

Finite-temperature Effective Field Theories for quarkonia

Jacopo Ghiglieri, Institute for Theoretical Physics
Albert Einstein Center, University of Bern

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Quarkonium as a hard probe

J/ψ SUPPRESSION BY QUARK-GLUON PLASMA FORMATION ☆

T. MATSUI

*Center for Theoretical Physics, Laboratory for Nuclear Science, Massachusetts Institute of Technology,
Cambridge, MA 02139, USA*

and

H. SATZ

*Fakultät für Physik, Universität Bielefeld, D-4800 Bielefeld, Fed. Rep. Germany
and Physics Department, Brookhaven National Laboratory, Upton, NY 11973, USA*

Received 17 July 1986

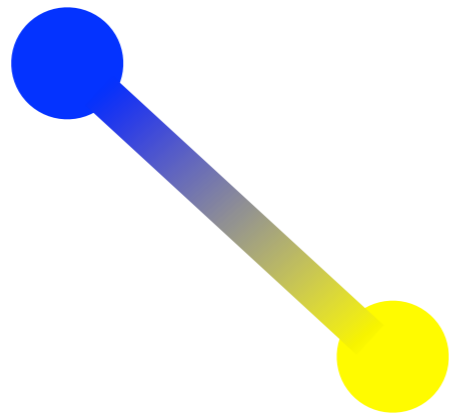
- Colour screening leads to the disappearance of the bound state
- A suppressed J/ψ yield is observed in the dilepton channel

Matsui Satz **PLB178** (1986)

Overview of dissociation

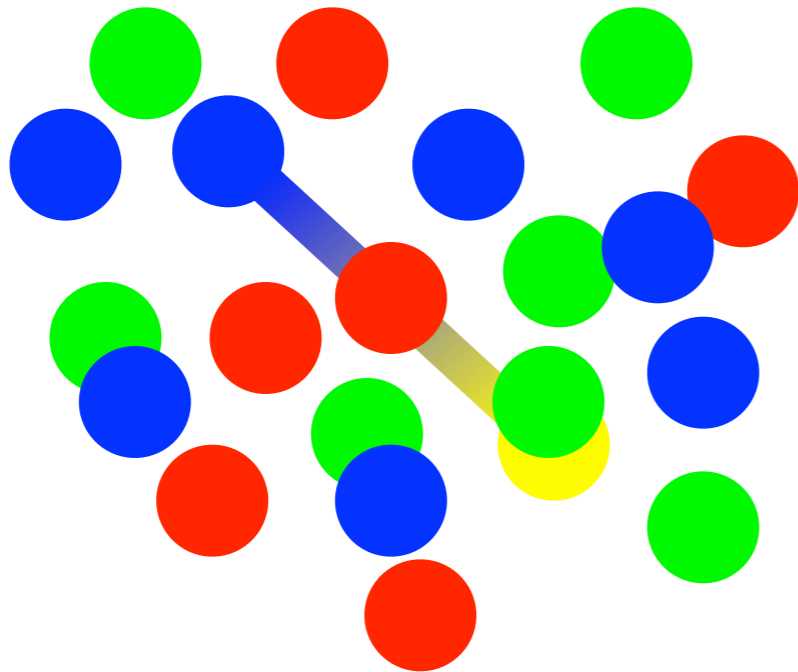
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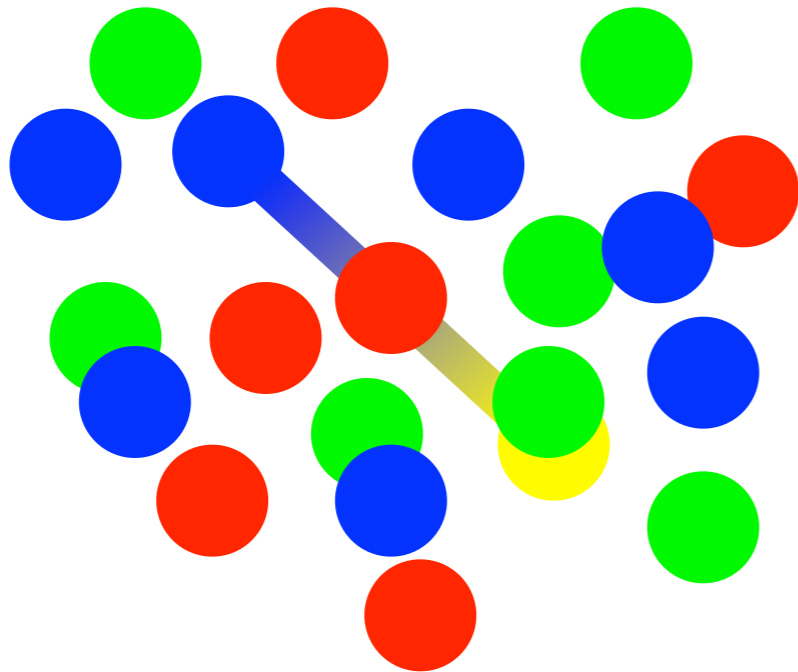
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$$V(r) \sim -\alpha_s \frac{e^{-m_D r}}{r}$$
$$r \sim \frac{1}{m_D} \longrightarrow \text{Bound state dissolves}$$

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- Since then, dissociation has been studied with potential models, lattice spectral functions, AdS / CFT and now with EFTs

Quarkonium suppression in experiments

- Typical observable: the nuclear modification factor

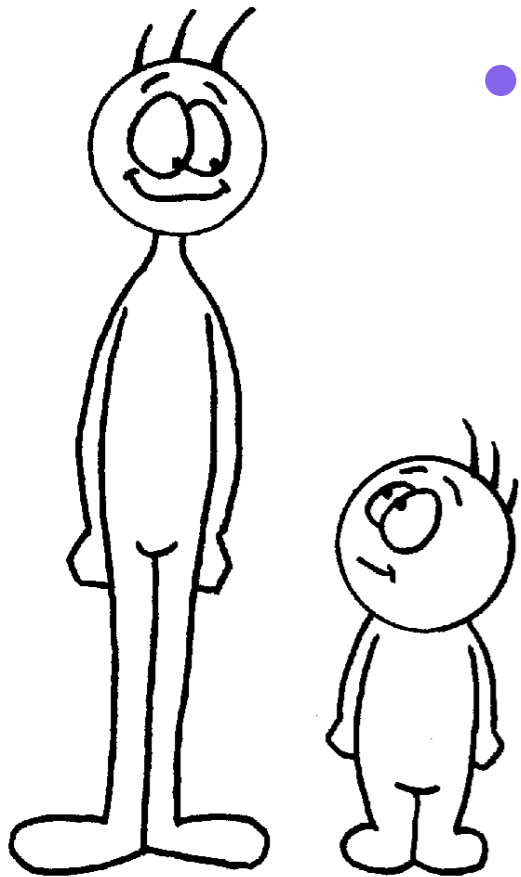
$$R_{AA} = \frac{\text{Yield}_{AA}}{\text{Yield}_{pp} \times N_{bin}}$$

- $R_{AA} \neq 1 \Rightarrow$ deviations from binary scaling. Causes:
 - Cold Nuclear Matter effects (affect production and early stages).
 - Hot Medium effects, such as screening. Reduce R_{AA}
 - Recombination effects. Increase R_{AA}

Why EFTs?

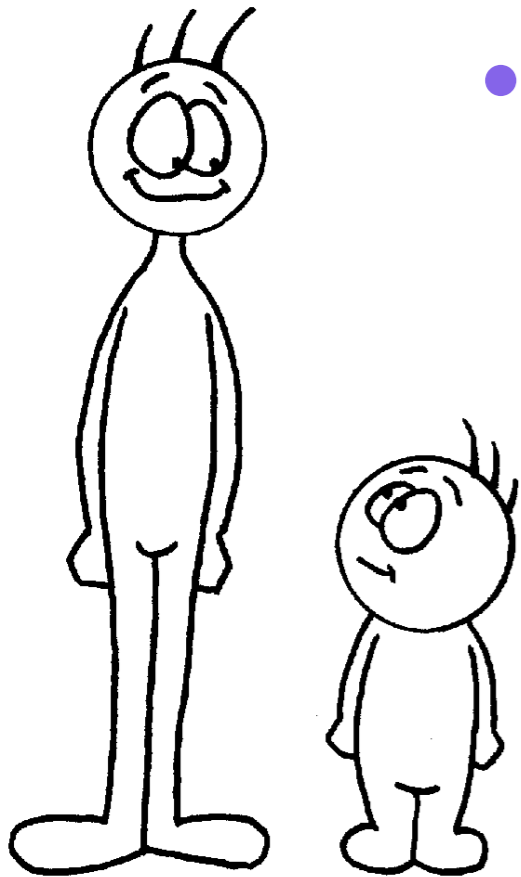
- We have a system characterized by many scales and degrees of freedom
- With EFTs we can
 - Have a clear counting
 - Integrate out unnecessary DOFs
 - Obtain an effective description with potentials rigorously obtained from QCD, including all relevant effects for the desired accuracy

Why EFTs?



- An EFT is constructed by integrating out modes of energy and momentum larger than the cut-off ($\mu \ll \Lambda$)

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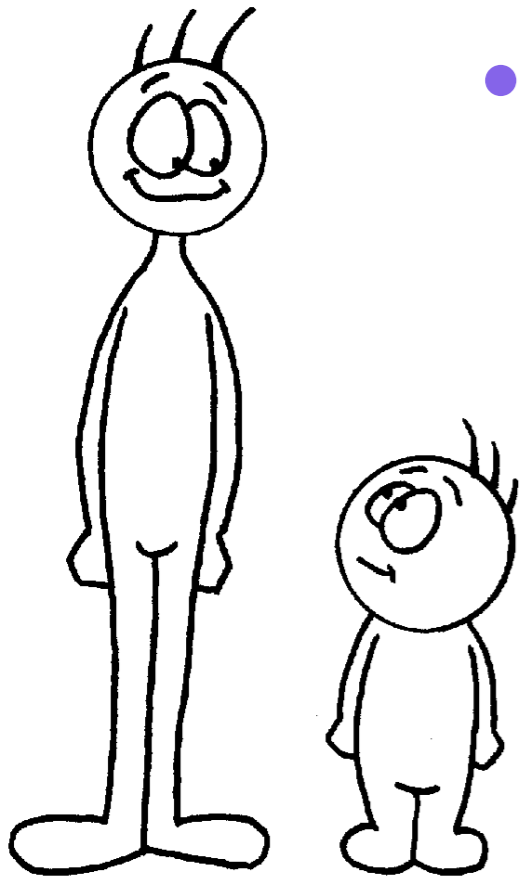
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$$\mathcal{L}_{\text{EFT}} = \sum_n c_n(\mu/\Lambda) \frac{O_n}{\Lambda^{d_n-4}}$$

Wilson coefficient (pointing to $c_n(\mu/\Lambda)$)

Low-energy operator / large scale (pointing to $\frac{O_n}{\Lambda^{d_n-4}}$)

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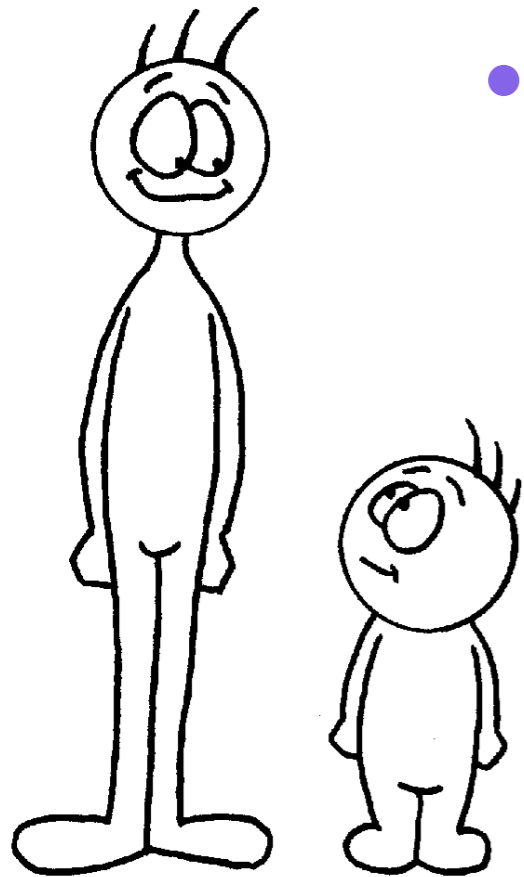
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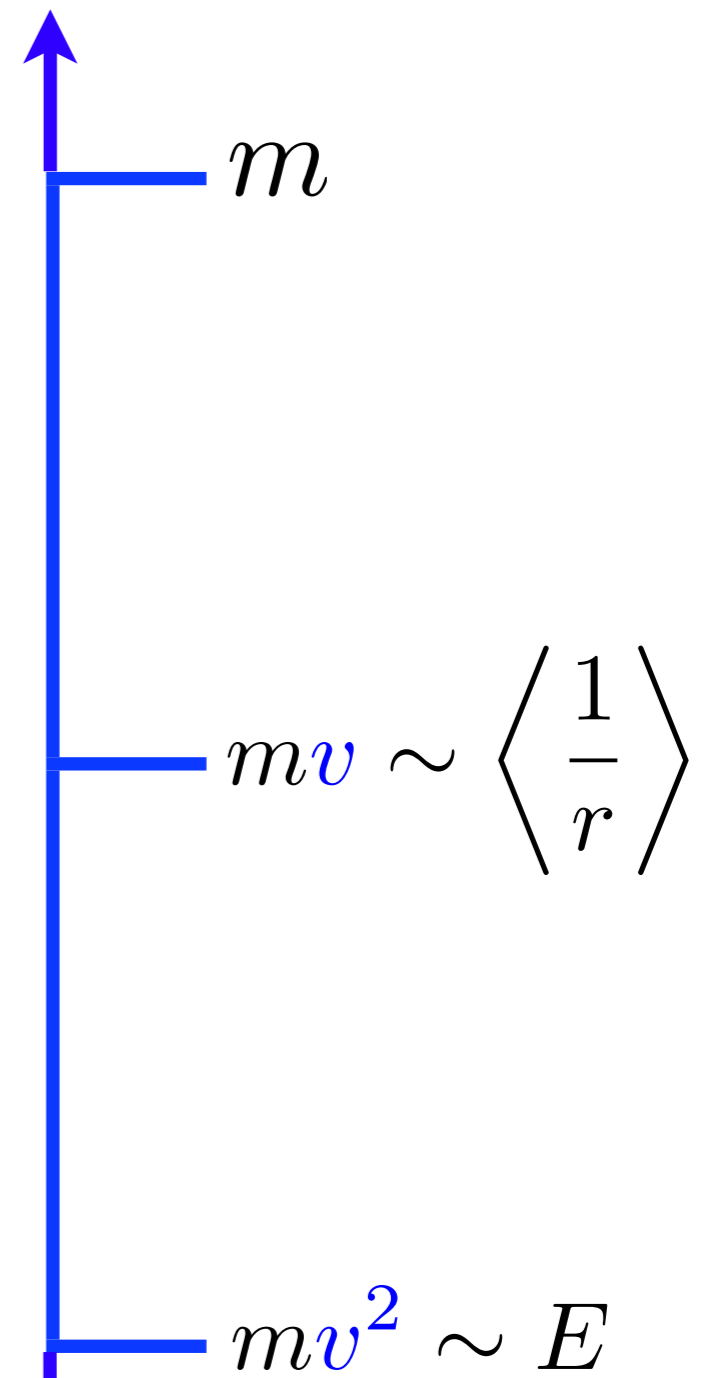
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- The procedure can be iterated $\dots \ll \mu_2 \ll \Lambda_2 \ll \mu_1 \ll \Lambda_1$

