Measurement of event-by-event fluctuations and chemical freeze-out conditions at LHC energies

A. Kalweit, CERN

Ab initio approaches in many-body QCD confront heavy-ion experiments | 2014-DEC-17 | Alexander Kalweit



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98% of all particles are produced with p_T < 2 GeV/c → thermal particle production in a non-perturbative regime => thermodynamics => LATTICE QCD calculations

- Measurement of the production *yields* of identified particles and chemical freeze-out conditions:
 - Hadron resonance gas approach in thermal-statistical *models*
 - Reducing the model dependence and towards ab initio approaches
- Measurement of event-by-event *fluctuations* of conserved quantities:
 - net-charge fluctuations
 - plans for future measurements
 - allows direct comparison of measurements to ab initio calculations

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Still a lot of work to do at LHC energies.. Many questions are still open.



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Drift Pixel Strip ACORDE ITS FMD EMCAL T0 & V0 V0 TRD T0 HMPID FMD TRACKING CHAMBERS PMD ZDC -116m from LP MUON FILTER V0 T0 TRIGGER TPC CHAMBERS/ ZDC -116m from I.P./ TOF DIPOLE MAGNET PHOS ABSORBER

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ALICE is ideally suited for the measurement of light flavor hadrons on an event-by-event basis.

Bulk particle production

 Investigate matter in local thermal equilibrium => Look at the hadrons made up of the most abundantly produced quarks: u,d,s.

π, K, p, Λ, Ξ, Ω, Φ, K^{*0}, d, ³He, ³_ΛH, ⁴He

 Decays of strange particles feed into the states with lower mass and need to be carefully subtracted for consistent data ↔ model comparisons:

 $\begin{array}{ll} \Lambda \rightarrow p \ \pi & (63.9 \ \%) \\ \Xi \rightarrow \Lambda \ \pi & (99.87 \ \%) \end{array}$



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(ALICE Definition) *Primary particles* are defined as prompt particles produced in the collision including all decay products, except products from weak decays of light flavor hadrons and of muons.



Chemical freeze-out and thermal model calculations

Fluctuations and chemical freeze-out at LHC energies



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Particle yields of light flavor hadrons are described over 7 orders of magnitude within 20% (except K^{*0}) with a common chemical freeze-out temperature of T_{ch} \approx 156 MeV (prediction from RHIC extrapolation was \approx 164 MeV).

[Wheaton et al, Comput.Phys.Commun, 180 84] [Petran et al, arXiv:1310.5108] [Andronic et al, PLB 673 142]

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Three different versions of thermal model implementations give similar results.

[Wheaton et al, Comput.Phys.Commun, 180 84] [Petran et al, arXiv:1310.5108] [Andronic et al, PLB 673 142]

The importance of (anti-)nuclei

- Also for nuclei, it is a priori not clear that they can be described in a thermal picture:
 - The binding energy of the deuteron is $E_{\rm B}$ = 2.2 MeV.. In principle, they should immediately dissociate in a medium with $T_{\rm ch} \approx 160$ MeV and be suppressed by a large factor.
 - However, it is the entropy per baryon which determines their production yield and this is fixed at chemical freeze-out [1].
- The model gives a very good description of the data. Nuclei yields are very sensitive to the freeze-out temperature due to their large mass:
 yield ~ exp(-m/T_{ch})
- Predictions from non-equilibrium models cannot describe the data (disagree up to a factor of five).

Anti-alpha

- Studies of light (anti-)nuclei production in ALICE have recently been extended to the anti-alpha.
- Also in this case the yield is found to be in agreement with thermal model expectations.



Light nuclei and resonance feed-down

$$\langle N_i \rangle = V \cdot \left(n_i^{th}(T, \vec{\mu}) + \sum_K \Gamma_{K \to i} n_i^{th - Res.}(T, \vec{\mu}) \right)$$

[A. Andronic, P.Braun-Munzinger, J. Stachel]

[1407.5003]

- In a thermal model approach, the production of light (anti-)nuclei is not affected by feed-down from (maybe unknown) resonances.
- Thermal-statistical analysis of light (anti-)nuclei yields reduces the model dependence!
- Can we go even further and use Lattice QCD calculations to deduce the particle yields?
 → No. But see talk by Krzysztof for what can be done in this respect.



