Particle Production and Currents from a Topological Domain

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Based on work in progress with Pablo Morales

Talk Plan

A Key Question

An Approach

Technical Details

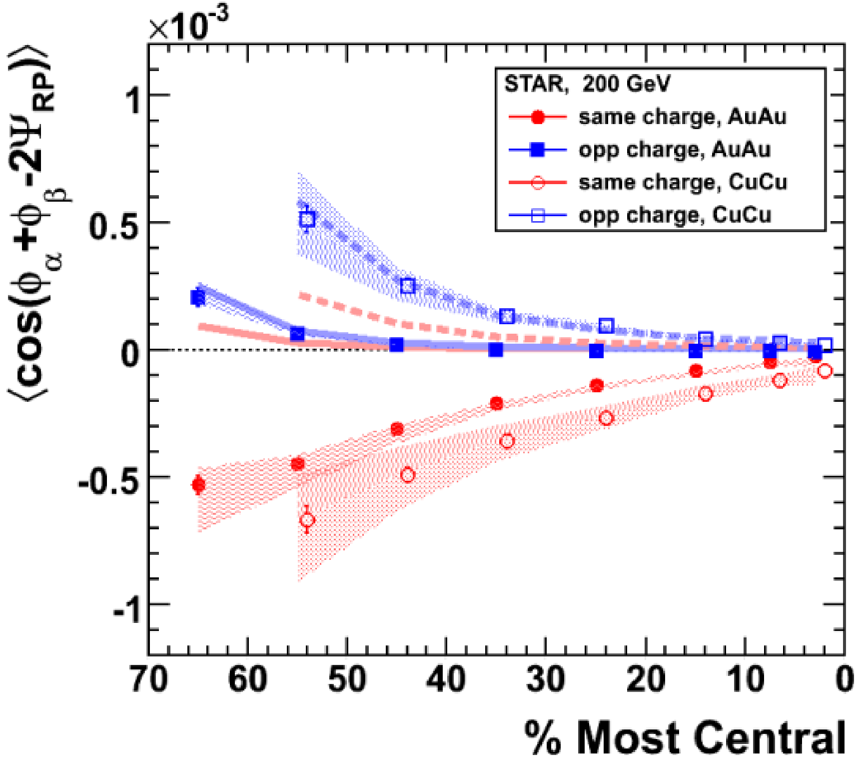
A Key Question

CME (and related phenomena) alive or dead?

A Key Question

CME (and related phenomena) alive or dead?

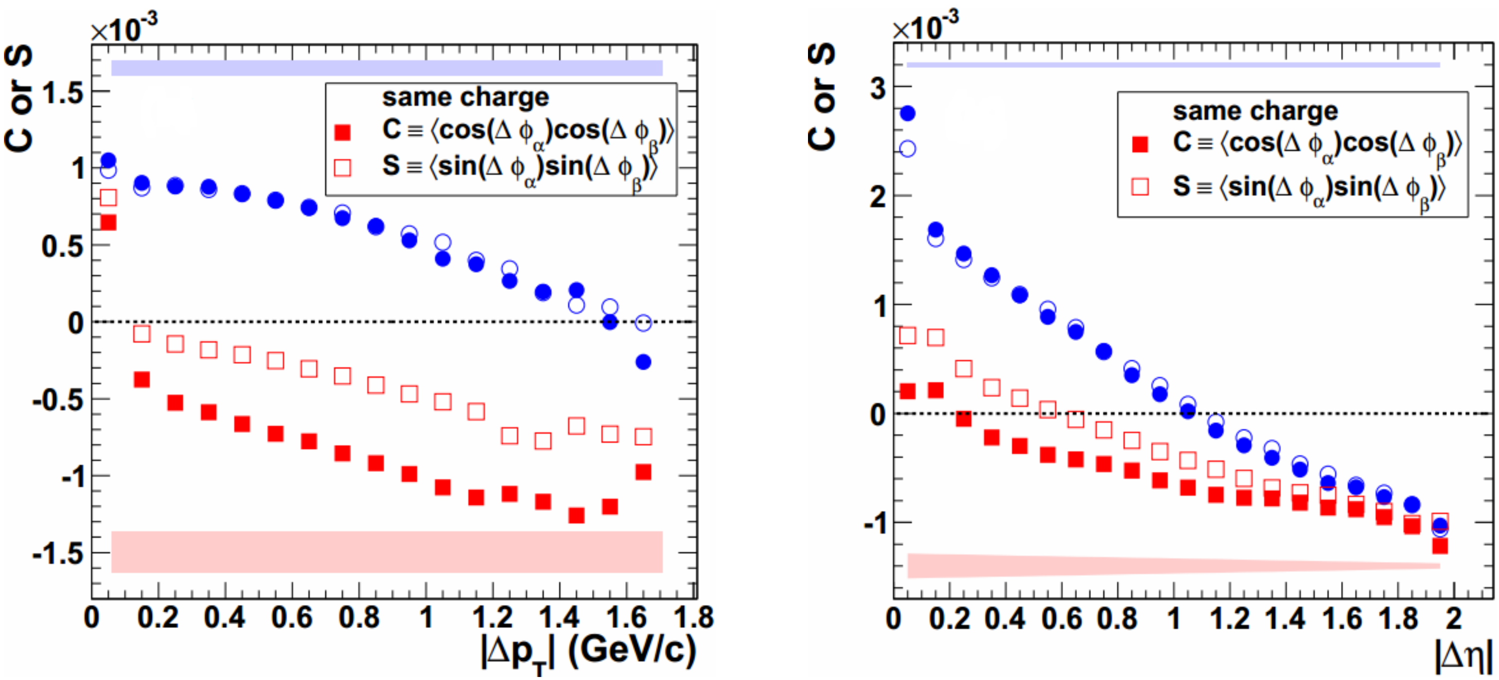
Famous (and confusing…) plot



A Key Question

CME (and related phenomena) alive or dead?

Fine structure of correlations



A Key Question

CME (and related phenomena) alive or dead?

Theory tells…

$$\boldsymbol{j} = N_c \sum_{f=\text{flavor}} \frac{q_f^2 \mu_5}{2\pi^2} \boldsymbol{B}$$

Useful for a practical purpose?

A Key Question

CME (and related phenomena) alive or dead?

Theory tells…

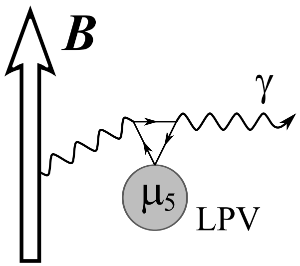
$$j = N_c \sum_{f=\text{flavor}} \frac{q_f^2 \mu_5}{2\pi^2} B$$

Useful for a practical purpose?

NO… unfortunately…

Example

WZW action in χPT



$$\mathcal{L}_{\mathrm{P}} = \frac{N_{\mathrm{c}}\, e^2\, \mathrm{tr}(Q^2)}{8 N_{\mathrm{f}}\, \pi^2}\, \epsilon^{\mu\nu\rho\sigma} \big[\mathcal{A}_\mu(\partial_\nu \mathcal{A}_\rho) + \mathcal{A}_\mu \bar{F}_{\nu\rho}\big]\partial_\sigma \theta$$

$$q_0 \frac{dN_\gamma}{d^3q} = \frac{q_z^2 + q_x^2}{2(2\pi)^3 \boldsymbol{q}^2} \cdot \frac{25\, \alpha_e\, \zeta(\boldsymbol{q})}{9\pi^3}$$

Source for anisotropy

$$\zeta(\boldsymbol{q}) \equiv \left| \int d^4x\, e^{-iq\cdot x} eB(x)\, \mu_5(x) \right|^2$$

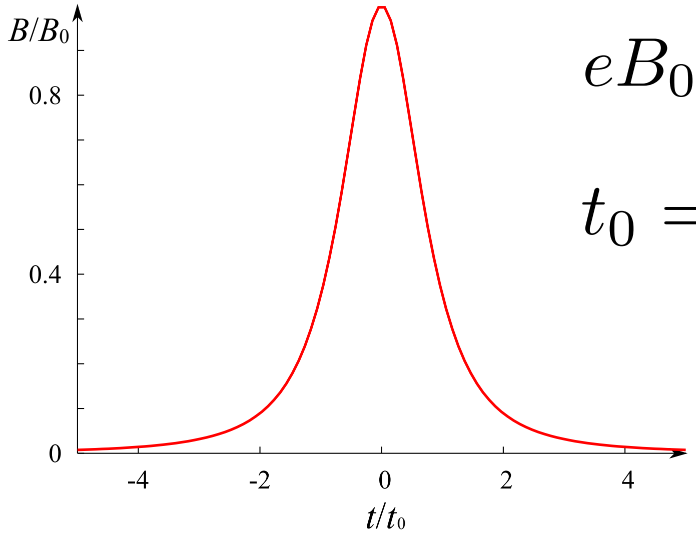
No concrete estimate…

Fukushima-Mameda (2010)
cf. Basar-Kharzeev-Skokov

A Key Question

CME (and related phenomena) alive or dead?