At zero temperature

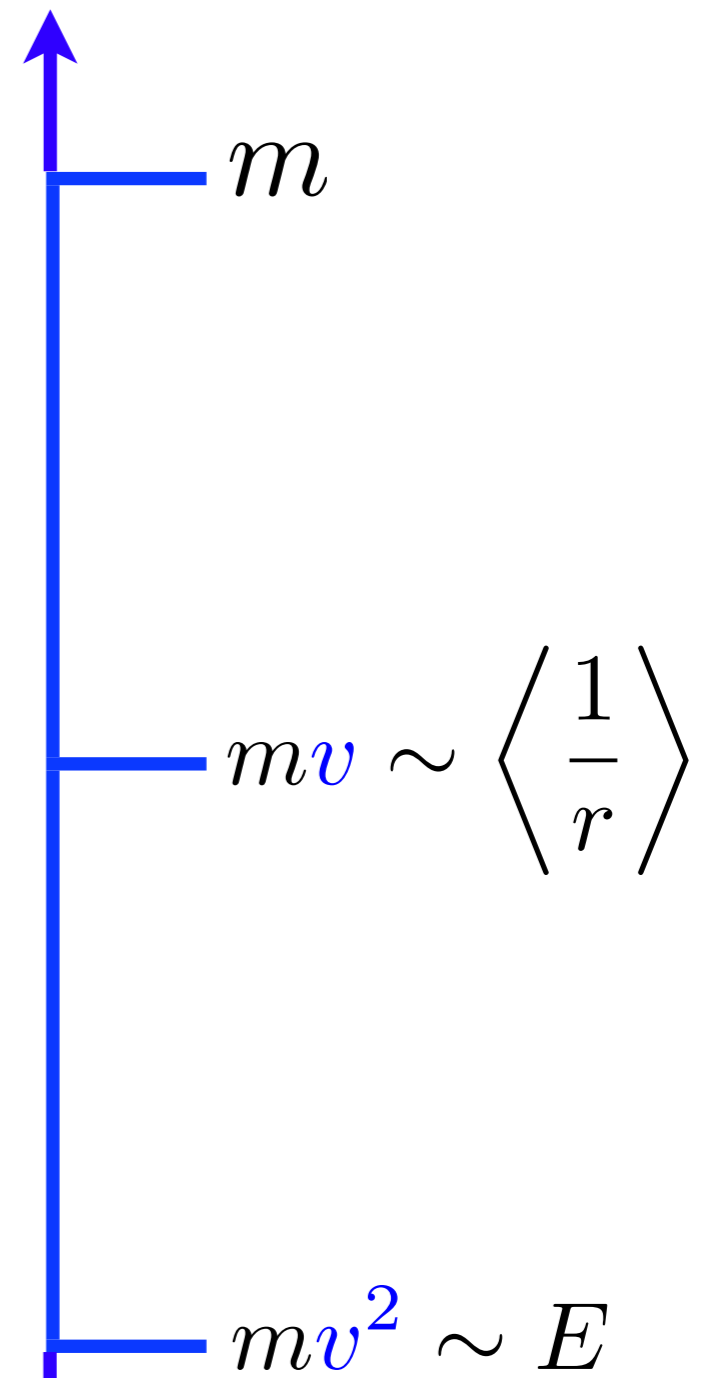
At zero temperature

- *Non-relativistic $Q\bar{Q}$ bound states are characterized by the hierarchy of the mass, momentum transfer and kinetic/binding energy scales*



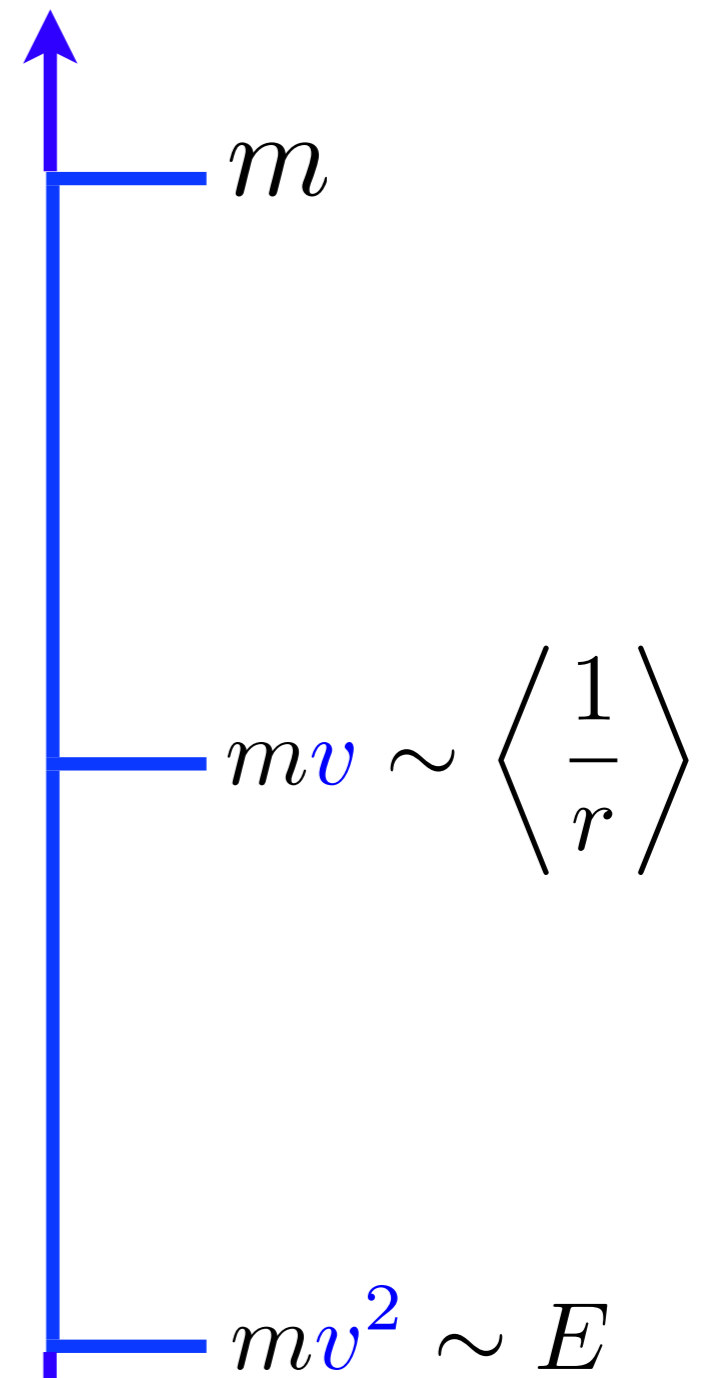
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At zero temperature

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- Expand observables in terms of the ratio of the scales, v
- Construct a *hierarchy of EFTs*.
Equivalent to QCD order-by-order in the expansion parameter



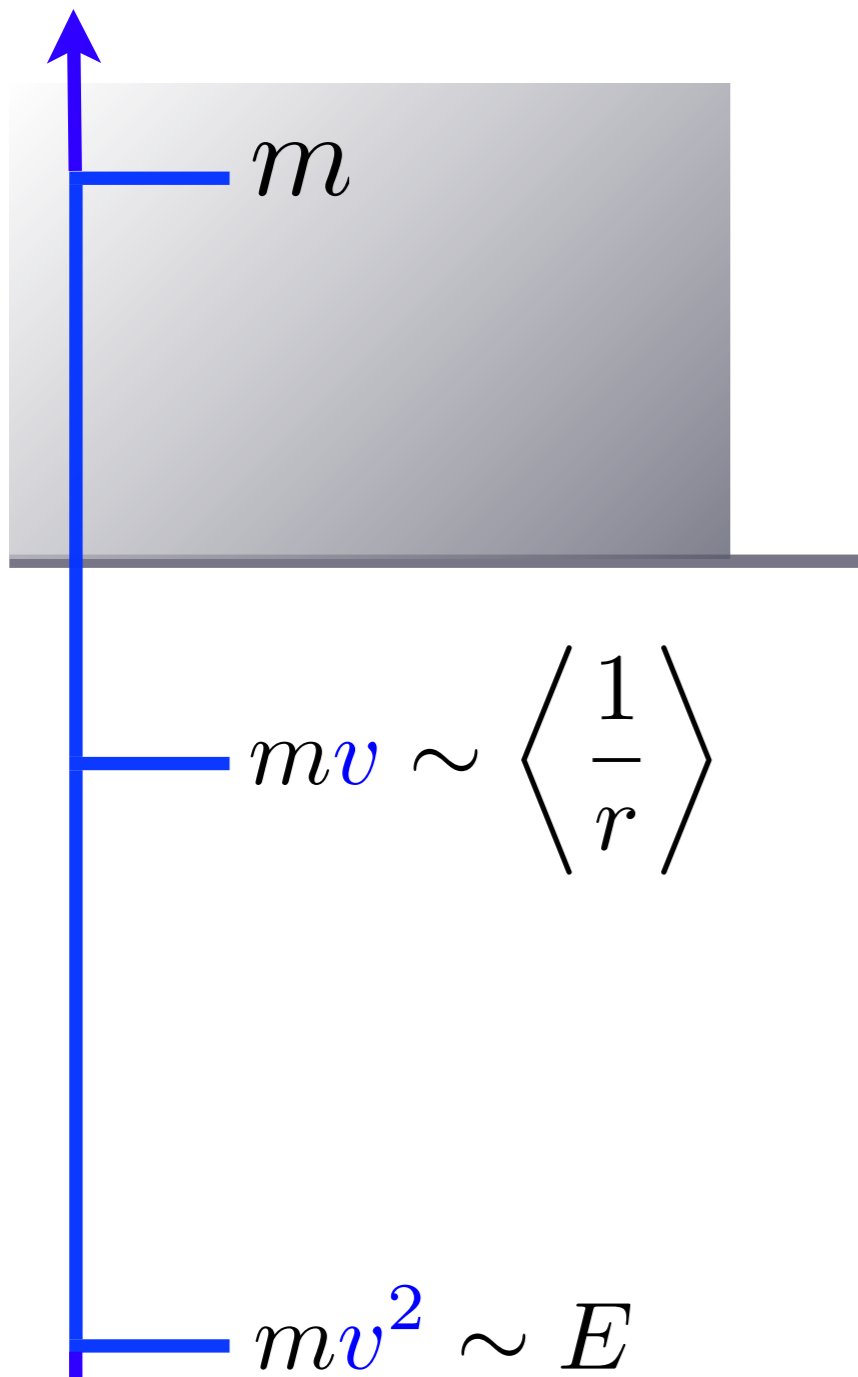
Integrating out the mass scale: Non-Relativistic QCD (NRQCD)

- The mass is integrated out and the theory becomes non-relativistic
- Factorization between contributions from the scale m and from lower-energies
- Ideal for production and decay studies

$$\mathcal{L}_{\text{NRQCD}} = \sum_n c_n(\mu/m) \frac{O_n}{m^{d_n-4}}$$

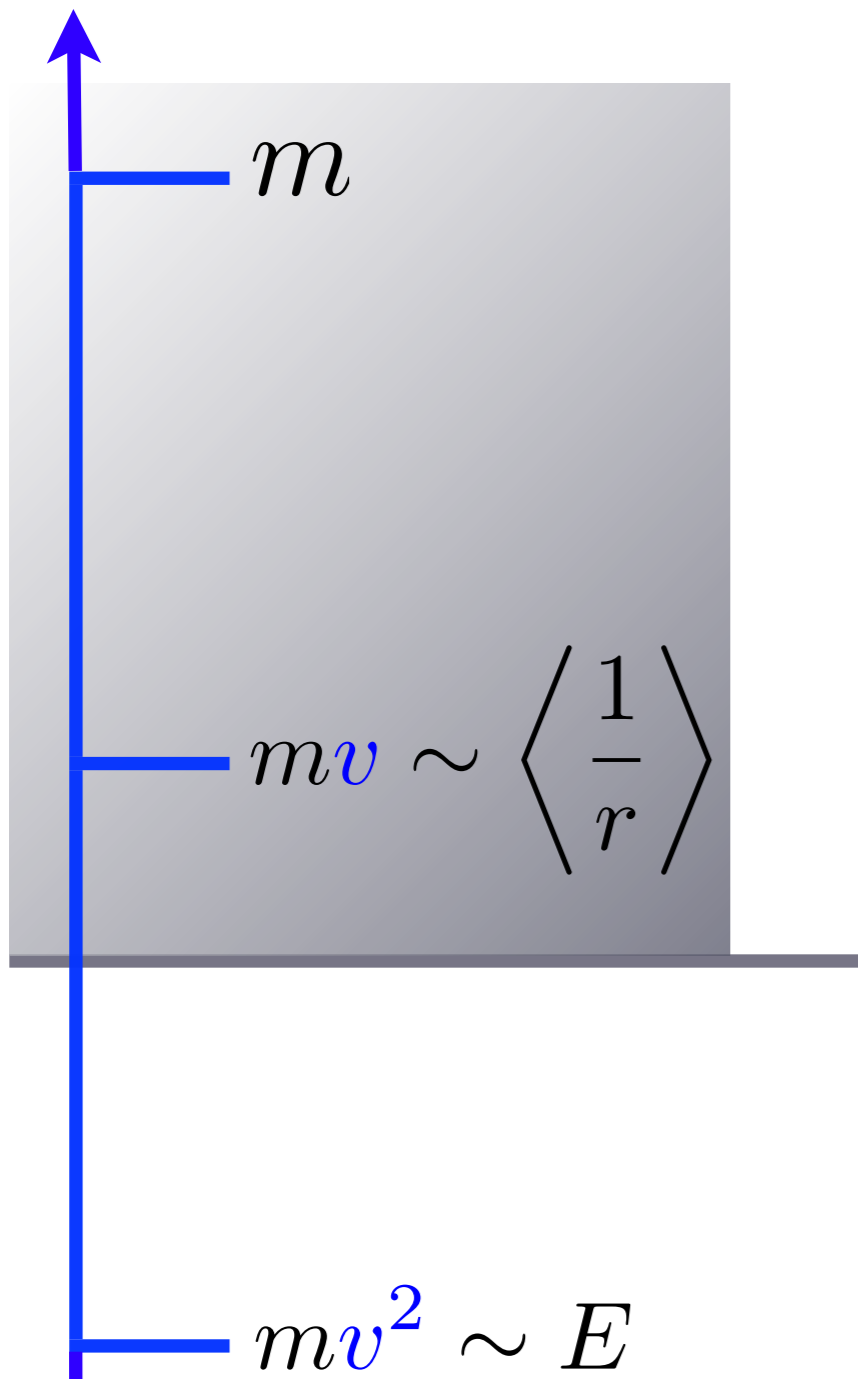
Caswell Lepage **PLB167** (1986)

