

# In-medium quarkonium properties and LQCD

Péter Petreczky

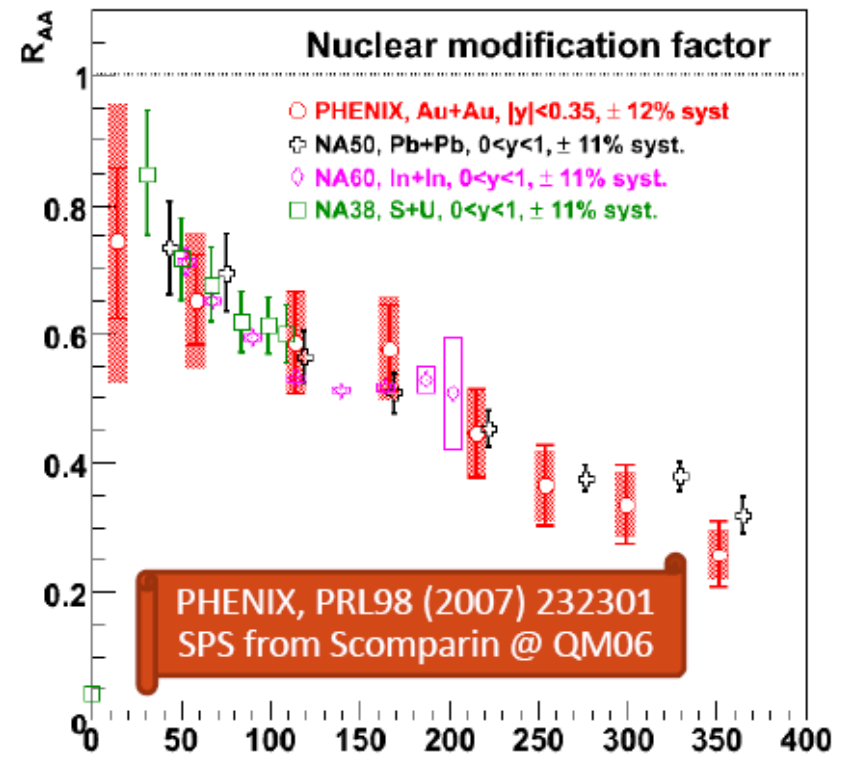


Quarkonium is signal for **deconfinement** and **color screening** (?)

To understand the experimental results it is necessary (**though not sufficient**) to know:

- What are the properties of quarkonia at high  $T$  in-medium masses and widths or melting:  
*Problem solved in weakly coupled case (see talk by Jacopo Ghiglieri)*

For the real world use  
LQCD +EFT and crosschecks from  
the spatial meson correlators



R. Granier de Cassagnac, Joint CATHIE-INT mini program Quarkonium in Hot Media 2009

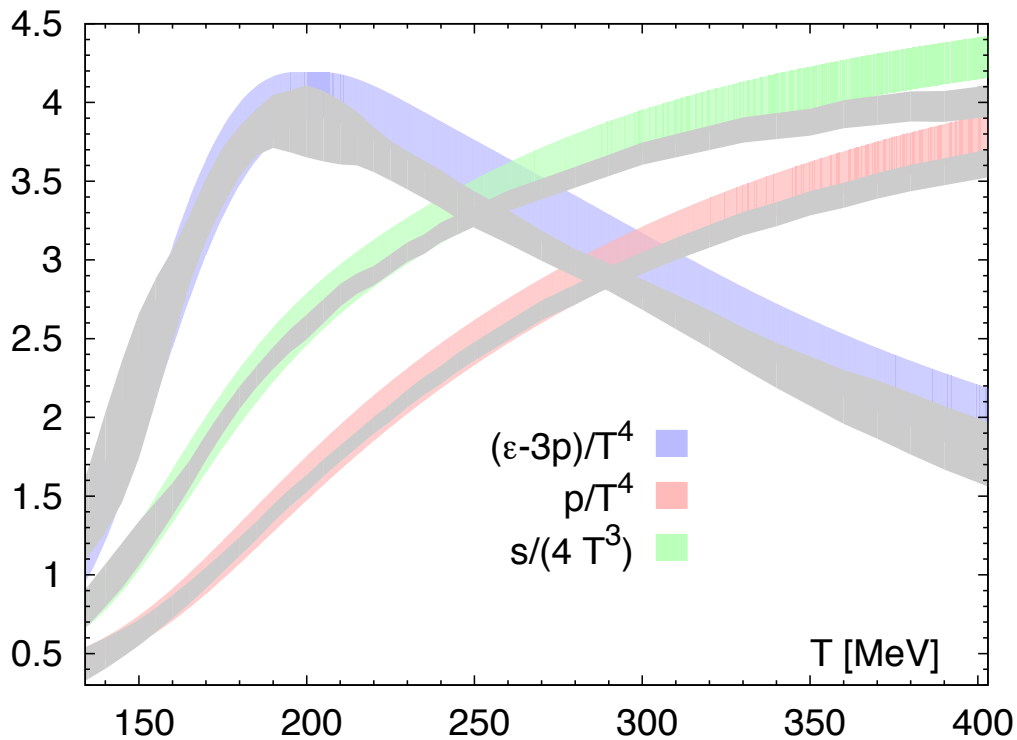
Ab initio approached in many body QCD confront HIE , Heidelberg, December 15-18, 2014

# Lattice QCD in 2014

Continuum results for physical quark masses for  $T_c$  and EoS :  
Chiral transition temperature

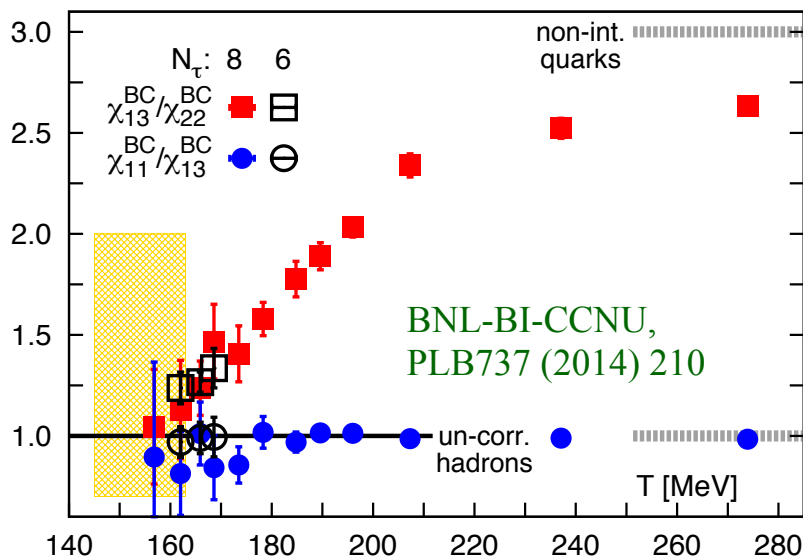
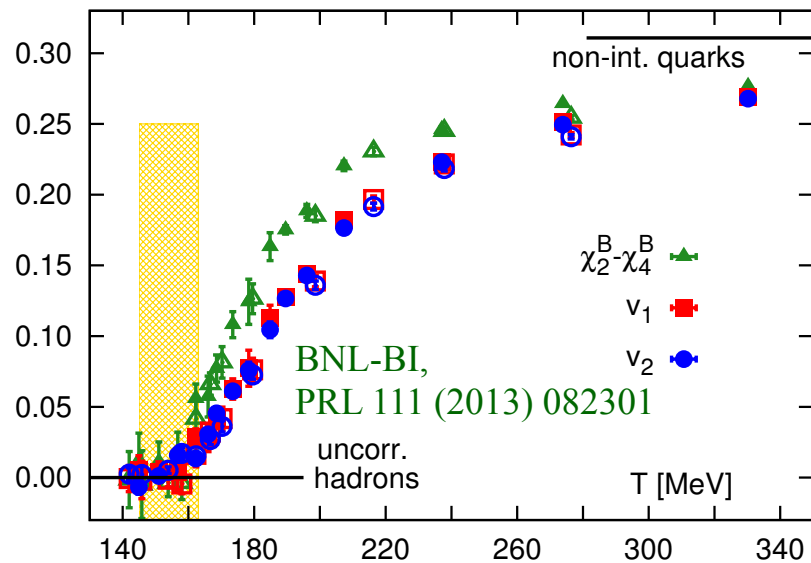
$$T_c = (154 \pm 9) \text{ MeV}$$

HotQCD, Bazavov et al, PRD90 (2014) 094503



Grey bands: WB, Borsanyi et al, PLB730 (2014) 99

Deconfinement of strange and charmed hadrons

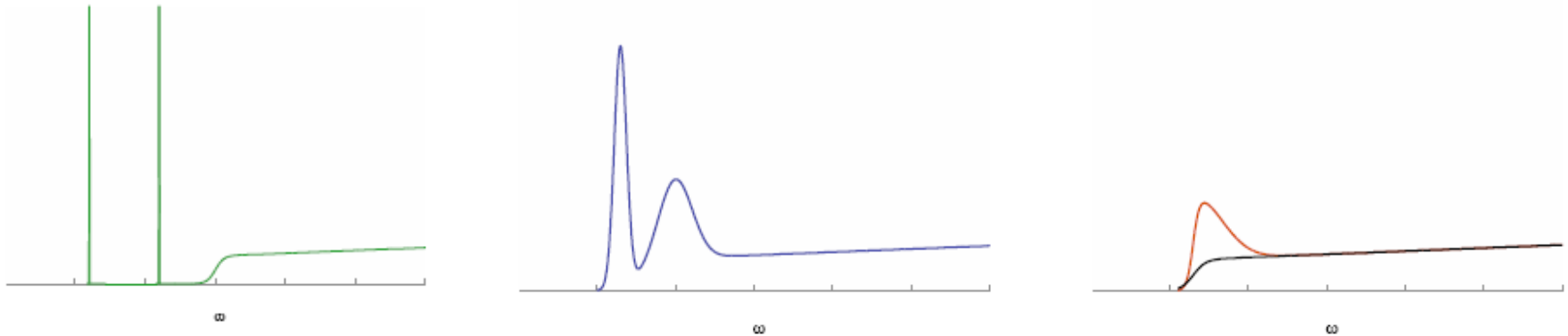


# Meson spectral functions

In-medium properties and/or dissolution of quarkonium states are encoded in the spectral functions

$$\sigma(\omega, p, T) = \frac{1}{2\pi} \text{Im} \int_{-\infty}^{\infty} dt e^{i\omega t} \int d^3x e^{ipx} \langle [J(x, t), J(x, 0)] \rangle_T$$

Melting is seen as progressive broadening and disappearance of the bound state peaks



Due to analytic continuation spectral functions are related to Euclidean time quarkonium correlators that can be calculated on the lattice

$$G(\tau, p, T) = \int d^3x e^{ipx} \langle J(x, -i\tau), J(x, 0) \rangle_T$$

$$G(\tau, p, T) = \int_0^{\infty} d\omega \sigma(\omega, p, T) \frac{\cosh(\omega \cdot (\tau - \frac{1}{2T}))}{\sinh(\omega/(2T))} \xrightarrow{\text{MEM}} \sigma(\omega, p, T)$$

*IS charmonium survives to  $1.6T_c$ ??*

Umeda et al, EPJ C39S1 (05) 9, Asakawa, Hatsuda, PRL 92 (2004) 01200, Datta, et al, PRD 69 (04) 094507, ...

Recent improvements: new Bayesian approach, Burnier and Rothkopf, PRL 111 (2013) 182003

# Quarkonium correlators at $T > 0$

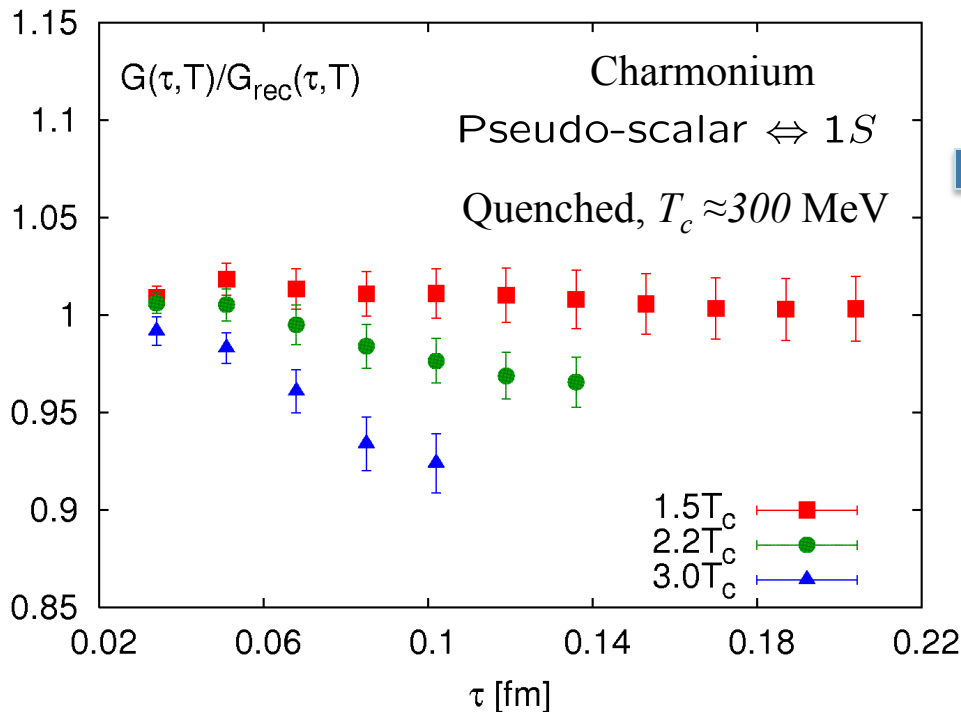
temperature dependence of  $G(\tau, T)$

$$G(\tau, T) = \int_0^\infty d\omega \sigma(\omega, T) \frac{\cosh(\omega(\tau - 1/(2T)))}{\sinh(\omega/(2T))}$$

If there is no  $T$ -dependence in the spectral function,  $G(\tau, T)/G_{rec}(\tau, T) = 1$

$$G_{rec}(\tau, T) = \int_0^\infty d\omega \sigma(\omega, T=0) \frac{\cosh(\omega \cdot (\tau - \frac{1}{2T}))}{\sinh(\omega/(2T))}$$

