Di–Jet Asymmetry and Wave Turbulence

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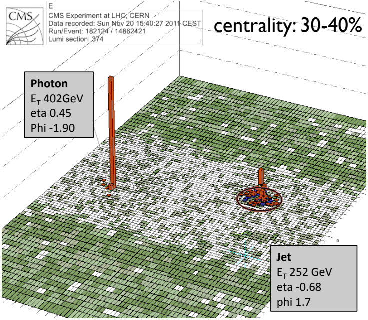
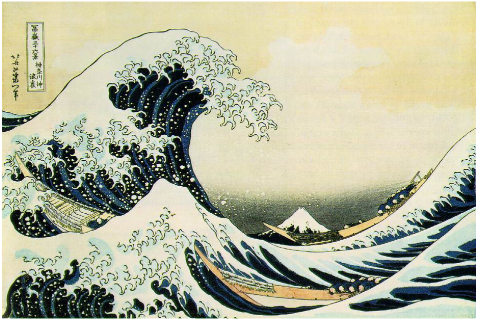
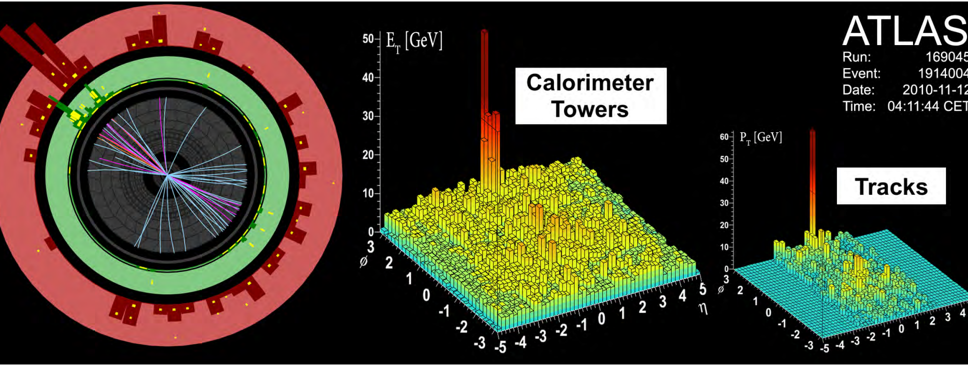
based on collabs. with Jean-Paul Blaizot, Fabio Dominguez, Leonard Fister, and Yacine Mehtar-Tani (since 2012)



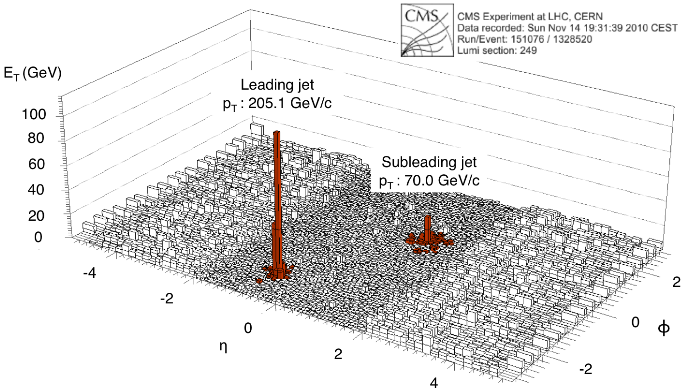


Di–jet asymmetry *(ATLAS)*



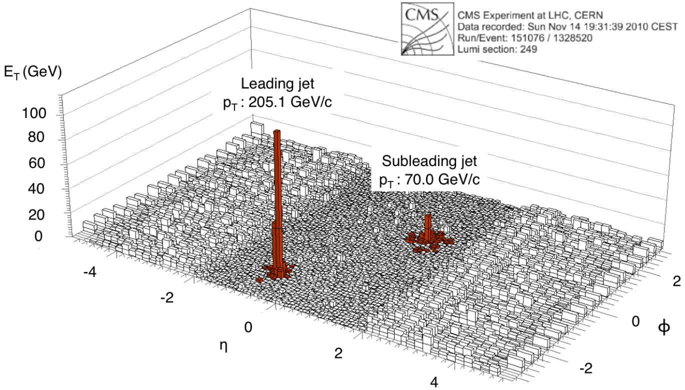
- Central Pb+Pb: 'mono–jet' events
- The secondary jet cannot be distinguished from the background: $E_{T1} \geq 100$ GeV, $E_{T2} > 25$ GeV