Bodwin Braaten Lepage **PRD51** (1995)



The scale mv : potential NRQCD (pNRQCD)

- Modes with momentum mv are integrated out
- This gives rise to non-local four-fermion operators. Their Wilson coefficients are the potentials, rigorously defined



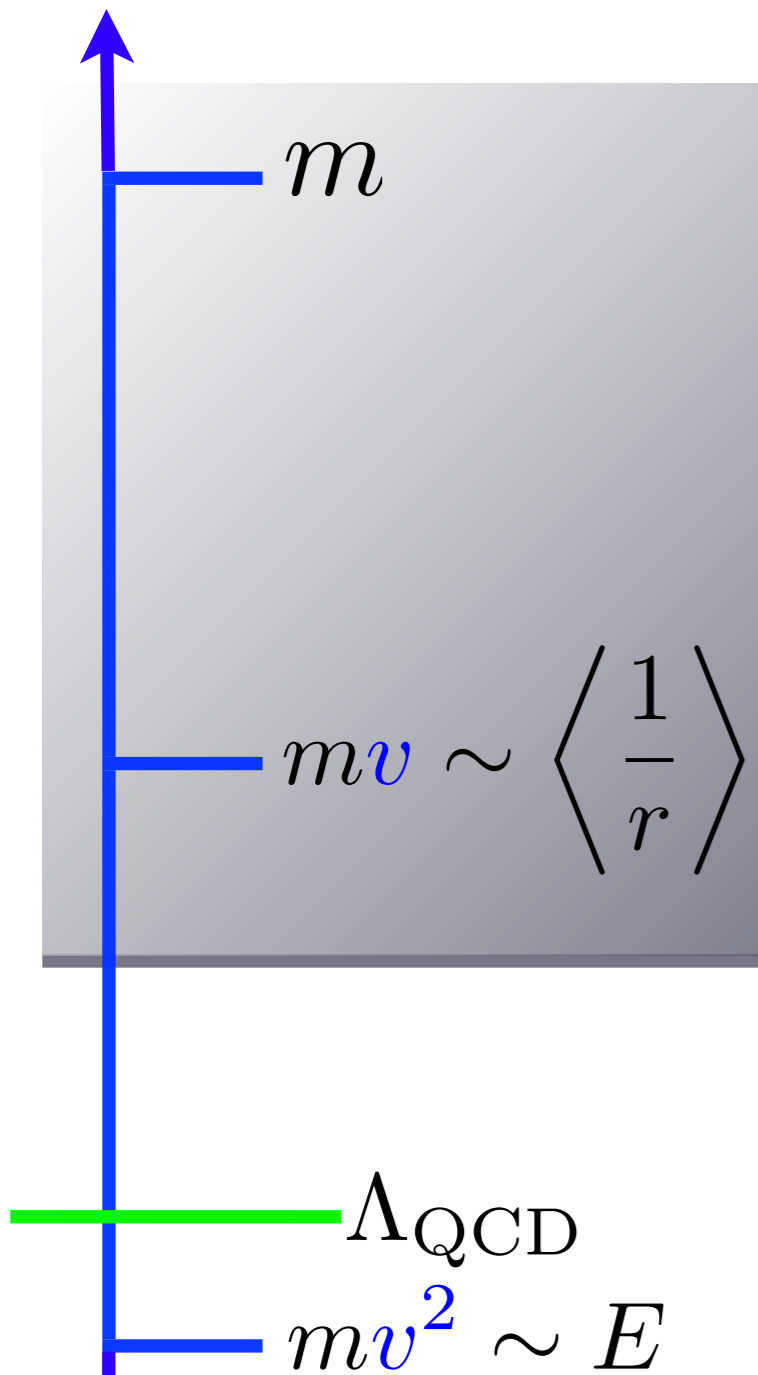
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$$\mathcal{L} = \mathcal{L}_{\text{light}} + \text{Tr} \left\{ \mathbf{S}^\dagger \left[i\partial_0 + \frac{\nabla^2}{m} - V_s \right] \mathbf{S} + \mathbf{O}^\dagger \left[iD_0 + \frac{\nabla^2}{m} - V_o \right] \mathbf{O} \right\} \\ + \text{Tr} \{ \mathbf{O}^\dagger \mathbf{r} \cdot g\mathbf{E} \mathbf{S} + \mathbf{S}^\dagger \mathbf{r} \cdot g\mathbf{E} \mathbf{O} \} + \frac{1}{2} \text{Tr} \{ \mathbf{O}^\dagger \mathbf{r} \cdot g\mathbf{E} \mathbf{O} + \mathbf{O}^\dagger \mathbf{O} \mathbf{r} \cdot g\mathbf{E} \} + \dots$$

Pineda Soto **NPPS64** (1998)

Brambilla Pineda Soto Vairo **NPB566** (2000)



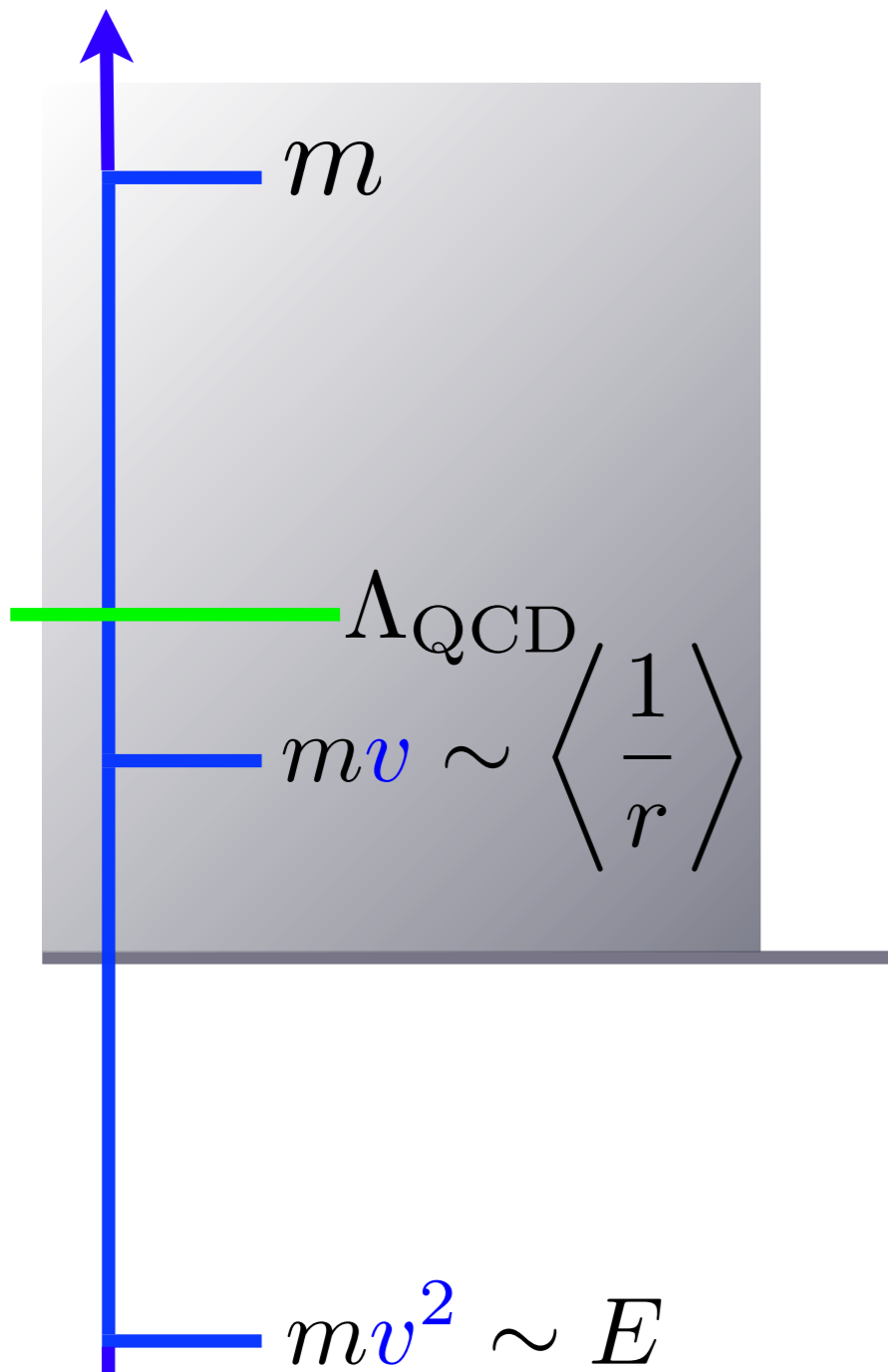
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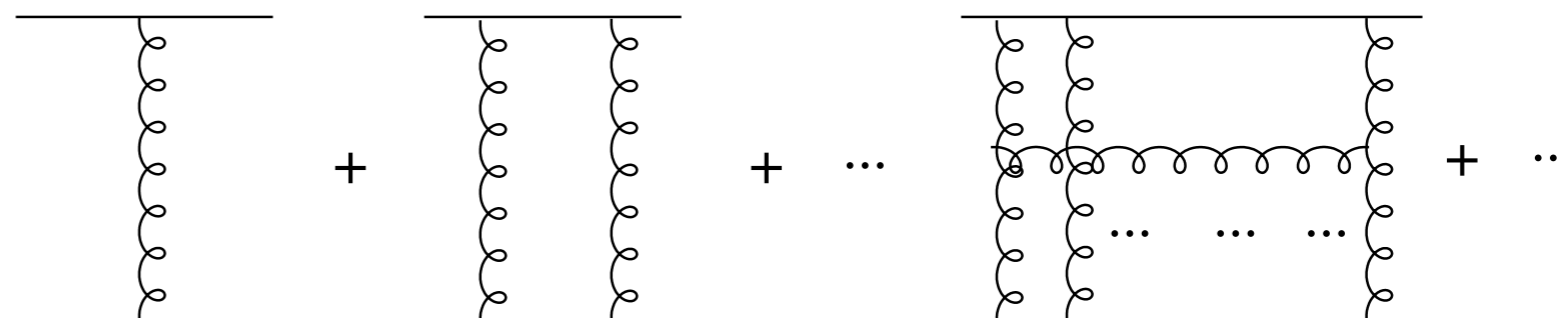
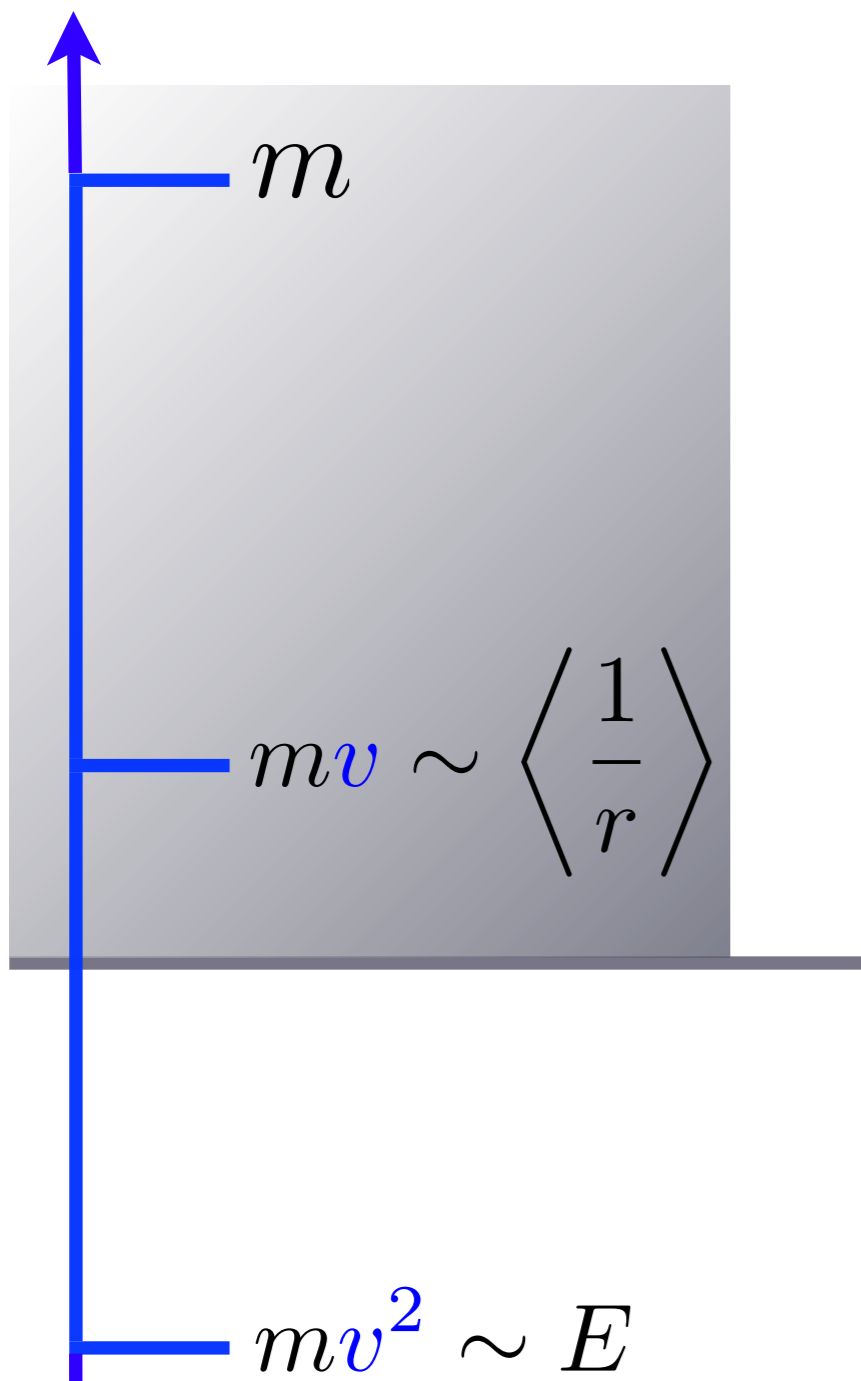
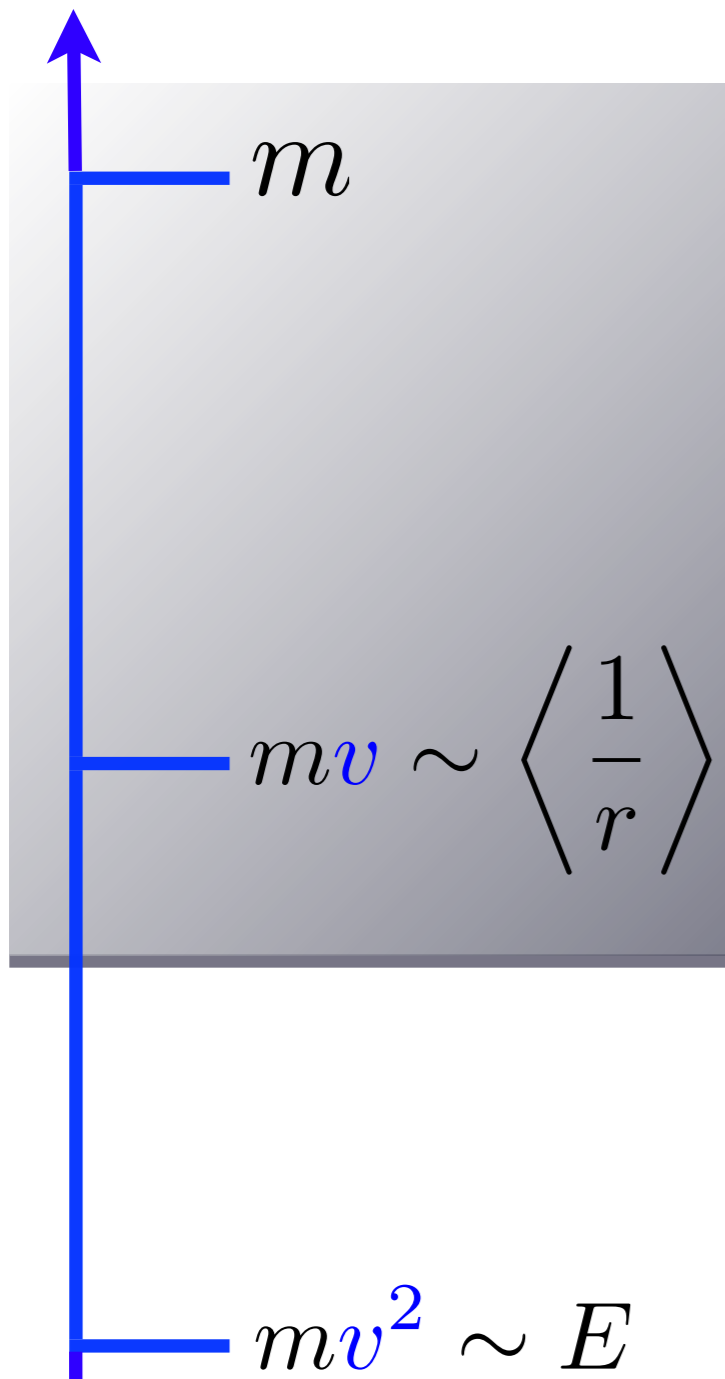