Datta, Karsch, P.P., Wetzorke, PRD 69 (04) 094507



Euclidean time charmonium correlation function show very mild  $T$ -dependence  
Limited sensitivity to the in-medium modification of the spectral function

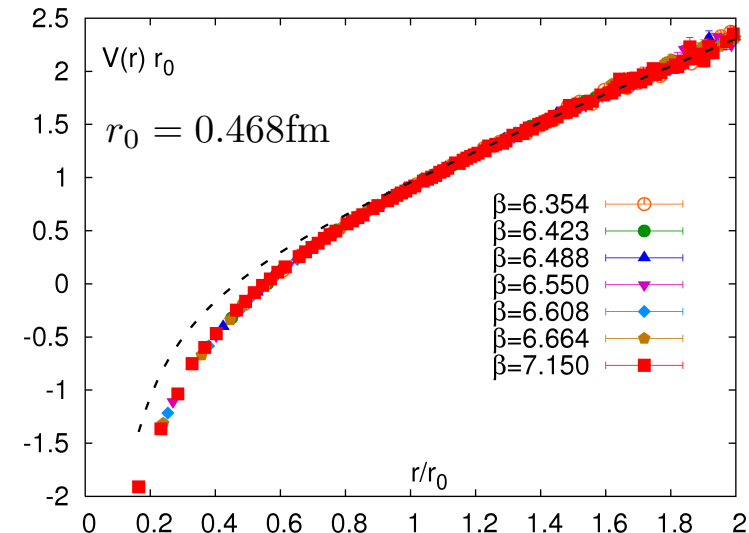
Large discretization errors for  $b$ -quarks,  $a M_b \sim 1$

LQCD +EFT for spectral function calculations + crosschecks from the spatial meson correlators

# Non-relativistic potential models for quarkonia

## $T=0$ quarkonium spectrum

state	$J/\psi$	$\chi_c$	$\psi'$	$\Upsilon$	$\chi_b$	$\Upsilon'$	$\chi'_b$	$\Upsilon''$
mass [GeV]	3.10	3.53	3.68	9.46	9.99	10.02	10.26	10.36
$\Delta E$ [GeV]	0.64	0.20	0.05	1.10	0.67	0.54	0.31	0.20
$\Delta M$ [GeV]	0.02	-0.03	0.03	0.06	-0.06	-0.06	-0.08	-0.07
$r_0$ [fm]	0.50	0.72	0.90	0.28	0.44	0.56	0.68	0.78



From Satz 2005

$$r_{J/\psi} \simeq r_{\chi_b} \simeq r_{\Upsilon'} \quad \Delta E_{J/\psi} \simeq \Delta E_{\chi_b} \simeq \Delta E_{\Upsilon'}$$

Medium effects on quarkonia depend on their size and or binding energy, e.g. in color screening picture dissolution is expected when  $r_{J/\psi} \approx r_D$



Sequential dissolution pattern:

$$T_d(\Upsilon) > T_d(J/\psi) > T_d(\chi_c)$$

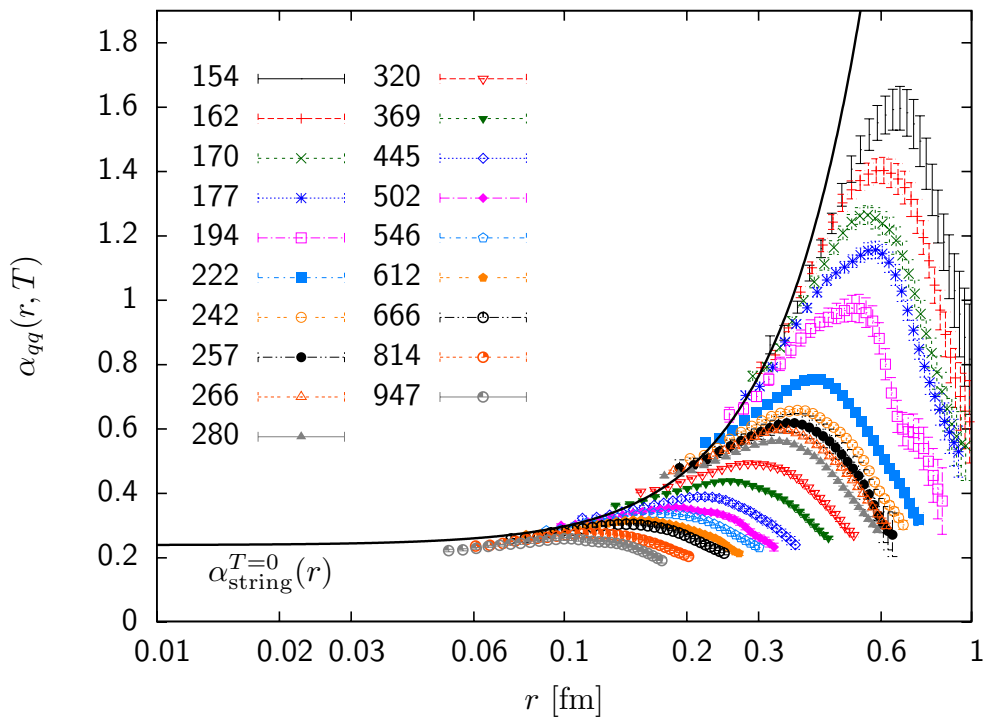
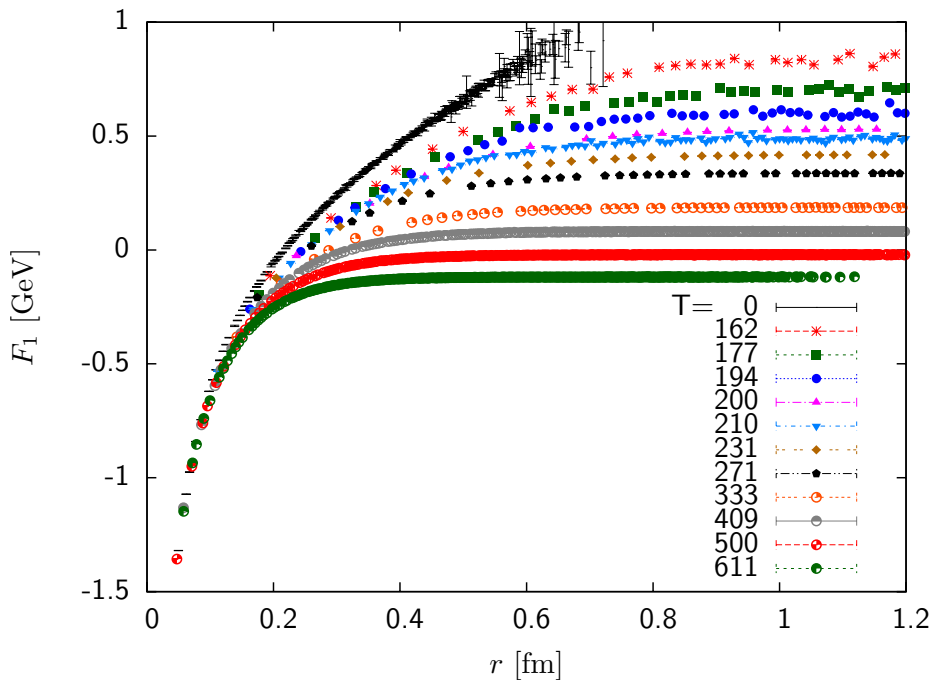
$$T_d(J/\psi) \simeq T_d(\chi_b) \simeq T_d(\Upsilon')$$

# Singlet free energy of static QQbar pair

Singlet free energy in Coulom gauge:

$$e^{-F_1(r,T)/T} = \text{Tr}\langle L(r)L^\dagger(0)\rangle, \quad L(x) = \prod_{x_0=1}^{N_\tau-1} U_0(x_0, x) \quad \alpha_{qq}(r, T) = r^2 \frac{dF_1(r, T)}{dr}$$

In collaboration with J. Weber



$\alpha_{qq}(r_{max}, T) > 0.5$  for  $T < 300\text{MeV}$



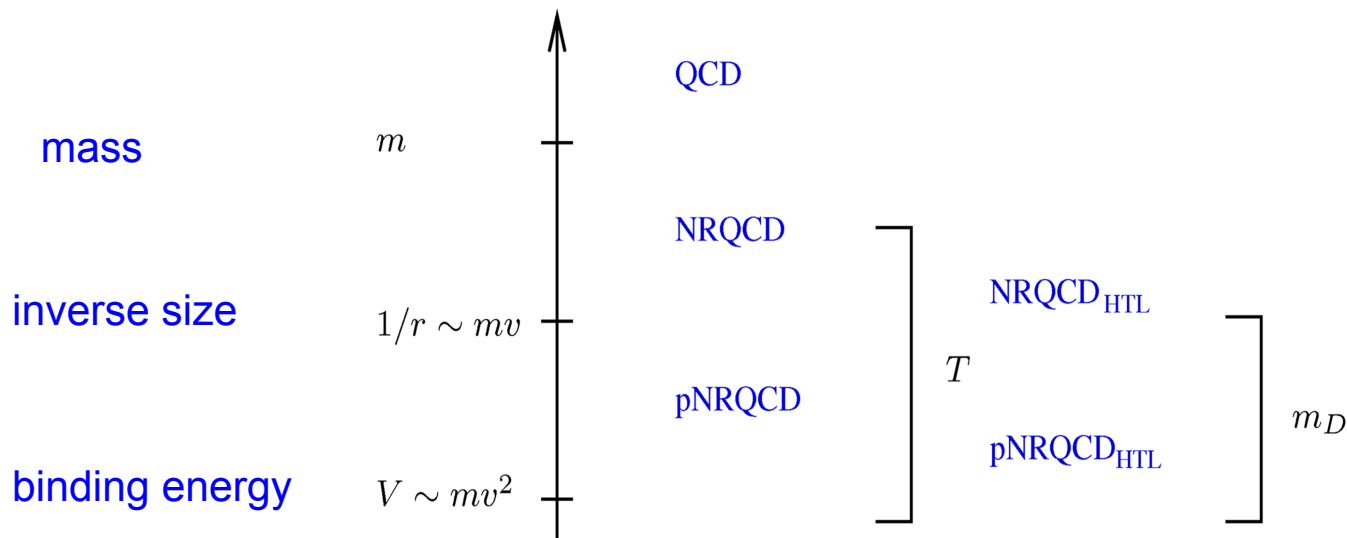
Strongly interacting QGP

$r_{max}(T) \simeq 0.5/T$  comparable to  $r_{J/\psi}, r_{\Upsilon'}, r_{\chi_b}, T \simeq 220\text{MeV}$

melting of 1S charmonia and excited bottomonia ?

# Effective field theory approach for heavy quark bound states and potential models

The heavy quark mass provides a hierarchy of different energy scales



The scale separation allows to construct sequence of effective field theories:  
NRQCD, pNRQCD

Potential model appears as the tree level approximation of the EFT  
and can be systematically improved

# NRQCD at finite temperature

EFT for energy scale  $m v$

$$\mathcal{L} = -\frac{1}{4} F_{\mu\nu}^a F^{a\mu\nu} + \sum_{i=1}^{n_f} \bar{q}_i i \not{D} q_i + \psi^\dagger \left( D_\tau - \frac{\mathbf{D}^2}{2m} \right) \psi + \chi^\dagger \left( D_\tau + \frac{\mathbf{D}^2}{2m} \right) \chi + \dots$$

Heavy quarks : non-relativistic Pauli spinors: no heavy quark pair creation, not part of the thermal medium, no boundary condition on the heavy quark fields



reconstruction of the spectral functions is simpler



No large discretization effects  $O(am)$



$\tau_{max} = 1/T$  instead of  $1/(2T)$   
In the relativistic formulation



Smaller contribution from the high frequency part, no transport peak

Bottomonium spectral functions:

Aarts et al, PRL 106 (2011) 061602; JHEP 1111 (2011) 103; arXiv:1402.6210

Kim, PP, Rothkopf, arXiv:1409.3630v1



# pNRQCD at finite temperature

EFT for energy scale  $E_{bind} \sim m v^2$

Brambilla, Ghiglieri, P.P., Vairo, PRD 78 (2008) 014017

Ultrasoft quark and gluons

$$\mathcal{L} = -\frac{1}{4} F_{\mu\nu}^a F^{a\mu\nu} + \sum_{i=1}^{n_f} \bar{q}_i i \not{D} q_i$$

Singlet  $Q\bar{Q}$  field

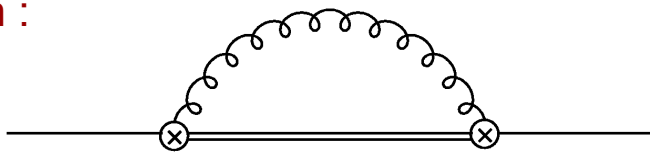
Octet  $Q\bar{Q}$  field

$$+ \int d^3r \text{Tr} \left\{ S^\dagger \left[ i\partial_0 - \frac{-\nabla^2}{m} - V_s(r, T) \right] S + O^\dagger \left[ iD_0 - \frac{-\nabla^2}{m} - V_o(r, T) \right] O \right\}$$

$$+ V_A \text{Tr} \left\{ O^\dagger \vec{r} \cdot g\vec{E} S + S^\dagger \vec{r} \cdot g\vec{E} O \right\} + \frac{V_B}{2} \text{Tr} \left\{ O^\dagger \vec{r} \cdot g\vec{E} O + O^\dagger O \vec{r} \cdot g\vec{E} \right\} + \dots$$

If  $E_{bind} < T$  there are thermal contribution to the potentials :  $V_s(r) \rightarrow V_s(r) + \delta V_s(r, T)$

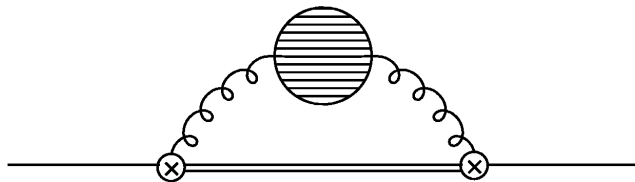
singlet-octet transition :



$$\text{Re} \delta V_s(r) \sim \alpha_s^2 T^2 r$$

$$\text{Im} \delta V_s(r) \sim \alpha_s^3 T$$

Landau damping :



$$\text{Re} \delta V_s(r, T) \sim \text{Im} \delta V_s(r, T)$$

$$\sim \alpha_s T^3 r^2 \times \left( \frac{m_D}{T} \right)^n$$

Free field equation:  $\left[ i\partial_0 - \frac{-\nabla^2}{m} - V_s(r, T) \right] S(r, t) = 0 \rightarrow$  potential model