Di–jet asymmetry *(CMS)*



- Additional energy imbalance as compared to p+p : 20 to 30 GeV
- Compare to the typical scale in the medium: $T \sim 1$ GeV (average $p_\perp$)
- Detailed studies show that the 'missing energy' is carried by many soft ($p_\perp < 2$ GeV) hadrons propagating at large angles

Di–jet asymmetry *(CMS)*

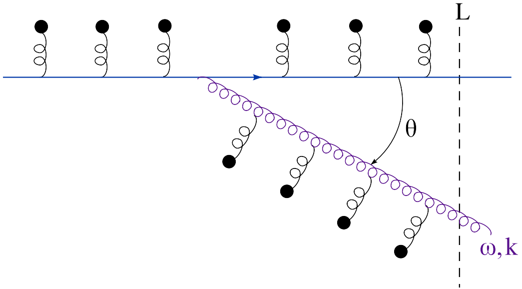


- A rather unexpected picture from the viewpoint of (in–vacuum) QCD
  ▷ collinear emissions, well collimated jets, little energy in soft hadrons
- Can we understand this *ab initio* (within QCD at weak coupling) ?
  ▷ main difficulty: efficiently transport energy down to soft quanta

pQCD : the BDMPSZ mechanism

Baier, Dokshitzer, Mueller, Peigné, and Schiff; Zakharov (96–97)
Wiedemann (2000); Arnold, Moore, and Yaffe (2002–03); ...

- Additional gluon radiation triggered by interactions in the medium



- Originally developed for a single gluon emission (energy loss by the LP)
- The LHC data call for a global understanding of the jet evolution
- Recent extension of the theory to multiple medium–induced emissions

Blaizot, Dominguez, E.I., Mehtar-Tani (2012–13)

Transverse momentum broadening

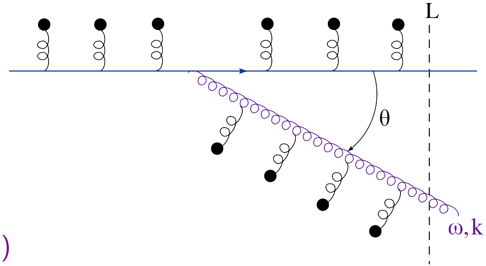
- Gluon emission is linked to transverse momentum broadening
  - transverse kicks provide acceleration and thus allow for radiation
  - they increase the emission angle θ
  - they occur randomly $\Longrightarrow$ Brownian motion in $k_\perp$

$$\langle k_\perp^2 \rangle \simeq \hat{q}\,\Delta t$$

$$\hat{q} \simeq \frac{m_D^2}{\lambda} = \frac{(\text{Debye mass})^2}{\text{mean free path}}$$

'jet quenching parameter'

(quasi–local transport coefficient)



- Gluon emissions require a formation time $\tau_f \simeq \omega/k_\perp^2$
- During formation, the gluon acquires a momentum $k_\perp^2 \sim \hat{q}\tau_f$

Formation time & emission angle

$$\tau_f \simeq \frac{\omega}{k_\perp^2} \quad \& \quad k_\perp^2 \simeq \hat{q}\tau_f \quad \Longrightarrow \quad \tau_f \simeq \sqrt{\frac{\omega}{\hat{q}}}$$

- Maximal ω for this mechanism: $\tau_f \leq L \Rightarrow \omega \leq \omega_c \equiv \hat{q}L^2$

  ▷ soft gluons $(\omega \ll \omega_c)$ have small formation times: $\tau_f \ll L$

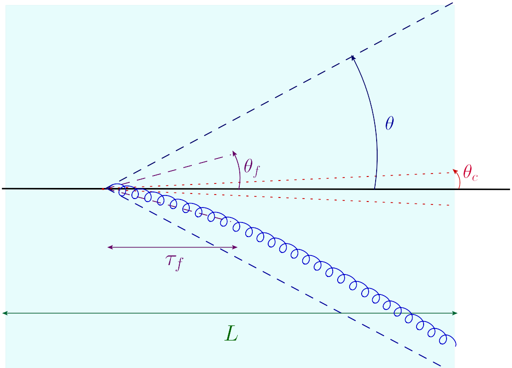
  ▷ ... and large emission angles $\theta \simeq k_\perp/\omega$: $\theta \gg \theta_c$

