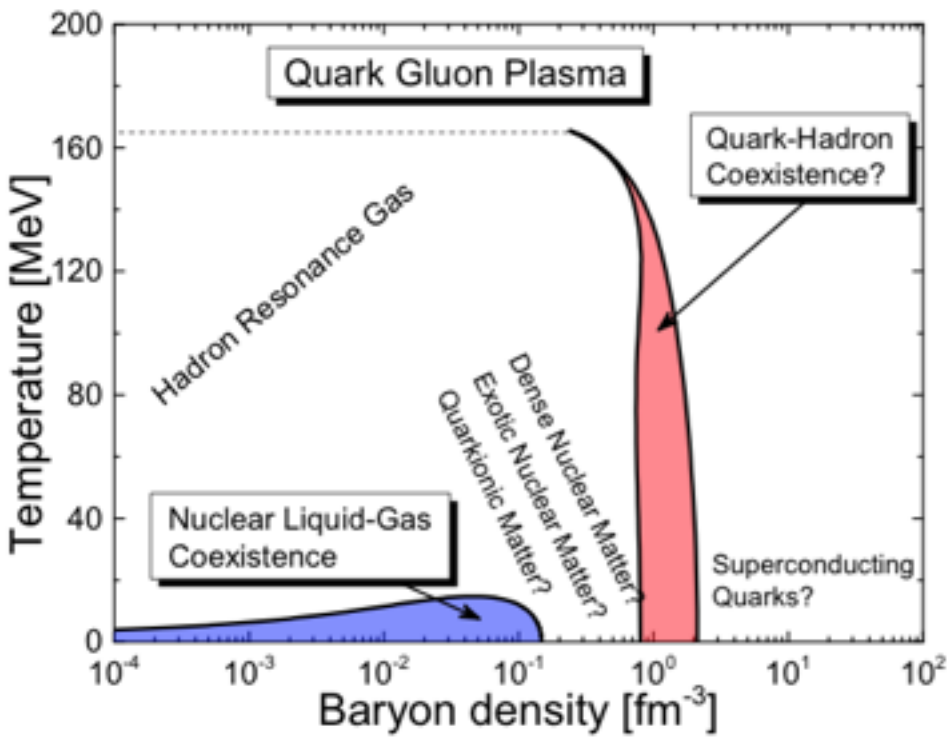


No stable droplets in vacuum

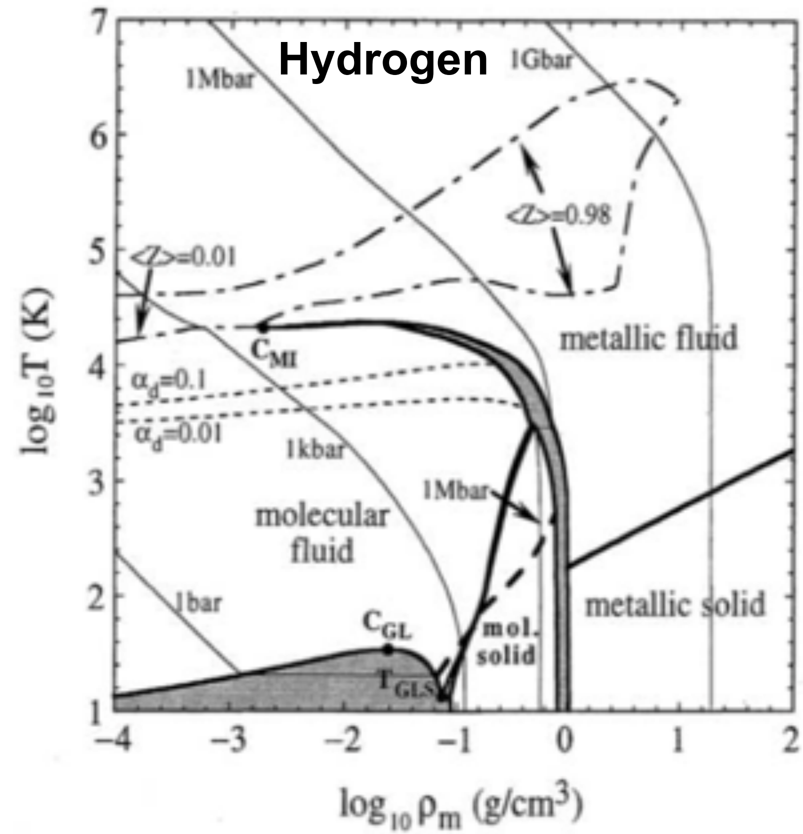
$$\frac{dP}{dT} < 0$$

Phase Diagrams

Maybe it's better to look at the Phase diagram in density.



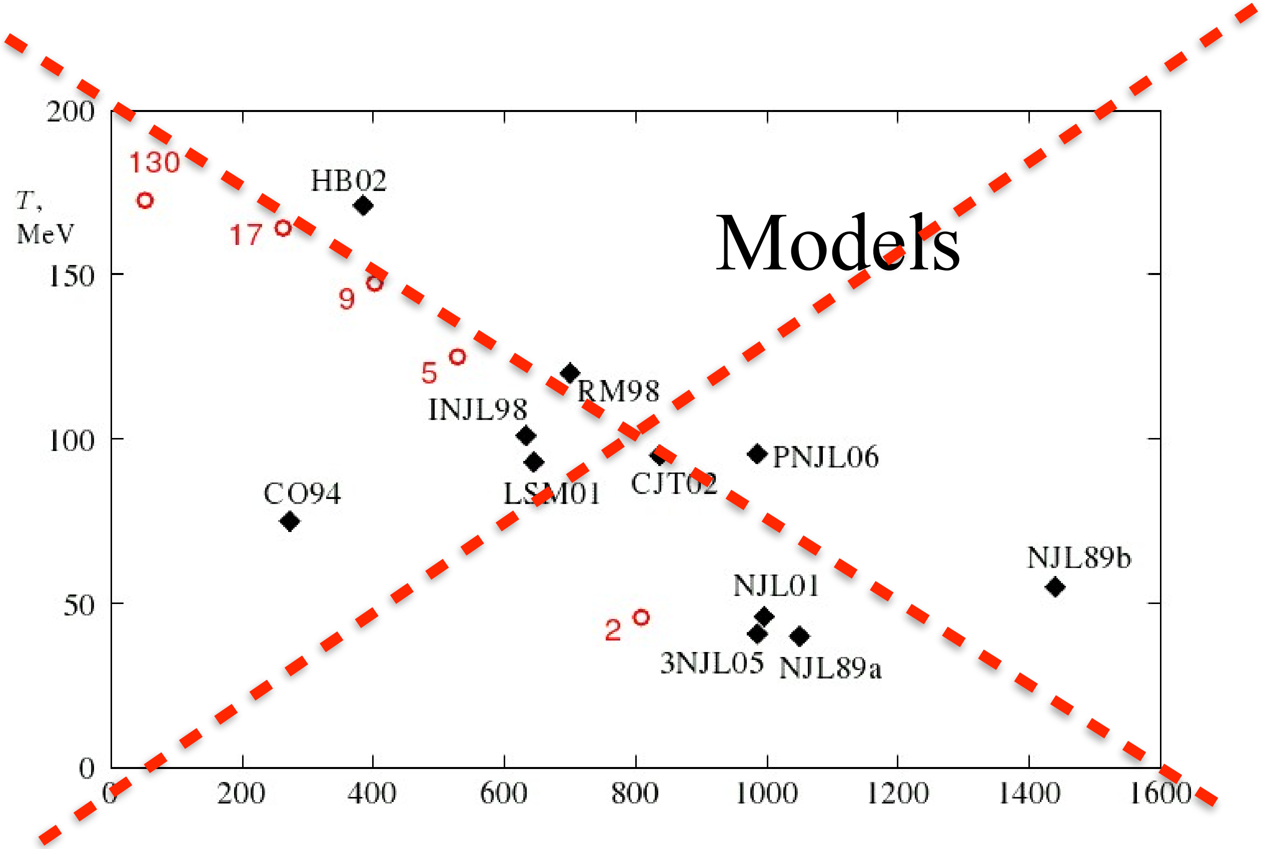
M. Hempel, V. Dexheimer, S. Schramm and I. Iosilevskiy, Phys. Rev. C 88, no. 1, 014906 (2013)



Kitamura H., Ichimaru S., J. Phys. Soc. Japan 67, 950 (1998).