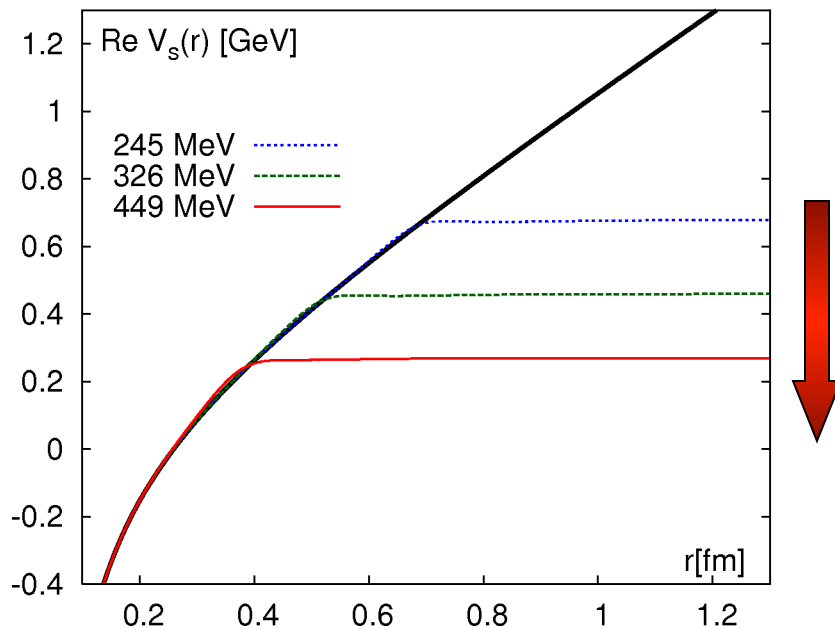
# pNRQCD beyond weak coupling and potential models

Above deconfinement the binding energy is reduced and eventually  $E_{bind} \sim mv^2$  is the smallest scale in the problem (zero binding)  $mv^2 \ll \Lambda_{QCD}, 2\pi T, m_D \Rightarrow$  most of medium effects can be described by a  $T$ -dependent potential

Determine the potential by non-perturbative matching to static quark anti-quark potential calculated on the lattice

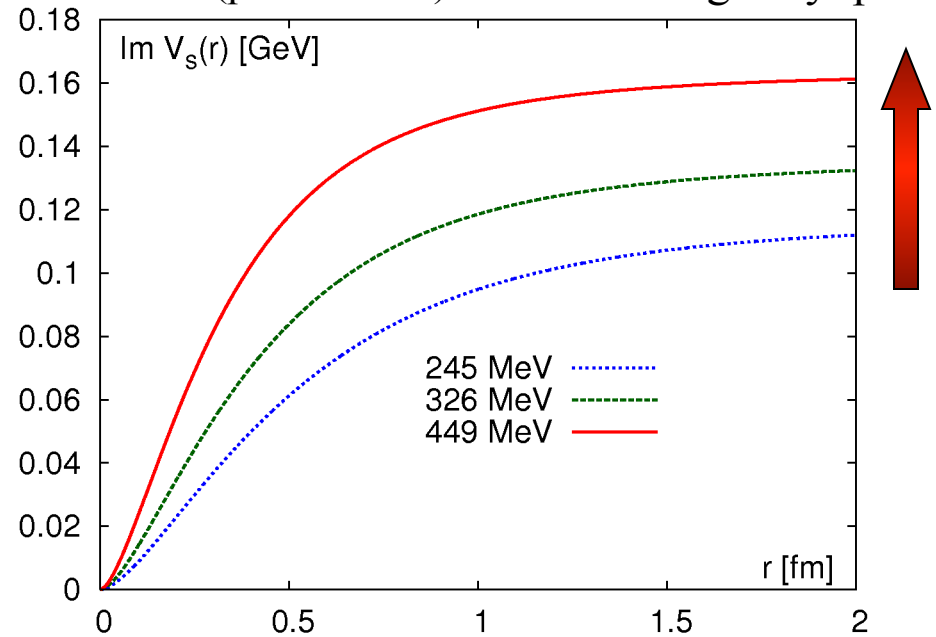
**Caveat** : it is difficult to extract static quark anti-quark energies from lattice correlators  $\Rightarrow$  constrain  $\text{Re}V_s(r)$  by lattice QCD data on the singlet free energy, take  $\text{Im}V_s(r)$  from pQCD calculations

“Maximal” value for the real part



Mócsy, P.P., PRL 99 (07) 211602

Minimal (perturbative) value for imaginary part



Laine et al, JHEP0703 (07) 054,

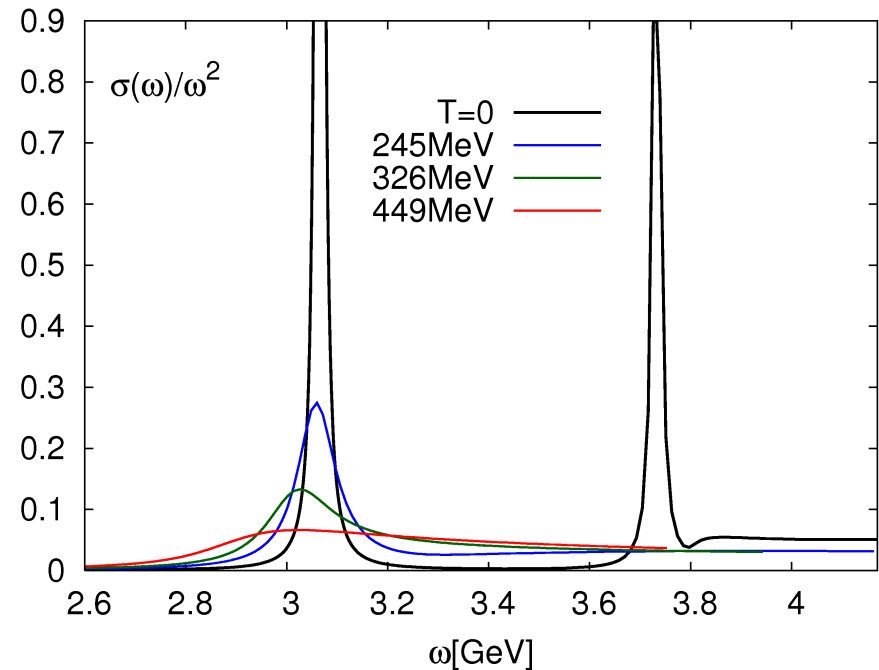
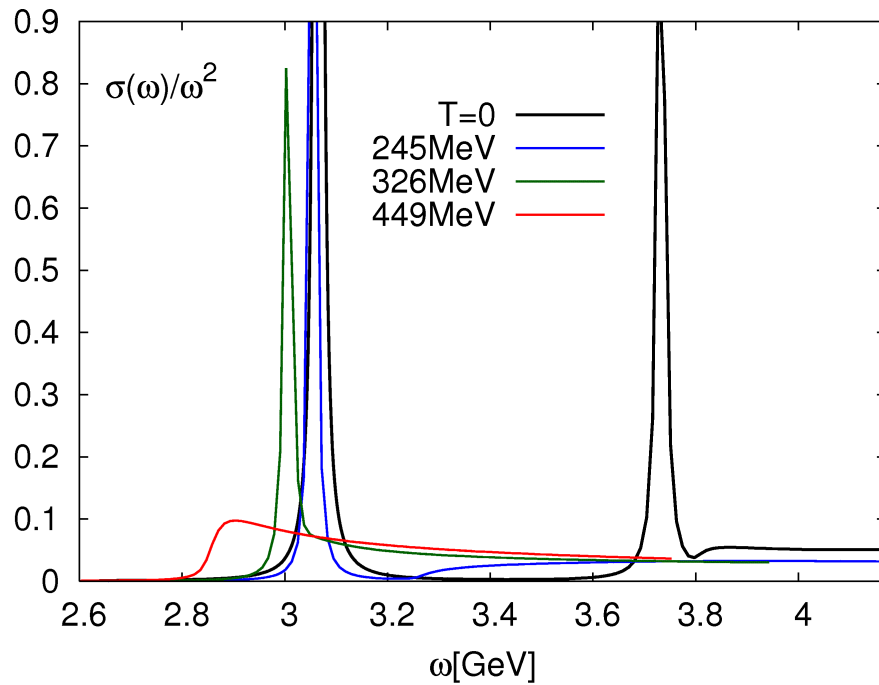
Beraudo et al, arXiv:0812.1130

# The role of the imaginary part for charmonium

Take the upper limit for the real part of the potential allowed by lattice calculations

Miao, Mocsy, PP, NPA855 (2011) 125

$Im V_s(r) = 0$  :  
1S state survives for  $T = 330$  MeV



imaginary part of  $V_s(r)$  is included :  
all states dissolves for  $T > 250$  MeV

Take the perturbative imaginary part  
of the potential and the code from  
Burnier, Laine, Vepsalainen JHEP 0801 (08) 043

no charmonium state could survive for  $T > 250$  MeV

# The role of the imaginary part for bottomonium

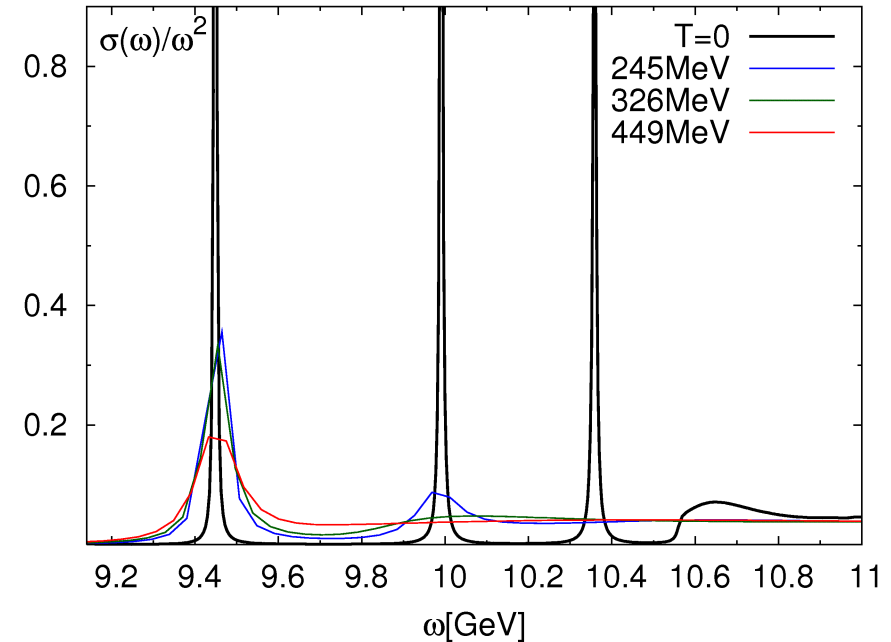
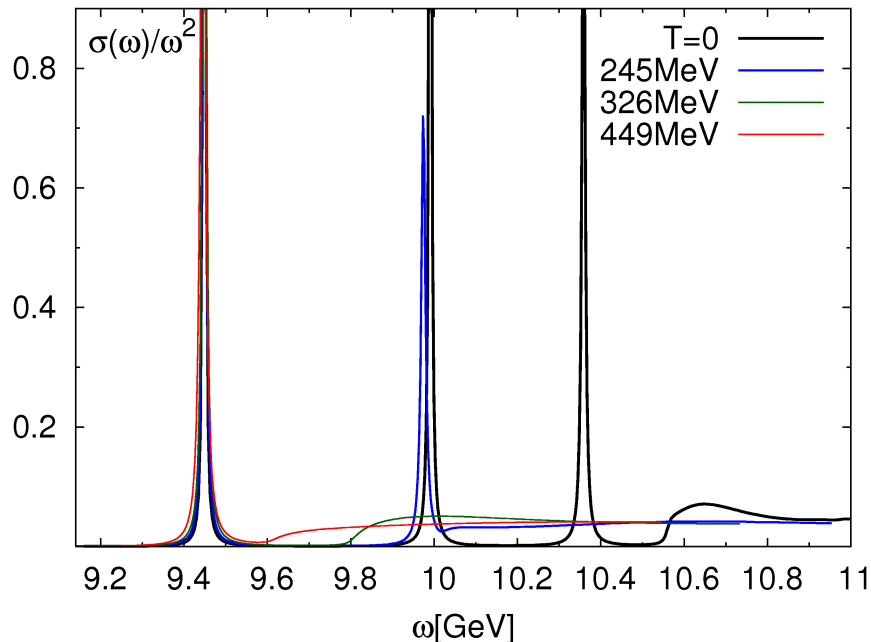
Take the upper limit for the real part of the potential allowed by lattice calculations

Miao, Mocsy, PP, NPA855 (2011) 125

$Im V_s(r) = 0:$

2S state survives for  $T > 245$  MeV

1S state could survive for  $T > 450$  MeV



with imaginary part:

2S state dissolves for  $T > 250$  MeV

1S states dissolves for  $T > 450$  MeV

Take the perturbative imaginary part  
the potential and the code from

Burnier, Laine, Vepsalainen JHEP 0801 (08) 043

no bottomonium state could survive for  $T > 450$  MeV

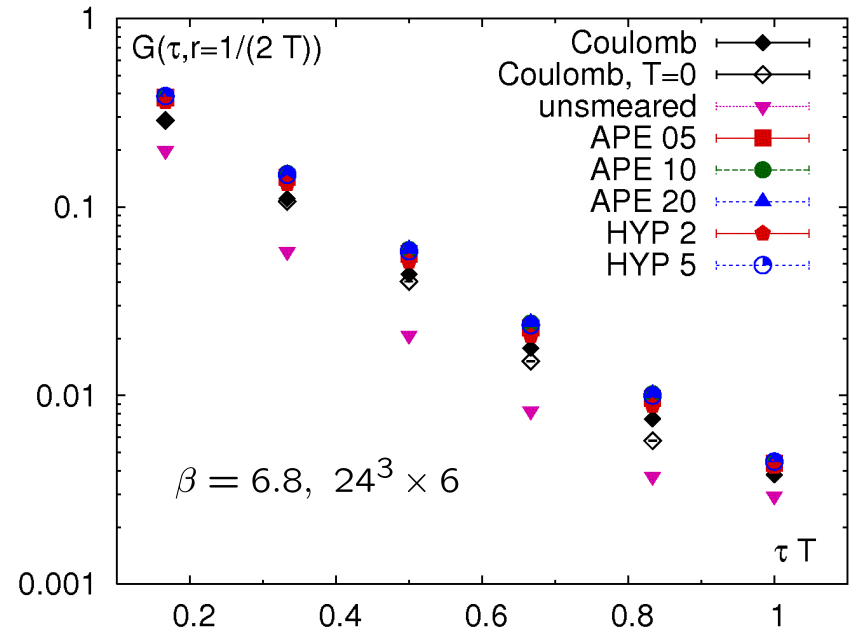
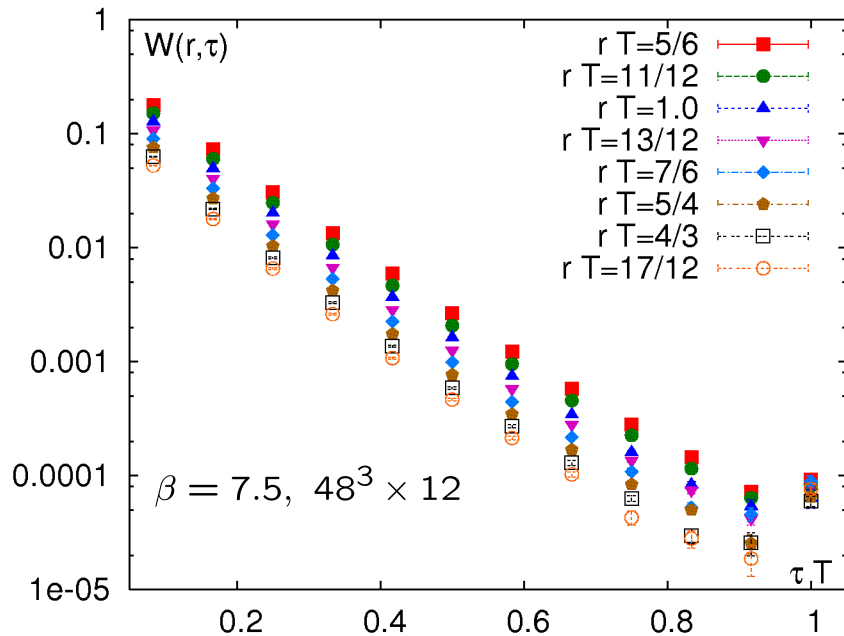
# Wilson loops and potential at $T > 0$

HISQ action,  $48^3 \times 12$ ,  $24^3 \times 6$  lattices,  $m_\pi \simeq 160$  MeV

$$W(r, \tau) = \int_{-\infty}^{\infty} d\omega \sigma(\omega, T) e^{-\omega \tau}, \tau < 1/T$$

Rothkopf 2009, Hatsuda, Rothkopf, Hatsuda 2011

not related to the free energy !



Choices of the spatial links:

Naïve= un-smearing



smearing



$$= \text{---} +$$

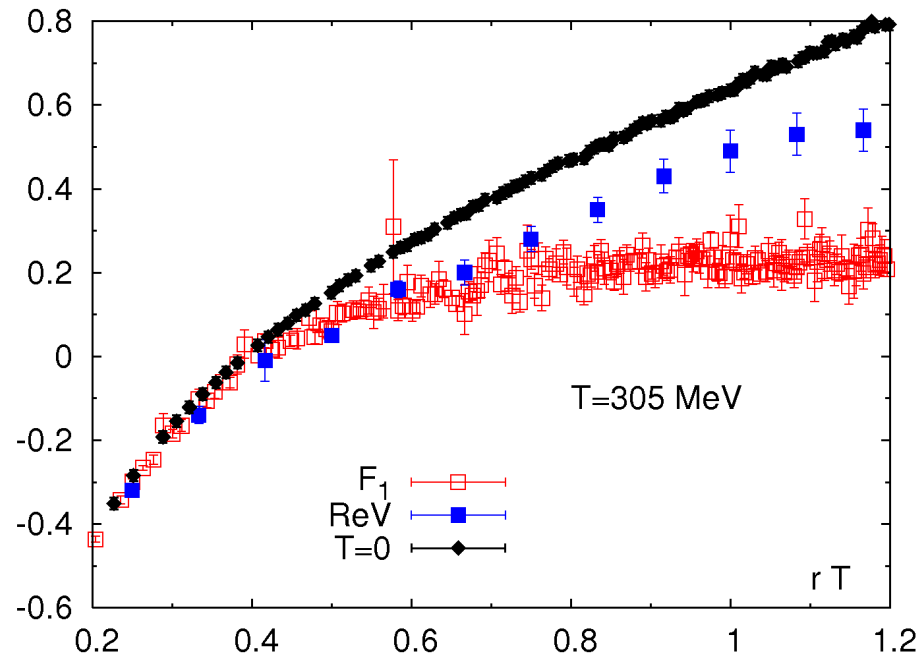
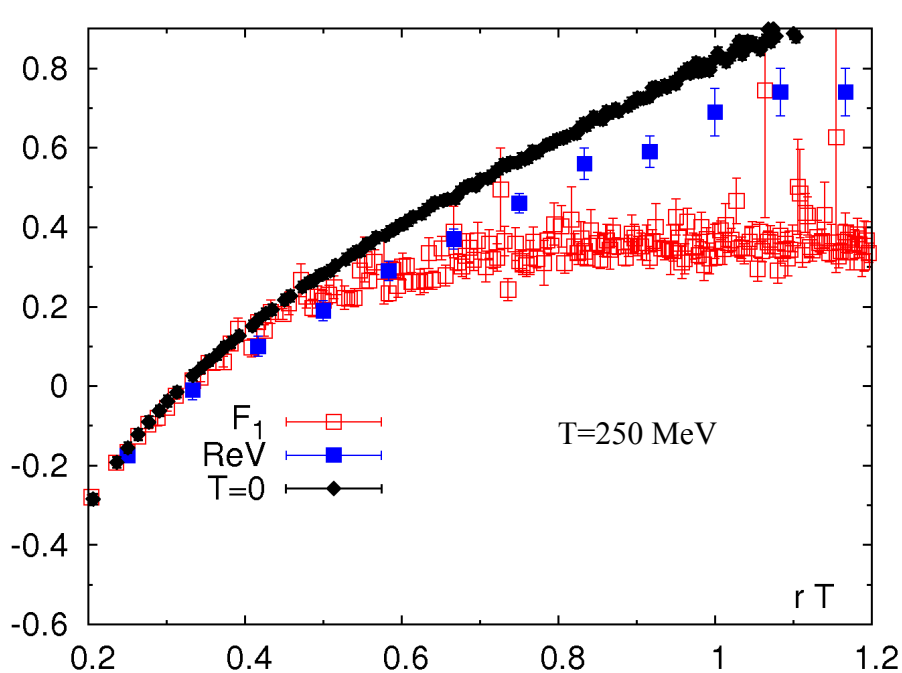


or use Coulomb gauge and  $U(x, y) = 1$

Un-smearing Wilson loops show non-exponential behavior and are suppressed compared to the smearing Wilson loops and Coulomb gauge corr. which decay exponentially, except for  $\tau T \sim 1$

# Real part of the potential above deconfinement

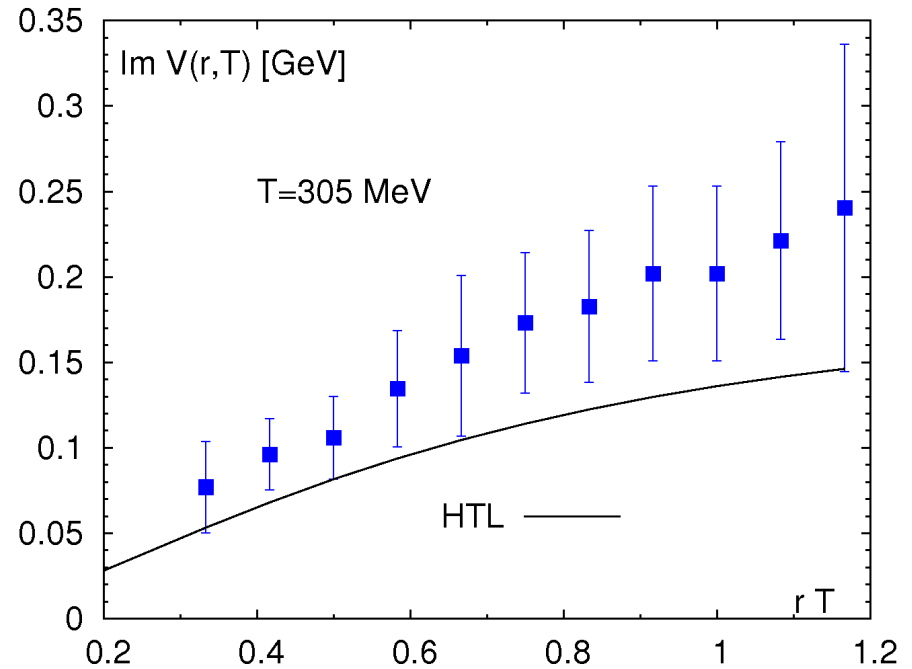
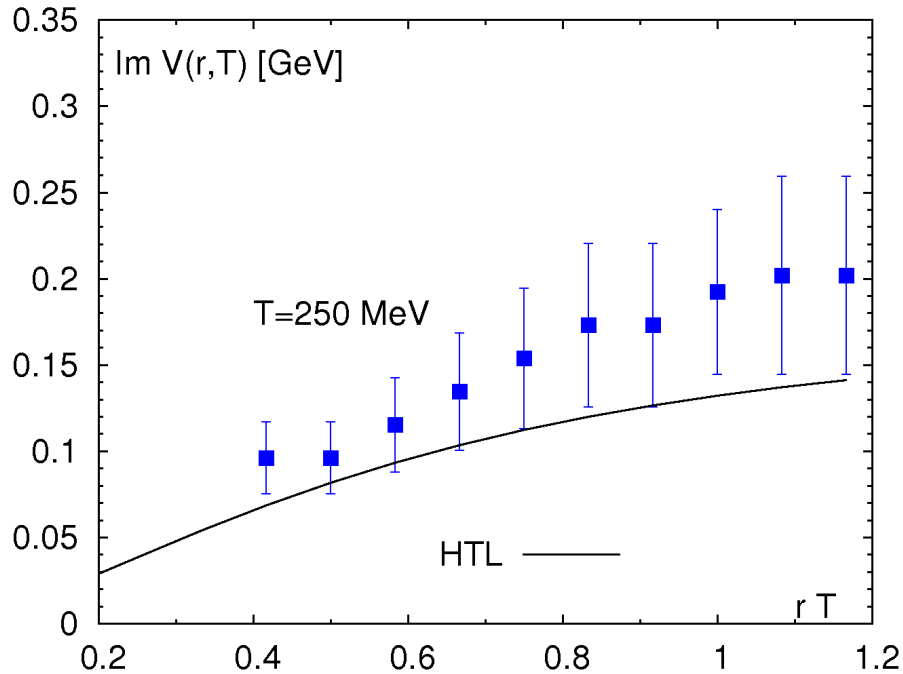
Results for  $N_\tau = 12$  lattices using  $\sigma^{HTL}((\omega-E)\lambda)$  as a 2-parameter fit Ansatz:



- For  $rT < 0.7$  the real part of the potential is roughly equal to the singlet free energy
- At larger distances it is between the singlet free energy and the  $T=0$  potential
- The real part of the potential saturates at  $rT \sim 1$  (screening) at a fairly large value (non-perturbative effects)

# Imaginary part of the potential above deconfinement

Results for  $N_\tau = 12$  lattices using  $\sigma^{HTL}((\omega-E)\lambda)$  as a 2-parameter fit Ansatz:



- The imaginary part of the potential has large errors as the width of the spectral functions is difficult to extract from the lattice correlators
- The imaginary part increases with  $r$  and saturates at  $r T \sim 1$
- The central value imaginary part of the potential is (1.5 - 2.0) larger than in HTL perturbation theory

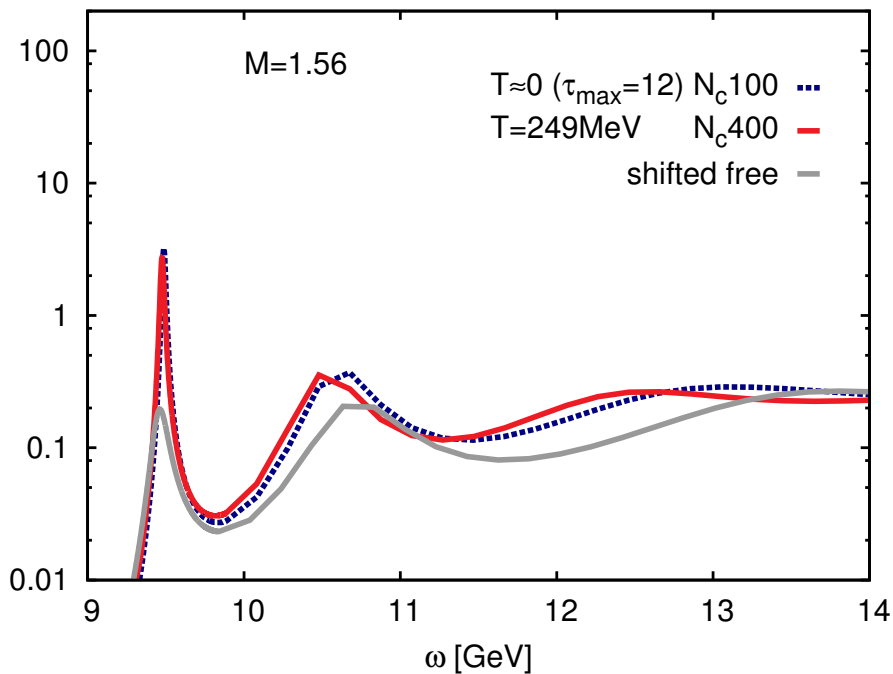
# Bottomonium spectral functions in NRQCD at $T > 0$