▷ final momentum $k_\perp^2 \sim \hat{q}L$

$$\theta(\omega) \simeq \frac{\sqrt{\hat{q}L}}{\omega} > \theta_f(\omega)$$

maximal energy ⇔ minimal angle

$$\theta_c \equiv \theta(\omega_c) \lesssim 0.1$$



Emission probability

- Spectrum : Bremsstrahlung $\times$ average number of emissions

$$\omega \frac{\mathrm{d}N}{\mathrm{d}\omega} \simeq \alpha \frac{L}{\tau_f(\omega)} \simeq \alpha \sqrt{\frac{\omega_c}{\omega}} \qquad (\omega_c = \hat{q}L^2)$$

- LPM effect : the emission rate decreases with increasing ω
  - coherence: many collisions contribute to a single, hard, emission
  - formation time $\tau_f(\omega) \gg$ mean free path λ
- Hard emissions : $\omega \sim \omega_c$
  - rare event : probability of $\mathcal{O}(\alpha)$
  - dominate the average energy loss by the leading particle

$$\Delta E = \int^{\omega_c} \mathrm{d}\omega\, \omega \frac{\mathrm{d}N}{\mathrm{d}\omega} \sim \alpha\omega_c$$

  - small emission angle $\theta_c \sim 0.1 \Rightarrow$ the energy remains inside the jet

Soft emissions at large angles

- Spectrum : Bremsstrahlung × average number of emissions

$$\omega \frac{\mathrm{d}N}{\mathrm{d}\omega} \simeq \alpha \frac{L}{\tau_f(\omega)} \simeq \alpha \sqrt{\frac{\omega_c}{\omega}} \qquad (\omega_c = \hat{q}L^2)$$

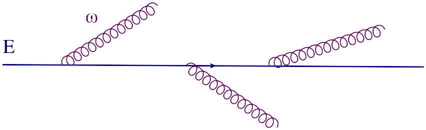
- Relatively soft emissions with $\omega \ll \omega_c$:
  - large emission probability : event–by–event physics

$$\omega \frac{\mathrm{d}N}{\mathrm{d}\omega} \sim 1 \qquad \text{when} \qquad \omega \sim \omega_s \equiv \alpha^2 \omega_c$$

  - a relatively smaller contribution to the energy loss : $\Delta E_s \sim \alpha^2 \omega_c$
  - ... but this can be lost at very large angles : $\theta \gtrsim \theta_s \equiv \theta_c/\alpha^2$
- This energy is lost by the jet $\Longrightarrow$ di–jet asymmetry
- When probability of $\mathcal{O}(1) \Longrightarrow$ multiple branchings become important

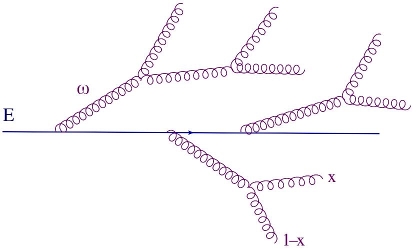
Multiple branchings

- Multiple 'primary' emissions with $\omega \lesssim \alpha^2 \omega_c$ by the leading particle



$$\omega \frac{\mathrm{d}N}{\mathrm{d}\omega} \simeq \alpha \sqrt{\frac{\omega_c}{\omega}}$$

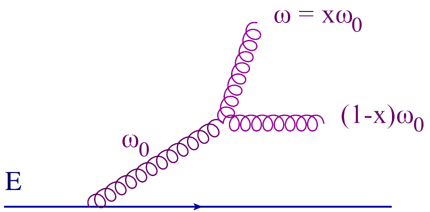
- Each primary gluon develops its own gluon cascade



- Branching probability of $\mathcal{O}(1)$: multiple branchings event-by-event

Quasi–democratic branchings

- The soft branchings are quasi–democratic
  - the daughter gluons carry comparable energy fractions: $x \sim 1/2$
- Non–trivial ! Not true for bremsstrahlung in the vacuum !



$$\mathrm{d}N \sim \alpha \frac{\mathrm{d}\omega}{\omega} \sim \alpha \frac{\mathrm{d}x}{x}$$

$$\Delta N \sim \alpha \int \frac{\mathrm{d}x}{x} \sim \alpha \ln \frac{1}{x}$$

  - probability of $\mathcal{O}(1)$ when $\alpha \ln(1/x) \sim 1 \Longrightarrow$ favors $x \ll 1$

