Classical gauge field picture of the initial stage in A+A collisions

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Ab-initio approaches in many-body QCD,
Heidelberg, December 2014

Outline

- Introduction: CGC, Glasma
- JIMWLK evolution in Langevin form
- Classical Yang-Mills fields in the initial stage
  T.L., [arXiv:1105.5511], PLB 2011
- Wilson loop in glasma with MV or JIMWLK initial conditions
  Dumitru, T.L., Nara [arXiv:1401.4124], PLB 2014

Comments:

- This talk is purely 2+1d boost-invariant.
- Only $Q_s\tau \lesssim 10$
- Starting point for isotropization

Gluon saturation, Glass and Glasma

Small x: the hadron/nucleus wavefunction is characterized by **saturation scale** $Q_s \gg \Lambda_{QCD}$.

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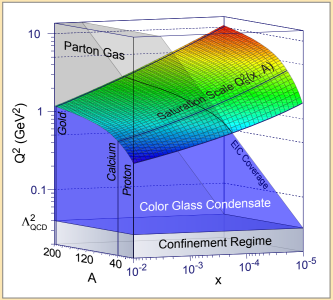
p $\sim Q_s$: strong fields $A_\mu \sim 1/g$

- occupation numbers $\sim 1/\alpha_s$
- classical field approximation.
- small α_s, but nonperturbative



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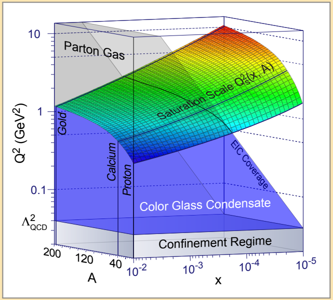

CGC: Effective theory for wavefunction of nucleus

- Large x = source ρ, **probability** distribution $W_y[\rho]$
- Small x = classical gluon field A_μ + quantum flucts.

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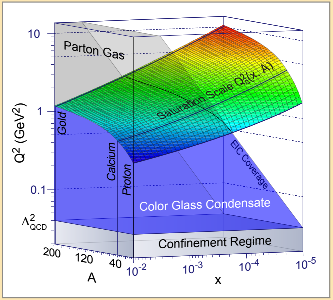

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- Small x = classical gluon field A_μ + quantum flucts.

Glasma: field configuration of two colliding sheets of CGC.
JIMWLK: y-dependence of $W_y[\rho]$; Langevin implementation

Wilson line

Classical color field described as Wilson line

Eikonal propagation of high energy probe in color field:

$$U(\mathbf{x}) = P \exp\left\{ ig \int \mathrm{d}x^- A^+_{\mathrm{cov}}(\mathbf{x}, x^-) \right\} \quad \in \quad \mathrm{SU}(3)$$

$$\text{Color charge } \rho : \quad \nabla^2 A^+_{\mathrm{cov}}(\mathbf{x}, x^-) = -g\rho(\mathbf{x}, x^-)$$

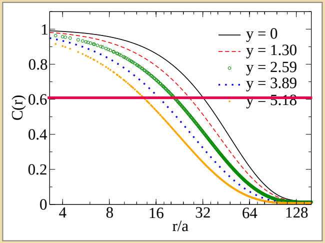
($x^\pm = \frac{1}{\sqrt{2}}(t \pm z)$; $A^\pm = \frac{1}{\sqrt{2}}(A^0 \pm A^z)$; $\mathbf{x}$ 2d transverse)

Q_s is characteristic momentum/distance scale

Precise definition used here:

$$\frac{1}{N_c} \left\langle \mathrm{Tr}\, U^\dagger(\mathbf{0}) U(\mathbf{x}) \right\rangle = e^{-\frac{1}{2}}$$

$$\Longleftrightarrow \mathbf{x}^2 = \frac{2}{Q_s^2}$$



JIMWLK evolution

Classical color field described as Wilson line

$$U(\mathbf{x}) = P\exp\left\{ig\int \mathrm{d}x^- A^+(\mathbf{x}, x^-)\right\} \in \mathrm{SU}(3)$$