Diagram illustrating the potential NRQCD Lagrangian. It shows a horizontal line representing the heavy quark, with a gluon loop (represented by a wavy line) attached to it. The loop is connected to the line by two vertices marked with a cross (⊗). The diagram is summed together, indicated by plus signs and ellipses. Below the diagram, the denominator of the potential is given as $E - p^2/m - V(r)$, where $V(r)$ is the potential energy function.

$$\frac{1}{E - p^2/m - V(r)}$$





Applications of pNRQCD

- Spectroscopy
- Extraction of SM parameters (m_c, m_b, α_s)
- Comparisons of lattice and perturbation theory
- $t\bar{t}$ threshold production
- Reviews [Brambilla et al. EPJC71 \(2011\)](#)
[EPJC74 \(2014\)](#)

Applications to quarkonia in HIC

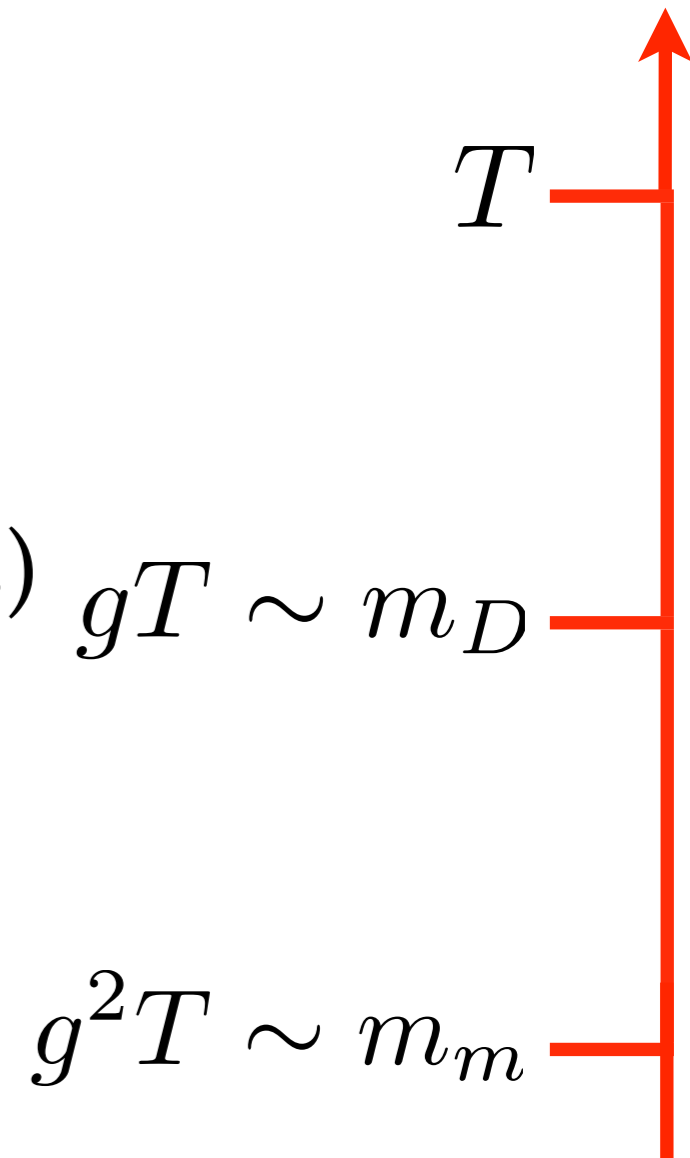
- Production (NRQCD [Vitev Sharma PRC87 2013](#), NRQCD+CGC with outlook to AA [Kang Ma Venugopalan 2013-14](#))
- In-medium evolution (NRQCD, pNRQCD and variants)
- Both perturbation theory and lattice studies

Bring in the medium

- The thermal medium introduces new scales in the physical problem
 - The temperature
 - The electric screening scale (Debye mass)
 - The magnetic screening scale (magnetic mass)
- In the weak coupling assumption these scales develop a hierarchy

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Finite-temperature NR EFT how-to

$$m \gg mv \sim m\alpha_s \sim \langle 1/r \rangle \gg mv^2 \sim m\alpha_s^2 \sim E$$

?

$$T \gg m_D \sim gT \gg m_m \sim g^2 T$$

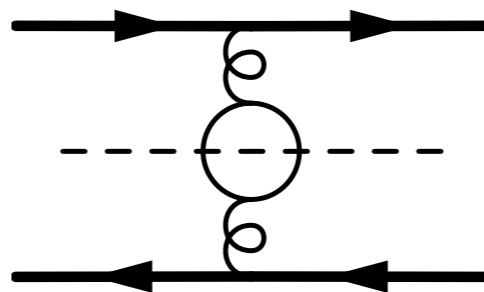
- Assume a global hierarchy between the bound-state and thermodynamical scales
- Many different possibilities have been considered in the relevant macroregions $T \ll mv$, $T \sim mv$ and $T \gg mv$ (with $T \ll m$)
- Proceed from the top to systematically integrate out each scale, creating a tower of EFTs. Make use of existing EFTs ($T=0$ NR EFTs, finite T EFTs such as HTL)
- Once the scale mv has been integrated out the colour singlet and octet potentials appear. They are always **complex**

The complex potential

- Laine Philipsen Romatschke Tassler [JHEP0703 \(2007\)](#) : analytical continuation of Wilson loop to large real time yields a complex potential in HTL-resummed PT

$$V_{\text{HTL}}(T \gg 1/r, m_D) = -C_F \alpha_s \left(\frac{e^{-m_D r}}{r} + m_D - i \frac{2T}{m_D r} f(m_D r) \right)$$

- $\text{Re } V \Rightarrow$ screening. $\text{Im } V \Rightarrow$ width induced by collisions with the medium. $\text{Im } V \gg \text{Re } V$



- In the EFT: compact real-time derivation, extension to other regimes [Brambilla JG Petreczky Vairo PRD78 \(2008\)](#)

The dissociation temperature

- Given the potential for $T \gg 1/r \gg m_D$

$$V_s(r) = -C_F \frac{\alpha_s}{r} - \frac{C_F}{2} \alpha_s r m_D^2 - i \frac{C_F}{6} \alpha_s r^2 T m_D^2 \left(-2\gamma_E - \ln(rm_D)^2 + \frac{8}{3} \right) + \dots$$

- When $T \sim m\alpha_s^{2/3} \Rightarrow \text{Im}V \sim \text{Re}V$ Dissociation temperature
Escobedo Soto **PRA78** (2008) Laine **0810.1112** (2008)
- Quantitatively, for the $\Upsilon(1S)$

m_c (MeV)	T_d (MeV)
∞	480
5000	480
2500	460
1200	440
0	420

Escobedo Soto **PRA82** (2010)

Below the dissociation temperature

- When $mv \gg T \gg mv^2$ the thermal medium acts as a perturbation to the potential.

Relevant for the ground states of bottomonium:

$$m\alpha_s \sim 1.5\text{GeV}, \quad T < 1\text{GeV}$$

- The EFT obtained by integrating out the temperature from pNRQCD is called pNRQCD_{HTL}

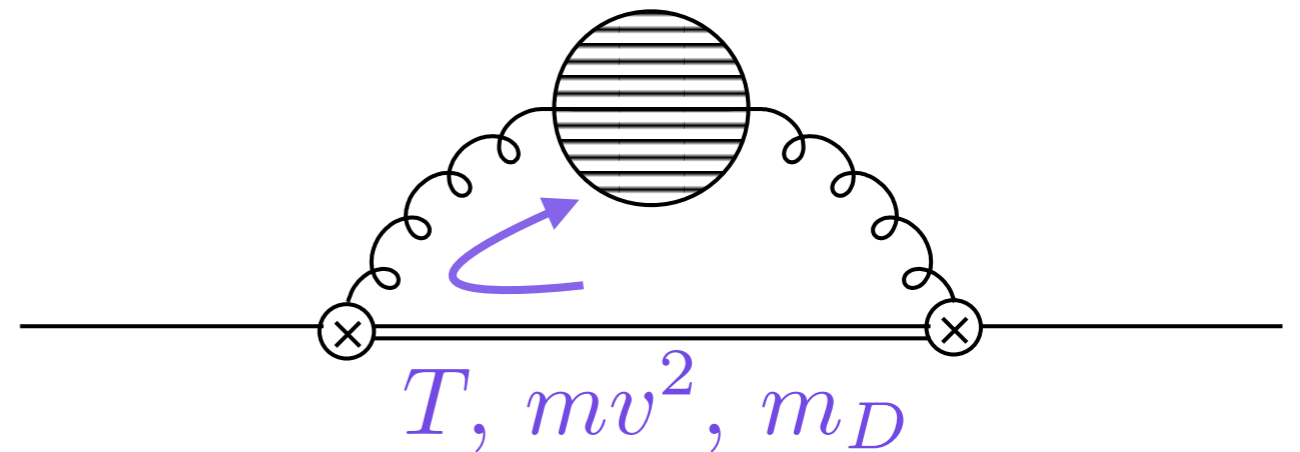
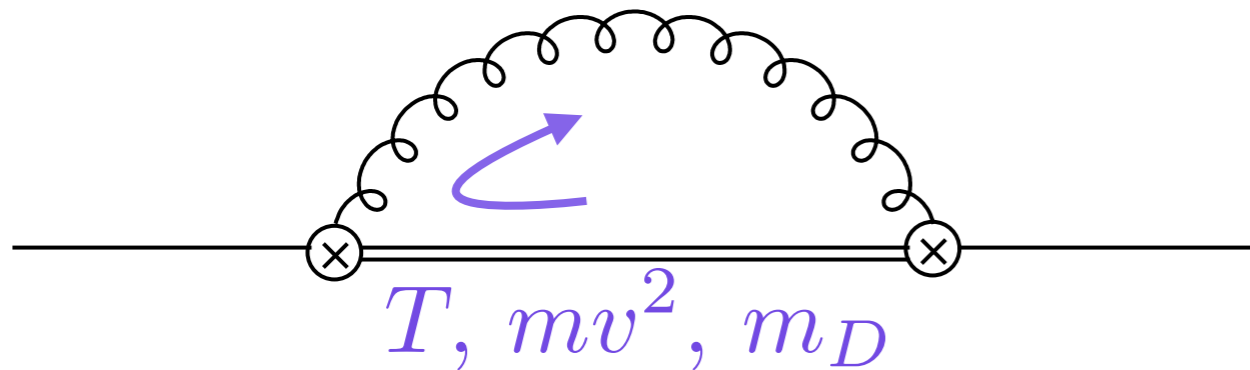
$$\mathcal{L}_{\text{pNRQCD}_{\text{HTL}}} = \mathcal{L}_{\text{HTL}} + \text{Tr} \left\{ \mathbf{S}^\dagger [i\partial_0 - h_s - \delta V_s] \mathbf{S} + \mathbf{O}^\dagger [iD_0 - h_o - \delta V_o] \mathbf{O} \right\} \\ + \text{Tr} \{ \mathbf{O}^\dagger \mathbf{r} \cdot g\mathbf{E} \mathbf{S} + \mathbf{S}^\dagger \mathbf{r} \cdot g\mathbf{E} \mathbf{O} \} + \frac{1}{2} \text{Tr} \{ \mathbf{O}^\dagger \mathbf{r} \cdot g\mathbf{E} \mathbf{O} + \mathbf{O}^\dagger \mathbf{O} \mathbf{r} \cdot g\mathbf{E} \} + \dots$$