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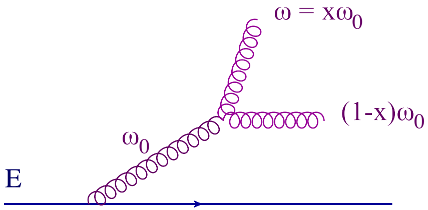

$$\mathrm{d}N \sim \alpha \frac{\mathrm{d}\omega}{\omega} \sqrt{\frac{\omega_c}{\omega}}$$

$$\sim \alpha \frac{\mathrm{d}x}{x} \sqrt{\frac{\omega_c}{x\omega_0}}$$

  - probability of $\mathcal{O}(1)$ when $\alpha \ln(1/x) \sim 1 \Longrightarrow$ favors $x \ll 1$
- In–medium radiation: rate also depends upon the parent energy ω_0
  - probability of $\mathcal{O}(1)$ when $\omega_0 \sim \alpha^2 \omega_c$ for any value of x

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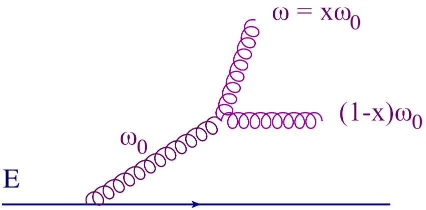


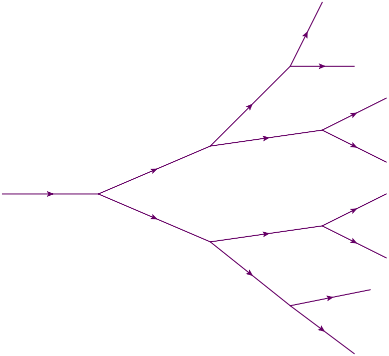
$$\mathrm{d}N \sim \alpha \frac{\mathrm{d}\omega}{\omega} \sqrt{\frac{\omega_c}{\omega}}$$

$$\sim \alpha \frac{\mathrm{d}x}{x} \sqrt{\frac{\omega_c}{x\omega_0}}$$

- Democratic branchings are unusual in gauge theories at weak coupling
- ... but become natural at strong coupling *(Y. Hatta, E.I., Al Mueller '08)*
- *N.B. weak coupling but high density can fake strong coupling*

The democratic cascade

- Democratic branchings imply an energy flux independent of ω (or x)
  - the energy flows from large x to small x without accumulating at any intermediate value of x



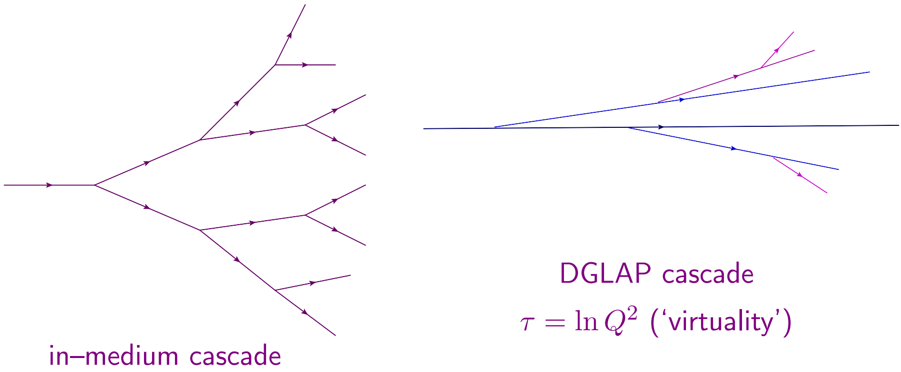
- maximally efficient energy transport to small x (all the energy can flow !)
- since very soft, such gluons propagate at very large angles w.r.t. jet axis
- the cascade stops when $\omega \sim T$
- gluons with $\omega \sim T$ thermalize

  (cf. talk by Bin Wu)