- Energy dependent **probability** distribution $W_y[U]$ $(y \sim \ln\sqrt{s})$
- Energy/rapidity dependence of $W_y[U]$ given by JIMWLK renormalization group equation

$$\partial_y W_y[U(\mathbf{x})] = \mathcal{H} W_y[U(\mathbf{x})]$$

- Then get all expectation values $\langle U \cdots U^\dagger \rangle$

JIMWLK Hamiltonian: (fixed coupling)

$$\mathcal{H} \equiv \frac{1}{2}\alpha_s \int_{\mathbf{xyz}} \frac{\delta}{\delta A_c^+(\mathbf{y})} \mathbf{e}^{ba}(\mathbf{x},\mathbf{z}) \cdot \mathbf{e}^{ca}(\mathbf{y},\mathbf{z}) \frac{\delta}{\delta A_b^+(\mathbf{x})},$$

$$\mathbf{e}^{ba}(\mathbf{x},\mathbf{z}) = \frac{1}{\sqrt{4\pi^3}} \frac{\mathbf{x}-\mathbf{z}}{(\mathbf{x}-\mathbf{z})^2} \left(1 - U^\dagger(\mathbf{x}) U(\mathbf{z})\right)^{ba}$$

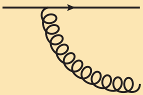
Langevin formulation

Fokker-Planck $\Longrightarrow$ Langevin in JIMWLK Blaizot, Iancu, Weigert 2002

Simple form for Langevin step

$$U_{\mathbf{x}}(y + \mathrm{d}y) = \exp\left\{ -i\frac{\sqrt{\alpha_s\,\mathrm{d}y}}{\pi} \int_{\mathbf{z}} \mathbf{K}_{\mathbf{x}-\mathbf{z}} \cdot (U_{\mathbf{z}}\xi_{\mathbf{z}}U^{\dagger}_{\mathbf{z}}) \right\}$$
$$\times\, U_{\mathbf{x}}(y) \exp\left\{ i\frac{\sqrt{\alpha_s\,\mathrm{d}y}}{\pi} \int_{\mathbf{z}} \mathbf{K}_{\mathbf{x}-\mathbf{z}} \cdot \xi_{\mathbf{z}} \right\},$$

$$K^i_{\mathbf{x}-\mathbf{z}} = \frac{(\mathbf{x}-\mathbf{z})^i}{(\mathbf{x}-\mathbf{z})^2}$$



$i = x, y$

Noise: $\langle \xi_{\mathbf{x}}(y_m)^a_i \xi_{\mathbf{y}}(y_n)^b_j \rangle = \alpha_s \delta^{ab}\delta^{ij}\delta^{(2)}_{\mathbf{xy}}\delta_{mn}, \quad \xi = \xi^a t^a$

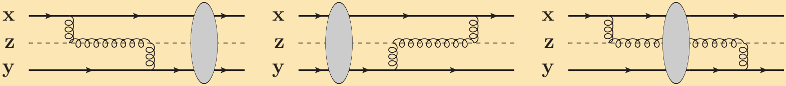
More recent developments not discussed here:

- Fixed $\Longrightarrow$ running α_s: proposal by T.L., Mäntysaari 2012
- Full NLO Balitsky, Chirilli 2013, Kovner, Lublinsky, Mulian 2014
  - NLO BFKL/BK problematic, treatment for JIMWLK not obvious.

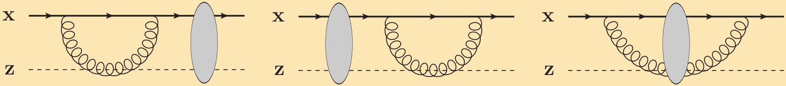
Interpreting JIMWLK: derive BK

$$U_{\mathbf{x}}(y + \mathrm{d}y) = e^{-i\frac{\sqrt{\alpha_s\,\mathrm{d}y}}{\pi}\int_{\mathbf{z}}\mathbf{K}_{\mathbf{x}-\mathbf{z}}\cdot(U_{\mathbf{z}}\boldsymbol{\xi}_{\mathbf{z}}U^{\dagger}_{\mathbf{z}})}\,U_{\mathbf{x}}\,e^{i\frac{\sqrt{\alpha_s\,\mathrm{d}y}}{\pi}\int_{\mathbf{z}}\mathbf{K}_{\mathbf{x}-\mathbf{z}}\cdot\boldsymbol{\xi}_{\mathbf{z}}},$$