Kim, PP, Rothkopf, arXiv:1409.3630v1

Gauge configurations from HotQCD,  $N_\tau = 12$  lattices

New Bayesian approach to reconstruction of the spectral functions

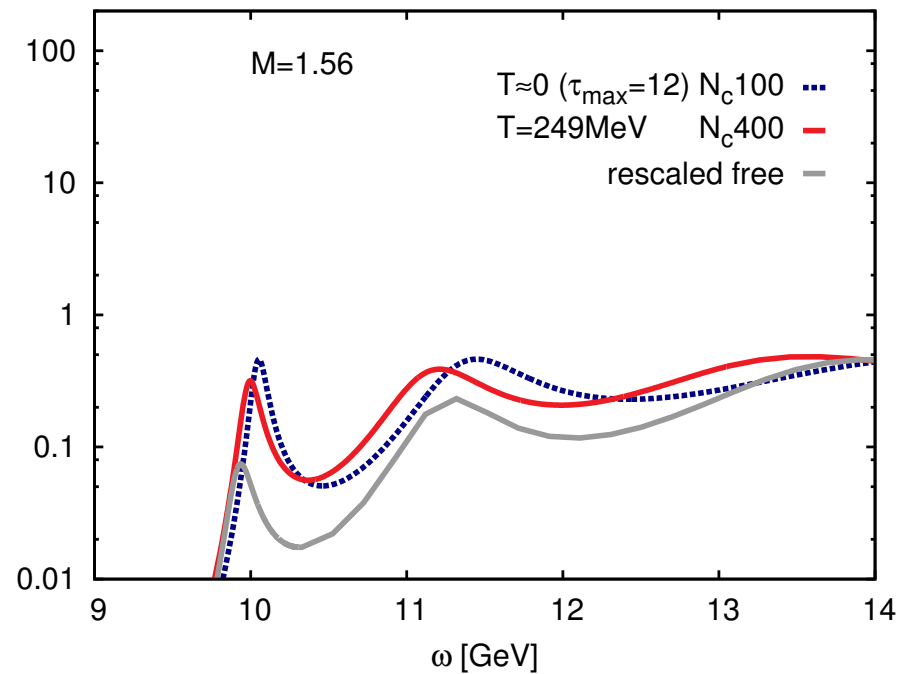
Need to take into account systematic effects when comparing to  $T=0$



Well reconstructed ground state peak for 1S bottomonium

Upper bounds on mass shift and width:

$$\Upsilon(1S) : \Delta m_T(249\text{MeV}) < 40\text{MeV}, \quad \Gamma_T(249\text{MeV}) < 21\text{MeV}$$



Some bound state-like 1P peak is visible for  $T = 249$  MeV



# Spatial vs. temporal meson correlators

Spatial correlation functions can be calculated for arbitrarily large separations  $z \rightarrow \infty$

$$G(z, T) = \int_0^{1/T} d\tau \int dx dy \langle J(\mathbf{x}, -i\tau), J(\mathbf{x}, 0) \rangle_T, \quad G(z \rightarrow \infty, T) \simeq A e^{-m_{scr}(T)z}$$

but related to the same spectral functions

$$G(z, T) = \int_{-\infty}^{\infty} e^{ipz} \int_0^{\infty} d\omega \frac{\sigma(\omega, p, T)}{\omega}$$

Low  $T$  limit :

$$\sigma(\omega, p, T) \simeq A_{mes} \delta(\omega^2 - p^2 - M_{mes}^2)$$

$$A_{mes} \sim |\psi(0)|^2 \rightarrow m_{scr}(T) = M_{mes}$$

$$G(z, T) \simeq |\psi(0)|^2 e^{-M_{mes}(T)z}$$

High  $T$  limit :

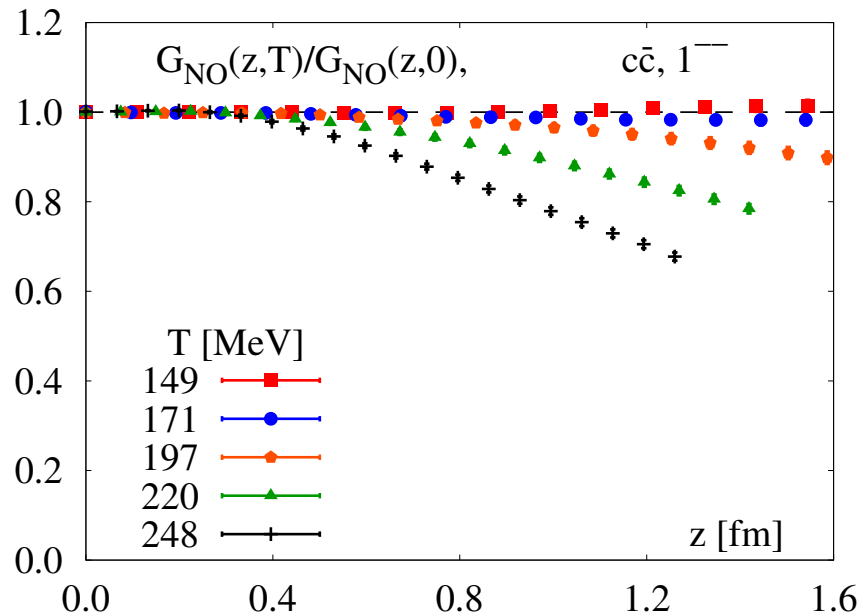
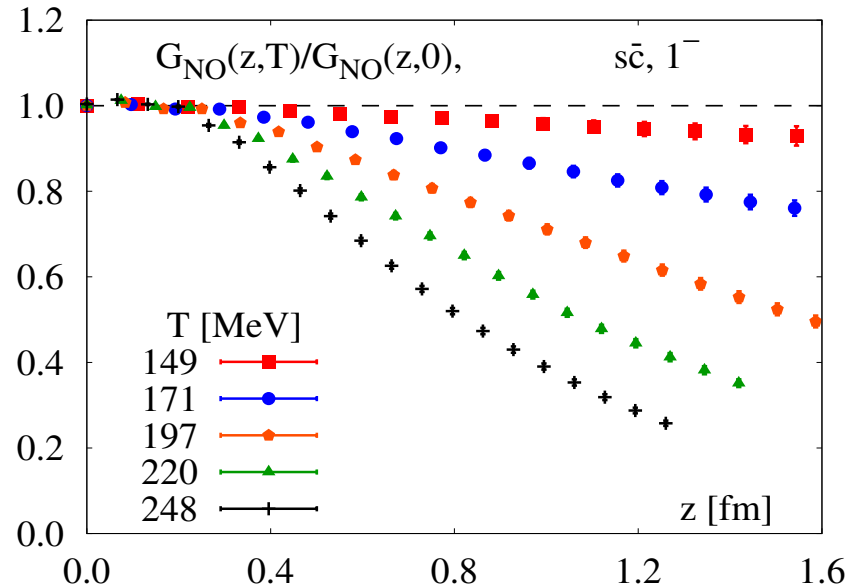
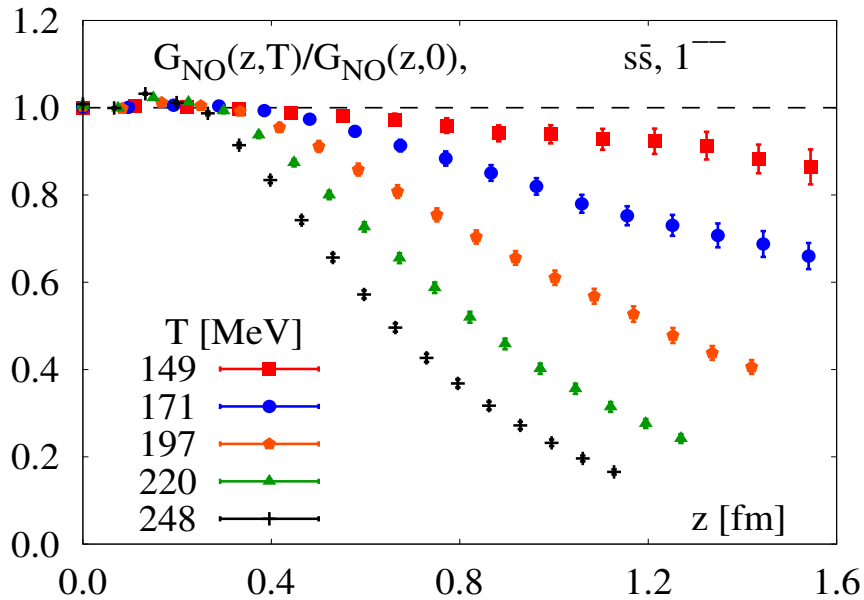
$$m_{scr}(T) \simeq 2\sqrt{m_c^2 + (\pi T)^2}$$

Temporal meson correlator only available for  $\tau T < 1/2$  and thus may not be very sensitive to In-medium modifications of the spectral functions; also require large  $N_\tau$  (difficult in full QCD)

Spatial correlators can be studied for arbitrarily large separations and thus are more sensitive to the changes in the meson spectral functions; do not require large  $N_\tau$  (easy in full QCD).

**Lattice calculations:** spatial meson correlators in 2+1 flavor QCD for  $s\bar{s}$ ,  $s\bar{c}$  and  $c\bar{c}$  sectors using  $48^3 \times 12$  lattices and highly improved staggered quark (HISQ) action (also suitable for charm quarks), physical  $m_s$  and  $m_\pi = 160$  MeV.

# Temperature dependence of spatial meson correlators

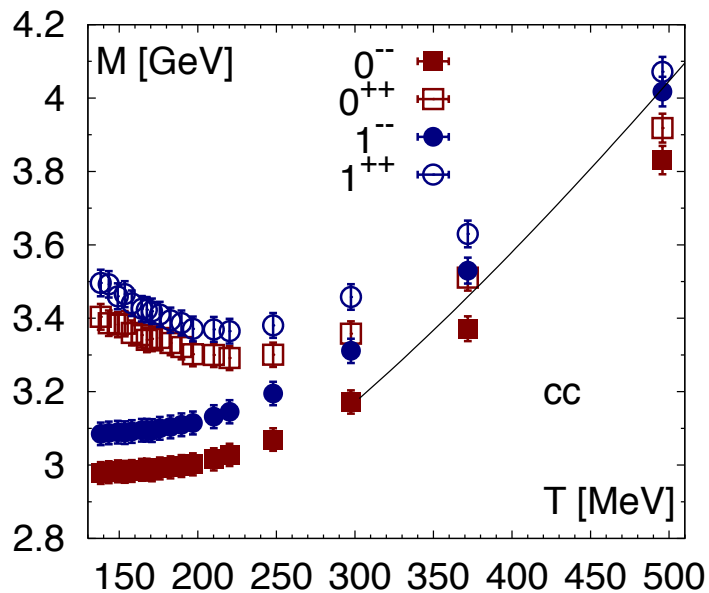
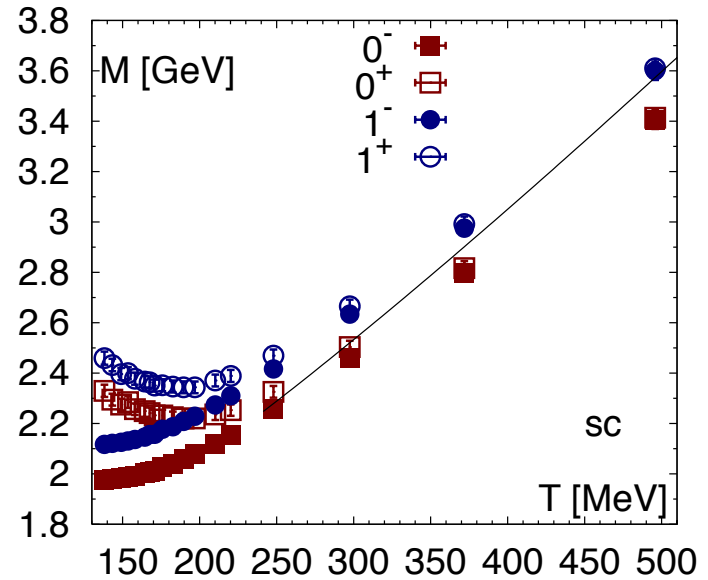
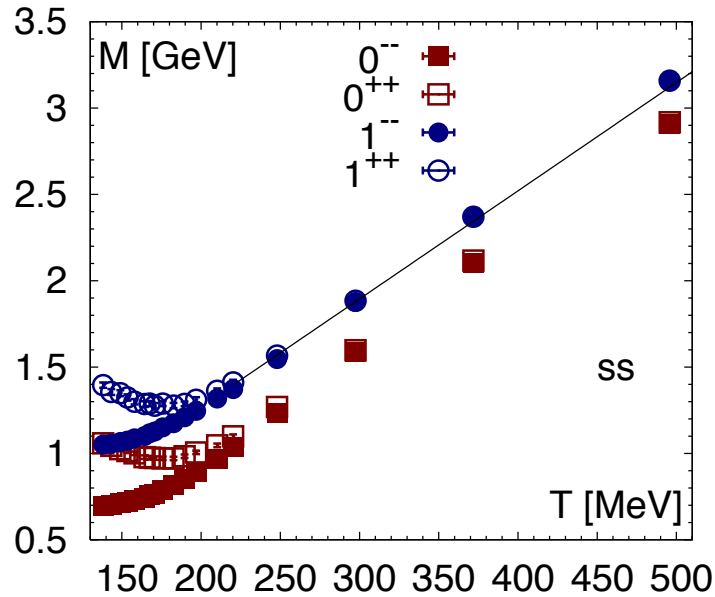


Medium modifications of meson correlators increase with  $T$ , but decrease with heavy quark content

Significant modifications for  $T > T_c = 154$  MeV

For  $z < 1/(2T)$  the  $T$ -dependence of the vector charmonium correlators is very small