Short-lived magnetic field

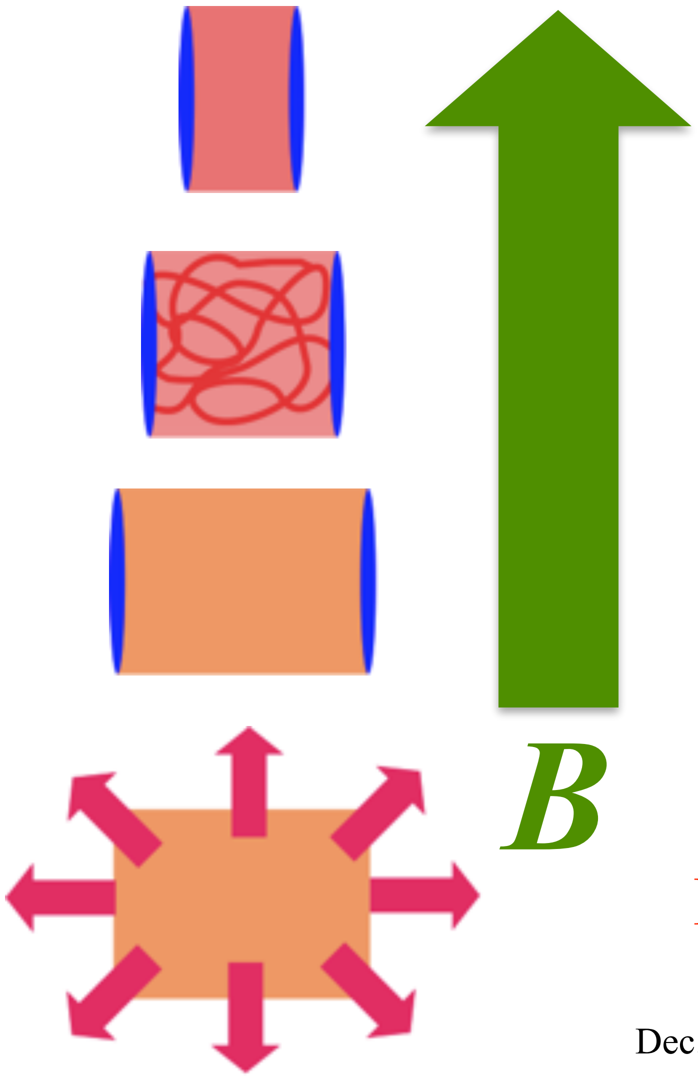


$$eB_0 = (47.6\ \mathrm{MeV})^2 \left(\frac{1\ \mathrm{fm}}{b}\right)^2 Z \sinh Y$$

$$t_0 = \frac{b}{2 \sinh Y}$$

Life-time < 0.1fm/c at most

Short-lived Magnetic Field



Color Glass Condensate (CGC)

$$\tau \lesssim 1/Q_s \sim 0.1 \mathrm{fm/c}$$

Color Glass + Plasma = Glasma

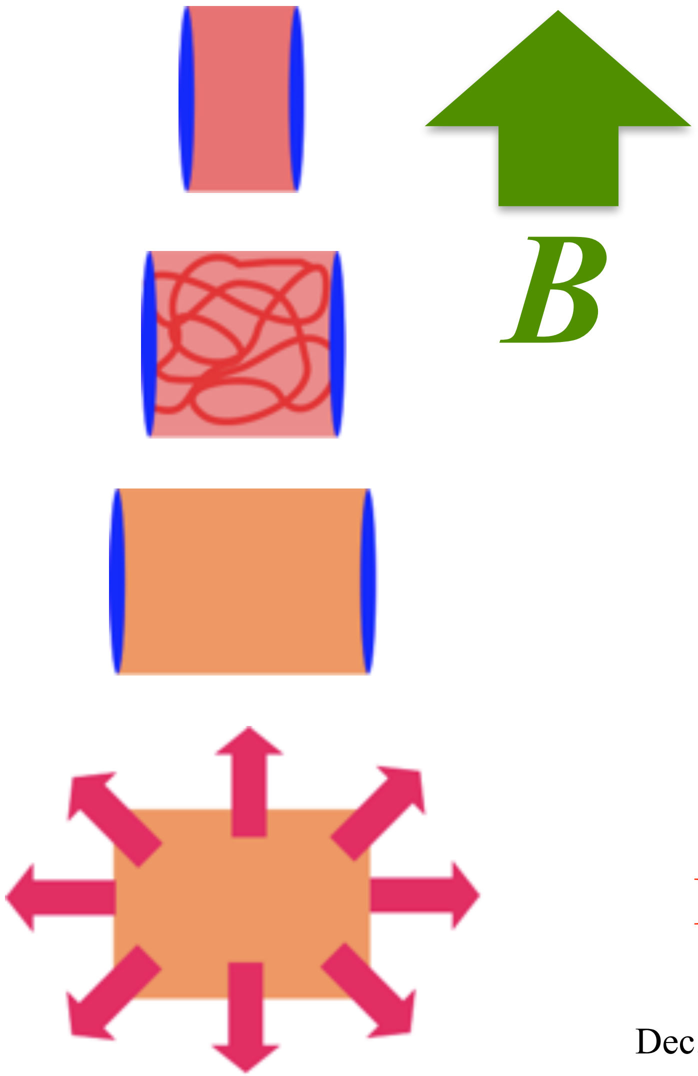
$$\tau \lesssim \tau_0 \sim 1 \mathrm{fm/c}$$

(s) Quark-Gluon Plasma

$$\tau \lesssim \tau_f \sim 10 \mathrm{fm/c}$$

Hadronization (quarks → hadrons)

Short-lived Magnetic Field



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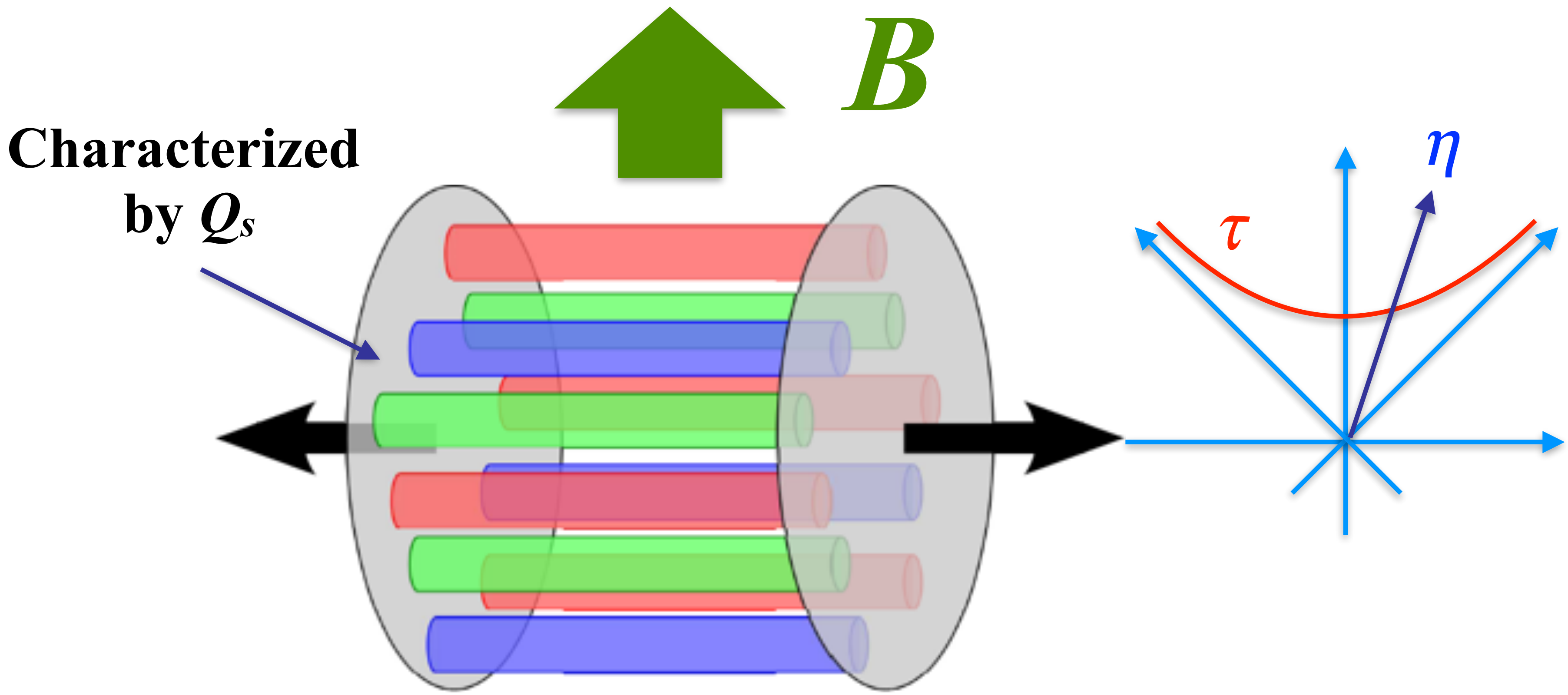
(s) Quark-Gluon Plasma

$$\tau \lesssim \tau_f \sim 10\mathrm{fm/c}$$

Hadronization (quarks → hadrons)