- Jet energy loss via many soft gluons propagating at large angles

  $\Longrightarrow$ a natural explanation for di–jet asymmetry ☺

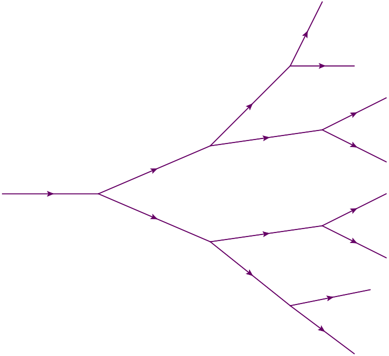
Compare to DGLAP cascade (jet in the vacuum)

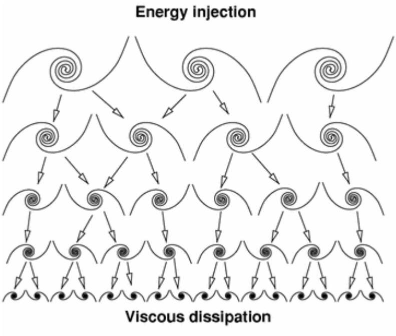


- The asymmetric splittings amplify the number of gluons at small x
- Yet, the energy remains in the few partons with larger values of x
- That is, the energy remains at small angles
- Di-jet asymmetry strongly supports a democratic cascade

Wave turbulence

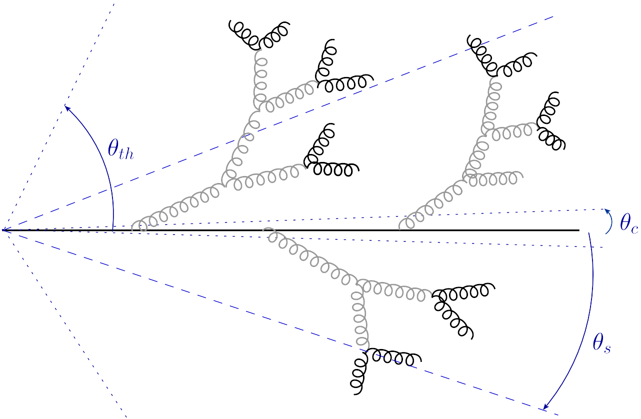
- Democratic branchings imply an energy flux independent of ω





- Uniform flux $\Longleftrightarrow$ turbulent cascade *(Kolmogorov, '41; Zakharov, '92)*
  - the prototype: Richardson cascade for breaking-up vortices
  - characteristic power–law spectrum: $D(\omega) \propto 1/\omega^{\nu}$ (here, $\nu = 1/2$)
  - non–thermal fixed point: 'gain' = 'loss'

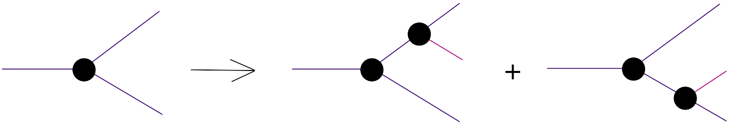
A typical gluon cascade



- The leading particle emits mostly soft gluons: $\omega \lesssim \omega_s \equiv \alpha^2 \omega_c$
- These primary gluons rapidly split into even softer ones
- The primary gluons propagate along typical angles $\theta_s \simeq \theta_c/\alpha^2 \sim 0.5$
- The final gluons $(\omega \sim T)$ make even larger angles $\theta_{\rm th} > \theta_s \gtrsim 1$

The rate equation

- Multiple branching $\approx$ a classical branching process (Markovian)
  - independent splittings with the rate given by BDMPSZ



  - interference effects are suppressed by scattering in the medium

    Mehtar-Tani, Salgado, Tywoniuk; Casalderrey-Solana, E. I. (10 –11)

    Blaizot, Dominguez, E.I., Mehtar-Tani (arXiv: 1209.4585)

- Evolution equation for the gluon spectrum ('rate equation')

$$D(x,t) \equiv x\frac{\mathrm{d}N}{\mathrm{d}x} \quad \text{where} \quad x = \frac{\omega}{E} \quad \text{and} \quad t \leq L$$

- Previously conjectured and used for phenomenological studies

  Baier, Mueller, Schiff, Son '01 ('bottom–up thermalization');

  Arnold, Moore, Yaffe, '03; Jeon, Moore '05; MARTINI (McGill)