Event-by-event fluctuations of conserved quantities

Thermodynamic susceptibilities (1)

 Event-by-event fluctuations of the conserved quantities in QCD (*charge Q*, *baryon number B*, *strangeness S*) correspond to thermodynamic susceptibilities *x* of the system which can be directly calculated in Lattice QCD or in the Hadron Resonance Gas (HRG) model:

$$\chi^{BSQ}_{lmn} = \frac{\partial^{l+m+n}(P/T^4)}{\partial(\mu_B/T)^l \,\partial(\mu_S/T)^m \,\partial(\mu_S/T)^n}$$

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LHC (ALICE) data allows the most direct comparison to Lattice QCD calculations which correspond to $\mu_B = 0$. No extrapolation needed at LHC energies!



Thermodynamic susceptibilities (2)

• Moments µ and cumulants K:

$$\begin{array}{rclcrcrcrcrc} M & = & K_1 & = & \mu & = & \langle N \rangle & = & VT^3 \cdot \chi_1 \\ \sigma^2 & = & K_2 & = & \mu_2 & = & \langle (\delta N)^2 \rangle & = & VT^3 \cdot \chi_2 \\ S & = & K_3/\sigma^3 & = & \mu_3/\sigma^3 & = & \langle (\delta N)^3 \rangle / \sigma^3 & = & VT^3 \cdot \chi_3 / (VT^3 \cdot \chi_2)^{3/2} \\ \kappa & = & K_4/\sigma^4 & = & (\mu_4 - 3\mu_2^2) / \mu_2^2 & = & \langle (\delta N)^4 \rangle / \sigma^4 - 3 & = & (VT^3 \cdot \chi_4) / (VT^3 \cdot \chi_2)^2 \end{array}$$

• In ratios of cumulants, the volume dependence cancels:

$$\begin{array}{rclrcrcrcrc} \chi_2/\chi_1 &=& K_2/K_1 &=& \mu_2/\mu &=& \sigma^2/M \\ \chi_3/\chi_1 &=& K_3/K_1 &=& \mu_3/\mu &=& S\cdot\sigma^2/M \\ \chi_3/\chi_2 &=& K_3/K_2 &=& \mu_3/\mu_2 &=& S\cdot\sigma \\ \chi_4/\chi_2 &=& K_4/K_2 &=& (\mu_4 - 3\mu_2^2)/\mu_2 &=& \kappa\cdot\sigma^2 \\ \chi_6/\chi_2 &=& K_6/K_2 &=& (\mu_6 - 15\mu_4\mu_2 - 10\mu_3^2 + 30\mu_2^3)/\mu_2 && . \end{array}$$

Fluctuations and lattice QCD

• Thermodynamic susceptibilities at $\mu_B = 0$ can be directly calculated in lattice QCD.



 The HRG is a very good approximation below T_c, but significant deviations at T_c are expected with increasing order of the moments due to remnants of the critical chiral behavior:

 $\chi_6/\chi_2 < 0$ at T_c in Lattice QCD and $\chi_6/\chi_2 = 1$ in HRG

Towards a measurement of net-baryon number at LHC... (1)

- The measurement is experimentally very challenging:
 - Correction for detector efficiency (N.B.: efficiencies differ for protons and anti-protons due to absorption).
 - Auto-correlations with centrality estimator.
 - Contamination from protons from material.
 - Contamination from weak decays:
 - •Does the inclusion of them bring us closer to the total baryon number *B*?
 - •Can one separate cleanly χ_B and χ_S ?
 - Misidentified particles.









Detector efficiencies
Towards a measurement of net-baryon number at LHC... (2)

• The choice of the rapidity and momentum window is crucial:

- It needs to be systematically studied (see recent STAR results): larger than typical correlation length, but small enough that requirements of a grand-canonical ensemble are still fulfilled (global conservation of conserved quantity suppresses the signal).
- The statistic requirements depend crucially on the maximum window. Assuming a Skellam-Distribution, we obtain in the delta-theorem that

$$\Delta K_{6} \propto \sigma_{4} \propto (\langle N_{p} \rangle + \langle N_{p} \rangle)^{2}$$

 Current estimates indicate that between 10⁸ and 10¹⁰ central events are needed to measure sixth order moments depending on the acceptance window and the size of the expected effect.