Brambilla Escobedo JG Soto Vairo [JHEP1009 \(2010\)](#)

Brambilla Escobedo JG Vairo [JHEP1107 \(2011\)](#)

Below the dissociation temperature

- Within this theory we computed the spectrum and the thermal width of the $\Upsilon(1S)$ to order $m\alpha_s^5$ in the power counting of the EFT
- We must evaluate loop diagrams in the EFTs



Below the dissociation temperature

- The width reads

$$\Gamma_{1S} = \frac{1156}{27} \alpha_s^3 T + \frac{7225}{162} E_1 \alpha_s^3 - \frac{4}{3} \alpha_s a_0^2 T m_D^2 \left(\ln \frac{E_1^2}{T^2} + 2\gamma_E - 3 - \log 4 - 2 \frac{\zeta'(2)}{\zeta(2)} - \frac{8}{3} I_{1S} \right) - \frac{32\pi}{3} \ln 2 a_0^2 \alpha_s^2 T^3$$

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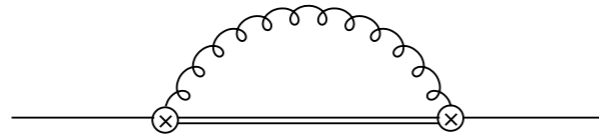
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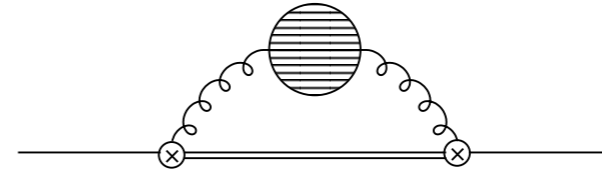
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- The leading contribution is linear in the temperature
- Two mechanisms: **singlet-to-octet thermal breakup** and **Landau damping**

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Singlet-to-octet



Landau Damping

perature

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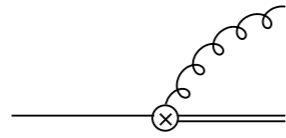
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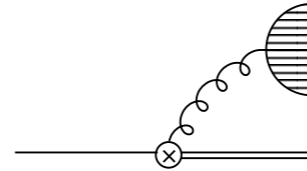
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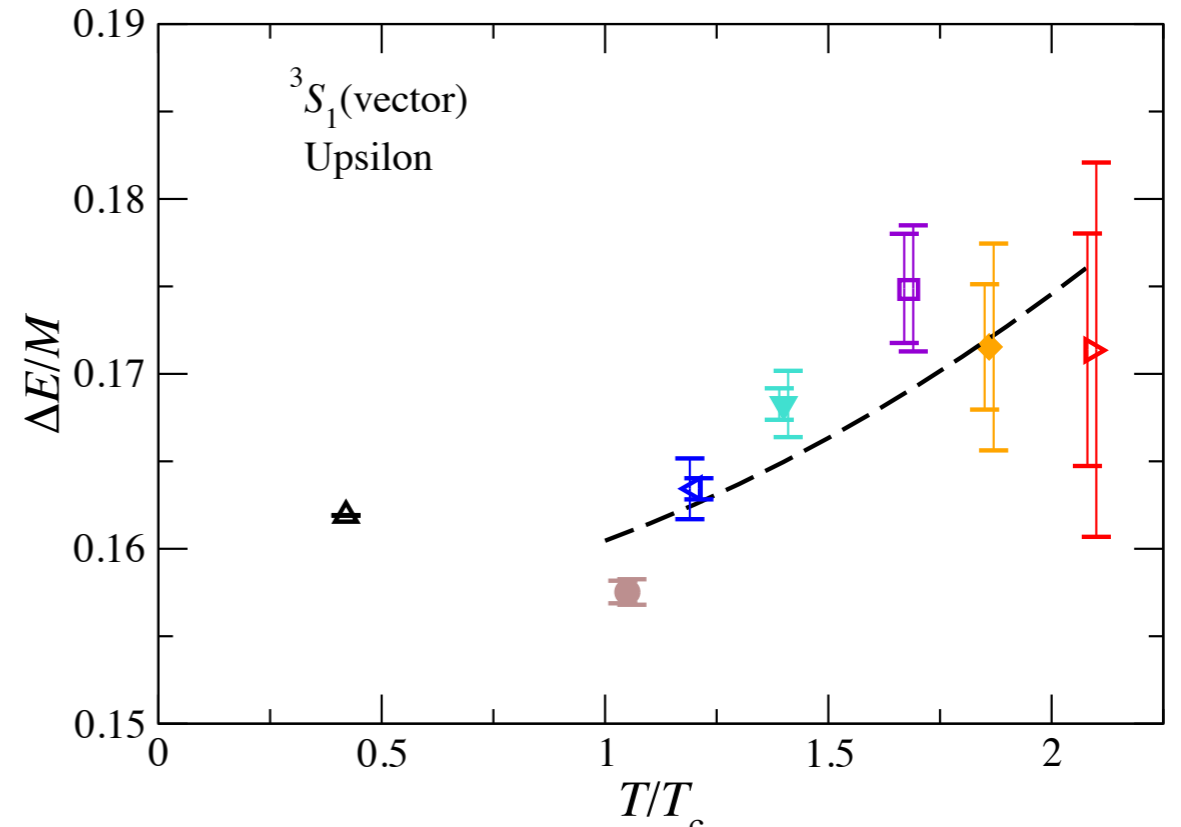
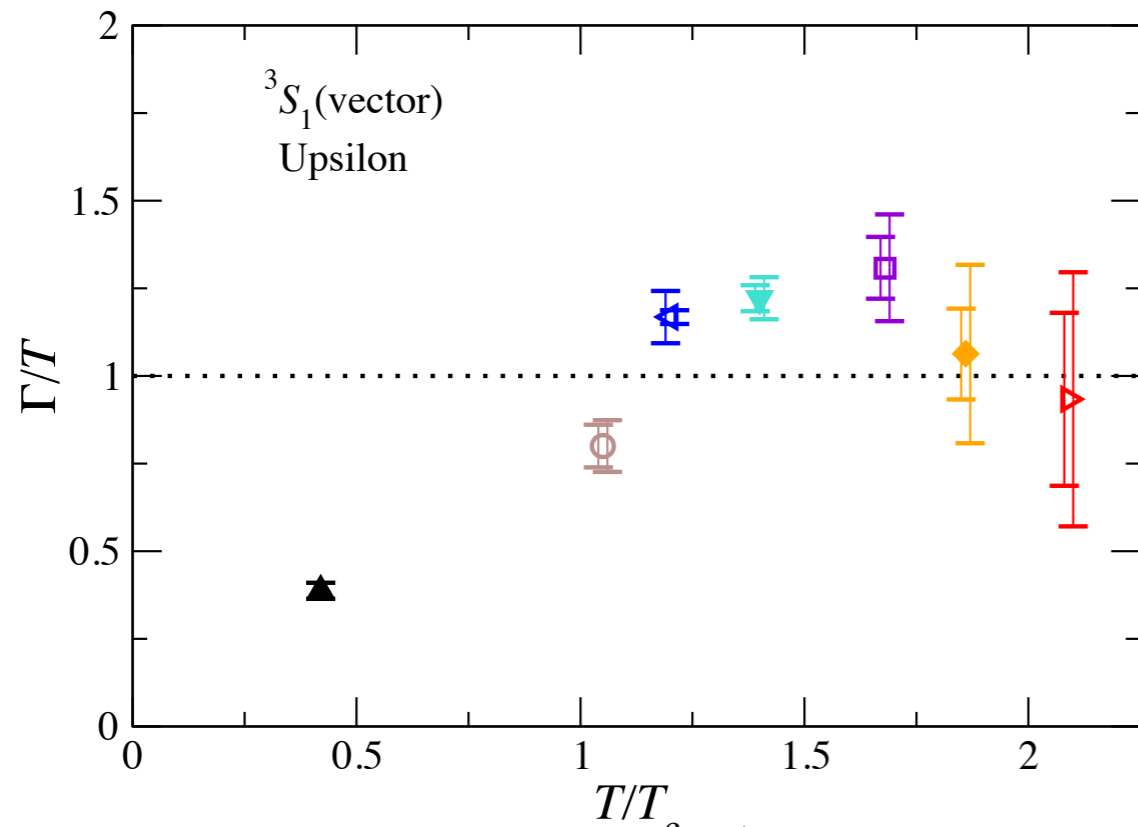
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NRQCD on the lattice

- Extraction of the $b\bar{b}$ spectral function from lattice NRQCD with MEM. Mass shifts and widths are obtained by fitting

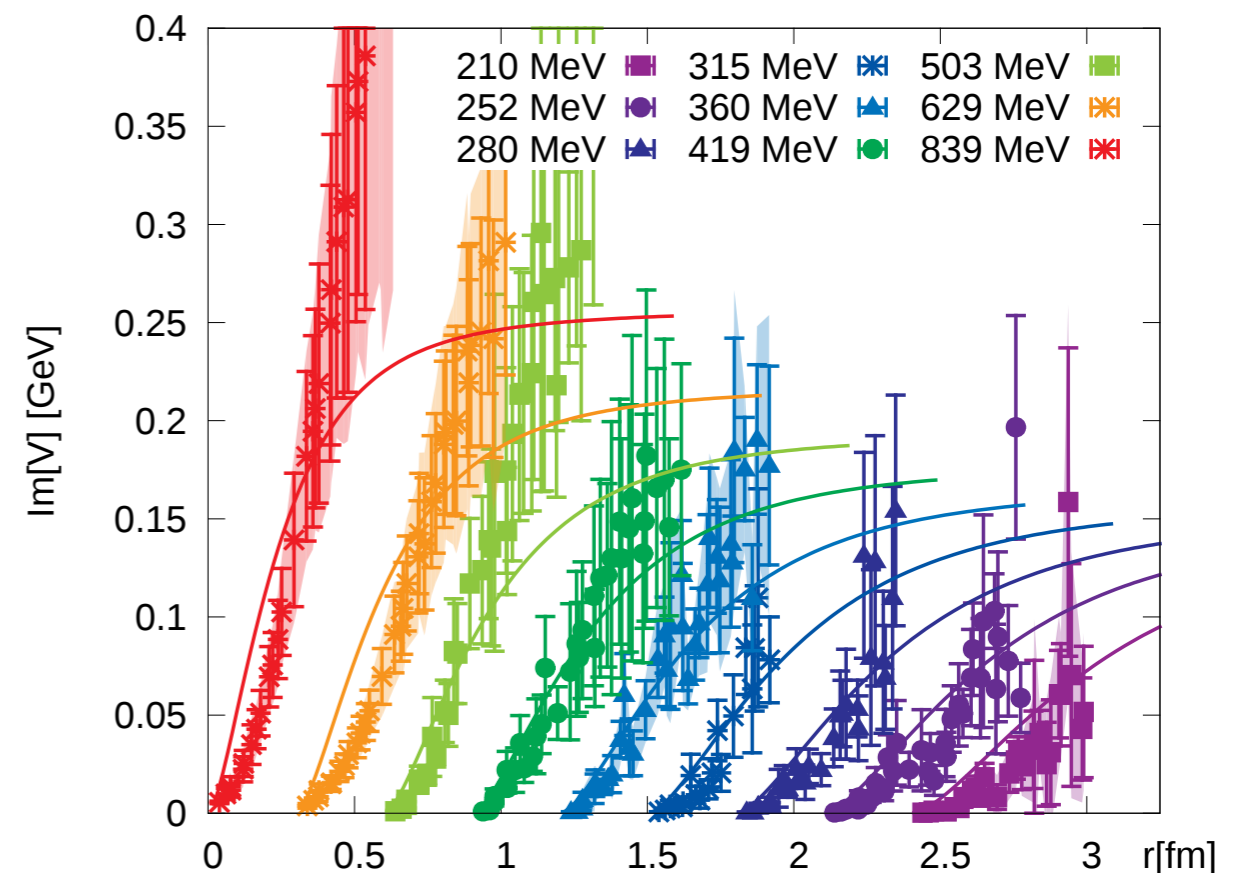
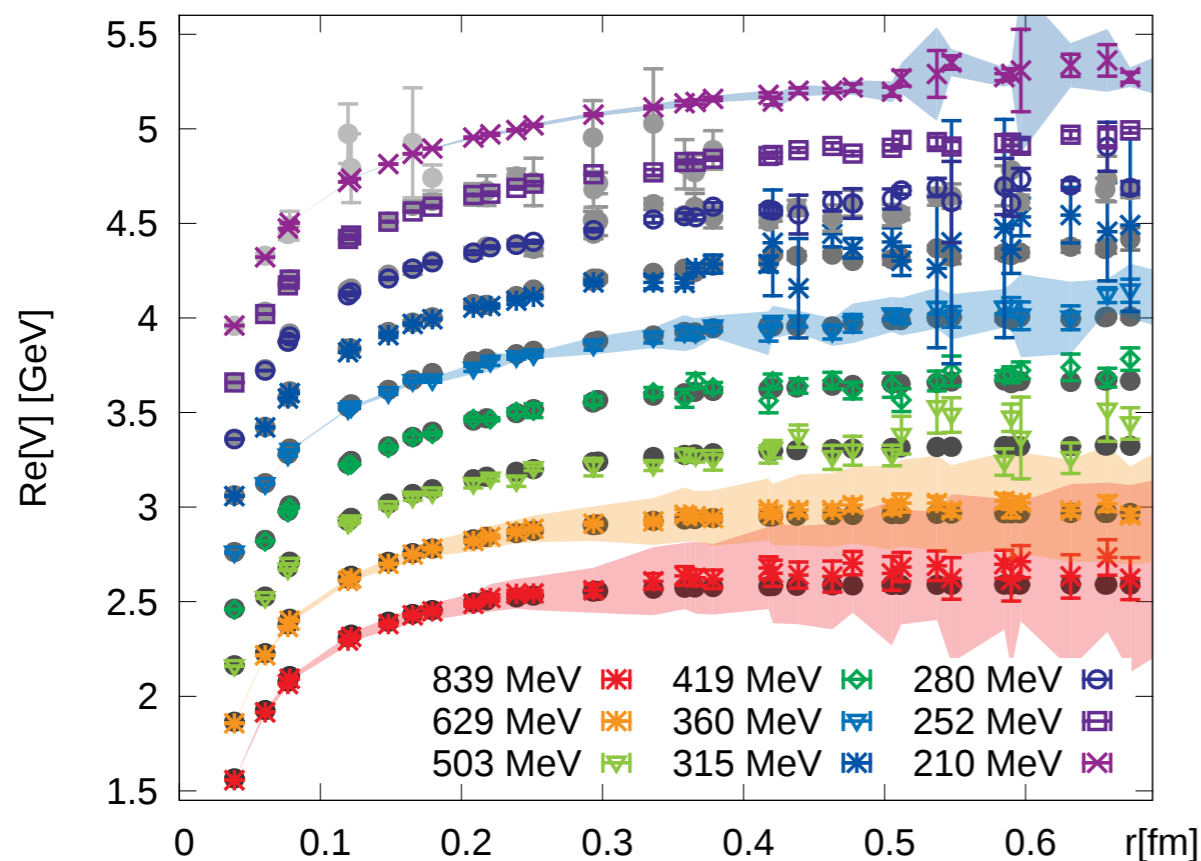