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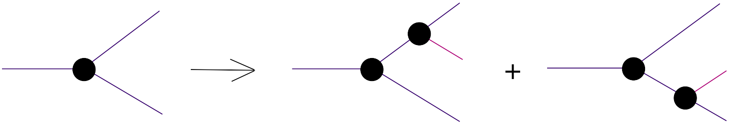

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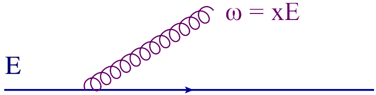
- Turbulence aspects only recently recognized (exact solutions)
  J.-P. Blaizot, E. I., Y. Mehtar-Tani, PRL 111, 052001 (2013)

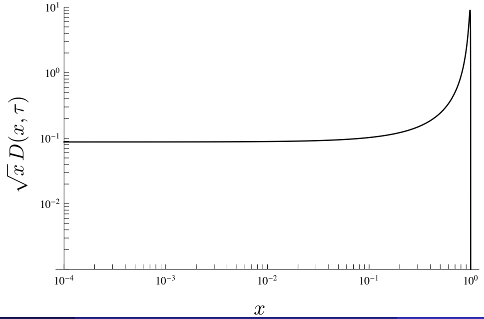
The gluon spectrum from multiple branchings

- The benchmark: the BDMPSZ spectrum for a single emission



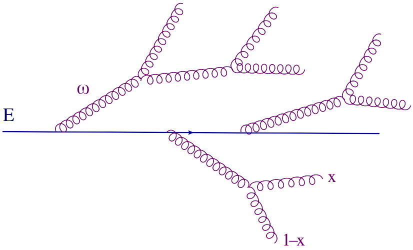
$$D^{(1)}(x,t) \simeq \alpha\sqrt{\frac{\hat{q}}{xE}}\,t \equiv \frac{\tau}{\sqrt{x}}$$

- Solution to the rate eq. with initial condition: $D(x,\tau=0)=\delta(x-1)$
- At small times: just one branching $\Longrightarrow$ BDMPSZ, as expected



Multiple branchings

- Arbitrarily many emissions $\Longrightarrow$ energy transfer towards small x



- one may expect the spectrum to rise faster at $x \ll 1$, like in DGLAP:

$$D(x,\tau) \propto \frac{1}{x^{\lambda(\tau)}} \quad \text{with} \quad \lambda(\tau) > 1/2$$

- "the energy moves from the leading particle into the bins at small x"

Multiple branchings

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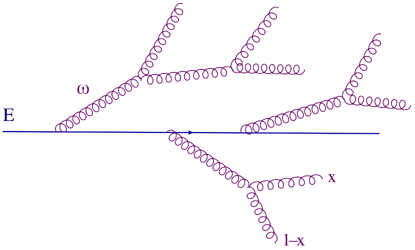

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$$D(x,\tau) \propto \frac{1}{x^{\lambda(\tau)}} \quad \text{with} \quad \lambda(\tau) > 1/2$$

  - "the energy moves from the leading particle into the bins at small x"

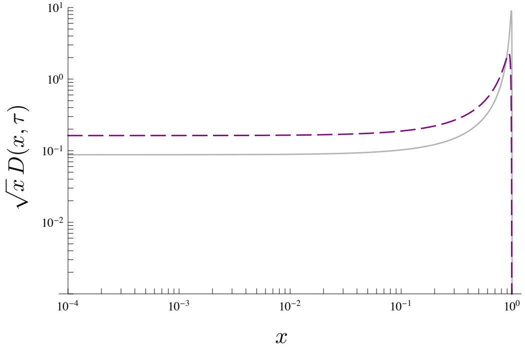
- But this is not what really happens !

The Kolmogorov–Zakharov fixed point

- The shape of the spectrum is preserved by multiple branchings

$$D(x,\tau) \simeq \frac{\tau}{\sqrt{x}}\,\mathrm{e}^{-\pi\tau^2}$$

  - KZ fixed point $\Longrightarrow$ energy flux uniform in ω $\Longrightarrow$ turbulence



- $\tau < 1$: broadening of the LP peak & growth at small x

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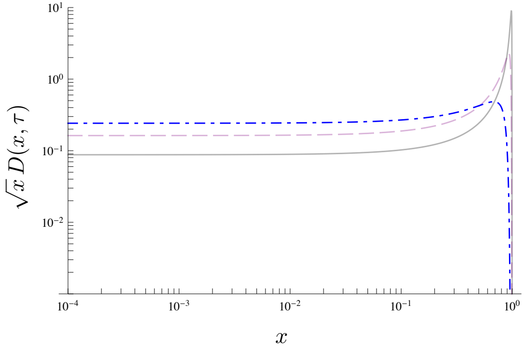