Net charge fluctuations

Net charge fluctuations — introduction

- So far, only a net-charge measurements corresponding has been finalized at LHC energies: [Phys. Rev. Lett. 110, 152301].
- Simplified picture:



• v_{dyn} as robust variable to quantify dynamical fluctuations and to identify relevant charge carriers:

$$\boldsymbol{v}_{(+-,dyn.)} = \frac{\left\langle N_{+} \left(N_{+} - 1 \right) \right\rangle}{\left\langle N_{+} \right\rangle^{2}} + \frac{\left\langle N_{-} \left(N_{-} - 1 \right) \right\rangle}{\left\langle N_{-} \right\rangle^{2}} - 2 \frac{\left\langle N_{+} N_{-} \right\rangle}{\left\langle N_{+} \right\rangle \left\langle N_{-} \right\rangle}$$

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Substantially smaller value of the correlation function is expected in the QGP phase than in the hadronic phase.

Hadronic phase: $q = \pm 1$ $\Rightarrow q^2 = \pm 1$ Partonic phase: q = $\pm(2/3), \pm(1/3)$ => q² = $\pm(4/9), \pm(1/9)$

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$$D = \langle N_{ch} \rangle \langle \delta R^2 \rangle$$
$$D \approx 4 \frac{\langle \delta Q^2 \rangle}{\langle N_{ch} \rangle} \approx \begin{cases} 3 & \text{HRG} \\ 1 - 1.5 & \text{QGF} \end{cases}$$
$$D - 4 \approx \langle N_{ch} \rangle v_{(+-,dyn)}^{corr}$$

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HIJING shows no centrality dependence and larger values than the data.



Energy and rapidity window dependence

- Results are shown for 0-5% most central collisions.
- Decreasing trend with increasing center-of-mass energy is observed.
- ALICE values significantly lower than the hadron gas expectation while RHIC measurements are still compatible.
- Strong dependence on rapidity window observed which saturates above Δη ≈ 2.3. Initial fluctuations are diluted by final state interactions and limited experimental acceptance.



Summary & conclusion

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- Light flavor hadron yields at LHC energies can be described in a thermal fit based on the hadron resonance gas model with a chemical freeze-out temperature of $T_{chem} = 156$ MeV.
- Production of light (anti-)nuclei is found to be in agreement with this temperature. Within thermal-statistical models, their yield is independent of feed-down from (unknown) resonances.
- In order to find deviations from HRG, the measurements of event-by-event fluctuations of conserved quantities (charge, baryon number, strangeness) are on their way...
- Measurements of net-charge fluctuations indicate a reduction of fluctuations from RHIC to LHC (as expected), but also emphasize the importance of systematic studies w.r.t. to the acceptance window etc.

SUPPORTING SLIDES

Missing strange resonances (Lattice QCD)





ALI-PREL-74481





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ALI-PREL-74481



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A combined blast-wave fit to the data (**simplified hydro model** $\rightarrow T_{kin}$, β) gives also a reasonable description allowing a systematic study of the evolution of the spectral shape versus centrality.



one finds: $T_{kin} \approx 100$ MeV significantly smaller than $T_{chem} \approx 156$ MeV and an average transverse expansion velocity around $<\beta_T> 0.65$ for most central Pb-Pb collisions. Characteristic hardening of the spectrum with increasing centrality. It is more pronounced for the heavier protons than for pions. → Mass ordering as expected from hydrodynamics.

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Baryon-to-meson and meson-to-meson have similar shapes in central Pb-Pb if the masses of the baryon and the meson are similar: p/Φ is flat as a function of p_T for $p_T < 3-4$ GeV/*c*.

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of the baryon and the meson are similar: p/Φ is flat as a function of p_T for $p_T < 3-4$ GeV/c.

Pb-Pb, consistent with the hydrodynamic picture.





Hydrodynamic models (EPOS, Krakow) show a better agreement than QCD inspired models (DPMJET).