Initial State of HIC



Topological charge density ~ $\mathcal{E}\cdot\mathcal{B}\sim Q_s^4$

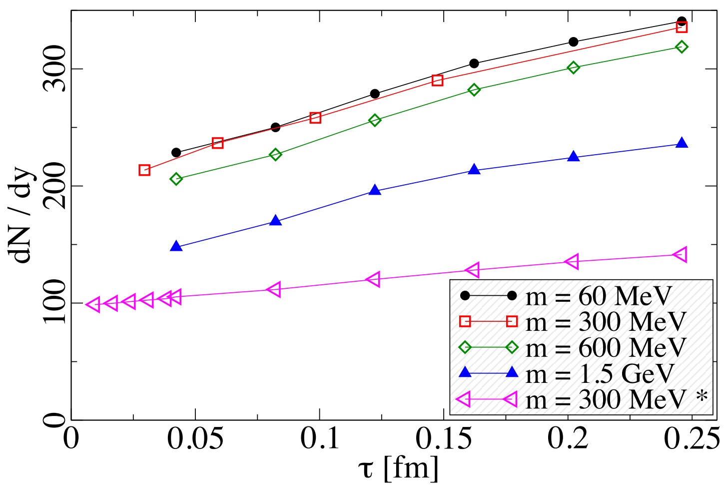
Particle Production in Glasma

Gelis-Kajantie-Lappi (2005)

$$M_\tau(p,q) \equiv \int \frac{\tau \mathrm{d}z \mathrm{d}^2\mathbf{x}_T}{\sqrt{\tau^2+z^2}}\, \phi^\dagger_{\mathbf{p}}(\tau,\mathbf{x})\gamma^0\gamma^\tau\psi_{\mathbf{q}}(\tau,\mathbf{x})$$

Amplitude from anti-particles to particles

$$\frac{dN}{dy} = \int \frac{\mathrm{d}y_p \mathrm{d}^2\mathbf{p}_T}{2\,(2\pi)^3}\frac{\mathrm{d}y_q \mathrm{d}^2\mathbf{q}_T}{2\,(2\pi)^3}\delta\left(y-y_p\right)\left|M_\tau(p,q)\right|^2$$