- At $\mathrm{d}y \to 0$ develop to $\mathcal{O}(\xi^2)$ and take expectation values.
- BK Balitsky-Kovchegov is equation for **dipole** $\hat{D}_{\mathbf{x},\mathbf{y}} = \mathrm{Tr}\, U^{\dagger}(\mathbf{x})U(\mathbf{y})/N_c$
- Contract ξ's from timestep of $U^{\dagger}(\mathbf{x})$ with one from $U(\mathbf{y})$: real terms



- Contract two ξ's from timestep of $U^{\dagger}(\mathbf{x})$ or $U(\mathbf{y})$: virtual terms

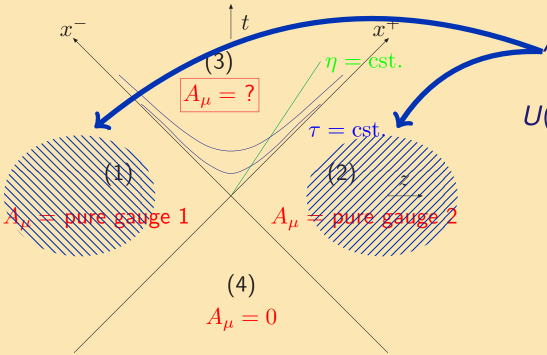


- Result is BK equation:

$$\partial_y \hat{D}_{\mathbf{x},\mathbf{y}}(y) = \frac{\alpha_s N_c}{2\pi^2}\int_{\mathbf{z}}\left(\mathbf{K}^2_{\mathbf{x}-\mathbf{z}} + \mathbf{K}^2_{\mathbf{y}-\mathbf{z}} - 2\mathbf{K}_{\mathbf{x}-\mathbf{z}}\cdot\mathbf{K}_{\mathbf{y}-\mathbf{z}}\right)\left[\hat{D}_{\mathbf{x},\mathbf{z}}\hat{D}_{\mathbf{z},\mathbf{y}} - \hat{D}_{\mathbf{x},\mathbf{y}}\right].$$

Gluon fields in AA collision

Classical **Y**ang-**M**ills



Change to LC gauge:

$$A^i_{(1,2)} = \frac{i}{g} U_{(1,2)}(\mathbf{x}) \partial_i U^\dagger_{(1,2)}(\mathbf{x})$$

$U(\mathbf{x})$ is the same Wilson line

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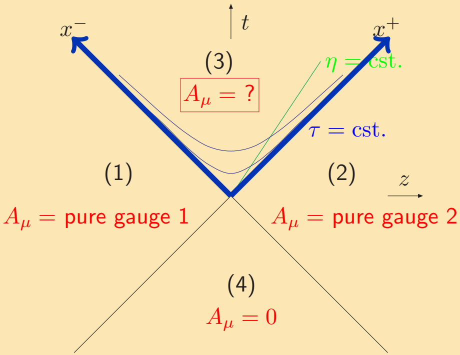

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At $\tau = 0$:

$$A^i\Big|_{\tau=0} = A^i_{(1)} + A^i_{(2)}$$

$$A^\eta\big|_{\tau=0} = \frac{ig}{2}[A^i_{(1)}, A^i_{(2)}]$$

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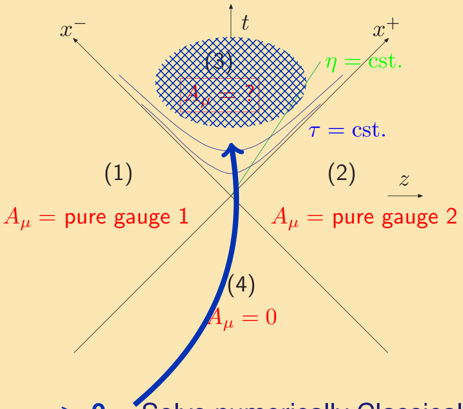

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$\boldsymbol{\tau > 0}$ Solve numerically Classical Yang-Mills **CYM** equations.
This is the **glasma** field $\Longrightarrow$ Then average over initial Wilson lines.