- Consistent with our LO predictions for $\alpha_s = 0.4$, $m_b = 5$ GeV
[Aarts et al. JHEP1111 \(2011\)](#).
- More lattice NRQCD in [Peter's talk](#)

The complex potential at strong coupling

- Extraction of a complex static potential from Euclidean Wilson loops or correlators through novel Bayesian methods Rothkopf Hatsuda Sasaki PRL108 Burnier Rothkopf 2012-14 Burnier Kaczmarek Rothkopf 2014

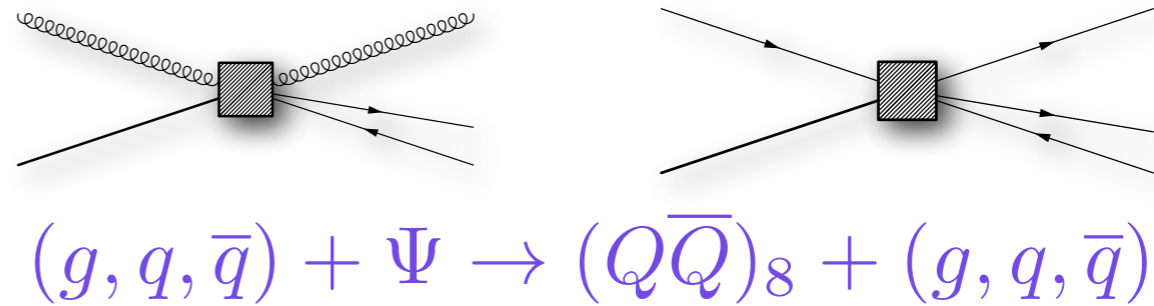
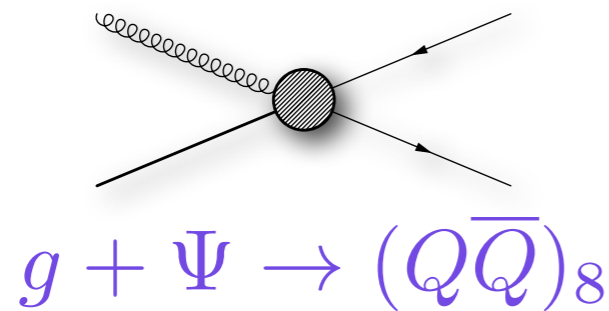
$$W(\tau) = \int d\omega e^{-\omega\tau} \rho(\omega) \leftrightarrow \int d\omega e^{-i\omega t} \rho(\omega) = W(t),$$



The imaginary part and the thermal width

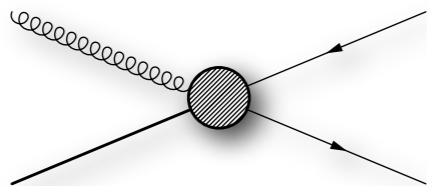
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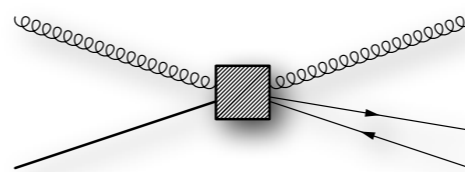


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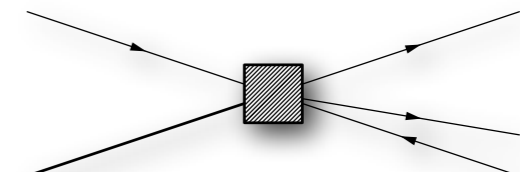
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$$g + \Psi \rightarrow (Q\bar{Q})_8$$



$$(g, q, \bar{q}) + \Psi \rightarrow (Q\bar{Q})_8 + (g, q, \bar{q})$$



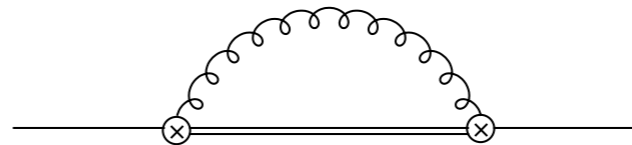
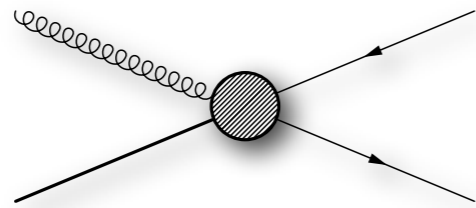
- In both cases the width is obtained by **convoluting** the (zero-temperature) *dissociation cross section* with a **thermal distribution** for the incoming light particle

$$\Gamma = \sum_i \int \frac{d^3 q}{(2\pi)^3} f_i(q, T) \sigma(q) v_{\text{rel}}$$

Kharzeev Satz **PLB334** (1994) Xu Kharzeev Satz Wang **PRC53** (1996)

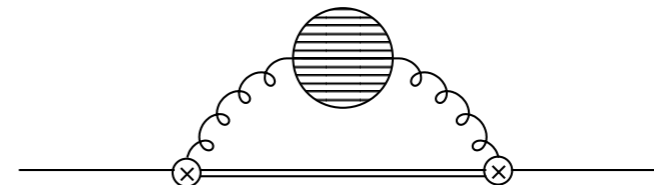
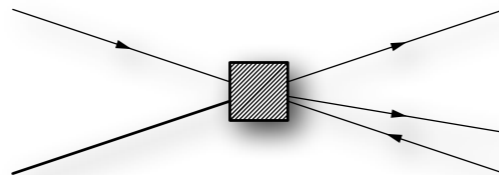
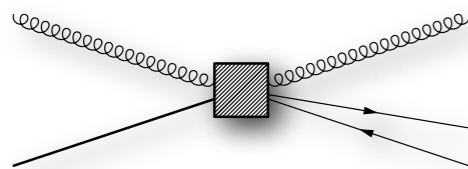
Grandchamp Rapp **PLB523** (2001)

The imaginary part and the thermal width



$$g + \Psi \rightarrow (Q\bar{Q})_8$$

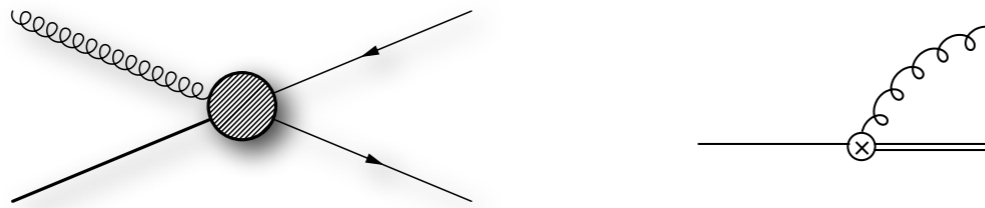
- Singlet-to-octet corresponds to *gluodissociation*. The old Bhanot-Peskin cross section is a limiting case of ours.
[Bhanot Peskin NPB156 \(1979\)](#) [Brambilla Escobedo JG Vairo JHEP1112 \(2011\)](#) [Brezinski Wolschin PLB707 \(2011\)](#)



$$(g, q, \bar{q}) + \Psi \rightarrow (Q\bar{Q})_8 + (g, q, \bar{q})$$

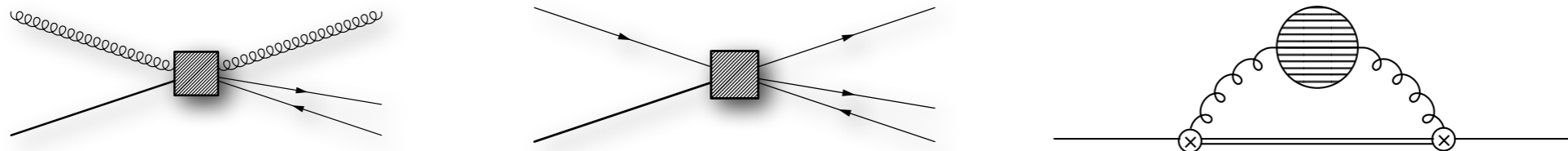
- Landau damping corresponds to elastic parton scattering. Clarification of validity regions in both cases
[Brambilla Escobedo JG Vairo JHEP1305 \(2013\)](#)

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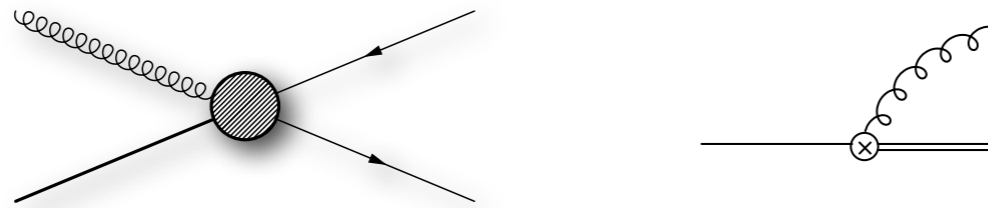
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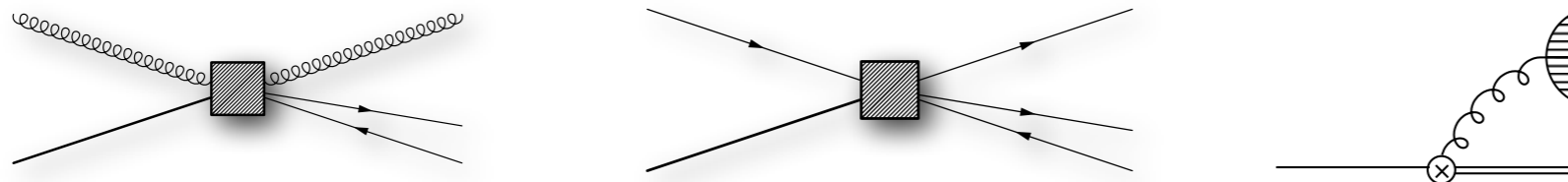
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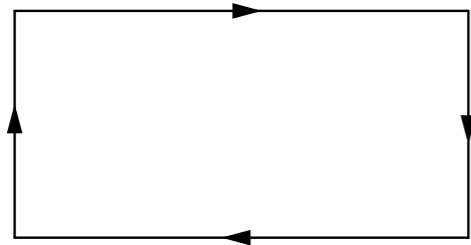


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Interpreting the imaginary part

- Irrespective of any perturbative mechanism, this thermal width is not for decay into light stuff.
Conserves HQ number and exists in the static limit
- It encodes the decoherence effect caused by the medium on the bound state wavefunction
- Open quantum system interpretation: correlators may decay exponentially, wave function (norm) may not
[Akamatsu, Rothkopf 2011-14](#)



$$W(r, t) \approx \langle S(r, t) S^\dagger(r, 0) \rangle \rightarrow Z \exp(-iV(r)t)$$