Curious similarity

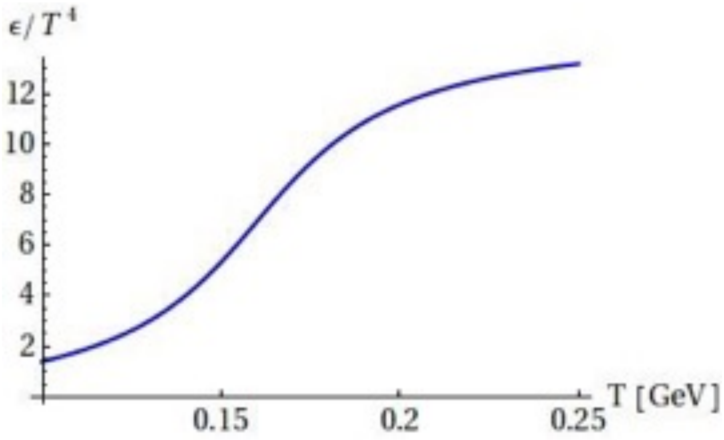
Most models are of liquid-gas type

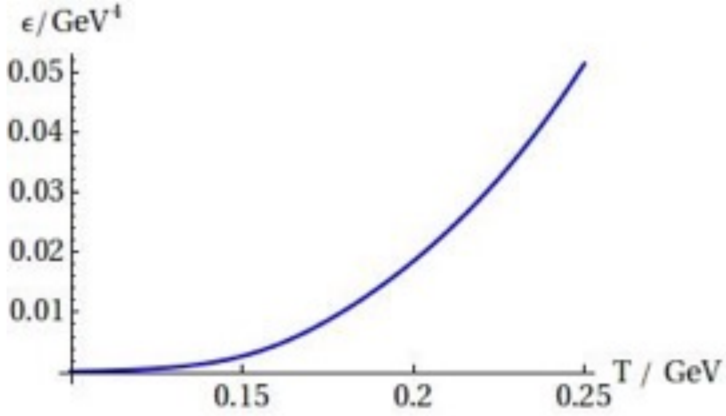


Not clear how useful the are

Guidance from Theory

The Lattice EOS



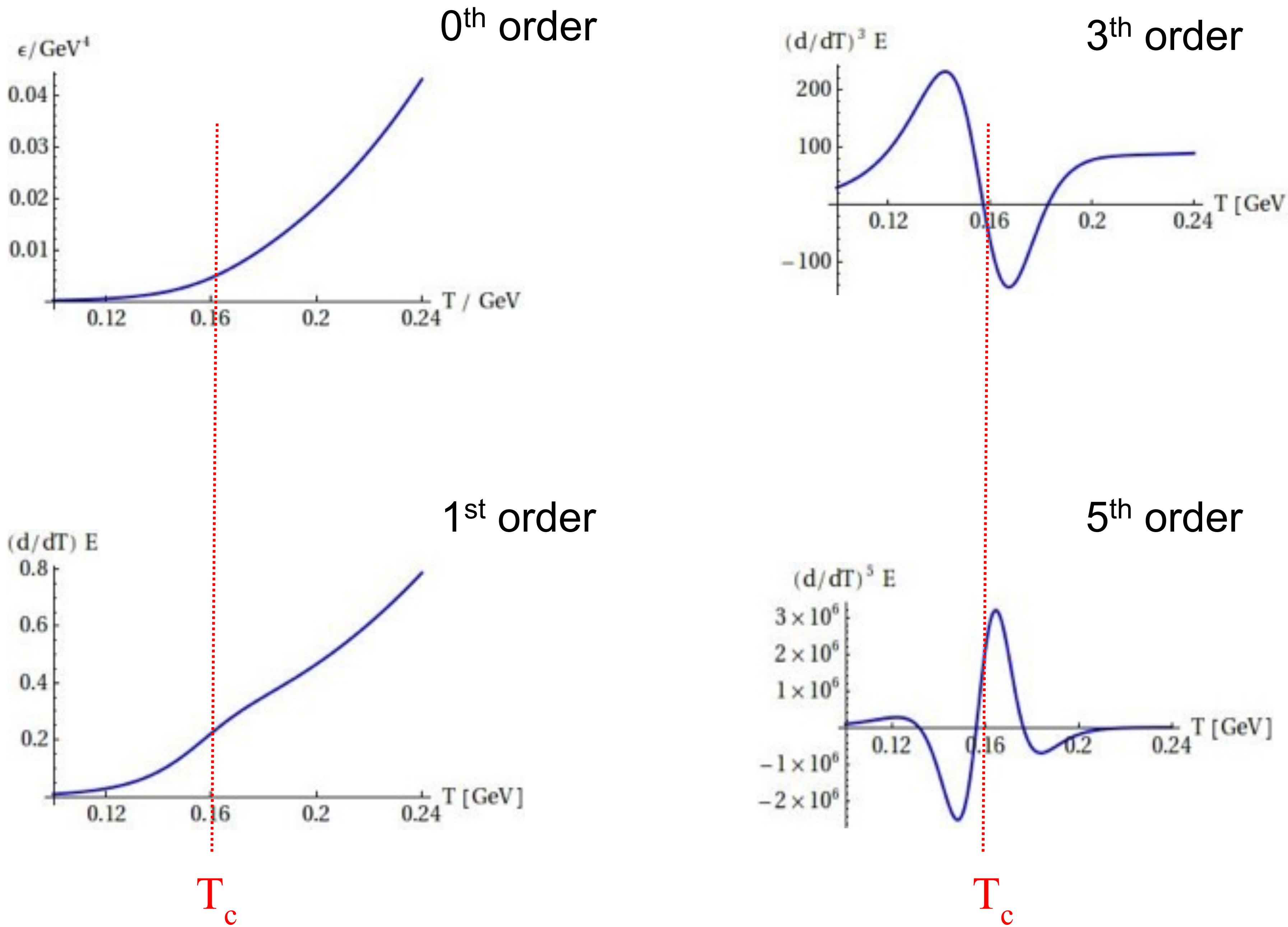


What we always see....

What it really means....

"T_c" ~ 160 MeV

Derivatives



How to measure derivatives

At μ = 0:

$$Z = tr\, e^{-\hat{E}/T + \mu/T \hat{N}_B}$$

$$\langle E \rangle = \frac{1}{Z}\, tr\, \hat{E}\, e^{-\hat{E}/T + \mu/T \hat{N}_B} = -\frac{\partial}{\partial 1/T} \ln(Z)$$

$$\langle (\delta E)^2 \rangle = \langle E^2 \rangle - \langle E \rangle^2 = \left(-\frac{\partial}{\partial 1/T} \right)^2 \ln(Z) = \left(-\frac{\partial}{\partial 1/T} \right) \langle E \rangle$$