Fix gauge, Fourier-decompose: gluon spectrum

Gluons with $p_T \sim Q_s$ — strings of size $R \sim 1/Q_s$

Gluon spectrum in the glasma

T.L., *Phys.Lett.* **B703** (2011) 325

Q_s is only dominant scale

Parametrically gluon spectrum $$\frac{dN_g}{dy\, d^2\mathbf{x}\, d^2\mathbf{p}} = \frac{1}{\alpha_s} f\left(\frac{p_T}{Q_s}\right)$$

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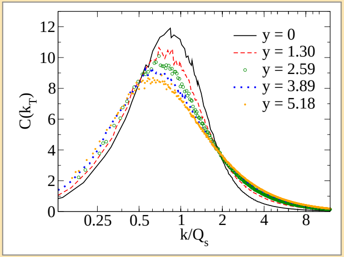

Unintegrated gluon distribution

$$C(\mathbf{k}) = \frac{{k_T}^2}{N_c} \mathrm{Tr} \left\langle U(\mathbf{k}) U^\dagger(\mathbf{k}) \right\rangle$$

becomes **harder** with evolution.

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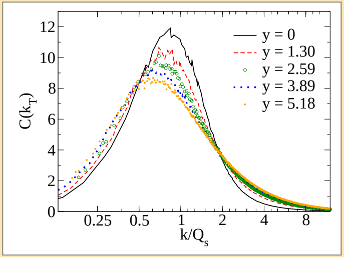

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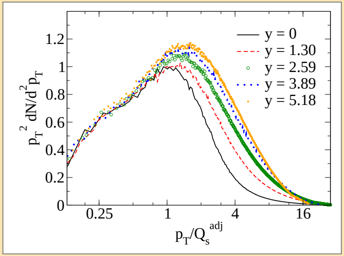
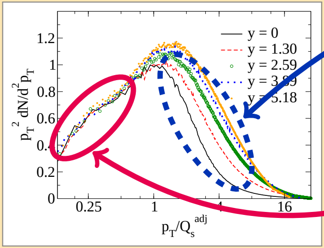


Produced gluon spectrum: harder at higher $\sqrt{s}$

(Here: midrapidity, $y \equiv \ln \sqrt{s/s_0}$)

Universality in the IR spectrum?



- Gluon spectrum in the UV depends on anomalous dimension $\Longrightarrow$ different for MV $(y=0)$, JIMWLK $(y>0)$
- IR seems to **scale**, close to

$$\frac{dN}{d^2\mathbf{p}} \sim \frac{1}{p_T}$$

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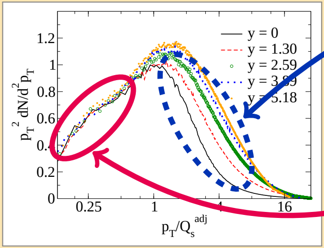

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Gauge inv. probe for $p_T \lesssim Q_s$?

Spatial Wilson loop

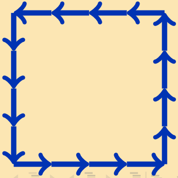
$$W(A) = \frac{1}{N_c} \operatorname{Tr} \mathbb{P} \exp\left\{ ig \oint_A d\mathbf{x} \cdot \mathbf{A} \right\}$$

A = area inside loop

2d lattice: transverse links:

$$\uparrow = U_i(\mathbf{x}) = \exp\{igaA_i\}$$

$$W(A) = \frac{1}{N_c} \operatorname{Tr}$$



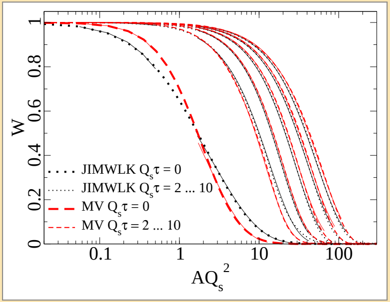
Measure Wilson loops

Dumitru, Nara, Petreska PRD 2013 & Dumitru, T.L., Nara PLB 2014

Calculation is simple:

- Construct initial glasma fields at $\tau = 0$ using e.g.
  - MV model
  - rcJIMWLK
  - fcJIMWLK

  (Try to have same $Q_s a$ to minimize lattice effects)
- Evolve forward in τ
- Measure $W(A)$ (A=area)



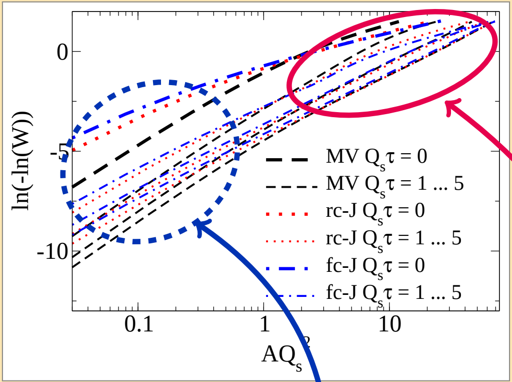
Behavior in both UV ($AQ_s^2 \lesssim 1$) and IR ($AQ_s^2 \gtrsim 1$) parametrized as

$$W = \exp\{-(\sigma A)^\gamma\}$$

Fit is quite good: solid lines in figure.