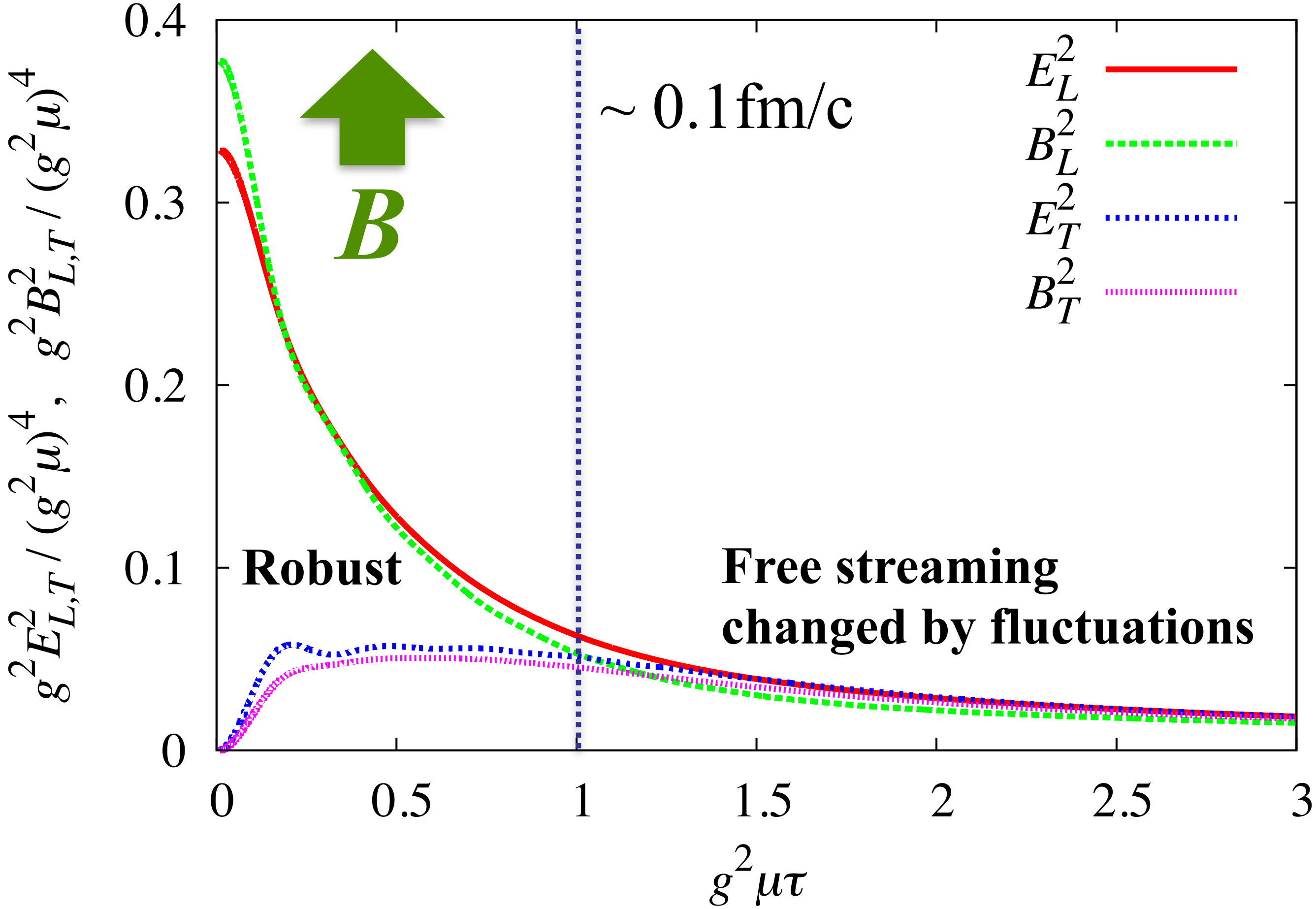


$$\psi_{\mathbf{q}}(t\to-\infty,\mathbf{x}) = e^{iq\cdot x}v(q)$$

$$\phi_{\mathbf{p}}(x) \;=\; e^{-ip\cdot x}u(p)$$

Dominated at $\tau < 0.1$fm/c

Everything happens at τ<0.1fm/c

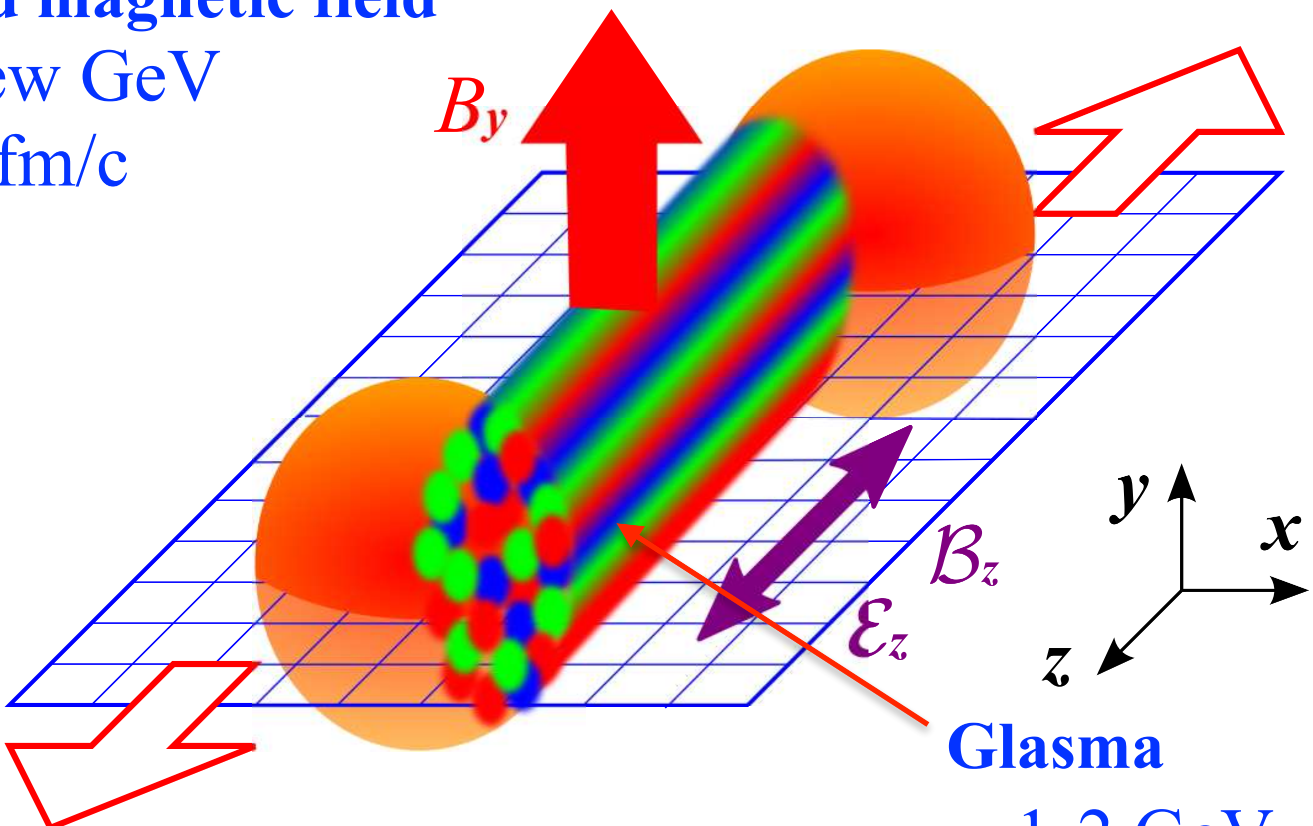


Most Relevant Picture

Pulsed magnetic field

~ a few GeV

< 0.1fm/c



Glasma

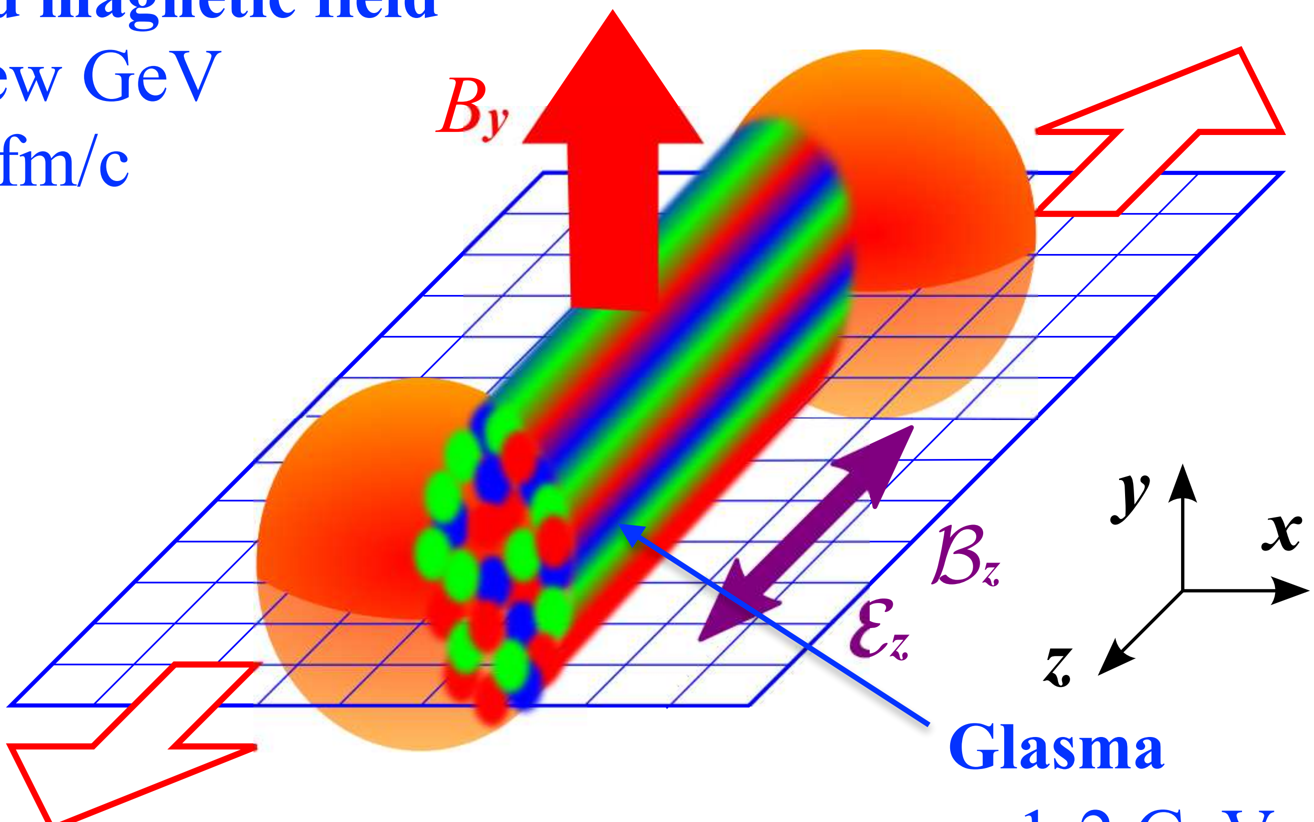
~ 1-2 GeV

< 0.1fm/c

Dominated at short-time scale

Most Relevant Picture

Pulsed magnetic field
~ a few GeV
< 0.1fm/c



Glasma
~ 1-2 GeV
< 0.1fm/c

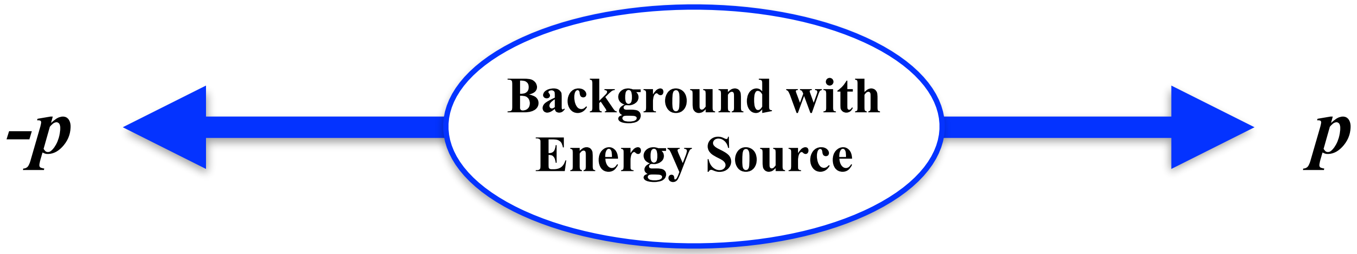
No need for μ_5 (case closed)

An Approach

Particle (current) production with strong fields

Electric Fields

$$\boldsymbol{E} = -\boldsymbol{\nabla}\phi - \partial_t \boldsymbol{A}$$



Pair production when energy conservation satisfied (Schwinger Mechanism)