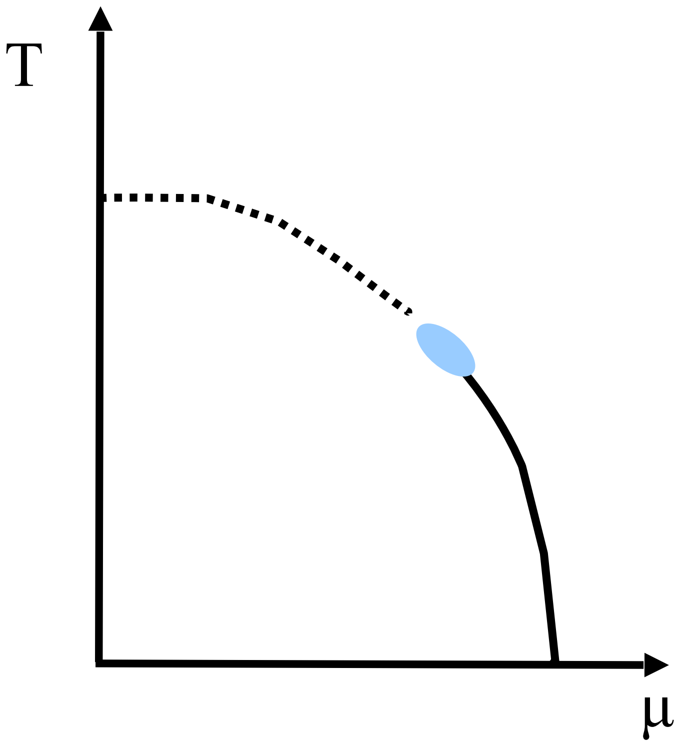
$$\langle (\delta E)^n \rangle = \left(-\frac{\partial}{\partial 1/T} \right)^{n-1} \langle E \rangle$$

Cumulants of Energy measure the temperature derivatives of the EOS

Another way

$$F = F(r), \quad r = \sqrt{T^2 + a\mu^2}$$

a ~ curvature of critical line



$$\partial_\mu^2 F(T,\mu)_{\mu=0} = \frac{a}{T}\partial_T F(T,0)$$

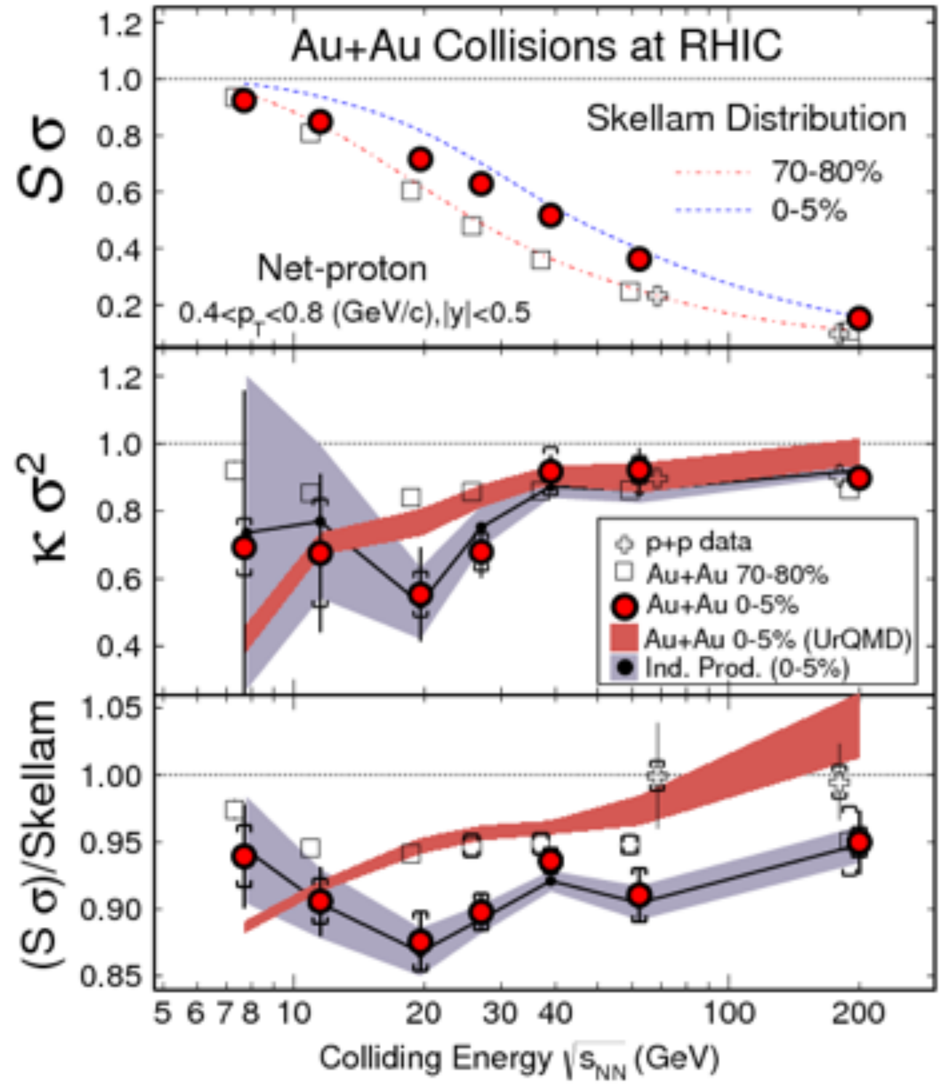
$$\partial_\mu^2 F(T,\mu)_{\mu=0} = 3\frac{a^3}{T}\left(T\partial_T^2 - \partial_T\right) F(T,0)$$

Baryon number cumulants give same info.
Less problem with flow etc.
Needs higher order cumulants (derivatives)
at μ ~ 0

Cumulants

STAR, Phys.Rev.Lett. 112 (2014) 032302

- High sensitivity to critical point
- Sensitive to any “wiggles” in the EoS. Also at μ=0
- Used to connect Lattice with data
  - freeze out parameters (HotQCD and Wuppertal/ Budapest)
- …



Things to consider

- Fluctuations of conserved charges ?!
- Higher cumulants probe the tails. Statistics!
- The detector “fluctuates” !
- Net-protons different from net-baryons
  - Isospin fluctuations
- Correlation length?
- “Stopping” Fluctuation

Detector induced Fluctuations a..k.a finite efficiency

A. Bzdak, VK; Phys.Rev. C86 (2012) 044904

Model with binomial distribution: $p_{1,2}$ = probability to see particle, antiparticle

True distribution

$$p(n_1, n_2) = \sum_{N_1=n_1}^{\infty} \sum_{N_2=n_2}^{\infty} P(N_1, N_2) \frac{N_1!}{n_1!(N_1-n_1)!} p_1^{n_1} (1-p_1)^{N_1-n_1} \times \frac{N_2!}{n_2!(N_2-n_2)!} p_2^{n_2} (1-p_2)^{N_2-n_2}.$$

Finite efficiency

True

$$F_{ik} \equiv \left\langle \frac{N_1!}{(N_1-i)!}\frac{N_2!}{(N_2-k)!} \right\rangle = \sum_{N_1=i}^{\infty}\sum_{N_2=k}^{\infty} P(N_1,N_2)\frac{N_1!}{(N_1-i)!}\frac{N_2!}{(N_2-k)!},$$

Measured

$$f_{ik} \equiv \left\langle \frac{n_1!}{(n_1-i)!}\frac{n_2!}{(n_2-k)!} \right\rangle = \sum_{n_1=i}^{\infty}\sum_{n_2=k}^{\infty} p(n_1,n_2)\frac{n_1!}{(n_1-i)!}\frac{n_2!}{(n_2-k)!}.$$