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Hyper-Triton



ALI-PREL-54321



ALI-PREL-81005

Hagedorn spectrum



Protons



Thermodynamics and hadron chemistry

Short introduction to statistical thermodynamics (B)

• We therefore distinguish three different *statistical ensembles*:

(i) micro-canonical: E, V, N fix

(ii) canonical: *T*, *V*, *N* fix \rightarrow given volume element is coupled to a heat bath

(iii) grand-canonical: *T*, *V*, μ fix \rightarrow given volume element can also exchange particles with its surrounding (heat bath and particle reservoir)



Short introduction to statistical thermodynamics (C)

- A small example: barometric formula (density of the atmosphere at a fixed temperature).
- Probability to find a particle on a given energy level *j*:



$$\frac{n(h_1)}{n(h_0)} = \frac{NP(h_1)}{NP(h_0)} = e^{-\frac{\Delta E_{pot}}{k_{\rm B}T}}$$
$$= e^{-\frac{Mg}{RT}\Delta h}$$

 Starting point: grand-canonical partition function for an *relativistic ideal quantum gas of hadrons* of particle type i (i = pion, proton,... → full PDG!):

$$\ln Z_{GK_i} = \pm g_i \frac{V}{2\pi^2 \hbar^3} \int_0^\infty dp \ p^2 \ln \left(1 \pm e^{-\beta(\epsilon(p) - \mu_i)}\right)$$

• Once the partition function is known, we can calculate all other thermodynamic quantities: $1 \partial (T \ln Z) = \partial (T \ln Z) = 1 \partial (T \ln Z)$

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spin
degeneracy

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Only two free parameters are needed: (T,μ_B) . Volume cancels if particle ratios n_i/n_j are calculated. If yields are fitted, it acts as the third free parameter.

 Starting point: grand-canonical partition function for an *relativistic ideal quantum gas of hadrons* of particle type i (i = pion, proton,... → full PDG!):



• Once the partition function is known, we can calculate all other thermodynamic quantities: $\frac{1}{2} \frac{\partial (T \ln Z)}{\partial T \ln Z} = \frac{1}{2} \frac{\partial (T \ln Z)}{\partial T \ln Z}$

$$n = \frac{1}{V} \frac{\partial (T \ln Z)}{\partial \mu} P = \frac{\partial (T \ln Z)}{\partial V} s = \frac{1}{V} \frac{\partial (T \ln Z)}{\partial T}$$

Only two free parameters are needed: (T,μ_B) . Volume cancels if particle ratios n_i/n_j are calculated. If yields are fitted, it acts as the third free parameter. Partition function shown here is only valid in the resonance gas limit (HRG), i.e. relevant interactions are mediated via resonances, and thus the non-interacting hadron resonance gas can be used as a good approximation for an interacting hadron gas.

• We thus arrive at a set of formulas:

$$\begin{pmatrix} n_i \\ \varepsilon_i \\ s_i \\ P_i \end{pmatrix} = \frac{g_i}{2\pi^2} \begin{pmatrix} \int_0^\infty \frac{p^2 dp}{e^{(E_i(p) - \mu_i)/T} \pm 1} \\ \int_0^\infty \frac{p^2 dp}{e^{(E_i(p) - \mu_i)/T} \pm 1} E_i(p) \\ \pm \int_0^\infty dp \ p^2 \left(\ln \left(1 \pm e^{-(E_i(p) - \mu_i)/T} \right) \pm \frac{E_i(p) - \mu_i}{T(e^{(E_i(p) - \mu_i)/T} \pm 1)} \right) \\ \pm \int_0^\infty dp \ p^2 \ln \left(1 \pm e^{-(E_i(p) - \mu_i)/T} \right) \end{pmatrix}$$

$$\begin{split} \mu_i &= \mu_B B_i + \mu_S S_i + \mu_{I_3} I_{3_i} + \mu_C C_i \\ E_i &= \sqrt{p^2 + m_i^2} \end{split}$$

- Baryon number conservation: $V \sum_{i} n_i B_i = Z + N$
- Initial system has no charm or strangeness:

$$\sum_i n_i(\mu_S)S_i = 0$$
 $\sum_i n_i(\mu_C)C_i = 0$

• Third component of isospin:

$$V\sum_i n_i(\mu_{I_3})I_{3_i}=rac{Z-N}{2}$$

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Sum over all known states from the PDG!

Final results of a thermal model calculation depend on the used hadron list (resonance spectrum)!

Thermal fits and comparison to data

• Non-stable particles (resonances) are decayed and the result is compared to the experimentally accessible particles. The procedure is iterated for different sets of (T, $\mu_{\rm B}$, V) until the residuals between data and model are minimized.