An Approach

Particle (current) production with strong fields

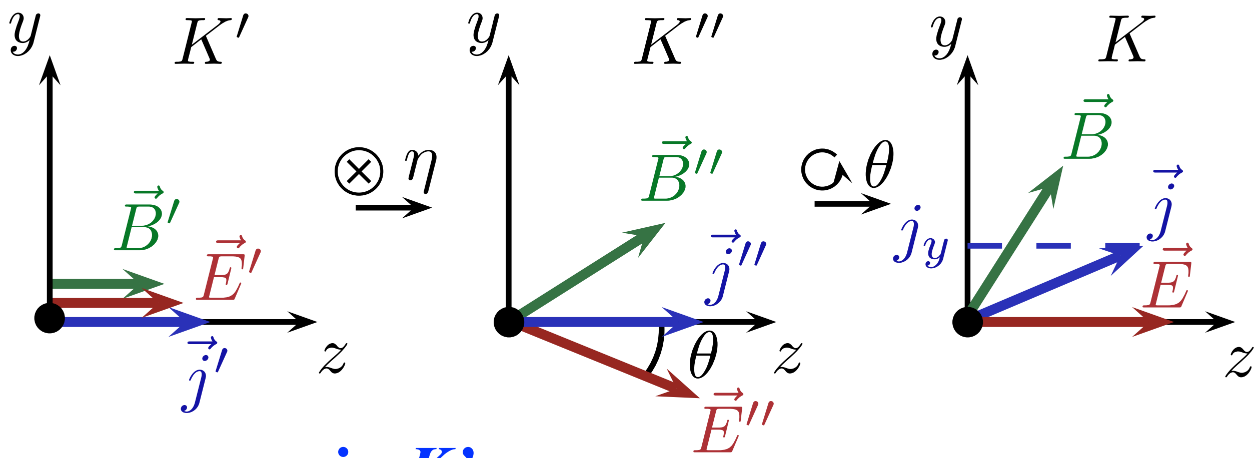
Electromagnetic Fields

$$\boldsymbol{E} = -\boldsymbol{\nabla}\phi - \partial_t \boldsymbol{A} \qquad \boldsymbol{B} = \boldsymbol{\nabla} \times \boldsymbol{A}$$

Net particle production for *R* and *L* fermions

“Carriers” for Hall and CME currents

Analytical Derivation



Schwinger process in *K'*

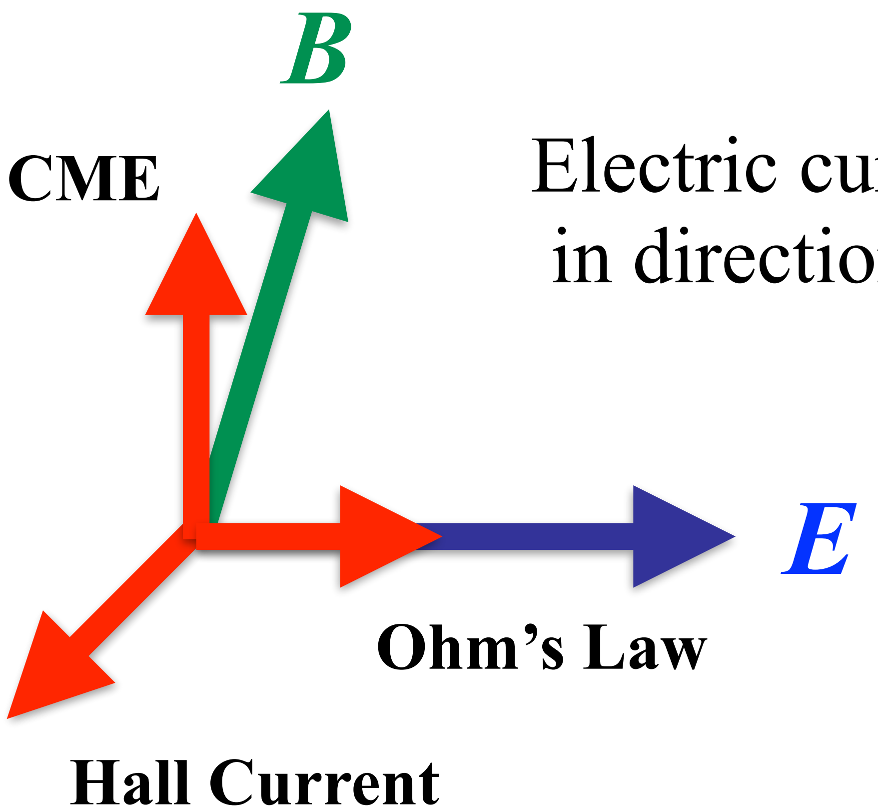
$$\Gamma = \frac{q^2 E'_z B'_z}{4\pi^2} \coth\left(\frac{B'_z}{E'_z}\pi\right) \exp\left(-\frac{m^2\pi}{|qE'_z|}\right)$$

Current generation rate

$$\partial_t j_y \simeq \frac{q^2 B_y}{2\pi^2} \frac{g\mathcal{E}_z \mathcal{B}_z^2}{\mathcal{B}_z^2 + \mathcal{E}_z^2} \coth\left(\frac{\mathcal{B}_z}{\mathcal{E}_z}\pi\right) \exp\left(-\frac{2m^2\pi}{|g\mathcal{E}_z|}\right)$$

Fukushima-Kharzeev-
-Warringa (2010)

Schematic Picture



Electric currents flow
in directions perpendicular to ***E***

Technical Details

Anomalous particle production on the lattice

Nielsen-Ninomiya Theorem
Chiral Symmetry → Doublers → No Anomaly

CME needs Chiral + Anomaly

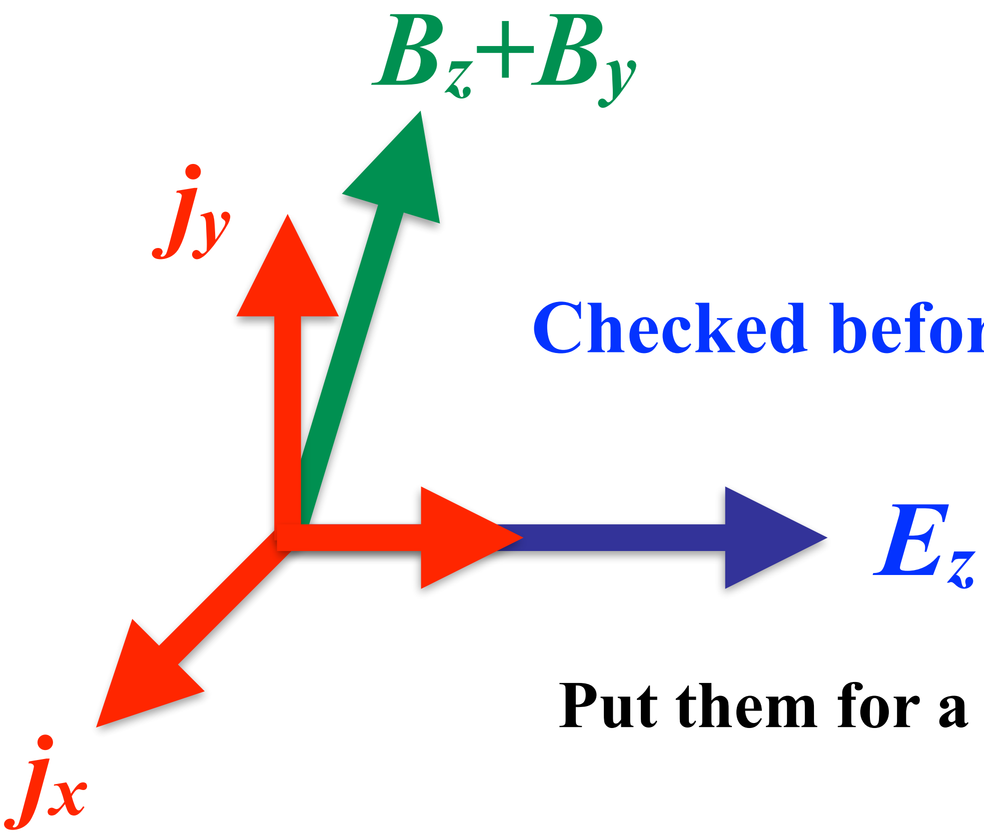
Schwinger pair production is insufficient
Net particle production is indispensable