# Temperature dependence of meson screening masses

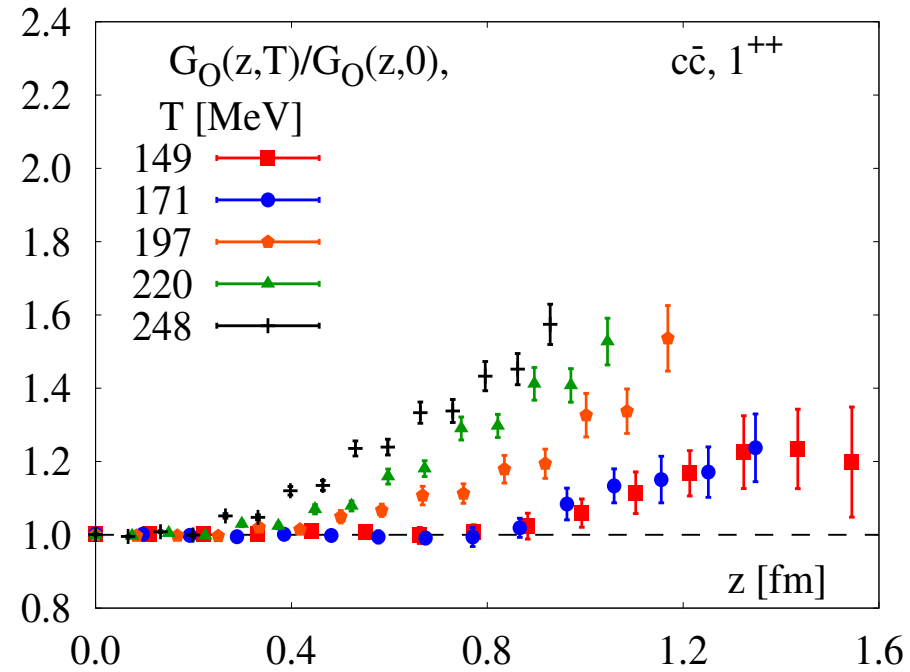
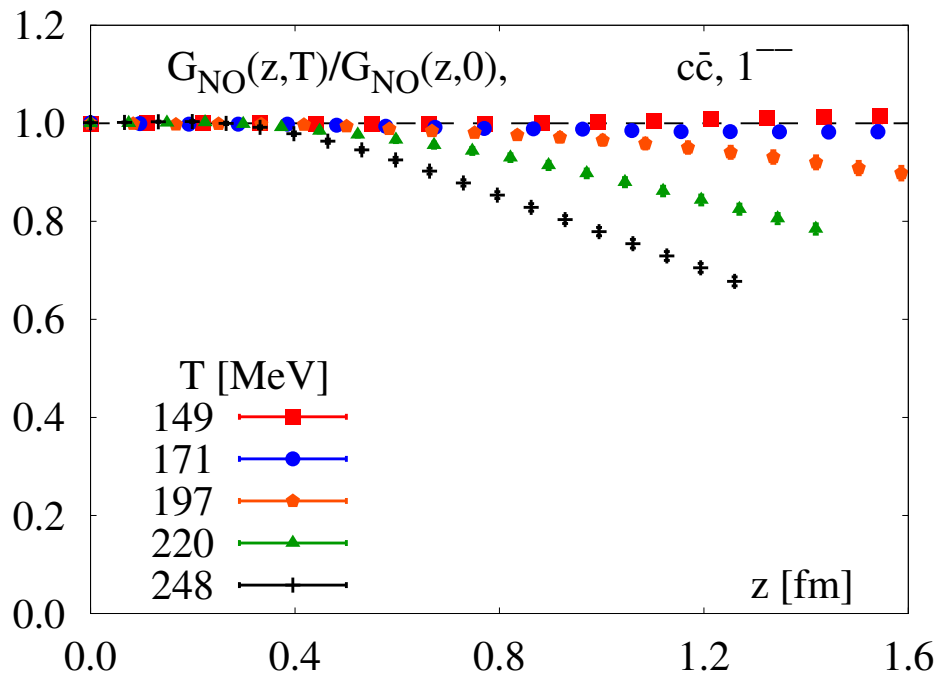


Qualitatively similar behavior of the screening masses for  $s\bar{s}$ ,  $s\bar{c}$  and  $c\bar{c}$  sectors

Screening Masses of opposite parity mesons become degenerate at high  $T$   
(restoration of chiral and axial symmetry)

Screening masses are close to the free limit  $2(m_q^2 + (\pi T)^2)^{1/2}$  at  $T > 200$  MeV,  $T > 250$  MeV,  $T > 300$  MeV for  $s\bar{s}$ ,  $s\bar{c}$  and  $c\bar{c}$  sectors, respectively.

# Temperature dependence of spatial charmonium correlators



Almost no medium modification of S-wave charmonium correlators across  $T_c \approx 154$  MeV,  
Medium modification of the correlators start to be visible for  $T > 197$  MeV

Significant medium modification of P-wave charmonium correlators already at  $T_c$   
and larger  $T$ -dependence than for 1S correlator for



Fits into the picture of sequential charmonium melting:  
 $\chi_c$  melts at smaller temperature than the more tightly bound  $J/\psi$

## Summary

So far we have only crude information on the quarkonium spectral functions: melting vs. survival, no quantitative results, upper bounds for mass shift and width for 1S and 1P bottomonium

Combination of LQCD and EFT approaches (lattice NRQCD, pNRQCD inspired Potential model + lattice ) and study of spatial charmonium correlators gives a consistent picture:

$$T_d(\chi_c) \approx T_c$$

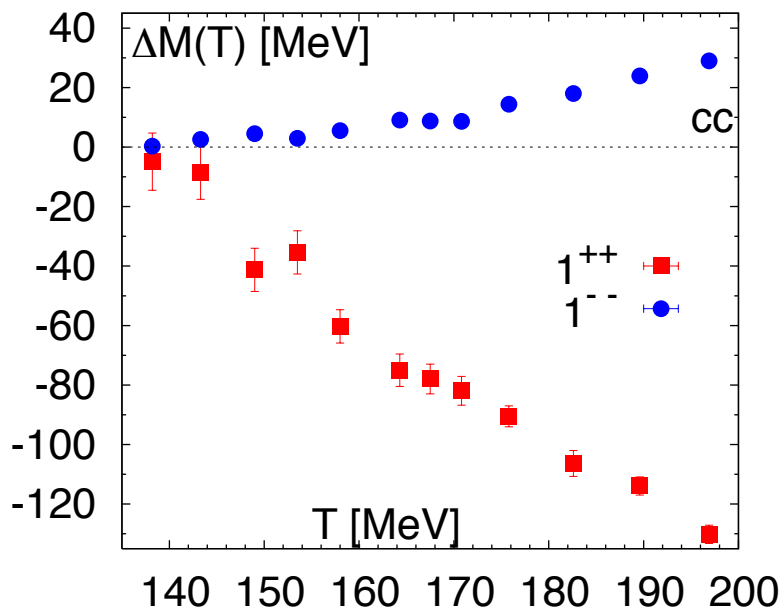
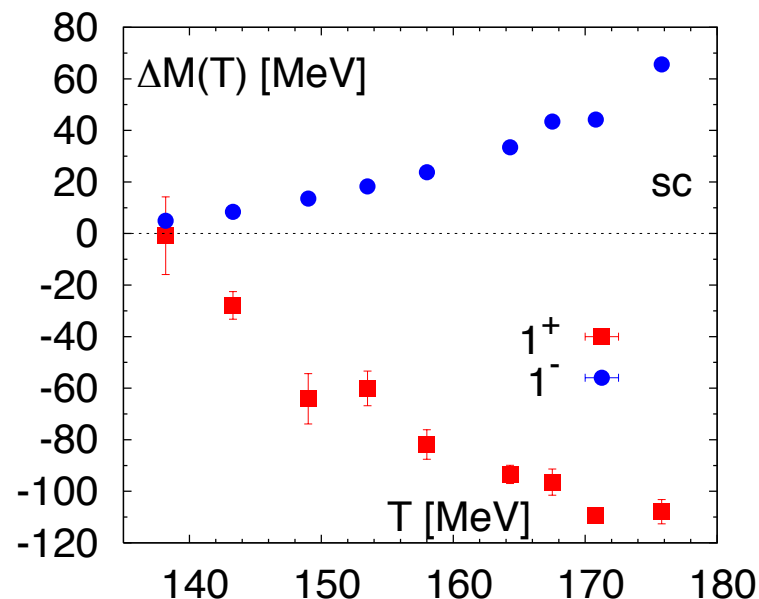
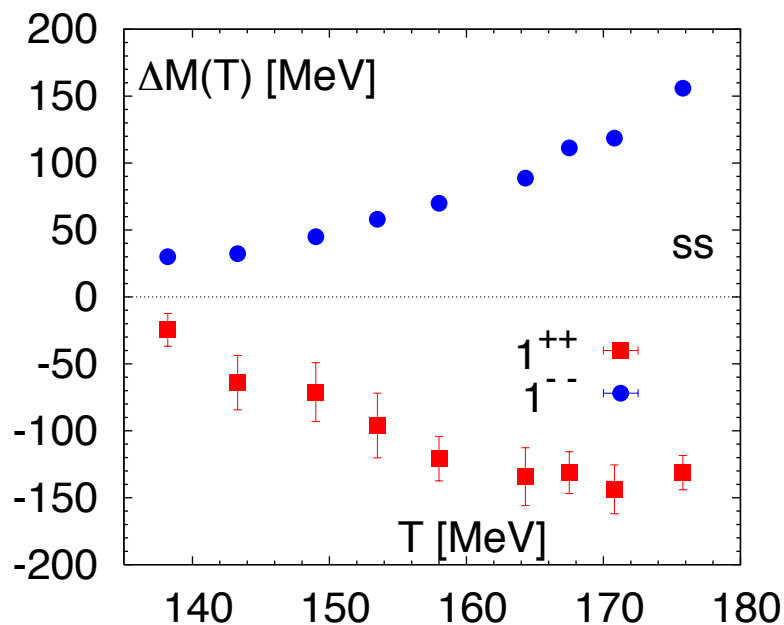
$$T_d(J/\psi) \approx T_d(\chi_b) > 250 \text{ MeV}$$

$$T_d(Y) > 450 \text{ MeV}$$

First direct lattice QCD indication of sequential melting

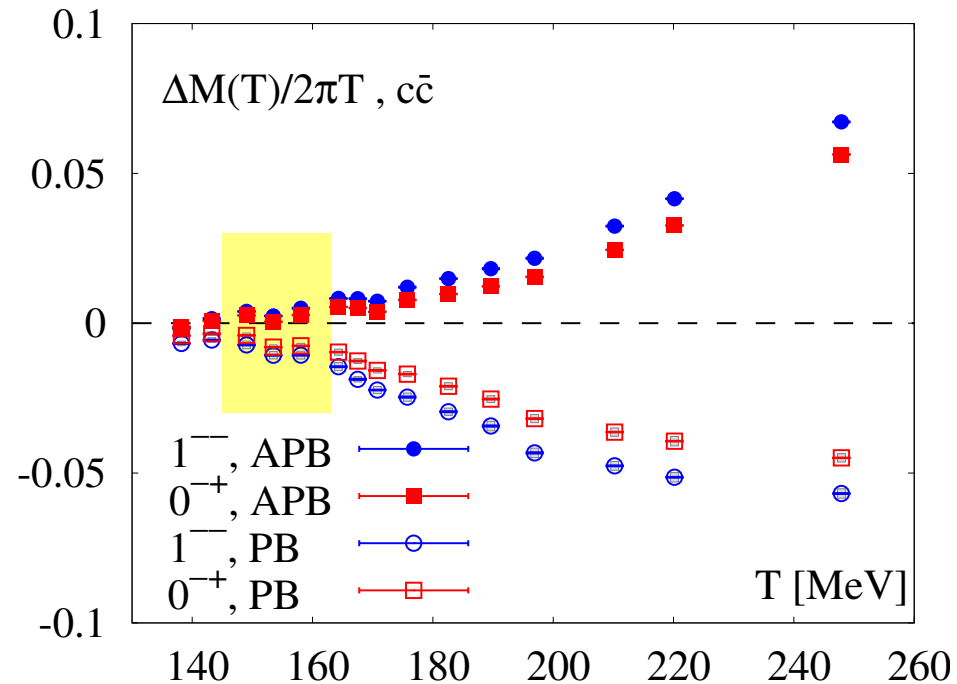
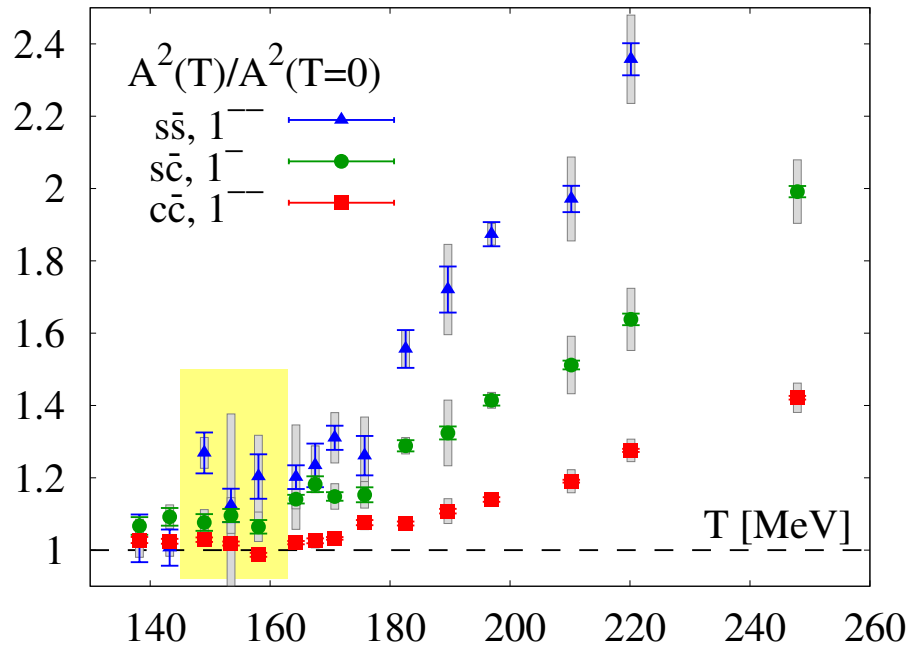
Back up slides

# Temperature dependence of meson screening masses (cont'd)



- At low  $T$  changes in the meson screening Masses  $\Delta M = M_{scr}(T) - M_{T=0}$  are indicative of the changes in meson binding energies
- $\Delta M$  is significant already below  $T_c = 154$  MeV
- Above the transition temperature the changes in  $\Delta M$  are comparable to the meson binding energy and are consistent with melting of meson states except for  $1S$  charmonium

# Temperature dependence of meson screening masses (cont'd)



$s\bar{s}$  and  $s\bar{c}$  mesons : Significant modifications of the squared amplitudes around  $T_c$

$c\bar{c}$  mesons : Similar medium modifications of the amplitudes only for  $T > 210$  MeV

If  $D_s$  and  $\phi$  melt just above  $T_c$  them  $J/\psi$  will melt around  $T > 210$  MeV

For small bound state like  $J/\psi$  the screening mass should not boundary condition  
 Large dependence on the temporal boundary conditions of  $S$ -wave charmonium  
 correlator for  $T > 200$  MeV