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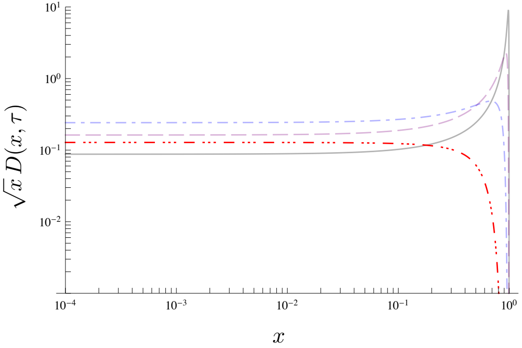

- For $\tau \gtrsim 1$, the spectrum is suppressed at any x

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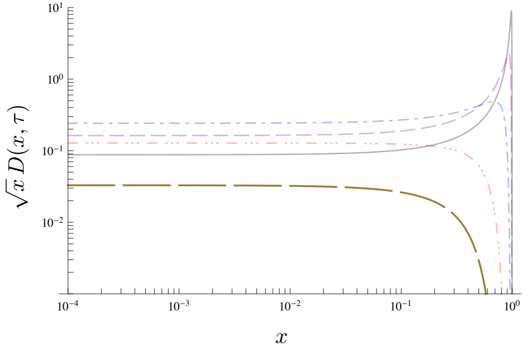

- The LP has disappeared $\Longrightarrow$ no source to feed the spectrum

The Kolmogorov–Zakharov fixed point

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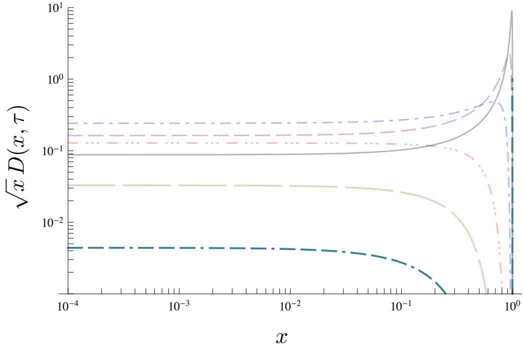
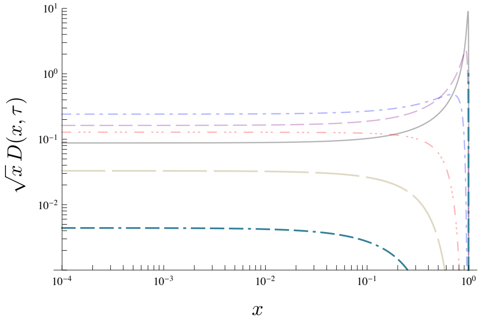


- The energy flows out of the spectrum

The turbulent flow

- The energy remaining in the spectrum dies exponentially fast :

$$\int_0^1 \mathrm{d}x\, D(x,\tau) = \mathrm{e}^{-\pi\tau^2} \implies \mathcal{E}_{\text{flow}}(\tau) = 1 - \mathrm{e}^{-\pi\tau^2}$$



- Formally, it accumulates into a condensate at $x = 0$
- Physically, it goes below $x_{\text{th}} = T/E \ll 1$, where it thermalizes

  ▷ the 'turbulent' picture ceases to be valid around T *(cf. talk by Bin Wu)*

Energy loss by the jet

$$\Delta E \equiv E\,\mathcal{E}_{\rm flow} = E\left(1 - {\rm e}^{-\pi\tau^2}\right)$$

- If $\tau \gtrsim 1$, all of the jet energy can be lost in this way !
- For the relevant kinematics at LHC (di–jet asymmetry) : $\tau \ll 1$

$$\tau \equiv \frac{L}{t_{\rm br}(E)}\,, \qquad t_{\rm br}(E) \sim \frac{1}{\alpha}\sqrt{\frac{E}{\hat q}}$$