"Zero" when it should be zero

Should be checked in an ideal (test) setup

Technical Details

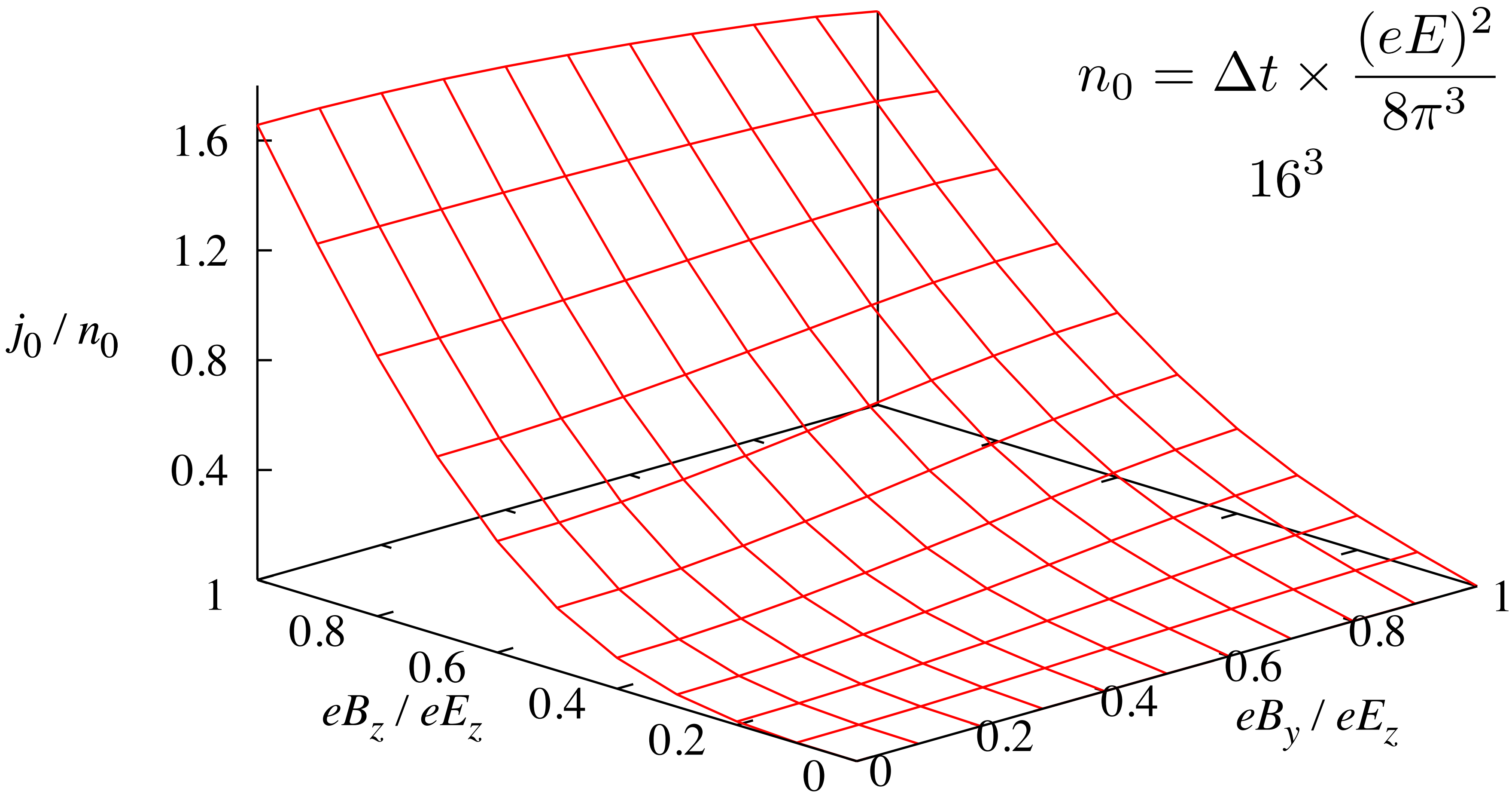
Anomalous particle production on the lattice



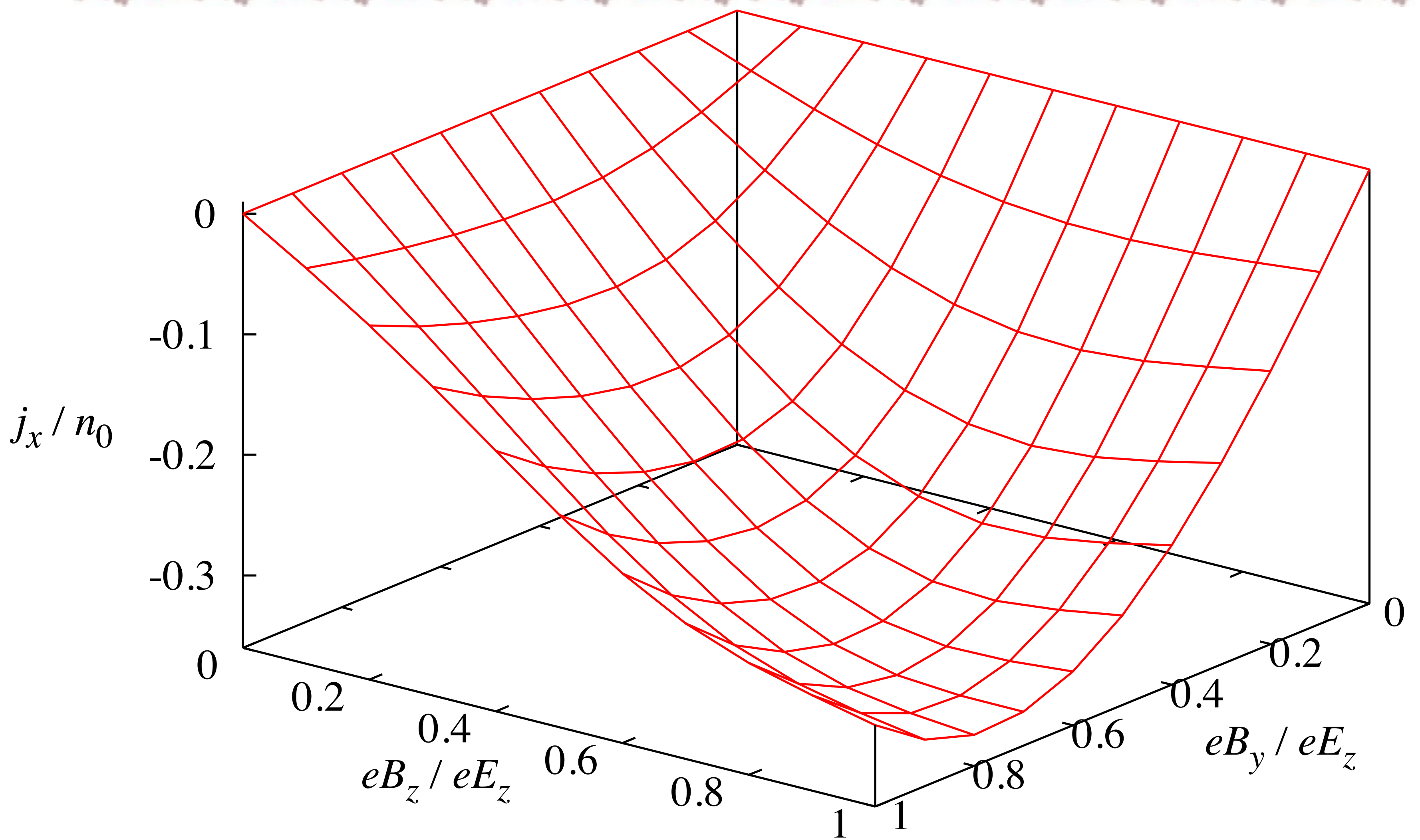
Checked before Glasma simulation

Put them for a finite period (pulse)