An EFT application

- Recent results by [Brambilla Escobedo Soto Vairo](#), presented by [M. Escobedo](#) at [ConfXI](#)
- Evolution equations for singlet and octet fields, with EFT Hamiltonians

$$f_s(x, y) = \text{Tr}(\rho S^\dagger(x) S(y))$$

$$H = \frac{H_{\text{eff}} + H_{\text{eff}}^\dagger}{2}$$

$$\Gamma = i(H_{\text{eff}} - H_{\text{eff}}^\dagger)$$

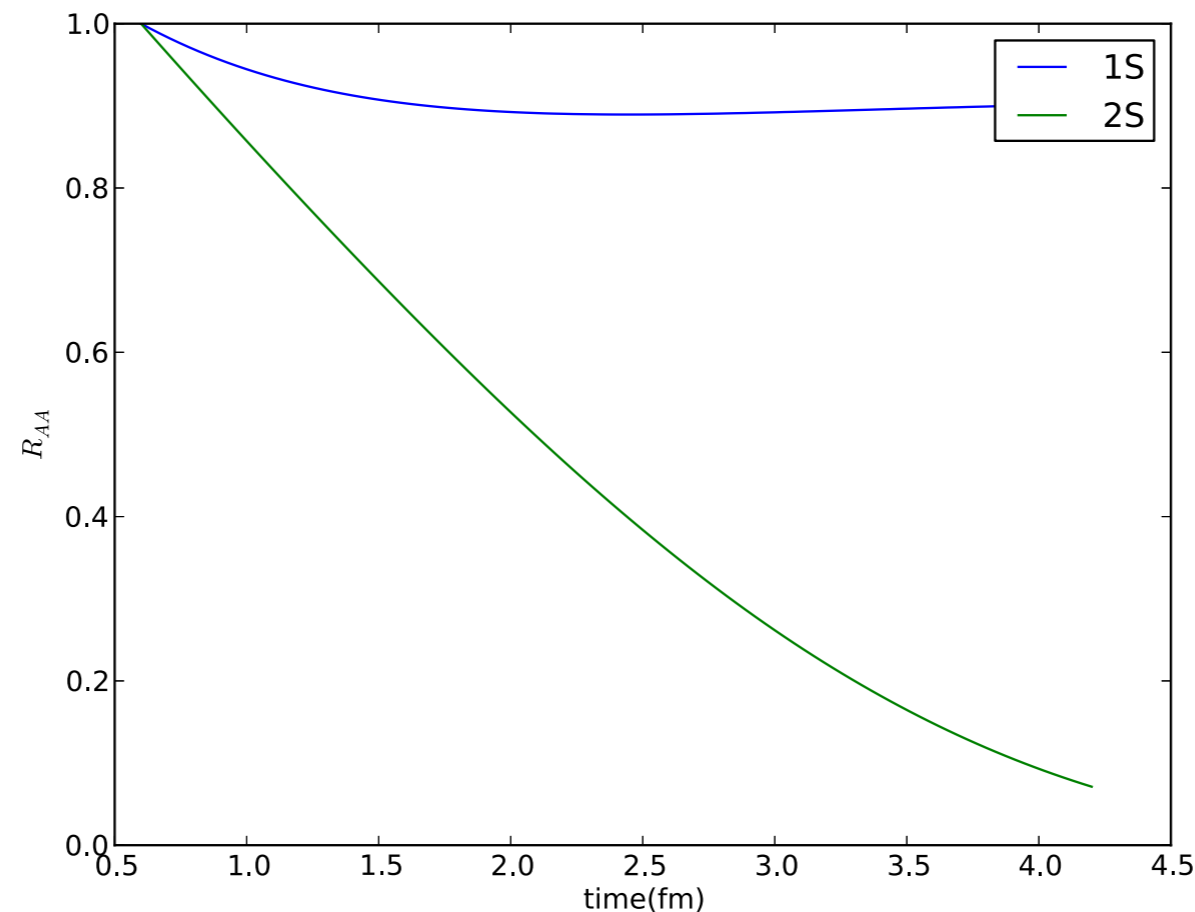
$$\partial_t f_s = -i[H, f_s] - \frac{1}{2}\{\Gamma, f_s\} + \mathcal{F}(f_o) \quad \partial_t f_o = -i[H_o, f_o] - \frac{1}{2}\{\Gamma, f_o\} + \mathcal{F}_1(f_s) + \mathcal{F}_2(f_o)$$

- HQ number conservation $\partial_t \text{Tr}(f_s) + \partial_t \text{Tr}(f_o) = 0$
- Numerical solution by using the Lindblad form [Akamatsu 2014](#)

$$\partial_t \rho = -i[H, \rho] + \sum_k (C_k \rho C_k^\dagger - \frac{1}{2}\{C_k^\dagger C_k, \rho\})$$

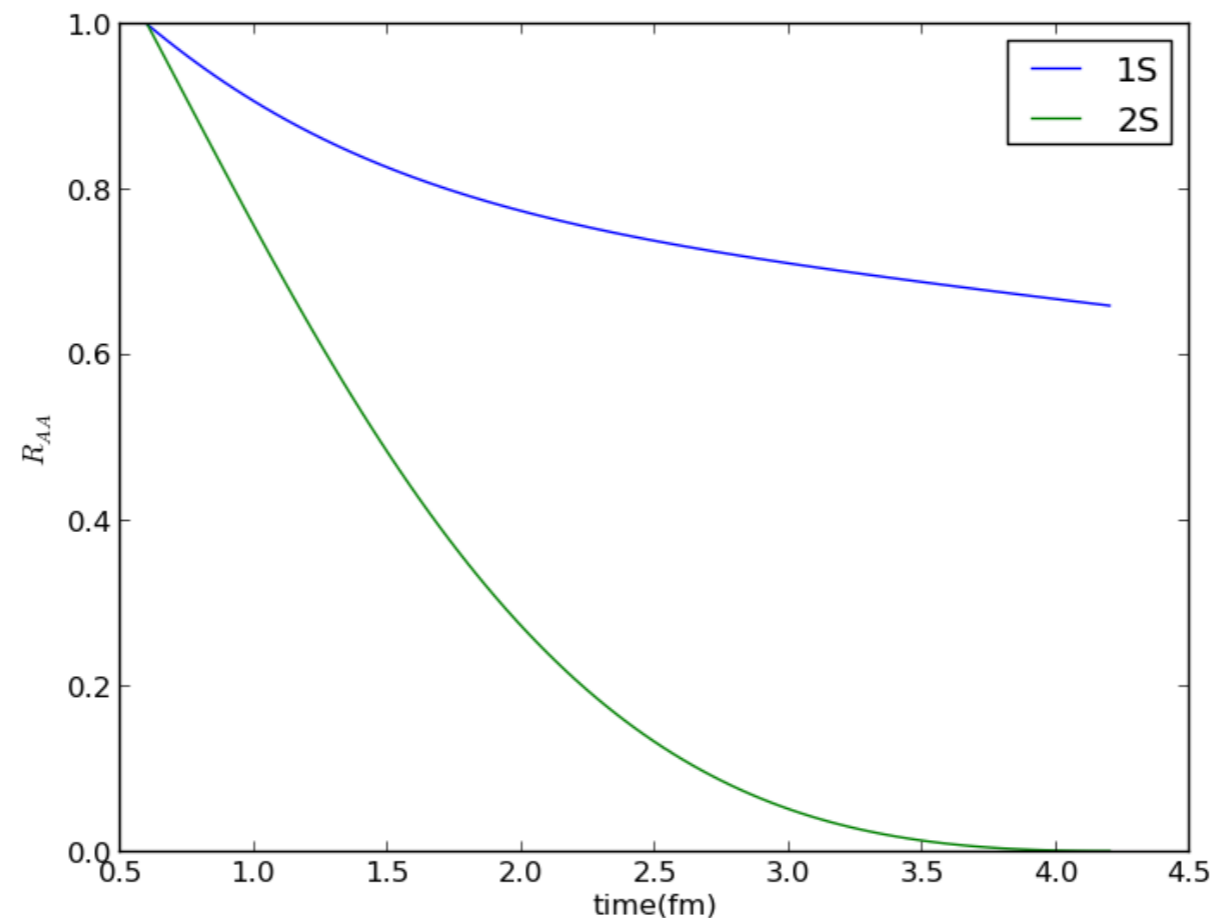
An EFT application

- First numerical results within a simple fireball, no CNM, no feeddown, simple initial conditions
- **Screening only**, R_{AA} for $\Upsilon(1S)$ (number of 1S states)



An EFT application

- First numerical results within a simple fireball, no CNM, no feeddown, simple initial conditions
- **Width and transitions, R_{AA} for $\Upsilon(1S)$**



Conclusions

- Lessons from the EFT framework
 - Systematically take into account corrections and include all medium effects
 - Give a rigorous QCD derivations of the potential, bridging the gap with potentials models which appear as leading-order picture here
 - Importance and interpretation of the complex potential
 - Nonperturbative extensions and applications (Peter's talk)

Discussion



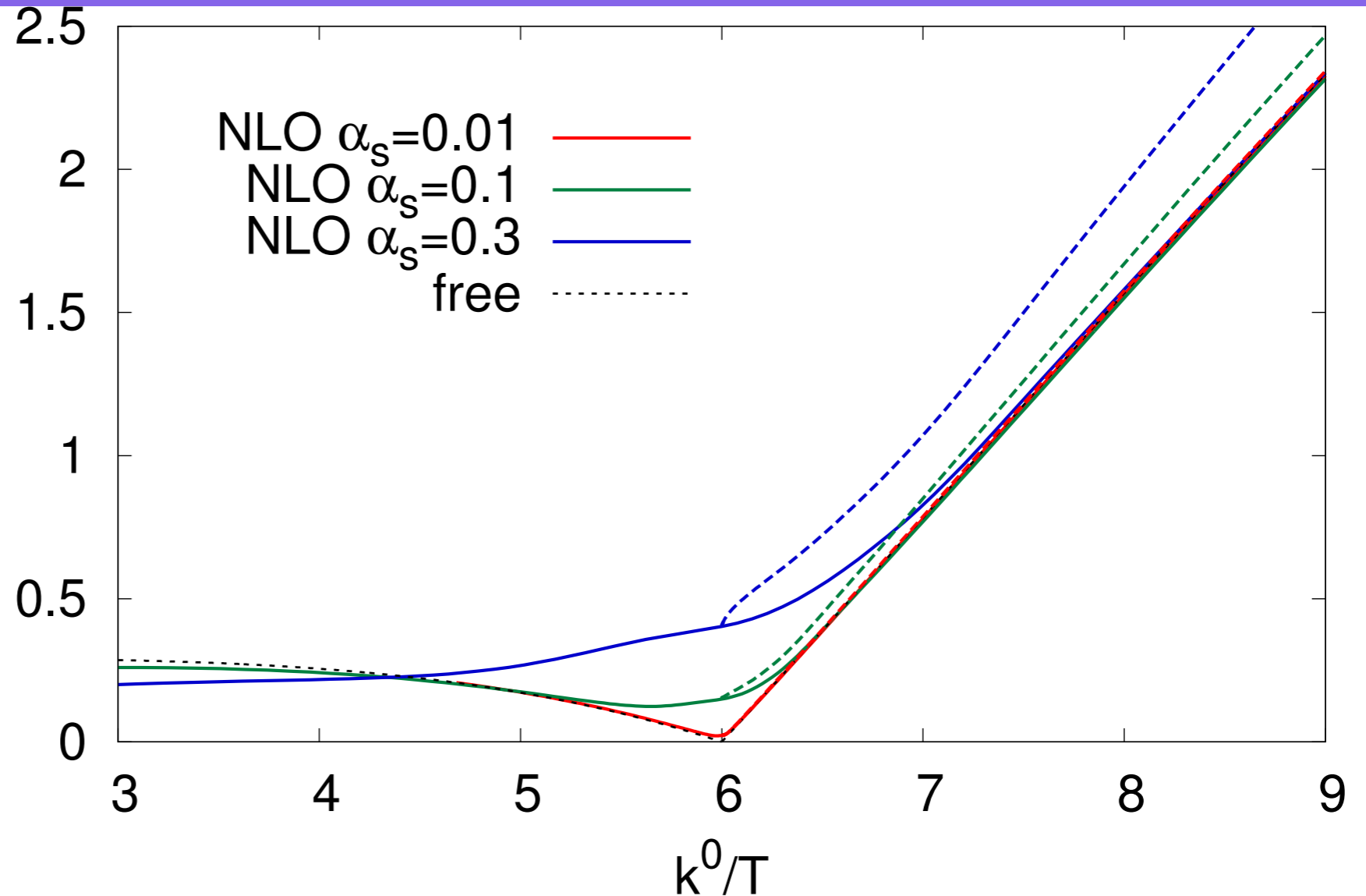
MAGNETIC DISCUSSION

brunDourneh

Photons and dileptons

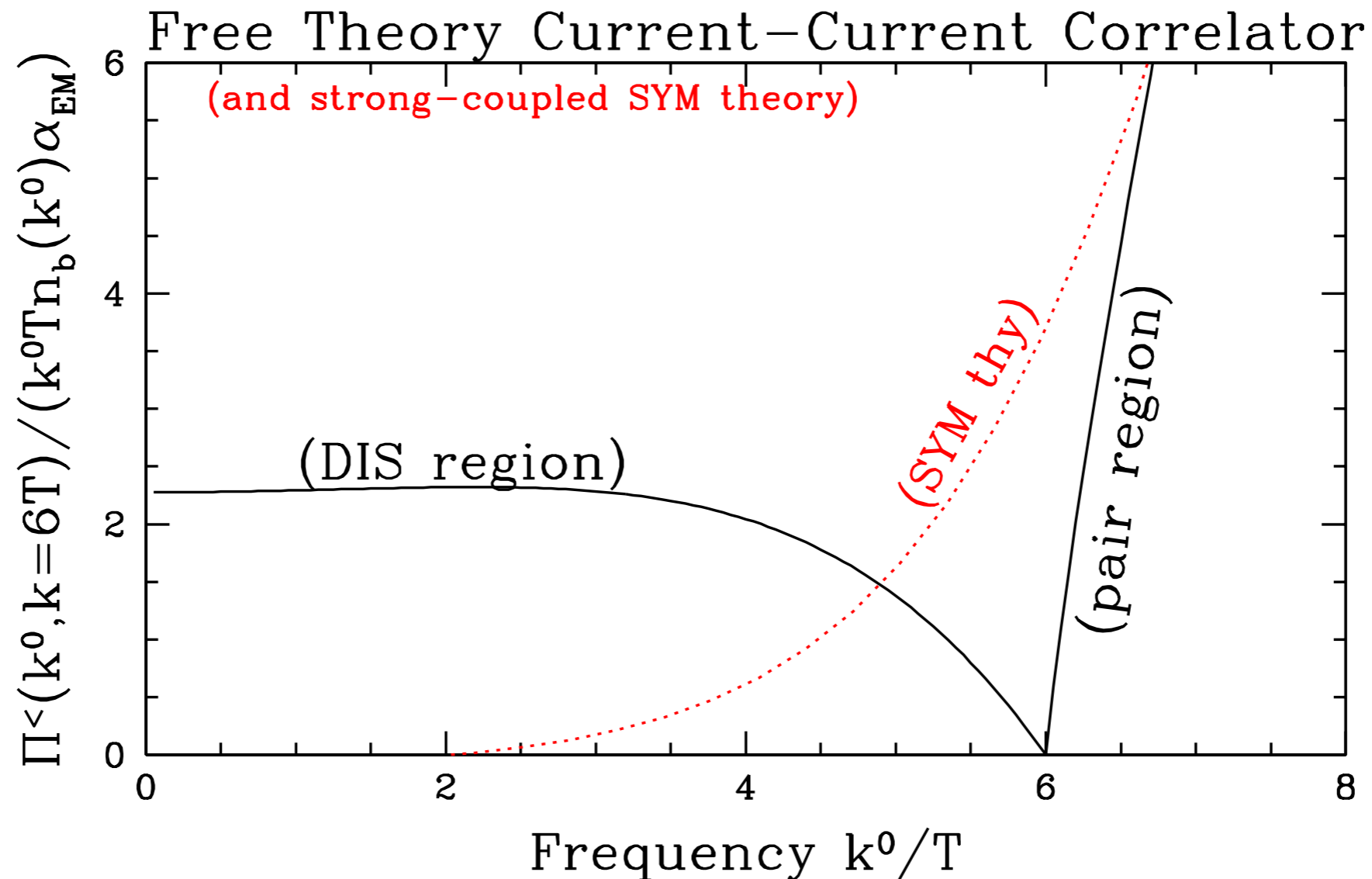
$$(2\pi)^4 \frac{dN_{l+l-}}{d^4K d^4X} \frac{3K^2 (\exp(k^0/T) - 1)}{16\alpha^2 k^0 T}$$

$$\rho_J(k^0, k=6T) / (4 \alpha_{EM} N_c \sum_s Q_s^2 k^0 T)$$



- Full lines: NLO calculation valid at small K^2 [JG Moore \(2014\)](#)
- Dashed lines: combination of the above with NLO calculation valid at large K^2 [Ghisoiu Laine \(2014\)](#) [Laine \(2013\)](#)