# Lattice QCD based potential model

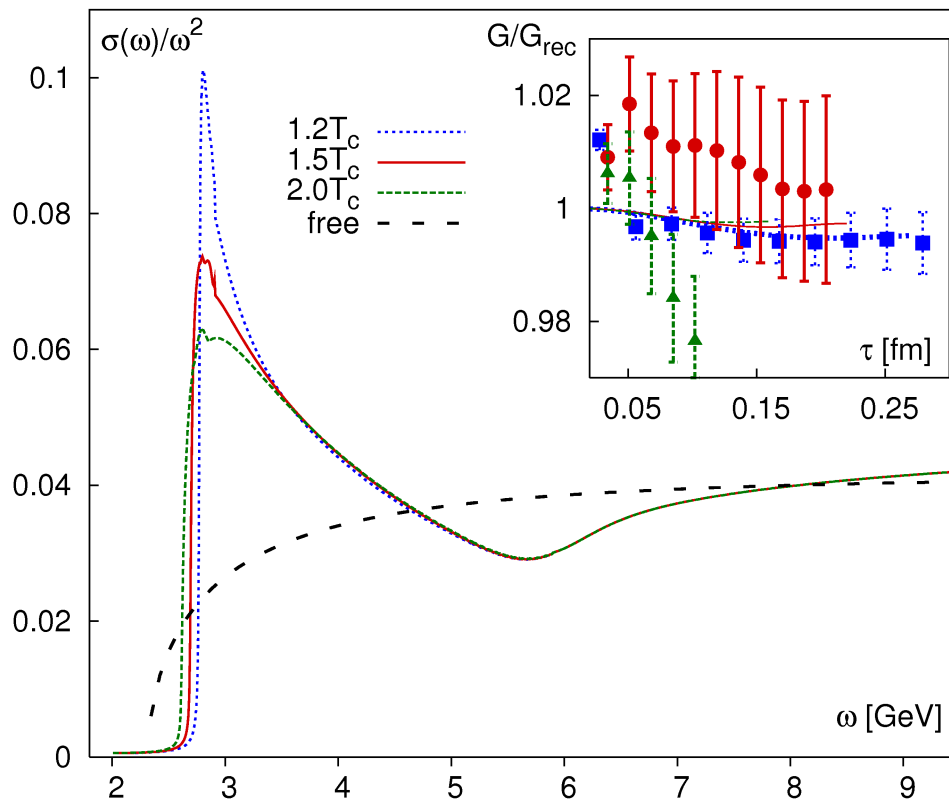
If the octet-singlet interactions due to ultra-soft gluons are neglected :

$$\left[ i\partial_0 - \frac{-\nabla^2}{m} - V_s(r, T) \right] S(r, t) = 0 \quad \Rightarrow \quad \sigma(\omega, T)$$

potential model is not a model but the tree level approximation of corresponding EFT that can be systematically improved

Test the approach vs. LQCD : quenched approximation,  $F_1(r, T) < \text{Re}V_s(r, T) < U_1(r, T)$ ,  $\text{Im}V(r, T) \approx 0$

Mócsy, P.P., PRL 99 (07) 211602, PRD77 (08) 014501, EPJC ST 155 (08) 101

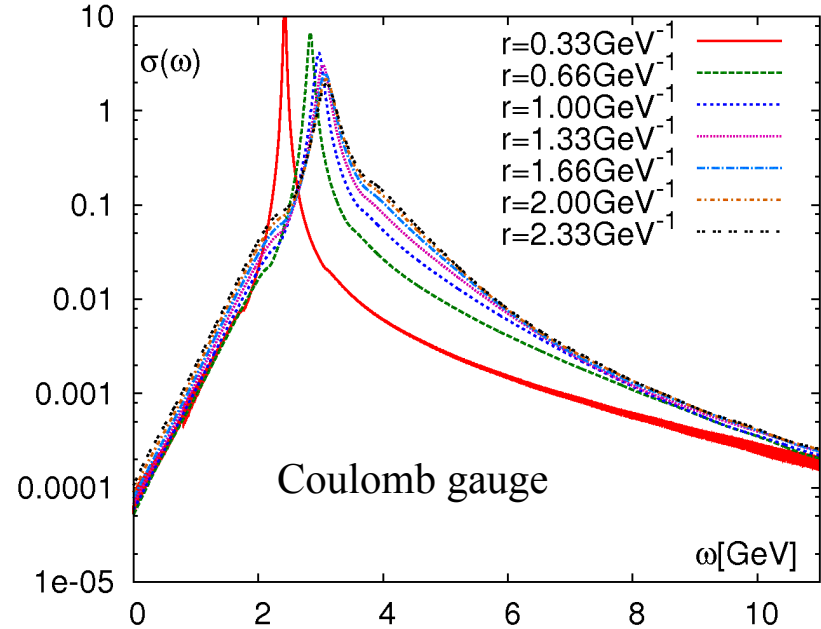
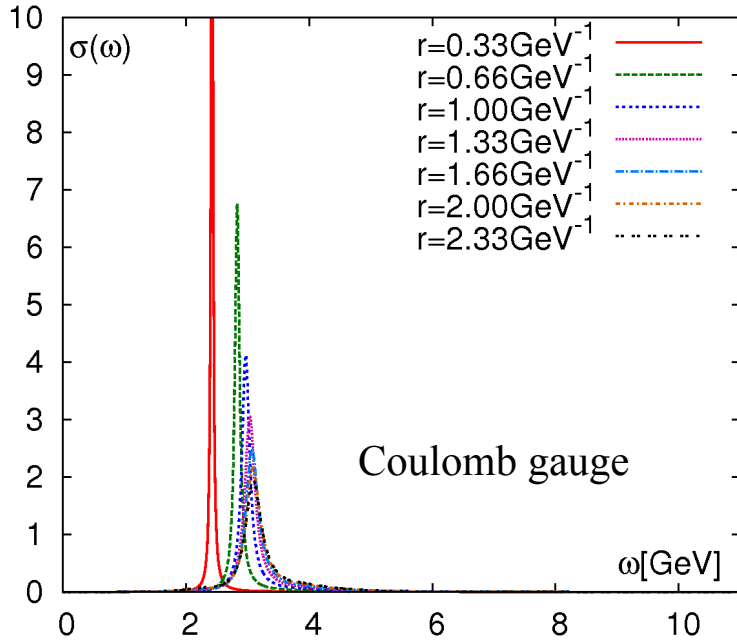


- resonance-like structures disappear already by  $1.2T_c$
- strong threshold enhancement above free case  
=> indication of correlations
- height of bump in lattice and model are similar
- The correlators do not change significantly despite the melting of the bound states => it is difficult to distinguish bound state from threshold enhancement in lattice QCD

# Static QQbar spectral function in perturbation theory

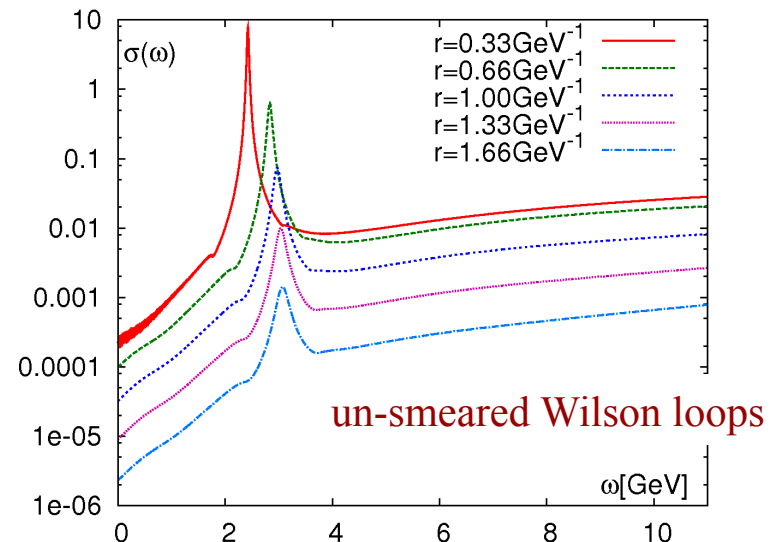
Perturbative hard thermal loop (HTL) calculations for  $T=2.33 T_c$ ,  $T_c=270$  MeV ( $N_f = 0$ ) :

Burnier, Rothkopf, 2013



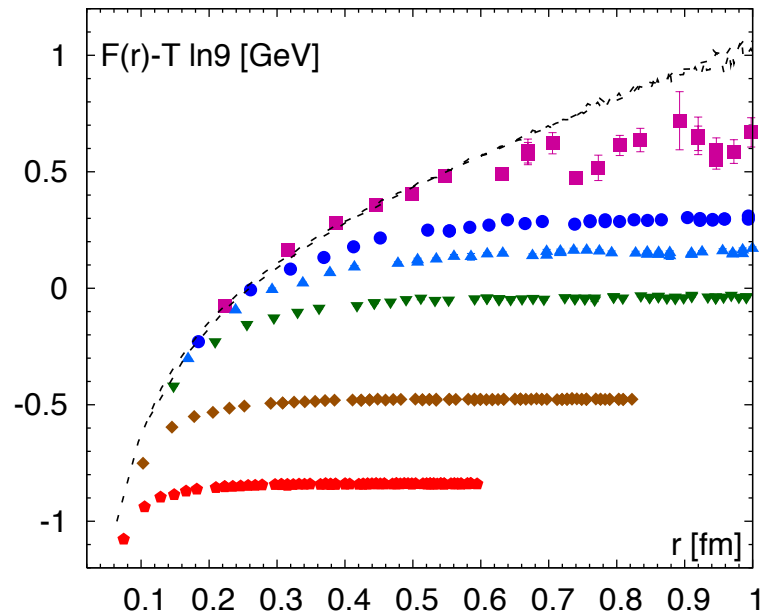
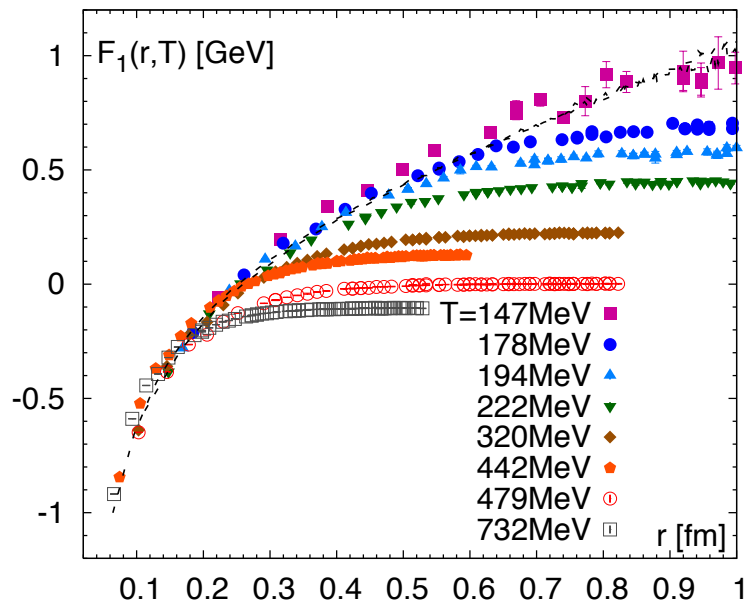
Spectral functions has long tails and non-Lorentzian away from the peak,  
 $\Rightarrow$  explanation for the behavior of the Wilson loops and  $V_{eff}$  at large times

For un-smearred Wilson loops the peak height is much suppressed compared to the Coulomb gauge case

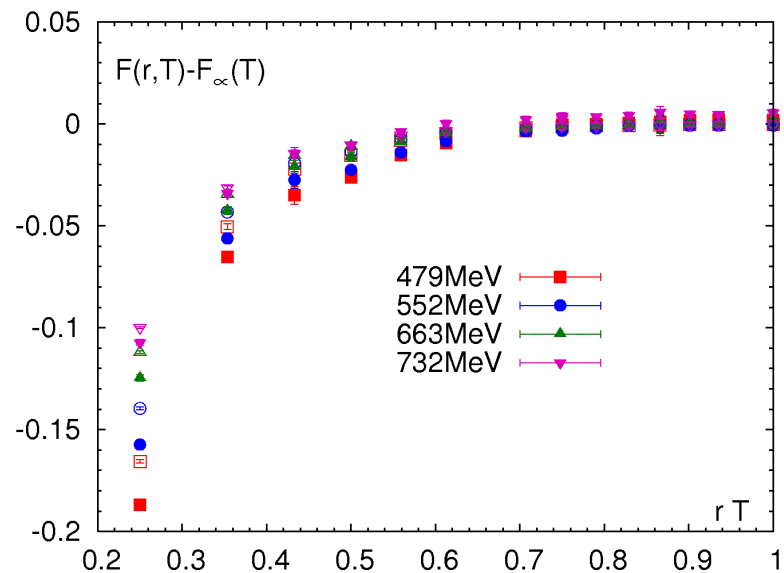


# Static quark anti-quark free energy in 2+1f QCD

HISQ action,  $24^3 \times 6$ ,  $16^3 \times 4$  (high T) lattices,  $m_\pi \simeq 160$  MeV