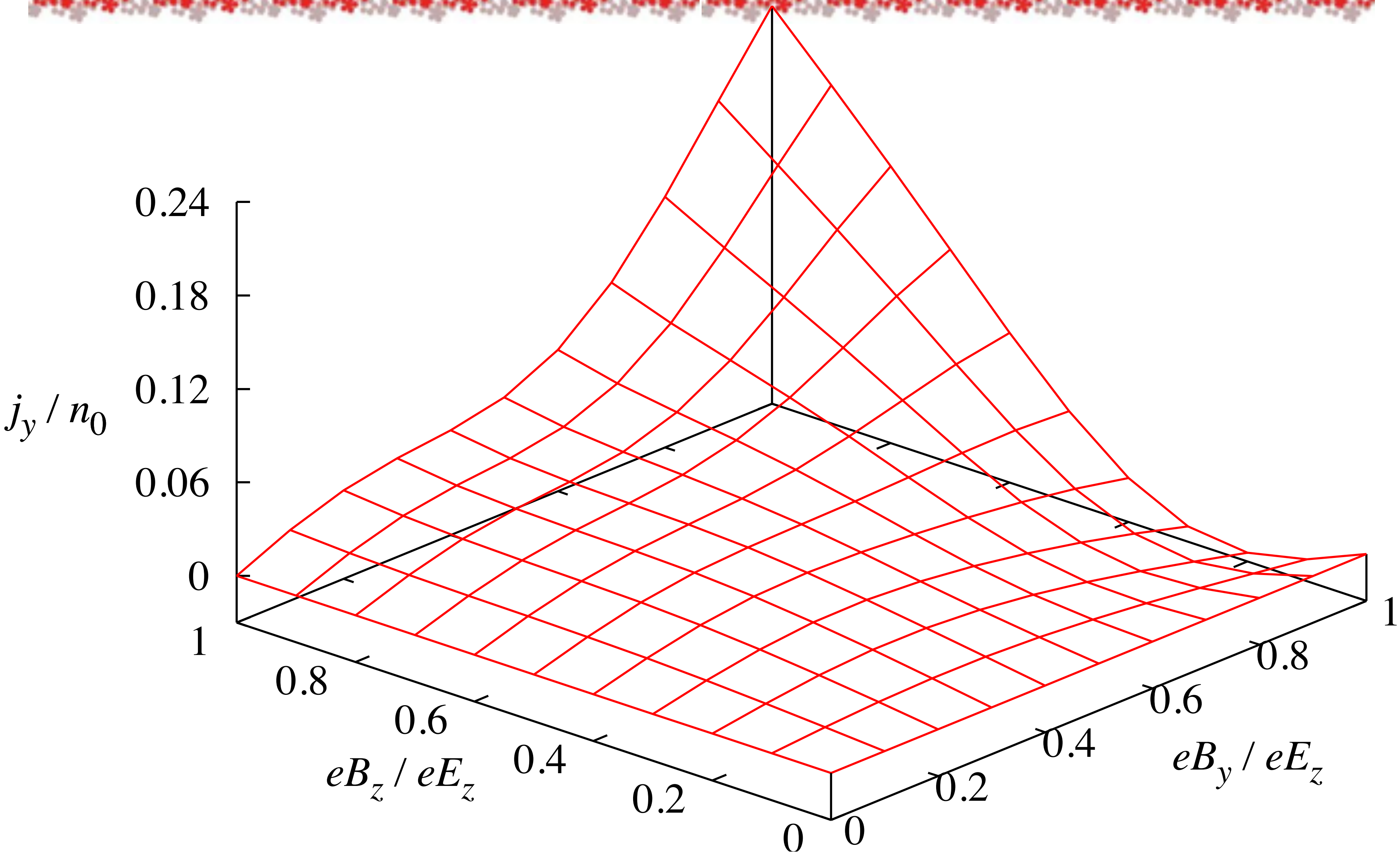
Produced Particle Density



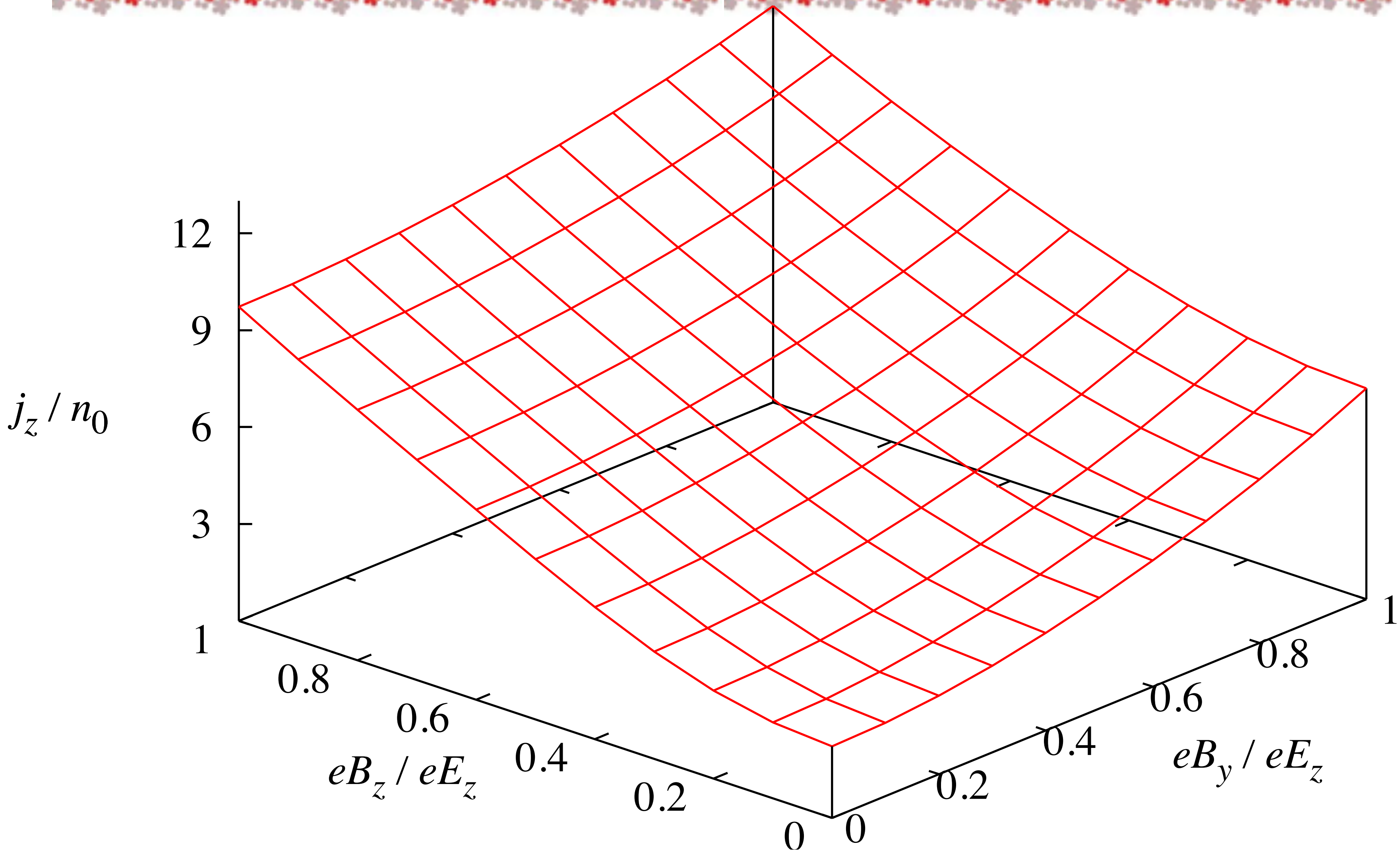
Hall current in the x-direction



CME current in the y-direction



Ohm's current in the z-direction



Implication

Reasonable estimate

- □ Chiral chemical potential ~ Q_s
- □ Particle production → Simultaneous current generation

Apply the formulation for Glasma (in progress)

- □ Particle production in an *expanding system*
- □ No need for sudden turn-off
- □ Singularity at the light cone (cumbersome…)

Observable ?

- □ Not currents but particle distribution in experiments
- □ Distribution in momentum space not gauge invariant
- □ Easy to introduce a baryon chemical potential (BES)

Weyl Fermions

Two-component chiral fermions

$$\left(i\sigma^\mu \partial_\mu - e\sigma^\mu A_\mu\right)\phi_R = 0$$

Free solution (with constant vector potentials)

$$e^{i\theta(\boldsymbol{p}_A)} = \frac{p_A^x + i p_A^y}{\sqrt{(p_A^x)^2 + (p_A^y)^2}}$$

$$u_R(\boldsymbol{p};\boldsymbol{A}) = u_R(\boldsymbol{p}_A = \boldsymbol{p} - e\boldsymbol{A}) = \begin{pmatrix} \sqrt{|\boldsymbol{p}_A| + p_A^z} \\ e^{i\theta(\boldsymbol{p}_A)}\sqrt{|\boldsymbol{p}_A| - p_A^z} \end{pmatrix}$$

Very singular at zero momentum — Berry's phase

Chiral anomaly from monopole singularity

Son-Yamamoto / Stephanov-Yin (2010)

Anti-Particles

$$\left(-i\bar{\sigma}^\mu\partial_\mu - e\bar{\sigma}^\mu A_\mu\right)(-i)\sigma^2\phi_R^* = 0$$

$$u_{\bar{R}}(\boldsymbol{p};\boldsymbol{A}) = u_R(-\boldsymbol{p}_{-A} = -\boldsymbol{p} - e\boldsymbol{A}) = \begin{pmatrix} \sqrt{|\boldsymbol{p}_{-A}| - p^z_{-A}} \\ -e^{i\theta(\boldsymbol{p}_{-A})}\sqrt{|\boldsymbol{p}_{-A}| + p^z_{-A}} \end{pmatrix}$$