$$f_{ik} = p_1^i \cdot p_2^k \cdot F_{ik}.$$

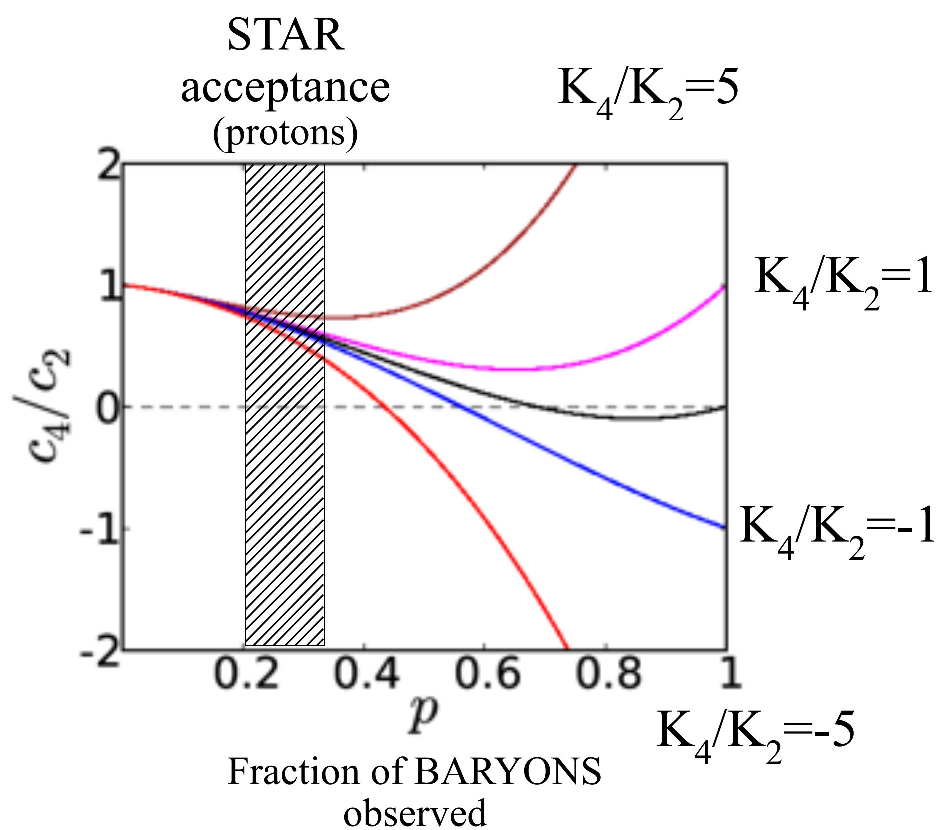
$$c_1 = pK_1,$$
$$c_2 = p\,(1-p)\,N + p^2 K_2,$$
$$c_3 = p(1-p^2)K_1 + 3p^2(1-p)\,(F_{20} - F_{02} - NK_1) + p^3 K_3,$$

Due to efficiency not only Cumulants of the the true distribution enter

$$K_1 = \langle N_1 \rangle - \langle N_2 \rangle\,,$$
$$K_2 = N - K_1^2 + F_{02} - 2F_{11} + F_{20},$$
$$K_3 = K_1 + 2K_1^3 - F_{03} - 3F_{02} + 3F_{12} + 3F_{20} - 3F_{21} + F_{30}$$
$$\quad - 3K_1(N + F_{02} - 2F_{11} + F_{20}),$$

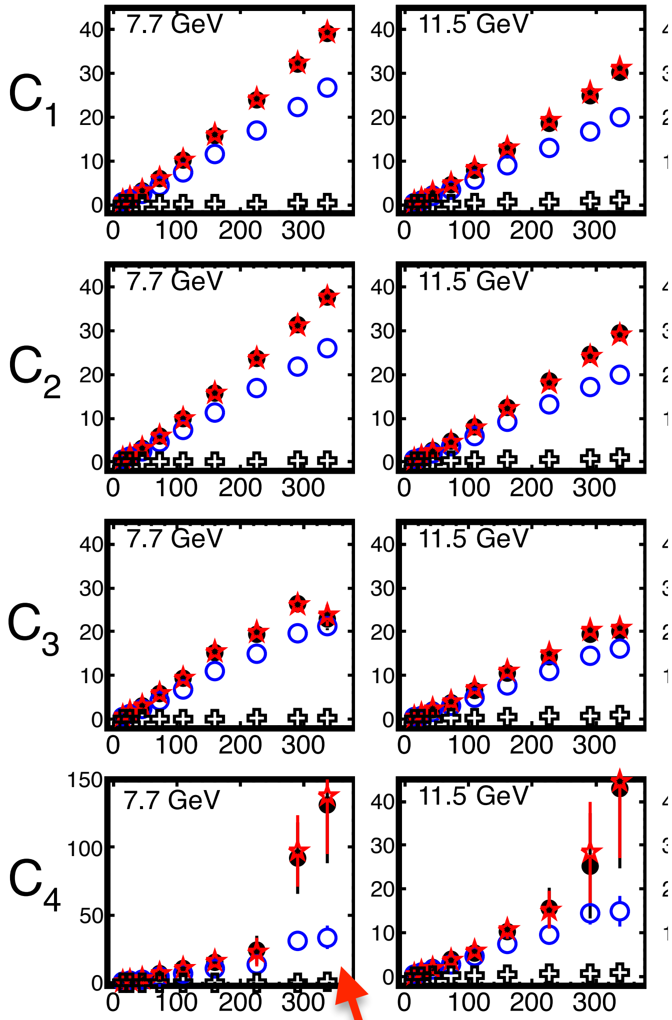
Unfolding is possible!

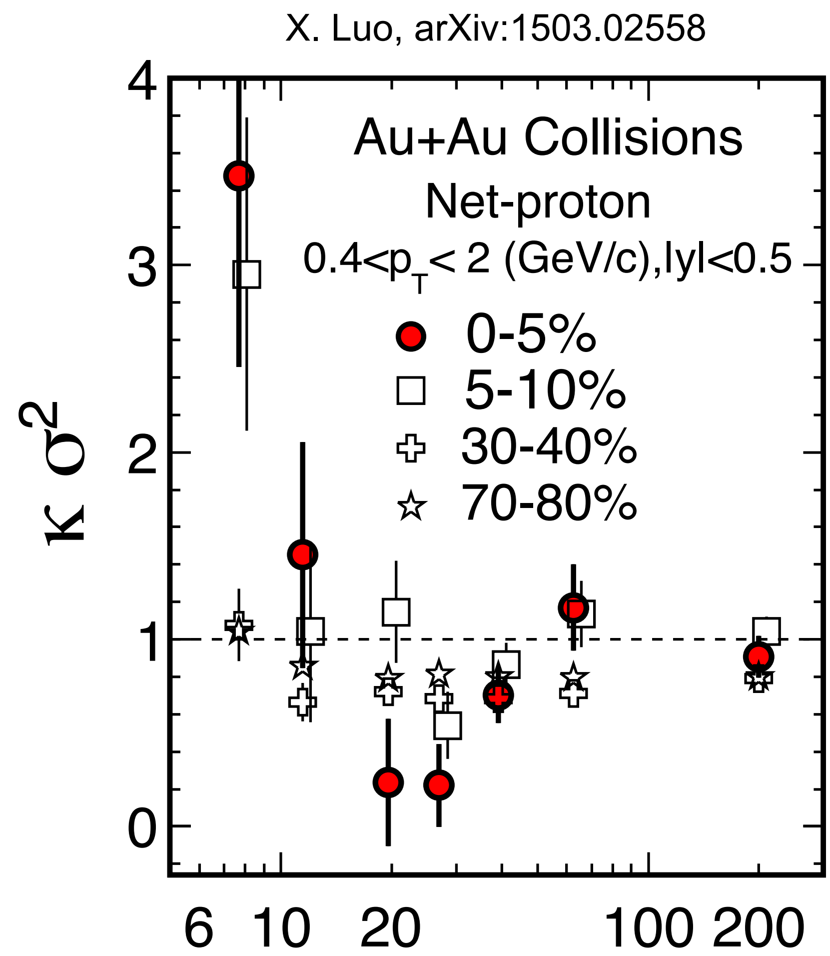
Finite efficiency



Unfolding needed if we want to know what the true cumulants are
Increases Errors!