Fit to Wilson loop area dependence

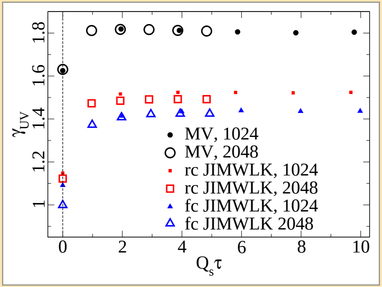
$$W = \exp\{-(\sigma A)^\gamma\} \iff \ln(-\ln W) = \gamma \ln(AQ_s^2) + \gamma \ln(\sigma/Q_s^2)$$

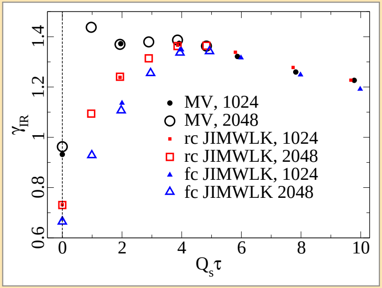


Main observations

- UV (small loop): initial slope γ stays
- IR (big loop): all init. conditions collapse to universal behavior

Wilson loop scaling exponents





UV ($e^{-3.5} < AQ_s^2 < e^{-0.5}$)

Remembers initial condition

IR ($e^{0.5} < AQ_s^2 < e^{5}$)

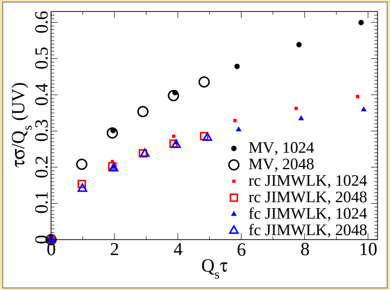
Initial conditions collapse to $\gamma_{IR} \approx 1.2$, decreasing slowly with τ

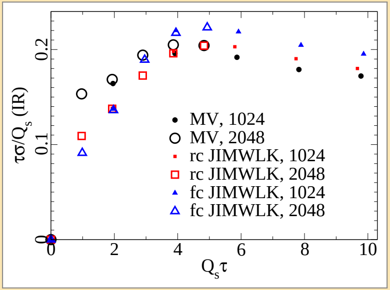
“String tension” coefficients

In expanding system fields naturally decrease as

$$\tau \gg 1/Q_s \implies A_\mu \sim 1/\sqrt{\tau} \implies \sigma/Q_s^2 \sim 1/(Q_s\tau)$$

Plot “string tension” σ as scaling variable $\sigma\tau/Q_s$





UV: initial conditions differ

IR: even σ universal within $\sim$ 10%

(Note: the numerical value of σ/Q_s^2 depends on the convention used to define Q_s)

At $\tau = 0$: $\sigma/Q_s^2 \approx 0.55 \ldots 0.6$ (UV) and $\sigma/Q_s^2 \approx 0.35 \ldots 0.45$ (IR)

Magnetic field correlator

Wilson loop measures magnetic flux:

$$W(A) = \frac{1}{N_c} \mathrm{Tr}\, \mathbb{P} \exp\left\{ ig \oint_A \mathrm{d}\mathbf{x} \cdot \mathbf{A} \right\} = \frac{1}{N_c} \mathrm{Tr} \exp\left\{ ig \int \mathrm{d}^2\mathbf{x} B_z(\mathbf{x}) \right\}$$

If magnetic field consists of **uncorrelated Gaussian domains**:

$$\langle W(A) \rangle = \exp\left\{ -\frac{1}{2}\frac{1}{N_c} \mathrm{Tr} \left\langle \left[\int \mathrm{d}^2\mathbf{x} g B_z(\mathbf{x}) \right]^2 \right\rangle \right\}$$

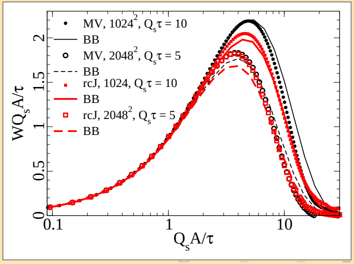
$\Longrightarrow$ $W(A)$ related to $\langle B(\mathbf{x}) B(\mathbf{y}) \rangle$