Negative-energy components

$$v_R(\boldsymbol{p};\boldsymbol{A}) = i\sigma^2 u^*_{\bar{R}}(\boldsymbol{p};\boldsymbol{A}) = -e^{-i\theta(\boldsymbol{p}_{-A})}u_R(\boldsymbol{p}_{-A})$$

$$v_{\bar{R}}(\boldsymbol{p};\boldsymbol{A}) = -i\sigma^2 u^*_R(\boldsymbol{p}) = -e^{-i\theta(\boldsymbol{p}_A)}u_{\bar{R}}(\boldsymbol{p}_A)$$

Bogoliubov Transformation

$$\hat{\phi}_R(x) = \int \frac{d^3\boldsymbol{p}}{(2\pi)^3} \left(\hat{a}_{\boldsymbol{p}} \, \frac{u_R(\boldsymbol{p}_A)\, e^{-i|\boldsymbol{p}_A|x^0 + i\boldsymbol{p}\cdot x}}{\sqrt{2|\boldsymbol{p}_A|}} + \hat{b}^\dagger_{\boldsymbol{p}} \, \frac{v_R(\boldsymbol{p}_{-A})\, e^{i|\boldsymbol{p}_{-A}|x^0 - i\boldsymbol{p}\cdot x}}{\sqrt{2|\boldsymbol{p}_{-A}|}} \right)$$

No Momentum Mixture (Schwinger Problem)

$$\frac{u_R(\boldsymbol{p}_A) e^{-i|\boldsymbol{p}_A|x^0 + i\boldsymbol{p}\cdot\boldsymbol{x}}}{\sqrt{2|\boldsymbol{p}_A|}} \longrightarrow \alpha_{\boldsymbol{p}} \frac{u_R(\boldsymbol{p}_{A'}) e^{-i|\boldsymbol{p}_{A'}|x^0 + i\boldsymbol{p}\cdot\boldsymbol{x}}}{\sqrt{2|\boldsymbol{p}_{A'}|}} - \beta^*_{-\boldsymbol{p}} \frac{v_R(-\boldsymbol{p}_{A'}) e^{i|\boldsymbol{p}_{A'}|x^0 + i\boldsymbol{p}\cdot\boldsymbol{x}}}{\sqrt{2|\boldsymbol{p}_{A'}|}}$$

$$\frac{v_R(\boldsymbol{p}_{-A}) e^{i|\boldsymbol{p}_{-A}|x^0 - i\boldsymbol{p}\cdot\boldsymbol{x}}}{\sqrt{2|\boldsymbol{p}_{-A}|}} \longrightarrow \alpha^*_{\boldsymbol{p}} \frac{v_R(\boldsymbol{p}_{-A'}) e^{i|\boldsymbol{p}_{-A'}|x^0 - i\boldsymbol{p}\cdot\boldsymbol{x}}}{\sqrt{2|\boldsymbol{p}_{-A'}|}} + \beta_{-\boldsymbol{p}} \frac{u_R(-\boldsymbol{p}_{-A'}) e^{-i|\boldsymbol{p}_{-A'}|x^0 - i\boldsymbol{p}\cdot\boldsymbol{x}}}{\sqrt{2|\boldsymbol{p}_{-A'}|}}$$

Positive- and Negative-energy Coefficients

$$\hat{a}_{\boldsymbol{p}} \longrightarrow \hat{a}'_{\boldsymbol{p}} = \alpha_{\boldsymbol{p}} \hat{a}_{\boldsymbol{p}} + \beta_{\boldsymbol{p}} \hat{b}^\dagger_{-\boldsymbol{p}}, \qquad \hat{b}^\dagger_{\boldsymbol{p}} \longrightarrow \hat{b}'^\dagger_{\boldsymbol{p}} = \alpha^*_{\boldsymbol{p}} \hat{b}^\dagger_{\boldsymbol{p}} - \beta^*_{\boldsymbol{p}} \hat{a}_{-\boldsymbol{p}}$$

Amplitude for Particle Production

Generalization

$$\frac{u_R(\boldsymbol{p}_A)e^{-i|\boldsymbol{p}_A|x^0+i\boldsymbol{p}\cdot\boldsymbol{x}}}{\sqrt{2|\boldsymbol{p}_A|}} \longrightarrow \int\frac{d^3\boldsymbol{q}}{(2\pi)^3}\left[\alpha_{\boldsymbol{q},\boldsymbol{p}}\frac{u_R(\boldsymbol{q}_{A'})e^{-i|\boldsymbol{q}_{A'}|x^0+i\boldsymbol{q}\cdot\boldsymbol{x}}}{\sqrt{2|\boldsymbol{q}_{A'}|}} - \beta^*_{-\boldsymbol{q},-\boldsymbol{p}}\frac{v_R(-\boldsymbol{q}_{A'})e^{i|\boldsymbol{q}_{A'}|x^0+i\boldsymbol{q}\cdot\boldsymbol{x}}}{\sqrt{2|\boldsymbol{q}_{A'}|}}\right],$$

$$\frac{v_R(\boldsymbol{p}_{-A})e^{i|\boldsymbol{p}_{-A}|x^0-i\boldsymbol{p}\cdot\boldsymbol{x}}}{\sqrt{2|\boldsymbol{p}_{-A}|}} \longrightarrow \int\frac{d^3\boldsymbol{q}}{(2\pi)^3}\left[\alpha^*_{\boldsymbol{q},\boldsymbol{p}}\frac{v_R(\boldsymbol{q}_{-A'})e^{i|\boldsymbol{q}_{-A'}|x^0-i\boldsymbol{q}\cdot\boldsymbol{x}}}{\sqrt{2|\boldsymbol{q}_{-A'}|}} + \beta_{-\boldsymbol{q},-\boldsymbol{p}}\frac{u_R(-\boldsymbol{q}_{-A'})e^{-i|\boldsymbol{q}_{-A'}|x^0-i\boldsymbol{q}\cdot\boldsymbol{x}}}{\sqrt{2|\boldsymbol{q}_{-A'}|}}\right]$$

$$f_{\boldsymbol{p}}(x^0\sim-\infty,\boldsymbol{x}) \longrightarrow \frac{v_R(\boldsymbol{p}_{-A})e^{i|\boldsymbol{p}_{-A}|x^0-i\boldsymbol{p}\cdot\boldsymbol{x}}}{\sqrt{2|\boldsymbol{p}_{-A}|}}$$

$$\beta_{\boldsymbol{q},\boldsymbol{p}} = \int d^3\boldsymbol{x}\,\frac{u_R^\dagger(\boldsymbol{q}_{A'})e^{i|\boldsymbol{q}_{A'}|x^0+i\boldsymbol{q}\cdot\boldsymbol{x}}}{\sqrt{2|\boldsymbol{q}_{A'}|}}\,f_{-\boldsymbol{p}}(x^0,\boldsymbol{x})$$

$$|\beta_{\boldsymbol{p}}|^2 = \int\frac{d^3\boldsymbol{q}}{(2\pi)^3}\,|\beta_{\boldsymbol{p},\boldsymbol{q}}|^2$$

cf. Gelis-Kajantie-Lappi

Currents

$$J^\mu = eV\int \frac{d^3\boldsymbol{p}}{(2\pi)^3}\frac{|\beta_{\boldsymbol{p}}|^2}{2|\boldsymbol{p}_{A'}|}u_R^\dagger(\boldsymbol{p}_{A'})\sigma^\mu u_R(\boldsymbol{p}_{A'}) - eV\int \frac{d^3\boldsymbol{p}}{(2\pi)^3}\frac{|\bar\beta_{\boldsymbol{p}}|^2}{2|\boldsymbol{p}_{-A'}|}u_{\bar R}^\dagger(\boldsymbol{p}_{-A'})\bar\sigma^\mu u_{\bar R}(\boldsymbol{p}_{-A'})$$

$$J^0/eV = \int \frac{d^3\boldsymbol{p}}{(2\pi)^3}\left(|\beta_{\boldsymbol{p}}|^2 - |\bar\beta_{\boldsymbol{p}}|^2\right), \qquad \boldsymbol{J}/eV = \int \frac{d^3\boldsymbol{p}}{(2\pi)^3}\left(\frac{\boldsymbol{p}_{A'}}{|\boldsymbol{p}_{A'}|}|\beta_{\boldsymbol{p}}|^2 - \frac{\boldsymbol{p}_{-A'}}{|\boldsymbol{p}_{-A'}|}|\bar\beta_{\boldsymbol{p}}|^2\right)$$

Use naive fermion with momentum integration only in one Brillouin zone (no doublers)

Chiral limit with Wilson fermion is very non-trivial (at the edge of the Aoki phase)