Latest STAR result



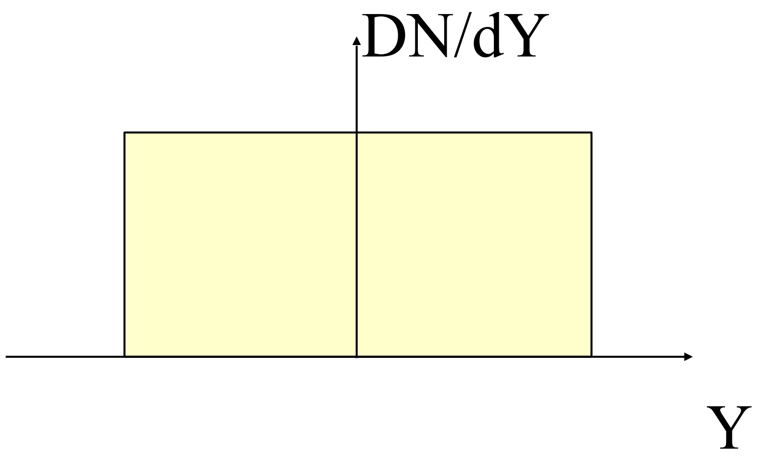


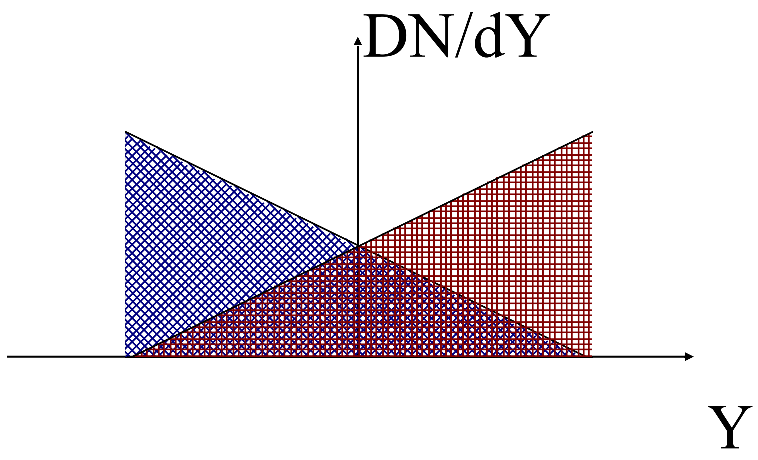
Unfolding makes huge difference in new STAR data!

“Stopping” Fluctuations

At low energy most of the baryon number (isospin) is brought in from the colliding nuclei.

Need to control the fluctuations to due baryon stopping



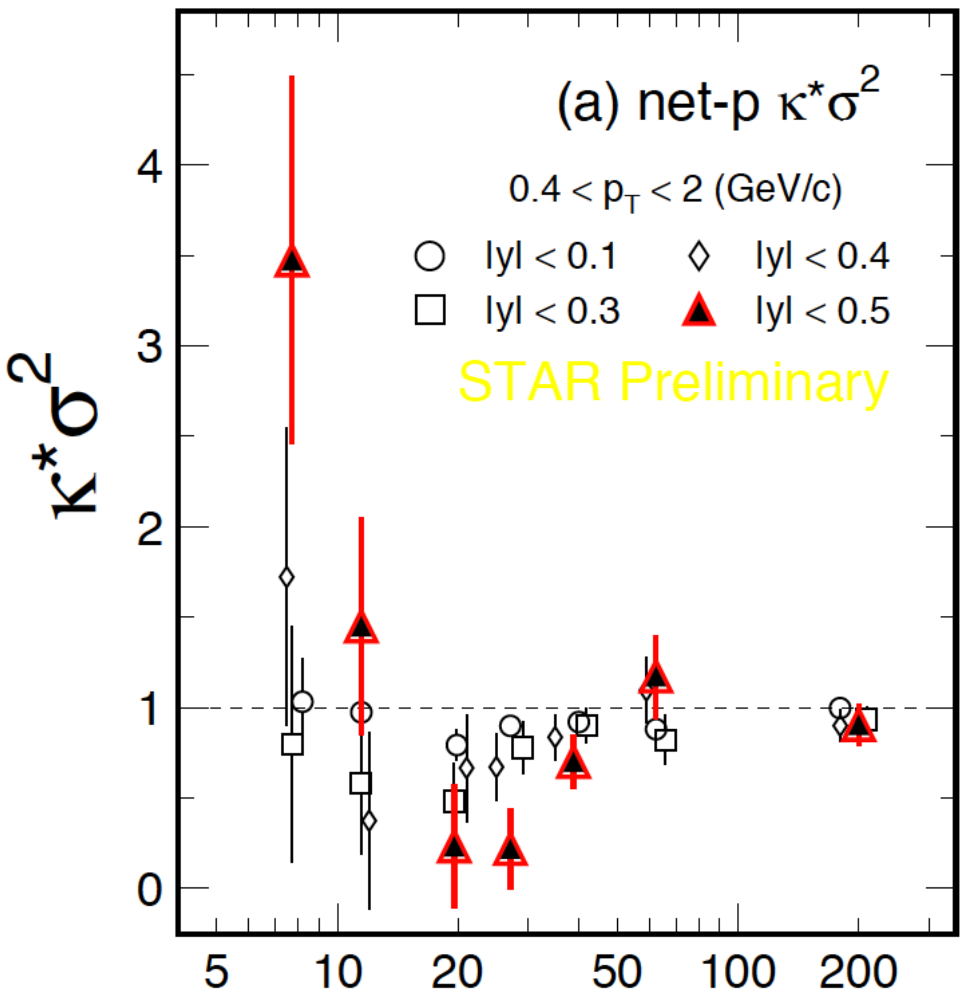


These fluctuations may also be biased by multiplicity selection.

Dependence on Rapidity window

- Kurtosis depends strongly on Rapidity window
- Comparison with Lattice:
  - Lattice catches the full correlation length
  - need to expand rapidity window until signal saturates

X. Luo, RBRC Workshop, Feb. 2015



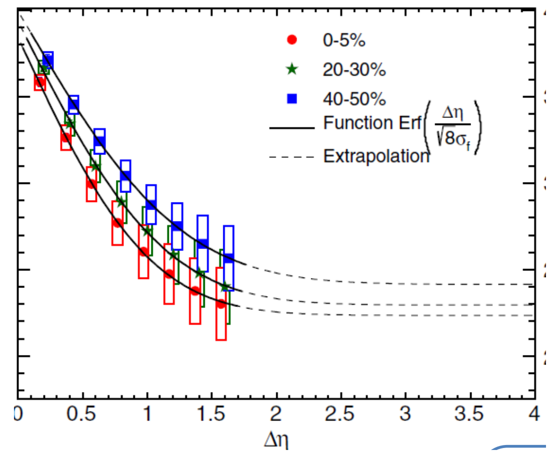
Correlations: Lattice vs Data

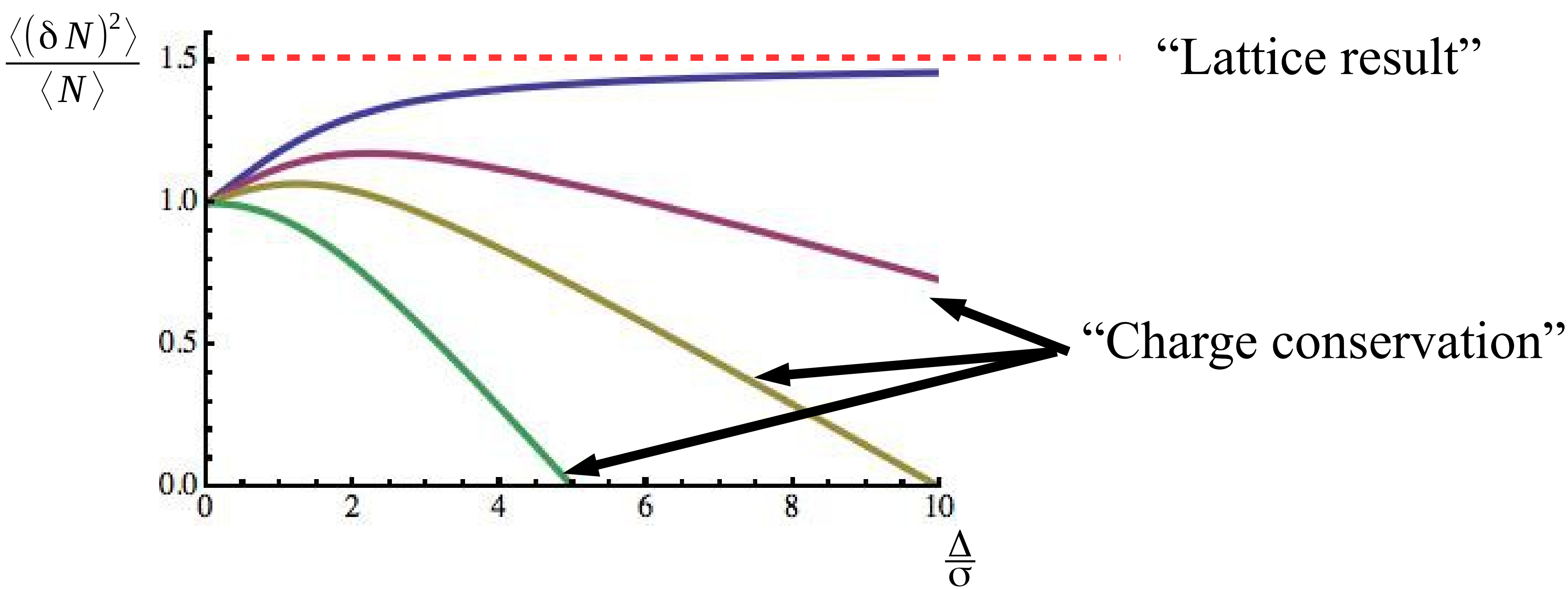
$$\langle n(y_1)(\, n(y_2)-\delta(y_1-y_2)\,)\rangle=\langle n(y_1)\rangle\langle n(y_2)\rangle\left(1+C(y_1,y_2)\right)$$