Here: no gauge fixing, but connect $B(\mathbf{x})$ and $B(\mathbf{y})$ with gauge link

Check: compare

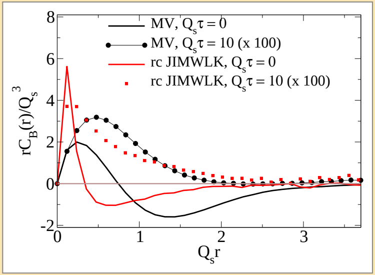
- Direct measurement of $W(A)$
- Reconstruction from BB-correlator

good agreement.



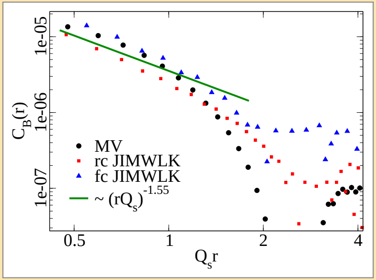
Magnetic field correlator

However: no obvious scaling seen in *BB*-correlator



Same on log plot

$$C(|\mathbf{x}-\mathbf{y}|) \equiv \mathrm{Tr}\left\langle [B(\mathbf{x})B(\mathbf{y})]_{\text{gauge link}} \right\rangle$$



Straight line: $\sim (rQ_s)^{-1.55}$.

(For $C(r) \sim (rQ_s)^{-\alpha}$ one would get $\gamma = 2 - \alpha/2 \iff \alpha = 4 - 2\gamma$;

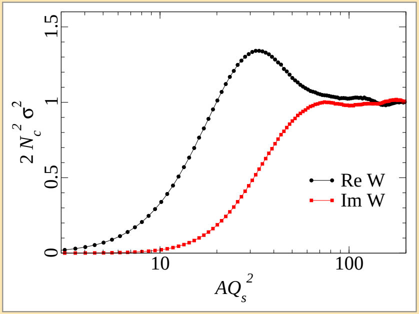
from $W(A)$ measured $\gamma = 1.22$)

Wilson loop fluctuations and eigenvalue distributions

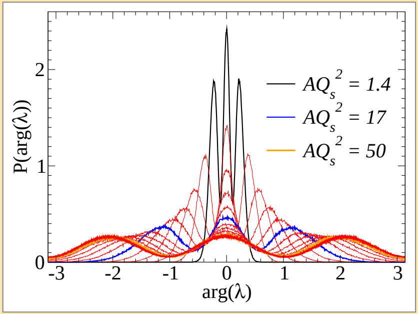
Work in progress, Dumitru, T.L., Nara

How are the Wilson lines distributed in SU(3)?

Fluctuations of Re W and Im W



Eigenvalue λ phase distribution:



For large areas A both look like random SU(3) matrices:

$$\sigma^2(\mathrm{Re}\,W) = \sigma^2(\mathrm{Im}\,W) = \frac{1}{2N_c^2} \qquad P(\varphi \equiv \arg(\lambda)) = \frac{1}{2\pi}\left(1 + \frac{2}{3}\cos 3\varphi\right)$$

Conclusions

- CYM initial state for AA collision
- Universal behavior in the for $p_T \ll Q_s$ seen in gluon spectrum
- Same universality seen in spatial Wilson loop
  - Slightly nontrivial area dependence $W \sim \exp\{-A^{1.2}\}$
- Note: this is still in the boost-invariant 2d theory.
  - Effect of instabilities, isotropization on the soft modes?

Fokker-Planck and Langevin

Textbook example: two descriptions of Brownian motion

- 1-d diffusion eq. ($\supset$ F.-P. eq.)

$$\partial_t P(x,t) = D\partial_x^2 P(x,t)$$

- $P(x,t)$=probability for particle to be at location x at time t.
- For $x = 0$ at $t = 0$ solution:

$$P(x,t) = \frac{1}{\sqrt{4\pi Dt}} \exp\left\{-\frac{x^2}{4Dt}\right\}$$

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- Langevin equation:

$$\begin{aligned} x(t) &= \sqrt{2D}\eta(t) \\ \langle \eta(t)\eta(t')\rangle &= \delta(t-t') \end{aligned}$$

- $$\begin{aligned} \langle x(t)\rangle &= 0 \\ \langle x^2(t)\rangle &= 2Dt \\ \langle x(t)x(t')\rangle &= 2D\min(t,t') \end{aligned}$$

  $\Longrightarrow$ same as F.-P.