Thermal model from: A.Andronic, P. Braun-Munzinger, J. Stachel, Nucl. Phys.A 772 (2006) 167

Implementations of statistical models

- Original ideas go back to Pomeranchuk (1950s) and Hagedorn (1970s).
- Precise implementations and also interpretations differ from group to group (no details in this lecture!):
 - K. Redlich
 - P. Braun-Munzinger, J. Stachel, A. Andronic (GSI)
 - •Eigen-volume correction: ideal gas \rightarrow Van-der-Waals gas
 - •emphasis on complete hadron list
 - F. Becattini
 - •non-equilibrium parameter $\gamma_{\rm S}{}^{\rm N}$
 - J. Rafelski (SHARE)
 - -non-equilibrium parameter $\gamma_{\text{S}}{}^{\text{N}}$ and $\gamma_{\text{q}}{}^{\text{N}}$
 - J. Cleymans (THERMUS)
 - •Allows also canonical suppression in sub-volumes of the fireball
 - W. Broniowski, W. Florkowski (THERMINATOR)
 - •space time evolution, p_T -spectra, HBT, fluctuations

Hypertriton branching ratio



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PHYSICAL REVIEW D

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Properties of ${}_{\Lambda}\mathbf{H}^{3}^{\dagger}$

G. KEYES

Argonne National Laboratory, Argonne, Illinois 60439 and Northwestern University, Evanston, Illinois 60201

AND

M. DERRICK, T. FIELDS,* AND L. G. HYMAN Argonne National Laboratory, Argonne, Illinois 60439

AN

J. G. FETKOVICH, J. MCKENZIE, B. RILEY, AND I.-T. WANG Carnegie-Mellon University, Pittsburgh, Pennsylvania 15213 (Received 29 September 1969)

The properties of the hypernucleus ${}_{A}$ H³ were measured in an analysis of helium-bubble-chamber pictures taken at the Argonne ZGS. Some 90 examples of the production reaction K^- +He⁴ $\rightarrow {}_{A}$ H³+p+ π^- were found in which the K^- stopped and the ${}_{A}$ H³ decayed via π^- emission. Twenty-seven events were observed to decay via the two-body mode ${}_{A}$ H³ $\rightarrow \pi^-$ He³, with the remaining events decaying to $\pi^- pd$ or $\pi^- pdn$. The production rate was measured to be $(1.8_{-0.6}^{+0.7}) \times 10^{-3}$ per stopping K^- . The mean life of the hyperfragment was measured from the two-body decay mode as $(2.64_{-0.52}^{+0.84}) \times 10^{-10}$ sec. The lifetime obtained from the three-body decays was consistent with this value, after the elimination of a serious source of background. The binding energy of ${}_{A}$ H³ was measured to be 0.25 ± 0.31 MeV. The decay ratio $R_3 = \Gamma({}_{A}$ H³ $\rightarrow \pi^-$ He³)/ $\Gamma({}_{A}$ H³ \rightarrow all π^-) was measured to be $0.36_{-0.06}^{+0.08}$, in agreement with values from previous experiments and with the value calculated for spin $\frac{1}{2}$ for the ${}_{A}$ H³ hypernucleus.

TABLE I. Partial and total mesonic and nonmesonic decay rates and corresponding lifetimes.

Channel	Γ [sec ⁻¹]	Γ/Γ_{Λ}	$\tau = \Gamma^{-1}$ [sec]
³ He $+\pi^-$ and ³ H $+\pi^0$	0.146×10^{10}	0.384	0.684×10^{-9}
$d+p + \pi^-$ and $d+n+\pi^0$	0.235×10^{10}	0.619	0.425×10^{-9}
$p + p + n + \pi^{-}$ and $p + n + n + \pi^{0}$	0.368×10^{8}	0.0097	0.271×10^{-7}
All mesonic channels	0.385×10^{10}	1.01	0.260×10^{-9}
d+n	0.67×10^{7}	0.0018	0.15×10^{-6}
p + n + n	0.57×10^{8}	0.015	0.18×10^{-7}
All nonmesonic channels	0.64×10^{8}	0.017	0.16×10^{-7}
All channels	0.391×10^{10}	1.03	2.56×10^{-10}
Expt. [6]			$2.64 + 0.92 - 0.54 \times 10^{-10}$
Expt. (averaged) [11]			$2.44 + 0.26 - 0.22 \times 10^{-10}$

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