$$C(y_{1,}\, y_2)\sim\exp\left(\frac{-(y_1-y_2)^2}{2\sigma^2}\right)$$

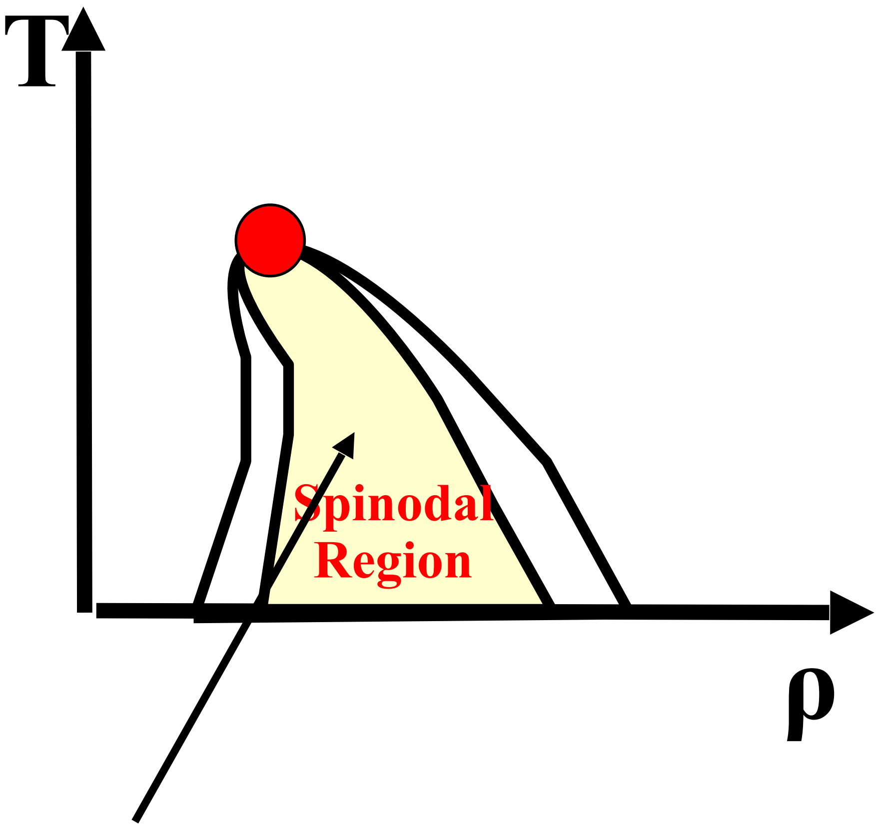
$$\frac{\langle(\delta N)^2\rangle}{\langle N\rangle}=1+\langle N\rangle\int_{\Delta/2}^{\Delta/2} C(y1,y2)dy1\; dy_2$$

Alice Charge Flucts





Co-existence region



Spinodal instability:
Mechanical instability

$$\frac{\partial p}{\partial \epsilon} < 0$$

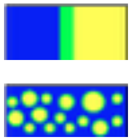
Exponential growth of clumping

Non-equilibrium phenomenon!

System should spent long time
in spinodal region

Phase-transition dynamics: Density clumping

Phase transition => { Phase coexistence: surface *tension*; Phase separation: *instabilities* }



Introduce a ***gradient*** term:

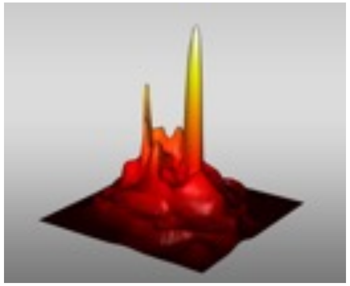
$$p(\mathbf{r}) = p_0(\varepsilon(\mathbf{r}), \rho(\mathbf{r})) - C\rho(\mathbf{r})\nabla^2\rho(\mathbf{r})$$

Insert the modified pressure into existing ideal finite-density fluid dynamics code

Use UrQMD for pre-equilibrium stage to obtain fluctuating initial conditions

Simulate central Pb+Pb collisions at ≈3 GeV/A beam kinetic energy on fixed target, using an Equation of State either ***with*** a phase transition or ***without*** (Maxwell partner):

With phase transition:



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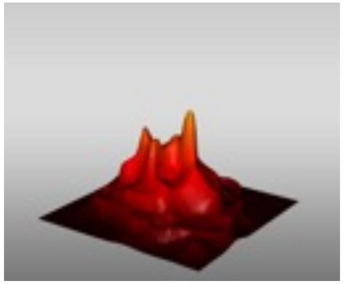
Without phase transition:



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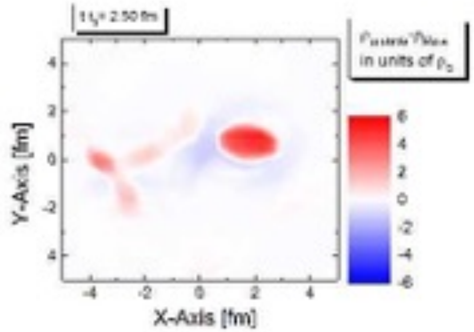
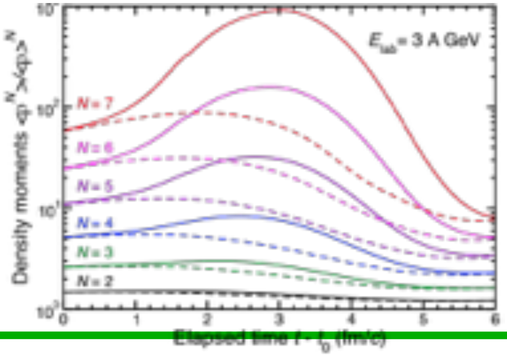
Density enhancement:

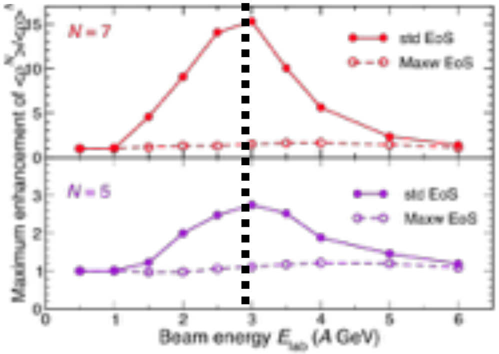




Evolution of density moments

$$\langle \rho^N \rangle \equiv \frac{1}{A}\int \rho(\mathbf{r})^N \rho(\mathbf{r}) d^3\mathbf{r}$$