- $\hat q \sim 1\,{\rm GeV}^2/{\rm fm}\,, \quad L \sim 5\,{\rm fm}\,, \quad E \gtrsim 100\,{\rm GeV} \Longrightarrow \tau \simeq 0.3$

$$\mathcal{E}_{\rm flow} \simeq \pi\tau^2 \Longrightarrow \Delta E \simeq \alpha^2 \hat q L^2 \simeq 10 \div 20\,{\rm GeV}$$

- independent of the energy E of the leading particle
- this is the typical energy loss at large angles (event–by–event)

Angular dependence of the energy loss

(L. Fister, E. I., arXiv:1409.2010)

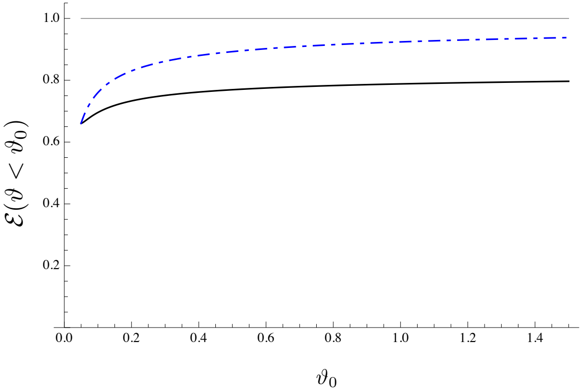
- $\mathcal{E}(\theta < \theta_0)$: the energy contained within a jet with opening angle θ_0

blue : BDMPSZ

black: multiple branchings

$\hat{q} \simeq 1$ GeV^2/fm

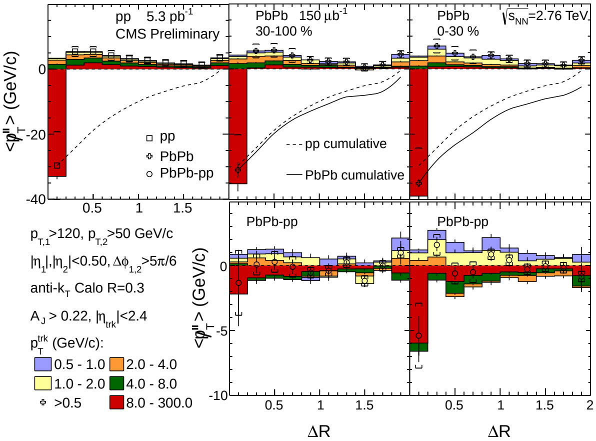
$L \simeq 4$ fm



- offset at $\theta_0 \sim \pi/2$: the energy $\mathcal{E}_{\rm flow}$ taken away by the flow
- almost flat in θ_0 : energy is lost directly at large angles

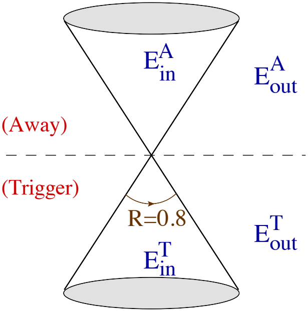
Angular distribution at the LHC *(CMS)*

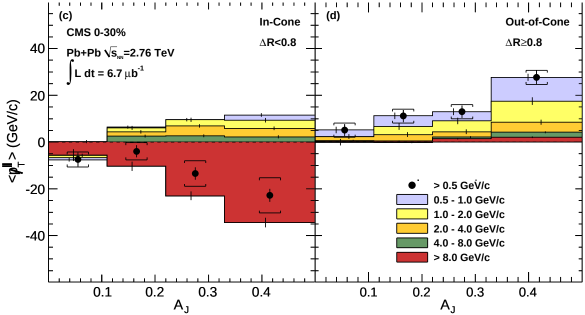
- For each bin in θ : energy difference between trigger jet and away jet



- the offset in Pb+Pb is clearly visible (larger than for p+p)
- the ΔR dependence looks stepper ... but is exactly the same in p+p

- The energy (im)balance for a jet with a wide opening : $R = 0.8$





- Di–jet asymmetry : $E^{\rm T}_{\rm in} > E^{\rm A}_{\rm in}$
- No missing energy : $E^{\rm T}_{\rm in} + E^{\rm T}_{\rm out} = E^{\rm A}_{\rm in} + E^{\rm A}_{\rm out}$
- The energy lost at large angles, $E^{\rm A}_{\rm out} - E^{\rm T}_{\rm out}$ ...

  ... is carried mostly by soft hadrons with $p_T < 2$ GeV