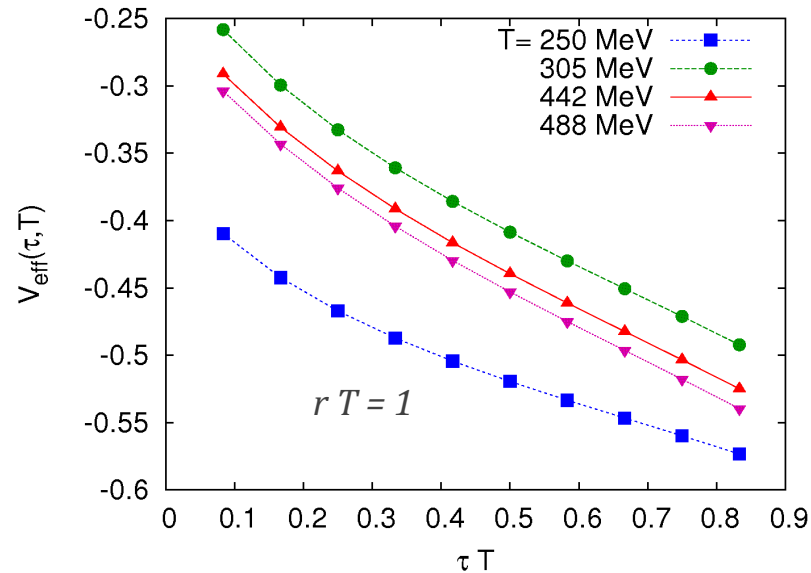
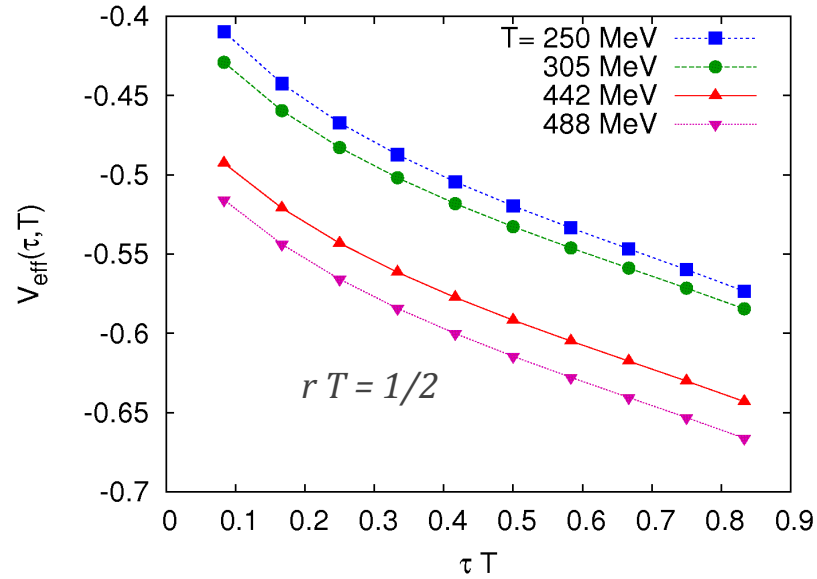


- The strong  $T$ -dependence for  $T < 200$  MeV is not necessarily related to color screening
- The free energy has much stronger  $T$ -dependence than the singlet free energy due to the octet contribution
- At high  $T$  the temperature dependence of the free energy can be entirely understood in terms of  $F_1$  and Casimir scaling  $F_1 = -8 F_8$



# Effective potential at high temperatures

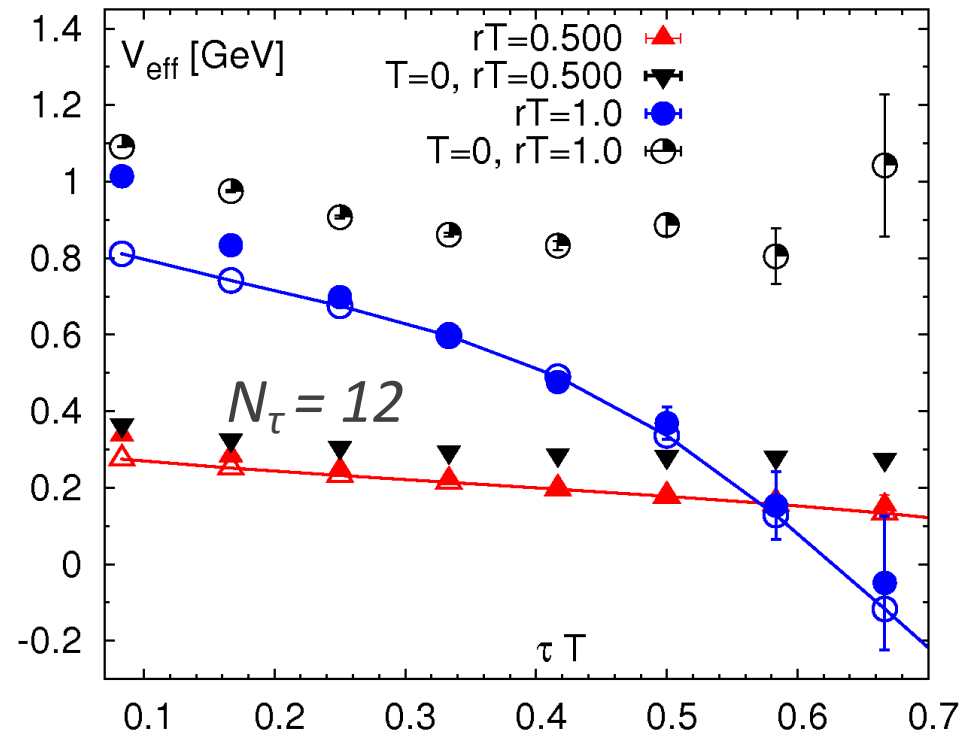
Effective potential  $V_{eff}$  in HTL perturbation theory:



$V_{eff}$  decreases with  $\tau$  due to the width of the spectral functions, its slope increases with  $T$  and the distance  $r$  as observed in the lattice calculations



Use  $\sigma^{HTL}((\omega-E)\lambda)$  as a 2-parameter fit Ansatz for the lattice results:

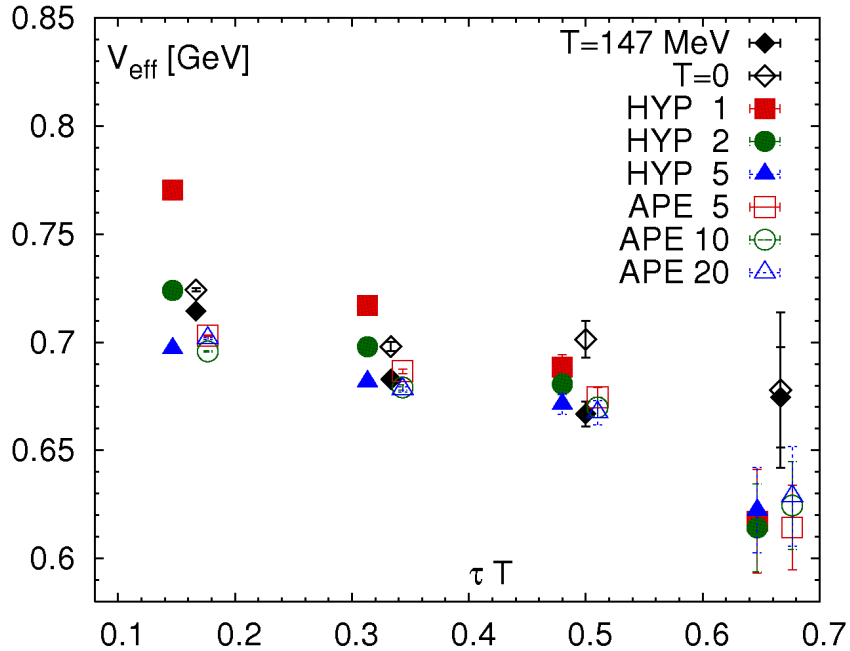


# The temperature dependence of the effective potentials

Assume single state dominance

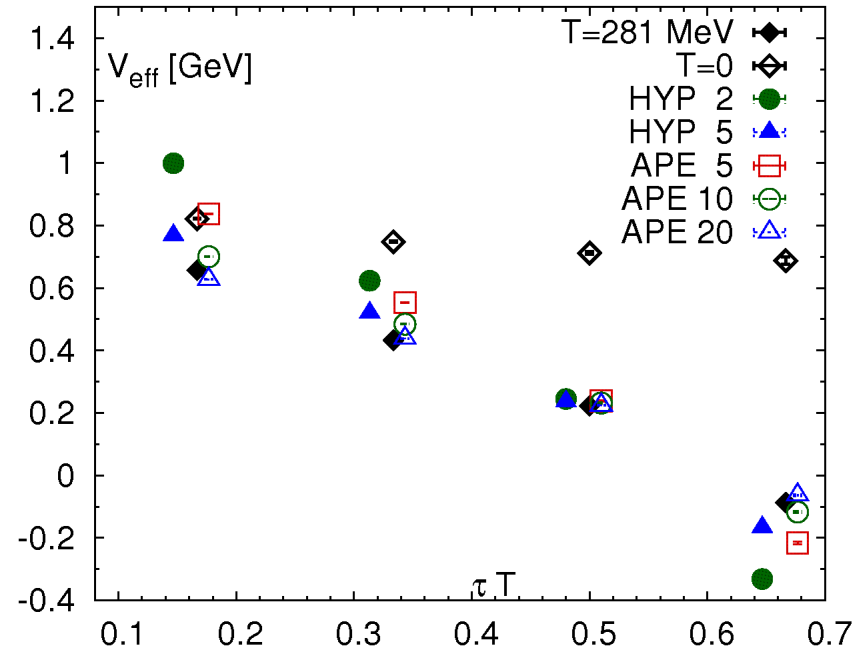
$$\sigma(\omega, T) \sim \delta(\omega - V(r, T))$$

$r T = 1/2$



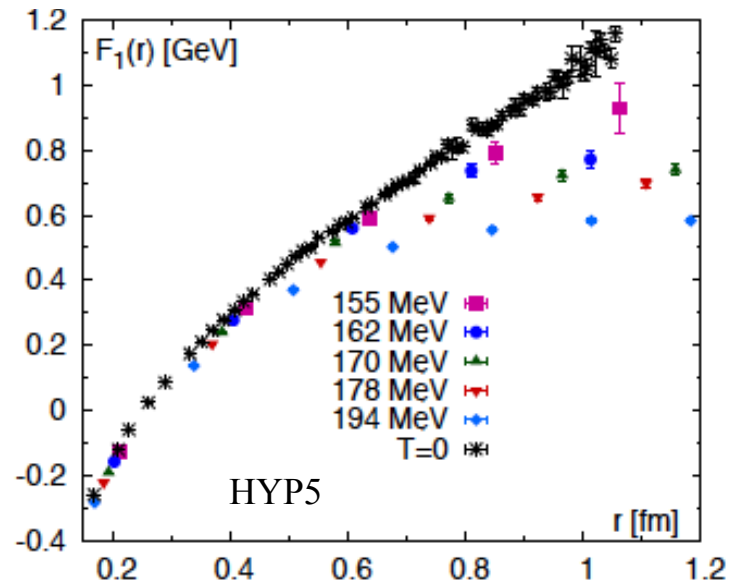
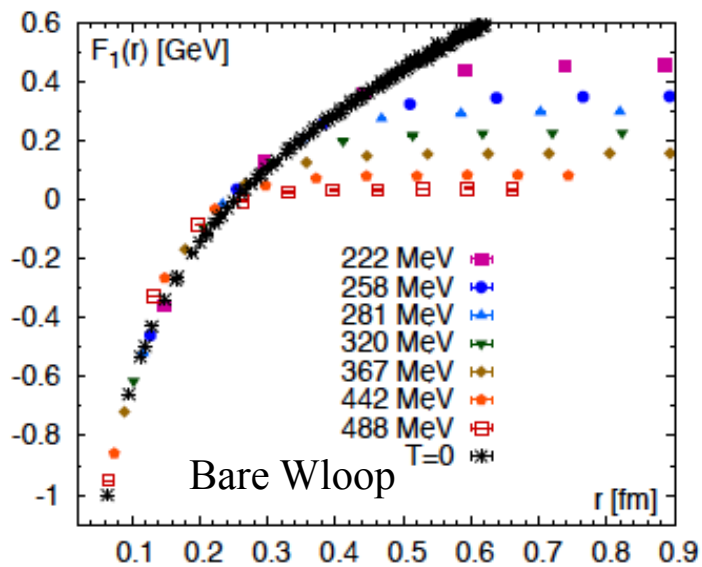
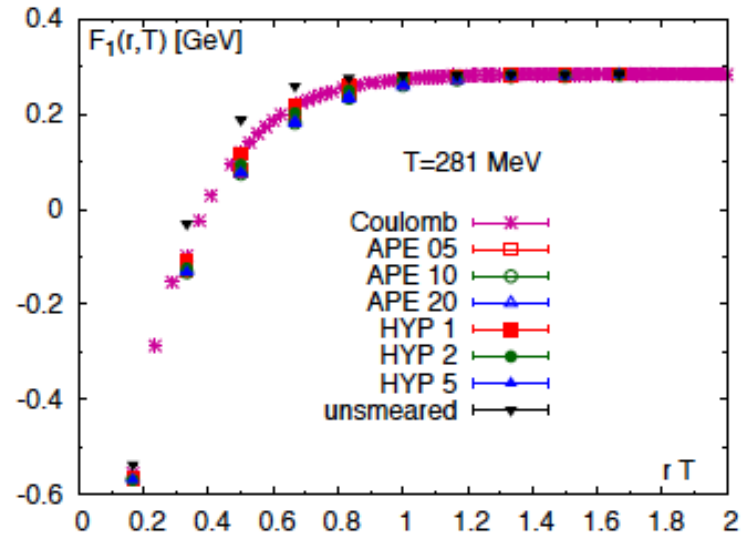
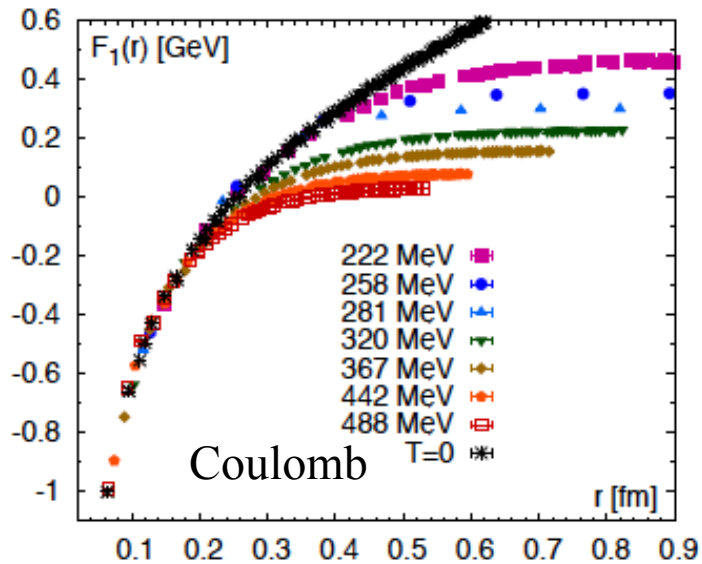
Can be seen at  $T=0$  in the considered range

$r T = 1$



# Different singlet free energies:

Bazavov, PP, Eur.Phys.J. A49 (2013) 85



# Burnier, Kaczmarek, Rothkopf, 1411.3141

## Quenched QCD, $N_\tau=24-96$