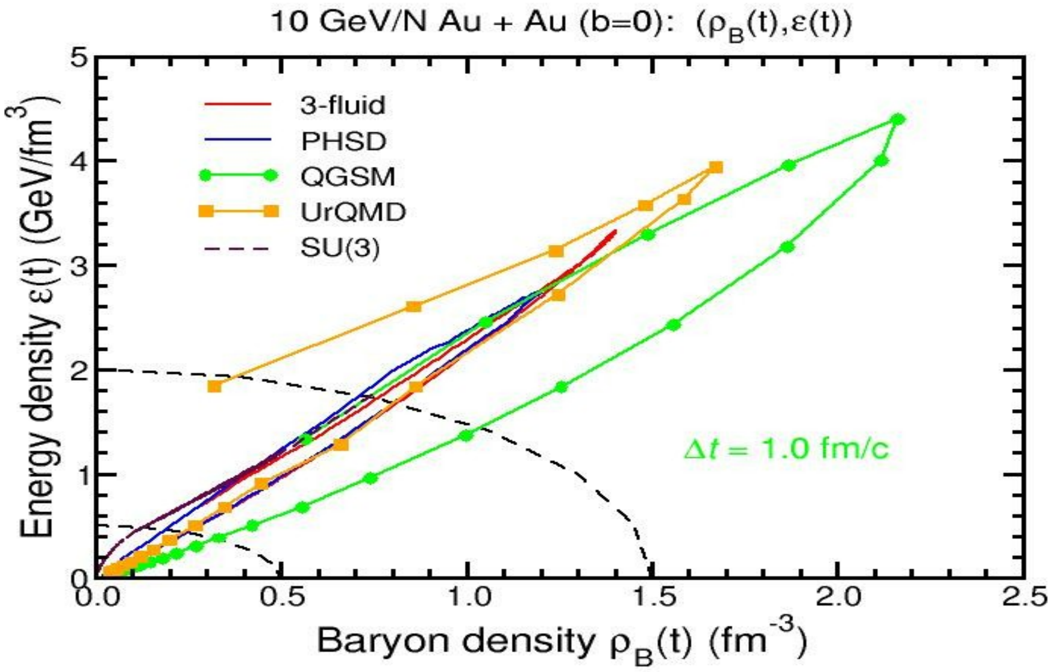
J. Steinheimer & J. Randrup,
PRL 109, 212301(2012)
PRC 87, 054903 (2013)



E_{Lab}=3 GeV

Phase trajectories

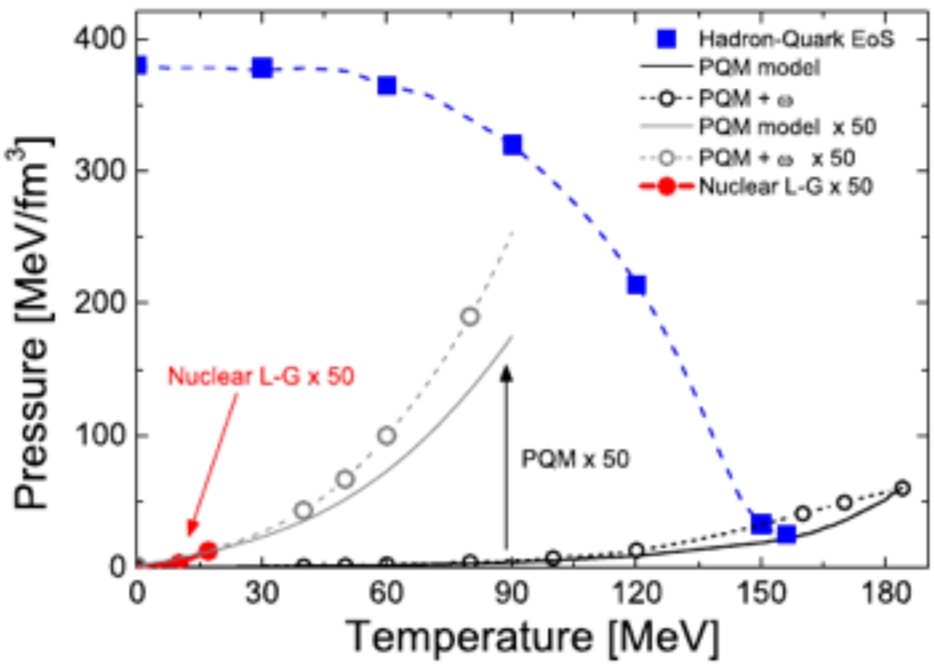
(J. Randrup et al)



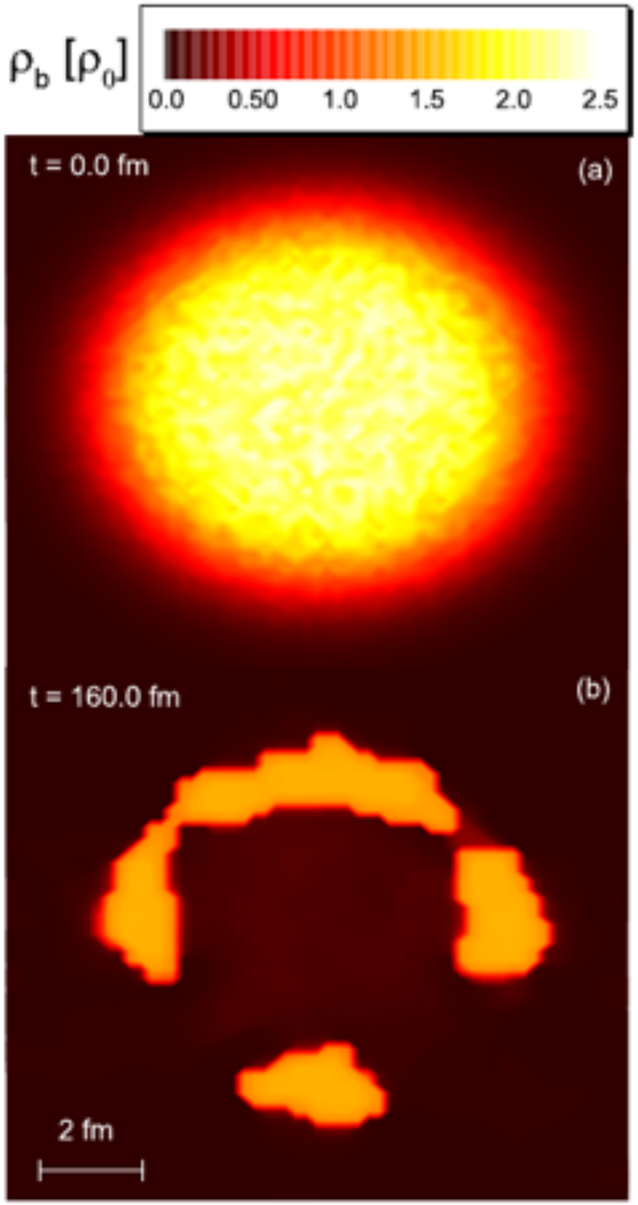
10 AGeV!!!!!