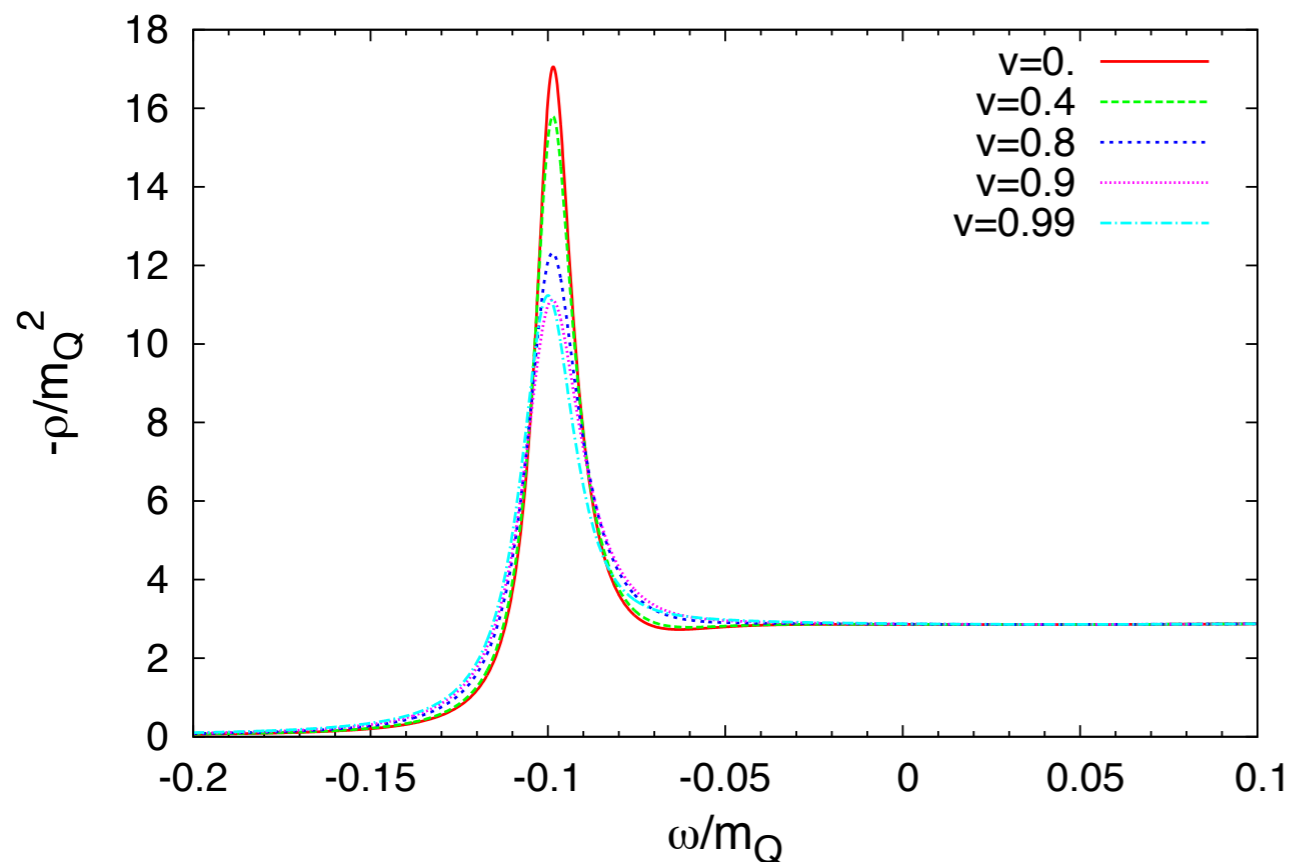
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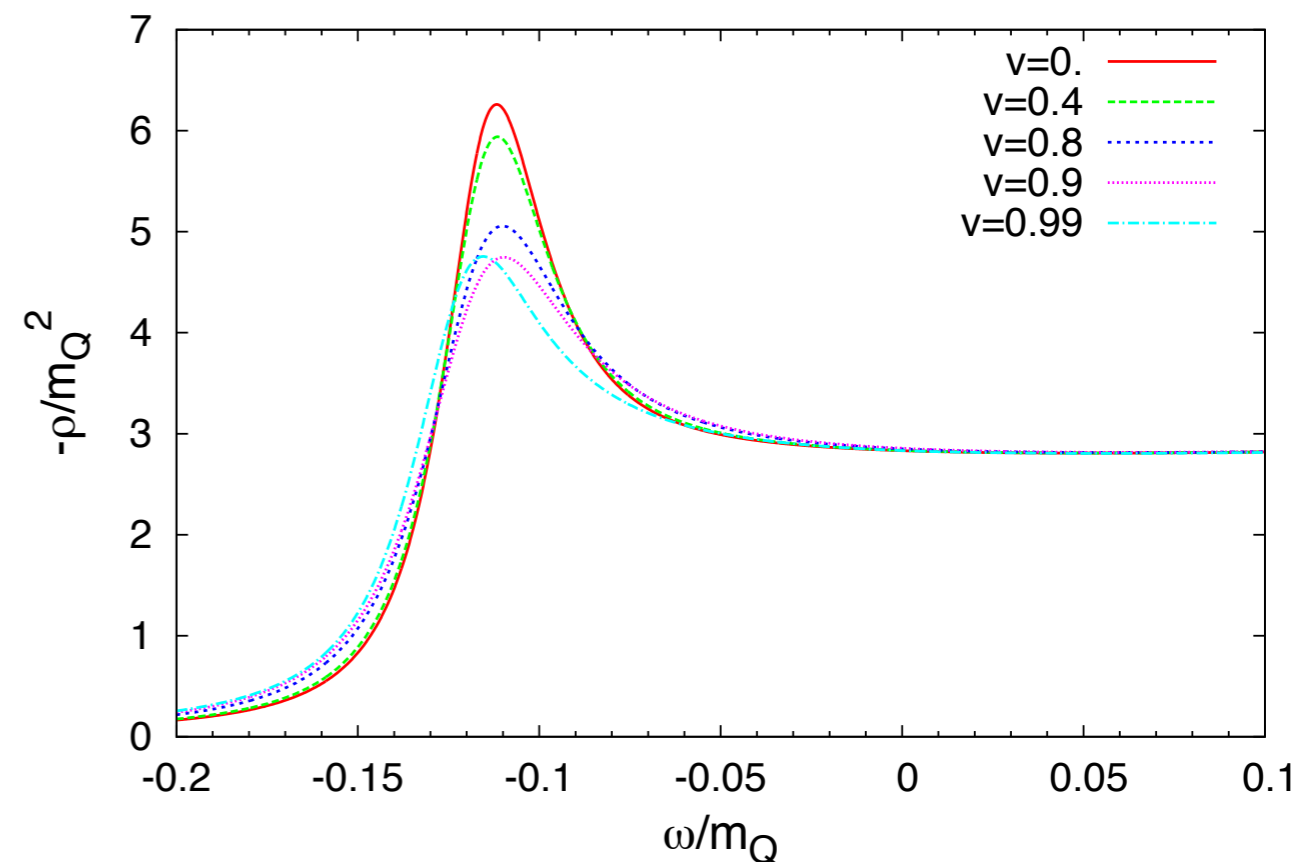
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EFTs at finite temperature

$\Upsilon(1S)$ $T = 250$ MeV



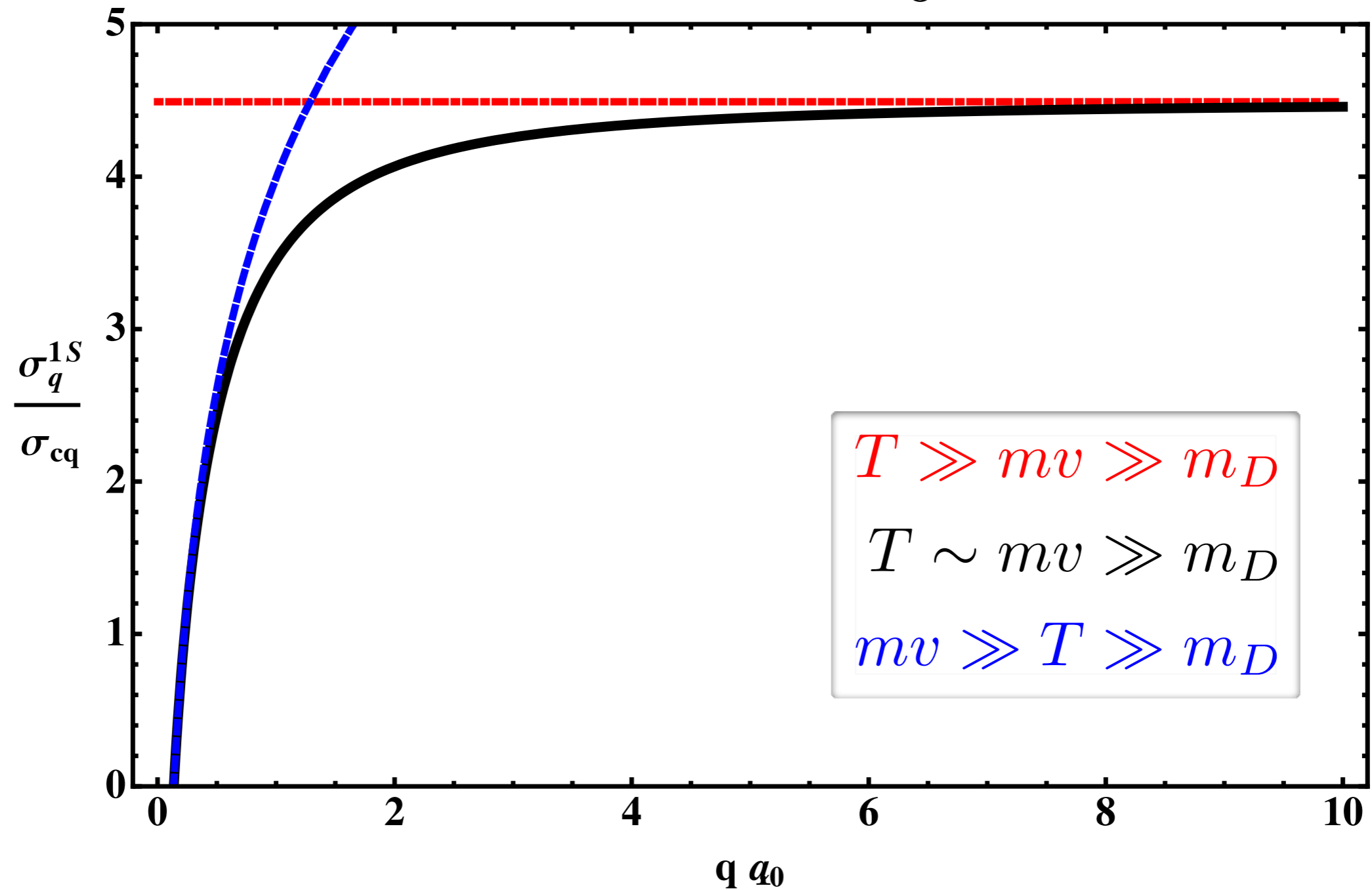
$\Upsilon(1S)$ $T = 400$ MeV



- Dependence on the relative velocity of quarkonium and medium Escobedo Giannuzzi Mannarelli Soto **PRD87** (2013)

Quasi-free cross section, quark contribution

$$m_D a_0 = 0.1 \quad \sigma_{cq} = \frac{32}{3} \pi \alpha_s^2 a_0^2$$



Potentials and free energies

- Thermodynamical free energies obtained from Polyakov loops are widely used and measured on the lattice
- We have studied the **correlator of Polyakov loops** and the associated **colour-average free energy** $\langle \text{Tr} L^\dagger(\mathbf{0}) \text{Tr} L(\mathbf{r}) \rangle = e^{-\frac{F_{Q\bar{Q}}(r,T)}{T}}$

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- We have shown that, with pNRQCD in imaginary time, it can be decomposed at short distances into gauge-invariant colour-singlet and octet free energies
- These free energies are quantitatively different from the real-time potentials

$$\text{Im}(F) = 0, \text{Im}(V) \neq 0 \quad \text{Re}(F) \neq \text{Re}(V)$$

Brambilla JG Petreczky Vairo **PRD82** (2010)

Potentials and free energies

- These free energies are quantitatively different from the potentials

$$\text{Im}(F) = 0, \text{Im}(V) \neq 0 \quad \text{Re}(F) \neq \text{Re}(V)$$

- Intuitively

$$t \rightarrow \infty \neq it = \frac{1}{T}$$



- Extra divergences can arise for observables spanning the entire imaginary axis

Berwein Brambilla JG Vairo **JHEP1303** (2013)