1d Brownian motion to JIMWLK

- Replace $x \Longrightarrow U(\mathbf{x})$ and $t \Longrightarrow y$.
- Constant $D \Longrightarrow$ nonlinearity (U-dependence) in kernel
- $({N_c}^2 - 1)N_\perp^2$-dimensional nonlinear diffusion equation.
  ($N_\perp^2$ = number of lattice points in transverse plane.)

Scale of running α_s in JIMWLK

BK for $\hat{D}_{\mathbf{x},\mathbf{y}}(y)$ describes dipole splitting $\mathbf{x}-\mathbf{y} \quad \longrightarrow \quad \mathbf{x}-\mathbf{z}\ ;\ \mathbf{z}-\mathbf{y}$

- α_s given by parent $\mathbf{x}-\mathbf{y}$: easy in BK, but funny in JIMWLK: Langevin is only for one Wilson line
- Daughter (scale in $\mathbf{K}$): easy to implement as $\sqrt{\alpha_s}$, but why?

$$\sqrt{\alpha_s}\mathbf{K}_{\mathbf{x}-\mathbf{z}} \to \sqrt{\alpha_s(\mathbf{x}-\mathbf{z})}\mathbf{K}_{\mathbf{x}-\mathbf{z}}$$

- Used in BK: combinations of these two.
- Suggestion T.L., H.Mäntysaari 2012 : natural scale is momentum of radiated gluon.
- Implemented by modifying momentum space noise correlator

$$\langle \xi_{\mathbf{x}}(m)_i^a \xi_{\mathbf{y}}(n)_j^b \rangle \sim \alpha_s \delta^{(2)}_{\mathbf{xy}} = \alpha_s \int \frac{d^2\mathbf{k}}{(2\pi)^2} e^{i\mathbf{k}\cdot(\mathbf{x}-\mathbf{y})}$$

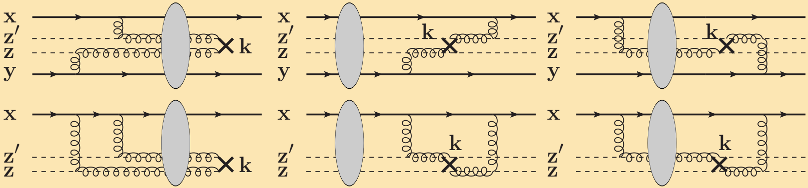
$$\Longrightarrow \int \frac{d^2\mathbf{k}}{(2\pi)^2} e^{i\mathbf{k}\cdot(\mathbf{x}-\mathbf{y})} \alpha_s(\mathbf{k})$$

Reinterpreting JIMWLK

$$U_{\mathbf{x}}(y+\mathrm{d}y) = \exp\left\{-i\frac{\sqrt{\mathrm{d}y}}{\pi}\int_{\mathbf{z}} \mathbf{K}_{\mathbf{x}-\mathbf{z}}\cdot(U_{\mathbf{z}}\xi_{\mathbf{z}}U^{\dagger}_{\mathbf{z}})\right\}$$

$$\times\, U_{\mathbf{x}}(y)\ \exp\left\{i\frac{\sqrt{\mathrm{d}y}}{\pi}\int_{\mathbf{z}'} \mathbf{K}_{\mathbf{x}-\mathbf{z}'}\cdot\xi_{\mathbf{z}'}\right\},$$

$$\langle \xi_{\mathbf{x}}(m)^a_i \xi_{\mathbf{y}}(n)^b_j\rangle \sim \int \frac{\mathrm{d}^2\mathbf{k}}{(2\pi)^2} e^{i\mathbf{k}\cdot(\mathbf{x}-\mathbf{y})}\alpha_{\mathrm{s}}(\mathbf{k}) \equiv \widetilde{\alpha}_{\mathbf{x}-\mathbf{y}}$$



- Breaks time-reversal-symmetry: choose scale as momentum of gluon either before or after the target
- Two gluon coordinates instead of one