SIS 100 territory

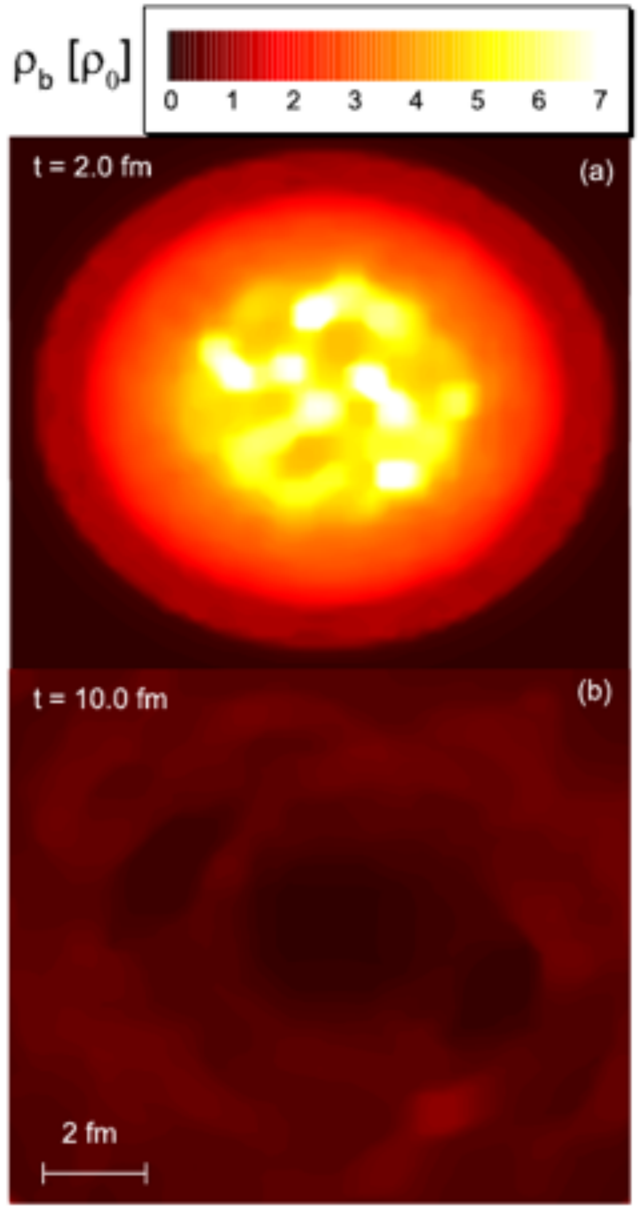
Consider two Equations of State



Steinheimer et al,
Phys.Rev. C89 (2014) 034901

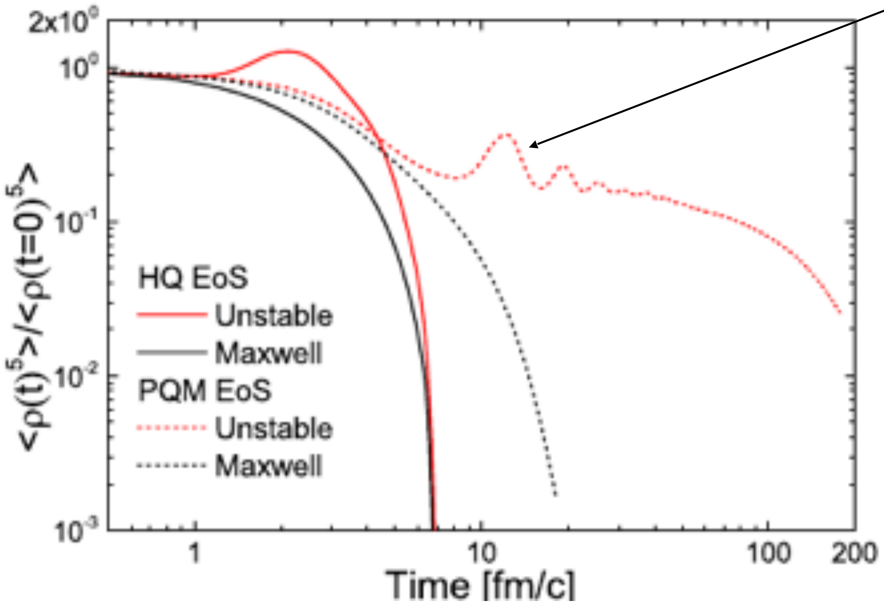


PQM (“liquid-gas”)



“QCD”

Time evolution

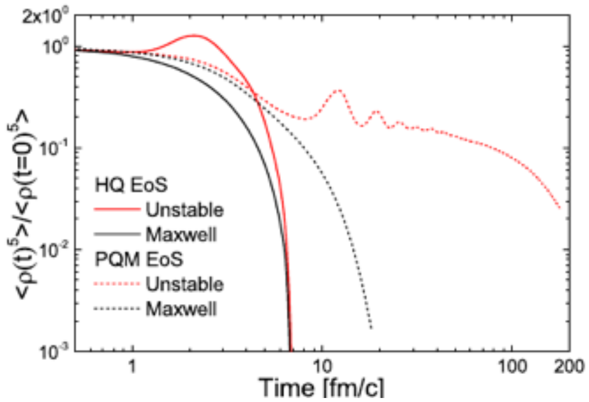


Oscillation of nearly stable droplets for "liquid-gas" EoS

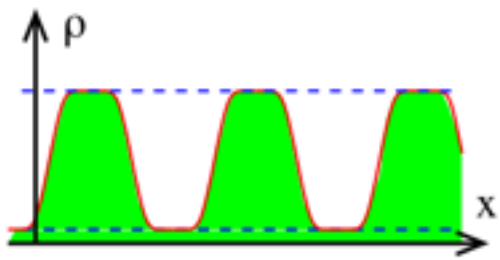
Higher pressure leads to faster evolution of "QCD" EoS.

Steinheimer et al,
Phys.Rev. C89 (2014) 034901

Cluster a.k.a. nuclei



Even if total baryon number does not fluctuate the baryon **density** does



Therefore measure production of NUCLEI: d, ^{3}He, ^{4}He, ^{7}Li....

$$\langle d \rangle \sim \langle \rho^2 \rangle \qquad \langle ^3He \rangle \sim \langle \rho^3 \rangle \qquad \langle ^7Li \rangle \sim \langle \rho^7 \rangle$$

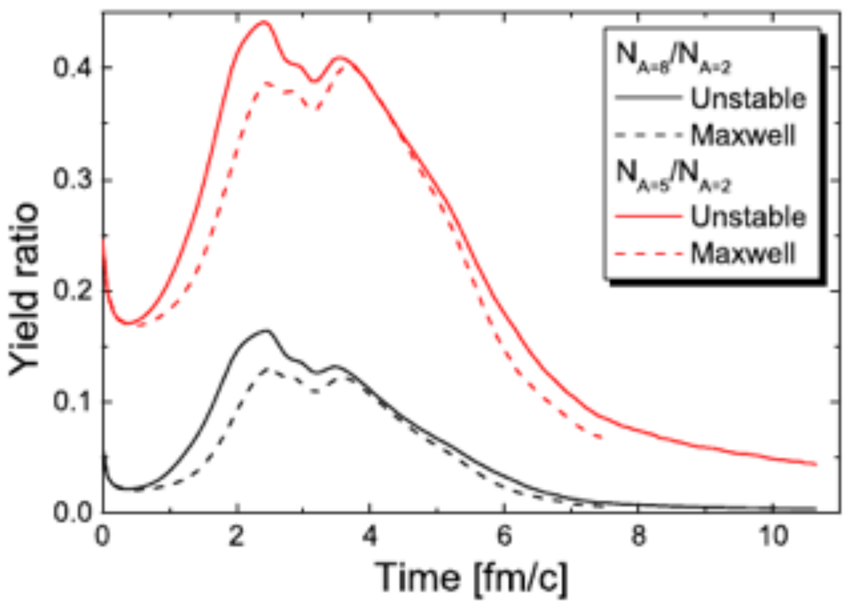
Extracts higher moments of the baryon **density** at freeze out

Nice Idea, but...

"Cluster" formation

"QCD" EoS

$$\left(\frac{S}{B}\right)_{hadron-gas} < \left(\frac{S}{B}\right)_{QGP-\text{liquid}}$$



Clumping in coordinate space is compensated by dilution in momentum space → tiny effect

Steinheimer et al,
Phys.Rev. C89 (2014) 034901

Summary

- Fluctuations sensitive to phase structure:
  - measure "derivatives" of EOS
- Phase diagram well known for small μ (Lattice)
  - No sign of phase transition there
- Little guidance from theory for large μ
  - most models predict phase co-existence between QM and vacuum
- BES I shows some very interesting results
  - better statistics (BES II)
  - need measurement at lower energies: SPS, SIS100, AGS?
  - other effects:
    - stopping fluctuations…

Summary

- Strong density fluctuations due to spinodal instability
  - So far no observable which is sensitive

Happy 40th, Johanna!