Recovering BK

- Equation for dipole now involves higher point functions:

$$\partial_y \hat{D} = \frac{N_c}{2\pi^2} \int_{\mathbf{u},\mathbf{v}} \widetilde{\alpha}_{\mathbf{u}-\mathbf{v}} \Big(\mathbf{K}_{\mathbf{x}-\mathbf{u}} \cdot \mathbf{K}_{\mathbf{x}-\mathbf{v}} + \mathbf{K}_{\mathbf{y}-\mathbf{u}} \cdot \mathbf{K}_{\mathbf{y}-\mathbf{v}} - 2\mathbf{K}_{\mathbf{x}-\mathbf{u}} \cdot \mathbf{K}_{\mathbf{y}-\mathbf{v}} \Big) \times \frac{1}{2} \left[\hat{D}_{\mathbf{x},\mathbf{u}} \hat{D}_{\mathbf{u},\mathbf{y}} + \hat{D}_{\mathbf{x},\mathbf{v}} \hat{D}_{\mathbf{v},\mathbf{y}} - \hat{D}_{\mathbf{x},\mathbf{y}} - \hat{D}_{\mathbf{v},\mathbf{u}} \hat{Q}_{\mathbf{x},\mathbf{v},\mathbf{u},\mathbf{y}} \right],$$

- But recall that α_s is a slowly varying function of the scale:

$$\widetilde{\alpha}_{\mathbf{x}-\mathbf{y}} \equiv \int \frac{d^2\mathbf{k}}{(2\pi)^2} e^{i\mathbf{k}\cdot(\mathbf{x}-\mathbf{y})} \alpha_s(\mathbf{k}) \sim \alpha_s \delta^2(\mathbf{x}-\mathbf{y})$$

$\Longrightarrow$ $\mathbf{u} \approx \mathbf{v}$ and structure simplifies to BK:

$$\frac{1}{2} \left[\hat{D}_{\mathbf{x},\mathbf{u}} \hat{D}_{\mathbf{u},\mathbf{y}} + \hat{D}_{\mathbf{x},\mathbf{v}} \hat{D}_{\mathbf{v},\mathbf{y}} - \hat{D}_{\mathbf{x},\mathbf{y}} - \hat{D}_{\mathbf{v},\mathbf{u}} \hat{Q}_{\mathbf{x},\mathbf{v},\mathbf{u},\mathbf{y}} \right] \approx \hat{D}_{\mathbf{x},\mathbf{u}} \hat{D}_{\mathbf{u},\mathbf{y}} - \hat{D}_{\mathbf{x},\mathbf{y}}$$

- Parametrically dominant length scale in coupling is “smallest dipole”, just like in Balitsky prescription for BK.

Gluon multiplicity and mean p_T

Q_s is only dominant scale

Parametrically $$\frac{\mathrm{d}N_g}{\mathrm{d}y\,\mathrm{d}^2\mathbf{x}} = c_N \frac{C_F}{2\pi^2\alpha_s} Q_s^2 \qquad \langle p_T \rangle \sim Q_s$$

Note: in full CYM total gluon multiplicity is IR finite, no cutoff.

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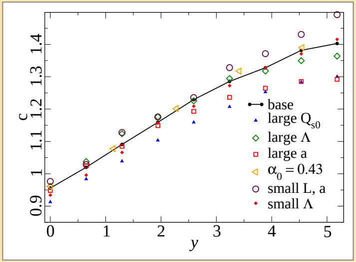

Scaled multiplicity increases with energy (Midrapidity, $y \equiv \ln \sqrt{s/s_0}$)

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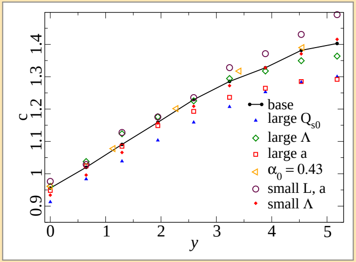

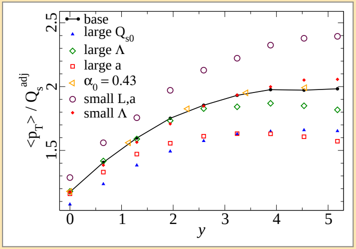

Scaled multiplicity increases with energy (Midrapidity, $y \equiv \ln \sqrt{s/s_0}$)

Harder gluon spectrum $\Longrightarrow$ higher $\langle p_T \rangle / Q_s$ as scaling regime sets in.

(Still very large lattice cutoff effects.)